UNIVERSITY OF IOANNINA
DEPARTMENT OF ECONOMICS

SAVING FOR RETIREMENT UNDER UNCERTAIN QUALITY OF INSTITUTIONS

Doctoral dissertation

by **Agathi Tsoka**

IOANNINA 2013

Ημερομηνία αίτησης της κα Τσόκα Αγαθούλας: 19-06-2007

Ημερομηνία ορισμού Τριμελούς Συμβουλευτικής Επιτροπής: 128/20-06-2007

Μέλη Τριμελούς Συμβουλευτικής Επιτροπής: 128/20-6-2007

Επιβλέπων:
Χατζηνικολάου Δημήτριος, Αναπληρωτής Καθηγητής του Τμήματος Οικονομικών Επιστημών του Π.Ι.

Μέλη:
Σίμος Θεόδωρος, Επίκουρος Καθηγητής του Τμήματος Οικονομικών Επιστημών του Π.Ι.
Χορταρέας Γεώργιος, Καθηγητής του Τμήματος Οικονομικών Επιστημών του Παν/μίου Κρήτης.

Ημερομηνία Ορισμού Θέματος: 5-06-2009
«Saving for retirement under uncertain quality of institutions»

ΔΙΟΡΙΣΜΟΣ ΕΠΤΑΜΕΛΟΥΣ ΕΞΕΤΑΣΤΙΚΗΣ ΕΠΙΤΡΟΠΗΣ: 194/24-4-2013

Χατζηνικολάου Δημήτριος	Αναπληρωτής Καθηγητής του Τμήματος Οικονομικών Επιστημών του Πανεπιστημίου Ιωαννίνων
Σίμος Θεόδωρος	Επίκουρος Καθηγητής του Τμήματος Οικονομικών Επιστημών του Πανεπιστημίου Ιωαννίνων
Απέργης Νικόλαος	Καθηγητής του Τμήματος Χρηματοοικονομικής και Τραπεζικής Διοικητικής του Παν/μίου Πειραιά
Νούλας Αθανάσιος	Καθηγητής του Τμήματος Λογιστικής και Χρηματοοικονομικής του Παν/μίου Μακεδονίας
Τζαβαλής Ηλίας	Καθηγητής του Τμήματος Οικονομικών Επιστημών του Ο.Π.Α.
Τσιριτάκης Εμμανουήλ	Αναπληρωτής Καθηγητής του Τμήματος Χρηματοοικονομικής και Τραπεζικής Διοικητικής του Παν/μίου Πειραιά
Φιλιππόπουλος Απόστολος	Καθηγητής του Τμήματος Οικονομικών Επιστημών του Ο.Π.Α.

Έγκριση Διδακτορικής Διατριβής με βαθμό «**ΑΡΙΣΤΑ**» στις 13-06-2013.

ΠΡΟΕΔΡΟΣ ΤΟΥ ΤΜΗΜΑΤΟΣ ΟΙΚΟΝΟΜΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

Ισαάκ Λαγαρής
Καθηγητής

Ο Γραμματέας του Τμήματος

Αλέξανδρος Κανδρέλης

ΠΑΝΕΠΙΣΤΗΜΙΟ ΙΩΑΝΝΙΝΩΝ 1990 ΤΜΗΜΑ ΟΙΚΟΝΟΜΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

ACKNOWLEDGEMENTS

I am grateful to my major supervisor, Associate Professor Dimitris Hatzinikolaou, for his guidance and support, for the excellent cooperation we had, and for his willingness to help me to get through this study. I would also like to thank Assistant Professor Theodore Simos for his valuable comments and for his invaluable help to carry out the calibration in Chapter 5. Last, but not least, I thank my parents for their understanding and support while I was working on this dissertation.

ABSTRACT

Using a two-period overlapping generations model, I examine the effects of several institutional and other variables, e.g., corruption, government stability, the debt-to-GDP ratio, etc., on the probability that the social-security system will grant pensions, and hence the effects of these variables on the relationship between social-security contributions and household saving. To my knowledge, these effects have not been studied in the literature. To address this issue, I derive theoretically a nonlinear Euler equation for household saving under the pay-as-you-go (PAYG) system. Using annual data from two panels of 11 and 25 countries, I estimate this equation by the Generalized Method of Moments (GMM) and by Nonlinear Least Squares (NLLS). I also linearize the Euler equation and estimate it by GMM. The results from the two methods do not differ considerably. The signs of the estimated parameters are compatible with the assumptions of the theoretical model. As well, the results are quite robust to substantial changes in the sample, i.e., from the 11-country to the 25-country panel. They suggest that social-security contributions reduce household saving. Also, the higher the level of corruption or the debt-to-GDP ratio the lower the probability that the PAYG system will grant pensions, and so the lower the reduction in household saving caused by an increase in social-security contributions. Moreover, I calibrate the model to match features of the U.S. economy under the PAYG system and of the Mexican economy under the fully-funded system. The results imply that the PAYG system reduces the steady-state values of household saving, capital stock, and real wage, and increases the steady-state value of the real interest rate, while the fully-funded system reduces the steady-state value of household saving with offset one-for-one and has no effect on the steady-state values of the other variables. A

transition from the PAYG system to the fully-funded system has the same effects on these variables, except for household saving, as would the elimination of the PAYG system without replacing it.

CONTENTS

CHAPTER 3: THE THEORETICAL MODEL

CHAPTER 4: ECONOMETRIC ANALYSIS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

1.1. Why social security?

Social security is an insurance program that has been introduced to ensure a reasonable level of income during retirement, because individuals may have not provided sufficiently for their old age. Social-security programs have become important, as they are the major source of financing the post-retirement consumption in most countries of the world. The need to provide old-age income security is growing with the aging of the population and the weakening of family ties. During the past several years, there has been much of political attention around social security, since many developed as well as developing countries confront crises in their retirement systems.

There are several reasons why a government may provide social security. First, market failure. A voluntary private retirement plan suffers from the adverse selection problem, as individuals have better information about their health and life expectancy than does a private insurance company. Individuals who expect to live a long life in retirement will be willing to pay higher insurance rates, and this will cause the insurance company to charge higher rates. As a result, many individuals will not be able to insure, and so a market voluntary retirement plan will fail to provide insurance effectively. As well, in the case that private plans offer flat benefits to all citizens, they may suffer from the moral hazard problem. Offering flat benefits to all citizens may cause some individuals who are able to work to retire earlier than they would otherwise. These asymmetric information problems explain why many governments provide mandatory social-security programs.

Another market failure occurs because of externalities. Some people fail to insure themselves against old-age poverty, so taxpayers must support them. This is a free-riding problem, according to which some individuals do not save sufficiently for retirement, knowing that the society will make up the difference. In this case, the government is required to establish a compulsory retirement plan in which all must contribute and all will benefit from it.

Second, paternalism. Many individuals fail to accumulate enough savings for retirement, due to their inability to look forward and to make decisions under uncertainty. They might also underestimate the length of their retirement life and do little saving. So a mandatory social-security program is required to force adequate saving for retirement.

Third, income redistribution. Social security programs provide guaranteed income to those who are economically vulnerable or unable to work, in two different redistributive ways: (1) redistribution *between* generations, where the government collects social-security taxes from the young working population and simultaneously pays benefits to the old retired population; and (2) redistribution *within* generations, where the government taxes high-income citizens and pays benefits to low-income citizens.

Fourth, macroeconomic risks. Private markets may not insure completely against the risk of inflation; accordingly, they may not guarantee constant real benefits. As well, the government can offer partial protection against capital-market risks. For instance, the capital-market crash of the 1930's in the U.S. caused private-pension plans to collapse and encourage the public provision of pensions. Under public social-security programs, however, benefit payments can be subject to fiscal pressure and political manipulation. Therefore, to insure against macroeconomic risks, some government intervention is required (Mitchell, 1993, p. 30).

Moreover, there exist some other factors that play a role in social-security programs. Sala-i-Martin (1996) suggests that older workers have lower than average skill and, because of interaction among workers, there is negative externality on the rest of the labor force. Thus, aggregate output would be larger if the older workers did not work, so the social-security program is required to encourage the older workers to leave the labor force.

Finally, in addition to social security, there exist other public-assistance programs. For example, disability insurance provides income payments to physically unable workers. As well, supplemental social-security programs provide income support to low-income retirees, especially in the U.S. These other programs interact with social security, and it is unclear whether there is a substitutional or complemental relation between them (Diamond and Gruber, 1997, pp. 9-14).

1.2. Characteristics of social-security programs

Social-security programs are complex and differ from country to country, because they depend upon individual countries' socioeconomic characteristics. As well, most countries have a mix of different schemes that provide old-age income support. Thus, it would be almost impossible to describe all of them in detail, so I will refer to their most common features.

In most countries, social-security programs cover citizens who work in the public or in the private sector, and the benefits received are linked to their previous wage history. Coverage rates vary across countries, and some workers may be excluded from retirement systems. For example, immigrants who do not hold citizenship may be excluded. Also, in certain economic sectors employees are covered by special retirement plans, as in the public sector and in the military.

In order to be eligible for receiving benefits, employees must have a certain number of years of work and attain a specific age. In most Organization for Economic Cooperation and Development (OECD) countries, the normal retirement age is 65 for men and somewhat lower for women, whereas the early retirement age is between 55 and 64. Those who choose to retire before the normal retirement age receive reduced benefits (Mitchell, 1993, pp. 18-22).

Some countries' retirement systems impose an "earnings test," that is, they reduce benefits for eligible recipients with earnings above a certain level. Others permit retirement benefits only to those with little or no earnings. In these cases, work among older population is discouraged.

In almost all countries, social-security programs are financed by levying wage taxes on both the employee and the employer. Pension benefits are adjusted to price inflation and wage growth to protect their purchasing power and fairness. Also, benefits are subject to taxation for retirees with adequately high income.

Further, it should be emphasized that social-security programs, public and private, are criticized for administrative inefficiency and inadequate integration, that is, their various components are managed as if they were separate agencies. In many developing countries, as in Latin America, and also in developed ones, social-security programs encounter procedural problems and costly bureaucratic management. As well, they do not keep adequate records for the employees' years of coverage, as in Eastern Europe (Mitchell, 1993, p. 32).

Finally, the increase in the proportion of retirees in the population causes the cost of social-security programs to accelerate, something that is expected to continue in the following years. In the OECD countries, the cost of the existing social-security programs has reached substantially high levels, thus social-security reforms are under way in many countries.

1.3. Types of social-security programs

There exist two types of social-security systems: a) pay-as-you-go (PAYG) systems, also called unfunded systems; and b) fully-funded systems. Most national social-security systems operate on a PAYG basis, where the contributions of the young working population in each period are used to pay benefits to the retired population in that specific period. The PAYG system works in a redistributional way, since it provides income support to the old-age individuals by taxing the young working ones.

Most private-pension systems are fully funded. The contributions of the young working population are invested in a diversified portfolio and are returned with interest upon retirement. The fully-funded system does not have redistributional purposes across generations and might entail some risky returns because of capital market variability.

Also, we can distinguish social-security plans between: a) defined-contribution plans, and b) defined-benefit plans. In a defined-contribution plan, each individual contributes a certain amount to a personal investment account, and most of the plans do not ensure a minimum pension or earnings replacement. Benefits depend upon the investment performance of the account. In a defined-benefit plan, benefits are determined by a benefit formula according to previous earnings and years of service, and ensure minimum pension payments and earnings replacement (Mitchell, 1993, pp. 23-24).

It is important to note that there are mixed systems as well. A defined contribution or benefit system can be either funded or unfunded. In an unfunded defined-contribution system, each individual puts an amount to a personal account. The rate of return is at a notional interest rate, which is specified by the government and is equal to the rate of growth of the contribution base. This fund is not invested in stocks and bonds, but is used by the government to pay current PAYG benefits. At retirement, each individual has an accumulated amount in his/her account augmented by the notional rate of return. This mixed system is recommended for the countries of the European Union (EU), with free labor mobility among them. Recently, Sweden and Italy shifted from an unfunded defined-benefit system to an unfunded defined-contribution one (Feldstein, 2001, pp. 10-12).

The U.S. social-security system is described as an unfunded defined-benefit one, since benefits are related to previous earnings and are paid to current retirees. In contrast, older U.S. social-security systems were of the funded defined-benefit type, where a worker's contributions were accumulated and upon retirement he received benefits, according to his previous earnings and years of work. Also, the benefits were defined in a way that was independent of investment returns.

Finally, some Latin American countries, like Chile, Mexico, Colombia, and Bolivia, use funded defined-contribution systems. Each employee (and/or employer) makes contributions to an individual investment account, and upon retirement he receives benefits according to his contributions and the accumulated investment returns.

1.4. The evolution of social-security programs

The first old-age pension system was established in Germany in 1889. The program was compulsory and both employees and employers made contributions. Denmark and New Zealand enacted old-age pension systems in 1891 and 1898, respectively. In Great Britain and Australia, old-age social security was adopted in 1908. Sweden introduced a compulsory old-age pension system in 1915, while Canada established a non-contributory system in 1927 (Feldstein, 2001, pp. 10-11).

In the U.S., the social-security program emerged after the Great Depression of the 1930's. The Social Security Act of 1935 established the Old-Age Assistance (OAA) program and the Old-Age Insurance (OAI) program, while there was a substantial amendment in 1939, when dependents' and survivors' benefits were introduced. The OAI program became recurrently the Old-Age Survivors and Disability Insurance (OASDI) program, which is now called Social Security. It was a compulsory contributory program, in which workers received benefits conditioned on contributions during their working years. Also, benefits were provided to dependents. In addition, the OAA program provided payments to poor elderly. The program has been replaced by the supplemental social-security program.

Since the beginning of the 20th century, social-security programs evolved and expanded all over the world. By 1989, 130 countries had some form of social-security program (Sala-i-Martin, 1996, p. 280). In most developed countries, they were extended to cover the major portion of the labor force (agricultural workers and self-employed are excluded in some countries) and other cohorts of individuals, such as the dependents of disabled workers. In contrast, in developing countries, social-security programs frequently cover few selected sectors. In most countries, there has been an increase in the benefits and in the share of government spending in percent of GDP. The rising cost of social-security programs and the anticipated declining social-security funding have caused many countries to reform their existing programs.

1.5. Current debates on social security

In the past few decades, the increased aging of the population has caused great pressure on social-security systems around the world and has stimulated growing interest in reform. Population aging has two sources: a) increased longevity, which extends the proportion of life spent in retirement and increases pre-retirement saving rates, and b) decreased fertility, which depresses the growth rate of the labor force and decreases the rate of return on capital (Kulish, Kent, and Smith, 2010, pp. 16-19). Providing a constant level of benefits to a growing number of retirees increases the burden of the public treasury and leads to deficit spending.

According to Demirguc-Kunt and Schwarz (1999), many current social-security systems are fiscally unsustainable. In their early years, most systems present a cash surplus, which disappears as they mature and turns into deficit. As population aging continues, deficits become much greater than can be covered by current taxes. Therefore, many countries undertake pension reforms. Such reforms are politically difficult, however, as they may lack the support of the majority of the voters (Galasso and Profeta, 2002, p. 26).

People are aware of the unsustainability of the public PAYG social-security systems. Many advocates of the social-security reform argue that the growth of the financial markets around the world could provide an opportunity for transferring social-security contributions into personal accounts and invest them in financial assets, turning from a PAYG system to a fully-funded one. Against this view is the argument that asset returns are risky.

Reforms can be classified into two categories: a) minor adjustments, and b) major reforms. Minor adjustments entail changes in the eligibility criteria, the contribution structure, and the benefit structure of the social-security system. First, eligibility criteria can be changed by shifting the retirement age and/or the years of service required for pension. Nearly all countries have increased the retirement age and some have also increased the necessary years of service for entitlement. Second, the contribution structure of the system can be changed by altering the contribution rate and/or base on which contributions are computed. The change in the contribution rate is the most common minor reform adopted across countries. The contribution base change is less common and has taken the form of increasing the income boundaries from which contributions are made. Finally, the benefit structure can be adjusted through modifications in the pension formula and/or the indexation to inflation. Older countries modified their pension formulae by increasing the number of years used to compute the pensionable earnings (Demirguc-Kunt and Schwarz, 1999, pp. 5-8).

Major reforms, on the other hand, have taken place in relatively fewer countries. Only about 25 percent of the reforms can be characterized as major. They include alteration of the social-security system from defined benefit to defined contribution, or vice versa, or from PAYG to fully funded, or vice versa. Starting up a new system or retaining a PAYG one and adding a new fully-funded defined contribution second pillar is also considered to be a major reform.

The type of reform varies significantly across countries in different regions. Several Latin American countries, like Chile, Mexico, Bolivia, El Salvador, Colombia, and Peru, have undertaken major reforms, switching from a PAYG defined-benefit to a fully-funded defined-contribution system. Uruguay has kept the PAYG defined-benefit system as its primary system, but has made a fully-funded defined contribution a mandatory second pillar for those of moderate income and optional for those of low income. Also, Argentina maintained the PAYG defined-benefit system as its primary system, but gave workers the choice between a PAYG defined benefit and a fully-funded defined contribution as a mandatory supplementary system (Demirguc-Kunt and Schwarz, 1999, pp. 9-11).

Another region where major reforms have been undertaken is Eastern Europe. Poland and Latvia have switched their PAYG defined-benefit system into a notional account PAYG defined-contribution one[1] with a fully-funded defined contribution second pillar. Italy and Sweden have undertaken this reform as well. Some Asian and African countries made the reverse reform from a fully-funded defined-contribution system to a PAYG defined-benefit one, due to abuses in the administration of the funds. An exception is Kazakhstan, which made the fully-funded defined-contribution system its primary system. In the industrialized countries, however, most of the reforms took the form of minor adjustments to their existing systems. An exception is Australia, where a fully-funded defined-contribution pillar was instituted as a complement to a general revenue-financed universal pension (Demirguc-Kunt and Schwarz, 1999, pp. 12-13).

Although the number of major reforms increases, the minor adjustments to the existing social-security systems are still the primary type of reform across countries. Most of the minor reforms, however, do not provide permanent solutions to the

[1] In this system, each individual contributes a certain amount to a personal account, which receives a rate of return at a notional interest rate legislated by the government. For example, Sweden uses the growth rate of nominal wage. This fund is used by the government to pay benefits to the elderly of the current period.

problems of pension systems, but simply postpone fiscal crises. While minor reforms improve the fiscal situation in the short-run, they might not be sufficient in the long-run and require considerable government funding. In contrast, major reforms are fully sustainable in the long-run, although they are costly in the short-run. Further, minor reforms will have to be repeated periodically, which undermines the reliability that a social-security system will provide retirement benefits. Finally, a major reform toward a fully-funded defined-contribution system can also advance economic efficiency, through enhancing capital market development and generating additional saving.

1.6. The motivation and the purpose of the study

As a rule, social-security systems operate under uncertain quality of institutions. This is particularly true for the less developed countries, where inefficient and corrupt governments administrate the retirement systems. As well, political instability and the lack of transparency impede their proper functioning. Developed countries also face fiscal problems and difficulties in funding their retirement systems. They may have difficulty in keeping their credibility, as they confront growing deficits in their systems. For instance, in the UK, the cost of paying pension benefits is expected to rise over the next years (quadruple over the next 50 years) and the retirement system will not be able to support its pensioners (O’Grady, 2012, p. 2). Also, many people face retirement poverty, since pension benefits are cut due to rising longevity and new EU rules, while their cost is rising (O’Grady, 2013, pp. 1-2). The task of making social-security systems more sustainable and credible is difficult and expensive considering the increased life expectancy and the lower fertility.

Low institutional quality undermines the ability of the social-security systems to honor their promises and also influences household saving. The impact of social security on household saving has occupied much of the literature in public finance. To my knowledge, however, previous research on social security has not addressed the issue of how uncertainty of the quality of institutions influences the relationship between social security and household saving. Therefore, the main purpose of this study is to incorporate several institutional and other variables, e.g., corruption, government stability, the debt-to-GDP ratio, etc., in a theoretical model derived from a typical household dynamic optimization problem under uncertain quality of social-security institutions. Specifically, I examine how institutional variables and the government debt-to-GDP ratio affect the probability that the social-security system (PAYG and fully funded) will grant pensions to the old at retirement; and, by extension, the relationship between social-security contributions and household saving.

1.7. Structure and results

The selection of the variables that influence household saving is crucial for explaining the accumulation of wealth and for policy. So far, the empirical findings on the relationship between social-security wealth and household saving are inconclusive, however. In the present study, in addition to socioeconomic characteristics, an important factor that affects the relationship between social security and household saving is the quality of institutions.

Chapter 2 presents the review of the literature on the effects of social security on private saving. First, I present various theoretical considerations on the effect of social security on private saving. Economic theory alone cannot establish the magnitude of the effect, however. Therefore, the conclusions of various empirical studies are briefly presented. These studies generally use reduced-form saving equations. They are categorized according to the nature of the data employed, i.e., time-series, cross-sections, cross-country, and time-series of cross-sections. Their results are inconclusive.

Then, I discuss the effects of political institutions and of economic and demographic factors on social security according to the positive theories of social security. Positive theories can be categorized into political theories and efficiency ones. Political theories suggest that political institutions are the most important determinants of social-security policy, while efficiency theories emphasize the economic and demographic factors. These studies do not provide clear evidence on the relationship between political institutions and social security. Their attention is limited to the institution of voting and to the form of the political system (democratic or nondemocratic) neglecting other institutional features, e.g., corruption, government stability, etc. In addition, they do not consider the effect of these institutional features on the relationship between social security and household saving. This study instead examines the effect of several institutional variables and of the debt-to-GDP ratio on social security and, by extension, on household saving.

In Chapter 3, I develop the theoretical model under the general framework of a life-cycle model. I employ a two-period Overlapping Generations Model (OGM) and solve the household maximization problem. The intertemporal budget constraint takes into account the fact that individuals contribute a certain amount of their labor income in the social-security system and expect to receive social-security benefits at retirement. I assume that the expected social-security benefits are affected by the probability that the social-security system will grant pensions to the old at retirement. This probability, which is determined by the use of a binary response model, is assumed to depend on institutional variables (e.g., corruption and government stability) and the government debt-to-GDP ratio. Maximizing the lifetime utility function under the intertemporal budget constraint gives an Euler equation for consumption. Moreover, I employ a stochastic production function, which comprises random productivity shocks. After solving the household and the firm maximization problems and after deriving the resource constraint, I present the system of equations that describe the competitive equilibrium.

The household maximization problem has no analytical (closed-form) solution. So, I take partial derivatives of the Euler equation to compute the effect of the institutional variables and of the debt-to-GDP ratio on the relationship between social-security contributions and household saving under the PAYG and the fully-funded system. First, I compute the effect of social-security contributions on household saving. Social-security contributions are expected to affect household saving negatively. In the fully-funded system, social-security contributions may not offset household saving one-for-one as suggested by the traditional life-cycle theory. The reduction in household saving is expected to be less than one-for-one.

Then, I compute the effect of the institutional variables and of the debt-to-GDP ratio on the relationship between social-security contributions and household saving. The higher the level of corruption or the debt-to-GDP ratio, the lower is expected to be the reduction in household saving caused by an increase in social-security contributions. In contrast, the higher the degree of government stability, the greater is expected to be the reduction in household saving caused by an increase in social-security contributions. The effect of the institutional variables and of the debt-to-GDP ratio on the probability that the social-security system will grant pensions at retirement is the channel through which these variables affect the relationship between social-security contributions and household saving. Finally, I extend the theoretical model by considering the possibility of a collapsing social-security system.

Chapter 4 presents the econometric investigation of the theoretical conclusions derived under the PAYG social-security system in Chapter 3. For the econometric analysis, I employ three panel data sets, a balanced panel of 11 OECD countries for the years 1984-2009, an unbalanced panel of 25 countries, which includes the previous 11 countries, and 14 more countries for the years 1995-2009, and a balanced panel of the 25 countries for the years 1995-2009. After describing the data sets, I use various panel unit-root tests to examine the stationarity properties of the variables. Since there is no closed-form solution to the household maximization problem, which implies that a household saving equation cannot be derived, I use the empirical counterpart of the Euler equation for household saving as the basic specification of the econometric analysis.

I estimate a fixed-effects Euler equation for household saving using the three panel data sets mentioned above and two estimation procedures, the Generalized Method of Moments (GMM) and the Nonlinear Least Squares (NLLS). I also linearize the Euler equation and estimate it by GMM. The estimates obtained from these panels do not differ dramatically, so it would not be unreasonable to argue that they are robust to substantial changes in the sample. Then, these estimates are used to compute the partial derivatives of the theoretical model. Generally, the empirical findings are compatible to a large extent with the implications of the theoretical model.

The GMM estimates generally support the assumption of the theoretical model that the higher the level of corruption or the debt-to-GDP ratio the lower the probability that the PAYG system will grant pensions to the old at retirement. In the case of the linearized Euler equation, the GMM estimates also support the assumption that the higher the degree of government stability the higher this probability. The variables that measure socioeconomic conditions and democratic accountability have no significant effect on this probability, however.

The effect of social-security contributions on household saving is found to be negative. In the case of the nonlinear Euler equation, this effect is found to be lower (in absolute value) than that found in the case of the linearized Euler equation.

As well, the GMM estimates generally suggest that the higher the level of corruption or the debt-to-GDP ratio the lower the reduction in household saving caused by an increase in social-security contributions. In the case of the linearized Euler equation, the GMM estimates also suggest that the higher the degree of government stability the greater the reduction in household saving caused by an increase in social-security contributions. Regarding the NLLS estimates, they are in accordance with the GMM ones, except for the coefficient of corruption, which is found to be statistically insignificant at conventional levels.

In Chapter 5, I calibrate the model (presented in Chapter 3) to examine its consistency with certain features of the actual data and then use it to examine the effects of the PAYG system as well as of the fully-funded system on the variables of interest. In particular, I transform the system of equations that describe the competitive equilibrium in per effective labor terms, so that this system is expressed in terms of stationary variables, and compute the steady-state equilibrium.

I calibrate the model by assigning values to the parameters and examine whether the model can replicate some features of the actual data from the U.S. economy reflected in the second-order sample moments, e.g., standard deviations and correlations. The results suggest that the model can replicate some features of the U.S. economy, but clearly not perfectly, since there are also some features of the actual data that have not been captured by the model. After computing the steady-state values of household saving, capital stock, real wage, and the real interest rate, I compute the general-equilibrium effects of the PAYG system on these values. Holding constant the real wage and the real interest rate, I compute the corresponding partial-equilibrium effects. The results suggest that the PAYG system causes a reduction in the steady-state values of household saving, capital stock, and real wage, and an increase in the steady-state value of the real interest rate. The general-equilibrium effects are smaller than the corresponding partial-equilibrium ones. The effect of the PAYG system on the steady-state value of capital stock is similar to that found by Kotlikoff (1979a), but lower (in absolute value) than that found by Hubbard and Judd (1987) using a life-cycle model and various parameter values. In addition, considering a possible collapse of the PAYG system, the state-state values of household saving, capital stock, and real wage increase, while the steady-state value of the real interest rate decreases. Examining the sensitivity of the results for different parameter values, they seem to be robust to changes in those values.

Then, I calibrate the model to match features of the Mexican economy, which uses a fully-funded social-security system, using the same procedure as that in the case of the U.S. economy. The results suggest that the model can capture some features of the Mexican economy, but some other features have not been captured by the model. The results imply that the fully-funded system causes a one-for-one reduction in the steady-state value of household saving, while it has no effect on the steady-state values of capital stock, real wage, and the real interest rate. These results are compatible with the traditional life-cycle ones (Feldstein and Pellechio, 1979, Kotlikoff, 1979b).

I also examine the effects of a transition from the steady-state equilibrium under the PAYG system to the steady-state equilibrium under the fully-funded system, which is financed by the government budget. The results imply that the transition to the steady-state equilibrium under the fully-funded system has the same effects on the steady-state values of capital stock, real wage, and the real interest rate as would the elimination of the PAYG system without replacing it. Chapter 6 presents a summary and some conclusions.

CHAPTER 2: REVIEW OF THE LITERATURE

2.1. Introduction

The literature on social security has grown substantially during the last few decades. The impact of social security on private saving has occupied much of the empirical research and is crucial for important policy issues. In this chapter, I present some theoretical effects of social security on private saving and various empirical models and results. The empirical studies are categorized according to the data type they use. I also present studies of two different approaches considering the relationship between political institutions and social security.

2.2. The effects of social security on private saving

For most economic research, the starting point in explaining private saving behavior is the neoclassical life-cycle theory of consumption and saving. The life-cycle theory is a forward-looking theory that assumes rational behavior. It explains saving and consumption by taking into account present and future recourses. Saving is considered as a way to smooth consumption when there are income fluctuations, and consumption is determined according to anticipated lifetime resources. An increase in expected future resources, like an increase in anticipated social-security benefits (not followed by an equal increase in taxes), raises lifetime income and consumption at every stage in life. Generally, the basic life-cycle theory suggests that individuals make their lifetime consumption and saving choice by maximizing their lifetime utility function subject to their lifetime budget constraint.

The life-cycle theory is considered by many researchers to be a suitable framework for discussing the effects of social security. It implies that social security affects saving and wealth accumulation. Feldstein (1974) argues that social security affects private saving in two different ways. First, for an actuarially fair social-security program, i.e., when the yield on social-security taxes equals the market interest rate, social-security benefits substitute for private wealth accumulation. This is called the “wealth replacement effect” of the traditional life-cycle model and implies that social-security benefits reduce private saving.

Second, he extends the life-cycle model by treating the retirement decision not as fixed, but as an endogenous variable. Social-security benefits are likely to induce early retirement, because benefits are subject to the earnings test, that is, reduced benefits for eligible recipients whose earnings exceed a certain threshold. But the resulting longer period of retirement requires more saving during the working years in order to maintain the consumption level after retirement. This is called the “induced retirement effect” suggested by the extended life-cycle model. The net effect of social security on saving depends on the relative strength of these two offsetting forces.

Barro (1974) suggests another extension of the life-cycle model by introducing intergenerational transfers. He claims that social security may not affect private saving through an offsetting change in social-security intergenerational transfers. Social security is a transfer from the children (who pay social security taxes) to the parents (who receive benefits). An increase in social-security benefits may cause parents to increase their bequests so as to offset the additional taxes that their children pay. Thus, the extra saving for these bequests offsets the reduction in private saving because of the increase in social-security taxes. Also, the effect of social security on saving is reduced to the extent that parents rely on their children to support them at retirement. Parents expect a reduction in children support when there is an increase in social-security benefits.

In considering an unfunded social-security system, Kotlikoff (1979a, 1979b) maximizes a lifetime utility function subject to a lifetime budget constraint. The introduction of social security gives rise to the following budget constraint:

$$\int_0^T C(t)e^{-rt}dt = \int_0^R E(t)(1-\tau)e^{-rt}dt + \int_R^T B(t)e^{-rt}dt\,, \tag{2.1}$$

where T is the fixed length of life, R is the retirement age, r is the market rate of interest, $C(t)$ is the consumption flow, $E(t)$ is the earnings flow, $\tau E(t)$ is the social-security tax, and $B(t)$ is the social-security benefit. If the retirement age is fixed and social security offers a yield on paid tax that is equal to the market interest rate, r, then the present value of lifetime social-security taxes is equal to the present value of lifetime social-security benefits. Thus, lifetime wealth is not affected by the social-security system. Under these assumptions, accumulated social-security taxes replace accumulated private savings one-for-one. This is the "wealth replacement effect" mentioned above, which implies the following expression:

$$\int_0^R E(t)\tau e^{-rt}dt = \int_R^T B(t)e^{-rt}dt\,. \tag{2.2}$$

Equation (2.2) implies that the present value of lifetime social-security taxes is equal to the present value of lifetime social-security benefits. Departure from this equality implies a lifetime wealth increment or decrement. For instance, a lifetime wealth increment occurs when the yield (e.g., the growth rate of population) on social-security taxes is in excess of the market interest rate, and thus the present value of lifetime benefits exceeds the present value of lifetime taxes. So, there is an increase in lifetime wealth and consumption, assuming that consumption in every period is a normal good. Because of this increase in lifetime consumption, accumulated private savings are reduced by more than the taxes paid. The opposite is true, when there is a lifetime wealth decrement. This is the "yield effect."

In this approach, social security affects individual saving through its impact on the intertemporal budget constraint. Disposable income falls by the amount of social-security tax. If the yield on social-security taxes is greater (lower) than the market interest rate, the present value of lifetime benefits is greater (lower) than the present value of lifetime taxes. Thus, an increase (decrease) in lifetime resources is generated, raising (reducing) consumption in every period, and so reducing individual saving by more (less) than the tax paid. Saving is reduced by more (less) than the amount of the tax because consumption in every period, and thus consumption of the young working individuals increases (decreases).

In the fully-funded system, the yield on social-security taxes equals the market interest rate, so the intertemporal budget constraint is unchanged by the introduction of social security. Social-security taxes offset individual saving one-for-one. The "wealth replacement effect" is valid here. Hubbard (1984) points out, however, that, under uncertainty over the length of life, even a fully-funded system reduces individual saving by more than the social-security tax. He considers that there is a difference in the discount rate under certainty and uncertainty, and this causes the present value of lifetime benefits to exceed the present value of lifetime taxes so that the lifetime resources and consumption increase. Thus, individual saving is reduced by more than the tax paid because of this increase in lifetime consumption.

An alternative version of the life-cycle hypothesis is the "buffer-stock" model of saving suggested by Carroll (1997), according to which individuals, particularly young workers, may maintain a “buffer stock” to protect themselves against uncertain events, as earnings downturn, medical expenses, etc. This is called precautionary saving.[2] To the extent that social security insures against uncertain events, it may be a substitute for precautionary saving.

From a psychological point of view, some writers, e.g., Cagan (1965) and Katona (1965), argue that pension plans make individuals realize the need of saving for their old age. Cagan (1965) interprets this as a “recognition effect,” that is, individuals recognize the significance of saving for retirement when they are obligated to participate in a retirement plan. As well, Katona’s (1965) interpretation of this result derives from the “goal gradient hypothesis,” according to which the closer is someone to achieve his retirement goal, the greater is his saving effort. It is important to note that psychological theories do not assume fixed preferences and objectives, as does the life-cycle theory, but are based on socioeconomic factors (Beverly and Sherraden, 1999, p. 460). These factors can also be taken into account when the life-cycle model is assumed.

Unlike the life-cycle theory, some studies assume that individuals do not always behave rationally or have perfect knowledge (Beverly and Sherraden, 1999, p. 461). Individuals with myopic behavior are unlikely to make optimal decisions on saving or adjust their saving in response to changes in social-security benefits (Feldstein and Pellechio, 1979, p. 361). Thaler and Shefrin (1981) suggest that individuals may choose to impose constraints on their own behavior, e.g., saving as a fixed fraction of disposable income, avoiding borrowing, etc. Thus, social-security benefits may not substitute for private saving. Dolde and Tobin (1981) note that

[2] Hubbard, Skinner, and Zeldes (1995) reconcile the life-cycle model with precautionary saving. They generalize the “buffer-stock” model to incorporate the U.S. social-security system and argue that social security depresses saving of households with low lifetime income. Insurance programs reduce the uncertainty facing households, thus decreasing precautionary saving.

individuals may be liquidity constrained and save for reasons other than retirement, e.g., to acquire a house. Hence, the replacement effect will be diminished. Also, Bernheim and Scholz (1993) imply that low-income individuals, with low education, may increase their saving by participating in private-pension plans.

Feldstein (1979) doubts, however, that such behavior is common. He notes that it may reduce the depressing effect of social security on saving, but it does not eliminate it. On the other hand, the life-cycle theory has been criticized, because individuals do not always behave perfectly rationally and may not be able to smooth lifetime consumption because of imperfect capital markets.

