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Chapter 3

Forecasting volatility and risk return trade off. An application on the Fama-French benchmark market return.

Preface

This Ph.D dissertation constitutes a collection of papers written during my studies (2008-2011) at the Economic Department of Ioannina University.I am mostly grateful to my supervisor Professor Theodoros Simos for showing patience, providing me with in depth guidance and support throughout my studies and for strengthening my confidence and self-esteem by encouraging the induction of personal elements and thinking into the present project. Finally, I would like to thank the other two members of the Ph.D committee, Spiros Symeonidis and Michael Hletsos, for their support.

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Summary of the dissertation

The present dissertation is oriented towards the empirical application of certain models and econometric techniques drawn from recent developments in the financial econometric literature. The aims of this project are targeted a) in testing the proposed financial models to financial data sets, b) in enriching and strengthen the analysis by inducing new aspects into the proposed methodologies, and finally c) in producing inferences and comparing the outcomes with other results existing in many related empirical applications.

Each chapter in the present dissertation corresponds a different section of applied econometrics and therefore three empirical projects are carried out. Those are : a) the estimation and test of the joint conditional CAPM model introduced by Morelli (2011), b) the detection of fractional cointegrating relations using the variance ratio approach introduced by Nielsen (2010), and finally c) a comparative analysis of different volatility models, aiming a) the comparison of their volatility forecasting potentials under various forecasting horizons, and b) the detection of possible statistically significant volatility - return relations. Specifically :

Chapter 1 follows the approach of Morelli (2011) and carries over the estimation and test of the joint conditional CAPM model. The analysis uses the monthly returns of the 25 Fama-French portfolios in the period from July 1926 to June 2008 to evolve in two phases. The first part of the analysis through the application of four different methodologies estimates corresponding versions of the time varying beta coefficients series, while the second based on those latter estimates tests the statistical significance of the beta - return relation, especially when the last is conditioned upon the sign of excess market returns.

Note that the above methodologies correspond a) the volatility approach, where conditional covariances and variances that define the notion of conditional beta are modeled as ARCH, GARCH, FIGARCH and FIEGARCH processes, b) the recursive OLS approach, and c) the Kalman filter analysis, where two different assumptions have been applied on the definition of the state equation. Those are a) the random walk approach and the b) AR(1) alternative.

In spite of differences existing in all four versions of the estimated time varying beta coefficients series, results in all four procedures reject the conditional and the joint conditional CAPM versions, while results appear robust either when the analysis examine the full sample case or two equal sub-samples.

The key feature of chapters 2 and 3 evolves around the idea of long memory that is detected both in cointegrating relations and volatility return series. From the initial work of Granger (1981) to nowadays there has been an increasing amount of evidence supporting the presence of long memory in different financial and macroeconomic series, with the list including data over exchange rates, interest rates, indexes of production, consumption, unemployment, estimated series on volatility and many others.[1]

Chapter 2 uses daily data from the European interbank money market to examine the term structure theory on four interest rates series. As it is well known expectations hypothesis suggests the existence of long run equilibrium relations among interest rates of different maturities. The relations imply the stationary nature of spreads, while traditionally the theory is verified through cointegration analysis. However, the restrictiveness of I(0)/I(1)

[1] See for example Diebold and Rudebusch (1989), Sowell (1992), Baillie (1996), Lobato and Velasco (2000), Andersen, Bollerslev, Diebold and Ebens (2001).

dichotomy that is followed in traditional cointegration analysis and the possibility that the time series in question may be fractionally integrated, forces the present application to examine the cointegration rank through fractionally integrated systems. Indeed chapter 2 follows the non parametric variance ratio test of Nielsen (2010) and applies such a fractional analysis, while at the same time and for comparative reasons the chapter expands with the estimation of parametric tests of Johansen (1998,1991) and the fractional alternative of Breitung and Hassler (2002).

Although results on the cointegration rank differ significantly between parametric and non parametric tests, however no specific outcome can be considered generally true for the parametric alternatives, since both procedures end up with different results when different lag augmentations are being applied. Finally the paper proceeds with an informal comparison of the estimated and hypothesized cointegrating space, given that the variance ratio procedure provides a consistent estimator of the last.

Chapter 3 deals with issues on volatility modeling and volatility forecasts. The chapter uses the daily returns of the Fama-French stock market index to estimate initially different volatility models (GARCH, EGACRH, FIGARCH, IGARCH, GARCH-M, EGARCH-M, FIGARCH-M, IGARCH-M) while aims the comparison of their volatility forecasting potentials and the detection of statistically significant volatility – return relations. As far as the volatility modeling part is concerned the chapter presents an application of the exponential fractional GARCH-M model (FIEGARCH-M) that extends the basic volatility FIEGARCH framework of Bollerslev and Mikkelsen (1996) by introducing a possible volatility in mean effect. However, the analysis of Christensen and Nielsen (2007) which claims that introducing volatility in the mean equation may generate long memory in returns, forces the present application to acknowledge existence of possible spillover effects and naturally the chapter extends by estimating the filtered long memory volatility models (FIEGARCH-MG and FIEGARCH-MH) of Christensen, Nielsen and Zhu (2010). Both enter the volatility forecast comparison and both are tested for the presence of statistically significant volatility-return relations. On the distributional assumption part the chapter explores all available options and alters the estimation settings of the estimated log likelihood functions by applying the following four distributional assumptions. These are a) normality, b) t-student, c) generalized error and c) skewed asymmetric distribution.

The results suggest the existence of a statistically significant volatility in mean effect when both filtered long memory volatility models are estimated under t student,[2] while on the other hand the volatility forecast comparison indicates a solid preference to the parsimonious FIEGARCH model, since the last dominates all other alternatives irrespective of the assumed forecasting horizon.

[2] Although this is not generally valid for the other competing volatility frameworks.

Summary in Greek

Η παρούσα διδακτορική διατριβή προσανατολισμένη στην εμπειρική εκτίμηση διαφόρων οικονομετρικών υποδειγμάτων που οριοθετούν τις πιο πρόσφατες εξελίξεις στα αντίστοιχα πεδία της χρηματοοικονομικής βιβλιογραφίας, αποβλέπει τόσο στον εμπλουτισμό και την ενδυνάμωση των ήδη υπάρχοντων εμπειρικών ερευνών με νέα στοιχεία, όσο και στην παραγωγή πορισμάτων και την σύγκριση τους με αποτελέσματα παρόμοιων εμπειρικών μελετών.

Κάθε ένα από τα τρία κεφάλαια της παρούσας διατριβής εμπίπτει σε ένα διακριτό ερευνητικό χωρίο της εφαρμοσμένης οικονομετρίας, και συνεπώς τα τρια αυτοτελή εμπειρικά αντικείμενα που πραγματεύεται το παρόν κείμενο είναι : α) ο υπό συνθήκη από κοινού έλεγχος του Morelli (2011) για την εμπειρική ισχύ του υποδείγματος αποτίμησης περιουσιακών στοιχείων (CAPM), β) ο κλασματικός έλεγχος της ύπαρξης συνολοκληρώσιμων σχέσεων σε Ευρωπαϊκά διατραπεζικά επιτόκια συγκεκριμένης διάρκειας χρησιμοποιώντας την αναλογία διακυμάνσεων του Nielsen (2009), και τέλος γ) η εκτίμηση διαφόρων υποδειγμάτων μεταβλητότητας χρησιμοποιώντας τον δείκτη αγοράς των Fama-French, επιδιώκοντας αφενός την συγκριτική ανάλυση της προβλεπτικής ικανότητας των υποδειγμάτων αυτών και αφετέρου τον εντοπισμό στατιστικά σημαντικών σχέσεων στην διαδικασιά ανταλλαγής του κινδύνου με την απόδοση.

Αναλυτικά :

Το πρώτο κεφάλαιο ακολουθώντας την εμπειρική προσέγγιση του Morelli (2011) εκτιμά την εγκυρότητα του υποδείγματος CAPM πραγματοποιώντας τον προτεινόμενο από τον Morelli από κοινού υπό συνθήκη έλεγχο του υποδείγματος. Η ανάλυση χρησιμοποιώντας τις μηνιαίες αποδόσεις των 25 χαρτοφυλακίων των Fama-French την περίοδο από τον

Ιούλιο του 1926 εως τον Ιούνιο του 2008 εκτυλίσσεται σε δύο φάσεις : στην πρώτη μέσω τεσσάρων προσεγγίσεων εκτιμώνται ισάριθμες εκδοχες των μεταβαλλόμενων χρονικά βήτα, ενώ στην δεύτερη ελέγχεται η στατιστική σημαντικότητας της σχέσης μεταξυ βήτα και αποδόσεων, ειδικά όταν η τελευταία τελεί υπό την συνθήκη προσήμου των υπερβαλλουσών αποδόσεων της αγοράς.

Να σημειωθεί ότι οι εκτιμήσεις του πρώτου σταδίου αυτές αντιστοιχούν η κάθε μια σε διακριτές μεθοδολογικές προσεγγίσεις. Αυτές αναλυτικά είναι : α) στην προσέγγιση της μεταβλητότητας, όπου οι υπό συνθήκη διακυμάνσεις και συνδιακυμάνσεις που χρησιμοποιούνται στην εξαγωγή των μεταβαλλόμενων χρονικά βήτα προκύπτουν από την εκτίμηση κατάλληλα εξειδικευμένων υποδειγμάτων μεταβλητότητας, β) η προσέγγιση των επαναληπτικών εκτιμήσεων με την μέθοδο των ελαχίστων τετραγώνων, και τέλος η μέθοδοσ του φίλτρου Kalman όπου διακρίνουμε την υπόθεση γ) του τυχαίου περίπατου και δ) την αυτοπαλίνδρομη εναλλακτική. Παρά τις προφανείς αντιθέσεις μεταξυ των τεσσάρων εκτιμήσεων τα αποτελέσματα συλλήβδην των ελέγχων απορρίπτουν αμφότερες τις υπό συνθήκη εκδοχές του CAPM, τόσο στην περίπτωση του πλήρους δείγματος, όσο και σε εκείνη των δύο υποπεριόδων ίσης χρονικής διάρκειας.

Το κυρίαρχο στοιχείο των κεφαλαίων 2 και 3 αφορά το χαρακτηριστικό της μακροχρόνιας μνήμης, που προσιδιάζει τόσο τις συνολοκληρώσιμες σχέσεις διαφόρων οικονομικών μεγεθών, όσο και την μεταβλητότητα που εκτιμάται στις αποδόσεις των μετοχών.[3] Αναλυτικά :

[3]Από την αρχική εργασία του Granger (1981) μέχρι και σήμερα πληθώρα στοιχείων υποστηρίζει την παρουσία μακροχρόνιας μνήμης σε διάφορες κατηγορίες δεδομένων είτε χρηματοοικονομικών είτε μακροοικονομικών. Τέτοια δεδομένα είναι συνήθως οι συναλλαγματικές ισοτιμίες, τα διατραπεζικά επιτόκια, οι δείκτες παραγωγής κατανάλωσης και ανεργίας. Βλ Diebold & Rudebusch (1989), Sowell (1992), Baillie (1996), Lobato & Velasco (2000), Andersen, Bollerslev, Diebold & Ebens.

Το κεφάλαιο 2 αξιοποιώντας ημερήσια δεδομένα της Ευρωπαϊκης διατραπεζικής αγοράς χρήματος ερευνά την ύπαρξη μακροχρόνιων σχέσεων στις αποδόσεις τεσσάρων επιτοκίων διαφορετικής χρονικής διάρκειας.

Η διαδικασία αποφαινόμενη την στασιμότητα των διαφορών στις αποδόσεις των εν λόγο σειρών παραδοσιακά πραγματοποιείται με συνολοκληρώσιμες μεθόδους, ενω οι περιορισμοί στην ανάλυση από την "κλασική" διχοτόμηση των σειρών σε στάσιμες και μη, όπως άλλωστε η πιθανότητα της ύπαρξης κλασματικά ολοκληρώσιμων μεταβλητών επιβάλλουν ευλόγως ελέγχους αναγνώρισης κλασματικά συνολοκληρώσιμων σχέσεων.

Υπό το πρίσμα αυτό το κεφάλαιο 2 εφαρμόζοντας τον μη παραμετρικό έλεγχο της αναλογίας διακυμάνσεων του Nielsen (2010) πραγματοποιεί μια εμπειρική εκτίμηση κλασματικού τύπου, ενώ τα εξαχθέντα πορίσματα αντιπαραβάλλονται με τα αποτελέσματα εκτίμησης των παραμετρικών ελέγχων του Johansen (1998,1991) και του Breitung and Hassler (2002).

Τα αποτελέσματα υπογραμμίζουν αφενός τα διαφορετικά πορίσματα των ελέγχων σχετικά με τον αριθμού των συνολοκληρώσιμών σχέσεων, ενω αφετέρου υπονομεύουν την αξιοπιστία των παραμετρικών μεθόδων κυρίως μέσω του υφιστάμενου πλουραλισμού των αποτελεσμάτων όταν επιβάλλονται διαφορετικές προσαυξήσεις στον αριθμό των υστερήσεων. Τελικά το κεφάλαιο 2 κλείνει με έναν άτυπο έλεγχο ο οποίος συγκρίνει τον εκτιμημένο και υποθετικό συνολοκληρώσιμο χώρο.

Το κεφάλαιο 3 ασχολείται με την υποδειγματοποίηση της μεταβλητότητας και την πραγματοποίηση προβλέψεων.[4] Χρησιμοποιώντας τις ημερήσιες αποδόσεις του δείκτη αγοράς των Fama-French, η ανάλυση εκτιμά αρχικά διάφορα υποδείγματα

[4] Οι προβλέψεις προφανώς αφορούν την μεταβλητότητα.

μεταβλητότητας,[5]αποβλέποντας τόσο στην σύγκριση της προβλεπτικής τους ικανότητας, όσο και τον εντοπισμό ενδεχομένως στατιστικά σημαντικών σχέσων ανταλλαγής κινδύνου-απόδοσης.

Ειδικά στην τελευταία περίπτωση ειδικό βάρος αποδίδεται στο FIEGARCH-M, το οποίο επεκτείνει το βασικο υπόδειγμα μεταβλητότητας των Bollerslev and Mikkelsen (1996) με την εξειδίκευση μιας εξίσωσης για τον υπό συνθήκη μέσο, η οποία συνδέει τις αποδόσεις των μετοχών με την προαναφερθείσα εξειδίκευση της μεταβλητότητας.