Overall, the theoretical considerations cannot determine the net effect of social security on private saving. The wealth replacement effect of the traditional life-cycle theory suggests that social-security benefits substitute for private saving. The yield effect suggests a greater (or lower, as I mentioned above) than one-for-one offset in private saving. The induced early retirement effect, the offsetting changes in intergenerational transfers, the special psychological characteristics, the irrational or myopic behavior, and the self-imposed constraints of some individuals imply that social-security benefits may not substitute for private saving, but may even raise it. Economic theory alone cannot establish the degree of substitutability between social security and private saving. Therefore, empirical research is required. Note, however, that empirical studies differ widely in model specification and results. The next section presents various empirical studies, which employ different models and data sets and find different results.

2.3. Empirical models and results

Most of the theoretical and empirical analyses on saving behavior employ some version of the life-cycle model. Numerous empirical studies have been conducted using time-series and/or cross-sectional data for single countries or for a group of countries, trying to quantify the relationship between social-security benefits and private saving and establish the degree of substitutability between them. They generally use reduced-form saving equations, however.

2.3.1. *Time-series analyses*

In his empirical model, Feldstein (1974) adopts the life-cycle consumption function of Ando and Modigliani (1963) and expands it to include social-security wealth and corporate retained earnings. He uses two definitions of social-security wealth: gross social-security wealth (GSSW) and net social-security wealth (NSSW). GSSW is the present value of the retirement benefits anticipated by individuals and NSSW equals GSSW minus the present value of social-security taxes to be paid by current workers. He estimates the consumption function with both definitions and finds the same implications. The basic specification is

$$C_t = a_0 + a_1 Y_t + a_2 RE_t + a_3 W_{t-1} + a_4 SSW_t \, , \tag{2.3}$$

where C_t is consumer expenditure, Y_t is permanent income, RE_t is corporate retained earnings, W_t is the stock of household wealth and SSW_t is social-security wealth. Feldstein (1974) argues that SSW_t reflects both the “wealth replacement effect” and the “induced retirement effect,” and does not include a distinct measure of the length of retirement, which reflects separately the “induced retirement effect.”

Feldstein (1974) estimates Equation (2.3) using aggregate U.S. time-series data for the period 1929-71, excluding the war years 1941-46, as well as for the postwar period 1947-71. He estimates several alternative specifications of Equation (2.3) by ordinary least squares.[3] In some cases, he includes the unemployment rate, but its coefficient is not statistically significant. Generally, the marginal propensity to consume out of social-security wealth is statistically significant, and the results support the conclusion that social security depresses personal saving.

Munnell (1974) employs a similar framework, but uses saving instead of consumer expenditure as the dependent variable. In her saving equation, she includes a measure of the length of retirement, and thus the coefficient of social-security wealth captures only the "wealth replacement effect." Her results imply a negative effect of social security on personal saving that is almost offset by a positive "induced retirement effect."[4]

Barro (1978) uses a consumption function that is similar to that of Feldstein (1974), but includes additional variables, such as the government surplus, the unemployment rate, and the stock of durable goods. His results are not strong, however, and imply a more ambiguous effect of social-security wealth on personal saving. Moreover, Darby (1979) includes measures of real money balances and concludes that the effects of social-security wealth on saving are negative in most cases, but positive in some others.

Leimer and Lesnoy (1982) find a computer programming error in Feldstein's (1974) algorithm for social-security wealth. They replicate correctly the algorithm and

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[3] He also estimates some alternative specifications of Equation (2.3) by the instrumental variable technique, which confirms his ordinary least squares results.

[4] In her 1976 cross-sectional study, Munnell finds that social security and private pensions coverage discourage saving at least of the older men aged 45-59, for whom retirement is the primary reason for saving.

also develop their own algorithm by considering a number of alternative individuals' perceptions of future benefits. They estimate Feldstein's specification of the consumption function for the period 1930-74, excluding the war yeas 1941-46, as well as for the postwar period 1947-74. Their results do not significantly support the hypothesis that social security reduces personal saving. In his reply, Feldstein (1982) corrects the computer programming error and accounts for the 1972 change in social-security benefit legislation by increasing his corrected variable by 20 percent, beginning in 1972. He argues that Leimer and Lensoy ignore this shift in the social-security wealth variable, and their alternative assumptions of individuals' perceptions of future benefits introduce biases in the estimated coefficient of social-security wealth. Re-estimating his original equation, he finds statistically significant effects of social-security wealth that support his earlier results.

The time-series studies presented above examine the effects of social-security wealth on consumer expenditure and by extension on personal saving.[5] They do not provide conclusive evidence, however. Feldstein (1974, 1982) finds negative effects, while Munnell (1974), Barro (1978), and Darby (1979) find contradictory effects, both positive and negative. Finally, Leimer and Lesnoy (1982) find no significant effect for the entire sample period, but a significant positive effect for the postwar sample period.

2.3.2. *Cross-sectional analyses using household data*

In his empirical specification, Kotlikoff (1979b) defines a linear wealth accumulation regression as follows:

[5] The effects of social-security wealth on consumer expenditure can be translated into effects on personal saving. Thus, an increase in social-security benefits may lead people to consume more and save less for retirement.

$$NW = \beta_0 + \beta_1 SST + \beta_2 LWI + \beta_3 RA + \beta_4 LI + \gamma' Z + \varepsilon \,, \qquad (2.4)$$

where *NW* is the net worth of households, i.e., assets minus liabilities, *SST* is the present value of accumulated social-security taxes, *LWI* stands for the lifetime wealth increment or the yield of the social-security system to households, *RA* is the expected retirement age, *LI* is the lifetime labor income gross of social-security taxes, *Z* is a vector of additional exogenous variables, and ε is the error term. *LWI* is equal to the present value of expected future social-security benefits minus the present value of expected future social-security taxes minus the accumulated taxes that have been paid for social security.

He also considers the following linear regression for the expected retirement:

$$RA = a_0 + a_1 SBL + a_2 LWI + a_3 LI + \delta' H + \zeta \,, \qquad (2.5)$$

where *SBL* is the ratio of social-security benefits lost at full-time work to earnings from full-time work, *H* is a vector of additional exogenous variables, and ζ is an error term. Both regressions include dummies for socioeconomic characteristics, e.g., race, marriage, age, education, pension, health, etc. Equation (2.5) is used to determine endogenously the retirement decision and capture the "induced retirement effect" mentioned above.

The study uses cross-sectional data of U.S. households for 1966. The life-cycle theory implies that in the wealth accumulation regression the coefficient of *SST* should equal -1, while the coefficient of *LWI* should be negative and less than 1 in absolute value. The empirical findings indicate, however, that accumulated social-security taxes substitute for accumulated household savings, but not one-for-one. Also, the coefficient of lifetime wealth increment is insignificant. Moreover, in the expected retirement age regression, Equation (2.5), Kotlikoff's (1979b) results imply that social security does not significantly affect the retirement age decision.

Feldstein and Pellechio (1979) estimate various specifications of the following regression equation:

$$A = a_0 + a_1 YL - \mu SSW + a_2 YL^2 + \varepsilon \,, \tag{2.6}$$

where A is "fungible" wealth, defined as household's total wealth minus social-security wealth, YL is net of tax labor income, SSW is social-security wealth, and ε is an error term. The estimated value of μ is crucial for the assessment of the effect of social security on household wealth accumulation. The traditional life-cycle hypothesis implies that $\mu = 1$, that is, there is a complete offset of social-security wealth for household wealth accumulation. The offsetting intergenerational transfers or the induced early retirement imply that $\mu < 1$ or even $\mu \leq 0$, while individuals' irrational and myopic behavior implies that $\mu = 0$.

The authors use cross-sectional U.S. data from the 1963 Survey of Financial Characteristics of Consumers. Their empirical results imply that social-security wealth reduces substantially household wealth. The implication of the traditional life-cycle model that there is one-for-one offset is not rejected, but the estimates are also consistent with the less than one-for-one offset. Overall, the findings support the life-cycle hypothesis.

King and Dicks-Mireaux (1982) examine the wealth holdings of households over the life-cycle and their dependence upon private pension and social-security wealth. Assuming that pension wealth is an imperfect substitute for other forms of wealth, they estimate a regression equation of households' net worth on social-security wealth, private-pension wealth, permanent income, and age. Using Canadian cross-sectional data for 1977 comprising both young and old households, their findings support the proposition that social security and pension wealth reduce household wealth. The estimated offset is an increasing function of wealth.

Hubbard (1986) employs a simple life-cycle model of wealth accumulation in the presence of social-security benefits and private-pension benefits. He defines a regression equation of nonpension wealth to permanent income ratio as a function of age, permanent income, social-security benefits to permanent income ratio, private-pension benefits to permanent income ratio, and a vector of individual characteristics. Using 1979 cross-sectional data for the U.S., the empirical results suggest that social-security and private-pension benefits affect negatively household nonpension wealth, but with offset less than one-for-one. For high income individuals the offset is predicted to be greater than one-for-one, however.[6]

Bernheim (1987) examines the effects of social security on individual wealth accumulation in the context of the life-cycle hypothesis. He employs cross-sectional U.S. data for the year 1969 and estimates a regression equation of accumulated wealth (exclusive of social security) on social-security wealth and other socioeconomic characteristics. Using the simple discounted value of social-security wealth (ignoring death) rather than the actuarial one (taking into account death), the empirical findings suggest that the depressive effects of social-security wealth on individual wealth may have been understated by previous research.[7]

Novos (1989) analyzes the sensitivity of the results of Feldstein and Pellechio (1979) on the effects of social-security wealth on household wealth. He employs the same empirical framework and data as Feldstein and Pellechio, but constructs a

[6] In his 1984 study, Hubbard points out that given uncertainty over the length of life, no bequest motive, and no discretion in pension participation, even a fully-funded social-security system can increase lifetime resources, raise lifetime consumption and so reduce individual saving by more than the tax paid. For high-income individuals with constrained participation in social security, the reduction in saving is smaller, however. In addition, the general-equilibrium impact of social security on the steady-state capital stock is likely to be smaller than the partial-equilibrium one.

[7] Bernheim (1987) points out that the use of actuarial valuation reflects the assumption that annuity markets are perfect. This assumption implies that consumers will have no positive annuity holdings at death, which is not true for the majority of consumers. Thus, retaining the life-cycle hypothesis one should relax the assumption of perfect annuity markets and use the simple discounted value of social-security benefits instead of the actuarial one.

number of different social-security wealth variables and makes changes in the sample composition.[8] His empirical findings question the robustness of the results of Feldstein and Pellechio by suggesting an insignificant effect of social-security wealth on household wealth both when farmers are included in the sample and when they are excluded.

Using a similar empirical model to that of Feldstein and Pellechio (1979) and more recent data from the 1983 Survey of Consumer Finances, Gullasson, Kolluri and Panik (1993) examine the effects of social-security wealth on household "fungible" wealth. Their results show that social-security wealth does not significantly affect household nonpension wealth. Unlike Feldstein and Pellechio (1979) and Novos (1989), they also consider the possibility that social-security wealth may affect other categories of "fungible" wealth, i.e., it may reduce "retirement saving" consisting of pensions or other forms of saving. Empirical evidence on this issue indicates that of all the categories of "fungible" wealth, only pension wealth is negatively and significantly affected by an increase in social-security wealth.

Kennickell and Sundén (1997) estimate the effects of pension wealth on nonpension wealth, considering a simple regression equation of households' nonpension net worth on various components of pension wealth and on other socioeconomic characteristics. Using data from the 1992 Survey of Consumer and Finances, they find that pension wealth from defined-benefit plans has a negative effect on net worth, while pension wealth from defined-contribution plans has no significant effect. Social-security wealth, as well, has an insignificant effect on nonpension net worth reflecting households' uncertainty about future benefits.

[8] The variations in sample composition refer to the inclusion and exclusion of farmers. In both cases, social-security wealth variables do not depress wealth accumulation.

In a cross-sectional analysis, Gale (1998) uses data from the 1983 Survey of Consumer Finances to estimate a regression equation of households' nonpension wealth on pension wealth and several other variables, e.g., age of the head of the household, years of education, marital status, family size, etc. He argues that previous empirical studies contain biases, which lead to underestimation of the offset between pension and nonpension wealth. Removing these biases, his findings suggest greater offset between pension and nonpension wealth. In particular, he includes employers' contributions in the measurement of pension wealth, but excludes the employees' contributions because they are already included in cash wages. He also uses a broader measure of nonpension wealth, other than financial assets alone. However, due to several other biases, e.g., pension wealth data of poor quality, the results may still understate the true offset between pensions and other forms of wealth.

Gustman and Steinmeier (1999) consider a variety of specifications of a household wealth equation estimated with different methods. The dependent variable is a comprehensive measure of household total wealth, such as total wealth including or excluding pension wealth, total wealth to lifetime earnings ratio, etc. The independent variables include various definitions of pension wealth, e.g., a pension coverage measure or the value of pension wealth, and many socioeconomic characteristics, e.g., lifetime earnings, age, health, type of employment, education, race, etc. Using cross-sectional data from the Health and Retirement Study of 1992, they find that in most specifications there is no substitution of pension for nonpension wealth, while in some others pensions may even increase nonpension wealth.

The above cross-sectional studies investigate the impact of social-security wealth on household wealth. To a large extent, cross-sectional empirical findings suggest that social-security wealth substitutes for household nonpension wealth, but the offset is not one-for-one. Some studies (Kotlikoff, 1979b, Feldstein and Pellechio, 1979, King and Dicks-Mireaux, 1982, Hubbard, 1986, Bernheim, 1987, Gale, 1998) find a statistically significant negative effect, while some others find no significant effect (Novos, 1989, Gullasson, et al., 1993, Kennickell and Sundén, 1997). Finally, Gustman and Steinmeier (1999) find no significant effect in some cases, but a significant positive effect in some others.

2.3.3. *Cross-country analyses*

Barro and MacDonald (1979) estimate a consumer spending equation using data for 16 industrial countries, for the period 1951-60. Their results are inconclusive, as they find both a negative and a positive relationship between social-security benefits and consumer expenditure. Therefore, there is no support for the proposition that social security depresses saving. Their findings differ from those of Feldstein (1977), who estimates a saving rate equation using time-averaged data for 15 countries for the period 1954-60 and finds a statistically significant negative effect of social-security benefits on the private saving rate. This divergence can be attributed to differences in the specification, the sample of countries, the variable definitions, and the time period.

Feldstein (1980) examines the relation between social-security programs and saving rates among the major OECD industrial countries. The traditional life-cycle model implies that the saving rate depends on the growth rate of income and on the demographic composition of the population. An economy's saving rate will be higher when the growth rate of income is higher and the working population is larger than the retirees and the young dependents. He specifies an equation for the private saving rate as follows:

$$\frac{S}{Y} = a_0 + a_1 G + a_2 AGE + a_3 DEP + a_4 \frac{B}{E} + a_5 LPAGED\,, \tag{2.7}$$

where *S*/*Y* is the private saving rate,[9] *G* is the growth rate of total private income, *AGE* is the ratio of retirees over age 65 to the population aged 20 to 65, *DEP* is the ratio of young dependents to the working population, *B*/*E* is the benefits to earnings replacement ratio,[10] and *LPAGED* is the labor force participation rate of the old-aged. He also specifies a separate equation for the labor force participation rate of the old-aged on the benefits to earnings replacement ratio and other variables, to determine endogenously the retirement decision, thus accounting for the "induced retirement effect." Moreover, he combines the coefficients of these two equations to calculate the reduced-form coefficients and find the net effect of the benefit replacement ratio on the saving rate, i.e., account for the relative strength of the "wealth replacement effect" and the "induced retirement effect."

The estimation is based on time-averaged data for 12 countries for the period 1960-75 and the methods used are two-stage least squares and ordinary least squares. Considering the retirement decision exogenous, the results indicate that a higher social-security benefit replacement ratio reduces the private saving rate. In contrast, considering the retirement decision endogenous, the labor force participation equation indicates that a higher social-security benefit replacement ratio lowers the labor force participation rate of the elderly. Thus, higher social-security benefits induce early retirement. On balance, the reduced-form estimated parameters imply that the net

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[9] Feldstein (1980) uses private saving that includes household and corporate saving.

[10] The benefits to earnings replacement ratio is defined as the ratio of the social-security benefits of a newly retired couple to the average earnings of a worker in manufacturing industry (Feldstein, 1980, p. 232).

effect of the social-security benefit replacement ratio on the private saving rate is significantly negative.

Modigliani and Sterling (1981) test the implications of the life-cycle theory regarding the impact of social-security benefits on private saving (the sum of household and corporate saving). They estimate various saving to income ratio equations using time-averaged data for the period 1960-70 for 21 OECD countries. The results support the life-cycle theory. For most countries, however, the net impact of social security on saving is close to zero, that is, the “wealth replacement effect” and the “induced retirement effect” almost offset each other.

Koskela and Virén (1983) provide further evidence on the relationship between social-security benefits and the household saving rate. They use a sample of 16 OECD countries over the period 1960-77 to estimate a saving-to-income ratio equation along with a participation rate equation.[11] The participation rate equation is estimated to capture the possible “induced retirement effect” of social security on the household saving rate. Their empirical findings indicate no support for the proposition that social security depresses the household saving rate.

Graham (1987) studies saving behavior in 24 OECD countries using time-averaged data for the period 1970-80. Estimating several regression equations of the household saving rate on social-security benefits, life expectancy, labor force participation rates of the aged males and young females, and other demographic variables, he finds no impact of social-security benefits on the saving rate. These findings support those of Koskela and Virén (1983). While in the national saving rate regressions, he finds limited evidence that social-security benefits reduces the saving rate.

[11] The participation rate refers to economically active population over age 65.

Some cross-country studies presented above consider the effects of social-security benefits on the private saving rate (Feldstein, 1977, 1980, Modigliani and Sterling, 1981), some others on consumer expenditure (Barro and MacDonald, 1979), and still others on the household saving rate (Koskela and Virén, 1983, Graham, 1987). The empirical evidence is mixed. Feldstein (1977, 1980) finds a significant negative effect, while Barro and MacDonald (1979) find both positive and negative effects. Modigliani and Sterling (1981) suggest that the net impact of social security on the private saving rate is close to zero. Finally, Koskela and Virén (1983) and Graham (1987) find no evidence that social-security benefits reduce the household saving rate.

2.3.4. *Time-series of cross-sections analyses using household data*

Diamond and Hausman (1984) use U.S. household data for the period 1966-76 to estimate a model with three components.[12] Estimating the first model of retirement behavior, they find that social security has a significant positive effect on retirement in that it encourages early retirement. In their second life-cycle specification of wealth accumulation, they find a significant wealth decumulation after retirement, though the higher the social-security benefits the lower the wealth decumulation. The findings of the third model of household saving behavior show that social-security benefits have a significant negative effect on the saving-to-income ratio with offset between 0.25 and 0.40 on a dollar. In general, their basic results do not contradict the life-cycle theory.

Within the framework of a simple life-cycle model, Attanasio and Brugiavini (2003) examine the relationship between pension wealth and the household saving

[12] The same households reinterviewed from 1966 to 1976 by the National Longitudinal Survey.

rate. They consider the 1992 reform of the Italian pension system because it caused changes in households' pension wealth. They analyze a four-period model, assuming that individuals work during the first three periods of their lives and then retire. Solving the household maximization problem yields expressions for each period's saving rate. They model the saving rate as follows:

$$SR_{it} = \beta' X + \phi(a_{it})FE_{it} + \theta(a_{it})PW_{it} + x_t + f_c + \varepsilon_{it}, \qquad (2.8)$$

where SR_{it} is the saving rate of household i at time t, X is a vector of demographic variables, $\phi(a_{it})$ is an age-dependent parameter, $\theta(a_{it})$ is a time-dependent parameter (time dependence is relative to the date of the reform), which varies with age,[13] FE_{it} is future to current earnings ratio, PW_{it} is pension wealth to current earnings ratio, x_t represents time effects, f_c represents individuals' groups effects,[14] and ε_{it} is an error term.

Using cross-sectional data from the Survey on Household Income and Wealth of Italy, for the years 1989, 1991, 1993, and 1995, they estimate various specifications of Equation (2.8) by the Instrumental Variable (IV) approach (using as instruments group dummies interacted with year dummies). With regard to the 1992 pension reform, which changed the eligibility criteria and the size of benefits, the empirical results show a significant offset of pension wealth on household saving. The degree of substitutability between pension wealth and household saving, however, varies among the different specifications.

Attanasio and Rohwedder (2003) investigate the relationship between pension wealth and household saving rates in a life-cycle model, in which individuals save for

[13] The effect of pension wealth depends on the age of the individual at the time of the reform.

[14] Attanasio and Brugiavini (2003) divide the sample on the basis of the year of birth into four groups and of the sector of activity of the household head into three groups (private-sector employees, public-sector employees, and self-employed). Dividing the sample into groups, they try to maximize the variation in pension wealth across groups caused by the reform.

retirement among other reasons. They use a four-period model and assume that individuals work and receive income in the first three periods of their life and then retire and receive benefits. They consider no uncertainty about the interest rate and income. They also assume that the individuals have no bequest motive and face no liquidity constraints. Maximizing a log-utility function under the budget constraint yields the optimal levels of consumption in each period. However, the empirical specification is based on the household saving rate as a function of the present value of expected pension wealth and a vector of control variables, including occupation groups and time effects, which capture other determinants of saving. Occupation groups are defined by the occupation of the head of household and categorized into professional employees, white collar employees, skilled workers and other occupations, and unoccupied.

They employ time-series of cross-sections data from the U.K. Family Expenditure Survey, for the period 1974-87. They consider the three major reforms of the U.K. pension system, including two indexation changes of the Basic State Pension (BSP) in 1975 and 1981, and the introduction of State Earnings Related Pension Scheme (SERPS) in 1978, to investigate the impact of changes in pension wealth on household saving rates. Using the IV approach, they try to determine the degree of substitutability between pension wealth and the household saving rate. Their findings suggest that an increase in SERPS pension wealth has a significant negative effect on the household saving rate with a considerable degree of substitutability between them, while the BSP wealth has no significant effect, except for the youngest individuals.

The time-series of cross-sections studies summarized above try to determine the impact of social-security wealth on household saving. The empirical findings suggest that social-security wealth affects negatively the household saving rate, with a degree of substitutability that varies among different specifications (Diamond and Hausman, 1984, Attanasio and Brugiavini, 2003, Attanasio and Rohwedder, 2003).

In sum, theoretical considerations have, on balance, been favorable to the proposition that social-security benefits depress private saving, although it is difficult to determine theoretically the size of the offset. Empirical specifications vary substantially, however, and the existing time-series and/or cross-sectional evidence from single countries or group of countries is mixed. Differences in the data, the empirical model, and the quantification of the variables may be the reasons for this variation in the results.

It should be noted that some forms of analysis presented above have some limitations, however. Time-series evidence is sensitive to the form of the regression equation and it may be difficult to distinguish between the effects of social-security benefits and those of other variables as they move closely together over time. Also, time-series data do not capture differences in individual household behavior. In the cross-sectional context, there are some identification problems considering the effects of a particular factor on saving and the difficulty in recognizing the time effects in the regression equation. As well, in the cross-country analyses, it is difficult to construct a homogenous measure of social-security wealth and contrast the pension systems among different countries.

Another basic problem in social-security studies is finding an adequate measure of the anticipated social-security benefits as individuals' preferences and expectations are difficult to measure. However, any measurement of social-security wealth can only be an approximation to the actual value. Also, it should be noted that the above studies examine the relationship between social-security benefits and household saving, but they do not consider the effect of institutional features on this relationship. The studies presented in the next section examine the relationship between political institutions and social-security spending, focusing on the institution of voting and the form of the political system. They do not consider, however, the effect of institutional features, such as corruption, government stability, etc., on the relationship between social security and household saving. This is the focus of the present study.

2.4. Political institutions and social security

Positive theories of social security can be categorized into political theories and efficiency ones. They consider political institutions and economic and demographic factors as determinants of social security. Political theories of social security can be classified into three categories: a) majority voting models, b) veto-power rules, and c) interest-group models. First, in a majority voting model, the political outcome is preferred to any other outcome by the majority of the voters. In the case of social security, agents vote on tax rates and the majority determines the policy outcome (Galasso and Profeta, 2002, pp. 10-12). The typical outcome is the policy preferred by the median voter. One way to set up social security is that the elderly win the political competition. But since the elderly are not the majority of the voters, they have to form a coalition with another group, e.g., the middle-aged or the poor, to support a policy taxing the losers of the political competition, e.g., the young and the rich. The young may also support social security because it might benefit them when they will become old, even though they pay taxes for a longer period until retirement (Mulligan and Sala-i-Martin, 2002, pp. 27-31). Voting models do not expect social security to emerge and grow without democracy and consider that democracies may have larger social-security budgets and different program designs than nondemocracies (Mulligan, Gil, and Sala-i-Martin, 2002, pp. 2-4).

Second, a political arrangement awards veto power over policy decisions to a powerful minority that can block any modification, which makes them worse off. Empirically, constitutional veto power has not been observed in social-security policy decisions, however. Third, interest-group models include influence-function and support-function models. In the influence-function models, political pressure is more important than voting in the determination of the political equilibrium for redistribution policies. The group that exerts more influence on policymakers wins the political competition and obtains a transfer from the other group. The existence of social security is explained by the political competition between two groups, the young and the old. In the support-function models, on the other hand, social security arises from a political process in which the government maximizes a political support-function that contains the utility of two currently living generations, the young and the old. In both models, social security is the equilibrium outcome, because the old win the political competition (Galasso and Profeta, 2002, pp. 12-15).

At the other extreme of the political theories of social security lie efficiency theories, which view social security as a way to regain optimality by reducing market inefficiency. They suppose that economic and demographic factors are more important determinants of social-security policy than voting and other political institutions. After holding constant the economic and demographic determinants of efficiency, they do not expect differences in the size or design of social-security programs between democracies and nondemocracies (Mulligan, Gil, and Sala-i-Martin, 2002, pp. 4-6).

Efficiency explanations of social security include the following. First, the theory that views social security as welfare for the elderly claims that the main purpose of social security is to reduce poverty among the elderly or insure against future labor-productivity shocks. Second, the optimal-retirement insurance theory suggests that the purpose of social security is to replace lost income through the retirement period. Third, the labor-market congestion theory argues that because of unemployment and other undesirable labor-market symptoms, social security should induce retirement and redistribute jobs from the old to the young. Fourth, the prodigal-father problem theory suggests that because individuals were not looking forward enough when they were young and saved too little for their old age, social security emerges to force individuals to save. Fifth, the misguided Keynesian theory suggests that social security was created to depress national saving when aggregate demand was low and consumption needed to be stimulated. Sixth, the optimal-longevity insurance theory argues that social security insures against uncertainty about the length of life. Seventh, the theory that argues that the government economizes on administration costs of social security as compared to private-pension plans. Finally, eighth, the theory that views social security as a return to the old of the human capital invested in the current workers when they were at schooling age (Mulligan and Sala-i-Martin, 2002, pp. 37-64).

As well, Sala-i-Martin (1996) develops a positive theory based on economic efficiency. He claims that the main reason for the introduction of social security is economic efficiency, because social security drives the elderly with lower than average skill out of the labor force. Since social-security programs are not related to a political system, they emerge in democratic countries as well as in nondemocratic ones.

2.4.1. *Voting models*

Browning (1975) develops a simple majority voting model, where individuals differ only in age, to analyze the determination of tax and transfer payments in a PAYG social-security system. He assumes that there is a once-and-for-all election to decide a policy. The main implications of his analysis are that social-security spending seems to be sensitive to the size of the elderly population in a democracy, and majority voting leads to an overexpansion of the size of the social-security system. The older the median voter, the higher the social-security tax he prefers to pay for the remaining shorter period until his/her retirement, in order to receive increased benefits at retirement.

Cooley and Soares (1999a) emphasize that a social-security system can be an equilibrium outcome of a voting process. The median voter prefers the economy with social security because of the general-equilibrium effects on the capital stock and on the rate of return. The introduction of social security tends to reduce the capital-to-output ratio, and thus reduce real wages and increase the real rate of return to capital. These effects may sustain social security as an economic and political equilibrium.

A departure from Browning's setting is provided by Tabellini (2000). In addition to the traditional differences in the age of the voters, he also introduces differences in their income. He assumes that social security redistributes income both across and within generations. Social-security contributions are proportional to wage income, while benefits are not, thus social security redistributes income from the rich to the poor. Another assumption is that there is no commitment of future majorities to preserve past social-security legislation. The political equilibrium is given by the policy preferred by the median voter.[15] The main implication of his analysis is that the equilibrium size of social security is larger the greater the income inequality and the larger the fraction of the elderly in the population. Empirical evidence on cross-country data supports both implications. In line with Tabellini (2000), Conde-Ruiz and Galasso (2005) argue that social security owes its political support to the political power of the elderly and its intragenerational redistribution component. Social security is sustained as a political equilibrium by a majority voting of retirees and low-income young.

In the majority voting model of Casamatta, Cremer, and Pestieau (2000), voters differ not only in age, as usual, but also in productivity. They assume no altruism and that the voters precommit their future decisions on social security legislation. The majority voting equilibrium level of social security is a positive tax rate, which is supported by a majority coalition consisting of the retirees and the medium-wage workers.

2.4.2. *Empirical studies*

Lindert (1994) investigates empirically the determinants of various kinds of social transfers, including pensions, in 21 OECD countries for the period 1880-1930. The results imply that democracy and the aging of the population raise social transfers more than the level of income per capita or its growth rate. Democracies with a higher voter turnout rate spend more on pensions than democracies with a lower one.[16] As

[15] The political equilibrium is the policy preferred by at least 50 percent of the voters to any other policy. Under the assumption that voters' preferences are single-peaked, that is, there is an ideal point as the top preference of each voter among different choices, the policy preferred by the majority of the voters is the policy preferred by the median voter.

[16] The voter turnout rate is defined as the ratio of voters to population over the age of 20.

well, the older is the adult population the greater is government spending on social transfers including pensions. The frequency of executive turnover also raises every kind of social transfer.[17] As well, in his 1996 study, Lindert examines the determinants of social and nonsocial expenditures in 19 OECD countries for the period 1960-1981. The findings suggest that a higher voter turnout rate raises social expenditures, but does not influence pensions significantly. By contrast, an executive turnover seems to raise nonsocial expenditures. Moreover, a rise in the share of adults over age 65 raises government spending on pensions.

Breyer and Craig (1997) examine empirically a subset of public-choice models of social security. They use data from the OECD countries for the years 1960, 1970, 1980, and 1990. Their estimates indicate a significant positive effect of median-voter age on the social-security benefits-to-GNP ratio, while the positive effect of income heterogeneity gets only weak support. As well, the similarity in family size affects positively the size of the public pension-systems (the benefits-to-GNP ratio). Holding constant the demographic variables and considering the efficiency of public-pension systems, the economy's growth rate and the inflation rate affect positively the size of the systems, whereas the real interest rate affects it negatively.

In a cross-section of 90 countries for the period 1960-90, Mulligan, Gil, and Sala-i-Martin (2002) examine the relation between democracy and social-security spending. They find no evidence that democracies spend a larger share of their GDP on social security than nondemocracies, holding constant economic and demographic variables, such as income per capita and the share of the population over age 65. The relationship between social-security spending and economic and demographic variables appears to be similar in democracies and nondemocracies. Case studies

[17] The executive turnover is defined as the number of changes in the executive post (president or prime minister) over the previous decade.

propose that countries with different political institutions, but similar economic and demographic conditions, have similar social-security systems. The empirical findings suggest that political institutions are only minor determinants of the size and design of social-security programs, whereas economic and demographic factors seem to be more important. In their 2004 study, Mulligan, Gil, and Sala-i-Martin estimate the effects of democracy on public-sector spending using cross-country data for the period 1960-90. They fail to find a significant effect of democracy on pension and other social spending, however. So democracies appear to be similar to nondemocracies in terms of social policy.

Pinotti (2009) investigates the relationship between financial development and social security using the legal origin as a proxy for financial frictions in 54 countries for the period 1990-2000.[18] The empirical evidence shows that higher levels of financial development due to its legal origin are associated with lower levels of social security. Also, the results suggest that the democracy index (as defined by the POLITY project) has no significant effect on social security.

The studies summarized above do not provide clear evidence on the relationship between political institutions and social security, and thus little is known about their interaction. As well, they do not consider the effect of institutional features on the relationship between social security and household saving. They limit their attention to the institution of voting and to the form of the political system (democratic or nondemocratic) and neglect other institutional features, e.g., corruption, government stability, etc. Moreover, institutions are incorporated in the empirical analysis rather than in the theoretical model. This study instead considers

[18] In economics, the legal-origin theory states that many aspects of a country's economic development are the result of its legal system, that is, where a particular country received its law from. The legal system may be based on common law or one of the different types of civil law, i.e., French law, German law, or Scandinavian law. Common law countries are characterized by higher financial development.

formally the effect of institutional variables and the debt-to-GDP ratio on social security and, by extension, on household saving.

CHAPTER 3: THE THEORETICAL MODEL

3.1. Introduction

According to the life-cycle model, individuals rationally plan their consumption and saving over their lifetime. In order to smooth out consumption, they accumulate savings in anticipation of their retirement and dissave in retirement. Their aim is to maximize their lifetime well-being, subject to the constraint that their lifetime consumption cannot exceed their lifetime wealth. In the context of an OGM (Diamond, 1965), individuals of different generations coexist and trade with one another. They save during their working lives to finance their consumption during retirement.

In this chapter, in the general framework of the life-cycle model, I employ a simple version of the OGM in which two generations of consumers coexist. Individuals are continually born and live for two periods. In the first period they belong to the young, while in the second period they belong to the old, and then they die. The intertemporal budget constraint takes into account that individuals contribute a certain amount of their labor income in the social-security system and expect to receive benefits at retirement. The expected social-security benefits depend on the probability that the social-security system will grant pensions to the old at retirement. I assume that this probability is determined by a binary response model and depends on institutional variables, e.g., corruption and government stability, and on the debt-to-GDP ratio. Maximizing the lifetime utility function under the intertemporal budget constraint yields the Euler equation for consumption. I also incorporate a stochastic production function, which comprises random productivity shocks. After solving the household and the firm maximization problems and deriving the resource constraint, the system of equations that describe the competitive equilibrium is presented.

Since there is no closed-form solution to the household maximization problem, I take partial derivatives of the Euler equation to examine the effects of the institutional variables and of the debt-to-GDP ratio on the relationship between social-security contributions and household saving under the PAYG as well as the fully-funded social-security system. After computing the effect of social-security contributions on household saving, I examine how corruption, government stability and the debt-to-GDP ratio affect it. The effect of these variables on the probability that the social-security system will grant pensions to the old at retirement is considered to be the channel through which their influence on the relationship between social-security contributions and household saving is transmitted. Finally, I consider the possibility that the social-security system (public or private) collapses.

3.2. The model

3.2.1. *Individual behavior*

The theoretical model I use is a standard discrete-time two-period OGM. Individuals are continually born, live for two periods and behave in the same way. In the first period of their life, they are young and work, offering inelastically one unit of labor[19] and receiving a real wage w_t. They consume part of their wage, contribute another part to the social-security system, and save the rest, in order to finance their

[19] The assumption of inelastic labor supply implies that labor supply, and thus leisure does not depend on real wage. So, leisure is not considered to be a choice variable in the model.

consumption in the next period. This saving is converted into capital, which is then jointly used with labor in the production function. In the second period of their life, individuals are old and do not work. They consume their saving, which they accumulated during the first period of their life, along with the interest, and the social-security benefits. Then they die, leaving no bequests.

For simplicity, I assume that each individual represents a household. I also assume discrete-time, where in period t there are L_t young individuals and L_{t-1} old ones. Employment grows at an exogenous rate n, so that $(L_t - L_{t-1})/L_{t-1} = n$, which can be written as:

$$L_t = (1+n)L_{t-1}. \tag{3.1}$$

Each one of the L_t young individuals offers one unit of labor in period t and receives a real wage w_t, which he/she disposes for consumption in period t, c_{1t}, social-security contributions, d_t, and saving, $s_t = w_t - c_{1t} - d_t$, in order to ensure consumption for the period t+1, c_{2t+1}, when he/she will be old. Under the assumption of a Constant Relative Risk Aversion (CRRA) utility function, the lifetime utility function of a young individual is given by[20]

$$U = \frac{c_{1t}^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho} E_t\left(\frac{c_{2t+1}^{1-\gamma}}{1-\gamma}\right), \ \gamma > 0, \ \frac{1}{1+\rho} > 0. \tag{3.2}$$

The parameter γ is the coefficient of relative risk aversion; the higher the value of γ the more cautious is the individual in undertaking economic risks. A larger γ implies a lower elasticity of intertemporal substitution in consumption, and thus a

[20] The CRRA utility function is required in order for the economy to converge to a balanced growth path. That is, in order to find a steady state in which the ratio c_{1t}/c_{2t+1} and the real interest rate r_t will be constant [see Equation (3.8)], the coefficient of relative risk aversion, γ, should also be constant (Barro and Sala-i-Martin, 1995, pp. 64-65). Hence, I follow the common practice and assume a CRRA utility function in which γ is constant.

lower interest sensitivity of saving.[21] As the elasticity of intertemporal substitution in consumption decreases, individuals care more about consumption smoothing. The parameter ρ is the rate of time preference. The higher the value of ρ the more impatient is the individual to consume in the present than in the future. The fraction $1/(1+\rho)$ is the discount factor, which converts a future value into a present value. The assumption that $1/(1+\rho)>0$ is necessary in order for the marginal utility of consumption of the second period to be positive. If $1/(1+\rho)<1$ ($\rho>0$), individuals value more the first-period than the second-period consumption; the opposite is true if $1/(1+\rho)>1$ ($-1<\rho<0$). For simplicity, I assume that the utility function is time-separable, that is, the marginal utility of one period's consumption is independent of another period's consumption. I also assume that individuals face uncertainty; E_t is the rational expectations operator conditional on information available up to time *t*.

The young save part of their wage for financing consumption in the next period, when they will be old. I assume that r_t is the real interest rate paid on saving held from period *t* to period *t*+1 (Hall, 1988, p. 341).[22] The old consume their entire capital, the return from it, and the social-security benefits, b_{t+1}. Thus, the expected consumption of the individual in period *t*+1 is given by

[21] For a time-separable lifetime utility function, the elasticity of intertemporal substitution σ between consumption at times *t* and *t*+1 is given by the reciprocal of the coefficient of relative risk aversion, that is, $\sigma = 1/\gamma$. The elasticity σ is defined as $d\ln(c_{t+1}/c_t)/d\ln MRS_{c_{t+1}}^{c_t}$, where $MRS_{c_{t+1}}^{c_t}$ is the marginal rate of substitution between c_t and c_{t+1}. Given $U=[c_t^{1-\gamma}/(1-\gamma)]+[1/(1+\rho)]c_{t+1}^{1-\gamma}/(1-\gamma)$, I get the $MRS_{c_{t+1}}^{c_t}=(\partial U/\partial c_t)/(\partial U/\partial c_{t+1})=(1+\rho)(c_{t+1}/c_t)^{\gamma}$. Taking logarithms and then differentiating yields $d\ln(MRS_{c_{t+1}}^{c_t})=\gamma d\ln(c_{t+1}/c_t)$. Thus, $\sigma = d\ln(c_{t+1}/c_t)/d\ln MRS_{c_{t+1}}^{c_t}=1/\gamma$. For $\gamma<1$ (or $\sigma>1$), there is high degree of substitution between consumption at any two points in time. The opposite is true for $\gamma>1$ (or $\sigma<1$). In the case of $\gamma = 1$(log utility function) these two forces cancel each other (Romer, 2001, p. 78).

[22] I assume inflation-protected securities so that r_t is known when the decision about consumption, c_{1t}, and saving, s_t, is made. Some researchers, however, consider that r_t becomes known at the beginning of period *t*+1, after the decision about saving is made (Deaton and Muellbauer, 1980, Sargent, 1987).

$$E_t(c_{2t+1}) = (1+r_t)(w_t - c_{1t} - d_t) + E_t(b_{t+1}) \tag{3.3}$$

or

$$c_{1t} + \frac{E_t(c_{2t+1})}{1+r_t} = w_t - d_t + \frac{E_t(b_{t+1})}{1+r_t}. \tag{3.4}$$

Equation (3.4) is the intertemporal budget constraint of the individual. The present value of consumption of the two periods equals the initial wealth of the individual, which is zero because the previous generation consumes all its capital without leaving any bequests, and the present value of income, which consists of the current real wage reduced by the amount of social-security contributions and the discounted value of the expected social-security benefits.

The individual maximizes his/her lifetime utility function (3.2) under the intertemporal budget constraint (3.4). The Lagrangian is

$$\ell = \frac{c_{1t}^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho} E_t\left(\frac{c_{2t+1}^{1-\gamma}}{1-\gamma}\right) + \lambda\left[w_t - d_t + \frac{E_t(b_{t+1})}{1+r_t} - c_{1t} - \frac{E_t(c_{2t+1})}{1+r_t}\right]. \tag{3.5}$$

Taking the partial derivatives of ℓ with respect to c_{1t} and c_{2t+1}, and setting them equal to zero yields

$$c_{1t}^{-\gamma} = \lambda \tag{3.6}$$

and

$$\frac{1}{1+\rho} E_t(c_{2t+1}^{-\gamma}) = \frac{\lambda}{1+r_t}. \tag{3.7}$$

Substituting Equation (3.6) into Equation (3.7) and rearranging gives the Euler equation for consumption (EEC):

$$E_t\left(\frac{c_{1t}}{c_{2t+1}}\right)^{\gamma} = \frac{1+\rho}{1+r_t}. \tag{3.8}$$

The greater the value of γ, and so the lower the elasticity of intertemporal substitution, $1/\gamma$, the lower the responsiveness of consumption to changes in the ratio

$(1+\rho)/(1+r_t)$. For $r_t > \rho$, the ratio c_{1t}/c_{2t+1} decreases, that is consumption increases with time. The opposite is true for $r_t < \rho$.

The empirical counterpart of the EEC emerges by removing the expectations operator, E_t, and by adding the rational expectations error, e_{t+1}:

$$\left(\frac{c_{1t}}{c_{2t+1}}\right)^{\gamma}\frac{1+r_t}{1+\rho}-1=e_{t+1}.[23] \tag{3.9}$$

3.2.2. *The production function*

The total product of the economy is given by $Y_t = e^{z_t}K_t^{\beta}(A_tL_t)^{1-\beta}$, where $0<\beta<1$, K_t is the amount of capital, L_t is the amount of labor, A_t is the level of knowledge, and z_t is a random productivity shock. This shock is a source of uncertainty for the economy and evolves according to the following equation of motion:

$$z_t = \mu z_{t-1} + \varepsilon_t,\ 0<\mu<1, \tag{3.10}$$

where ε_t is normally distributed with mean zero and standard deviation σ_ε (Cooley and Prescott, 1995, p. 13). Also, the level of knowledge is exogenous and evolves according to the following equation of motion:

$$A_{t+1} = (1+r_A)A_t, \tag{3.11}$$

where r_A is the growth rate of knowledge.

[23] Equation (3.9) can be written as $(c_{2t+1}/c_{1t})[(1+\rho)/(1+r_t)]^{1/\gamma} = 1/(1+e_{t+1})^{1/\gamma}$. Note that since $(c_{2t+1}/c_{1t}) = r_c + 1$, where r_c is the growth rate of consumption per worker, and $(1+\rho)/(1+r_t)\approx 1$, it follows that the right-hand side term, $1/(1+e_{t+1})^{1/\gamma}$, should be close to 1, which implies that the values of e_{t+1} should be close to 0. My empirical results (see section 4.3.2) confirm this conjecture.

Considering a competitive economy, firms maximize their profits so that labor and capital are paid according to their marginal products. That is,

$$w_t = (1-\beta) y_t \tag{3.12}$$

and

$$r_t = \beta \frac{y_t}{k_t}, \tag{3.13}$$

where $y_t = Y_t / L_t$ is product per worker and $k_t = K_t / L_t$ is capital per worker.

3.2.3. *The resource constraint*

The equilibrium condition for the goods and services market requires that in each period total demand for goods and services be equal to total supply. I assume that the economy's resource constraint in period *t* is given by $C_t + I_t = Y_t$, where C_t is total consumption and I_t is total gross investment. Total consumption is the sum of total consumption of the young individuals, $c_{1t} L_t$, and that of the old individuals, $c_{2t} L_{t-1}$, which can be written as $c_{2t} L_t /(1+n)$ using Equation (3.1), $L_t = (1+n) L_{t-1}$. Hence, $C_t = c_{1t} L_t + c_{2t} L_t /(1+n)$. In addition, total gross investment is the sum of total net investment, $K_{t+1} - K_t$, and of the depreciation of capital, δK_t, where δ is the depreciation rate. Thus, $I_t = K_{t+1} - K_t + \delta K_t$. Therefore, the economy's resource constraint can be written as follows:

$$c_{1t} L_t + c_{2t} \frac{L_t}{1+n} + K_{t+1} + (\delta - 1) K_t = Y_t. \tag{3.14}$$

Diving both sides of Equation (3.14) by L_t; using Equation (3.1), $L_{t+1} = (1+n) L_t$; and setting $c_t = c_{1t} + c_{2t} /(1+n)$; yields

$$c_t + (1+n)k_{t+1} + (\delta - 1)k_t = y_t \tag{3.15}$$

or

$$(1+n)k_{t+1} - k_t = y_t - \delta k_t - c_t\,. \tag{3.16}$$

According to Equation (3.16), net investment per worker, $(1+n)k_{t+1} - k_t$, is equal to net saving per worker, $y_t - \delta k_t - c_t$ (Barro and Sala-i-Martin, 1995, pp. 130-33).

Next, the total saving of the L_t young individuals is the economy's capital in period t+1, that is $K_{t+1} = L_t s_t$. Dividing this equation by L_{t+1} and using Equation (3.1), $L_{t+1}/L_t = 1+n$, yields

$$k_{t+1} = \frac{s_t}{1+n}\,. \tag{3.17}$$

Equation (3.17) says that the formation of capital per worker depends upon saving per worker and the growth rate of employment (Barro and Sala-i-Martin, 1995, pp. 130-31). Solving Equation (3.17) for s_t, and subtracting k_t from both sides of the resulting equation yields

$$(1+n)k_{t+1} - k_t = s_t - k_t\,. \tag{3.18}$$

Since s_t is saving per worker of the young and $-k_t$ is the dissaving of the old, the right-hand side of Equation (3.18) is net saving per worker, while the left-hand side is net investment per worker (Blanchard and Fischer, 1989, p. 94). Also, Equation (3.18) can be derived from Equation (3.16) by setting $y_t - c_t = s_t$ and δ = 1 since the previous generation consumes all its capital without leaving any bequests to the next generation.

3.2.4. *The system of equations of the competitive economy*

The social-security system ensures a certain level of income at retirement and has an effect on the path of income received by individuals. Hence, it is likely to have an effect on household saving. As well, since I examine the effect of the quality of institutions on social security and by extension on household saving, the Euler equation is modified to include household saving rather than consumption. The system of equations that describe the competitive economy and can be used to compute the equilibrium is[24]

$$\frac{(1+r_t)s_t + E_t(b_{t+1})}{w_t - s_t - d_t}\left(\frac{1+\rho}{1+r_t}\right)^{\frac{1}{\gamma}} = E_t\left[\frac{1}{(1+e_{t+1})^{\frac{1}{\gamma}}}\right], \tag{3.19}$$

$$k_{t+1} = \frac{s_t}{1+n}, \tag{3.17}$$

$$w_t = (1-\beta)y_t, \tag{3.12}$$

$$r_t = \beta\frac{y_t}{k_t}, \tag{3.13}$$

$$y_t = e^{z_t}k_t^{\beta}A_t^{1-\beta}, \tag{3.20}$$

and

$$z_t = \mu z_{t-1} + \varepsilon_t. \tag{3.10}$$

Equation (3.19), which is the Euler equation for saving, is obtained by substituting the definition $c_{1t} = w_t - s_t - d_t$ and Equation (3.3), $E_t(c_{2t+1}) = (1+r_t)s_t + E_t(b_{t+1})$, into Equation (3.9) (see Appendix A).

[24] Blanchard and Fischer (1989, pp. 110-11) describe the conditions that characterize the equilibrium in a competitive economy and examine how they are affected by the introduction of social security.

To investigate the influence of the quality of institutions on social security and by extension on household saving to finance retirement I consider: 1) a PAYG social-security system and 2) a fully-funded system. I assume that the expected social security-benefits are affected by the probability that the social-security system will grant pensions.

Let $p(\boldsymbol{x}_t)$ be this probability, where $\boldsymbol{x}_t$ is a column vector that contains variables like an index of corruption, IC_t, an index of government stability, PS_t, the government debt-to-GDP ratio, DG_t, etc. Here, IC_t is corruption within the political system, where high (low) values of IC_t represent low (high) corruption; PS_t is the government's ability to stay in office, where the higher the value of PS_t the greater the degree of government stability. I assume that the better the quality of institutions, i.e., the lower the level of corruption, the higher the degree of government stability, etc., or the lower the debt-to-GDP ratio, the greater the probability that the social-security system will grant pensions to the old at retirement. That is, I assume that $\partial p(\boldsymbol{x}_t)/\partial(IC_t) > 0$, $\partial p(\boldsymbol{x}_t)/\partial(PS_t) > 0$, and $\partial p(\boldsymbol{x}_t)/\partial(DG_t) < 0$.

3.2.5. *The determination of* $p(\boldsymbol{x}_t)$

The probability $p(\boldsymbol{x}_t)$ is determined in the context of a binary response model. A binary response model is a regression model in which the dependent variable, e.g., Z, is a binary random variable that takes on only two values, zero and one. The conditional probability, $p(Z=1 \mid \boldsymbol{x}_t)$, is described as a function of the explanatory variables, i.e., $p(Z=1 \mid \boldsymbol{x}_t) = G(\boldsymbol{\beta}'\boldsymbol{x}_t)$, where G is a function taking on values between zero and one and $\boldsymbol{\beta}'$ is a row vector of parameters.[25] In the logit model, *G* is the standard logistic cumulative distribution function, while in the probit model it is the standard normal.

An alternative approach to derive logit and probit models is the latent-variable model. Let Z^* be the level of institutional quality that is required for a positive probability that an individual will receive a pension at retirement. Then, consider the binary variable *Z*, which takes on the value of one if $Z^* > 0$, and zero otherwise. Also, *Z* is a function of the variables IC_t, PS_t, and DG_t, that is, $Z = h(\boldsymbol{x}_t)$. In practice, Z^* is an unobservable or latent variable defined by a regression relationship, which is called the latent-variable model (Wooldridge, 2009, pp. 575-77). That is,

$$Z^* = \boldsymbol{\beta}' \boldsymbol{x}_t + u \,,\; Z = 1[Z^* > 0] \,. \tag{3.21}$$

The function $Z = 1[Z^* > 0]$ is an indicator function, i.e., $Z = 1$ if $Z^* > 0$ and $Z = 0$ if $Z^* \leq 0$. The error *u* has either the standard logistic or the standard normal distribution with cumulative distribution function *G*(.). Thus, the response probability for *Z* is

$$\begin{aligned} p(\boldsymbol{x}_t) &= p(Z = 1 | \boldsymbol{x}_t) = p[u > -\boldsymbol{\beta}' \boldsymbol{x}_t | \boldsymbol{x}_t] = 1 - p[u < -\boldsymbol{\beta}' \boldsymbol{x}_t | \boldsymbol{x}_t] = \\ &= 1 - G[-\boldsymbol{\beta}' \boldsymbol{x}_t] = G(\boldsymbol{\beta}' \boldsymbol{x}_t) \,, \end{aligned} \tag{3.22}$$

since $1 - G(-v) = G(v)$ for any real number *v*. In the logit model, $G(v) = \dfrac{e^v}{1 + e^v}$.

Therefore,

$$p(\boldsymbol{x}_t) = p(Z = 1 | \boldsymbol{x}_t) = \frac{e^{\boldsymbol{\beta}' \mathbf{x}_t}}{1 + e^{\boldsymbol{\beta}' \mathbf{x}_t}} = \frac{1}{1 + e^{-\boldsymbol{\beta}' \mathbf{x}_t}} \,, \tag{3.23}$$

[25] In the linear probability model (LPM), where *Z* is a binary variable taking on two values, zero and one, the conditional probability, $p(Z = 1 | \boldsymbol{x}_t)$, equals the conditional expectation, $E(Z | \boldsymbol{x}_t) = \boldsymbol{\beta}' \boldsymbol{x}_t$. Therefore, the conditional probability is a linear function, i.e., $p(Z = 1 | \boldsymbol{x}_t) = \boldsymbol{\beta}' \boldsymbol{x}_t$. One drawback of the LPM is that the estimated probability may lie outside the internal [0, 1] (Horowitz and Savin, 2001, pp. 43-44).

whereas in the probit model, $G(v)=\int_{-\infty}^{v}\frac{1}{(2\pi)^{1/2}}e^{-m^2/2}dm$. Thus,

$$p(\boldsymbol{x}_t)=p(Z=1|\boldsymbol{x}_t)=\int_{-\infty}^{\beta'\mathbf{x}_t}\frac{1}{(2\pi)^{1/2}}e^{-m^2/2}dm.^{26} \tag{3.24}$$

3.2.6. *The PAYG social-security system*

In a PAYG system, the social-security contributions paid by the young finance the social-security benefits paid to the old in the same period. Under the assumption that the expected social-security benefits are affected by the probability $p(\boldsymbol{x}_t)$; and provided that $L_{t+1}/L_t=1+n$ [Equation (3.1)], that is, to each old individual there correspond $1+n$ young individuals; one can write

$$E_t(b_{t+1})=p(\boldsymbol{x}_t)(1+n)d_{t+1}. \tag{3.25}$$

Since d_t is the social-security contribution paid by the young, it follows that $c_{1t}=w_t-s_{t,PAYG}-d_t$, where $s_{t,PAYG}$ is the chosen level of saving under the PAYG system. Also, since the income of the old is expected to increase by $p(\boldsymbol{x}_t)(1+n)d_{t+1}$, it follows that Equation (3.3) can be written as $E_t(c_{2t+1})=(1+r_t)s_{t,PAYG}+p(\boldsymbol{x}_t)(1+n)d_{t+1}$.

In what follows, I will calculate the effects of IC_t, PS_t, and DG_t on the relationship between social-security contributions and household saving under the PAYG system. Substituting Equation (3.25) into (3.19) and rearranging yields

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[26] The standard logistic and the standard normal cumulative distribution functions are close to each other, except for the extreme tails. Thus, it is not likely to get different results, unless the samples are large and have enough observations at the tails. The estimates of the coefficients β of the two models are not directly equal, however. Multiplying the estimated coefficients of the probit model by 1.6, yields approximately the estimated coefficients of the logit model (Maddala, 1986, pp. 22-23).

$$(1+r_t)s_{t,PAYG} + p(\boldsymbol{x}_t)(1+n)d_{t+1} = \left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}(w_t - s_{t,PAYG} - d_t)u_{t+1}, \tag{3.26}$$

where $u_{t+1} = E_t[1/(1+e_{t+1})^{1/\gamma}]$.

First, I calculate the effect of social-security contributions on household saving by applying the implicit-function theorem to Equation (3.26). For simplicity, I set e_{t+1} equal to its expected value, which is zero, so $u_{t+1} = 1$.[27] Under the assumption of constant rate of growth of d_t at steady state, I replace d_{t+1} by $(1+r_d)d_t$, where r_d is the growth rate of d_t. Also, holding r_t, w_t, IC_t, PS_t, and DG_t constant, the result is

$$\frac{\partial s_{t,PAYG}}{\partial d_t} = -\frac{p(\boldsymbol{x}_t)(1+n)(1+r_d) + \left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}}{1+r_t + \left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}} < 0\,. \tag{3.27}$$

According to Equation (3.27), social-security contributions affect negatively household saving. The effect on household saving is lower the lower the probability $p(\boldsymbol{x}_t)$ and the lower the expected social-security benefits. For instance, the higher the level of corruption (the lower the value of IC_t) the lower will be the probability $p(\boldsymbol{x}_t)$, because of the assumption $\partial p(\boldsymbol{x}_t)/\partial(IC_t) > 0$ (see the end of section 3.2.4). Thus, the lower is the index IC_t the lower the expected social-security benefits. Therefore, when social-security contributions increase, a rational individual, who wants to secure a certain level of income for retirement, will reduce his/her saving by less when corruption is high than when it is low.

[27] Without this simplification, the term u_{t+1} will appear in the derivatives (3.27)-(3.30). In the empirical part of the study (Chapter 4), however, it turns out that the presence of the estimated value of u_{t+1} in these derivatives does not affect their sign, so the simplifying assumption $u_{t+1} = 1$ in Equation (3.26) is empirically justified.

I now turn to the effect of the institutional variables and of the debt-to-GDP ratio on the relationship between social-security contributions and household saving. To start with, taking the partial derivative of $\partial s_{t,PAYG} / \partial d_t$ with respect to IC_t yields

$$\frac{\partial\left(\dfrac{\partial s_{t,PAYG}}{\partial d_t}\right)}{\partial(IC_t)} = -\frac{\dfrac{\partial p(\boldsymbol{x}_t)}{\partial(IC_t)}(1+n)(1+r_d)}{1+r_t+\left(\dfrac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}} < 0, \tag{3.28}$$

because of my earlier assumption that $\partial p(\boldsymbol{x}_t)/\partial(IC_t) > 0$. Equation (3.28) implies that the index of corruption affects negatively the impact of social-security contributions on household saving. According to Equation (3.27), the impact of social-security contributions on household saving is negative; therefore, the reduction in household saving caused by an increase in social-security contributions will be greater when the level of corruption is low (i.e., the value of IC_t is high). An interpretation of this result is that the higher the index of corruption the higher the probability $p(\boldsymbol{x}_t)$ [since $\partial p(\boldsymbol{x}_t)/\partial(IC_t) > 0$], and thus the higher the expected social-security benefits. So, when social-security contributions increase individuals will reduce their saving by more when corruption is low than when it is high.

Next, I calculate the partial derivative of $\partial s_{t,PAYG} / \partial d_t$ with respect to PS_t as follows:

$$\frac{\partial\left(\dfrac{\partial s_{t,PAYG}}{\partial d_t}\right)}{\partial(PS_t)} = -\frac{\dfrac{\partial p(\boldsymbol{x}_t)}{\partial(PS_t)}(1+n)(1+r_d)}{1+r_t+\left(\dfrac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}} < 0, \tag{3.29}$$

because of my earlier assumption that $\partial p(\boldsymbol{x}_t)/\partial(PS_t) > 0$. Equation (3.29) suggests that the index of government stability affects negatively the impact of social-security contributions on household saving. According to Equation (3.27), the impact of social-security contributions on household saving is negative; hence, the reduction in household saving caused by an increase in social-security contributions will be greater when the degree of government stability is high. This is because the higher the index of government stability the higher the probability $p(\boldsymbol{x}_t)$ [since $\partial p(\boldsymbol{x}_t)/\partial(PS_t) > 0$], and thus the higher the expected social-security benefits. Therefore, the reduction in household saving when social-security contributions increase will be greater when the index of government stability is higher.

Finally, taking the partial derivative of $\partial s_{t,PAYG}/\partial d_t$ with respect to DG_t yields

$$\frac{\partial\left(\dfrac{\partial s_{t,PAYG}}{\partial d_t}\right)}{\partial(DG_t)} = -\frac{\dfrac{\partial p(\boldsymbol{x}_t)}{\partial(DG_t)}(1+n)(1+r_d)}{1+r_t+\left(\dfrac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}} > 0, \tag{3.30}$$

because of my earlier assumption that $\partial p(\boldsymbol{x}_t)/\partial(DG_t) < 0$. According to Equation (3.30), the debt-to-GDP ratio affects positively the impact of social-security contributions on household saving. Since the impact of social-security contributions on household saving is negative [see Equation (3.27)], the reduction in household saving caused by an increase in social-security contributions will be lower when DG_t increases. An interpretation of this result is that the higher the debt-to-GDP ratio the lower the probability $p(\boldsymbol{x}_t)$ [since $\partial p(\boldsymbol{x}_t)/\partial(DG_t) < 0$], and hence the lower the expected social-security benefits. Thus, the reduction in household saving when social-security contributions increase will be lower when the debt-to-GDP ratio is higher. Generally, the effect of the institutional variables and of the debt-to-GDP ratio on the probability $p(\boldsymbol{x}_t)$ is the channel through which their influence on the relationship between social-security contributions and household saving is transmitted.

3.2.7. *The fully-funded social-security system*

In a fully-funded system, the contributions made by the young individuals are accumulated in pension funds, are invested, and are returned with interest to the same individuals when they become old. Given the assumption that the expected social-security benefits are affected by the probability $p(\boldsymbol{x}_t)$, one can write

$$E_t(b_{t+1}) = p(\boldsymbol{x}_t)(1+r_t)d_t\,. \tag{3.31}$$

Since the contribution d_t is subtracted from the current income of the young, we have that $c_{1t} = w_t - s_{t,ff} - d_t$, where $s_{t,ff}$ is the chosen level of saving under the fully-funded system. The income of the old in period t+1 is expected to increase by $p(\boldsymbol{x}_t)(1+r_t)d_t$, so (3.3) can be written as $E_t(c_{2t+1}) = (1+r_t)s_{t,ff} + p(\boldsymbol{x}_t)(1+r_t)d_t$.

In what follows, I will compute the effects of IC_t, PS_t, and DG_t on the relationship between social-security contributions and household saving under the fully-funded system. Substituting Equation (3.31) into (3.19) and rearranging yields

$$(1+r_t)s_{t,ff} + p(\boldsymbol{x}_t)(1+r_t)d_t = \left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}(w_t - s_{t,ff} - d_t)u_{t+1} \tag{3.32}$$

or

$$(1+r_t)[s_{t,ff} + p(\boldsymbol{x}_t)d_t] = \left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}(w_t - s_{t,ff} - d_t)u_{t+1}\,, \tag{3.33}$$

where $u_{t+1} = E_t[1/(1+e_{t+1})^{1/\gamma}]$.

To begin with, I apply the implicit-function theorem to Equation (3.33) to compute the effect of social-security contributions on household saving. Again, for simplicity, I set e_{t+1} equal to its expected value, which is zero, so $u_{t+1} = 1$. Holding r_t, w_t, IC_t, PS_t, and DG_t constant yields

$$\frac{\partial s_{t,ff}}{\partial d_t} = -\frac{p(\boldsymbol{x}_t)(1+r_t) + \left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}}{1+r_t + \left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}} < 0\,. \tag{3.34}$$

Equation (3.34) suggests that under a fully-funded system, social-security contributions affect negatively household saving as in the case of the PAYG system. Household saving decreases by less than the increase in social-security contributions the lower the probability $p(\boldsymbol{x}_t)$, and hence the lower the expected social-security benefits. For example, the higher the debt-to-GDP ratio, the lower the probability that the social-security system will grant pensions at retirement and the lower the expected social-security benefits. Thus, lifetime resources and consumption decrease, under the assumption that consumption in every period is a normal good [see Equation (3.4)]. Hence, household saving is reduced by less than the increase in social-security contributions. Consumption will decrease instead.

If $p(\boldsymbol{x}_t) = 1$, that is, if there is certainty that the fully-funded system will grant pensions at retirement, then an increase in social-security contributions reduces household saving one-for-one. This is because of the fact that the yield of social-security contributions equals the market interest rate, r_t, and hence lifetime resources remain unchanged by the introduction of the fully-funded system. If $0 \le p(\boldsymbol{x}_t) < 1$, however, then the reduction in household saving is less than one-for-one. In this case, although the yield of social-security contributions is still equal to the market interest rate, the expected social-security benefits decrease due to risk regarding the viability of the social-security system. Lifetime resources, and hence lifetime consumption fall [see Equation (3.4)]. So, household saving is reduced by less than the increase in social-security contributions.

Note that in the existing literature, the theoretical analyses do not take into account the quality of institutions and its effect on the probability that the social-security system will grant pensions at retirement. The above result differs from the traditional life-cycle one, according to which in the fully-funded system the social-security contributions reduce household saving one-for-one (Feldstein and Pellechio, 1979, Kotlikoff, 1979b) or even greater than one-for-one (Hubbard, 1984).

Next, I examine how the institutional variables and the debt-to-GDP ratio affect the relationship between social-security contributions and household saving. Firstly, from Equation (3.34), the effect of IC_t on $\partial s_{t,ff} / \partial d_t$ is calculated as follows:

$$\frac{\partial\left(\dfrac{\partial s_{t,ff}}{\partial d_t}\right)}{\partial(IC_t)} = -\frac{\dfrac{\partial p(\boldsymbol{x}_t)}{\partial(IC_t)}(1+r_t)}{1+r_t+\left(\dfrac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}} < 0, \tag{3.35}$$

because of my assumption that $\partial p(\boldsymbol{x}_t)/\partial(IC_t) > 0$. According to Equation (3.35), the index of corruption affects negatively the impact of social-security contributions on household saving. Since the impact of social-security contributions on household saving is negative [see Equation (3.34)], the reduction in household saving caused by an increase in social-security contributions will be greater the lower the level of corruption (the higher the value of IC_t). The interpretation of this result is similar to that of Equation (3.28).

Next, taking the partial derivative of $\partial s_{t,ff} / \partial d_t$ with respect to PS_t yields

$$\frac{\partial\left(\dfrac{\partial s_{t,ff}}{\partial d_t}\right)}{\partial(PS_t)} = -\frac{\dfrac{\partial p(\boldsymbol{x}_t)}{\partial(PS_t)}(1+r_t)}{1+r_t+\left(\dfrac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}} < 0, \tag{3.36}$$

because of my earlier assumption that $\partial p(\boldsymbol{x}_t)/\partial(PS_t) > 0$. Equation (3.36) implies that the index of government stability affects negatively the impact of social-security contributions on household saving. According to Equation (3.34), the impact of social-security contributions on household saving is negative; hence, the reduction in household saving caused by an increase in social-security contributions will be greater the higher the index of government stability. The interpretation of this result is similar to that of Equation (3.29).

Finally, I calculate the partial derivative of $\partial s_{t,ff} / \partial d_t$ with respect to DG_t as follows:

$$\frac{\partial\left(\dfrac{\partial s_{t,ff}}{\partial d_t}\right)}{\partial(DG_t)} = -\frac{\dfrac{\partial p(\boldsymbol{x}_t)}{\partial(DG_t)}(1+r_t)}{1+r_t+\left(\dfrac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}} > 0, \tag{3.37}$$

because of my earlier assumption that $\partial p(\boldsymbol{x}_t)/\partial(DG_t) < 0$. Equation (3.37) suggests that the debt-to-GDP ratio affects positively the impact of social-security contributions on household saving. According to Equation (3.34), the impact of social-security contributions on household saving is negative; thus, if there is an increase in the debt-to-GDP ratio, the reduction in household saving caused by an increase in social-security contributions will be lower. The interpretation of this result is similar to that of Equation (3.30).

3.2.8. *Private-pension plans and the possibility of collapse*

There are three possible sources of income at retirement: public-pension schemes, private-pension schemes, and individual saving. Private-pension schemes play an important role in many OECD countries, where there is a need to supplement the PAYG public pensions and ensure an adequate level of income during retirement for a large share of the population. The share of the working age population (15 to 64 years of age) enrolled in private-pension plans is lower in countries where participation in these plans is voluntary (Antolin, Payet, and Yermo, 2012, pp. 6-10). In this subsection, I take into account the possibility that some working individuals participate in private-pension plans. If the pension system (public or private) collapses, because the contributions are lower than the benefits, the government will step in and cover the difference, thus increasing its budget deficit.

I consider two possible states of the world. First, the pension system will grant pensions to the old at retirement with probability $p(\boldsymbol{x}_t)$ using the workers' contributions. Second, the pension system will collapse, i.e., the workers' contributions will not be sufficient, and so the government will finance the benefits, with probability $1 - p(\boldsymbol{x}_t)$. In the latter case, the government's period-by-period budget constraint is given by[28]

$$T_t + D_t + \Delta Q_{t+1} = B_t + r_t Q_t + G_t, \tag{3.38}$$

where T_t is general-government revenue, excluding social-security contributions, D_t; Q_t is general-government debt; and G_t is general-government purchases of goods and services. The left-hand side of Equation (3.38) represents the sources of general-government income, i.e., taxes collected by general government, social-security contributions, and the issuance of new debt, whereas the right-hand side represents the uses of general-government income, i.e., social-security benefits, interest payments on the public debt, and general-government purchases of goods and services. Equation

[28] I am grateful to Professor Costas Azariadis for making this suggestion during my presentation at the *Ioannina Meeting on Applied Economics and Finance* (IMAEF) on 22 June 2012, thus making the model of section 3.2.6 more general.

(3.38) can be expressed in per-worker terms by dividing both sides by L_t and using Equation (3.1), $L_{t+1} = L_t(1+n)$, as follows:

$$\tau_t + d_t + (1+n)q_{t+1} = b_t + (1+r_t)q_t + g_t, \tag{3.39}$$

where the lower-case variables are the corresponding variables of Equation (3.38) in per-worker terms, e.g., $\tau_t = T_t/L_t$.