Ωστόσο η ανάλυση των Christensen and Nielsen (2007) που κάνει λόγο για την διάχυση των ιδιοτήτων της μακροχρόνιας μνήμης από την μεταβλητότητα στις αποδόσεις των μετοχών, “υποχρεώνει” εκ νέου την ανάλυση να συμπεριλάβει στις εκτιμήσεις της τα υποδείγματα FIEGARCH-MG- FIEGARCH-MH των Christensen, Nielsen and Zhu (2010), τα οποία χρησιμοποιούν φίλτρα για την ακύρωση των προαναφερθέντων αποτελεσμάτων διάχυσης.

Αμφότερα αξιολογούνται τόσο για την προβλεπτική τους ικανότητας όσο και για τον εντοπισμό ανταλλακτικών σχέσεων κινδύνου - απόδοσης. Να σημειωθεί ότι αναφορικά με τις χρησιμοποιούμενες κατανομές στις εκάστοτε εξειδίκευσεις των λογαριθμικών συναρτήσεων πιθανοφάνειας, το κεφάλαιο διερευνά όλες τις διαθέσιμες επιλογέ και χρησιμοποιεί κατά περίπτωση την α) η κανονική, β) την t-student, γ) την γενικευμένη κατανομή των λαθών και δ) την ασύμμετρη με κύρτωση t-student κατανομή.

Σχολιάζοντας τέλος τα εμπειρικά αποτελέσματα του κεφαλαίου αξίζει να σημειωθεί ότι μόνο η εκτίμηση των υποδειγμάτων FIEGARCH-MG και FIEGARCH-MH υπο την

[5] GARCH, EGACRH, FIGARCH, IGARCH, GARCH-M, EGARCH-M, FIGARCH-M, IGARCH-M.

υπόθεση της t-student αναγνωρίζει στατιστικά σημαντικές σχέσεις ανταλλαγής κινδύνου-απόδοσης, ενώ σε ότι αφορά την σύγκριση της προβλεπτικής ικανότητας, όλα ανεξαιρέτως τα υποδείγματα μειονεκτούν σε σχέση με το FIEGARCH, που υπερέχει έναντι οποιουδήποτε άλλου σε οποιοδήποτε εκ των εξεταζομένων χρονικό ορίζοντα προβλεψης.

Chapter1

Testing Conditional CAPM using Time Varying Beta.

An Application to the Fama-French Portfolios data set.

Testing Conditional CAPM with Time Varying Beta :

An Application to the Fama-French Portfolios

A B S T R A C T

Conditional versions of CAPM utilize the idea of time varying beta coefficients, while recently Morelli (2011) following Pettengill et al. (1995) introduced a joint conditional test that explores the relation between the varying beta coefficients and the return series. His approach basically evolves in two phases : a) the estimation of time varying beta and b) the test of beta-return relation conditioned upon the sign of excess market returns. The present analysis in an attempt to access the roll of varying beta and explain the monthly excess returns of the 25 Fama-French portfolios, in a period from July 1926 to June 2008, applies Morelli's (2011) joint conditional test. However, the approach innovates by introducing various assumptions on the formation of the time varying beta coefficients series, and moreover the analysis beside applying the volatility approach of Morelli (2011), which models conditional covariances and variances through known volatility models, such as ARCH, GARCH, FIGARCH and FIEGARCH, applies three more methodologies. Those are the recursive OLS beta estimates and two Kalman filter approaches, with each introducing different assumptions on the definition of the state equation. Those are the random walk hypothesis and the AR(1) alternative. In spite of pronounced differences existing in all four versions of the estimated time varying beta series, results in all procedures reject the simple conditional and the joint conditional CAPM, while results appear robust either when examining the full sample or the two equal spread in time sub-periods. The above results are further strengthen by a monthly seasonality analysis that indicates a consistent throughout the whole year non statistically significant beta-return relation. The last is valid irrespective of the apllied methodology for the estimation of time varying beta.

Keywords: Conditional Beta, Market Risk Premium, ARCH, FIGARCH, Kalman Filter, Joint Conditionality testing

1. Introduction.

The Sharpe (1964)-Litner (1965) Capital Asset Pricing Model (CAPM) is among the most well established models in financial empirical literature postulating a linear tradeoff between expected returns and betas. The model states that the expected return of an asset is exclusively related to the market return through the estimation of the beta coefficient that defines the nature of link between the returns of an asset and the risk of the market as a whole.[6]

The model armed with the ideas of diversification and division of risk into systematic and unsystematic components, implies the existence of reward solely for the non diversifiable component of risk.[7] Therefore CAPM under simplifying assumptions,[8]concerning mainly the behavior of investors and the presence of a single common risk factor, attempts linearly to quantify the relation between the beta of an asset and its corresponding expected return.

However, the usefulness of beta as the single risk measure has been challenged in different ways. For example Chen, Roll and Ross (1986) argue that beta is not the most efficient measure of systematic risk and favor instead several macroeconomic variables that can jointly assess it.

[6]The CAPM model estimates the asset's sensitivity to the market risk, also known as non diversifiable or systematic risk. This measurement is represented by the quantity beta. The fundamental equation of CAPM is based upon the following idea, the reward to risk ratio of any individual security in the market is equal to the market reward to risk ratio.

$$(\mathrm{E}(R_i)-R_F)/\beta_i = E(R_M)-R_F \Leftrightarrow E(R_i)=R_F+\beta_i(E(R_M)-R_F)$$

where E(Ri) is the expected return of asset i, R_F is the return of the risk free asset, $E(R_M)$ is the expected return of the market portfolio and β_i denotes asset's beta.

[7]Unsystematic risk does not co-vary with the market as a whole, and therefore is considered as the additional random noise added in every asset's return equation.

[8]The first assumption of CAPM states that investors are only interested in expected returns and risk, and so if they are rational they will always try to maximize expected returns for any given level of risk. Another assumption of the model assumes that standard deviation of past returns is a perfect proxy for the future risk associated with a given security. A third assumption states that all investors have homogeneous beliefs in the process of the risk-reward trade off, while a fourth one defines the systematic risk of the market as the determinant element of the non diversifiable part of risk.

Another view mainly attributed to Lakonishok and Shapiro (1986) challenges the solitary presence of beta and states that various measures of unsystematic risk affect securities returns, while another dispute stimulated by the empirical evidence of Fama and French (1992,1993,1996) introduces in the return equation various explanatory variables beside the excess market returns and indicates that beta does not measure risk[9] and hence in terms of CAPM there is not risk-return trade off.

Despite these arguments the wide spread of the models is an undisputed fact. This preference probably stems from the convenience of using a model with a single measure of risk, although there seems to be no consensus among professionals and academics of how his key parameter should be modeled.[10]

One quest mainly triggered by the empirical evidence of Ferson (1989), Ferson and Harvey (1991,1993), Ferson and Korajczyk (1995), Lettau and Ludvigson (2001), Fama and French (1997, 2004), Lewellen and Nagel (2006) and Ang and Chen (2007) argues that beta coefficients and market risk premiums vary over time and hence unconditional CAPM is improper in producing correct empirical inferences.[11] In this direction Jagannathan and Wang (1996) endorse that unconditional CAPM tend to estimate statistically significant

[9]Fama-French examined CAPM with constant betas (i.e., the unconditional CAPM) and found that the model is inadequate in explaining specific asset pricing anomalies. In particular Fama-French found that unconditional CAPM cannot explain a) why portfolios of small firms outperform those of large firms, that is the size effect, b) why portfolios of firms with high book to market (B/M) ratios outperform those of firms with low (B/M) ratios, that is the B/M effect, and c) why portfolios of firms with relatively high returns in the past year outperform those of firms with relatively low past returns, that is the momentum effect.

[10]This discussion includes various aspects of the estimation procedure, while some of the most popular debates cover issues over indexes, time frames and data frequencies. See Blume (1975), Carleton and Lakonishok (1985), Klemkosky and Martin (1975), Reilly and Wright (1988).

[11]In unconditional versions of CAPM beta estimates are generated after regressing an asset's return on the return to the market portfolio. Using stationary time series this process generally produces estimates where their distributions have also time invariant properties.

alpha coefficients,[12] and hence conclude that relevant CAPM frameworks tend to create biases that favor the model rejection.

In fact the majority of empirical studies[13] provide weak or no support in favor of a stable linear relation, while on the other hand, increasing evidence suggest that expected returns and corresponding risks vary over time, and hence conditional CAPM frameworks are essential in incorporating such variations.[14]

Note that an insightful advocacy in favor of conditional CAPM is offered by Hansen and Richards (1987) and underlines two things. First, that tests incorporating conditional moments will be more powerful, and second, that absence of conditional information, as in the unconditional CAPM, will often lead to incorrect inferences about the mean variance efficiency of the market portfolio.

In spite the latter undisputed conditional CAPM endorsement the latter is not clear how it should be pursued. For example many studies on conditional CAPM such as Shanken (1990), Jagannathan and Wang (1996), Clare, O' Brien, Smith and Thomas (1996), Ferson and Harvey (1999), Lettau and Ludvigson (2001), Petkova and Zhang (2005) and Avramov and Chordia (2006) depend on instrumental variables for modeling time-varying betas and market risk premiums,[15]while another view recently demonstrated by Morelli (2011) estimates time varying beta using appropriate autoregressive conditional heteroskedastic forms (ARCH).

[12] Alpha coefficient corresponds the part of the expected excess return that is not predictable by the unconditional CAPM. The empirical evidence speculate that those estimates are possibly related to covariances generated from possible time varying betas and hence time varying risk premiums.

[13]See Banz (1981), Basu (1983), Bhandari (1988), Fama and French (1992), Grinold (1993), Davis (1994), Chan and Chui (1996), Fletcher (1997), Hung et al (2004).

[14] See Bodurtha and Mark (1991), Ng (1991), Petkova and Zhang (2005), Lewellen and Nagel (2006).

[15]Harvey (2001) states that in this case results are probably sensitive to the choice of instrumental variables, while Lewellen and Nagel (2006) argue that tests based on cross sectional regressions for the conditional CAPM, do not impose theoretical restrictions on the covariance corresponding the beta of an asset and the market risk premium.

In the latter case the volatility frameworks that model conditional functions of variances and covariances become the essential mechanisms in computing the conditional beta coefficients.[16]The conditional nature of the process guarantees incorporation of information at every discrete moment and hopefully promises a functional version of the conditional CAPM approach.

However, the entrepreneurial element in Morelli's work is concentrated upon what he calls joint conditional test. His idea basically combines the estimation of conditional betas with the methodology presented in Pettengill et.al (1995) that examines the roll of beta conditional upon the sign of realized market excess returns.

Indeed Pettengill et al (1995) underlines a fundamental contrast in the CAPM logic. They interestingly note that although CAPM is a model based upon expectations, however it uses out of necessity realized returns instead of missing expected data. This substitution is obviously instigated by the lack of expected evidence, while is entrenched in the critical assumption that realized returns accurately reflect the missing expected data.

The above choice conceals an inevitable transformation of the model since drastically alters its fundamental properties. Moreover the statement of CAPM for a positive relation between betas and expected returns that naturally leads to the implication of expected market returns always exceed the risk free rate,[17] is no longer valid. In fact the use of realized returns creates an actual possibility for the appearance of negative realized market risk premiums, an acknowledgment ultimately utilized in the return equation induced by

[16]Modeling conditional versions of CAPM using appropriate volatility frameworks is not a new approach. See Bollerslev et al. (1988), Ng (1991), Hanson and Hordahl (1998).
[17]The market risk premium in the unconditional CAPM is always assumed to be positive.

Pettengill et al (1995).[18] In this particular case the estimated betas were linked with the realized returns in statistically significant positive and negative relations, an outcome obviously underlying the importance of separating first market returns, into up and down markets.[19]

The present paper using the monthly realized excess returns of the 25 Fama-French portfolios in a period of 82 years from July 1926 to June 2008, follows the approach of Morelli (2011) and produces in it's first part of the analysis the necessary time varying beta estimates for all 25 Fama-French portfolios. When this phase is completed, the estimated coefficients are tested upon the sign of excess market returns and this process consummates the second stage of the joint conditional CAPM test.

However, there is no reason to assume as Morelli (2011) that conditional variances will necessarily follow an ARCH or GARCH process, and in this track of thinking a decision was taken for strengthening and enriching the joint conditional test by allowing other possibilities to enter the specification of conditionally heteroskedastic frameworks. Such alternatives are the long memory volatility models, FIGARCH and FIEGARCH, which both are estimated alongside traditional volatility processes.

Although returns on all 25 Fama-French portfolios are stationary processes,[20] volatility of those returns incorporates long memory characteristics and this sensibly justifies the entrance in the analysis of long memory volatility models. On the other hand the fact that excess returns on the majority of cases create leptokyrtic, positive skewed distributions

[18]Other studies following Pettengill's et al (1995) approach are Fletcher (1997), Hung et al. (2004), Faff (2001), Elsas et al. (2003), Ho et al (2006).

[19]Pettengill et al. (1995) conclude that when realized market return outperforms the risk free rate, a case which is refer to as up market, there exists a positive relation between beta and returns. On the other hand when realized market return is negative, a case which is referred to as down market, the beta - return relation turns out being a negative one.

[20] See in the appendix the results presented in table B.

forces the analysis to consider distributional assumptions beside normality and indeed at some point the paper uses t-student as an alternative assumption for the formation of the corresponding log-likelihood functions.

Using standard criteria for the volatility model selection two decision were taken : a) to model the conditional variance of the market excess returns upon the framework of FIEGARCH (1,d,1) under the assumption of t student, and b) to estimate the GARCH (1,1) for all conditional covariances, applying constantly the assumption of normality.

However, in the present analysis this latter approach is not the only one applied, since modeling time varying beta eventually implicates other two known possibilities. Those are a) the recursive OLS beta estimates and b) the kalman filter approach,[21] where two possibilities are explored when formulating the state equation. Those are the i) random walk hypothesis and ii) the AR(1) alternative.

Although the estimated beta coefficients differ significantly from case to case,[22] however, the final outcome is common in all alternatives and rejects both the simple conditional and the joint conditional CAPM, either when examining the full sample or the two equal spread in time sub-periods.

Furthermore the results of a monthly seasonality analysis that are consistent throughout the whole year, reveal a non statistically significant beta-return relation irrespective of the applied methodology for the estimation of time varying beta.

Section 2 briefly discusses the fundamental equations of the joint conditional test, while section 3 provides the mathematical tools for the estimation of Kalman filtered betas, while

[21]See Wells (1996) and Bucland and Fraser (2001) for an application of kalman filter on time varying beta.
[22] In the appendix table (k) reports the correlation matrixes over all estimated betas.

at the same time presentes empirical results on all four estimated risk premiums. Finally, section 4 concludes.

2. The joint conditional modeling.

The CAPM can be expressed as in equation (1)

$$R_{it} - R_F = a_{it} + \beta_{it}(R_{mt} - R_F) + \varepsilon_{it} \quad (1)$$

with R_{it}, R_{mt}, R_f present respectively returns of portfolio i, returns of the market portfolio m and returns of the risk free asset at time t, with β_i standing for the beta coefficient of portfolio i, defined as in the following equation

$$\hat{\beta}_i = \text{cov}(R_{it}, R_{mt}) / var(R_{mt}) \quad (2)$$

The CAPM can also be written in terms of cross sectional returns and this expression is called the security market line. The latter is presented in equation (3)

$$E(R_{it} - R_F) = \gamma_0 + \gamma_1 \beta_i \quad (3)$$

Equation (3) stands for a linear constant relation among the excess returns of portfolio i and it's beta estimate. This version is called the static or unconditional CAPM, since β_i is by default a constant term and hence conditional information play no role in determining the excess returns of portfolio i.

As was stated in the introduction the majority of empirical evidence suggest the existence of time-varying risk premiums, and if true unconditional versions of CAPM will unlikely hold. Therefore conditional CAPM expressions as in equation (4) may be more appropriate

$$E(r_{it} \mid I_{t-1}) = \beta_{i,I_{t-1}}(E(r_{mt} \mid I_{t-1})) \quad (4)$$

In equation (4) r_{it} and r_{mt} present respectively the excess returns of portfolio i and the excess returns of the market portfolio, $E(.|I_{t-1})$ states the expectation operator conditional upon the available set of econometric information at time t-1, and $\beta_{i,It-1}$ measures the systematic risk of the market. The latter is defined as in the following equation

$$\beta_{i,I_{t-1}} = \mathrm{cov}(r_{it}, r_{mt} \mid I_{t-1}) / \mathrm{var}(r_{mt} \mid I_{t-1}) \tag{5}$$

Following Morelli (2011) the analysis assumes next that the excess returns of portfolio i and the excess returns of the market portfolio can be modeled under appropriate autoregressive forms, with their general presentations given in equations (6) and (7) following

$$r_{it} = a_0 + \sum_{j=1}^{g} a_j r_{it-j} + \varepsilon_{it} \tag{6}$$

$$r_{mt} = a_0 + \sum_{j=1}^{g} a_j r_{mt-j} + \varepsilon_{it} \tag{7}$$

Morelli (2011) then states that the above excess returns are further decomposed into their expected and unexpected counterparts and these separations are expressed in equations (8) and (9)

$$r_{it} = E(r_{it} \mid I_{t-1}) + e_{it} \tag{8}$$

$$r_{mt} = E(r_{mt} \mid I_{t-1}) + e_{mt} \tag{9}$$

Next the disturbance terms of the estimated AR models provide equations (10) and (11) which finally incorporate the desired conditional expressions of covariances and variances

$$Cov(\varepsilon_{it},\varepsilon_{mt}) = Cov(\varepsilon_{it},\varepsilon_{mt} \mid I_{t-1}) + h_{it} \tag{10}$$

$$E(\varepsilon_{mt}^2) = E(\varepsilon_{mt}^2 \mid I_{t-1}) + h_{mt} \tag{11}$$

Using the conditional parts of equations (10) and (11) and the residuals generated from equations (6) and (7) the analysis proceeds with the estimation of appropriate[23]conditional heteroskedastic frameworks for both conditional variance and conditional covariances.[24] Once those estimates are finally completed the time varying betas are then generated using the following ratio

$$\hat{\beta}_{i,\mathrm{I}_{t-1}} = Cov(\varepsilon_{it},\varepsilon_{mt} \mid I_{t-1}) \big/ E(\varepsilon_{mt}^2 \mid I_{t-1}) \tag{12}$$

So, equation (4) which expresses the risk–return relation conditional upon information set I can be recast as in equation (13). In the last case the conditional information are incorporated into the estimated time varying betas through modeling conditional variances and conditional covariances as appropriate volatility forms.

$$E(r_{it} \mid I_{t-1}) = (\frac{Cov(\varepsilon_{it},\varepsilon_{mt} / I_{t-1})}{E(\varepsilon_{mt}^2 / I_{t-1})})(E(r_{mt} \mid I_{t-1}) \tag{13}$$

The conditional relation among time varying betas and returns is then tested through the following cross-sectional regression

$$r_i = a_0 + \gamma_1 \hat{\beta}_i + \xi_i \tag{14}$$

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[23]The competing conditional heteroskedastic frameworks are ARCH (1,1), GARCH (1,1), FIGARCH (1,1) and FIEGARCH (1,1). All models are estimated under either normal or t-student distribution. Using standard criteria for volatility model selection the best volatility frameworks are chosen and these frameworks model conditional variance and conditional covariances.

[24]Conditional covariances and conditional variance are denoted respectively as $E(\varepsilon_{it},\varepsilon_{mt}|I_{t-1})$ and $E(\varepsilon^2_{mt}|I_{t-1})$.

If conditional CAPM model holds then the constant term in equation (14) should be equal to zero, while the market risk premium, that is γ_1 coefficient, must be positive and statistically significant. If these conditions are met then beta is considered a statistically significant pricing risk factor.

Adjusting equation (14) to Pettengill's et al (1995) approach generates Morelli's (2011) joint conditional test. The latter requires the additional separation of market excess returns into up and down categories and doing so introduces in equation (14) a dummy variable. In this last case the cross sectional regressions are stated as in the following equation

$$r_i = a_0 + \delta\gamma_1^+\hat{\beta}_i + (1-\delta)\gamma_1^-\hat{\beta}_i + \xi_i \tag{15}$$

where δ presents a dummy variable,[25]and positive and negative symbols stand respectively for positive and negative market excess returns.

Under Pettengill's methodology beta is a significant price factor if the following conditions are met a) both γ_1 variables are statistically significant, b) both corresponding coefficients have the expected signs, and c) the constant term in equation (15) is equal to zero. The above requirements in mathematical terms are expressed in the following statement

$$\alpha_0 = 0, \bar{\gamma}_1^+ > 0, \bar{\gamma}_1^- < 0 \tag{16}$$

As Grauer and Janmaat (2010) state the econometrician in equations (14) and (15) typically tests whether the intercept is equal to zero against a two sided alternative. However these tests may be substituted by those examining the slope of the corresponding equations and particularly those approaches test whether the slope coefficients are equal to zero.

[25] $\delta=1$ if $r_{mt}>0$ and $\delta=0$ if $r_{mt}<0$

The last statement defines on each occasion the null hypothesis. However, the alternative in equation (14) is that of a positive and negative slope, while in equation (15) the alternative is that of a positive and negative slope, corresponding respectively the cases of up and down markets. Obviously when the null hypothesis are rejected the conditional and joint conditional notions of CAPM are valid.[26] So the t-statistics for the constant term and for γ_1 in equation (14) constitute a two tailed test, whereas the t-statistics for the slopes in equation (15) correspond each to one tailed test.

3. Empirical Application.

Table (C) in the appendix presents summary statistics on the excess returns of the market portfolio and the excess returns of each of the 25 Fama-French portfolios.[27] The results confirm two features that are common in all cases. Those are a) the leptokytric distribution in all excess returns and b) the statistically significant autocorrelations found even at 300 lags at some occasions.

In table (1) the analysis model the excess returns of all 25 Fama-French portfolios as appropriate autoregressive processes. The specification procedure estimates initially an AR(20) model and then tests using the likelihood ratio statistic whether the individually non significant variables can jointly be dropped. For example, the initially estimated AR (20) model that corresponds portfolio "1" estimates non statistically significant variables at different lags.[28] Looking at the likelihood ratio column and the corresponding p-values, those lagged variables are jointly dropped and so the remaining variables that determine the exact specified autoregressive framework of portfolio 1 are all reported in table (1).

[26] It is important to clarify here that any tests adopting Pettengill's et al. (1995) methodology can not be considered as tests of the CAPM model, since the relationships tested focus on realized and not expected returns.
[27] The analysis corresponds at each of the 25 Fama-French portfolios a number from 1 to 25. For details about definitions see the appendix.
[28] Those lags are 2,4,5,6,7,8,9,10,14,17,18,19,20.

Table (1) Specified autoregressive models for the market and the 25 Fama-French portfolios.

Portfolio	Lagged Variables												Subset	L.R	F-test
1	1	3	11	12	13	15	16	-	-	-	-	-	Chi^2(13)	14.985 [0.308]	7.545 [0.000]**
2	1	3	6	8	9	13	14	16	17	-	-	-	Chi^2(11)	12.509 [0.252]	10.51 [0.000]**
3	1	3	6	7	9	12	13	15	16	17	20	-	Chi^2(9)	6.484 [0.690]	9.147 [0.000]**
4	1	9	13	15	16	17	20	-	-	-	-	-	Chi^2(13)	18.8702 [0.1272]	22.54 [0.000]**
5	1	3	7	8	9	12	13	17	20	-	-	-	Chi^2(11)	2.725 [0.974]	11.85 [0.000]**
6	1	3	14	15	17	-	-	-	-	-	-	-	Chi^2(14)	14.074 [0.444]	8.132 [0.000]**
7	1	3	9	14	15	16	17	20	-	-	-	-	Chi^2(12)	9.551 [0.655]	12.4 [0.000]**
8	1	3	6	9	13	14	15	20	-	-	-	-	Chi^2(9)	8.129 [0.521]	15.1 [0.000]**
9	1	3	5	6	9	12	13	14	15	16	17	20	Chi^2(8)	2.731 [0.950]	12.84 [0.000]**
10	1	3	6	7	9	12	13	15	16	17	20	-	Chi^2(9)	7.824 [0.551]	10.4 [0.000]**
11	1	3	9	14	16	17	20	-	-	-	-	-	Chi^2(13)	8.243 [0.827]	11.65 [0.000]**
12	9	14	15	16	17	20	-	-	-	-	-	-	Chi^2(13)	10.897 [0.619]	5.401 [0.000]**
13	1	3	5	6	9	14	15	17	-	-	-	-	Chi^2(12)	11.039 [0.525]	11.73 [0.000]**
14	1	3	5	6	9	14	15	17	-	-	-	-	Chi^2(12)	11.039 [0.525]	11.73 [0.000]**
15	1	3	5	6	9	12	14	15	16	17	20	-	Chi^2(9)	5.073 [0.827]	11.35 [0.000]**
16	1	3	14	16	17	-	-	-	-	-	-	-	Chi^2(15)	9.694 [0.838)	5.858 [0.000]**
17	1	3	4	6	9	14	15	17	-	-	-	-	Chi^2(12)	13.704 [0.320]	7.56 [0.000]**
18	1	3	5	9	12	14	15	16	-	-	-	-	Chi^2(10)	16.995 [0.074]	7.147 [0.000]**
19	1	3	5	6	9	12	14	16	20	-	-	-	Chi^2(11)	8.678 [0.651]	10.25 [0.000]**
20	1	3	6	9	12	14	16	17	20	-	-	-	Chi^2(10)	9.951 [0.444]	10.62 [0.000]**
21	1	3	5	8	14	16	17	-	-	-	-	-	Chi^2(13)	10.263 [0.672]	5.481 [0.000]**
22	1	3	5	17	-	-	-	-	-	-	-	-	Chi^2(16)	17.022 [0.384]	6.928 [0.000]**
23	1	3	5	7	9	12	14	-	-	-	-	-	Chi^2(13)	14.545 [0.336]	10.02 [0.000]**
24	1	3	5	6	7	9	12	14	16	20	-	-	Chi^2(10)	12.618 [0.245]	11.7 [0.000]**
25	1	2	3	8	9	12	13	14	15	17	-	-	Chi^2(9)	8.298 [0.504]	97.92 [0.000]**
market	1	3	5	9	14	16	17	-	-	-	-	-	Chi^2(13)	12.479 [0.488]	7.166 [0.000]**

Note : Table (1) reports the lagged variables that create appropriate autoregressive forms for each Fama-French portfolio. All estimated models use a constant term. Column (L.R) reports the results on the likelihood ratio statistic. The last tests whether the missing lagged variables of the initially estimated AR(20) models can jointly be dropped. P-values are reported in the brackets. The last column reports the F-statistic for the joint significance test for the selected variables.*(**) denotes rejection at 5% (1%) significance level.

Further evidence in favor of the suggested AR specifications are found in the F-test column that assesses the joint significance of the remaining non excluded variables. The p-values of the corresponding F- statistics are reported in the brackets. The results clearly indicate that the chosen lagged variables of all AR models are jointly significant at both conventional levels of significance.

Furthermore table (D) in the appendix that reports the log-likelihood values and the Ljung-Box Q statistics on the estimated residuals at different number of lags, provide solid support for the chosen autoregressive models, since the results strongly indicate that no statistically significant autocorrelations are found at 5% significant level in all 26 AR models even at very distant lags.

So, as in Morelli (2011) a well fitted autoregressive process is what it takes to create an uncorrelated sequence of residuals from the initial excess return series. However, the statistically significant autocorrelations found on the squared residuals series at all selected number of lags, clearly indicate the presence of ARCH errors and apparently imply that residuals series may well be uncorrelated but they are not independent.

3.1 Estimating time varying beta using volatility models.

In order to proceed with the estimation of time varying beta coefficients using volatility models a two stage process is followed. The first stage estimates the conditional variance of the AR residuals correspond the excess returns of the market portfolio, and the second, estimates conditional covariances for the cross products of the latter residuals and the ones correspond each AR model of table (1). Evidently this approach assumes that both components of conditional beta, conditional variances and conditional covariances, follow a known volatility process.

Using the square error terms of the market portfolio the analysis estimates three volatility models with results reported in table (3). The competing volatility frameworks are GARCH (1,1), FIGARCH (1,d,1) and FIEGARCH (1,d,1). Note that tables (3i) and (3ii) which both are nested in table (3) estimate the above volatility models under the assumption of normality and t student respectively. Finally table (2) concentrates the fundamental mathematical expressions of all competing volatility frameworks.

Table (2) Mathematical frameworks of GARCH, FIGARCH, FIEGARCH models.