Under a PAYG system, the expected pension benefits are the weighted average of the benefits financed by workers' contributions, $b_{t+1} = (1+n)d_{t+1}$ (if the system is sustainable) and those financed by the government, in accordance with Equation (3.39), i.e., $b_{t+1} = \tau_{t+1} + d_{t+1} + (1+n)q_{t+2} - (1+r_{t+1})q_{t+1} - g_{t+1}$ (if the system collapses), where the weights are $p(\boldsymbol{x}_t)$ and $1 - p(\boldsymbol{x}_t)$, respectively. Thus,

$$E_t(b_{t+1}) = p(\boldsymbol{x}_t)(1+n)d_{t+1} + [1-p(\boldsymbol{x}_t)][\tau_{t+1} + d_{t+1} + (1+n)q_{t+2} - (1+r_{t+1})q_{t+1} - g_{t+1}]. \tag{3.40}$$

Equation (3.40) extends Equation (3.25) by considering the possibility of a collapsing pension system. Substituting Equation (3.40) into Equation (3.19) and rearranging yields

$$\begin{aligned}(1+r_t)s_{t,PAYG} &+ p(\boldsymbol{x}_t)[nd_{t+1} - \tau_{t+1} - (1+n)q_{t+2} + (1+r_{t+1})q_{t+1} + g_{t+1}] + \tau_{t+1} \\ &+ d_{t+1} + (1+n)q_{t+2} - (1+r_{t+1})q_{t+1} - g_{t+1} = \left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}(w_t - s_{t,PAYG} - d_t)u_{t+1}.\end{aligned} \tag{3.41}$$

Equation (3.41) is a modification of Equation (3.26), the Euler equation for saving.[29]

Similarly, under a fully-funded system, the expected benefits are the weighted average of the benefits financed by workers' contributions, $b_{t+1} = (1+r_t)d_t$ (if the

[29] I estimated Equation (3.41) by GMM and NLLS, but could not obtain statistically significant and correctly signed coefficients. A possible explanation for this result is that, to my knowledge, in no country of the sample has the pension system collapsed. So, I only present the results from the estimation of Equation (3.26) transformed in logs [see Equation (4.4) in Chapter 4].

system is sustainable), and those financed by the government (if the system collapses), where, as before, the weights are $p(\boldsymbol{x}_t)$ and $1 - p(\boldsymbol{x}_t)$, respectively. Hence,

$$E_t(b_{t+1}) = p(\boldsymbol{x}_t)(1+r_t)d_t + [1-p(\boldsymbol{x}_t)][\tau_{t+1} + d_{t+1} + (1+n)q_{t+2} - (1+r_{t+1})q_{t+1} - g_{t+1}]. \quad (3.42)$$

Substituting (3.42) into (3.19) and rearranging yields a modification of Equation (3.32), that is,

$$(1+r_t)s_{t,ff} + p(\boldsymbol{x}_t)[(1+r_t)d_t - \tau_{t+1} - d_{t+1} - (1+n)q_{t+2} + (1+r_{t+1})q_{t+1} + g_{t+1}]$$

$$+\tau_{t+1} + d_{t+1} + (1+n)q_{t+2} - (1+r_{t+1})q_{t+1} - g_{t+1} = \left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}}(w_t - s_{t,ff} - d_t)u_{t+1}. (3.43)$$

3.3. Conclusion

The impact of social-security contributions on household saving is decreased (increased) when the probability that the social-security system will grant pensions to the old at retirement is decreased (increased). In the fully-funded system, if the probability that the social-security system will grant pensions to the old at retirement is less than one, the reduction in household saving due to an increase in social-security contributions is expected to be less than one-for-one. This differs from the implications of the traditional life-cycle model. In the PAYG system, the reduction in household saving caused by an increase in social-security contributions cannot be implied to be less (or more) than one-for-one, because it is not known whether the yield on social-security contributions is lower (or greater) than the real interest rate. For example, if the yield on social-security contributions is lower than the real interest rate, the expected social-security benefits decrease, and hence lifetime resources and consumption fall. Thus, household saving is reduced by less than the increase in social-security contributions. The opposite is true if the yield on social-security contributions is greater than the real interest rate.

In considering the effect of the institutional variables and of the debt-to-GDP ratio on the relationship between the social-security contributions and household saving, the theoretical results suggest that the debt-to-GDP ratio affects this relationship positively, while the index of corruption and the index of government stability affect it negatively. Thus, the reduction in household saving caused by an increase in social-security contributions is expected to be lower the higher the debt-to-GDP ratio or the higher the level of corruption (the lower the index of corruption) or the lower the degree of government stability. The effect of the institutional variables and of the debt-to-GDP ratio on the probability that the social-security system will grant pensions is the channel through which these variables affect the relationship between social-security contributions and household saving. The effects computed under the PAYG system have the same sign as those computed under the fully-funded one. Empirical investigation, however, is required to quantify the theoretical implications. This task is undertaken in the next chapter.

CHAPTER 4: ECONOMETRIC ANALYSIS

4.1. Introduction

This chapter investigates econometrically the theoretical conclusions derived under the PAYG social-security system in Chapter 3.[30] After describing the data sets, I employ various panel unit-root tests to examine the stationarity properties of the variables. Since there is no analytical solution to the household maximization problem discussed in Chapter 3, the empirical counterpart of the Euler equation for saving is used as the basic specification of the econometric analysis.

I estimate the resulting regression equation using the GMM and the NLLS estimation procedures and three panel data sets. Using the estimated coefficients, I estimate the partial derivatives of the theoretical model given by Equations (3.27)-(3.30). In particular, I estimate the effect of the debt-to-GDP ratio and of the index of corruption on the probability that the PAYG system will grant pensions at retirement. After examining the relationship between social-security contributions and household saving, I estimate the effect of the debt-to-GDP ratio and of the index of corruption on this relationship. I also linearize the regression equation and estimate it by GMM. In this case, I also estimate the effect of the index of government stability on the probability that the PAYG system will grant pensions and, by extension, on the relationship between social-security contributions and household saving.

[30] The limited availability of the data for the countries that use a fully-funded social-security system as their primary system, e.g., Chile, Bolivia, etc., impedes the econometric analysis of the theoretical results derived in Chapter 3 under the fully-funded system.

4.2. Data description

The econometric analysis is based on the following three panel data sets: (1) a balanced panel of annual data from 11 OECD countries, namely, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, the United Kingdom, and the United States, for the period 1984-2009; (2) an unbalanced panel of annual data from 25 countries, which includes the previous 11 countries, and 14 more countries, namely, Austria, Cyprus, Czech Republic, Estonia, Greece, Hungary, Latvia, Lithuania, Norway, Poland, Portugal, Slovakia, Spain, and Sweden, for the period 1995-2009; and (3) a balanced panel of annual data from all the above 25 countries for the period 1995-2009. The inclusion of the countries in the samples depends on the availability of the data and on the fact that these countries generally use a PAYG social-security system.[31] The sources of the data are as follows: (1) AMECO, which is the annual macroeconomic database of the European Commission's directorate for economic and financial affairs;[32] (2) the World Development Indicators (WDI); (3) the International Financial Statistics (IFS); and (4) the International Country Risk Guide (ICRG).

The variables used here are household saving per employee (s_t), social-security contributions per employee (d_t), compensation per employee (w_t), gross domestic product per employee ($gdpe_t$), general-government debt per employee (dbe_t), general-government deficit per employee (dfe_t), the $dbe_t/gdpe_t$ ratio (DG_t), the

[31] Their pension systems are a mix of different schemes and work primarily on a PAYG basis. These schemes are categorized into two tiers. The first includes public-pension schemes, which focus on income adequacy during the retirement period. The second includes public or private schemes, which focus on replacing some level of previous earnings from work. The first tier comprises of the basic scheme, which pays flat-rate benefits, the income-tested scheme, and the minimum-pension scheme. The second tier comprises of the earnings-related and the defined-contribution schemes (OECD, 2009).

[32] AMECO contains data for the European Union countries, candidate for entry countries, and other OECD countries.

$dfe_t/gdpe_t$ ratio (FG_t), total employment (TE_t), total unemployment (TU_t, total number of workers currently unemployed), the growth rate of TE_t (n_t), the growth rate of TU_t (nu_t), exchange rate (ER_t) (number of units of national currency per Euro), the Consumer Price Index (CPI) with base year 2005 (P_t), the percentage change of CPI (PE_t), *ex-post* real interest rate constructed by deflating the nominal interest rate by the series PE_t (r_t), and the GDP price deflator with base year 2005 (GP_t). The variables s_t, d_t, w_t, dbe_t, dfe_t, and $gdpe_t$ are expressed in thousands of Euros. The variables s_t, d_t, and w_t are deflated by P_t, while the variables dbe_t, dfe_t, and $gdpe_t$ are deflated by GP_t.

Regarding the institutional variables, I use some political risk components of the ICRG, namely, the index of corruption (IC_t), the index of government stability (PS_t), the index of socioeconomic conditions (SC_t), and the index of democratic accountability (DA_t). IC_t and DA_t take on values between 0 and 6, while PS_t and SC_t take on values between 0 and 12. The higher is their value the lower is the political risk. (For the definitions of the variables and the sources of the data, see Appendix B.)

4.3. Econometric methodology and results

4.3.1. *Panel unit-root tests*

To begin with, in the case where the number of observations T in each cross-section, i.e., country, is small, the time-series properties of the panel data are usually a side issue, but when T is growing, these properties become a central issue of the analysis (Greene, 2008, p. 767). Before proceeding to the estimation procedure, I apply various panel unit-root tests to examine the stationarity properties of the variables. The estimated regressions and hypothesis tests can be distorted by nonstationarity in the data and the causal relationships can be spurious. Thus, the implementation of unit-root tests is an important consideration (Greene, 2008, p. 243).

I use the following six panel unit-root tests: (1) the Levin, Lin, and Chu (2002) (LLC) test; (2) the Breitung (2000) test; (3) the Hadri (2000) test; (4) the Im, Pesaran, and Shin (2003) (IPS) test; (5) the Fisher-type Augmented Dickey-Fuller (ADF) test; and (6) the Fisher-type Phillips-Peron (PP) test (Maddala and Wu, 1999). The LLC and Breitung tests assume that there is a common unit-root process across cross-sections, while the IPS, Fisher-ADF and Fisher-PP tests allow for individual unit-root processes that vary across cross-sections.

Specifically, the LLC and Breitung tests consider the basic ADF specification from which an estimate of the autoregressive coefficient is derived after some auxiliary estimates. The lag length of the difference terms may vary across cross-sections, while the autoregressive coefficient is assumed to be identical. The null hypothesis of a common unit-root is tested against the alternative of stationarity. In the LLC test, under the null hypothesis, a modified *t*-statistic (t^*) for the resulting estimate of the autoregressive coefficient is asymptotically normally distributed. As well, in the Breitung test the *t*-statistic for the resulting estimator has asymptotically a standard normal distribution.

In contrast to the LLC and Breitung tests, the Hadri test has a null hypothesis of stationarity for all cross-sections in the panel and an alternative of a unit-root. This test is similar to the Kwiatkowski, Phillips, Schmidt, and Shin (1992) (KPSS) unit-root test, that is, it is based on the residuals from the regression of the variable of interest on a constant, or on a constant and a trend. Two Lagrange Multiplier (LM) statistics are formed, which are asymptotically normally distributed. The Z_1-statistic is based on LM_1, which assumes homoskedastic errors, while the Z_2-statistic is based on LM_2, which is heteroskedasticity consistent. In the presence of autocorrelation, however, the Hadri test appears to over reject the null hypothesis of stationarity.

The IPS test averages the *t*-statistics of the autoregressive coefficient from the ADF regression, which is estimated for each cross-section separately; the resulting statistic is known as *t*-bar test. In the general case, where the lag length in the ADF regression may not be zero for some cross sections, the asymptotic distribution of the standardized *t*-bar statistic (*W*) is the standard normal. Under the null hypothesis, there are unit roots in all cross-sections, while under the alternative, there are no unit roots for some cross-sections. It is also allowed for some cross-sections (but not all) to have unit roots under the alternative hypothesis.

Also, the Fisher-ADF and Fisher-PP tests combine the *p*-values from a unit-root test applied to each cross-section in the panel. The asymptotic distribution of the test statistics is chi-square (χ^2) with $2N$ degrees of freedom, where N is the number of cross-sections. The null and alternative hypotheses are formed as in the IPS test.

Table 4.1 reports the results from the unit-root tests for the 25-country unbalanced panel produced by the econometric program *EViews* 6.[33] The tests are allowed to include individual constants or individual constants and time trends. In the Breitung test, both individual constants and time trends are included. In the Hadri test, the Z_1 and Z_2 statistics give similar results, so I only present the results for the Z_2-statistic. The *p*-values are used to indicate the statistical significance of the tests.

According to Table 4.1, the results of the tests are not in agreement. Overall, for each variable, stationarity is supported by at least one unit-root test, so I take all variables to be I(0).

[33] The results from the unit-root tests for the 11-country panel and for the 25-country balanced panel are similar, so I do not report them.

Table 4.1. Panel unit-root tests

Test	LLC		Breitung	Hadri		IPS		Fisher-ADF		Fisher-PP		Decision
Variable	t^*_μ	t^*_τ	t	$Z_{2\mu}$	$Z_{2\tau}$	W_μ	W_τ	χ^2_μ	χ^2_τ	χ^2_μ	χ^2_τ	I(0) or I(1)?
s_t	-3.5***	0.5	2.2	6.4***	10***	-3.9***	-1.8**	104***	72.1**	106***	66.7*	I(0)
d_t	-1.5*	-3.1***	0.9	10***	8.9***	1.0	-2.5***	58.7	82.3***	58.4	49.9	I(0)
w_t	-2.3**	-1.2	3.4	11***	9.1***	1.4	-1.2	49.1	79.2***	61.1	42.5	I(0)
r_t	-4.7***	-4.0***	-0.5	9.6***	9.7***	-3.2***	-4.3***	85.1***	97.9***	84***	112***	I(0)
DG_t	2.6	0.1	0.2	8.2***	8.2***	0.7	-2.4***	38.1	72.8**	27.6	34.8	I(0)
FG_t	-1.9**	5.1	7.1	3.9***	6.6***	-4.2***	1.3	99.8***	47.5	85.7***	39.4	I(0)
$gdpe_t$	-5.9***	6.6	9.6	16***	9.3***	0.0	4.1	54.3	31.4	51.7	11.6	I(0)
n_t	0.3	1.8	4.4	1.4*	4.2***	-3.7***	-1.5*	97.3***	73.2**	73.2**	47.1	I(0)
nu_t	1.6	5.6	6.6	1.2	4.6***	-3.5***	0.0	101***	68.6**	75.4**	41.8	I(0)
IC_t	-2.2**	-2.7***	-3.1***	9.4***	7.1***	0.2	-1.0	40.7	61.3*	31.8	29.2	I(0)
PS_t	-4.5***	-3.2***	-2.4***	1.4*	6.4***	-4.5***	-4.3***	99.6***	106***	86.9***	186***	I(0)
SC_t	-4.1***	-9.2***	-1.3*	5.3***	8.9***	-2.2**	-2.9***	70.7**	82.9***	77.1***	39.9	I(0)
DA_t	-3.6***	-2.4***	-0.6	3.6***	6.1***	-3.9***	-0.4	62.5***	32.1	73.4***	28.3	I(0)

Notes: (1) the subscripts μ and τ indicate the presence of individual constant and individual constant and time trend, respectively; (2) in the LLC, Breitung, IPS, and Fisher-ADF tests, the lag length in each cross-section ADF regression is chosen by the Schwartz criterion; (3) in the LLC, Hadri, and Fisher-PP tests, a kernel-based consistent estimator of the residual covariance is obtained using the lag truncation parameter selection method of Newey and West (1994); (4) ***, **, and * indicate statistical significance at the 1-percent, 5-percent, and 10-percent level, respectively.

4.3.2. *Empirical specification of the Euler equation*

Since the hypothesis of I(0) process is supported by at least one unit-root test for all the variables of the empirical analysis, I proceed to the estimation procedure. As there is no closed-form solution to the household maximization problem described in section 3.2, the empirical counterpart of the Euler equation for saving is used as the basic specification of the econometric analysis, under the PAYG system. Thus, Equation (3.26) can be written as follows:

$$\frac{(1+r_{it})s_{it} + p(\boldsymbol{x}_{it})(1+n_{it})d_{it+1}}{w_{it} - s_{it} - d_{it}} \left(\frac{1+\rho}{1+r_{it}}\right)^{\frac{1}{\gamma}} = u_{it+1}, \tag{4.1}$$

where i = 1, 2, …, N (N is the number of countries) and t = 1, 2, …, T (T is the number of observations for each country i).

To simplify to some extent the form of this nonlinear equation I transform it by taking logarithms as follows:

$$\ln[(1+r_{it})s_{it} + p(\boldsymbol{x}_{it})(1+n_{it})d_{it+1}] - \ln(w_{it} - s_{it} - d_{it}) + \frac{1}{\gamma}\ln(1+\rho) - \frac{1}{\gamma}\ln(1+r_{it}) = \ln u_{it+1}. \tag{4.2}$$

Substitute Equation (3.23), $p(\boldsymbol{x}_t) = \dfrac{1}{1+e^{-\boldsymbol{\beta}'\boldsymbol{x}_t}}$, into Equation (4.2) and take one period lag, so that the variables are expressed in past or current values. The result is

$$\beta_0^* + r_{it-1}^* + \gamma c_{it-1}^* - \gamma \ln[(1+r_{it-1})s_{it-1} + \frac{1}{1+e^{-\boldsymbol{\beta}'\boldsymbol{x}_{it-1}}}(1+n_{it-1})d_{it}] = u_{it}^*, \tag{4.3}$$

where $\beta_0^* = -\ln(1+\rho)$, $r_{it}^* = \ln(1+r_{it})$, $c_{it}^* = \ln(w_{it} - s_{it} - d_{it})$,[34] and $u_{it}^* = -\gamma \ln u_{it}$. The parameter β_0^* is approximately the rate of time preference with negative sign, -ρ,

[34] Note that in the data $w_{it} - s_{it} - d_{it} > 0$ for each i and t, so that the variable $c_{it}^* = \ln(w_{it} - s_{it} - d_{it})$ is a finite number for each i and t.

r_{it}^* is approximately the real interest rate, r_{it}, and c_{it}^* is the logarithm of the first-period consumption per employee, $c_{1it} = w_{it} - s_{1it} - d_{it}$.

To account for the effects of unobserved country characteristics that are assumed to be constant over time, I add country specific dummy variables, F_i, to Equation (4.3) as follows:

$$\beta_0^* + \sum_{i=1}^{N-1} \delta_i F_i + r_{it-1}^* + \gamma c_{it-1}^* - \gamma \ln[(1 + r_{it-1}) s_{it-1} + \frac{1}{1 + e^{-\boldsymbol{\beta}'\mathbf{x}_{it-1}}} (1 + n_{it-1}) d_{it}] = u_{it}^*. \quad (4.4)$$

Equation (4.4) incorporates N - 1 dummy variables, one for each country, except for the last, which is taken to be the reference country, along with the intercept, β_0^*. Thus, each parameter δ_i represents the difference in the negative of the rate of time preference between country i and the reference country, for which the dummy, F_N, is omitted from Equation (4.4). Equation (4.4) is the basic specification of the estimating equation.[35]

4.3.3. *The GMM estimation procedure using the* 11-*country panel*

In this section, I estimate Equation (4.4) by GMM for the 11-country panel. The moment conditions are derived under the assumption that the error term is orthogonal to the 1×M row vector of the instrumental variables (IVs), $\boldsymbol{V}$, that is, $E[\boldsymbol{V}' u^*] = \mathbf{0}$, where $\mathbf{0}$ is a M×1 column vector. The vector $\boldsymbol{V}$ contains a constant, the dummy variables F_1, F_2, …, and F_{10}, the once-lagged exogenous variables IC_{it}, SC_{it},

[35] According to the comment made on Equation (3.9) of section 3.2.1 (see footnote 23), the error term e_{t+1} should take on values that are close to zero. Given that $u_{t+1} = E_t[1/(1+e_{t+1})^{1/\gamma}]$, it follows that $u_{t+1} \approx 1$, and hence $\ln(u_{t+1}) = u_{t+1}^* \approx 0$. The data confirm this approximation. In the case of the 11-country panel, the GMM residuals from my preferred regression (see third regression of Table 4.2) range from -0.053 to 0.084; in the case of the 25-country unbalanced panel (see third regression in Part A of Table 4.3), they range from -0.073 to 0.149; and in the case of the 25-country balanced panel (see third regression in Part B of Table 4.3), they range from -0.064 to 0.039.

PS_{it}, and DA_{it} [since in Equation (4.4) these variables are also once-lagged], and variables r^*_{it-q}, c^*_{it-q}, s_{it-q}, n_{it-q}, d_{it-q}, IC_{it-q}, SC_{it-q}, PS_{it-q}, DA_{it-q}, DG_{it-q}, FG_{it-q}, $gdpe_{it-q}$, and nu_{it-q}, where q = 2, 3, 4 (Hall, 1988, pp. 347-48).[36] The vector of IVs includes lagged variables that are included in Equation (4.4) as well as lags of variables that are not included. These IVs are correlated with the variables employed in Equation (4.4). The values of R^2 from the regressions of each of the endogenous variables c^*_{it}, r^*_{it}, s_{it}, and d_{it} on the IVs are 0.96, 0.75, 0.94, and 0.98, respectively.

Note that the literature in dynamic panel-data models is concerned with the consequences of using too many moment conditions (Baltagi, 2008, pp. 164-66). Using time-series data (a sample of 50 or 75 observations), Tauchen (1986) demonstrates that there is a bias/efficiency trade-off as the number of moment conditions increases, and thus he recommends the use of suboptimal instrument sets in small samples. This problem, however, becomes more pronounced with panel data, because the number of moments conditions increases considerably as the number of exogenous variables and the number of time-series observations increase. Although it is desirable from an asymptotic point of view to use as many moment conditions as possible, it may be impractical to do so in many cases. Using a life-cycle labor-supply model, Ziliak (1997) finds that the same trade-off between bias and efficiency exists for panel data. In particular, he finds that the downward bias in GMM is quite severe as the number of moment conditions increases, outweighing the efficiency gains.

As well, in panel data sets with long time series, the number of instruments can increase by including instruments dated far into the past. The quality of these

[36] The use of time-aggregated variables, like variables measured on a yearly basis, may introduce first-order serial correlation not present in the original error term (Working, 1960, pp. 916-18). In particular, the error term may become a first-order moving average process and be correlated with once-lagged instruments. This problem is avoided by lagging the instruments more than one period (Campbell and Mankiw, 1990, p. 268).

instruments, however, is probably poor because they may be weakly correlated with the endogenous variables in the equation. This weak correlation between the instruments and the endogenous variables can lead to large standard errors and bias in GMM (Ziliak, 1997, pp. 419-20). Overall, there is no clear evidence in the literature regarding the number of instruments used in GMM in order to achieve the best empirical performance in terms of the bias/efficiency trade-off.

The GMM estimators are defined by replacing the moment conditions by their sample counterparts, given by $(1/N)\sum_{i=1}^{N}\boldsymbol{V}_i^{'}u_i^{*}=\boldsymbol{0}$. These are the empirical moment conditions, which can be written as $(1/N)\sum_{i=1}^{N}\boldsymbol{m}_i=\overline{\boldsymbol{m}}=\boldsymbol{0}$. The intuition of the GMM estimation is that it provides parameter estimates such that the empirical moment conditions, which correspond to the number of IVs, are as close as possible to zero. If there are more empirical moment conditions than parameters to be estimated, the system is over-identified and may not have a unique solution (Greene, 2008, pp. 443-45). The GMM estimates, $\widehat{\boldsymbol{\theta}}=(\widehat{\beta}_0^{*},\widehat{\gamma},\widehat{\delta}_i,\widehat{\boldsymbol{\beta}}^{'})$, where i = 1, 2, …, 10, are obtained as the solution to the following minimization problem:

$$\min_{\boldsymbol{\theta}}\Pi(\boldsymbol{\theta})=\overline{\boldsymbol{m}}^{'}\boldsymbol{\Omega}\overline{\boldsymbol{m}}\ , \qquad (4.5)$$

where $\boldsymbol{\Omega}$ is a positive-definite weighting matrix, which determines the relative importance of the empirical moment conditions.[37] Any positive-definite matrix $\boldsymbol{\Omega}$ will produce a consistent estimator of $\boldsymbol{\theta}$ (Dejong and Dave, 2007, pp. 152-58, Greene, 2008, pp. 474-76). The weighting matrix, $\boldsymbol{\Omega}$, employed here is computed to be robust to heteroscedasticity and serial correlation. Specifically, clustered standard errors are

[37] According to Hansen (1982), an optimal estimate of the weighting matrix is given by the inverse of the covariance matrix of the empirical moment conditions. This matrix is computed iteratively. The algorithm may fail to converge, however. Thus, a sub-optimal weighting matrix can be computed, which is adjusted so as to be robust to heteroscedasticity and serial correlation (*Rats User's Guide*, pp. 279-83).

used, which allow for arbitrary patterns of serial correlation and heteroscedasticity. So, a consistent estimator of $\boldsymbol{\Omega}$ is obtained (*Rats User's Guide*, pp. 184-87, *Rats Reference Manual*, p. 298).

The estimates are produced by the econometric computer program *WinRATS Pro* 7.0. The Gauss-Newton iterative algorithm is employed for the nonlinear estimation. As starting value for each parameter, I use zero. The estimates seem to be robust to the choice of starting values. In particular, using 30 different combinations of the values 0, 0.5, 1, and 1.5 for each parameter yields the same estimates.

To evaluate further the results, I test the validity of the over-identifying restrictions by using the well-known *J*-statistic, suggested by Hansen (1982). Under the null hypothesis that the over-identifying restrictions are satisfied, that is, the empirical moment conditions are close to zero, the *J*-statistic is asymptotically distributed as χ^2 with degrees of freedom equal to the number of instruments minus the number of estimated parameters.

I mainly employ the following three vectors of IVs:[38]

$\boldsymbol{V}_1$ = (Constant, F_1, F_2, …, F_{10}, r^*_{it-3}, c^*_{it-3}, $s_{it\text{-}3}$, $n_{it\text{-}4}$, $d_{it\text{-}2}$, $d_{it\text{-}3}$, $d_{it\text{-}4}$, $IC_{it\text{-}1}$, $SC_{it\text{-}1}$, $SC_{it\text{-}3}$, $PS_{it\text{-}1}$, $PS_{it\text{-}4}$, $DA_{it\text{-}1}$, $DA_{it\text{-}2}$, $DA_{it\text{-}3}$, $DA_{it\text{-}4}$, $DG_{it\text{-}2}$, $DG_{it\text{-}3}$, $FG_{it\text{-}2}$, $FG_{it\text{-}3}$, $FG_{it\text{-}4}$, $gdpe_{it\text{-}2}$, $gdpe_{it\text{-}3}$, $gdpe_{it\text{-}4}$, $nu_{it\text{-}2}$, $nu_{it\text{-}3}$), which contains M = 37 IVs;

$\boldsymbol{V}_2$ = (Constant, F_1, F_2, …, F_{10}, r^*_{it-3}, c^*_{it-3}, $s_{it\text{-}2}$, $s_{it\text{-}3}$, $n_{it\text{-}4}$, $d_{it\text{-}3}$, $IC_{it\text{-}1}$, $SC_{it\text{-}1}$, $SC_{it\text{-}2}$, $DA_{it\text{-}1}$, $DA_{it\text{-}3}$, $DA_{it\text{-}4}$, $DG_{it\text{-}2}$, $DG_{it\text{-}3}$, $FG_{it\text{-}2}$, $FG_{it\text{-}3}$, $gdpe_{it\text{-}2}$, $gdpe_{it\text{-}3}$, $gdpe_{it\text{-}4}$, $nu_{it\text{-}3}$), which contains M = 31 IVs; and

[38] I have chosen the IVs so as to achieve empirical identification (i.e., correct signs and statistical significance) of as many parameters as possible.

$\boldsymbol{V}_3$ = (Constant, F_1, F_2, …, F_{10}, r^*_{it-3}, c^*_{it-3}, s_{it-2}, s_{it-3}, s_{it-4}, n_{it-3}, d_{it-2}, d_{it-3}, IC_{it-1}, IC_{it-2}, SC_{it-1}, SC_{it-2}, PS_{it-1}, PS_{it-2}, DG_{it-3}, FG_{it-3}, $gdpe_{it-2}$, $gdpe_{it-3}$, $gdpe_{it-4}$, nu_{it-2}), which contains M = 31 IVs.

The results are reported in Table 4.2. To begin with, defining $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, SC_{it}, PS_{it})'$ and $\boldsymbol{\beta}' = (\beta_1, \beta_2, \beta_3, \beta_4)$, I estimate Equation (4.4) using the vector of IVs $\boldsymbol{V}_1$.[39] This is the first regression reported in Table 4.2. Note that the parameter ρ is estimated by $\widehat{\rho} = e^{-\widehat{\beta}^*_0} - 1$ [since $\widehat{\beta}^*_0 = -\ln(1+\widehat{\rho})$, see below Equation (4.3)] and its standard error (se) is estimated by $se(\widehat{\rho}) \approx e^{-\widehat{\beta}^*_0} se(\widehat{\beta}^*_0)$, which is obtained using a first-order Taylor expansion. Moreover, all the coefficients of interest are correctly signed and statistically significant except for the coefficient of SC_{it}, $\widehat{\beta}_3$, and the coefficient of PS_{it}, $\widehat{\beta}_4$, which are wrongly signed and statistically insignificant.

Then, I exclude the variable PS_{it} (since its coefficient is found to be insignificant and with lower t-statistic than that of $\widehat{\beta}_3$) and re-estimate Equation (4.4) using the vector of IVs $\boldsymbol{V}_2$. This is the second regression reported in Table 4.2. The coefficient of SC_{it}, $\widehat{\beta}_3$, is wrongly signed and statistically insignificant, whereas the other coefficients are correctly signed and statistically significant. Note that the coefficient $\widehat{\beta}_2$ would be statistically significant at the 5-percent level, if the alternative hypothesis is considered to be one-sided.[40]

[39] I also used alternative definitions of $\boldsymbol{x}_{it}$, e.g., $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, SC_{it}, PS_{it}, DA_{it})'$, $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, PS_{it}, DA_{it})'$, and $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, SC_{it}, DA_{it})'$, and alternative vectors of IVs. These alternatives failed to yield statistically significant and correctly signed coefficients, so I do not report them in Table 4.2.

[40] The signs of $\widehat{\beta}_2$, $\widehat{\beta}_3$, and $\widehat{\beta}_4$ are expected to be positive, while that of $\widehat{\beta}_1$ is expected to be negative (see the end of section 3.2.4). The reason is that an increase in the value of IC_{it} or SC_{it} or PS_{it} (i.e., lower political risk) is expected to increase $p(\boldsymbol{x}_{it})$, while an increase in DG_{it} is expected to reduce it. Thus, according to the assumptions of the theoretical model, the tests of significance for the coefficients β_1, β_2, β_3, and β_4 could be viewed as one-sided.

Table 4.2. GMM and NLLS estimation results for the 11-country panel

	$\hat{\rho}$	$\hat{\gamma}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	J
GMM($\boldsymbol{V}_1$)	0.098***	0.043***	-17.31**	2.07*	-0.45	-0.28	0.022
	(6.83)	(6.23)	(-2.37)	(1.78)	(-0.65)	(-0.41)	(1.0)
GMM($\boldsymbol{V}_2$)	0.098***	0.043***	-15.33**	1.78*	-0.62	–	0.019
	(7.36)	(6.70)	(-2.03)	(1.65)	(-1.00)		(1.0)
GMM($\boldsymbol{V}_3$)	0.098***	0.044***	-10.82***	0.73**	–	–	0.027
	(6.55)	(5.80)	(-2.61)	(1.98)			(1.0)
NLLS	0.071***	0.030***	-10.87***	0.67	–	–	–
	(6.13)	(5.08)	(-2.61)	(1.21)			

Notes: (1) ***, **, and * indicate statistical significance at the 1-percent, 5-percent, and 10-percent level, respectively, assuming a two-sided alternative hypothesis; (2) the values in parentheses below coefficient estimates are *t*-statistics, while those below the *J*-statistic are *p*-values.

To deal with these problems of empirical identification, I also exclude the variable SC_{it} (since its coefficient is generally found to be insignificant) and re-estimate Equation (4.4) using the vector of IVs $\boldsymbol{V}_3$. This is the third regression reported in Table 4.2, where all the coefficients have the correct sign and are statistically significant. The *J*-statistic, which is distributed as χ^2_{21}, χ^2_{16}, and χ^2_{17} in the first, second, and third regression of Table 4.2, respectively, does not reject the hypothesis of a correct model at any level of significance.

Now consider the third regression of Table 4.2, which is my preferred regression for the 11-country panel. The estimated value of ρ, 0.098, is somewhat larger than that found in the literature.[41] In contrast, the estimated value of γ, 0.044, is smaller than that usually found in the literature, implying a higher interest sensitivity of household saving.[42]

[41] In a different model of consumption with mortality risk and bequests, Hurd (1989) finds an estimate of the rate of time preference of 0.05, which is somewhat large, but still smaller than what I find.

[42] Note, however, that Hansen and Singleton (1984) and Campbell and Mankiw (1989) find estimates of γ to be less than one and close to zero, using different models and datasets.

I now use the GMM estimates from the third regression of Table 4.2 to estimate the derivatives of interest (see section 3.2.6). To find the effect of the debt-to-GDP ratio on the probability that the PAYG system will grant pensions at retirement, I estimate the partial derivative $\partial p(\boldsymbol{x}_{it})/\partial(DG_{it})$, using the coefficients $\widehat{\beta}_1$ and $\widehat{\beta}_2$. This derivative has the sign of $\widehat{\beta}_1$ for each i and t. The result implies that DG_{it} affects negatively $p(\boldsymbol{x}_{it})$, as expected (see the end of section 3.2.4). Thus, the lower the debt-to-GDP ratio, the higher the probability that the PAYG system will grant pensions at retirement.

As well, to find the effect of the index of corruption on the probability $p(\boldsymbol{x}_{it})$, I estimate the partial derivative $\partial p(\boldsymbol{x}_{it})/\partial(IC_{it})$, using the coefficients $\widehat{\beta}_1$ and $\widehat{\beta}_2$. This derivative has the sign of $\widehat{\beta}_2$ for each i and t. The result suggests that IC_{it} affects positively $p(\boldsymbol{x}_{it})$, that is, the lower the index of corruption (higher corruption), the lower the probability that the PAYG system will grant pensions at retirement. These results are compatible with the corresponding assumptions of the theoretical model (see the end of section 3.2.4). Estimating these partial derivatives at the sample means of the variables yields the values of -0.16 and 0.01, respectively. The effect of corruption on the probability $p(\boldsymbol{x}_{it})$ is lower (in absolute value) than that of the debt-to-GDP ratio.

Next, consider the effect of social-security contributions on household saving. I estimate the partial derivative $\partial s_{it}/\partial d_{it}$ [Equation (3.27)] using the estimated coefficients from the third regression of Table 4.2, my preferred regression for the 11-country panel. As expected, this derivative is found to be negative for each i and t. Estimating it at the sample means of the variables yields -0.20. Thus, a *ceteris paribus* increase in social-security contributions (from its sample mean) by 1 euro is expected to decrease household saving by 0.20 euros. This finding indicates that social-security contributions reduce household saving with offset less than one-for-one. The size of this offset is similar to that found by some previous empirical studies (King and Dicks-Mireaux, 1982, Diamond and Hausman, 1984, Hubbard, 1986), but is lower than that found by some other studies (Feldstein and Pellechio, 1979, Kotlikoff, 1979b, Bernheim, 1987, Attanasio and Rohwedder, 2003). These studies use different data sets and models, which do not consider the effect of institutional variables, however.

Turning to the effect of DG_{it} on $\partial s_{it}/\partial d_{it}$, I estimate the partial derivative $\partial(\partial s_{it}/\partial d_{it})/\partial(DG_{it})$ [Equation (3.30)]. As expected, this derivative is found to be positive for each *i* and *t*. Estimating this partial derivative at the sample means yields 0.13. Thus, a *ceteris paribus* increase in the debt-to-GDP ratio by one percentage point renders the impact of social-security contributions on household saving less negative by 0.13 (it becomes -0.07 from -0.20). This result implies that the reduction in household saving in response to an increase in the social-security contributions is lower the higher the debt-to-GDP ratio.

Finally, I estimate the effect of IC_{it} on $\partial s_{it}/\partial d_{it}$ by estimating the partial derivative $\partial(\partial s_{it}/\partial d_{it})/\partial(IC_{it})$ [Equation (3.28)]. As expected, this derivative is found to be negative for each *i* and *t*. Estimating this partial derivative at the sample means yields -0.01. Hence, a *ceteris paribus* decrease in the index of corruption (higher corruption) by one unit renders the impact of social-security contributions on household saving less negative by 0.01 (it becomes -0.19 from -0.20). This result suggests that the reduction in household saving caused by an increase in social-security contributions is lower when corruption is high than when it is low. At the sample means, the effect of the debt-to-GDP ratio on the impact of social-security contributions on household saving is greater than that of corruption.