GARCH	$\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_t \sigma_{t-i}^2 = a_0 + a(L)\varepsilon_t^2 + B(L)\sigma_t^2$	$\alpha(L)=a_1L+\ldots+a_qL^q$ $B(L)=\beta_1L+\ldots+\beta_pL^P$
FI GARCH	$\sigma_t^2 = \omega\{1-\theta(L)]^{-1} + \{1-\Phi(L)(1-L)^d[1-\theta(L)]^{-1}\}\varepsilon_t^2$	$\Phi(L)=(1-\varphi_1L-\ldots-\varphi_pL^P)$ $\Theta(L)=(1+\theta_1L+\ldots+\Theta_qL^q)$
FIE GARCH	$\log(\sigma_t^2) = \omega + \Phi(L)^{-1}(1-L)^{-d}[1+\theta(L)]g(v_{t-1})$	$g(v_t) = \theta_1 v_t + \theta_2[\lvert v_t\rvert - E(\lvert v_t\rvert)]$

Note : For the general GARCH (p,q) case presented here, p refers to the number of GARCH terms, while q numbers the ARCH terms and a_0 denotes the constant term of the volatility equation. For FIEGARCH ω stands for the mean of the logarithmic conditional variance, while Φ(L) and Ψ(L) are polynomials in the lag operator, $\Phi(L)=(1-\varphi_1L)x\ldots x(1-\varphi_pL)$ and $\Psi(L)=(1+\psi_1L)x\ldots(1+\psi_qL)$ and $(1-L)^d$ denotes the fractional difference operator, with d reporting the order of fractional intergration for the log variance. The presence of long memory implies stronger persistence of shocks to volatility than the one correspond by the GARCH type model. Note that modeling log of σ^2 instead of just σ^2 implies that FIEGARCH does not require any constraints for assuring the positive sign of the expected conditional volatility. This stems EGARCH model of Nelson (1991). In contrast to GARCH, that requires non-negative coefficients in order to ensure a positive sign for the expected conditional volatility, the EGARCH model of Nelson does not impose such constraints on the parameters, since models the logarithm of the conditional variance. Furthermore the exponential or asymmetry feature of FIEGARCH is ensured by the presence of the news impact function g(.) that defines the manner in which past returns affect the current levels of volatility. Note that $v_t=\varepsilon_t/\sigma_t$ is the normalized innovation, θ_2 is the rate at which the magnitude of the normalized innovations in deviations from the mean enter into current volatility levels. θ_1 coefficient is the one generates an asymmetry on the news impact on volatility. So, if $\theta_1<0$ then negative innovations cause higher volatility than positive innovations of the same magnitude. Note that the above asymmetrical reaction to innovations does not induce unconditional skewness in returns, which instead is produced by the incorporation of an in mean feature. For the last notation see He et al (2008).

The existence of many empirical evidence endorsing the presence of long memory in volatility[29]makes natural at this point the choice of long memory specifications alongside traditional options in volatility modeling.[30] Such frameworks are the FIEGARCH and FIGARCH models that frequently have been used in the volatility literature.

[29] See Crato and de Lima (1994), Baillie et al (1996), Robinson (1991), Baillie and Morana (2007).
[30] Presumably GARCH and ARCH models are considered here the traditional volatility specifications. See Morelli (2011), Bollerslev et al (1998).

Furthermore since the empirical distribution Fama-French returns is characterized by a severe leptokyrtic shape, choosing of normality as the solitary distributional assumption may not be congruent for the properties inherited to the estimated conditional volatility levels, and therefore the analysis estimates the above volatility models under an alternative distributional assumption.

Indeed table (3ii) re-estimates GARCH (1,1), FIGARCH (1,d,1) and FIEGARCH (1,d,1) under the t-student assumption. The latter sets an interesting alternative especially if returns are characterized by a fat tail distribution.

Although all volatility models initially are estimated using one ARCH and one GARCH term, however the long memory volatility specifications under the assumption of t student turn over a non statistically significant ARCH term at both conventional levels of significance.[31]Deciding to drop the ARCH terms and continue with the restricted versions of the models is a decision eventually based upon likelihood ratio test. The related statistics and the corresponding p-values for FIGARCH and FIEGARCH models are $X^2(1)$ = 0.675919 [0.4110] and $X^2(1)$ = 0.978886 [0.3225] respectively. The p-values are reported in the brackets. Obviously both tests accept the restricted frameworks and so the reported in table (3ii) FIGARCH and FIEGARCH models use alone the GARCH term and therefore the estimated long memory volatility models become the FIGARCH(1,d,0) and FIEGARCH(1,d,0) respectively.

Once the competing volatility models are estimated a decision must taken about the framework that will ultimately model the conditional variance. This is the next step in the

[31] Those are 1% and 5%.

analysis and is equivalent as seeking the best volatility model among those presented in tables (3i) and (3ii). The comparison of results in table (3) indicate that the best volatility model is FIEGARCH (1,d,0) estimated under the t-student assumption. The model estimates a statistically significant long memory parameter, d=0.442, while it's log-likelihood value and both information criteria, Akaike and Schwarz, report respectively the highest and smallest values among all estimated volatility models.

However it seems true that the rest frameworks also exhibit well conditional heteroskedastic properties. Those properties are seen in the high p-values of the Ljung-Box Q statistics, the acceptance of null in all negative and positive size bias tests and the outcomes presented in all individual Nyblom statistics, clearly implying the stability of estimated coefficients in time.

As it has been said most of the results in table (3) endorse FIEGARCH as the best volatility model. However the Engle and Ng (1993) sign bias tests and the statistical significance of θ_2 coefficient, that denotes the rate at which the magnitude of normalized innovations in deviations from the mean enter the current volatility levels, at 1% significant level question somewhat the evident superiority of the model.

Specifically the statistical significance of the sign bias tests in all frameworks imply the strong presence of asymmetric phenomena in the Fama-French returns. The belief is strengthen by the fact that both θ_1 coefficients, estimated under either the normality or t-student assumptions are not statistically significant, although this holds true

Table (3) Estimations of GARCH (1,1), FIGARCH (1,d,1), FIEGARCH(1,d,1) using ε_{Mt}^{2}

	Table (3i) - Normal Distribution			Table (3ii)-t-student Distribution		
	GARCH (1,1)	FIGARCH (1,d,1)	FIEGARCH (1,d,1)	GARCH (1,1)	FIGARCH (1,d,0)	FIEGARCH (1,d,0)
ω	0.582 [0.021]*	0.469 [0.010]*	4.175 [0.000]**	0.929 [0.003]**	0.730 [0.011]*	3.838 [0.000]**
α_1	0.106 [0.000]**	-0.033 [0.664]	-0.558 [0.002]**	0.104 [0.000]**	-	-
β_1	0.875 [0.000]**	0.775 [0.000]**	0.796 [0.000]**	0.859 [0.000]**	0.716 [0.000]**	0.710 [0.000]**
Θ_1	-	-	-0.205 [0.057]	-	-	-0.117 [0.046]*
Θ_2	-	-	0.209 [0.000]**	-		0.147 [0.000]**
d	-	0.809 [0.000]**	0.456 [0.000]**	-	0.762 [0.000]**	0.442 [0.000]**
Logl	-2877.1	-2873.3	-2857.8	-2863.6	-2862.6	-2845.44
Alaike	5.938	5.930	5.904	5.912	5.910	5.879
Schwarz	5.953	5.945	5.934	5.932	5.930	5.909
Q(50)	36.003 [0.931]	35.585 [0.938]	40.466 [0.829]	36.349 [0.925]	35.848 [0.934]	40.096 [0.840]
Q(100)	97.339 [0.556]	97.001 [0.566]	93.361 [0.667]	96.296 [0.586]	96.270 [0.586]	93.100 [0.674]
Q(150)	145.298 [0.593]	146.139 [0.573]	135.673 [0.792]	143.625 [0.631]	145.239 [0.594]	135.915 [0.788]
Q(200)	193.862 [0.608]	192.777 [0.630]	185.610 [0.759]	192.781 [0.630]	192.601 [0.633]	184.061 [0.783]
Q(250)	242.935 [0.613]	244.064 [0.593]	235.992 [0.728]	240.833 [0.649]	243.767 [0.599]	235.113 [0.741]
S.B.T	3.629 [0.000]**	3.429 [0.000]**	3.422 [0.000]**	3.603 [0.000]**	3.453 [0.000]**	3.470 [0.000]**
N.S.B.T	1.254 [0.209]	0.173 [0.862]	2.666 [0.007]**	1.108 [0.267]	0.416 [0.676]	1.785 [0.074]
P.S.B.T	0.785 [0.432]	0.388 [0.697]	0.092 [0.926]	0.573 [0.566]	0.423 [0.671]	0.100 [0.920]
Joint	24.613 [0.000]**	25.326 {0.000]**	14.838 [0.001]**	23.359 [0.000]**	24.221 [0.000]**	15.542 [0.001]**
N_ω	0.093	0.045	0.098	0.181	0.048	0.166
N_a	0.144	-	0.189	0.258	-	-
N_d	-	0.086	0.227	-	0.072	0.074
N_β	0.106	0.061	0.132	0.210	0.040	0.078
$N_{\theta 1}$	-	-	0.295	-	-	0.070
$N_{\theta 2}$	-	-	0.213	-	-	0.147
Student DF	-	-	-	8.782 [0.000]**	9.125 [0.000]**	10.301 [0.004]**

Note : Tables reports the estimated coefficients on each estimated volatility model. P-values are reported in the brackets. Results are reported : a) for the Akaike and Schwarz information criteria, b) for the Ljung Box Q statistic at different lags, c) for the Engle and Ng (1993) ci) S.B.T, (sign bias test), cii) N.S.B.T (negative size bias test), ciii) P.S.B.T (positive size bias test), civ) joint (joint sign and size bias test), d) for the Nyblom statistic in all estimated coefficients, where for instance (N_a) corresponds to Nyblom statistic for coefficient a, whereas (N_b) corresponds to Nyblom statistic for b coefficient, e) for the Log-likelihood value. * (**) denotes rejection at

5%(1%) significance level. For Nyblom statistic note : Asymptotic 1% critical value for individual statistics = 0.75, Asymptotic 5% critical value for individual statistics = 0.47.

for different levels of significance. Particularly, if FIEGARCH is estimated using the normality assumption θ_1 turns out being not statistically significant at both conventional levels, whereas if FIEGARCH is estimated under t-student the same variable is not statistically significant at only 1% significant level.

On the other hand θ_2 parameter which is statistically significant in both versions of FIEGARCH it is probably of no use, since the results clearly indicate that the negative and positive size bias tests are not statistically significant in all volatility models.[32] So, there is actually a conflict between the statistical significance of θ_2 coefficient and it's actual contribution in improving the other two volatility models, FIGARCH and GARCH, that do not incorporate terms for eliminating the size biases.

Hence, all evidence conduce that the news impact function is not the competitive edge of FIEGARCH. However the analysis, principally motivated by the competitive values reported for the log-likelihood function and the corresponding estimated information criteria, decides to model the conditional variance upon the FIEGARCH framework, using the t-student distribution.

As for the cross products of error terms [33] the paper in all 25 cases estimates a GARCH (1,1) model under the assumption of normality. The estimated coefficients and other corresponding results are all reported in tables (E) and (F) in the appendix. Note that using alternative distributions or estimating long memory volatility models for these cross

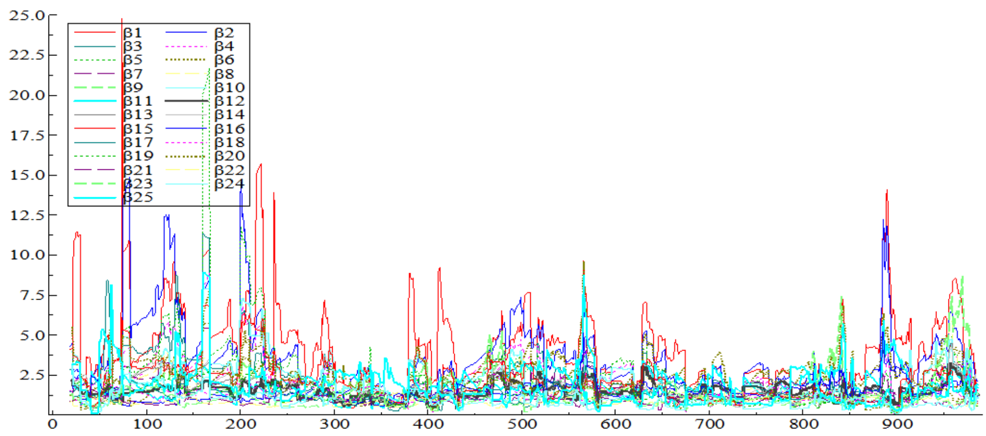
[32]This result is true for all cases except the one correspond to negative size bias test for FIEGARCH under normal distribution

[33]The errors terms here refer to residuals generated from the autoregressive model correspond the market and each individual Fama-French portfolio. The cross products are estimated using the R program and particularly the cross-covariance function of package “NCF”. Modeling these cross products through a volatility model generates the conditional covariances.

products generally delivers irrational outcomes and that's why the paper avoids at this point the use of such complex volatility forms.

However, the results presented in tables E and F in the appendix provide strong evidence that GARCH (1,1) may in fact be the correct specified form, since results at the Ljung-Box Q statistics indicate the absence of serial correlation in the residuals at all selected lags, while the sign and size bias tests imply the absence of relative asymmetric phenomena[34] and the individual Nyblom statistics indicate the absence of structural breaks. The next step in the analysis uses equation (12) to estimate the 25 time varying betas. Table (G) in the appendix presents their summary statistics, while graph (1) below prints their graphical representation. The feature in table (G) that intrigues the most is the characteristic of long memory that designates certain estimated beta series. Indeed looking at the column reporting the long memory estimates the analysis detects 9 cases where these estimates are above the threshold point of 0.5 and at the same time are statistically significant.[35]

Graph (1) Time varying beta of the 25 Fama-French portfolios : "The volatility approach".



[34] If asymmetries were present then GARCH (1,1) should estimate statistically significant sign and size bias tests The fact that in all cases the asymmetrical subtests are statistically insignificant provides strong evidence against this hypothesis.

[35]The presence of long memory can be defined can be defined in terms of persistence in the observed autocorellations. Fractional integration in a series y_t can be described as $(1-L)^d(y_t-\mu)=u_t$ where L is the lag operator, d is the fractional integration parameter, μ is the expectation of y_t and u_t is considered a stationary short memory disturbance with zero mean. If $|d|>1/2$ y_t is non stationary and has long memory. If $0<d<1/2$ y_t is stationary, while for $-1/2<d<0$ y_t is stationary and is referred to as anti-persistent.

Note that returns in all 25 Fama-French portfolios constitute stationary processes, as clearly can be seen in the results presented in table (B) in the appendix.

So now the analysis is in a position to test whether the beta coefficient is indeed a pricing risk factor. In order to do that we estimate cross sectional regressions as in equation (14). The regression is estimated over every month and a total of 965 regressions are performed. Summary statistics on the estimated γ_1 coefficients are presented in table (G) in the appendix under the reference name "risk premium (I)", while graph (2) below presents it's graphical evolution.

Graph (2) Estimated risk premium over the full sample period using the "volatility approach".

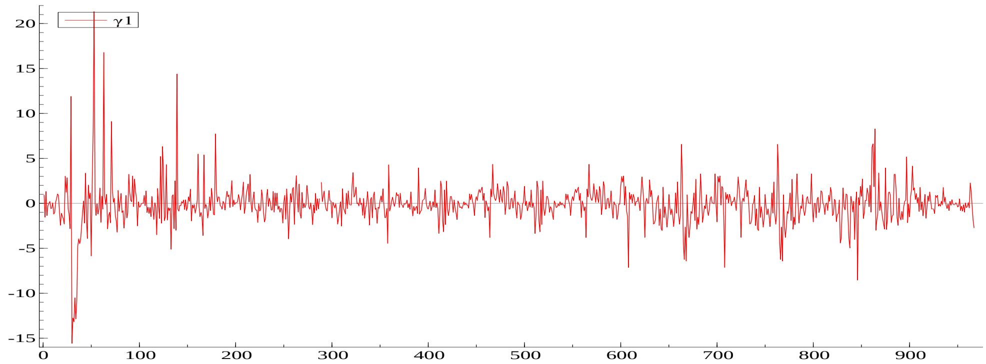


Table (3) estimates average risk premiums over the total period and the two equal spread in time sub-periods, each extending over 41 years.[36] The brackets below the estimated means report the corresponding t-statistics.[37] Irrespective of the sample period the results clearly indicate a non statistically significant negative risk premium and hence the

[36] Splitting the sample into two equal sub-periods provides more robust testing.

[37] The statistical significance here refers to the outcome of the following hypothesis test : $H_0 : \overline{\gamma}_1 = 0$ against the alternative $H_0 : \overline{\gamma}_1 \neq 0$. Note that γ_1 denotes stands for the average risk premium.

outcomes imply the rejection of conditional CAPM.[38]Note that the null hypothesis $H_0 : \overline{\gamma}_1 = 0$ is against the two sided alternative $H_1 : \overline{\gamma}_1 \neq 0$.

Table (3) Average risk premiums and t statistics over the total period and two equal sub-periods

Average risk premium	Full Sample	Sub-period 1	Sub-period 2
$\overline{\gamma}_1$	-0.106 [-0.046]	-0.050 [-0.019]	-0.164 [-0.086]

Note : The table reports average risk premiums of the full sample and two equal sub-periods, obtained from monthly cross section regressions. In the brackets the t-statistic values are reported. *(**) denotes rejection at 5% (1%) significance level.

So, under the volatility approach the estimated time varying beta coefficients have no explanatory power over the formation of Fama-French portfolios returns.[39] However, as was stated in the introduction using realized returns instead of real expected data tends to violate certain fundamental aspects of the CAPM model and since the absence of expected returns is a rather insurmountable issue, it is useful at this point to apply the methodology presented in Pettengill.et al (1995).

Under the dichotomy of market excess returns into up and down categories table (4) reports the estimated average risk premiums[40] and hence presents outcomes on the joint conditional test.

Table (4) Average risk premium (I) following the methodology of Pettengill et al (1995).

Average risk premium	Full Sample	Sub-period 1	Sub-period 2
$\overline{\gamma}_1^+$	0.400 [0.172]	0.578 [0.218]	0.207 [0.110]
$\overline{\gamma}_1^-$	-0.856 [-0.422]	-1.089 [-0.482]	-0.651 [-0.368]

Note: Both significance tests are based on one–tailed test. In the brackets t-statistics are reported.*(**) denotes rejection at 5% (1%) significance level.

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[38]These findings are consistent with a number of studies. See for example Davis (1994), Fama and French (1992), Pettengill et al (1995)

[39]Pettengill et al. (1995) notes that an insignificant beta can be attributed to the aggregation of data during periods where the excess market return is positive and negative.

[40]The excess market returns are divided in two categories. The ones correspond the up market and the rest which belong to the down market. The risk premiums are estimated separately using cross sectional regressions and the monthly estimates are averaged so that the following two hypotheses can be tested : a) $H_0 : \overline{\gamma}_1^+ = 0$ against the alternative $H_1 : \overline{\gamma}_1^+ > 0$, and b) $H_0 : \overline{\gamma}_1^- = 0$ against the alternative $H_1 : \overline{\gamma}_1^- < 0$.

The results in contrast with the outcomes reported in Pettengill et al. (1995) present an insignificant positive and negative relation in the up and down markets respectively. The results are valid both in the full sample and the two equal extended in time sub-periods, and so clearly results suggest that time varying beta coefficients cannot be regarded statistically significant pricing risk factors even when conditioning on the sign of excess market returns. Therefore the notion of Pettengill et al (1995) as this is integrated in the joint conditional test is clearly rejected.

3.2 Estimating time varying beta using i) the recursive OLS estimates and ii) the Kalman filter approach.

The next section estimates time varying conditional betas using two different methodologies. Those are a) the recursive regression approach and b) the Kalman filter analysis.

3.2.1 The recursive OLS approach.

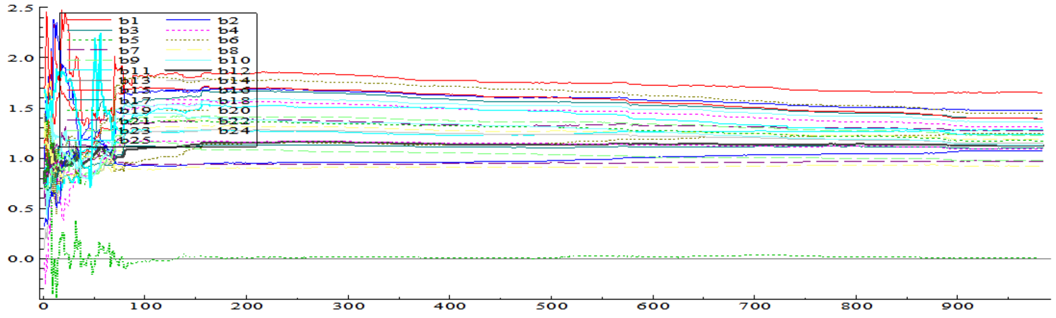
Graph (3) presents graphical representation of all 25 recursively estimated betas, while graph (4) after having applied cross sectional regressions, for every single available[41] time period, generates the corresponding market risk premiums.[42] Obviously the above estimations differ significantly from the ones presented in the previous procedure.[43]

Table (5) resumes results in all aspects of the estimated γ_1 coefficient which the analysis refer to as risk premium (II). As usual γ_1 stands for the estimated risk premium of the simple conditional CAPM model, while $\overline{\gamma}_1^+$ and $\overline{\gamma}_1^-$ both refer to the joint conditional

[41] Availability here refers to the availability of data at certain points in time.
[42] The recursive OLS betas are estimated using the Eviews 6 program.
[43] Table (H) in the appendix concentrates the descriptive statistics of the 25 recursively OLS estimated betas.

CAPM approach. Again the results show that the null hypothesis,[44] on either the conditional or the joint conditional tests cannot be rejected, while this is true irrespective of the examined sample period.

Graph (3) Time varying beta of the 25 Fama-French portfolios using the recursive OLS approach**.**



Graph (4) The estimated risk premiums over the full sample period using the recursive OLS beta approach.

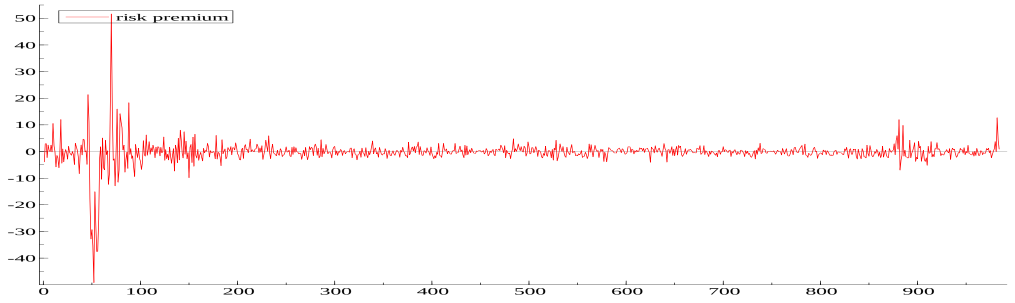


Table (5) Average risk premiums (II) following the methodology of Pettengill et al (1995).

Average risk premium	Full Sample	Sub-period 1	Sub-period2
$\overline{\gamma}_1$	-0.291 [-0.064]	-0.488 [-0.079]	-0.094 [-0.053]
$\overline{\gamma}_1^+$	0.560 [0.135]	0.754 [0.139]	0.289 [0.168]
$\overline{\gamma}_1^-$	-1.561 [-0.329]	-2.648 [-0.406]	-0.599 [-0.359]

Note: Significance tests on γ_1(+) and γ_1(-) are both based on a one–tailed probability. Significance test on γ_1 is based on a two tailed probability.*(**) denotes rejection at 5% (1%) significance level.

Hence the recursively estimated time varying betas does not have any explanatory power over the formation of returns of the 25 Fama-French portfolios.

[44] $H_0 : \overline{\gamma}_1^- = 0, H_0 : \overline{\gamma}_1^+ = 0, H_0 : \overline{\gamma}_1 = 0$

3.2.2 The kalman filter approach

As mentioned earlier an alternative procedure in the estimation of time varying beta coefficients is the Kalman filter analysis. The latter constitutes a recursive algorithm for the estimation of the systematic risk of the market and generally operates through the induction of new information every time the fundamental regression is repeating itself.[45] In general lines Kalman filter is consider being a dynamic system that follows a state space regression that is briefly discussed in the following lines.

Suppose we have n different observations at time t that contain k different signals with additive noise such as in equation (17)

$$Y_t = CX_t + \varepsilon_t \tag{17}$$

where Y_t is the (nx1) observation vector, X_t is the (kx1) signal vector, C is the (nxk) coefficient matrix,[46]that describes the relationship between signals and observations while ε_t is a (nx1) vector of observation noise for which we accept the following relations

$$E(e_t) = 0 \tag{18}$$

$$E(e_t e_t^{'}) = 0 \tag{19}$$

Let assume that the signal vector X_t follows a first order vector autoregression (VAR) as in the following equation[47]

$$X_t = AX_{t-1} + v_t \tag{20}$$

[45]In recursive estimation methodology, a new estimate is obtained when adding a correction term to the previous estimate. The correction is such that if the new observation is higher than the previous estimate the last is corrected upwards and vice versa. Naturally, if the new observation is equal to the previous estimate there is no meaning to change the estimate since there exist no new information.
[46] C is broadly known as the observation matrix.
[47] Equation (20) is known as state space representation.

where X_t is a (kx1) signal vector and A defines a (kxk) coefficient matrix that describes the dynamics of the system.[48]The system noise ν_t is a (kx1) vector, that exhibits similar properties as the observation noise vector. Those are resumed in the following two equations

$$E(\nu_t) = 0 \tag{21}$$

$$E(\nu_t \nu_t^{'}) = Q_t \tag{22}$$

Given N observations the problem of finding an optimum estimator is obtained by minimizing the mean variance-covariance matrix of the estimated errors. That is

$$P_N^e = E(e_N e_N^{'}) \tag{23}$$

Finally, the estimator at time N may be written as

$$\hat{X}_N = A\hat{X}_{N-1} + k_N[Y_N - CA\hat{X}_{N-1}] \tag{24}$$

where k_N is the (kxn) Kalman gain matrix defined as in (25)

$$k_N = S_N C^T [CS_N C' + R_N]^{-1} \tag{25}$$

and S_N is the (kxk) matrix defined as in the following equation

$$S_N = AP_{N-1}^e A' + Q_N \tag{26}$$

Suppose now that coefficients in a K variable regression vary across time according to the following equation

$$a_{t+1} = A_t a_t + \nu_t \tag{27}$$

where α_t is the (kx1) coefficients vector, A_t is the (kxk) system matrix similar to the one presented in equation (20), while ν_t is a (kx1) serially uncorrelated vector for which we assume that

[48] This matrix is usually called system matrix.

$$\nu_t \sim N(0, Q_t) \tag{28}$$

The regression equation is defined as

$$Y_t = X_t a_t + \varepsilon_t \tag{29}$$

where Y_t is a (nx1) observation vector, X_t is a (nxk) matrix of independent variables and the disturbance term is serially uncorrelated vector for which we assume

$$\varepsilon_t \sim N(0, R_t) \tag{30}$$

Kalman filter sets a recursive process of three steps. The algorithm starts with the prediction of the signal. At time N-1 the best estimate of the vector coefficient α_N before the observation arrives at time N is given by equation (31) which constitutes the prediction step.

$$\hat{a}_N = A_{N-1}\hat{a}_{N-1} \tag{31}$$

After arrival of observation Y_N three quantities exist : a) the estimate of the signal at time N-1, b) the estimate of observation and finally c) the difference between the actual observation and its estimate which is named innovation.

The new information used in kalman filtering is not the observation itself but the difference between the observation and its estimate. This step is called innovation accounting. Finally, the last step which is called update, weights innovation by the Kalman gain and adds the latter to the estimate of the signal in order to provide an updated estimate. The last is presented in equation (32)

$$\hat{a}_N = A_{N-1}\hat{a}_{N-1} + k_N (Y_N - X_N A_{N-1}\hat{a}_{N-1}) \tag{32}$$

where k_N (kxn) matrix is the Kalman gain defined in equation (33)

$$k_N = S_N X_N^{'} [X_N S_N X_N^{'} + R_N]^{-1} \qquad (33)$$

$$S_N = A_{N-1} P_{N-1}^{e} A_{N-1}^{'} + Q_{N-1} \qquad (34)$$

Equation (31) is flexible and allows the OLS estimates of the coefficients in the regression.[49] However, the system matrix can be specified in more than one ways and therefore we have a range of choices over this matter. In regards with our application two alternatives are performed. Those are : a) the random walk hypothesis and b) the autoregressive alternative.[50]

In terms of CAPM these choices imply two things. First that the observation equation is basically the market line

$$y_{it} = \beta_{it} x_{it} + \varepsilon_{it} \qquad (35)$$

where $y_{it}= r_{it}-r_{0t}$ stands for the excess return of asset or portfolio i at time t, $x_{it}=r_m - r_{ot}$ denotes the excess return of the market portfolio also at time t, and ε_{it} is without serial correlation residual that follows the normal distribution with zero mean and $\sigma^2_{\varepsilon i}$ variance.