4.3.4. *NLLS estimation using the* 11-*country panel*

For comparison and to avoid the criticism of achieving empirical identification (i.e., correct signs and statistical significance) of the parameters by "appropriately" choosing the IVs, I also estimate Equation (4.4) by the NLLS estimation method using the 11-country panel. Again, the Gauss-Newton iterative algorithm is employed, where as starting value for each parameter I use zero. As in the case of the GMM, I obtain clustered standard errors, which allow for arbitrary patterns of serial correlation and heteroscedasticity.

To start with, I estimate Equation (4.4) by defining $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, SC_{it})'$.[43] All coefficients are correctly signed and statistically significant, except for $\hat{\beta}_3$, which is wrongly signed and insignificant at conventional levels (*t*-statistics = -1.11). This regression is not reported in Table 4.2. Thus, I exclude the variable SC_{it} and re-estimate Equation (4.4) by setting $\boldsymbol{x}_{it} = (DG_{it}, IC_{it})'$.. This is the last regression of Table 4.2. The estimates are similar to those obtained by the GMM. All coefficients have the correct sign and are statistically significant, except for $\hat{\beta}_2$, which is not significant at conventional levels. I now use these NLLS estimates to estimate the derivatives of interest.

First, the estimated value of the derivative $\partial p(\boldsymbol{x}_{it})/\partial(DG_{it})$ is negative and that of the derivative $\partial p(\boldsymbol{x}_{it})/\partial(IC_{it})$ is positive for each *i* and *t*, as expected. Evaluated at the sample means, the estimated values of these two derivatives are -0.12 and 0.007, respectively, which are similar to their GMM counterparts (-0.16 and 0.01).

[43] I also used alternative definitions of $\boldsymbol{x}_{it}$, e.g., $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, SC_{it}, PS_{it})'$, $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, PS_{it}, DA_{it})'$, and $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, PS_{it})'$, but could not obtain statistically significant and correctly signed coefficients, so I do not report them in Table 4.2.

Second, the estimated value of the derivative $\partial s_{it}/\partial d_{it}$ [Equation (3.27)] is negative for each i and t, as expected. Estimating this derivative at the sample means yields the value of -0.21, which is similar to that estimated by GMM (-0.20).

Third, the estimated value of the derivative $\partial(\partial s_{it}/\partial d_{it})/\partial(DG_{it})$ [Equation (3.30)] is positive and that of the derivative $\partial(\partial s_{it}/\partial d_{it})/\partial(IC_{it})$ [Equation (3.28)] is negative for each i and t, as expected. Evaluated at the sample means, the estimated values of these two derivatives are 0.09 and -0.006, respectively, which are also similar to their GMM counterparts (0.13 and -0.01).

4.3.5. *Estimation using the 25-country panels*

4.3.5.1. *GMM estimates from the 25-country unbalanced panel*

In order to check the robustness of the results presented in section 4.3.3 to substantial changes in the sample, I estimate Equation (4.4) by GMM using the 25-country unbalanced panel. To begin with, I use the following three vectors of IVs:

$\boldsymbol{V}_1'$ = (Constant, F_1, F_2, …, F_{24}, r^*_{it-2}, r^*_{it-3}, c^*_{it-2}, c^*_{it-3}, $s_{it\text{-}2}$, $n_{it\text{-}3}$, $d_{it\text{-}2}$, $IC_{it\text{-}1}$, $IC_{it\text{-}4}$, $SC_{it\text{-}1}$, $SC_{it\text{-}2}$, $PS_{it\text{-}1}$, $PS_{it\text{-}2}$, $PS_{it\text{-}3}$, $PS_{it\text{-}4}$, $DA_{it\text{-}1}$, $DA_{it\text{-}2}$, $DG_{it\text{-}2}$, $FG_{it\text{-}2}$, $FG_{it\text{-}3}$, $FG_{it\text{-}4}$), which contains M = 46 IVs;

$\boldsymbol{V}_2'$ = (Constant, F_1, F_2, …, F_{24}, r^*_{it-2}, r^*_{it-3}, c^*_{it-2}, c^*_{it-3}, $s_{it\text{-}2}$, $n_{it\text{-}3}$, $d_{it\text{-}2}$, $d_{it\text{-}3}$, $IC_{it\text{-}1}$, $SC_{it\text{-}1}$, $SC_{it\text{-}2}$, $PS_{it\text{-}1}$, $PS_{it\text{-}3}$, $DA_{it\text{-}1}$, $DA_{it\text{-}3}$, $DG_{it\text{-}3}$, $FG_{it\text{-}3}$, $nu_{it\text{-}3}$), which contains M = 43 IVs; and

$\boldsymbol{V}_3'$ = (Constant, F_1, F_2, …, F_{24}, r^*_{it-2}, r^*_{it-3}, c^*_{it-2}, c^*_{it-3}, $s_{it\text{-}2}$, $s_{it\text{-}3}$, $n_{it\text{-}2}$, $d_{it\text{-}2}$, $IC_{it\text{-}1}$, $IC_{it\text{-}3}$, $SC_{it\text{-}1}$, $SC_{it\text{-}2}$, $PS_{it\text{-}1}$, $PS_{it\text{-}3}$, $DA_{it\text{-}1}$, $DA_{it\text{-}3}$, $DG_{it\text{-}2}$, $DG_{it\text{-}3}$, $FG_{it\text{-}2}$, $FG_{it\text{-}3}$), which contains M = 45 IVs.

The standard errors are robust to serial correlation and heteroscedasticity. I use zero as starting value for each parameter. As before (see section 4.3.3), I tried 30 different combinations of the starting values 0, 0.5, 1, and 1.5 for each parameter and obtained the same estimates. As well, I test the overidentifying restrictions using the *J*-statistic. The results are reported in Part A of Table 4.3.

First, I estimate Equation (4.4) using the vectors $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, SC_{it}, PS_{it})'$, $\boldsymbol{\beta}' = (\beta_1, \beta_2, \beta_3, \beta_4)$, and $\boldsymbol{V}_1'$.[44] This is the first regression reported in Part A of Table 4.3. Again, I estimate ρ by $\hat{\rho} = e^{-\hat{\beta}_0^*} - 1$ and its standard error by $se(\hat{\rho}) \approx e^{-\hat{\beta}_0^*} se(\hat{\beta}_0^*)$. All coefficients are correctly signed and statistically significant, except for $\hat{\beta}_3$ and $\hat{\beta}_4$, which are insignificant. (The coefficient $\hat{\beta}_3$ is also wrongly signed.)

Second, I exclude the variable PS_{it} (since its coefficient, $\hat{\beta}_4$, is found to be insignificant and with lower *t*-statistic than that of $\hat{\beta}_3$) and re-estimate Equation (4.4) using the vector of IVs $\boldsymbol{V}_2'$. This is the second regression reported in Part A of Table 4.3. The coefficients $\hat{\rho}$, $\hat{\gamma}$, and $\hat{\beta}_1$ are correctly signed and statistically significant, while the coefficient $\hat{\beta}_3$ is wrongly signed and insignificant. Note that the coefficient $\hat{\beta}_2$ would be statistically significant at the 10-percent level, if the alternative hypothesis is stated as one-sided (see section 4.3.3).

Third, I also exclude the variable SC_{it} (since its coefficient is found to be wrongly signed and insignificant) and re-estimate Equation (4.4) using the vector of IVs $\boldsymbol{V}_3'$. This is the third regression reported in Part A of Table 4.3, where all coefficients are correctly signed and statistically significant, except for $\hat{\beta}_2$, which is

[44] As in the case of the 11-country panel (see section 4.3.3), I also used alternative definitions of $\boldsymbol{x}_{it}$, e.g., $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, SC_{it}, PS_{it}, DA_{it})'$, $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, PS_{it}, DA_{it})'$, and $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, SC_{it}, DA_{it})'$, and alternative IVs, but these alternatives failed to yield statistically significant and correctly signed coefficients.

not significant at conventional levels. As well, the *J*-statistic, which is distributed as χ^2_{16}, χ^2_{14}, and χ^2_{17} in the first, second, and third regression in Part A of Table 4.3, respectively, does not reject the overidentifying restrictions in any case.

Now consider the third regression in Part A of Table 4.3, which is my preferred regression for the 25-country unbalanced panel. The estimates of ρ and γ are similar to their GMM counterparts in the case of the 11-country panel (see section 4.3.3). Also, the estimates of β_1 and β_2 do not differ considerably from their GMM counterparts in the case of the 11-country panel, except for the fact that in this case the estimate of β_2 is not significant at conventional levels.

Table 4.3. GMM estimation results for the 25-country panels

	$\widehat{\rho}$	$\widehat{\gamma}$	$\widehat{\beta}_1$	$\widehat{\beta}_2$	$\widehat{\beta}_3$	$\widehat{\beta}_4$	J
Part A. The 25-country unbalanced panel							
GMM($\boldsymbol{V}_1'$)	0.053***	0.032***	-5.98*	3.02*	-1.03	0.25	0.052
	(6.46)	(5.02)	(-1.76)	(1.66)	(-1.26)	(0.37)	(1.0)
GMM($\boldsymbol{V}_2'$)	0.078***	0.037***	-4.55***	0.61	-0.15	–	0.054
	(4.33)	(3.79)	(-6.10)	(1.38)	(-1.07)		(1.0)
GMM($\boldsymbol{V}_3'$)	0.084***	0.041***	-5.09***	0.42	–	–	0.066
	(6.59)	(7.21)	(-3.27)	(1.14)			(1.0)
Part B. The 25-country balanced panel							
GMM($\boldsymbol{V}_1''$)	0.059*	0.041*	-2.79	7.05	-1.55	-0.37	0.019
	(1.93)	(1.78)	(-1.30)	(1.60)	(-1.24)	(-0.53)	(1.0)
GMM($\boldsymbol{V}_2''$)	0.081***	0.048***	-3.38*	0.30*	0.11	–	0.014
	(3.58)	(2.75)	(-1.66)	(1.65)	(0.49)		(1.0)
GMM($\boldsymbol{V}_3''$)	0.078***	0.042***	-2.85*	0.28**	–	–	0.013
	(4.90)	(3.83)	(-1.67)	(2.06)			(1.0)

Notes: (1) ***, **, and * indicate statistical significance at the 1-percent, 5-percent, and 10-percent level, respectively, assuming a two-sided alternative hypothesis; (2) the values in parentheses below coefficient estimates are *t*-statistics, while those below the *J*-statistic are *p*-values; (3) these results have been produced by the computer econometric program *WinRATS Pro* 7.0.

Then, I estimate the derivatives of interest using the estimates from my preferred regression in this case. First, I estimate the derivatives $\partial p(\boldsymbol{x}_{it})/\partial(DG_{it})$ and $\partial p(\boldsymbol{x}_{it})/\partial(IC_{it})$. The first of these derivatives is negative for each i and t, as expected, and at the sample means it is -0.91. The second of these derivatives is positive for each i and t, as expected, and at the sample means it is 0.08. In absolute value, these estimates are larger than their GMM counterparts in the case of the 11-country panel (-0.16 and 0.01).

Second, I estimate the derivative $\partial s_{it}/\partial d_{it}$, which is negative for each i and t, as expected. At the sample means of the variables, this derivative is estimated as -0.38, which is larger (in absolute value) than its GMM counterpart in the case of the 11-country panel (-0.20).

Third, I estimate the derivatives $\partial(\partial s_{it}/\partial d_{it})/\partial(DG_{it})$ and $\partial(\partial s_{it}/\partial d_{it})/\partial(IC_{it})$. As expected, the first of these derivatives is positive for each i and t, and at the sample means it is 0.73. The second of these derivatives is negative for each i and t and at the sample means it is -0.06. In absolute value, these estimates are larger than their GMM counterparts in the case of the 11-country panel (0.13 and -0.01). For example, a *ceteris paribus* increase in the debt-to-GDP ratio by one percentage point causes the impact of social-security contributions on household saving to increase from -0.38 to 0.35.

4.3.5.2. *NLLS estimates from the* 25-*country unbalanced panel*

I also estimate Equation (4.4) by NLLS using the 25-country unbalanced panel, since I have chosen the IVs so as to obtain empirical identification of as many parameters as possible. I begin by defining $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, PS_{it})'$, but PS_{it} turns out to be insignificant (t-statistic = -0.52). Thus, I re-estimate Equation (4.4) by setting $\boldsymbol{x}_{it}$ = $(DG_{it}, IC_{it})'$. This regression is not reported in Table 4.3. All the estimated coefficients are correctly signed and statistically significant, except for the coefficient of IC_{it}, $\widehat{\beta}_2$, which is not significant at conventional levels (t-statistic = 1.18). The NLLS estimates are similar to their GMM counterparts.

Then, I estimate the derivatives of interest using the NLLS estimates. First, as expected, the estimated value of $\partial p(\boldsymbol{x}_{it})/\partial(DG_{it})$ is negative and that of $\partial p(\boldsymbol{x}_{it})/\partial(IC_{it})$ is positive for each i and t. At the sample means of the variables, these two derivatives are estimated as -0.77 and 0.12, respectively. Second, as expected, the estimated value of $\partial s_{it}/\partial d_{it}$ is negative for each i and t, and at the sample means it is -0.69 [similar to that found by Kotlikoff (1979b)]. Third, as expected, the estimated value of $\partial(\partial s_{it}/\partial d_{it})/\partial(DG_{it})$ is positive and that of $\partial(\partial s_{it}/\partial d_{it})/\partial(IC_{it})$ is negative for each i and t. At the sample means of the variables, these two derivatives are estimated as 0.58 and -0.09, respectively. The estimates of these derivatives are similar to their GMM counterparts in this case and are larger (in absolute value) than their NLLS counterparts in the case of the 11-country panel.

4.3.5.3. *GMM estimates from the* 25-*country balanced panel*

In this section, I estimate Equation (4.4) by GMM using the 25-country balanced panel. I employ the following three vectors of IVs:

$\boldsymbol{V}_1''$ = (Constant, F_1, F_2, …, F_{24}, r^*_{it-2}, c^*_{it-3}, s_{it-2}, s_{it-3}, n_{it-2}, d_{it-2}, IC_{it-1}, SC_{it-1}, SC_{it-2}, SC_{it-3}, PS_{it-1}, PS_{it-2}, DA_{it-1}, DA_{it-2}, DG_{it-2}, FG_{it-2}, FG_{it-3}, $gdpe_{it-2}$, nu_{it-2}, nu_{it-3}), which contains M = 45 IVs;

$\boldsymbol{V}_2''$ = (Constant, F_1, F_2, …, F_{24}, r^*_{it-2}, r^*_{it-3}, c^*_{it-2}, c^*_{it-3}, $s_{it\text{-}3}$, $n_{it\text{-}2}$, $n_{it\text{-}4}$, $d_{it\text{-}2}$, $IC_{it\text{-}1}$, $IC_{it\text{-}3}$, $IC_{it\text{-}4}$, $SC_{it\text{-}1}$, $SC_{it\text{-}2}$, $PS_{it\text{-}1}$, $PS_{it\text{-}2}$, $PS_{it\text{-}3}$, $PS_{it\text{-}4}$, $DA_{it\text{-}1}$, $DA_{it\text{-}2}$, $DG_{it\text{-}2}$, $FG_{it\text{-}2}$, $nu_{it\text{-}3}$), which contains M = 47 IVs; and

$\boldsymbol{V}_3''$ = (Constant, F_1, F_2, …, F_{24}, r^*_{it-2}, r^*_{it-3}, c^*_{it-2}, c^*_{it-3}, $s_{it\text{-}3}$, $n_{it\text{-}2}$, $n_{it\text{-}4}$, $d_{it\text{-}2}$, $IC_{it\text{-}1}$, $IC_{it\text{-}2}$, $IC_{it\text{-}4}$, $PS_{it\text{-}1}$, $PS_{it\text{-}2}$, $PS_{it\text{-}3}$, $PS_{it\text{-}4}$, $DA_{it\text{-}1}$, $DA_{it\text{-}2}$, $DG_{it\text{-}2}$, $FG_{it\text{-}2}$), which contains M = 44 IVs.

The results are reported in Part B of Table 4.3. First, I estimate Equation (4.4) using the definitions $\boldsymbol{x}_{it} = (DG_{it}, IC_{it}, SC_{it}, PS_{it})'$, $\boldsymbol{\beta}' = (\beta_1, \beta_2, \beta_3, \beta_4)$, and $\boldsymbol{V}_1''$.[45] This is the first regression reported in Part B of Table 4.3. The estimates of ρ and γ are correctly signed and statistically significant; those of β_1 and β_2 are correctly signed and statistically significant only at the 10-percent level and only if the alternative hypothesis is considered to be one-sided (see section 4.3.3); whereas those of β_3 and β_4 are wrongly signed and insignificant.

Second, since the estimate of β_4 is found to be insignificant and with lower *t*-statistic than that of the estimate of β_3, I exclude PS_{it} and re-estimate Equation (4.4) using the vector of IVs $\boldsymbol{V}_2''$. This is the second regression in Part B of Table 4.3. In this case, all the estimated coefficients are correctly signed and statistically significant, except for the estimates of β_3, which is insignificant.

Third, to achieve empirical identification, I also exclude SC_{it} (since its coefficient is found to be insignificant) and re-estimate Equation (4.4) using the vector of IVs $\boldsymbol{V}_3''$. This is the third regression in Part B of Table 4.3, where all coefficients are correctly signed and statistically significant. As before, the *J*-statistic, which is distributed as χ^2_{15}, χ^2_{18}, and χ^2_{16} in the first, second, and third regression in Part B of

[45] As before, I also used alternative definitions of $\boldsymbol{x}_{it}$, alternative IVs, and an alternative estimation method (NLLS), but failed to achieve empirical identification of the parameters.

Table 4.3, respectively, does not reject the overidentifying restrictions at any level of significance.

Now consider the third regression in Part B of Table 4.3, which is my preferred regression for the 25-country balanced panel. These estimates do not differ dramatically from their 25-country unbalanced panel counterparts as well as from their 11-country panel counterparts. Thus, it would not be unreasonable to argue that the estimates are robust to substantial changes in the sample.

Using the estimates from my preferred regression in this case, I estimate the derivatives of interest, which have the expected sign for each *i* and *t*. Evaluated at the sample means, the estimated values of $\partial p(\boldsymbol{x}_{it})/\partial(DG_{it})$ and $\partial p(\boldsymbol{x}_{it})/\partial(IC_{it})$ are -0.67 and 0.07, respectively; that of $\partial s_{it}/\partial d_{it}$ is -0.50; and those of $\partial(\partial s_{it}/\partial d_{it})/\partial(DG_{it})$ and $\partial(\partial s_{it}/\partial d_{it})/\partial(IC_{it})$ are 0.54 and -0.05, respectively. These estimates are similar to their GMM and NLLS counterparts in the case of the 25-country unbalanced panel and are larger (in absolute value) than their GMM and NLLS counterparts in the case of the 11-country panel.

4.3.6. *Estimation of the linearized Euler equation using the* 11-*country panel*

The results obtained from the nonlinear Equation (4.4) may depend on the choice of the starting values for the parameters. Although the estimates seem to be robust to the choice of starting values (see sections 4.3.3-4.3.5), I linearize Equation (4.4) to deal with this potential problem.

To begin with, leading Equation (4.4) by one period and rearranging yields

$$c_{it}^{*} = -\frac{\beta_0^{*}}{\gamma} - \sum_{i=1}^{N-1}\frac{\delta_i}{\gamma}F_i - \frac{1}{\gamma}r_{it}^{*} + \ln[(1+r_{it})s_{it} + \frac{1}{1+e^{-\boldsymbol{\beta}'\boldsymbol{x}_{it}}}(1+n_{it})d_{it+1}] + \upsilon_{it+1}, \qquad (4.6)$$

where $\upsilon_{it+1} = \frac{1}{\gamma} u^*_{it+1}$. In Equation (4.6), I set

$$h(\boldsymbol{\xi}_{it}, \boldsymbol{a}) = -\frac{\beta_0^*}{\gamma} - \sum_{i=1}^{N-1} \frac{\delta_i}{\gamma} F_i - \frac{1}{\gamma} r_{it}^* + \ln[(1 + r_{it}) s_{it} + \frac{1}{1 + e^{-\boldsymbol{\beta}' \mathbf{x}_{it}}} (1 + n_{it}) d_{it+1}], \tag{4.7}$$

where $\boldsymbol{\xi}_{it} = (F_1, ..., F_{N-1}, r_{it}^*, s_{it}, n_{it}, d_{it+1}, \boldsymbol{x}_{it}')$ and $\boldsymbol{a} = (\beta_0^*, \gamma, \delta_i, \boldsymbol{\beta}')$. Thus, Equation (4.6) can be written as follows:

$$c_{it}^* = h(\boldsymbol{\xi}_{it}, \boldsymbol{a}) + \upsilon_{it+1}. \tag{4.8}$$

Next, linearize the function $h(\boldsymbol{\xi}_{it}, \boldsymbol{a})$ in Equation (4.8) using a first-order Taylor expansion (Greene, 2008, pp. 288-90). Define the following variables:

$$\xi_{1,it}^0 = \frac{\partial h(\boldsymbol{\xi}_{it}, \boldsymbol{a})}{\partial \beta_0^{*0}} = -\frac{1}{\gamma^0}, \tag{4.9a}$$

$$\xi_{2,it}^0 = \frac{\partial h(\boldsymbol{\xi}_{it}, \boldsymbol{a})}{\partial \gamma^0} = \frac{\beta_0^{*0}}{(\gamma^0)^2} + \frac{1}{(\gamma^0)^2} \sum_{i=1}^{N-1} \delta_i^0 F_i + \frac{1}{(\gamma^0)^2} r_{it}^*, \tag{4.9b}$$

$$\xi_{3i}^0 = \frac{\partial h(\boldsymbol{\xi}_{it}, \boldsymbol{a})}{\partial \delta_i^0} = -\frac{1}{\gamma^0} F_i, \ i = 1, \ldots, N - 1, \tag{4.9c}$$

and

$$\xi_{4k,it}^0 = \frac{\partial h(\boldsymbol{\xi}_{it}, \boldsymbol{a})}{\partial \beta_k^0} = \frac{1}{(1 + r_{it}) s_{it} + \frac{(1 + n_{it}) d_{it+1}}{1 + e^{-\boldsymbol{\beta}^{0\prime} \mathbf{x}_{it}}}} \frac{e^{-\boldsymbol{\beta}^{0\prime} \mathbf{x}_{it}} x_{kit} (1 + n_{it}) d_{it+1}}{(1 + e^{-\boldsymbol{\beta}^{0\prime} \mathbf{x}_{it}})^2}, \tag{4.9d}$$

where k = 1, …, K (K is the number of the variables included in the vector $\boldsymbol{x}_{it}$). Considering the first-order Taylor expansion around the parameter values: $\beta_0^{*0} = 0$, $\gamma^0 = 1$, $\delta_i^0 = 0$, and $\boldsymbol{\beta}^{0\prime} = 0$, the definitions (4.9a)-(4.9d) become

$$\xi_{1,it}^0 = -1, \tag{4.10a}$$

$$\xi_{2,it}^0 = r_{it}^*, \tag{4.10b}$$

$$\xi_{3i}^0 = -F_i, \ i = 1, \ldots, N - 1, \tag{4.10c}$$

and

$$\xi^0_{4k,it} = \frac{x_{kit}(1+n_{it})d_{it+1}}{4(1+r_{it})s_{it} + 2(1+n_{it})d_{it+1}}, \; k = 1, \ldots, K. \tag{4.10d}$$

As well, the definition (4.7) at $\boldsymbol{a}^0$ becomes

$$h(\boldsymbol{\xi}_{it}, \boldsymbol{a}^0) = -r^*_{it} + \ln[(1+r_{it})s_{it} + \frac{1}{2}(1+n_{it})d_{it+1}]. \tag{4.11}$$

Thus, for the given value of $\boldsymbol{a}^0$, the definitions of $\xi^0_{1,it}$, $\xi^0_{2,it}$, ξ^0_{3i}, $\xi^0_{4k,it}$, and $h(\boldsymbol{\xi}_{it}, \boldsymbol{a}^0)$ are functions only of the data, not of the unknown parameters.

Now define

$$c^{*0}_{it} = c^*_{it} - h(\boldsymbol{\xi}_{it}, \boldsymbol{a}^0) + \beta^{*0}_0 \xi^0_{1,it} + \gamma^0 \xi^0_{2,it} + \sum_{i=1}^{N-1} \delta^0_i \xi^0_{3i} + \sum_{k=1}^{K} \beta^0_k \xi^0_{4k,it}\,. \tag{4.12}$$

Given the definitions (4.10a)-(4.10d) and (4.11), Equation (4.12) at $\boldsymbol{a}^0$ becomes

$$c^{*0}_{it} = c^*_{it} + 2r^*_{it} - \ln[(1+r_{it})s_{it} + \frac{1}{2}(1+n_{it})d_{it+1}]. \tag{4.13}$$

Finally, I obtain the following linear equation:

$$c^{*0}_{it} = \beta^*_0 \xi^0_{1,it} + \gamma \xi^0_{2,it} + \sum_{i=1}^{N-1} \delta_i \xi^0_{3i} + \sum_{k=1}^{K} \beta_k \xi^0_{4k,it} + \upsilon^0_{it+1} \tag{4.14}$$

or

$$c^{*0}_{it} = -\beta^*_0 + \gamma r^*_{it} - \sum_{i=1}^{N-1} \delta_i F_i + \sum_{k=1}^{K} \beta_k \frac{x_{kit}(1+n_{it})d_{it+1}}{4(1+r_{it})s_{it} + 2(1+n_{it})d_{it+1}} + \upsilon^0_{it+1}\,, \tag{4.15}$$

where υ^0_{it+1} contains both the error term υ_{it+1} and the error from the first-order Taylor expansion of the function $h(\boldsymbol{\xi}_{it}, \boldsymbol{a})$.

I estimate Equation (4.15) by GMM using the 11-country panel; setting $x_{1it} = DG_{it}$, $x_{2it} = IC_{it}$, $x_{3it} = SC_{it}$, and $x_{4it} = PS_{it}$;[46] and employing the following three vectors of IVs, where the IVs are lagged at least once (see section 4.3.3):

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[46] I also estimated Equation (4.15) by NLLS using the 11-country panel, and by GMM and NLLS using the 25-country unbalanced panel as well as the 25-country balanced panel, but failed to achieve empirical identification of most of the parameters.

$\boldsymbol{V}_1''' =$ (Constant, F_1, F_2, …, F_{10}, r^*_{it-1}, r^*_{it-2}, r^*_{it-3}, $\xi^0_{41,it-1}$, $\xi^0_{42,it-2}$, $\xi^0_{42,it-3}$, $\xi^0_{43,it-1}$, $\xi^0_{44,it-2}$, $\xi^0_{45,it-1}$), which contains $M = 20$ IVs;

$\boldsymbol{V}_2''' =$ (Constant, F_1, F_2, …, F_{10}, r^*_{it-1}, r^*_{it-2}, r^*_{it-3}, $\xi^0_{41,it-1}$, $\xi^0_{43,it-3}$, $\xi^0_{44,it-3}$, $\xi^0_{45,it-2}$), which contains $M = 18$ IVs; and

$\boldsymbol{V}_3''' =$ (Constant, F_1, F_2, …, F_{10}, r^*_{it-1}, r^*_{it-2}, $\xi^0_{41,it-1}$, $\xi^0_{41,it-2}$, $\xi^0_{42,it-3}$, $\xi^0_{44,it-3}$), which contains $M = 17$ IVs.

The results are reported in Table 4.4. To start with, I estimate Equation (4.15) using the vector of IVs $\boldsymbol{V}_1'''$. This is the first regression reported in Table 4.4. All coefficients are correctly signed. The coefficient of IC_{it}, $\hat{\beta}_2$, and that of PS_{it}, $\hat{\beta}_4$, are statistically significant (t-statistic = 2.40 and 1.69, respectively), whereas the coefficient of DG_{it}, $\hat{\beta}_1$, and that of SC_{it}, $\hat{\beta}_3$, are not significant (t-statistic = -1.11 and 0.14, respectively).

Then, I exclude $\xi^0_{43,it}$ (since its coefficient $\hat{\beta}_3$ is insignificant and with a lower t-statistic than that of $\hat{\beta}_1$) and re-estimate Equation (4.15) using the vector of IVs $\boldsymbol{V}_2'''$. This is the second regression reported in Table 4.4. Again, all coefficients have the correct sign. The coefficients $\hat{\beta}_1$ and $\hat{\beta}_4$ are statistically significant, whereas the coefficient $\hat{\beta}_2$ is not significant at conventional levels. Note, however, that the coefficient $\hat{\beta}_2$ would be statistically significant at the 10-percent level if the alternative hypothesis is stated as one-sided (see section 4.3.3).

To achieve empirical identification, I also exclude $\xi^0_{42,it}$ (since its coefficient $\hat{\beta}_2$ is insignificant at conventional levels) and re-estimate Equation (4.15) using the vector of IVs $\boldsymbol{V}_3'''$. This is the third regression of Table 4.4, where all coefficients are correctly signed and statistically significant. The J-statistic, which is distributed as

χ_4^2, χ_3^2, and χ_3^2 in the first, second, and third regression of Table 4.4, respectively, does not reject the overidentifying restrictions in any of these regressions.

Now consider the third regression of Table 4.4, which is my preferred regression in the case of the linearized Euler equation. The estimate of ρ is larger than that usually found in the literature, while the estimate of γ is similar to that found by Campbell and Mankiw (1989) using a different model and dataset, however. As well, the estimates of ρ and γ are much larger than their GMM counterparts in the case of the nonlinear Euler equation (4.4) (see the third regression of Table 4.2 in section 4.3.3). In contrast to their GMM counterparts in the case of the nonlinear Euler equation, the estimate of β_4 is significant even at the 1-percent level, while the estimate of β_2 is not significant at conventional levels. The size of these estimates is somewhat different from that of their GMM counterparts in the case of the nonlinear Euler equation (4.4).

Table 4.4. GMM estimation results of the linearized Euler equation

	$\hat{\rho}$	$\hat{\gamma}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	J
GMM($\boldsymbol{V}_1'''$)	0.76^{***}	0.29	-0.76	0.35^{**}	0.03	0.44^{*}	0.24
	(3.22)	(0.13)	(-1.11)	(2.40)	(0.14)	(1.69)	(0.99)
GMM($\boldsymbol{V}_2'''$)	0.43^{**}	2.15	-1.25^{*}	0.39	–	0.61^{***}	0.36
	(2.01)	(0.82)	(-1.72)	(1.56)		(4.29)	(0.95)
GMM($\boldsymbol{V}_3'''$)	0.39^{*}	4.82^{**}	-1.57^{**}	–	–	0.83^{***}	0.27
	(1.68)	(2.50)	(-2.15)			(8.13)	(0.97)

Notes: (1) ***, **, and * indicate statistical significance at the 1-percent, 5-percent, and 10-percent level, respectively, assuming a two-sided alternative hypothesis; (2) the values in parentheses below coefficient estimates are *t*-statistics, while those below the *J*-statistic are *p*-values; (3) these results have been produced by the computer econometric program *WinRATS Pro* 7.0.

In what follows, I estimate the derivatives of interest using the estimates from my preferred regression in this case. First, I estimate the derivatives $\partial p(\boldsymbol{x}_{it})/\partial(DG_{it})$ and $\partial p(\boldsymbol{x}_{it})/\partial(PS_{it})$. The first of these derivatives is negative for each i and t, as expected, and at the sample means it is -0.006, which is much smaller (in absolute value) than its GMM counterpart in the case of the nonlinear Euler equation (4.4) (see section 4.3.3). The second of these derivatives is positive for each i and t, as expected, and at the sample means it is 0.003.

Second, I estimate the derivative $\partial s_{it}/\partial d_{it}$. As expected, this derivative is negative for each i and t, and at the sample means it is -0.99, which is larger (in absolute value) than its GMM counterpart in the case of the nonlinear Euler equation (see section 4.3.3). Hence, a *ceteris paribus* increase in social-security contributions (from its sample mean) by 1 euro is expected to decrease household saving by about 1 euro. This result suggests that social-security contributions reduce household saving with offset almost one-for-one. The size of this offset is similar to that found by Feldstein and Pellechio (1979) using different data and a different model.

Third, I estimate the derivatives $\partial(\partial s_{it}/\partial d_{it})/\partial(DG_{it})$ and $\partial(\partial s_{it}/\partial d_{it})/\partial(PS_{it})$. As expected, the first of these derivatives is positive for each i and t, and at the sample means it is 0.003, which is much smaller (in absolute value) than its GMM counterpart in the case of the nonlinear Euler equation (see section 4.3.3). The second of these derivatives is negative for each i and t and at the sample means it is -0.002.

4.4. Summary

In this chapter, I have used three panel data sets and two estimation procedures, GMM and NLLS, to estimate the coefficients and the derivatives of interest (see Chapter 3). The results generally confirm the implications of the theoretical model. Estimating the nonlinear Euler equation (4.4) by GMM, the debt-to-GDP ratio and the index of corruption are found to be statistically significant, while some other institutional variables, e.g., the index of government stability and the index of socioeconomic conditions, are insignificant. The NLLS estimation procedure yields almost the same results as the GMM, except that the index of corruption turns out to be insignificant at conventional levels. Moreover, estimating the linearized Euler equation by GMM, the debt-to-GDP ratio and the index of government stability turn out to be statistically significant, while the index of corruption is insignificant at conventional levels. The estimates obtained from the three panel data sets do not differ dramatically, so it would not be unreasonable to argue that they pass the robustness test. In the next chapter, since the household maximization problem (described in Chapter 3) has no analytical solution, I calibrate the model to obtain a numerical solution and examine the relationships between the variables of interest.

CHAPTER 5: CALIBRATION

5.1. Introduction

The household maximization problem described in Chapter 3 has no analytical solution. Hence, in this chapter, I employ a numerical solution method, namely calibration. Calibration of a model involves the setting of specific values for the parameters to replicate a benchmark data set as the model solution (Dawkins, Srinivasan, and Whalley, 2001, p. 3656). Calibration, however, remains an imprecise term and no single set of calibration procedures exist.

In general, calibration involves the following steps. First, the choice of the model is usually based on the theoretical literature and depends on the question that the researcher is seeking to answer as well as on its feasibility. In particular, the choice of functional form is influenced by the feasibility of computing the equilibrium process of the model (Kydland and Prescott, 1996, pp. 72-73). Convenient functional forms for the production function (e.g., a Cobb-Douglas production function), the utility function, and for the processes that describe the evolution of the capital stock and of productivity shocks are commonly used.

Second, the choice of values for the model parameters is based on the literature and/or on the model's fit to the actual data, that is, setting values to the parameters so that the behavior of the model matches features of the actual data (Cooley and Prescott, 1995, p. 15).[47] The use of econometric estimates of elasticities from the literature may face some problems, i.e., estimates may differ widely or may

[47] This can be done by substituting into the equilibrium conditions of the model the sample means of the variables from the actual data and then by solving these conditions for the parameter values.

be contradictory. As well, the use of parameter estimates from microeconometric studies in dynamic macroeconomic models can be misleading when the economic environment for the two models is different (Hansen and Heckman, 1996, pp. 97-98). These problems may be a source of uncertainty in model specification, which can be dealt with by carrying out sensitivity analysis to see how the results are affected by different choices of parameter values.

Third, after choosing functional forms and assigning values to the parameters, the equilibrium process of the model is computed and artificial time series of the desired length are generated. Then, a set of statistics that summarize certain features of the actual data, such as second-order sample moments, are computed. As well, the same statistics are computed for the artificial data (Kydland and Prescott, 1996, p. 75). Finally, the statistics of the artificial data are compared with the statistics of the actual data. The adequacy of the model can be evaluated by the degree to which the statistics of the artificial data match with those of the actual data.

In this chapter, I calibrate the model (presented in Chapter 3) to examine its consistency with certain features of the actual data and then use it to examine the relationships between the variables of interest. In particular, I transform the system of equations presented in section 3.2.4 in per effective labor terms, so that this system is expressed in terms of stationary variables, and compute the steady-state equilibrium. I calibrate the model by assigning values to the parameters and examine whether the model can replicate some features of the actual data from the U.S. economy reflected in the second-order sample moments, e.g., standard deviations and correlations. After computing the state-state values of household saving, capital stock, real wage, and the real interest rate, I compute the general-equilibrium effects of the PAYG social-security system on these values. Holding constant the real wage and the real interest rate, I also compute the corresponding partial-equilibrium effects. In addition, considering that the PAYG system may not be sustainable, I examine the effects of a collapse of the PAYG system on the state-state values of household saving, capital stock, real wage, and the real interest rate. This is because of the fact that I could not achieve empirical identification in this case (see section 3.2.8). Since the steady-state values of the variables depend on the calibrated parameters, I examine the sensitivity of the results to changes in the values of the parameters.

I also calibrate the model to match features of the Mexican economy, which employs a fully-funded social-security system, using the same procedure as that in the case of the U.S. economy. Finally, I examine the effects of a transition from the PAYG system to the fully-funded system, which is financed by the government budget, on the steady-state values of household saving, capital stock, real wage, and real interest rate.

5.2. The model

To begin with, I express the system of equations presented in section 3.2.4 in terms of stationary variables (without trend), so that these variables converge to the steady-state equilibrium (Hansen and Prescott, 1993, p. 283). Therefore, I express the system of equations in per effective labor terms as follows:

$$E_t\left(\frac{\tilde{c}_{1t}}{\tilde{c}_{2t+1}}\right)^{\gamma} = \frac{(1+r_A)^{\gamma}(1+\rho)}{1+r_t}, \tag{5.1}$$

$$\tilde{c}_{1t} = \tilde{w}_t - \tilde{s}_t - \tilde{d}_t, \tag{5.2}$$

$$E_t(\tilde{c}_{2t+1}) = \frac{1+r_t}{1+r_A}\tilde{s}_t + E_t(\tilde{b}_{t+1}), \tag{5.3}$$

$$\tilde{k}_{t+1} = \frac{\tilde{s}_t}{(1+n)(1+r_A)}, \tag{5.4}$$

$$\tilde{w}_t = (1-\beta)\tilde{y}_t, \tag{5.5}$$

$$r_t = \beta \frac{\tilde{y}_t}{\tilde{k}_t}, \tag{5.6}$$

$$\tilde{y}_t = e^{z_t} \tilde{k}_t^{\beta}, \tag{5.7}$$

and

$$z_t = \mu z_{t-1} + \varepsilon_t, \tag{3.10}$$

where the variables with the tilde are the corresponding variables in per worker terms divided by A_t, e.g., $\tilde{c}_{1t} = c_{1t} / A_t$, $\tilde{w}_t = w_t / A_t$, $\tilde{s}_t = s_t / A_t$, etc. Equation (5.1) is the Euler equation for consumption per effective labor, which corresponds to Equation (3.8). Equations (5.2) and (5.3) are the budget constraints of the first and of the second period, respectively, in per effective labor terms. Equations (5.4), (5.5), and (5.6) are the corresponding Equations (3.17), (3.12) and (3.13), respectively, in per effective labor terms. Also, Equation (5.7) gives product per effective labor. (For the derivation of Equations (5.1)-(5.7), see Appendix C.)

Then, I compute the steady-state value of the capital stock per effective labor (see Appendix D). In the steady state, in the absence of expectational and productivity shocks, the variables (expressed in per effective labor terms) are constant. Thus, setting expectational and productivity shocks equal to zero, and $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$, the steady-state value of the capital stock per effective labor, $\tilde{k}^*$, is given by

$$\tilde{k}^* = \left\{ \left[(1-\beta)\tilde{k}^{*\beta} - \tilde{d} \left[\frac{(1+\beta\tilde{k}^{*\beta-1})^{\frac{1-\gamma}{\gamma}}}{(1+\beta\tilde{k}^{*\beta-1})^{\frac{1-\gamma}{\gamma}} + (1+\rho)^{\frac{1}{\gamma}}} \right] - \tilde{b} \left[\frac{(1+r_A)(1+\rho)^{\frac{1}{\gamma}}}{(1+\beta\tilde{k}^{*\beta-1})^{\frac{1}{\gamma}} + (1+\beta\tilde{k}^{*\beta-1})(1+\rho)^{\frac{1}{\gamma}}} \right] \right] \right\} / [(1+n)(1+r_A)]. \tag{5.8}$$

Solving Equation (5.8) for $\tilde{k}^*$,[48] substituting the resulting equation into Equation (5.7), and eliminating the productivity shock z_t yields the steady-state value of product per effective labor, $\tilde{y}^* = \tilde{k}^{*\beta}$.

5.3. Implementation and results

5.3.1. *Introducing the PAYG social-security system*

In this section, I calibrate the model to match features of the U.S. economy. After computing the steady-state values of household saving per effective labor, capital stock per effective labor, real wage per effective labor, and the real interest rate, I examine the effects of the PAYG social-security system on these values. As I described in section 3.2.6, under the PAYG system the expected benefits per worker are given by $E_t(b_{t+1}) = p(\boldsymbol{x}_t)(1+n)d_{t+1}$ [see Equation (3.25)]. In addition, I assume that the PAYG system is self-financing, that is, it is financed by workers' contributions and not by the government. The only role of the government is to administer the PAYG system, e.g., it chooses the contribution rate, collects the contributions, and pays the benefits (Imrohoroglu, Imrohoroglu, and Joines, 1995, pp. 87-88).

I also consider two possible states of the world (see section 3.2.8). First, the PAYG system will be sustainable, i.e., it will grant pensions to the old at retirement using the workers' contributions. Second, the PAYG system will collapse,[49] i.e. the

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[48] Equation (5.8) is solved numerically for $\tilde{k}^*$ (see section 5.3), due to difficulty in obtaining an analytical solution.

[49] Cooley and Soares (1996) consider that the aging of the baby-boom generation (born in the late 1940s and the early 1950s) and the increase in the share of the population over the age of 65 would cause the PAYG system to collapse (that is, individuals would abandon the public PAYG system in favor of a private-pension system), assuming that the PAYG system is simply a tax and transfer system.

workers' contributions will not be sufficient to finance the pension benefits.[50] So, I consider the dummy variable $dum_{pg,t}$ [in the place of the probability $p(\boldsymbol{x}_t)$], which takes on the value of one if the PAYG system is sustainable and zero otherwise. Thus, the expected benefits per worker are given by

$$E_t(b_{t+1}) = dum_{pg,t}(1+n)d_{t+1}. \tag{5.9}$$

Substituting the definitions $b_{t+1} = \tilde{b}_{t+1}A_{t+1}$ and $d_{t+1} = \tilde{d}_{t+1}A_{t+1}$ into Equation (5.9); dividing both sides of the resulting equation by A_t; and using Equation (3.11), $A_{t+1} = (1+r_A)A_t$; gives the expected benefits per effective labor:

$$E_t(\tilde{b}_{t+1}) = dum_{pg,t}(1+n)\tilde{d}_{t+1}. \tag{5.10}$$

Substituting Equation (5.10) into (5.3) yields

$$E_t(\tilde{c}_{2t+1}) = \frac{1+r_t}{1+r_A}\tilde{s}_t + dum_{pg,t}(1+n)\tilde{d}_{t+1}. \tag{5.11}$$

Equation (5.11) is the second-period budged constraint in per effective labor terms under the PAYG system.

In order to obtain a numerical solution (compute the equilibrium) for the model [which consists of Equations (5.1)-(5.2), (5.4)-(5.7), (5.11), and (3.10)] under the PAYG system, I calibrate the model to match features of the U.S. economy. I use annual data from the U.S. for the period 1980-2010.[51] Note that the U.S. mainly employs a defined-benefit PAYG system financed by a contribution rate of 12.4

If, however, workers must honor their obligations to the current retirees, then the PAYG system would not collapse.

[50] In the case of collapse, I assume that individuals will replace the PAYG system with a private-pension system, e.g., a private fully-funded system (see section 5.3.3).

[51] As in the econometric analysis (see Chapter 4), most of the series are obtained from AMECO; the interest rate is obtained from the IFS; and the CPI inflation rate is obtained from the WDI. The variables are expressed in thousands of U.S. dollars, in constant 2005 prices. The definitions of the variables are given in Appendix B.

percent on labor income (Gern, 2002, p. 466). The parameter values are chosen as follows.

First, I assume that individuals are born and enter the workforce at the age of 21. A period in the model corresponds to 14 years (Cooley and Soares, 1999a, p. 150). The growth rate of employment, n, is set to be 0.011 per year (the average growth rate of employment in the U.S. for the period 1980-2010). Thus, since $(1+0.011)^{14} \approx 1.17$, it follows that the growth rate for the 14-year period is about 17 percent.

Second, the growth rate of technology, r_A, which is approximated by the average growth rate of real GDP per employee in the U.S. for the period 1980-2010, is set to 0.016 per year (Cooley and Prescott, 1995, p. 20). For the two-period model, 14 years per period, this value corresponds to the value of 0.25. Third, following Hubbard and Judd (1987), the share of capital in the production function, β, is set to be 0.3.

Fourth, in the literature, there is a wide range of empirical estimates of the coefficient of relative risk aversion, γ. For example, Hansen and Singleton's (1984) estimates range from -1.26 to 1.59. I set γ to 0.1, as a benchmark case. Fifth, following Hubbard and Judd (1987), the rate of time preference, ρ, is set to 0.015 per year, which corresponds to the value of 0.23 for the two-period model.

Sixth, in order to set values for the parameters μ and σ_ε in the process that generates the productivity shock, $z_t = \mu z_{t-1} + \varepsilon_t$ [see Equation (3.10)], I consider the properties of the Solow residuals, which are calculated as follows (Cooley and Prescott, 1995, pp. 21-22). Taking logs in Equation (5.7), $\tilde{y}_t = e^{z_t}\tilde{k}_t^{\beta}$; then taking first-differences in the resulting equation; and rearranging yields

$$z_t - z_{t-1} = \ln \tilde{y}_t - \ln \tilde{y}_{t-1} - \beta(\ln \tilde{k}_t - \ln \tilde{k}_{t-1})\,. \tag{5.12}$$

Using the value of $\beta = 0.3$ and the series $\tilde{y}_t$ and $\tilde{k}_t$,[52] I construct the series $z_t - z_{t-1}$, the Solow residuals. Setting $\Delta z_t = z_t - z_{t-1} = \omega_t$, where Δ is the difference operator, I recover the series z_t from the series ω_t using the transformation $z_t = \sum \omega_t = \omega_t + \omega_{t-1} + \omega_{t-2} + \ldots$, where $\sum$ is the summation operator, with initial value for z_t zero ($z_0 = 0$) (Box and Jenkins, 1976, p. 12). Assuming a first-order autoregressive process [AR(1)] for the constructed series z_t, and estimating it, the estimates of μ and σ_ε are 0.90 and 0.013, respectively (Hartley, Salyer, and Sheffrin, 1997, p. 8). The choice of the parameter values for the benchmark case of the model are summarized in Table 5.1.

In Equations (5.2) and (5.11), I set $\tilde{d}_t = 0.124\tilde{w}_t$, assuming that under the PAYG system the contribution rate is 12.4 percent of $\tilde{w}_t$ (see above). So, the variable $\tilde{d}_t$ is endogenously determined. As well, in Equation (5.11), I set $dum_{pg,t} = 1$, assuming that the PAYG system has not collapsed.

Then, to compute the equilibrium, I solve the certainty version of the model (ignoring E_t) using the Newton's iterative method for nonlinear equations. The computations are produced by the econometric program *EViews* 6. Hence, I obtain artificial time series for the variables $\tilde{y}_t$, $\tilde{k}_t$, $\tilde{s}_t$, $\tilde{c}_{1t}$, $\tilde{c}_{2t}$, $\tilde{w}_t$, and r_t, which are endogenously determined. Only the variable z_t is exogenously determined.

Table 5.1. Benchmark parameter values under the PAYG system

Parameter	n	r_A	β	γ	ρ	μ	σ_ε
Value	0.17	0.25	0.3	0.1	0.23	0.9	0.013

[52] The series $\tilde{y}_t$ and $\tilde{k}_t$ are calculated by using actual data and by using the definitions $\tilde{y}_t = y_t / A_t = y_t / A_0(1+r_A)^t$ and $\tilde{k}_t = k_t / A_t = k_t / A_0(1+r_A)^t$, respectively, where A_0 is set to be 1, r_A is set to be 0.016, y_t is real GDP per employee, and k_t is real net capital stock per employee.

I examine whether the model can replicate some features of the actual data reflected in the second-order sample moments, e.g., standard deviations and correlations. Thus, I first calculate these second-order sample moments of the actual data from the U.S. economy for the variables of interest, i.e., $\tilde{y}_t$, $\tilde{k}_t$, $\tilde{s}_t$, $\tilde{w}_t$, and r_t.[53] In particular, I take logarithms to the variables $\tilde{y}_t$, $\tilde{k}_t$, $\tilde{s}_t$, and $\tilde{w}_t$ and apply the Hodrick-Prescott (H-P) filter to the resulting variables and to r_t in order to represent the growth and the cyclical component of these variables (Cooley and Prescott, 1995, pp. 27-29).[54] I calculate the standard deviations of the cyclical components of these variables and the correlations of the cyclical components of these variables with the cyclical component of $\tilde{y}_t$. Then, I calculate the corresponding second-order sample moments of the same variables ($\tilde{y}_t$, $\tilde{k}_t$, $\tilde{s}_t$, $\tilde{w}_t$, and r_t) for the artificial data. Finally, I compare these second-order sample moments for the actual data with the corresponding ones for the artificial data. The results are reported in Table 5.2.

In the model, $\tilde{y}_t$, $\tilde{k}_t$, and $\tilde{w}_t$ fluctuate about as much as they do in the U.S. economy, while $\tilde{s}_t$ and r_t fluctuate less than they do in the U.S. economy. Also, in the model, all variables except for r_t fluctuate about as much as $\tilde{y}_t$, implying that these fluctuations are accounted for by the process that generates the productivity shock z_t, which is the only source of uncertainty. In the model, however, r_t fluctuates much less than it does in the U.S. economy, suggesting that the effect of z_t on r_t is canceled out [see Equation (5.6)].

[53] The series $\tilde{s}_t$ and $\tilde{w}_t$ are calculated in the same way as the series $\tilde{y}_t$ and $\tilde{k}_t$, that is, using the definitions $\tilde{s}_t = s_t / A_t = s_t / A_0(1+r_A)^t$ and $\tilde{w}_t = w_t / A_t = w_t / A_0(1+r_A)^t$, respectively, where s_t is real household saving per employee and w_t is real compensation per employee.

[54] In the H-P filter, the smoothing parameter λ, which reflects the relative variance of the growth component to the cyclical component, is set to be 100 for annual data. The higher the value of λ, the smoother will be the growth component (Hodrick and Prescott, 1997, pp. 3-6).

Table 5.2. Standard deviations and correlations – U.S. economy and model

	U.S. economy		Model	
Variable	Standard deviation	Correlation with $\tilde{y}_t$	Standard deviation	Correlation with $\tilde{y}_t$
$\tilde{y}_t$	0.010	1.000	0.022	1.000
$\tilde{k}_t$	0.010	0.263	0.024	0.855
$\tilde{s}_t$	0.092	-0.405	0.023	0.993
r_t	0.012	0.163	0.0004	0.148
$\tilde{w}_t$	0.013	0.765	0.022	1.000

Note: The standard deviations and correlations with $\tilde{y}_t$ are calculated for the period 1985-2010 (instead of the period 1980-2010) to avoid the influence of the starting values.

Regarding the correlations of $\tilde{k}_t$, $\tilde{s}_t$, r_t, and $\tilde{w}_t$ with $\tilde{y}_t$, these variables, with the exception of r_t are highly correlated with $\tilde{y}_t$ in the model. This result also suggests that there is only one exogenously determined shock (z_t) in the model economy, which affects these endogenously determined variables. In addition, in the model, all the variables are positively correlated with $\tilde{y}_t$, as they are in the U.S. economy, except for $\tilde{s}_t$, which is negatively correlated.

Generally, the results suggest that the model can replicate some features of the U.S. economy reflected in the reported standard deviations and correlations with $\tilde{y}_t$, but clearly not perfectly. There are also some features of the actual data that have not been captured by the model. A problem is that the variables of the model may not correspond exactly to the variables of the actual data from the U.S. economy (Cooley and Prescott, 1995, p. 35).

In what follows, I compute the general-equilibrium effects of the PAYG system on the steady-state values of household saving per effective labor and capital stock per effective labor and, by extension, on the steady-state values of the real wage per effective labor and the real interest rate. As well, holding constant the real wage per effective labor and the real interest rate, I compute the corresponding partial-equilibrium effects.

To start with, I compute the steady-state values $\tilde{k}^*$, $\tilde{s}^*$, $\tilde{w}^*$, and r^* under the PAYG system. First, I compute the steady-state value of the capital stock per effective labor, $\tilde{k}^*$, substituting into Equation (5.8) where $\tilde{d} = 0.124(1-\beta)\tilde{k}^{*\beta}$[55] and where $\tilde{b} = dum_{pg}(1+n)0.124(1-\beta)\tilde{k}^{*\beta}$,[56] where dum_{pg} is set equal to one, assuming that the PAYG system has not collapsed. Then, I compute the steady-state value of household saving per effective labor, $\tilde{s}^*$, by substituting the value of $\tilde{k}^*$ into Equation (5.4), $\tilde{s}_t = (1+n)(1+r_A)\tilde{k}_{t+1}$. Finally, I compute the steady-state values of the real wage per effective labor, $\tilde{w}^*$, and the real interest rate, r^*, by substituting Equation (5.7) into (5.5) and (5.6), respectively, setting z_t equal to zero, and substituting the value of $\tilde{k}^*$ into the resulting equations.

Then, I compute the (proportional) change in the steady-state values $\tilde{k}^*$, $\tilde{s}^*$, $\tilde{w}^*$, and r^* from a steady-state equilibrium with no PAYG system to a steady-state equilibrium with a PAYG system financed by a contribution rate of 12.4 percent. Note that the PAYG system causes a reduction in the steady-state value of household saving, and hence in the steady-state value of the capital stock [see Equation (5.8)], which in turn causes a reduction in the steady-state value of the real wage and an increase in the steady-state value of the real interest rate (see above). In addition, the

[55] This equation is obtained by setting $\tilde{d}_t = 0.124\tilde{w}_t$; substituting Equation (5.7), $\tilde{y}_t = e^{z_t}\tilde{k}_t^{\beta}$, into (5.5), $\tilde{w}_t = (1-\beta)\tilde{y}_t$; and the resulting equation into $\tilde{d}_t = 0.124\tilde{w}_t$; setting $z_t = 0$ and $\tilde{k}_t = \tilde{k}^*$.

[56] This equation is obtained by substituting into Equation (5.10) (ignoring E_t) where $\tilde{d} = 0.124(1-\beta)\tilde{k}^{*\beta}$.

reduction in the steady-state value of the real wage causes a further reduction in the steady-state value of household saving, while the increase in the steady-state value of the real interest rate has an ambiguous effect on the steady-state value of household saving (see Appendix E).

The general-equilibrium effect of the PAYG system on the steady-state values of household saving and capital stock can be either larger or smaller than the corresponding partial-equilibrium one. This is because, in the general-equilibrium model, the effects of the PAYG system on the steady-state values of the real wage and the real interest rate feed back into the model and affect the steady-state values of household saving and capital stock (Kotlikoff, 1979a, p. 241). The effect of the real wage on household saving is positive (see Appendix E), and thus the general-equilibrium effect of the PAYG system on the steady-state values of household saving and capital stock becomes more negative than the corresponding partial-equilibrium one. In contrast, since I assume that $\gamma < 1$ (see Table 5.1), the effect of the real interest rate on household saving is positive (see Appendix E), and thus the general-equilibrium effect of the PAYG system on the steady-state values of household saving and capital stock becomes less negative than the corresponding partial-equilibrium one. Therefore, whether the general-equilibrium effect is larger or smaller than the partial-equilibrium one depends on the relative size of these two effects. Table 5.3 reports the results.

Regarding the partial-equilibrium effects, the introduction of the PAYG system causes each of $\tilde{s}^*$ and $\tilde{k}^*$ to decrease by 26.5 percent. Turning to the general-equilibrium effects, the introduction of the PAYG system causes each of $\tilde{s}^*$ and $\tilde{k}^*$ to decrease by 14.9 percent; it causes $\tilde{w}^*$ to decrease by 4.7 percent; and it increases r^* by 0.4 percentage points. Also, the general-equilibrium effects of the PAYG system on $\tilde{s}^*$ and $\tilde{k}^*$ are smaller (in absolute value) than the corresponding partial-equilibrium ones.

The general-equilibrium effect of the PAYG system on the steady-state value of capital stock is similar to that found by Kotlikoff (1979a). Using a life-cycle model with certain longevity, a Cobb-Douglas production function, and various parameter values, he suggests that this effect ranges from 10 to 21 percent. This effect, however, is lower (in absolute value) than that found by Hubbard and Judd (1987), who used a life-cycle model with uncertain longevity and liquidity constraints.

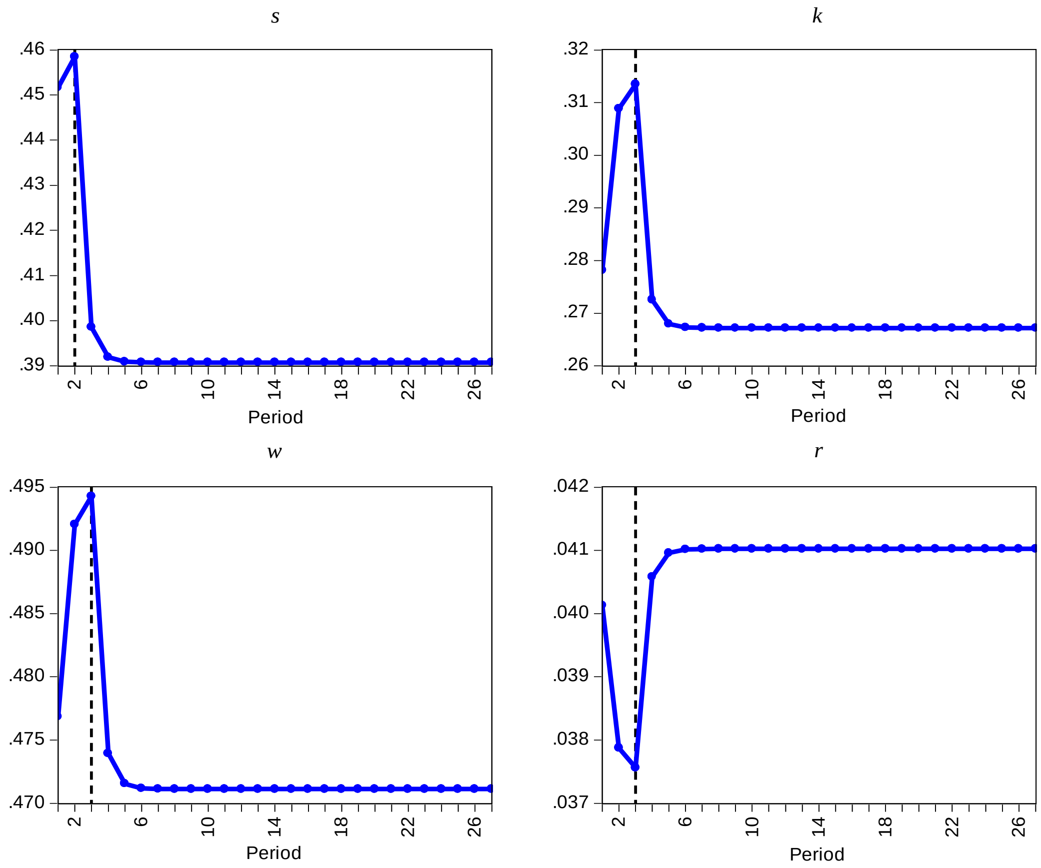
Figure 5.1 shows the transition path of $\tilde{s}^*$, $\tilde{k}^*$, $\tilde{w}^*$, and r^* from a steady-state equilibrium with no PAYG system to a steady-state equilibrium with a PAYG system. The steady-state values of household saving per effective labor, capital stock per effective labor, and real wage per effective labor decrease with the introduction of the PAYG system, while the steady-state value of the real interest rate increases.

Table 5.3. Partial- and general-equilibrium effects of the PAYG system

Variable	Partial-equilibrium effect	General-equilibrium effect	General-equilibrium effect ($dum_{pg,t}$ = 0)
$\tilde{s}^*$	-0.265	-0.149	0.176
$\tilde{k}^*$	-0.265	-0.149	0.176
$\tilde{w}^*$	_	-0.047	0.050
r^*	_	0.004	-0.004

Note: In order to compute the partial-equilibrium effects, I set the real wage per effective labor equal to 49.054 and the real interest rate equal to 0.017, which are the average real wage per effective labor and the average real interest rate, respectively, computed using annual data from the U.S. for the period 1980-2010.

Figure 5.1. Transition path for the introduction of the PAYG system



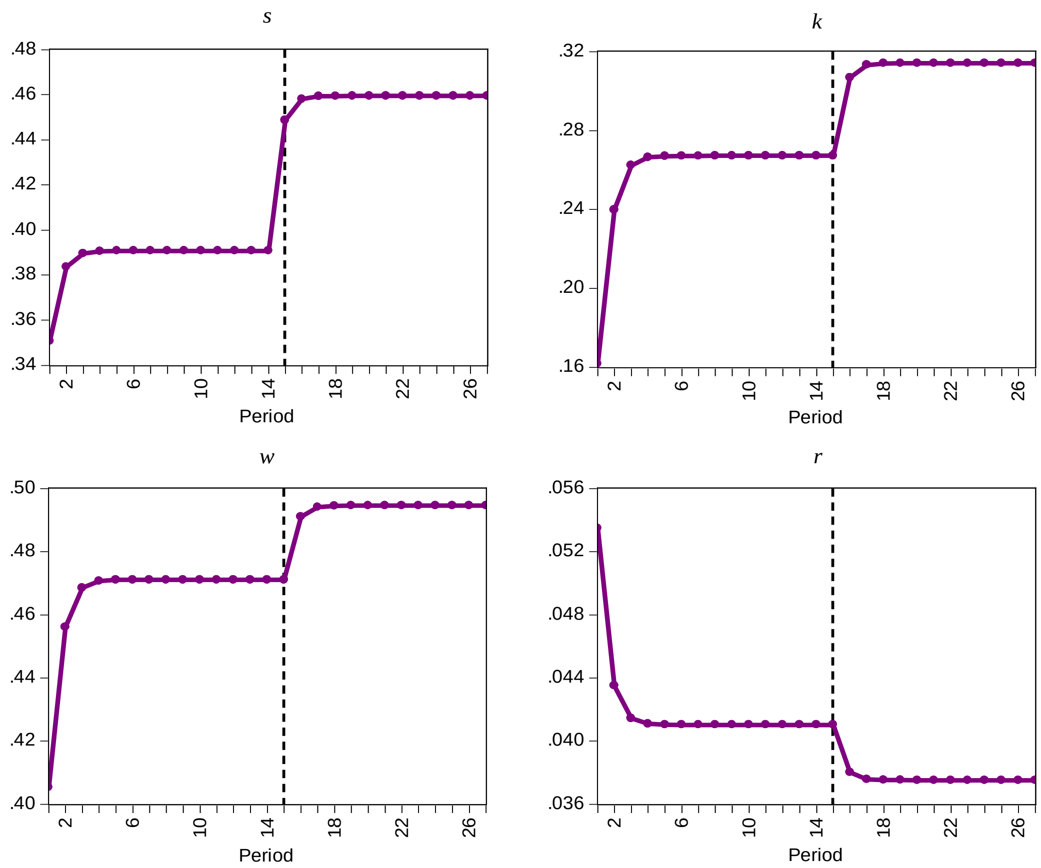
Demographic changes, such as increasing life expectancy and declining fertility, may cause uncertainty about the sustainability of the PAYG system. An increasing number of retirees combined with a decreasing number of workers may reduce the PAYG system's capacity to collect enough contributions for the provision of future benefits. As well, the 1983 pension reform in the U.S. has not been sufficient to stabilize the PAYG system and there is a significant deficit projection for the next seventy five years. Some researchers expect that from the year 2021 onward, social-security benefits will exceed contributions and the system will not be sustained without financing from the government budget (Gern, 2002, p. 466, Velloso, 2006, pp. 19-22). So, considering that the PAYG system may not be sustainable, I examine the effects of a collapse of the PAYG system on $\tilde{s}^*$, $\tilde{k}^*$, $\tilde{w}^*$, and r^*, by setting $dum_{pg,t} = 0$ in any period t (e.g., t = 15).

Note that if the PAYG system collapses, the steady-state value of household saving per effective labor, and thus the steady-state value of the capital stock per effective labor increases [see Equation (5.8)], since individuals try to secure a certain level of income for retirement. Consequently, the increase in the steady-state value of the capital stock per effective labor causes an increase in the steady-state value of the real wage per effective labor and a decrease in the steady-state value of the real interest rate. The results are reported in the last column of Table 5.3.

The collapse of the PAYG system causes each of $\tilde{s}^*$ and $\tilde{k}^*$ to increase by 17.6 percent; it causes $\tilde{w}^*$ to increase by 5 percent; and it reduces r^* by 0.4 percentage points. The increase in $\tilde{s}^*$ and $\tilde{k}^*$ caused by the collapse of the PAYG system is higher than the decrease in these values caused by the introduction of the PAYG system. Cooley and Soares (1996) suggest that the steady-state value of the capital-to-output ratio increases, while the steady-state value of the real interest rate decreases as the economy moves to a steady-state equilibrium without a PAYG system (assuming that the prolonged increase in the size of the older population would cause the PAYG system to collapse, i.e., individuals abandon the PAYG system in favor of a private-pension system).

Figure 5.2 shows the transition path of $\tilde{s}^*$, $\tilde{k}^*$, $\tilde{w}^*$, and r^* from a steady-state equilibrium with the PAYG system to a steady-state equilibrium without the PAYG system. The steady-state values of household saving per effective labor, capital stock per effective labor, and real wage per effective labor increase with the collapse of the PAYG system, while the steady-state value of the real interest rate decreases.

Figure 5.2. Transition path for a collapse of the PAYG system



Next, I examine the sensitivity of the general-equilibrium effects of the PAYG system on $\tilde{s}^*$, $\tilde{k}^*$, $\tilde{w}^*$, and r^*, for different values of the parameters γ, ρ, β, n, and r_A. First, as was noted earlier, the PAYG system causes a reduction in household saving and the capital stock, which consequently causes a reduction in the real wage and an increase in the real interest rate. (With the exception of the real interest rate, all the other variables are expressed in per effective labor terms.) The reduction in the real wage causes a further reduction in household saving, while the increase in the real interest rate has an ambiguous effect on household saving (see Appendix E). Recall that a higher value of γ implies a lower elasticity of intertemporal substitution, $\sigma = 1/\gamma$, and thus a lower interest sensitivity of saving. Hence, the higher the value of γ the lower the effect of the real interest rate on household saving [see Equation (E.2) in

Appendix E] and the greater the reduction in household saving (and thus in the capital stock) caused by the introduction of the PAYG system.

Second, the lower the value of the rate of time preference, ρ, the less impatient the individual to consume in the present than in the future. Thus, household saving (and capital stock) increases. Consequently, the real wage increases, while the real interest rate decreases. Furthermore, as was noted earlier, the increase in the real wage causes an increase in household saving, while the reduction in the real interest rate causes a reduction in household saving, since I assume that $\gamma < 1$. If the effect of the real wage on household saving is greater (lower) than the effect of the real interest rate, the reduction in household saving (and in the capital stock) caused by the introduction of the PAYG system is lower (greater) when the value of ρ decreases.

Third, the higher the value of the share of capital, β, the higher the real interest rate [see Equation (5.6)] and the lower the real wage [see Equation (5.5)]. In addition, as before, the increase in the real interest rate causes an increase in household saving, while the reduction in the real wage causes a reduction in household saving. If the effect of the real interest rate on household saving is greater (lower) than the effect of the real wage, the reduction in household saving (and in the capital stock) caused by the introduction of the PAYG system is lower (greater) when the value of β increases.

Table 5.4. Sensitivity analysis under the PAYG system

Variable	Benchmark	$\gamma = 0.75$	$\gamma = 1.25$	$\rho = 0.15$	$\beta = 0.35$	$n = 0.32$	$r_A = 0.32$
$\tilde{s}^*$	-0.149	-0.244	-0.310	-0.158	-0.169	-0.155	-0.152
$\tilde{k}^*$	-0.149	-0.244	-0.310	-0.158	-0.169	-0.155	-0.152
$\tilde{w}^*$	-0.047	-0.081	-0.105	-0.050	-0.063	-0.049	-0.048
r^*	0.004	0.009	0.013	0.004	0.004	0.004	0.004

Fourth, the higher the value of the growth rate of employment, n, the higher the social-security benefits under the PAYG system, and thus the higher the first- and the second-period consumption [see the intertemporal budget constraint, Equation (D.2) in Appendix D] and the lower household saving (and the capital stock). Hence, the higher is the real interest rate and the lower is the real wage. In addition, as was noted earlier, the increase in the real interest rate causes an increase in household saving, while the reduction in the real wage causes a reduction in household saving (see Appendix E). If the effect of the real interest rate on household saving is greater (lower) than the effect of the real wage, the reduction in household saving (and in the capital stock) caused by the introduction of the PAYG system is lower (greater) when the value of n increases.

Fifth, the higher the value of the growth rate of technology, r_A, the lower the capital stock [see Equation (5.4)]. Thus, the lower is the real wage [see Equation (5.5)] and the higher is the real interest rate [see Equation (5.6)]. Also, as before, the reduction in the real wage causes a reduction in household saving, while the increase in the real interest rate causes an increase in household saving. If the effect of the real wage on household saving is greater (lower) than the effect of the real interest rate, the reduction in household saving (and in the capital stock) caused by the introduction of the PAYG system is greater (lower) when the value of r_A increases. The results are reported in Table 5.4.

For a value of γ of 0.75 the introduction of the PAYG system causes $\tilde{s}^*$, and thus $\tilde{k}^*$ to decrease by 24.4 percent, while for a value of γ of 1.25 it causes $\tilde{s}^*$, and thus $\tilde{k}^*$ to decrease by 31 percent. So, the higher the value of γ the greater the reduction in $\tilde{s}^*$, and thus in $\tilde{k}^*$ caused by the introduction of the PAYG system. Also, the lower the value of ρ (0.15 instead of 0.23) the greater the reduction in $\tilde{s}^*$, and thus in $\tilde{k}^*$ caused by the introduction of the PAYG system, implying that the effect of the real interest rate on household saving per effective labor is greater than the effect of the real wage per effective labor. The higher the values of β (0.35 instead of 0.30), n (0.32 instead of 0.17), and r_A (0.32 instead of 0.25) the greater the reduction in $\tilde{s}^*$, and thus in $\tilde{k}^*$ caused by the introduction of the PAYG system, implying that the effect of the real wage per effective labor on household saving per effective labor is greater than the effect of the real interest rate. Overall, the results for the general-equilibrium effects of the PAYG system on the steady-state values of household saving per effective labor, capital stock per effective labor, real wage per effective labor, and the real interest rate seem to be robust to changes in the parameter values.