The second thing implied is that the beta of asset i will follow either the random walk model

$$\beta_{it} = \beta_{i,t-1} + v_{it} \qquad (36)$$

or the first order autoregressive alternative

$$\beta_{it} - \beta = p_{\beta}(\beta_{i,t-1} - \beta) + v_{it} \qquad (37)$$

---

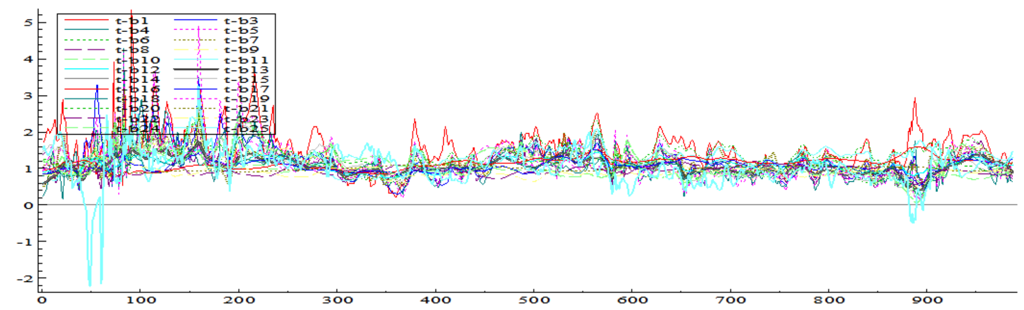
[49] The last requires setting A_t=I and Q_t=0

[50] Note that estimation of the time varying beta coefficients using the Kalman filter approach is carried out with Stamp 8.2 program.

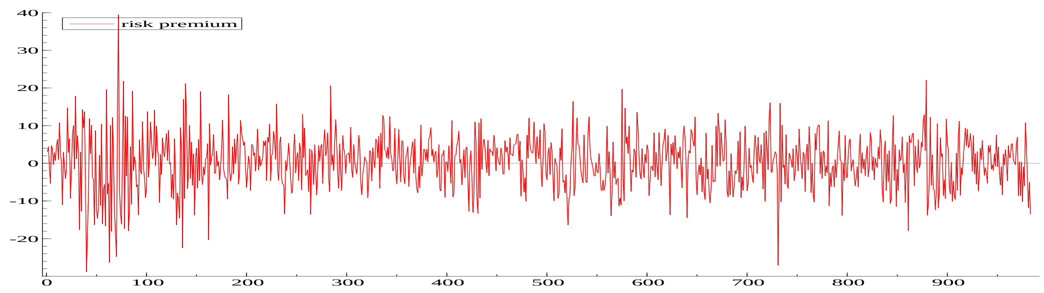
with v_{it}~NID(0,σ^2_v) and β defined as a long term coefficient.[51]

Using the Kalman filter approach and assuming that time varying beta coefficients follow the random walk model as in equation (36), the analysis estimates next all 25 recursively estimated betas. Note that graph (5) below generates their graphical output, while graph (6) presents their corresponding market risk premiums which are denoted as risk premiums (III). These are generated after estimating the cross sectional regressions of equations (14) and (15) for every time period that there exists availability of data.

Graph (5) Time varying beta using the Kalman Filter approach and assuming a random walk state equation.



Graph (6) Risk premium (III) using the kalman filtered beta and assuming a random walk model for the state equation.



[51] Given an initial estimate of beta β_0 and an initial prediction error variance p^e_o the beta can be estimated recursively through Kalman Filtering. Note that an initial beta estimate and a prediction of the observation equation disturbance variance are regularly obtained through the full sample OLS estimation of the market line equation.

Using the estimated risk premiums printed in graph 6 the analysis moves on with the assessment of the statistical significance of the average risk premiums. Table (6) presents the corresponding results under the simple conditional CAPM approach and the methodology presented in Morelli (2011). Note that the table presents the estimated average risk premiums for the full sample period and the two sub-periods, while brackets report the estimated t statistics that correspond the null hypothesis that the average risk premium is equal to zero.

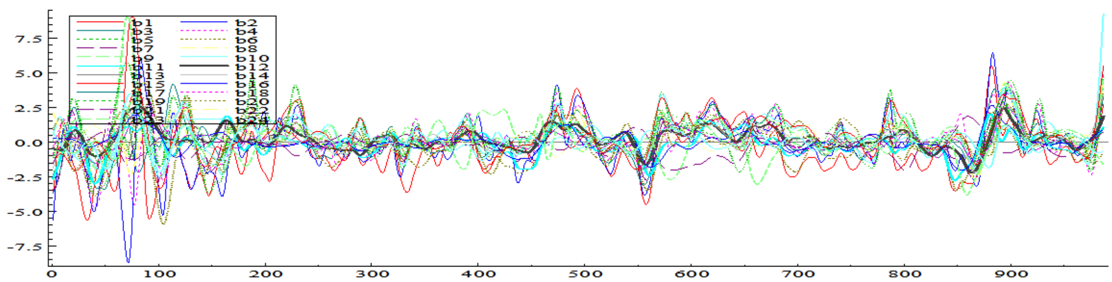
Estimated risk premiums in all 9 blocks of the table are not statistically significant at 5% significant level, although the sign of the up market risk premium in the full sample case and in sub-period 1 is not what is expected and hence this fact alone rejects the joint conditional framework. Conversely the signs of the down market, both in the full sample and in the two sub-periods are the ones expected, although again both conditional tests find no explanatory power over the time varying betas.

Table (6) Estimated risk premiums (III) using the kalman filter approach and assuming the random walk state equation.

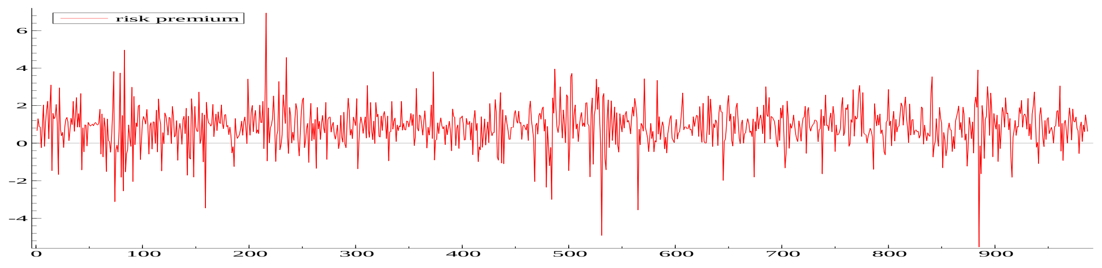
Average risk premium	Full Sample	Sub-period 1	Sub-period2
$\bar{\gamma}_1$	0.814 [0.107]	1.564 [0.018]	0.067 [0.010]
$\bar{\gamma}_1^+$	-4.875 [-0.021]	-12.924 [-0.040]	3.765 [0.754]
$\bar{\gamma}_1^-$	-4.660 [-0.769]	-4.398 [-0.589]	-4.887 [-1.092]

Note: Significance tests on $\gamma_1(+)$ and $\gamma_1(-)$ are based on a one–tailed probability. Significance test on γ_1 is based on a two tailed probability.*(**) denotes rejection at 5% (1%) significance level.

Graph (7) Time varying beta using the Kalman filtering and assuming AR(1) equation for the state equation modeling.



Graph (8) Risk premium using kalman filtered beta and assuming an AR(1) model for the state equation.



Assuming that the state equation forms a first order autoregressive framework, graphs (7) and (8) in the previous page present respectively the kalman filter estimated betas and the corresponding market risk premiums which are denoted as risk premiums (IV).[52]

Finally table (7) concentrates results on conditional and joint conditional CAPM under the AR (1) hypothesis for the state equation. Again the same conclusion drawn from all previous approaches is repeat it here, and that is all estimated risk premiums, either when examining the full sample case or the two equal spread in time sub-samples, either when conditioning cross sectional regressions on the dichotomy of up and down market excess returns or not, they are not statistically significant at 5% significant level, and therefore time varying beta measurements of risk cannot be considered as statistically significant pricing risk factors. Impression cause the unexpected positive signs of the average risk premiums reported in all three blocks of the table that correspond the down market case.

Table (7) Estimated average risk premium (IV) using the Kalman filter betas and assuming an AR(1) state equation.

Average risk premium	Full Sample	Sub-period 1	Sub-period2
$\bar{\gamma}_1$	0.887 [0.836]	0.853 [0.786]	0.920 [0.889]

[52] Note that the descriptive statistics of the Kalman filter estimated betas and the corresponding risk premiums are reported in table (J) in the appendix.

$\overline{\gamma}_1^{+}$	0.854 [0.745]	0.886 [0.750]	0.819 [0.742]
$\overline{\gamma}_1^{-}$	0.950 [1.032]	0.796 [0.887]	1.083 [1.179]

Note: Significance tests on $\gamma_1(+)$ and $\gamma_1(-)$ are all based on a one–tailed probability. Significance test on γ_1 is based on a two tailed probability.*(**) denotes rejection at 5% (1%) significance level.

Note that the issue of seasonality is extensively explored in table (L) in the appendix. The table reports a) the monthly average risk premiums and b) the corresponding t-statistics over the full sample period after re-estimating equations (14) and (15) for every month[53]and using all previous methodologies for the estimation of time varying beta. The results indicate a consistent throughout the whole year non statistically significant beta-return relation, a fact that strengthens considerably the outcomes presented in the previous parts of the analysis.[54]

4. Conclusions.

The present paper attempts to estimate and test four versions of the conditional CAPM, which are all based on the idea of time varying beta coefficients. Those versions introduce corresponding methodologies of how conditional beta can be estimated, and briefly mentioned those alternatives are : a) the volatility approach, estimates time varying beta after modeling conditional variances and conditional covariances as appropriate volatility frameworks, b) the recursive OLS approach, which estimates conditional betas from OLS regressions after adjustments made on the sample size, and finally c) the kalman filter approach, which estimates betas using recursive analysis and inducing two different assumptions on the state equation. Those are a) the random walk hypothesis and b) the AR(1) alternative.

[53]Obviously these monthly estimations require the previous separation of data according to the months of the year.

[54] The presence of strong relations found only in particular months of the years can bias results when the overall analysis is conducted. A representative example is found in Pettengill et al. (1995) . Their statistically significant unconditional beta–return relation found solely in the months of January and February tend to affect the unconditional beta-return relation through out the hall year.

Furthermore, the paper following Morelli (2011) attempts to test these conditional CAPM forms upon the sign of excess market returns, following at this point Pettengill et al. (1995) who suggest the division of excess market returns into up and down markets. This joint conditional test constitutes the core interest of the presents analysis. Although all four methods estimate completely different sets on the time varying beta coefficients series, however all methods agree that the beta measurement of risk cannot be considered a statistically significant pricing risk factor, either when the analysis examines the simple conditional CAPM or when testes the joint conditional approach. These results are further supported a) by a monthly seasonality analysis and b) by the split of the full sample into two sub-periods. In both situations the estimated risk premiums in conditional and joint conditional tests are not statistically significant and therefore time varying betas can not be regarded as statistically significant pricing risk factors.

Appendix

1.Naming the Fama-French portfolios.

The 25 Fama-French portfolios contain equal weighted returns generated from the intersection of 5 ME portfolios and 5 BE/ME portfolios. ME stands for market equity or (size) and is defined as price times number of shares, with breakpoints for every month t using all NYSE stocks for which Fama-Frech have availability over the market equity. Price is taken from CRSP, while number of shares is taken from Compustat (if available) or alternatively from CRSP. The five ME portfolios are ranked and named as “small”, “2”, “3”, “4” and “big”.

BE is defined as the book value of stockholders equity plus balance sheet deferred taxes and investment tax credit (if available) minus the book value of preferred stock.[55]The five BE/ME portfolios are ranked and named as “low”, “2”, “3”, “4”, “high”.

The intersection of ME and BE/ME portfolios naturally creates 25 new portfolios. For convenience the paper applies the following notation. The intersection of “small” and “low” portfolio create a new asset which the paper refers to as “1”. Combining “small” with portfolios “2”, “3”, “4” and “high” from the BE/ME segmentation creates respectively portfolios “2”, “3”,”4” and “5” of the present analysis. Combining now “low” with portfolios “2”, “3”, “4” and “big” which are generated from the segregation of NYSE stocks based upon the market equity generates new assets which the paper name as “6”, “11”, “16”and “21”. The same logic is applied in all cases and therefore all names can be seen in the inputs of table (A) in the next page.

[55] For definitions see the http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/variable_definitions.html

Table (A) The intersection of 5 ME portfolios and 5 BE/ME portfolios. The creation of the 25 Fama-French portfolios.

	low	**2**	**3**	**4**	**High**
small	1	2	3	4	5
2	6	7	8	9	10
3	11	12	13	14	15
4	16	17	18	19	20
big	21	22	23	24	25

Graph (1) Graphical representation of excess returns of the 25 Fama-French portfolios.

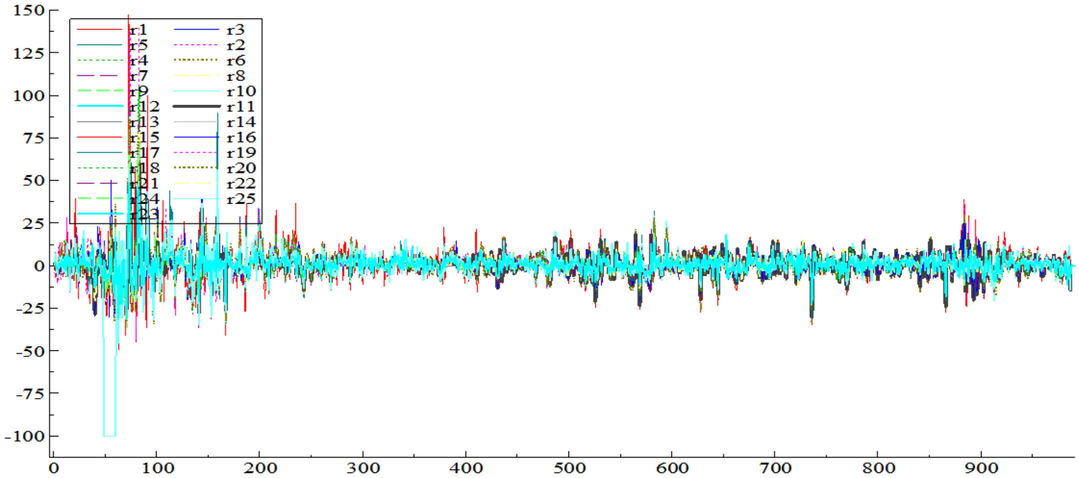


Table (B) Augmented Dickey-Fuller test for the 25 Fama-French portfolios.

Augmented Dickey-Fuller test									
1	-28.472 [0.000]**	6	-28.037 [0.000]**	11	-26.233 [0.000]**	16	-28.171 [0.000]**	21	-28.228 [0.000]**
2	-19.587 [0.000]**	7	-25.976 [0.000]**	12	-27.171 [0.000]**	17	-26.348 [0.000]**	22	-29.130 [0.000]**
3	-27.138 [0.000]**	8	-24.890 [0.000]**	13	-25.808 [0.000]**	18	-26.853 [0.000]**	23	-27.169 [0.000]**
4	-24.623 [0.000]**	9	-25.113 [0.000]**	14	25.808 [0.000]**	19	-26.788 [0.000]**	24	-19.726 [0.000]**
5	-25.410 [0.000]**	10	-25.589 [0.000]**	15	-19.297 [0.000]**	20	-19.476 [0.000]**	25	-8.241 [0.000]**

Note : *(**) denotes rejection at 5% (1%) significance level.