5.3.2. *Introducing the fully-funded social-security system*

In this section, I calibrate the model to match features of the Mexican economy. The choice of this country depends on the fact that it employs a fully-funded social-security system and on the availability of the data. After computing the state-state values of household saving per effective labor, capital stock per effective labor, real wage per effective labor, and the real interest rate, I examine the effects of the fully-funded system on these values. As was noted in section 3.2.7, in the fully-funded system the expected social-security benefits per worker are given by $E_t(b_{t+1}) = p(\boldsymbol{x}_t)(1+r_t)d_t$ [see Equation (3.31)]. Also, as in the case of the PAYG system (see section 5.3.1), I assume that the fully-funded system is financed only by workers' contributions and not by the government. The contributions are accumulated in pension funds, are invested, and are returned with interest upon retirement.

Moreover, as in the case of the PAYG system (see section 5.3.1), I assume two possible states of the world. First, the fully-funded system will be sustainable, i.e., it will grant pensions using the accumulated pension funds. Second, the fully-funded system will collapse, i.e., the accumulated pension funds will not be sufficient to finance the pension benefits. Define the dummy variable $dum_{ff,t}$ [in the place of the probability $p(\mathbf{x}_t)$], which takes on the value of one if the fully-funded system is sustainable and zero otherwise. Therefore, the expected benefits per worker are given by

$$E_t(b_{t+1}) = dum_{ff,t}(1+r_t)d_t\,. \tag{5.13}$$

Substituting the definitions $b_{t+1} = \tilde{b}_{t+1}A_{t+1}$ and $d_t = \tilde{d}_t A_t$ into Equation (5.13); dividing both sides of the resulting equation by A_t; and using Equation (3.11), $A_{t+1} = (1+r_A)A_t$; yields the expected benefits per effective labor under the fully-funded system:

$$E_t(\tilde{b}_{t+1}) = dum_{ff,t}\frac{1+r_t}{1+r_A}\tilde{d}_t\,. \tag{5.14}$$

Substituting Equation (5.14) into (5.3) and rearranging yields

$$E_t(\tilde{c}_{2t+1}) = \frac{1+r_t}{1+r_A}(\tilde{s}_t + dum_{ff,t}\tilde{d}_t)\,. \tag{5.15}$$

Equation (5.15) is the second-period budged constraint in per effective labor terms under the fully-funded system. As well, solving Equation (5.2) for $\tilde{s}_t$, that is, $\tilde{s}_t = \tilde{w}_t - \tilde{c}_{1t} - \tilde{d}_t$; substituting this equation into (5.15); and rearranging yields the intertemporal budget constraint under the fully-funded system:

$$\tilde{c}_{1t} + E_t(\tilde{c}_{2t+1})\frac{1+r_A}{1+r_t} = \tilde{w}_t + (dum_{ff,t} - 1)\tilde{d}_t\,. \tag{5.16}$$

According to Equation (5.16), if $dum_{ff,t}$ = 1 (sustainable fully-funded system), the introduction of the fully-funded system will not affect lifetime resources and lifetime consumption per effective labor (the second term in the right-hand side becomes zero). Social-security contributions per effective labor reduce household saving per effective labor by an equal amount, since under the fully-funded system the yield on social-security contributions equals the market interest rate. The young individuals are indifferent with respect to who does the saving, the fully-funded system or themselves (Blanchard and Fisher, 1989, p. 111).

As well, under the fully-funded system, the capital stock per effective labor comprises household saving per effective labor and social-security contributions per effective labor, and thus Equation (5.4) is modified as follows:

$$\tilde{k}_{t+1} = \frac{\tilde{s}_t + \tilde{d}_t}{(1+n)(1+r_A)}. \tag{5.17}$$

Given that under the fully-funded system $K_{t+1} = L_t s_t + L_t d_t$; dividing this equation by $A_{t+1}L_{t+1}$; using Equation (3.1), $L_t / L_{t+1} = 1/(1+n)$, and Equation (3.11), $A_{t+1} = (1+r_A)A_t$; and then using the definitions $\tilde{k}_{t+1} = k_{t+1} / A_{t+1} = (K_{t+1} / L_{t+1}) / A_{t+1}$, $\tilde{s}_t = s_t / A_t$, and $\tilde{d}_t = d_t / A_t$; yields Equation (5.17). Note that since under the fully-funded system social-security contributions per effective labor reduce household saving per effective labor one-for-one, the introduction of the fully-funded system has no effect on the capital stock per effective labor.

In order to obtain a numerical solution for the model [which consists of Equations (5.1)-(5.2), (5.5)-(5.7), (5.15), (5.17), and (3.10)] under the fully-funded system, I calibrate the model to match features of the Mexican economy. I use annual data from Mexico for the period 1993-2009.[57] In Mexico, demographic trends (population growing slowly and aging rapidly) cause an increase in the ratio of the elderly in the population, and so the insufficiency of contributions to finance the benefits has rendered the PAYG system financially unsustainable (Sales-Sarrapy, Solis-Soberon, and Villagomez-Amezcua, 1998, p. 142). Thus, Mexico reformed its pension system in 1995 (the reform was implemented in 1997), replacing the former public PAYG system with a mandatory fully-funded one, where contributions are accumulated in pension funds managed by private companies. Under the fully-funded system, the contribution rate is 6.5 percent of labor income (Gern, 2002, p. 472, Tapia, 2008, p. 30). The parameter values are chosen as follows.

First, as in section 5.3.1, I assume that individuals are born as workers at the age of 21 and a period in the model corresponds to 14 years. I set the growth rate of employment, n, to 0.02 per year (the average growth rate of employment in Mexico for the period 1993-2009), which corresponds to a value of 0.32 for the two-period model.

Second, I set the growth rate of technology, $r_{A,}$ to 0.0035 per year (approximated by the average growth rate of real GDP per employee in Mexico for the period 1993-2009), which corresponds to a value of 0.05 for the two-period model. Third, the share of capital, β, is set to 0.35. In a cross-country analysis of the determinants of growth in Latin American countries (including Mexico), Solimano and Soto (2005) use the same value for β.[58]

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[57] With the exception of the data for the interest rate, which have been obtained from the IFS, and those for the percentage change in CPI, which have been obtained from the WDI, all the other data have been obtained from AMECO. The variables are expressed in thousands of pesos, in constant 2005 prices (for the variables definitions see Appendix B).

[58] In a cross-country analysis, Gollin (2002) suggests that factor shares are approximately constant across time and across countries. For most countries, the calculations of the labor share range from 0.65 to 0.80.

Table 5.5. Benchmark parameter values under the fully-funded system

Parameter	n	r_A	β	γ	ρ	μ	σ_ε
Value	0.32	0.05	0.35	0.1	0.23	0.74	0.029

Fourth, as in section 5.3.1, I set the coefficient of relative risk aversion, γ, to 0.1. Fifth, I set the rate of time preference, ρ, to 0.015 per year (as in section 5.3.1), which corresponds to the value of 0.23 for the two-period model.[59]

Sixth, in order to set values for the parameters μ and σ_ε in the process that generates the productivity shock, $z_t = \mu z_{t-1} + \varepsilon_t$ [see Equation (3.10)], I consider the properties of the Solow residuals, which are calculated as described in section 5.3.1 [see Equation (5.12)]. As before, assuming an AR(1) process for the constructed series z_t, and estimating it, the estimate of μ is 0.74 and that of σ_ε is 0.029. The parameter values for the benchmark case of the model are summarized in Table 5.5.

In Equations (5.2), (5.15), and (5.17), I set $\tilde{d}_t = 0.065\tilde{w}_t$, assuming that under the fully-funded system the contribution rate is 6.5 percent of $\tilde{w}_t$ (see above). Thus, $\tilde{d}_t$ is endogenously determined, as in section 5.3.1. Also, in Equation (5.15), I set $dum_{ff,t} = 1$, assuming that the fully-funded system has not collapsed.

Next, in order to compute the equilibrium, I solve the certainty version of the model (ignoring E_t) using Newton's iterative method. Thus, I obtain artificial time series for the endogenously determined variables $\tilde{y}_t$, $\tilde{k}_t$, $\tilde{s}_t$, $\tilde{c}_{1t}$, $\tilde{c}_{2t}$, $\tilde{w}_t$, and r_t.

I examine whether the model can replicate the second-order sample moments (standard deviations and correlations) of the actual data from the Mexican economy. Hence, I compute the second-order sample moments of the variables of interest, i.e.,

[59] Calculating the social discount rate (defined as the rate at which a society is willing to trade present for future consumption) for nine Latin America countries (including Mexico), Lopez (2008) uses the value 0.01 for the rate of time preference.

$\tilde{y}_t$, $\tilde{k}_t$, $\tilde{s}_t$, $\tilde{w}_t$, and r_t for the actual data as well as for the artificial data. In particular, in the same way as in section 5.3.1, I compute the standard deviations of the cyclical components of these variables and their correlations with the cyclical component of $\tilde{y}_t$ (the variables are transformed in logarithms except for r_t). Then, I compare these second-order sample moments of the actual data with the corresponding ones of the artificial data. The results are reported in Table 5.6.

In the model, $\tilde{y}_t$ fluctuates about as much as it does in the Mexican economy, while $\tilde{k}_t$ fluctuates more. Moreover, $\tilde{s}_t$, r_t, and $\tilde{w}_t$ in the model fluctuate less than they do in the Mexican economy. In the model, all the variables fluctuate about as much as $\tilde{y}_t$ except for r_t, which fluctuates much less than does $\tilde{y}_t$. This result implies that these fluctuations are caused by the productivity shock, z_t, which is the only (exogenously determined) shock in the model economy (as in section 5.3.1).

Table 5.6. Standard deviations and correlations – Mexican economy and model

	Mexican economy		Model	
Variable	Standard deviation	Correlation with $\tilde{y}_t$	Standard deviation	Correlation with $\tilde{y}_t$
$\tilde{y}_t$	0.015	1.000	0.014	1.000
$\tilde{k}_t$	0.007	0.231	0.016	0.454
$\tilde{s}_t$	0.116	0.005	0.016	0.987
r_t	0.024	0.476	0.0005	0.438
$\tilde{w}_t$	0.038	0.622	0.014	1.000

Note: The standard deviations and correlations with $\tilde{y}_t$ are calculated for the period 1997-2009 (instead of the period 1993-2009) to avoid the influence of the starting values.

Turning to the correlations of $\tilde{k}_t$, $\tilde{s}_t$, r_t, and $\tilde{w}_t$ with $\tilde{y}_t$ in the model, these variables are positively correlated with $\tilde{y}_t$, as they are in the Mexican economy. In the model, however, $\tilde{s}_t$ is highly correlated with $\tilde{y}_t$, while in the Mexican economy $\tilde{s}_t$ is almost uncorrelated with $\tilde{y}_t$. Overall, the results suggest that the model can replicate some features of the Mexican economy, but some other features have not been captured by the model.

Then, I compute the general-equilibrium effects of the fully-funded system on the steady-state values of household saving per effective labor and the capital stock per effective labor, and consequently on the steady-state values of the real wage per effective labor and the real interest rate. Holding constant the real wage per effective labor and the real interest rate, I compute the corresponding partial-equilibrium effects.

To begin with, I compute the steady-state values $\tilde{k}^*$, $\tilde{s}^*$, $\tilde{w}^*$, and r^* under the fully-funded system. I first compute the steady-state value of the capital stock per effective labor, $\tilde{k}^*$ (see Appendix F), which is given by

$$\tilde{k}^* = \left[(1-\beta)\tilde{k}^{*\beta} \frac{(1+\beta\tilde{k}^{*\beta-1})^{\frac{1-\gamma}{\gamma}}}{(1+\beta\tilde{k}^{*\beta-1})^{\frac{1-\gamma}{\gamma}} + (1+\rho)^{\frac{1}{\gamma}}} \right] / \left[(1+n)(1+r_A)\right]. \tag{5.18}$$

Then, I compute the steady-state value of household saving per effective labor, $\tilde{s}^*$, substituting where $\tilde{d}_t = 0.065(1-\beta)\tilde{k}_t^{\beta}$ [60] into Equation (5.17), $\tilde{s}_t = (1+n)(1+r_A)\tilde{k}_{t+1} - \tilde{d}_t$, and substituting the value of $\tilde{k}^*$ into the resulting equation. Finally, in the same way as in section 5.3.1, I compute $\tilde{w}^*$ and r^*.

[60] This equation is obtained by setting $\tilde{d}_t = 0.065\tilde{w}_t$; substituting Equation (5.7), $\tilde{y}_t = e^{z_t}\tilde{k}_t^{\beta}$, into (5.5), $\tilde{w}_t = (1-\beta)\tilde{y}_t$; and the resulting equation into $\tilde{d}_t = 0.065\tilde{w}_t$; and setting $z_t = 0$.

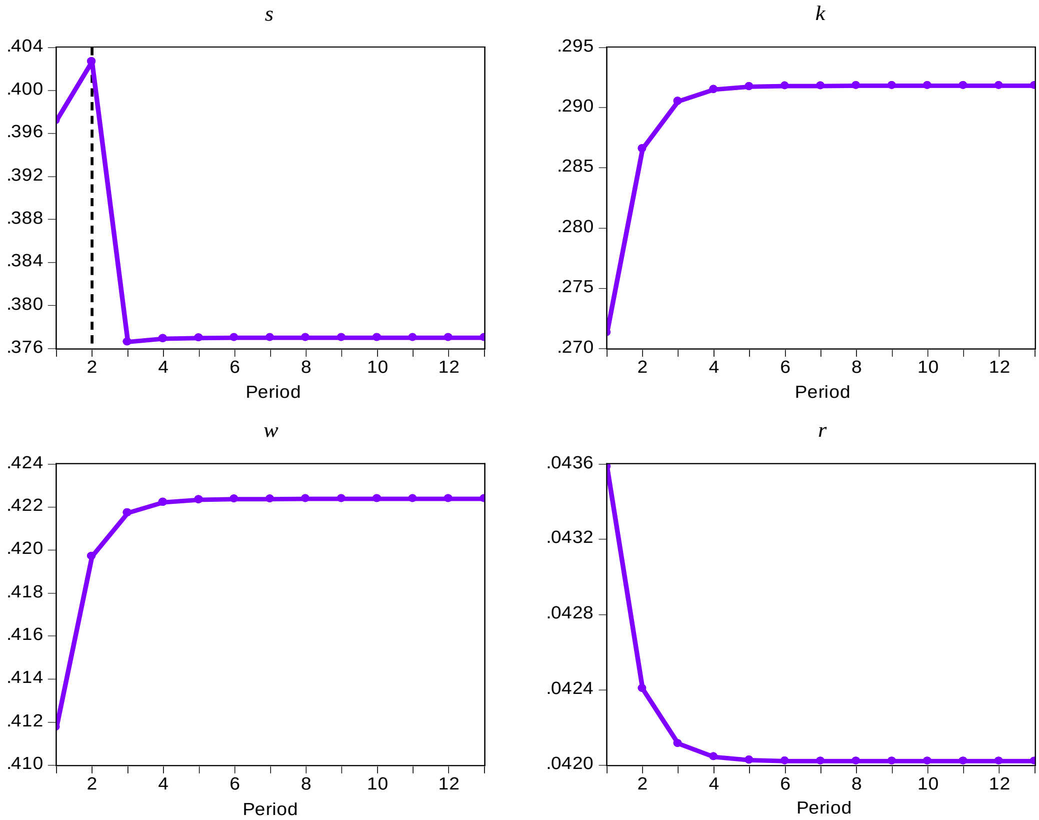
Note that under the fully-funded system, social-security contributions per effective labor reduce the steady-state value of household saving per effective labor by an equal amount. Because the yield on social-security contributions equals the market interest rate (see above). Thus, the fully-funded system has no effect on the steady-state value of the capital stock per effective labor [see Equation (5.18)], and consequently on the steady-state values of the real wage per effective labor and the real interest rate.

Next, I calculate the (percentage) change in $\tilde{s}^*$ from a steady-state equilibrium with no fully-funded system to a steady-state equilibrium with a fully-funded system financed by a contribution rate of 6.5 percent. Regarding the partial-equilibrium effect, the introduction of the fully-funded system causes $\tilde{s}^*$ to fall by 6.9 percent.[61] Also, regarding the general-equilibrium effect, the introduction of the fully-funded system causes $\tilde{s}^*$ to fall by 6.8 percent, while it has no effect on $\tilde{k}^*$, $\tilde{w}^*$, and r^*. These results are in accordance with the traditional life-cycle ones, according to which under an actuarially fair fully-funded system social-security contributions reduce household saving with offset one-for-one (Feldstein and Pellechio, 1979, Kotlikoff, 1979b).

Figure 5.3 shows the transition path of $\tilde{s}^*$, $\tilde{k}^*$, $\tilde{w}^*$, and r^* from a steady-state equilibrium with no fully-funded system to a steady-state equilibrium with a fully-funded system. The introduction of the fully-funded system reduces the steady-state value of household saving per effective labor, while it has no effect on the steady-state values of the capital stock per effective labor, real wage per effective labor, and the real interest rate.

[61] In order to compute the partial-equilibrium effect, I set the real wage per effective labor equal to 165.449 and the real interest rate equal to 0.038, which are the average real wage per effective labor and the average real interest rate, respectively, computed using annual data from Mexico for the period 1993-2009.

Figure 5.3. Transition path for the introduction of the fully-funded system



Next, I examine the sensitivity of the general-equilibrium effect of the fully-funded system on $\tilde{s}^*$, for different values of the parameters γ, ρ, β, n, and r_A. Note that for different values of the parameters from those of the benchmark case the steady-state value of the capital stock per effective labor differs from its corresponding value for the benchmark case, and thus the steady-state value of social-security contributions per effective labor also differs from their corresponding value for the benchmark case, assuming that social-security contributions per effective labor are endogenously determined [$\tilde{d}_t = 0.065(1-\beta)\tilde{k}_t^{\beta}$, see below Equation (5.18)]. Hence, the steady-state value of household saving per effective labor changes for different parameter values. The results are reported in Table 5.7. The interpretation of the results is similar to that provided in section 5.3.1 (see above and below Table 5.4).

Table 5.7. Sensitivity analysis under the fully-funded system

Variable	Benchmark	$\gamma = 0.75$	$\gamma = 1.25$	$\rho = 0.15$	$\beta = 0.40$	$n = 0.23$	$r_A = 0.32$
$\tilde{s}^*$	-0.068	-0.128	-0.159	-0.067	-0.066	-0.069	-0.066

The results suggest that the higher the value of γ (0.75 and 1.25 instead of 0.1) the greater the reduction in $\tilde{s}^*$ caused by the introduction of the fully-funded system. Using different values for ρ (0.15 instead of 0.23), β (0.40 instead of 0.35), n (0.23 instead of 0.32), and r_A (0.32 instead of 0.05), however, the effect of the fully-funded system on $\tilde{s}^*$ does not differ much from that in the benchmark case, implying that the effect of the real interest rate and the real wage per effective labor on household saving per effective labor almost cancel each other out. Overall, the results for the general-equilibrium effect of the fully-funded system on the steady-state value of household saving per effective labor seem to be robust to changes in the parameter values.

5.3.3. *Transition from the PAYG system to the fully-funded system*

In the U.S., as in many other countries, demographic changes (decreased fertility and increased longevity) have caused an increase in the ratio of retirees to workers in the population. As a result, social-security contributions paid by workers grow more slowly than social-security benefits received by retirees. This situation creates financial stress to the U.S. PAYG system, which may face sustainability problems. To address these problems many proposals for reform have been made, one of which considers the transition from the public PAYG system to a private fully-funded one (Feldstein and Samwick, 1998, pp. 215-16, Cooley and Soares, 1999b, p. 732).

Many authors argue that the transition from the public PAYG system to a private fully-funded one is too costly to be politically feasible. There are, however, several alternative ways to finance the transition cost. Huang, Imrohoroglu, and Sargent (1997) consider two such alternative ways. First, the government suddenly terminates the PAYG system and compensates all individuals, who have been expecting to receive benefits under the PAYG system, by a one-time increase in government debt. Second, the government acquires claims on private physical capital and uses the returns from this publicly held private capital to pay social-security benefits. Feldstein and Samwick (1998) suggest that during the transition to the fully-funded system, current workers would have to pay contributions to finance the benefits of current retirees along with contributions to accumulate a fund to finance their own benefits. Also, Cooley and Soares (1999b) examine several transition policies, such as the imposition of taxes on labor income, on consumption, and the issuance of debt, and suggest that in order for the transition to be politically feasible, it would have to rely on the use of debt to finance the cost.

In this section, I examine the effects of a transition from the PAYG system to a fully-funded system on the steady-state values of household saving per effective labor, capital stock per effective labor, real wage per effective labor, and the real interest rate. I consider that the transition to the fully-funded system is financed by the government budget. To begin with, I express the government's period-by-period budget constraint [Equation (3.39), see section 3.2.8] in per effective labor terms by dividing both sides of Equation (3.39) by A_t and using Equation (3.11), $A_{t+1} = (1 + r_A)A_t$, as follows:

$$\tilde{\tau}_t + \tilde{d}_t + (1+n)(1+r_A)\tilde{q}_{t+1} = \tilde{b}_t + (1+r_t)\tilde{q}_t + \tilde{g}_t\,, \tag{5.19}$$

where $\tilde{\tau}_t = \tau_t / A_t$, $\tilde{q}_t = q_t / A_t$, and $\tilde{g}_t = g_t / A_t$.

Considering that the PAYG system is not sustainable, the government eliminates the PAYG system and compensates individuals, who were expecting to receive benefits from the PAYG system during the transition to the fully-funded system, by issuing debt. Thus, the expected benefits per effective labor are given by

$$E_t(\tilde{b}_{t+1}) = dum_{pg,t}(1+n)\tilde{d}_{t+1} + dum_{ff,t}\frac{1+r_t}{1+r_A}\tilde{d}_t + dum_{bc,t}[\tilde{\tau}_t + \tilde{d}_t + (1+n)(1+r_A)\tilde{q}_{t+1} - (1+r_t)\tilde{q}_t - \tilde{g}_t]. \tag{5.20}$$

Regarding Equation (5.20), the first term on the right-hand side represents the benefits under the PAYG system [see Equation (5.10)], the second term represents the benefits under the fully-funded system [see Equation (5.14)], and the third term represents the financing of the transition to the fully-funded system from the government budget. If the PAYG system is sustainable, $dum_{pg,t}$ takes on the value of one, while $dum_{ff,t}$ and $dum_{bc,t}$ take on the value of zero. If the PAYG system is not sustainable and is to be replaced by the fully-funded system, $dum_{pg,t}$ and $dum_{ff,t}$ take on the value of zero and of one, respectively, while $dum_{bc,t}$ takes on the value of one during the transition period.

Moreover, Equation (5.17) is modified as follows:

$$\tilde{k}_{t+1} = \frac{\tilde{s}_t + dum_{ff,t}\tilde{d}_t}{(1+n)(1+r_A)}\,, \tag{5.21}$$

where $dum_{ff,t}$ takes on the value of one, if there is a transition to the fully-funded system and zero otherwise.

Next, I compute the steady-state values $\tilde{k}^*$, $\tilde{s}^*$, $\tilde{w}^*$, and r^* considering a transition from the PAYG system to a fully-funded system (in any period *t*). First, in order to compute the steady-state value of the capital stock per effective labor, $\tilde{k}^*$, I substitute Equation (D.4) (see Appendix D) into Equation (5.21) and into the resulting equation I substitute Equation (5.20) (ignoring E_t),[62] where $\tilde{d} = 0.124(1-\beta)\tilde{k}^{*\beta}$,[63] $\tilde{w} = (1-\beta)\tilde{k}^{*\beta}$, and $r = \beta\tilde{k}^{*\beta-1}$.[64] Then, I compute the steady-state value of household saving per effective labor, $\tilde{s}^*$, by substituting $\tilde{d}_t = 0.124(1-\beta)\tilde{k}_t^{\beta}$ into Equation (5.21), $\tilde{s}_t = (1+n)(1+r_A)\tilde{k}_{t+1} - dum_{ff,t}\tilde{d}_t$, and the value of $\tilde{k}^*$ into the resulting equation. Finally, in the same way as in section 5.3.1, I compute $\tilde{w}^*$ and r^*.

As was pointed out in section 5.3.1, the PAYG system reduces the steady-state values of household saving, capital stock, and real wage, while it increases the steady-state value of the real interest rate. Also, the fully-funded system reduces the steady-state value of household saving, while it has no effect on the steady-state values of capital stock, real wage, and the real interest rate (see section 5.3.2). Thus, the transition from the PAYG system to the fully-funded system would have the same effects on the steady-state values of capital stock, real wage, and the real interest rate as would the elimination of the PAYG system without replacing it (see section 5.3.1).[65]

The results suggest that the transition from the PAYG system to the fully-funded system causes $\tilde{k}^*$ to increase by 17.6 percent; it causes $\tilde{w}^*$ to increase by 5

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[62] I assume that under the PAYG system as well as under the fully-funded system the contribution rate is 12.4 percent of $\tilde{w}_t$. Feldstein and Samwick (1998) suggest that shifting from the PAYG system to a fully-funded system the contribution rate of 12.4 percent would be replaced in the long run by a contribution rate of about 2 percent because the rate of return under the fully-funded system is higher than that under the PAYG system. Also, I set $\tilde{\tau}_t$, $\tilde{g}_t$, and $\tilde{q}_t$ equal to their sample averages for the period 1980-2010.

[63] This equation is obtained by setting $\tilde{d}_t = 0.124\tilde{w}_t$; substituting Equation (5.7), $\tilde{y}_t = e^{z_t}\tilde{k}_t^{\beta}$, into (5.5), $\tilde{w}_t = (1-\beta)\tilde{y}_t$; and the resulting equation into $\tilde{d}_t = 0.124\tilde{w}_t$; setting $z_t = 0$ and $\tilde{k}_t = \tilde{k}^*$.

[64] These two equations are obtained by substituting Equation (5.7) into (5.5) and (5.6), respectively, setting $z_t = 0$ and $\tilde{k}_t = \tilde{k}^*$.

[65] Cooley and Soares (1996) suggest that the privatization of the social-security system causes the capital-to-output ratio to increase to pre-social-security levels and the real interest rate to decrease.

percent; and it reduces r^* by 0.4 percentage points. Hence, the transition to the fully-funded system has the same effects on $\tilde{k}^*$, $\tilde{w}^*$, and r^* as the abandonment of the PAYG system without replacing it (see section 5.3.1, the last column of Table 5.3). Also, the transition to the fully-funded system causes $\tilde{s}^*$ to increase by 1.9 percent, implying that the reduction in $\tilde{s}^*$ caused by the fully-funded system is lower than that caused by the PAYG system.

Figure 5.4. Transition path from the PAYG system to the fully-funded system

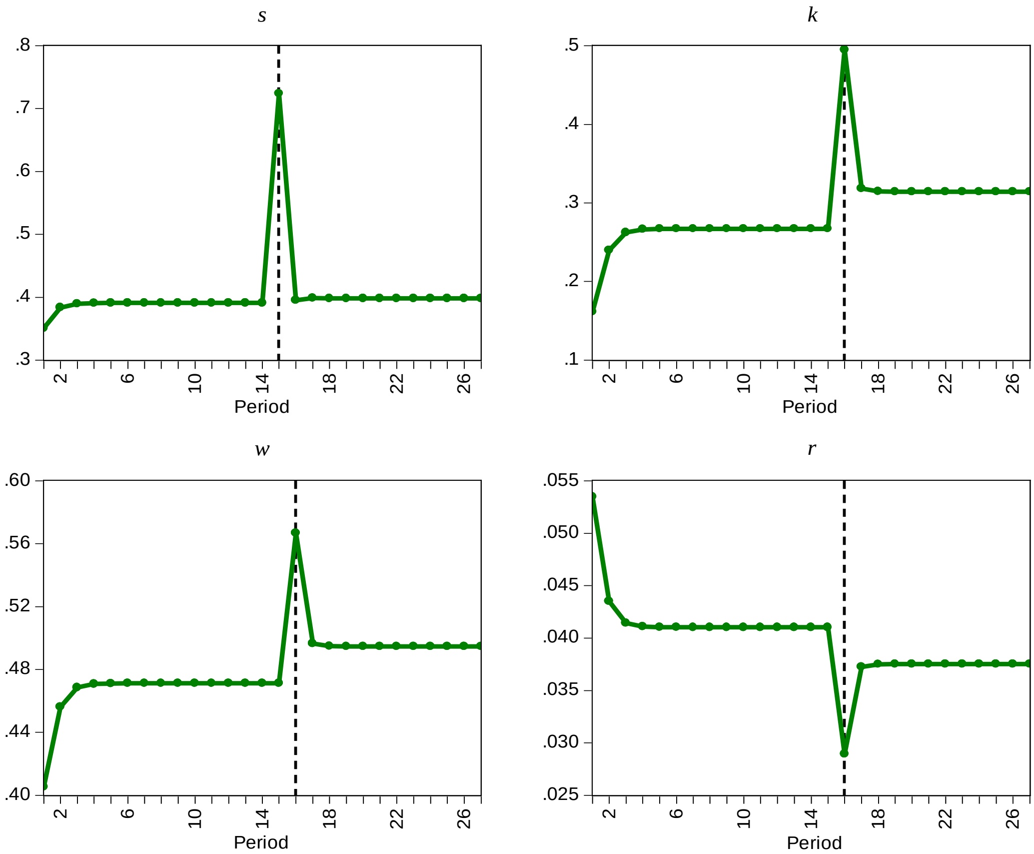

Figure 5.4 shows the transition path of $\tilde{s}^*$, $\tilde{k}^*$, $\tilde{w}^*$, and r^* from a steady-state equilibrium under the PAYG system to a steady-state equilibrium under the fully-funded system. During the transition period (e.g., t = 15), $\tilde{s}^*$, $\tilde{k}^*$, and $\tilde{w}^*$ increase, while r^* decreases, considering that the government uses new debt to compensate individuals entitled to retirement benefits under the previous pension system. Also, after the transition to the steady-state equilibrium under the fully-funded system, $\tilde{k}^*$ and $\tilde{w}^*$ increase to pre-social-security levels, while r^* decreases.

CHAPTER 6: CONCLUSION

The main purpose of the social-security systems is to ensure an "adequate" level of income during retirement. Their credibility, however, has been a global issue during the past few decades. In many developed countries, retirement systems may face sustainability problems as they confront growing deficits. As well, in the less developed countries, retirement systems may operate under uncertain quality of institutions, which undermines their reliability. Low institutional quality impedes the proper functioning of the retirement systems, and thus affects household saving.

In this study, I examine the effects of several institutional and other variables, e.g., corruption, government stability, the debt-to-GDP ratio, etc., on the probability that the social-security system (PAYG and fully funded) will grant pensions to the old at retirement. Through this channel, I examine the effects of these variables on the relationship between social-security contributions and household saving. To my knowledge, these effects have not been studied in the literature. Previous studies limit their attention to the institution of voting and to the form of the political system (democratic or nondemocratic), neglecting other institutional features, e.g., corruption, government stability, etc. I employ a two-period OGM and maximize a lifetime CRRA utility function under the intertemporal budget constraint, which takes into account that individuals contribute a certain amount of their labor income in the social-security system and expect to receive benefits at retirement. The expected benefits depend on the probability that the social-security system will grant pensions to the old at retirement. This probability is determined by a logit model and depends on institutional variables, e.g., corruption and government stability, and on the debt-to-GDP ratio. Since the household maximization problem has no analytical solution, I take partial derivatives of the Euler equation for household saving to examine the effects mentioned above.

The empirical findings generally support the implications of the theoretical model. To check the robustness of the estimates to substantial changes in the sample, I use three panel data sets, a balanced panel of 11 OECD countries for the period 1984-2009, an unbalanced panel of 25 countries, which includes the previous 11 countries, and 14 more countries for the period 1995-2009, and a balanced panel of the 25 countries for the period 1995-2009. The estimates obtained from these panels do not differ dramatically, so it would not be unreasonable to argue that they pass the robustness test.

Estimating a fixed-effects (nonlinear) Euler equation for household saving under the PAYG system by GMM and by NLLS, the results suggest that social-security contributions reduce household saving in a less than one-for-one manner. High levels of corruption or the debt-to-GDP ratio reduce the probability that the PAYG system will grant pensions at retirement. As well, the higher the level of corruption or the debt-to-GDP ratio the lower the reduction in household saving caused by an increase in social-security contributions, as individuals try to self-insure themselves against the higher uncertainty induced by corruption and indebtedness. The NLLS estimates are similar to the GMM ones, except for the coefficient of the index of corruption, which is found to be statistically insignificant at conventional levels.

Moreover, linearizing the Euler equation for household saving and estimating it by GMM, the results suggest that social-security contributions reduce household saving with offset almost one-for-one. In contrast to the GMM estimates in the case of the nonlinear Euler equation, the index of government stability turns out to be statistically significant.

The above findings may be useful in evaluating policy proposals that aim to improve the viability of the PAYG systems in the countries considered here. Along with the reforms of the PAYG systems that have been taking place in these countries, their governments may be able to improve the viability and credibility of their PAYG systems by reducing corruption and the debt-to-GDP ratio.

Calibrating the model to match features of the U.S. economy under the PAYG system and of the Mexican economy under the fully-funded system, the results imply that the model can replicate some features of the actual data, but there are also some features that have not been captured by the model. Using a general-equilibrium analysis, the PAYG system reduces the steady-state values of household saving, capital stock, and real wage, while it increases the steady-state value of the real interest rate. As well, the fully-funded system reduces the steady-state value of household saving with offset one-for-one, while it has no effect on the steady-state values of the other variables. The partial-equilibrium effect of the social-security system (PAYG and fully funded) on the steady-state value of household saving is larger (in absolute value) than the corresponding general-equilibrium one.

A transition from the steady-state equilibrium under the PAYG system to the steady-state equilibrium under the fully-funded system, which is financed by the government budget, has the same effects on the steady-state values of capital stock, real wage, and the real interest rate as would the elimination of the PAYG system without replacing it. That is, the steady-state values of capital stock and real wage increase to pre-social-security levels, while the steady-state value of the real interest rate decreases. Thus, policy makers should consider the above macroeconomic effects on capital stock, real wage, and the real interest rate, and also potential efficiency gains, i.e., making some individuals better off without making some others worse off, when addressing proposals to privatize social security.

REFERENCES

Ando, A. and Modigliani, F., 1963. The Life Cycle Hypothesis of Saving: Aggregate Implications and Tests. *The American Economic Review*, 53(1), 55-84.

Antolin, P., Payet, S., and Yermo, J., 2012. Coverage of Private Pension Systems: Evidence and Policy Options. OECD Working Papers on Finance, Insurance and Private Pensions No. 20. OECD Publishing.

Attanasio, O.P. and Brugiavini, A., 2003. Social Security and Households' Saving. *The Quarterly Journal of Economics*, 118(3), 1075-119.

Attanasio, O.P. and Rohwedder, S., 2003. Pension Wealth and Household Saving: Evidence from Pension Reforms in the United Kingdom. *The American Economic Review*, 93(5), 1499-521.

Baltagi, B.H., 2008. *Econometric Analysis of Panel Data*. 4th ed. John Wiley and Sons Ltd.

Barro, R.J., 1974. Are Government Bonds Net Wealth? *Journal of Political Economy*, 82(6), 1095-117.

---, 1978. *The Impact of Social Security on Private Saving: Evidence from the U.S. Time Series*. Washington, D.C.: American Enterprise Institute for Public Policy Research.

Barro, R.J. and MacDonald, G.M., 1979. Social Security and Consumer Spending in an International Cross Section. *Journal of Public Economics*, 11(3), 275-89.

Barro, R.J. and Sala-i-Martin, X., 1995. *Economic Growth*. New York: McGraw-Hill.

Bernheim, B.D., 1987. The Economic Effects of Social Security: Toward a Reconciliation of Theory and Measurements. *Journal of Public Economics*, 33(3), 273-304.

Bernheim, B.D. and Scholz, J.K., 1993. Private Saving and Public Policy. *In*: Poterba, J., ed. *Tax Policy and the Economy*, Vol. 7. MIT Press, 73-110.

Beverly, S.G. and Sherraden, M., 1999. Institutional Determinants of Saving: Implications for Low-Income Households and Public Policy. *Journal of Socio-Economics*, 28(4), 457-73.

Blanchard, O.J. and Fischer, S., 1989. *Lectures in Macroeconomics*. Cambridge, MA: MIT Press.

Box, G.E.P. and Jenkins, G.M., 1976. *Time Series Analysis, Forecasting and Control*. 3rd ed. San Francisco: Holden-Bay.

Breitung, J., 2000. The Local Power of Some Unit Root Tests for Panel Data. *In*: Baltagi, B.H., Fomby, T.B. and Hill, R.C., eds. *Nonstationary Panels, Panel Cointegration, and Dynamic Panels*, *Advances in Econometrics*, Vol. 15. Emerald Group Publishing Limited, 161-77.

Browning, E.K., 1975. Why the Social Insurance Budget is Too Large in a Democracy. *Economic Inquiry*, 13(3), 373-88.

Cagan, P., 1965. The Effect of Pension Plans on Aggregate Saving: Evidence from a Sample Survey. NBER Occasional Paper No. 95. New York: Columbia University Press.

Campbell, J.Y. and Mankiw, N.G., 1989. Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence. *In*: Blanchard, O. and Fischer, S., eds. *NBER Macroeconomic Annual 1989*, Vol. 4. MIT Press, 185-246.

---, 1990. Permanent Income, Current Income and Consumption. *Journal of Business and Economic Statistics*, 8(3), 265-79.

Carroll, C.D., 1997. Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis. *The Quarterly Journal of Economics*, 112(1), 1-55.

Casamatta, G., Cremer, H., and Pestieau, P., 2000. The Political Economy of Social Security. *The Scandinavian Journal of Economics*, 102(3), 503-22.

Conde-Ruiz, J.I. and Galasso, V., 2005. Positive Arithmetic of the Welfare State. *Journal of Public Economics*, 89(5-6), 933-55.

Cooley, T.F. and Prescott, E.C., 1995. Economic Growth and Business Cycles. *In*: Cooley, T.F., ed. *Frontiers of Business Cycle Research*. Princeton University Press, 1-38.

Cooley, T.F. and Soares, J., 1996. Will Social Security Survive the Baby Boom? *Carnegie-Rochester Conference Series on Public Policy*, 45(1), 89-121.

---, 1999a. A Positive Theory of Social Security Based on Reputation. *Journal of Political Economy*, 107(1), 135-60.

---, 1999b. Privatizing Social Security. *Review of Economic Dynamics*, 2(3), 731-55.

Dawkins, C., Srinivasan, T.N., and Whalley, J., 2001. Calibration. *In*: Heckman, J.J. and Leamer, E.E., eds. *Handbook of Econometrics*, Vol. 5. Elsevier, 3654-84.

Deaton, A. and Muellbauer, J., 1980. *Economics and Consumer Behavior*. New York: Cambridge University Press.

Dejong, D.N. and Dave, C., 2007. *Structural Macroeconometrics*. New Jersey: Princeton University Press.

Demirguc-Kunt, A. and Schwarz, A.M., 1999. Taking Stock of Pension Reforms around the World. Social Protection Discussion Papers No. 20533. Washington, D.C.: The World Bank.

Diamond, P.A., 1965. National Debt in a Neoclassical Growth Model. *The American Economic Review*, 55(5), 1126-50.

Diamond, P.A. and Gruber, J., 1997. Social Security and Retirement in the U.S. NBER Working Paper No. 6097. Cambridge, MA: National Bureau of Economic Research.

Diamond, P.A. and Hausman, J.A., 1984. Individual Retirement and Savings Behavior. *Journal of Public Economics*, 23(1-2), 81-114.

Dolde, W. and Tobin, J., 1981. Mandatory Retirement Saving and Capital Formation. Cowles Foundation Discussion Paper No. 594. Cowles Foundation for Research in Economics.

Durby, M., 1979. *The Effects of Social Security on Income and the Capital Stock*. Washington, D.C.: American Enterprise Institute.

Estima, 2007. *Rats Reference Manual*. Ver. 7, Evanston, IL.

---, 2007. *Rats User's Guide*. Ver. 7, Evanston, IL.

Feldstein, M., 1974. Social Security, Induced Retirement, and Aggregate Capital Accumulation. *Journal of Political Economy*, 82(5), 905-26.

---, 1977. Social Security and Private Savings: International Evidence in an Extended Life-Cycle Model. *In*: Feldstein, M. and Inman, R., eds. *The Economics of Public Services*, An International Economic Association Conference Volume. London: Macmillan Press.

---, 1979. The Effect of Social Security on Saving. NBER Working Paper No. 334. Cambridge, MA: National Bureau of Economic Research.

---, 1980. International Differences in Social Security and Saving. *Journal of Public Economics*, 14(2), 225-44.

---, 1982. Social Security and Private Saving: Reply. *Journal of Political Economy*, 90(3), 630-42.

---, 2001. The Future of Social Security Pensions in Europe. NBER Working Paper No. 8487. Cambridge, MA: National Bureau of Economic Research.

Feldstein, M. and Pellechio, A., 1979. Social Security and Household Wealth Accumulation: New Microeconometric Evidence. *The Review of Economics and Statistics*, 61(3), 361-68.

Feldstein, M. and Samwick, A. 1998. The Transition Path in Privatizing Social Security. *In*: Feldstein, M., ed. *Privatizing Social Security*. University of Chicago Press, 215-64.

Galasso, V. and Profeta, P., 2002. The Political Economy of Social Security: a Survey. *European Journal of Political Economy*, 18(1), 1-29.

Gale, W.G., 1998. The Effects of Pensions on Household Wealth: A Reevaluation of Theory and Evidence. *Journal of Political Economy*, 106(4), 706-23.

Gern, K-J., 2002. Recent Developments in Old Age Pension Systems: An International Overview. *In*: Feldstein, M. and Siebert, H., eds. *Social Security Pension Reform in Europe*. University of Chicago Press, 439-78.

Gollin, D., 2002. Getting Income Shares Right. *Journal of Political Economy*, 110(21), 458-74.

Graham, J.W., 1987. International Differences in Saving Rates and the Life Cycle Hypothesis. *European Economic Review*, 31(8), 1509-29.

Greene, W.H., 2008. *Econometric Analysis.* 6th ed. New Jersey: Pearson Prentice Hall.

Gullasson, E.T., Kolluri, B.R., and Panik, M.J., 1993. Social Security and Household Wealth Accumulation: Refined Microeconometric Evidence. *The Review of Economics and Statistics*, 75(3), 548-51.

Gustman, A.L. and Steinmeier, T.L., 1999. Effects of Pensions on Savings: Analysis with Data from the Health and Retirement Study. *Carnegie-Rochester Conference Series on Public Policy*, 50(1), 271-324.

Hadri, K., 2000. Testing for Stationarity in Heterogeneous Panel Data. *Econometrics Journal*, 3(2), 148-61.

Hall, R.E., 1988. Intertemporal Substitution in Consumption. *Journal of Political Economy*. 96(2), 339-57.

Hansen, G.D. and Prescott, E.C., 1993. Did Technology Shocks Cause the 1990-1991 Recession? *The American Economic Review*, 83(2), 280-6.

Hansen, L.P. and Heckman, J.J., 1996. The Empirical Foundations of Calibration. *Journal of Economic Perspectives*, 10(1), 87-104.

Hansen, L.P. and Singleton, K.J., 1984. Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models. *Econometrica*, 52(1), 267-8.

Hartley, J., Salyer, K., and Sheffrin, S., 1997. Calibration and Real Business Cycle Models: An Unorthodox Experiment. *Journal of Macroeconomics*, 19(2), 1-17.

Hodrick, R.J. and Prescott, E.C., 1997. Postwar U.S. Business Cycles: An Empirical Investigation. *Journal of Money, Credit and Banking*, 29(1), 1-16.

Horowitz, J.L. and Savin, N.E., 2001. Binary Response Models: Logits, Probits and Semiparametrics. *Journal of Economic Perspectives*, 15(4), 43-56.

Huang, H., Imrohoroglu, S., and Sargent, T.J., 1997. Two Computations to Fund Social Security. *Macroeconomics Dynamics*, 1(01), 7-44.

Hubbard, R.G., 1984. Uncertain Lifetimes, Pensions, and the Individual Saving. NBER Working Paper No. 1363. Cambridge, MA: National Bureau of Economic Research.

---, 1986. Pension Wealth and Individual Saving: Some New Evidence. *Journal of Money, Credit and Banking*, 18(2), 167-78.

Hubbard, R.G. and Judd, K.L., 1987. Social Security and Individual Welfare: Precautionary Saving, Borrowing Constraints, and the Payroll Tax. *The American Economic Review*, 77(4), 630-46.

Hubbard, R.G., Skinner, J., and Zeldes, S.P., 1995. Precautionary Saving and Social Insurance. *Journal of Political Economy*, 103(2), 360-99.

Hurd, M.D., 1989. Mortality Risk and Bequests. *Econometrica*, 57(4), 779-813.

Im, K.S., Pesaran, M.H., and Shin, S., 2003. Testing for Unit Roots in Heterogeneous Panels. *Journal of Econometrics*, 115(1), 53-74.

Imrohoroglu, A., Imrohoroglu, S., and Joines, D.H., 1995. A Life Cycle Analysis of Social Security. *Economic Theory*, 6(1), 83-114.

Katona, G., 1965. Private Pensions and Individual Saving. Monograph No. 40. Survey Research Center, Institute for Social Research.

Kennickell, A.B. and Sundén, A.E., 1997. Pensions, Social Security, and the Distribution of Wealth. Finance and Economics Discussion Series No. 1997-55. Federal Reserve Board.

King, M.A. and Dicks-Mireaux, L-D.L., 1982. Asset Holdings and the Life-Cycle. *The Economic Journal*, 92(366), 247-67.

Kotlikoff, L.J., 1979a. Social Security and Equilibrium Capital Intensity. *The Quarterly Journal of Economics*, 93(2), 233-53.

---, 1979b. Testing the Theory of Social Security and the Life Cycle Accumulation. *The American Economic Review*, 69(3), 396-410.

Koskela, E. and Virén, M., 1983. Social Security and Household Saving in an International Cross Section. *The American Economic Review*, 73(1), 212-217.

Kulish, M., Kent, C., and Smith, K., 2010. Aging, Retirement, and Savings: A General Equilibrium Analysis. *The B.E. Journal of Macroeconomics*, 10(1), 1-30.

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P., and Shin, Y., 1992. Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root: How Sure Are We that Economic Time Series Have a Unit Root? *Journal of Econometrics*, 54(1-3), 159-78.

Kydland, F.E. and Prescott, E.C., 1996. The Computational Experiment: An Econometric Tool. *Journal of Economic Perspectives*, 10(1), 69-85.

Leimer, D.R. and Lesnoy, S.D., 1982. Social Security and Private Saving: New Time-Series Evidence. *Journal of Political Economy*, 90(3), 606-29.

Levin, A., Lin, C-F., and Chu, C-S.J., 2002. Unit Root Tests in Panel Data: Asymptotic and Finite-sample Properties. *Journal of Econometrics*, 108(1), 1-24.

Lindert, P.H., 1994. The Rise of Social Spending, 1880-1930. *Explorations in Economic History*, 31(1), 1-37.

---, 1996. What Limits Social Spending? *Explorations in Economic History*, 33(1), 1-34.

Lopez, H., 2008. The Social Discount Rate: Estimates for Nine Latin American Countries. Policy Research Working Paper No. 4639. Washington, D.C.: The World Bank.

Maddala, G.S., 1986. *Limited-Dependent and Qualitative Variables in Econometrics*. Cambridge University Press.

Maddala, G.S. and Wu, S., 1999. A Comparative Study of Unit Root Tests with Panel Data and a New Simple Test. *Oxford Bulletin of Economics and Statistics*, 61(1), 631-52.

Mitchell, O.S., 1993. Retirement Systems in Developed and Developing Countries: Institutional Features, Economic Effects, and Lessons for Economies in Transition. NBER Working Paper No. 4424. Cambridge, MA: National Bureau of Economic Research.

Modigliani, F. and Sterling, A., 1981. Determinants of Private Saving with Special Reference to the Role of Social Security: Cross Country Tests. MIT Working Paper No. 1209-81.

Mulligan, C.B., Gil, R., and Sala-i-Martin, X., 2002. Social Security and Democracy. NBER Working Paper No. 8958. Cambridge, MA: National Bureau of Economic Research.

---, 2004. Do Democracies Have Different Public Policies than Nondemocracies? *Journal of Economic Perspectives*, 18(1), 51-74.

Mulligan, C.B. and Sala-i-Martin, X., 2002. Social Security in Theory and Practice with Implications for Reform. Discussion Paper No. 0203-01. Columbia University.

Munnell, A.H., 1974. *The Effect of Social Security on Personal Saving*. Cambridge, MA: Ballinger Publishing Company.

---, 1976. Private Pensions and Savings: New Evidence. *Journal of Political Economy*, 84(5), 1013-32.

Newey, W.K. and West, K.D., 1994. Automatic Lag Selection in Covariance Matrix Estimation. *Review of Economic Studies*, 61(4), 631-53.

Novos, I.E., 1989. Social Security Wealth and Wealth Accumulation: Further Microeconomic Evidence. *The Review of Economics and Statistics*, 71(1), 167-71.

O'Grady, S., 2012. Pension Bill to Quadruple in Next 50 Years. *Daily Express Clippings*, Friday 28 September, 2.

---, 2013. Pensions Crisis as Payouts Slashed. *Daily Express Clippings*, Wednesday 24 April, 1-2.

Organization for Economic Cooperation and Development, 2009. *Pension at Glance 2009*. Paris: OECD.

Pinotti, P., 2009. Financial Development and the Pay-As-You-Go Social Security. *The B. E. Journal of Macroeconomics*, 9(1), 1-19.

Romer, D., 2001. *Advanced Macroeconomics*. 2nd ed. New York: McGraw-Hill.

Sala-i-Martin, X., 1996. A Positive Theory of Social Security. *Journal of Economic Growth*, 1(2), 277-304.

Sales-Sarrapy, C., Solis-Soberon, F., and Villagomez-Amezcua, A., 1998. Pension System Reform: The Mexican Case. *In*: Feldstein, M., ed. *Privatizing Social Security*. University of Chicago Press, 135-75.

Sargent, T.J., 1987. *Dynamic Macroeconomic Theory*. Cambridge, MA: Harvard University Press.

Solimano, A. and Soto, R., 2005. Economic Growth in Latin America in the Late 20th Century: Evidence and Interpretation. ECLAC Macroeconomics of Development Series No. 33. Santiago, Chile: United Nations.

Tabellini, G., 2000. A Positive Theory of Social Security. *The Scandinavian Journal of Economics*, 102(3), 523-45.

Tapia, W., 2008. Description of Private Pension Systems. OECD Working Papers on Insurance and Private Pensions No. 22. OECD Publishing.

Tauchen, G., 1986. Statistical Properties of Generalized Method-of-Moments Estimators of Structural Parameters Obtained from Financial Market Data. *Journal of Business and Economic Statistics*, 4(4), 397-416.

Thaler, R.H. and Shefrin, H.M., 1981. An Economic Theory of Self-Control. *Journal of Political Economy*, 89(2), 392-406.

Velloso, H., 2006. Social Security in the United States: Overview and Outlook. ECLAC Project Document No. 89. Santiago, Chile: United Nations.

Wooldridge, J.M., 2009. *Introductory Econometrics: A modern Approach*. 4th ed. South-Western.

Working, H., 1960. Note on the Correlation of First Differences of Averages in a Random Chain. *Econometrica*, 28(4), 916-18.

Ziliak, J.P., 1997. Efficient Estimation with Panel Data when Instruments Are Predetermined: An Empirical Comparison of Moment-Condition Estimators. *Journal of Business and Economic Statistics*, 15(4), 419-31.

Appendix A: Proof of Equation (3.19)

In order to be able to substitute Equation (3.3), $E_t(c_{2t+1}) = (1+r_t)s_t + E_t(b_{t+1})$, into Equation (3.9), $\left(\frac{c_{1t}}{c_{2t+1}}\right)^{\gamma} \frac{1+r_t}{1+\rho} = 1 + e_{t+1}$, I first rewrite Equation (3.9) as follows:

$$\frac{c_{1t}}{c_{2t+1}}\left(\frac{1+r_t}{1+\rho}\right)^{\frac{1}{\gamma}} = (1+e_{t+1})^{\frac{1}{\gamma}} \tag{A1}$$

or

$$\frac{c_{2t+1}}{c_{1t}}\left(\frac{1+\rho}{1+r_t}\right)^{\frac{1}{\gamma}} = \frac{1}{(1+e_{t+1})^{\frac{1}{\gamma}}}. \tag{A2}$$

Then, taking expectations in Equation (A2) yields

$$\frac{E_t(c_{2t+1})}{c_{1t}}\left(\frac{1+\rho}{1+r_t}\right)^{\frac{1}{\gamma}} = E_t\left[\frac{1}{(1+e_{t+1})^{\frac{1}{\gamma}}}\right]. \tag{A3}$$

Finally, substituting Equation (3.3) and the definition $c_{1t} = w_t - s_t - d_t$ into Equation (A3) yields Equation (3.19), which is the Euler equation for saving.

Appendix B: The data

The definitions and sources of the data are as follows:

a) Household saving. This is the gross saving of households and non-profit institutions serving households (NPISHs).[66] It measures the part of households and

[66] NPISHs are non-market producers, which are separate legal entities. They are not primarily financed and controlled by the government, and provide goods or services to households free of charge or at prices that are not economically significant. Their main resources, apart from those derived from occasional sales, are obtained from voluntary contributions in cash or in kind from households, from

NPISHs' disposable income that is not used for final consumption expenditure. For Canada, only the series net saving is available. Net saving equals gross saving less consumption of fixed capital. The source is AMECO.

b) Social-security contributions. These are the actual social contributions received by general government. Actual social contributions are payments to social-insurance schemes, in order to secure the entitlement of social benefits. They consist of contributions of employers, employees, self-employed, and non-employed persons. The source is AMECO.

c) Compensation of employees. It includes wages, salaries, and employers' social contributions. The source is AMECO.

d) Interest rate. It is either the interbank offer rate attached to loans given and taken amongst banks, or the rate associated with treasury bills, certificates of deposit or comparable instruments, each of three month maturity. For Belgium, Canada, Cyprus, Czech Republic, France, Hungary, Italy, Latvia, Lithuania, Mexico, Poland, Spain, Sweden, the United Kingdom, and the United States the source is IFS, while for Austria, Denmark, Estonia, Finland, Germany, Greece, Japan, the Netherlands, Norway, Portugal, and Slovakia the source is AMECO.

e) General-government debt. This refers to total gross debt outstanding at the end of the year and owed by the general government. It comprises all financial liabilities of general government mainly in the form of government bills and bonds. The source is AMECO.

f) General-government deficit. It is the difference between total general-government expenditures, including interest payments, and total general-government revenues. The source is AMECO.

payments made by general governments, and from property income. Some examples are churches and religious societies, sports and other clubs, trade unions, and political parties.

g) Gross domestic product (AMECO).

h) Net capital stock. It is the difference between gross capital stock and consumption of fixed capital. The source is AMECO.

i**)** Exchange rate (number of units of national currency per Euro, AMECO).

j) Total employment. It includes employees and self-employed persons (AMECO).

k) Total unemployment. It is the total number of individuals that are without work, available for paid employment or self-employment, and actively seeking to work (AMECO).

l) CPI. It measures changes in the cost of living of the average consumer (WDI).

m) The annual percentage change in CPI (WDI).

n) GDP deflator (AMECO).

o) The index of corruption. This is an assessment of corruption within the political system, according to the International Country Risk Guide (ICRG). Such corruption distorts the economic and financial environment, reduces the efficiency of government and business, and introduces instability into the political system. This variable takes on values (risk points) between 0 and 6, with the value 0 indicating the highest political risk and the value 6 indicating the lowest political risk. The source is the ICRG.

p) The index of government stability. This represents the government's ability to implement its program and stay in office. It consists of three subcomponents: government unity, legislative strength, and popular support. It takes on values between 0 and 12, with the highest value indicating the lowest political risk (ICRG).

q) The index of socioeconomic conditions. It describes the socioeconomic pressure at work in society, which could constrain government action and stimulate social dissatisfaction. It contains three subcomponents, which are unemployment, consumer confidence, and poverty. It takes on values between 0 and 12, with the highest value indicating the lowest political risk (ICRG).

r) The index of democratic accountability. This measures how responsive is the government to its people. The less responsive it is, the more likely it becomes that it will fall. It takes on values between 0 and 6, with the highest value indicating the lowest political risk (ICRG).

Appendix C: Proofs of Equations (5.1)-(5.7)

First, I derive Equation (5.1), the Euler equation for consumption per effective labor. To begin with, I express the lifetime utility function (3.2) in per effective labor terms. Given that c_{1t} is consumption per worker in period *t*, $\tilde{c}_{1t} = c_{1t} / A_t$ is consumption per effective labor in period *t*. As well, $\tilde{c}_{2t+1} = c_{2t+1} / A_{t+1}$ is consumption per effective labor in period *t*+1. Thus, $c_{1t} = \tilde{c}_{1t} A_t$ and $c_{2t+1} = \tilde{c}_{2t+1} A_{t+1} = \tilde{c}_{2t+1}(1 + r_A) A_t$ using Equation (3.11), $A_{t+1} = (1 + r_A) A_t$. Substituting these definitions into the lifetime utility function (3.2) yields

$$U = \frac{(\tilde{c}_{1t} A_t)^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho} E_t \left\{ \frac{[\tilde{c}_{2\tau+1}(1 + r_A) A_t]^{1-\gamma}}{1-\gamma} \right\}. \tag{C.1}$$

Then, I express the intertemporal budget constraint (3.4) in per effective labor terms. Given that w_t is wage per worker, $\tilde{w}_t = w_t / A_t$ is wage per effective labor. Also, given that d_t is social-security contributions per worker and b_{t+1} is social-security benefits per worker, $\tilde{d}_t = d_t / A_t$ and $\tilde{b}_{t+1} = b_{t+1} / A_{t+1}$ are social-security contributions per effective labor and social-security benefits per effective labor, respectively. Hence, $w_t = \tilde{w}_t A_t$, $d_t = \tilde{d}_t A_t$, and $b_{t+1} = \tilde{b}_{t+1} A_{t+1} = \tilde{b}_{t+1}(1 + r_A) A_t$ using

Equation (3.11). Substituting these definitions along with the definitions of c_{1t} and c_{2t+1} [see above Equation (C.1)] into the intertemporal budget constraint (3.4) yields

$$\tilde{c}_{1t} A_t + \frac{E_t[\tilde{c}_{2t+1}(1+r_A)A_t]}{1+r_t} = \tilde{w}_t A_t - \tilde{d}_t A_t + \frac{E_t[\tilde{b}_{t+1}(1+r_A)A_t]}{1+r_t}. \tag{C.2}$$

Dividing both sides of Equation (C.2) by A_t yields the intertemporal budget constraint in per effective labor terms:

$$\tilde{c}_{1t} + \frac{E_t[\tilde{c}_{2t+1}(1+r_A)]}{1+r_t} = \tilde{w}_t - \tilde{d}_t + \frac{E_t[\tilde{b}_{t+1}(1+r_A)]}{1+r_t} \tag{C.3}$$

or

$$E_t(\tilde{c}_{2t+1}) = \frac{1+r_t}{1+r_A}(\tilde{w}_t - \tilde{c}_{1t} - \tilde{d}_t) + E_t(\tilde{b}_{t+1}). \tag{C.4}$$

The individual maximizes his/her lifetime utility function (C.1) under the intertemporal budget constraint (C.3). The Lagrangian is

$$\ell = \frac{(\tilde{c}_{1t}A_t)^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho} E_t\left\{\frac{[\tilde{c}_{2t+1}(1+r_A)A_t]^{1-\gamma}}{1-\gamma}\right\} + \lambda\left\{\tilde{w}_t - \tilde{d}_t + \frac{E_t[\tilde{b}_{t+1}(1+r_A)A_t]}{1+r_t} - \tilde{c}_{1t} - \frac{E_t[\tilde{c}_{2t+1}(1+r_A)]}{1+r_t}\right\}. \tag{C.5}$$

Taking the partial derivatives of ℓ with respect to $\tilde{c}_{1t}$ and $\tilde{c}_{2t+1}$, and setting them equal to zero yields

$$(\tilde{c}_{1t}A_t)^{-\gamma} A_t = \lambda \tag{C.6}$$

and

$$\frac{1}{1+\rho} E_t[\tilde{c}_{2t+1}(1+r_A)A_t]^{-\gamma} A_t = \frac{\lambda}{1+r_t}. \tag{C.7}$$

Finally, substituting Equation (C.6) into Equation (C.7) and rearranging yields Equation (5.1), the Euler equation for consumption per effective labor.

Second, I derive Equations (5.2) and (5.3), the budget constraints of the first and of the second period, respectively, in per effective labor terms. Given that s_t is saving per worker, $\tilde{s}_t = s_t / A_t$ is saving per effective labor. Dividing both sides of the first-period budget constraint, $c_{1t} = w_t - s_t - d_t$, by A_t and using the definitions of $\tilde{c}_t$, $\tilde{w}_t$, $\tilde{s}_t$, and $\tilde{d}_t$ yields Equation (5.2). Moreover, solving Equation (5.2) for $\tilde{s}_t$, that is, $\tilde{s}_t = \tilde{w}_t - \tilde{c}_{1t} - \tilde{d}_t$, and substituting this equation into Equation (C.4) yields Equation (5.3).

Third, I derive Equation (5.4). Given that k_{t+1} is capital per worker, $\tilde{k}_{t+1} = k_{t+1} / A_{t+1}$ is capital per effective labor. Dividing both sides of Equation (3.17), $k_{t+1} = s_t /(1+n)$, by A_{t+1}, using Equation (3.11), $A_{t+1} = (1+r_A)A_t$, and using the definitions of $\tilde{k}_{t+1}$ and $\tilde{s}_t$ yields Equation (5.4).

Fourth, I derive Equations (5.5) and (5.6). Given that y_t is product per worker, $\tilde{y}_t = y_t / A_t$ is product per effective labor. Substituting the definitions $w_t = \tilde{w}_t A_t$ and $y_t = \tilde{y}_t A_t$ into Equation (3.12), $w_t = (1-\beta) y_t$, and dividing both sides of the resulting equation by A_t yields Equation (5.5). Also, substituting the definitions $y_t = \tilde{y}_t A_t$ and $k_t = \tilde{k}_t A_t$ into Equation (3.13), $r_t = \beta y_t / k_t$, and rearranging yields Equation (5.6).

Fifth, I derive Equation (5.7). Dividing both sides of Equation (3.20), $y_t = e^{z_t} k_t^{\beta} A_t^{1-\beta}$, by A_t and using the definitions of $\tilde{y}_t$ and $\tilde{k}_t$ yields Equation (5.7).

Appendix D: Proof of Equation (5.8)

Since I assume that in the steady-state equilibrium expectational and productivity shocks are absent (see section 5.2), to compute the steady-state value of the capital stock per effective labor, $\tilde{k}^*$, I first remove the expectations operator, E_t, from Equation (5.1), $E_t(\tilde{c}_{1t}/\tilde{c}_{2t+1})^{\gamma} = (1+r_A)^{\gamma}(1+\rho)/(1+r_t)$, and solve the resulting equation for $\tilde{c}_{2t+1}$ as follows:

$$\tilde{c}_{2t+1} = \tilde{c}_{1t}\,\frac{1}{(1+r_A)}\left(\frac{1+r_t}{1+\rho}\right)^{1/\gamma} \quad . \tag{D.1}$$

Then, I remove E_t from Equation (C.4), $E_t(\tilde{c}_{2t+1}) = [(1+r_t)/(1+r_A)](\tilde{w}_t - \tilde{c}_{1t} - \tilde{d}_t) + E_t(\tilde{b}_{t+1})$, thus obtaining the equation

$$\tilde{c}_{2t+1} = \frac{1+r_t}{1+r_A}(\tilde{w}_t - \tilde{c}_{1t} - \tilde{d}_t) + \tilde{b}_{t+1}. \tag{D.2}$$

Substituting Equation (D.1) into (D.2) and solving the resulting equation for $\tilde{c}_{1t}$ yields

$$\tilde{c}_{1t} = \left(\tilde{w}_t - \tilde{d}_t + \tilde{b}_{t+1}\frac{1+r_A}{1+r_t}\right)\left[\frac{(1+\rho)^{1/\gamma}}{(1+r_t)^{(1-\gamma)/\gamma} + (1+\rho)^{1/\gamma}}\right]. \tag{D.3}$$

Also, substituting Equation (D.3) into Equation (5.2), $\tilde{c}_{1t} = \tilde{w}_t - \tilde{s}_t - \tilde{d}_t$, and solving the resulting equation for $\tilde{s}_t$ yields

$$\tilde{s}_t = \left(\tilde{w}_t - \tilde{d}_t\right)\left[\frac{(1+r_t)^{(1-\gamma)/\gamma}}{(1+r_t)^{(1-\gamma)/\gamma} + (1+\rho)^{1/\gamma}}\right] - \tilde{b}_{t+1}\left[\frac{(1+r_A)(1+\rho)^{1/\gamma}}{(1+r_t)^{1/\gamma} + (1+r_t)(1+\rho)^{1/\gamma}}\right]. \tag{D.4}$$

Using Equation (5.7), $\tilde{y}_t = e^{z_t}\tilde{k}_t^{\beta}$, Equations (5.5), $\tilde{w}_t = (1-\beta)\tilde{y}_t$, and (5.6), $r_t = \beta\tilde{y}_t/\tilde{k}_t$, can be written as $\tilde{w}_t = (1-\beta)e^{z_t}\tilde{k}_t^{\beta}$ and $r_t = \beta e^{z_t}\tilde{k}_t^{\beta-1}$, respectively. Finally, substituting these equations into Equation (D.4), and the resulting equation into Equation (5.4), $\tilde{k}_{t+1} = \tilde{s}_t/[(1+n)(1+r_A)]$, setting z_t = 0 and $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$ yields Equation (5.8).

Appendix E: Computation of the partial derivatives $\partial\tilde{s}_t / \partial\tilde{w}_t$ and $\partial\tilde{s}_t / \partial r_t$

In order to compute the effects of the real wage per effective labor, $\tilde{w}_t$, on household saving per effective labor, $\tilde{s}_t$, I take the partial derivative of Equation (D.4) (see Appendix D) with respect to $\tilde{w}_t$. Thus, assuming $1 + \rho > 0$ and $1 + r_t > 0$,

$$\frac{\partial\tilde{s}_t}{\partial\tilde{w}_t} = \frac{1}{1+[(1+\rho)^{1/\gamma}/(1+r_t)^{(1-\gamma)/\gamma}]} > 0\,. \tag{E.1}$$

Equation (E.1) implies that the real wage per effective labor affects positively household saving per effective labor.

As well, to compute the effect of the real interest rate, r_t, on $\tilde{s}_t$, I take the partial derivative of Equation (D.4) with respect to r_t as follows:

$$\frac{\partial\tilde{s}_t}{\partial r_t} = (\tilde{w}_t - \tilde{d}_t)\frac{\frac{1-\gamma}{\gamma}(1+r_t)^{\frac{1-\gamma}{\gamma}-1}(1+\rho)^{\frac{1}{\gamma}}}{\left[(1+r_t)^{\frac{1-\gamma}{\gamma}}+(1+\rho)^{\frac{1}{\gamma}}\right]^2} + \tilde{b}_{t+1}\frac{(1+r_A)(1+\rho)^{\frac{1}{\gamma}}\left[\frac{1}{\gamma}(1+r_t)^{\frac{1-\gamma}{\gamma}}+(1+\rho)^{\frac{1}{\gamma}}\right]}{\left[(1+r_t)^{\frac{1}{\gamma}}+(1+r_t)(1+\rho)^{\frac{1}{\gamma}}\right]^2}\,. \tag{E.2}$$

According to Equation (E.2), the effect of the real interest rate on household saving per effective labor is ambiguous. Since the sign of $\tilde{w}_t - \tilde{d}_t$ is positive by definition and the sign of the second term on the right-hand side is also positive, the sign of this derivative depends on the sign of $1-\gamma$. Thus, if $\gamma \le 1$, then $\partial\tilde{s}_t / \partial r_t > 0$, while if $\gamma > 1$, then the sign of $\partial\tilde{s}_t / \partial r_t$ is ambiguous (the first term on the right-hand side is negative, while the second term is positive).

Appendix F: Proof of Equation (5.18)

In order to compute the steady-state value of the capital stock per effective labor, $\tilde{k}^*$, under the fully-funded system, I first substitute Equation (D.4) (see Appendix D) into Equation (5.17), $\tilde{k}_{t+1} = \tilde{s}_t + \tilde{d}_t / [(1+n)(1+r_A)]$, as follows:

$$\tilde{k}_{t+1} = \left\{ \left(\tilde{w}_t - \tilde{d}_t \right) \left[\frac{(1+r_t)^{(1-\gamma)/\gamma}}{(1+r_t)^{(1-\gamma)/\gamma} + (1+\rho)^{1/\gamma}} \right] - \tilde{b}_{t+1} \left[\frac{(1+r_A)(1+\rho)^{1/\gamma}}{(1+r_t)^{1/\gamma} + (1+r_t)(1+\rho)^{1/\gamma}} \right] + \tilde{d}_t \right\} / [(1+n)(1+r_A)]. \quad \text{(F.1)}$$

Then, substituting Equation (5.14), $E_t(\tilde{b}_{t+1}) = dum_{ff,t} \tilde{d}_t (1+r_t)/(1+r_A)$ (ignoring E_t and setting $dum_{ff,t} = 1$), into Equation (F.1) yields

$$\tilde{k}_{t+1} = \left\{ \left(\tilde{w}_t - \tilde{d}_t \right) \left[\frac{(1+r_t)^{(1-\gamma)/\gamma}}{(1+r_t)^{(1-\gamma)/\gamma} + (1+\rho)^{1/\gamma}} \right] - \tilde{d}_t \left[\frac{(1+r_t)(1+\rho)^{1/\gamma}}{(1+r_t)^{1/\gamma} + (1+r_t)(1+\rho)^{1/\gamma}} \right] + \tilde{d}_t \right\} / [(1+n)(1+r_A)]. \quad \text{(F.2)}$$

After some algebraic calculations, Equation (F.2) can be written as follows:

$$\tilde{k}_{t+1} = \tilde{w}_t \left[\frac{(1+r_t)^{(1-\gamma)/\gamma}}{(1+r_t)^{(1-\gamma)/\gamma} + (1+\rho)^{1/\gamma}} \right] / [(1+n)(1+r_A)]. \quad \text{(F.3)}$$

Finally, substituting Equation (5.7), $\tilde{y}_t = e^{z_t} \tilde{k}_t^{\beta}$, into (5.5), $\tilde{w}_t = (1-\beta)\tilde{y}_t$, and (5.6), $r_t = \beta \tilde{y}_t / \tilde{k}_t$, and the resulting equations into Equation (F.3), setting $z_t = 0$ and $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$ yields Equation (5.18).