Table (C) Summary statistics of the market and the 25 Fama-French portfolios.

Portfolio	Mean	Skewness	Kurtosis	st.devi	Q(50)	Q(100)	Q(150)	Q(200)	Q(250)	Q(300)
1	0.433	2.762	27.912	12.358	103.496 [0.000]**	103.496 [0.000]**	205.313 [0.001]**	235.694 [0.042]*	256.573 [0.374]	284.077 [0.737]
2	0.794	4.496	57.633	10.666	154.773 [0.000]**	242.400 [0.000]**	261.578 [0.000]**	288.205 [0.000]**	304.031 [0.010]*	314.750 [0.267]
3	1.007	1.859	15.913	9.264	172.161 [0.000]**	230.168 [0.000]**	261.737 [0.000]**	290.797 [0.000]**	308.353 [0.006]**	323.406 [0.168]
4	1.176	2.866	31.341	8.671	223.280 [0.000]**	284.875 [0.000]**	323.638 [0.000]**	356.342 [0.000]**	379.028 [0.000]**	394.259 [0.000]
5	1.393	3.263	31.687	9.573	149.614 [0.000]**	214.870 [0.000]**	256.491 [0.000]**	294.120 [0.000]**	314.813 [0.003]**	332.024 [0.098]
6	0.550	0.388	5.116	8.002	82.0538 [0.002]**	136.308 [0.009]**	166.226 [0.172]	200.553 [0.475]	227.162 [0.847]	256.014 [0.968]
7	0.928	1.995	22.039	7.882	134.570 [0.000]**	189.086 [0.000]**	206.984 [0.001]**	237.540 [0.035]*	259.812 [0.321]	280.727 [0.781]
8	1.025	2.220	23.399	7.32	188.874 [0.000]**	236.402 [0.000]**	262.631 [0.000]**	293.582 [0.000]**	313.660 [0.003]**	330.700 [0.107]
9	1.090	1.843	19.084	7.586	195.143 [0.000]**	249.196 [0.000]**	279.801 [0.000]**	318.457 [0.000]**	339.871 [0.000]**	358.409 [0.011]
10	1.219	1.949	18.820	8.689	164.483 [0.000]**	219.237 [0.000]**	253.955 [0.000]**	287.976 [0.000]**	311.109 [0.005]**	330.993 [0.105]
11	0.645	1.084	10.803	7.667	139.213 [0.000]**	173.993 [0.000]**	192.576 [0.010]**	226.672 [0.094]	247.130 [0.539]	267.869 [0.908]
12	0.854	0.325	6.983	6.559	91.425 [0.000]**	136.397 [0.009]**	161.668 [0.243]	200.535 [0.476]	227.280 [0.845]	251.241 [0.981]
13	0.972	1.115	15.052	6.734	157.960 [0.000]**	202.511 [0.000]**	225.633 [0.000]**	255.069 [0.005]**	274.291 [0.139]	295.866 [0.556]
14	0.972	1.115	15.052	6.734	157.960 [0.000]**	202.511 [0.000]**	225.633 [0.000]**	255.069 [0.005]**	274.291 [0.139]	295.866 [0.556]
15	1.117	2.020	20.286	8.635	186.327 [0.000]**	253.056 [0.000]**	289.327 [0.000]**	309.493 [0.000]**	332.456 [0.000]**	351.673 [0.021]
16	0.655	-0.190	3.563	6.232	69.5623 [0.035]*	108.090 [0.272]	133.038 [0.836]	167.461 [0.954]	202.399 [0.987]	234.302 [0.998]
17	0.724	0.934	12.857	6.265	108.650 [0.000]**	147.101 [0.001]**	173.668 [0.090]	207.146 [0.349]	230.044 [0.812]	256.512 [0.967]
18	0.833	1.109	15.572	6.341	118.137 [0.000]**	166.107 [0.000]**	200.258 [0.003]**	233.848 [0.050]	252.480 [0.444]	278.634 [0.806]
19	0.941	1.959	21.410	7.002	124.827 [0.000]**	180.514 [0.000]**	205.994 [0.001]**	226.715 [0.094]	245.111 [0.575]	270.389 [0.889]
20	1.051	2.156	22.778	8.979	144.467 [0.000]**	198.368 [0.000]**	236.450 [0.000]**	259.599 [0.002]**	274.279 [0.139]	293.656 [0.592]
21	0.576	0.018	5.439	5.484	85.713 [0.001]**	134.115 [0.012]**	168.145 [0.147]	199.431 [0.498]	232.153 [0.784]	266.452 [0.918]
22	0.589	-0.044	5.314	5.236	81.791 [0.003]**	142.331 [0.003]**	180.354 [0.046]	218.500 [0.175]	245.269 [0.572]	278.760 [0.805]
23	0.647	0.926	15.177	5.728	125.412 [0.000]**	168.748 [0.000]**	202.174 [0.002]**	227.470 [0.088]	249.361 [0.499]	280.213 [0.787]
24	0.702	2.003	24.430	6.901	175.102 [0.000]**	223.899 [0.000]**	246.992 [0.000]**	267.655 [0.000]**	283.815 [0.069]	311.627 [0.310]
25	-0.26	-4.829	36.287	13.375	1939.10 [0.000]**	2000.73 [0.000]**	2014.94 [0.000]**	2029.78 [0.000]**	2041.12 [0.000]**	2061.26 [0.000]**
market	0.617	0.220	7.891	5.411	102.589 [0.000]**	155.636 [0.000]**	186.428 [0.023]*	218.700 [0.173]	242.496 [0.621]	273.336 [0.863]

Note : Q stands for Ljung-Box statistic. In the brackets the corresponding p values are reported. *(**) denotes rejection at 5% (1%) significance level.

Table (D) Logl, Ljung-Box Q and Q* statistics for the chosen autoregressive models.

P	Logl	Q(50)	Q(100)	Q(150)	Q(200)	Q*(50)	Q*(100)	Q*(150)	Q*(200)
1	-3797.54	50.305 [0.461]	108.425 [0.265]	145.576 [0.586]	173.812 [0.909]	326.403 [0.000]**	377.006 [0.000]**	386.837 [0.000]**	390.647 [0.000]**
2	-3630.51	55.594 [0.272	130.069 [0.023]	154.263 [0.388]	180.237 [0.838]	245.451 [0.000]**	275.819 [0.000]**	279.160 [0.000]**	279.794 [0.000]**
3	-3487.92	75.236 [0.012]*	144.806 [0.002]**	177.746 [0.060]	200.293 [0.480]	636.580 [0.000]**	1011.96 [0.000]**	1048.97 [0.000]**	1053.10 [0.000]**
4	-3403.94	56.7407 [0.238]	125.083 [0.045]*	168.819 [0.139]	201.943 [0.448]	543.101 [0.000]**	786.334 [0.000]**	800.229 [0.000]**	804.114 [0.000]**
5	-3517.09	53.718 [0.333]	53.718 [0.333]	171.846 [0.106]	200.645 [0.473]	301.145 [0.000]**	609.136 [0.000]**	623.970 [0.000]**	626.219 [0.000]**
6	-3380.61	37.816 [0.897]	91.755 [0.709]	129.441 [0.886]	166.027 [0.961]	373.384 [0.000]**	581.029 [0.000]**	601.953 [0.000]**	626.883 [0.000]**
7	-3331.27	30.829 [0.984]	83.132 [0.888]	106.317 [0.997]	135.779 [0.999]	512.550 [0.000]**	741.880 [0.000]**	745.177 [0.000]**	750.963 [0.000]**
8	-3249.4	37.265 [0.908]	77.078 [0.956]	106.888 [0.996]	137.892 [0.999]	646.942 [0.000]**	856.127 [0.000]**	863.812 [0.000]**	869.768 [0.000]**
9	-3274.93	38.525 [0.881]	85.401 [0.850]	116.102 [0.981]	152.877 [0.994]	885.673 [0.000]**	1224.43 [0.000]**	1237.66 [0.000]**	1245.44 [0.000]**
10	-3420.78	49.7663 [0.482]	105.150 [0.342]	140.157 [0.706]	164.879 [0.966]	557.495 [0.000]**	886.145 [0.000]**	905.566 [0.000]**	918.105 [0.000]**
11	-3313.3	40.376 [0.832]	83.516 [0.882]	107.475 [0.996]	142.509 [0.999]	591.118 [0.000]**	757.217 [0.000]**	767.148 [0.000]**	782.100 [0.000]**
12	-3189.87	61.038 [0.136]	99.874 [0.484]	128.453 [0.897	167.590 [0.953]	613.319 [0.000]**	895.443 [0.000]**	909.175 [0.000]**	920.314 [0.000]**
13	-3193.2	39.272 [0.862]	91.941 [0.704]	117.732 [0.975]	146.991 [0.998]	531.082 [0.000]**	798.170 [0.000]**	803.269 [0.000]**	807.713 [0.000]**
14	-3187.68	38.669 [0.877]	89.948 [0.754]	115.441 [0.983]	144.648 [0.998]	541.917 [0.000]**	811.589 [0.000]**	817.456 [0.000]**	821.902 [0.000]**
15	-3407.56	47.079 [0.591]	112.446 [0.186]	148.427 [0.520]	168.076 [0.951]	510.610 [0.000]**	768.755 [0.000]**	784.451 [0.000]**	788.285 [0.000]**
16	-3148.12	34.800 [0.949]	69.068 [0.992]	96.132 [0.999]	132.604 [0.999]	364.148 [0.000]**	440.829 [0.000]**	477.006 [0.000]**	497.832 [0.000]**
17	-3137.45	39.028 [0.868]	80.223 [0.927]	107.415 [0.996]	141.785 [0.999]	402.047 [0.000]**	581.619 [0.000]**	586.693 [0.000]**	593.260 [0.000]**
18	-3155.3	51.052 [0.432]	99.189 [0.504]	131.054 [0.865]	164.079 [0.970]	364.807 [0.000]**	483.449 [0.000]**	486.723 [0.000]**	490.815 [0.000]**
19	-3227.12	33.373 [0.966]	95.080 [0.620]	117.816 [0.975]	140.792 [0.999]	561.452 [0.000]**	741.105 [0.000]**	745.852 [0.000]**	748.733 [0.000]**
20	-3462.06	40.547 [0.827]	101.377 [0.442]	140.954 [0.689]	162.789 [0.974]	756.564 [0.000]**	1021.08 [0.000]**	1031.13 [0.000]**	1034.78 [0.000]**
21	-3014.96	46.411 [0.618]	92.239 [0.697]	126.818 [0.915]	161.266 [0.979]	444.836 [0.000]**	593.825 [0.000]**	615.093 [0.000]**	627.009 [0.000]**
22	-2977.15	53.517 [0.340]	104.815 [0.351]	138.885 [0.732]	180.080 [0.840]	600.027 [0.000]**	807.140 [0.000]**	831.081 [0.000]**	841.899 [0.000]**
23	-3061.54	50.289 [0.461]	95.119 [0.619]	131.672 [0.856]	158.589 [0.986]	760.161 [0.000]**	832.365 [0.000]**	835.879 [0.000]**	839.919 [0.000]**
24	-3195.96	39.166 [0.865]	94.006 [0.649]	118.066 [0.974]	141.310 [0.999]	696.672 [0.000]**	850.311 [0.000]**	852.104 [0.000]**	853.768 [0.000]**
25	-3556.83	63.379 [0.096]	128.081 [0.030]*	162.129 [0.235]	183.000 [0.800]	792.423 [0.000]**	957.573 [0.000]**	1036.71 [0.000]**	1037.30 [0.000]**
market	-2998.96	44.974 [0.674]	94.870 [0.626]	127.473 [0.908]	164.735 [0.967]	591.678 [0.000]**	787.614 [0.000]**	799.688 [0.000]**	807.480 [0.000]**

Note : Q and Q* stands respectively for the Ljung-Box statistic of residuals and squared residuals. In the brackets the corresponding p-values are reported. *(**) denotes rejection at 5% (1%) significance. Logl refers to the value of the log-likelihood value function.

Table (E) Estimating conditional covariances using the GARCH (1,1) model.

GARCH (1,1)-Normal Distribution									
	Logl	α_1	β_1	Akaike	Schwarz	S.B.T	N.S.B	P.S.B	Joint
CP1	-5702.75	0.438 [0.000]**	0.847 [0.000]**	11.762	11.772	1.116 [0.264]	0.041 [0.967]	0.691 [0.489]	1.708 [0.635]
CP2	-5634.89	0.457 [0.007]**	0.853 [0.000]**	11.622	11.632	0.453 [0.650]	0.163 [0.870]	0.119 [0.905]	0.322 [0.955]
CP3	-5408.51	0.424 [0.000]**	0.840 [0.000]**	11.155	11.165	0.523 [0.600]	0.086 [0.931]	0.331 [0.740]	0.440 [0.931]
CP4	-5369.47	0.387 [0.001]**	0.855 [0.000]**	11.109	11.119	0.294 [0.768]	0.121 [0.903]	0.477 [0.632]	0.349 [0.950]
CP5	-5411.58	0.387 [0.001]**	0.851 [0.000]**	11.196	11.206	2.362 [0.018]*	1.076 [0.281]	0.101 [0.919]	5.757 [0.124]
CP6	-5457.55	0.310 [0.004]**	0.867 [0.000]	11.256	11.266	0.558 [0.576]	0.184 [0.853]	0.729 [0.465]	0.737 [0.864]
CP7	-5338.06	0.397 [0.000]**	0.840 [0.000]**	11.044	11.054	0.649 [0.516]	0.082 [0.934]	0.537 [0.591]	0.785 [0.852]
CP8	-5260.17	0.358 [0.000]**	0.850 [0.000]**	10.883	10.893	35.902 [0.933]	91.031 [0.727]	0.569 [0.569]	0.581 [0.900]
CP9	-5290.55	0.382 [0.001]**	0.844 [0.000]**	10.946	10.956	2.168 [0.030]*	0.997 [0.318]	0.243 [0.807]	0.243 [0.807]
CP10	-5406.44	0.409 [0.000]**	0.840 [0.000]**	11.186	11.1960	0.631 [0.527]	0.288 [0.772]	0.575 [0.564]	0.995 [0.802]
CP11	-5361.43	0.481 [0.000]**	0.814 [0.000]**	11.092	11.103	0.739 [0.459]	0.117 [0.906]	0.578 [0.563]	0.908 [0.823]
CP12	-5261.59	0.380 [0.001]**	0.841 [0.000]**	10.886	10.896	0.804 [0.421]	0.634 [0.525]	0.614 [0.538]	1.216 [0.749]
CP13	-5252.21	0.347 [0.001]**	0.854 [0.000]**	10.867	10.877	0.005 [0.995]	0.532 [0.594]	0.597 [0.550]	0.684 [0.876]
CP14	-5270.66	0.361 [0.006]**	0.857 [0.000]**	10.905	10.915	0.696 [0.486]	0.190 [0.849]	0.636 [0.524]	1.177 [0.758]
CP15	-5366.74	0.399 [0.000]**	0.840 [0.000]**	11.103	11.113	0.662 [0.507]	0.291 [0.770]	0.674 [0.500]	1.191 [0.755]
CP16	-5244.2	0.377 [0.000]**	0.835 [0.000]**	10.850	10.860	0.963 [0.335]	0.046 [0.962]	1.197 [0.231]	2.224 [0.527]
CP17	-5231.73	0.393 [0.000]**	0.837 [0.000]**	10.791	10.801	0.877 [0.380]	1.128 [0.259]	0.633 [0.526]	1.864 [0.601]
CP18	-5190.41	0.433 [0.000]**	0.824 [0.000]**	10.739	10.749	0.118 [0.905]	0.033 [0.973]	0.349 [0.726]	0.149 [0.985]
CP19	-5156.77	0.298 [0.000]**	0.852 [0.000]**	10.669	10.679	0.751 [0.452]	0.015 [0.987]	0.664 [0.506]	1.065 [0.785]
CP20	-5312.31	0.339 [0.000]**	0.839 [0.000]**	10.991	11.001	1.669 [0.095]	0.384 [0.700]	0.453 [0.650]	3.354 [0.340]
CP21	-5092.09	0.381 [0.000]**	0.829 [0.000]**	10.535	10.545	1.583 [0.123]	0.571 [0.567]	0.773 [0.439]	3.374 [0.337]
CP22	-5025.93	0.316 [0.000]**	0.847 [0.000]**	10.399	10.409	0.192 [0.847]	0.051 [0.959]	0.775 [0.437]	0.689 [0.875]
CP23	-5034.75	0.329 [0.000]**	0.848 [0.000]**	10.417	10.427	0.951 [0.341]	0.108 [0.913]	0.477 [0.633]	1.587 [0.662]
CP24	-5023.36	0.300 [0.003]**	0.843 [0.000]**	10.393	10.403	1.036 [0.299]	0.031 [0.975]	0.676 [0.498]	1.518 [0.678]
CP25	-5247.8	0.353 [0.006]**	0.837 [0.000]**	10.857	10.867	1.255 [0.209]	0.052 [0.958]	0.966 [0.333]	2.200 [0.531]

Note : Q and Q* stand respectively for Ljung-Box statistic of residuals and squared residuals. In the brackets the corresponding p values are reported : *(**) denotes rejection at 5% (1%) significance level. Table reports the results on the Engle and Ng (1993) sign bias test (SBT), negative size bias test (NSBT), positive size bias test (PSBT), Joint test (Joint).

Table (F) Estimating conditional covariances using the GARCH (1,1) model.

	N_{α}	N_{β}	Q(50)	Q(100)	Q(150)	Q(200)	Q(250)
CP1	0.158	0.190	28.260 [0.994]	58.325 io[0.999]	142.670 [0.652]	175.969 [0.888]	225.366 [0.866]
CP2	0.097	0.130	29.254 [0.991]	58.584 [0.999]	134.843 [0.807]	178.410 [0.861]	220.756 [0.908]
CP3	0.141	0.189	33.315 [0.966]	78.894 [0.941]	157.683 [0.317]	212.676 [0.256]	264.729 [0.249]
CP4	0.168	0.192	32.026 [0.977]	68.589 [0.993]	148.711 [0.514]	209.176 [0.313]	254.566 [0.407]
CP5	0.151	0.159	32.466 [0.974]	73.764 [0.977]	153.098 [0.414]	206.676 [0.358]	255.006 [0.400]
CP6	0.080	0.097	30.649 [0.985]	68.930 [0.992]	140.356 [0.702]	174.529 [0.902]	222.649 [0.892]
CP7	0.126	0.163	30.961 [0.984]	76.569 [0.960]	146.350 [0.569]	200.502 [0.476]	248.962 [0.506]
CP8	0.117	0.132	35.902 [0.933]	91.031 [0.727]	160.327 [0.267]	229.890 [0.072]	284.174 [0.067]
CP9	0.152	0.164	35.998 [0.931]	80.940 [0.918]	144.667 [0.607]	219.256 [0.166]	264.257 [0.256]
CP10	0.133	0.156	30.630 [0.985]	78.002 [0.949]	150.185 [0.480]	226.689 [0.094]	269.183 [0.193]
CP11	0.109	0.193	30.721 [0.985]	70.806 [0.988]	133.595 [0.827]	171.695 [0.927]	211.300 [0.963]
CP12	0.162	0.175	34.294 [0.955]	88.744 [0.782]	159.815 [0.276]	211.535 [0.274]	255.450 [0.392]
CP13	0.150	0.174	30.185 [0.988]	83.513 [0.882]	151.270 [0.455]	223.654 [0.120]	264.022 [0.259]
CP14	0.163	0.179	28.159 [0.994]	76.183 [0.963]	150.359 [0.476]	217.574 [0.187]	253.629 [0.424]
CP15	0.144	0.182	32.076 [0.977]	86.462 [0.830]	140.239 [0.704]	213.296 [0.247]	257.480 [0.359]
CP16	0.064	0.073	28.976 [0.992]	74.160 [0.975]	132.369 [0.846]	173.028 [0.916]	215.864 [0.942]
CP17	0.141	0.170	33.305 [0.966]	81.482 [0.911]	144.171 [0.618]	192.092 [0.643]	226.532 [0.854]
CP18	0.147	0.149	34.141 [0.957]	86.507 [0.829]	151.822 [0.443]	212.753 [0.255]	251.367 [0.463]
CP19	0.099	0.162	38.227 [0.888]	102.402 [0.414]	150.516 [0.472]	214.721 [0.226]	259.317 [0.329]
CP20	0.074	0.112	31.828 [0.978]	109.791 [0.236]	166.394 [0.170]	235.938 [0.041]	298.627 [0.018]
CP21	0.038	0.046	39.188 [0.864]	95.281 [0.614]	143.509 [0.633]	195.089 [0.584]	242.217 [0.626]
CP22	0.101	0.119	34.743 [0.950]	83.240 [0.887]	141.484 [0.678]	192.409 [0.637]	235.843 [0.730]
CP23	0.059	0.052	38.835 [0.873]	90.299 [0.745]	147.089 [0.551]	203.783 [0.412]	242.117 [0.627]
CP24	0.047	0.138	45.119 [0.669]	105.933 [0.323]	154.727 [0.378]	208.990 [0.316]	257.987 [0.350]
CP25	0.074	0.173	41.519 [0.797]	105.225 [0.340]	166.601 [0.167]	225.924 [0.100]	269.440 [0.190]

Note : Q and Q* stand respectively for Ljung-Box statistic of residuals and squared residuals. In the brackets the corresponding p values are reported : *(**) denotes rejection at 5% (1%) significance level. For the individually estimated Nyblom statistic note that the 1% critical value is equal to 0.75 while the asymptotic 5% critical value is equal to 0.47. N_a and N_b stands respectively for the individually estimated Nyblom statistics of coefficient a and b.

Table (G) Descriptive statistics of a) betas estimated using volatility models and b) corresponding risk premiums (I).

Conditionally Variance/covariance beta	mean	St. dev	kurtosis	Skewness	GPH test
1	4.079	2.604	6.340	1.805	0.324 [0.018]*
2	3.504	2.316	6.093	2.162	0.480 [0.000]**
3	2.262	1.559	8.913	2.302	0.344 [0.012]*
4	2.138	1.342	3.516	1.501	0.552 [0.000]**
5	2.498	2.344	30.202	4.569	0.380 [0.005]**
6	2.347	1.239	1.169	1.169	0.441 [0.001]**
7	1.806	0.777	3.392	1.259	0.447 [0.001]**
8	1.601	0.827	2.235	1.224	0.594 [0.000]**
9	1.720	1.122	7.798	2.385	0.537 [0.000]**
10	2.234	1.294	3.631	1.499	0.554 [0.000]**
11	1.918	0.893	5.945	1.594	0.405 [0.003]**
12	1.485	0.487	0.678	0.579	0.405 [0.003]**
13	1.508	0.643	1.321	0.960	0.577 [0.000]**
14	1.579	0.695	0.455	0.829	0.615 [0.000]**
15	2.075	1.211	2.669	1.370	0.635 [0.000]**
16	1.450	0.586	5.884	1.789	0.618 [0.000]**
17	1.343	0.409	0.612	0.471	0.346 [0.011]*
18	1.284	0.418	2.478	0.983	0.322 [0.018]*
19	1.278	0.619	0.807	1.007	0.518 [0.000]**
20	1.836	1.040	1.931	1.239	0.425 [0.001]**
21	1.022	0.281	1.558	1.022	0.440 [0.001]**
22	0.895	0.245	0.023	0.533	0.337 [0.013]*
23	0.936	0.332	0.832	0.672	0.428 [0.001]**
24	0.987	0.508	1.235	1.058	0.459 [0.000]**
25	1.647	1.140	13.840	2.934	0.345 [0.011]*
Risk premium (I)	-0.106	2.290	14.006	0.919	0.295 [0.032]*

Note : In the brackets the corresponding p values are reported : *(**) denotes rejection at 5% (1%) significance level. GPH test reports the slope coefficient of the Log periodogram regression. The number of periodogram points is 31 and the bandwidth parameter is set to 0.50.

Table (H) Descriptive statistics of a) estimated betas using the recursive OLS approach and b) corresponding risk premiums (II).

Recursively generated beta	mean	St. Dev	kurtosis	Skewness	GPH test
1	1.744	0.116	10.383	1.065	0.614 [0.000]**
2	1.579	0.125	10.848	-0.231	0.350 [0.010]**
3	1.510	0.141	6.731	-2.205	1.121 [0.000]**
4	1.404	0.201	19.328	-3.819	0.973 [0.000]**
5	0.032	0.144	74.028	8.367	0.093 [0.497]
6	1.148	0.101	7.097	-2.317	0.934 [0.000]**
7	1.306	0.096	14.118	-3.413	0.828 [0.000]**
8	1.242	0.100	7.364	-1.435	0.677 [0.000]**
9	1.318	0.096	3.194	-1.673	1.470 [0.000]**
10	1.459	0.139	10.164	-2.707	0.857 [0.000]**
11	1.236	0.094	17.421	-4.125	1.018 [0.000]**
12	1.127	0.046	16.476	-3.760	0.929 [0.000]**
13	1.197	0.120	24.421	-4.256	0.558 [0.000]**
14	1.197	0.120	24.421	-4.256	0.558 [0.000]**
15	1.556	0.127	2.034	-1.046	0.811 [0.000]**
16	0.994	0.081	21.511	-3.195	0.195 [0.154]
17	1.099	0.060	16.413	-3.902	0.670 [0.000]**
18	1.119	0.071	9.188	-3.007	1.167 [0.000]**
19	1.261	0.111	6.152	-2.181	1.119 [0.000]**
20	1.609	0.172	4.252	-1.689	1.085 [0.000]**
21	0.962	0.041	14.127	3.543	0.835 [0.000]**
22	0.916	0.032	52.861	4.741	-0.061 [0.654]
23	1.027	0.062	4.645	-0.943	0.868 [0.000]**
24	1.267	0.119	2.293	-1.092	1.053 [0.000]**
25	1.410	0.134	5.322	0.220	0.138 [0.313]
Risk premium (II)	-0.291	4.506	51.685	-2.098	-0.163 [0.233]

Note : In the brackets the corresponding p values are reported : *(**) denotes rejection at 5% (1%) significance level. GPH test reports the slope coefficient of the Log periodogram regression. The number of periodogram points is 31 and the bandwidth parameter is set to 0.50.

Table (I) Descriptive statistics of a) the kalman filter estimated betas and the b) corresponding risk premiums (III) using the random walk assumption.

Kalman filter Beta Random Walk choice	mean	St. Dev	kurtosis	Skewness	GPH test
1	1.534	0.563	3.779	1.105	0.487 [0.000]**
2	1.331	0.532	5.628	1.170	0.385 [0.005]**
3	1.220	0.463	1.770	1.004	0.525 [0.000]**
4	1.124	0.468	2.616	1.126	0.457 [0.000]**
5	1.229	0.545	5.431	1.597	0.430 [0.001]**
6	1.325	0.321	0.188	0.057	0.696 [0.000]**
7	1.183	0.314	1.860	0.638	0.237 [0.084]
8	1.102	0.326	1.249	0.650	0.564 [0.000]**
9	1.114	0.318	0.807	0.524	0.485 [0.000]**
10	1.247	0.386	0.802	0.644	0.632 [0.000]**
11	1.232	0.267	-0.014	-0.053	0.572 [0.000]**
12	1.105	0.171	-0.906	-0.035	0.533 [0.000]**
13	1.060	0.213	0.335	0.102	0.540 [0.000]**
14	1.060	0.213	0.335	0.102	0.540 [0.000]**
15	1.202	0.390	0.287	0.335	0.676 [0.000]**
16	1.141	0.154	0.310	0.182	1.062 [0.000]**
17	1.059	0.149	0.513	0.087	0.472 [0.000]**
18	1.046	0.174	2.73	-0.822	0.473 [0.000]**
19	1.075	0.274	0.546	0.168	0.516 [0.000]**
20	1.217	0.375	0.523	0.527	0.694 [0.000]**
21	0.999	0.064	-1.303	-0.090	1.113 [0.000]**
22	0.941	0.107	0.448	-0.371	0.710 [0.000]**
23	0.895	0.116	1.900	0.204	0.833 [0.000]**
24	0.967	0.245	1.884	0.110	0.849 [0.000]**
25	1.060	0.517	7.475	-0.975	0.463 [0.000]**
Risk Premium (III)	-4.7805	175.96	890.86	-29.855	-0.000 [0.995]

Note : In the brackets the corresponding p values are reported : *(**) denotes rejection at 5% (1%) significance level. GPH reports the slope coefficient of the Log periodogram regression. The number of periodogram points is 31 and the bandwidth parameter is set to 0.50.

Table (J) Descriptive statistics of a) the kalman filter estimated betas and the b) corresponding risk premiums (IV) using the AR (1) assumption.

Kalman filter Beta –AR(1) choice	mean	St. Dev	kurtosis	Skewness	GPH
1	-0.547	2.061	3.137	1.043	0.084 [0.539]
2	-0.156	1.921	2.274	-0.261	0.131 [0.338]
3	0.221	1.369	0.494	0.230	0.194 [0.155]
4	0.265	1.454	1.066	-0.219	0.023 [0.864]
5	0.552	1.592	0.150	-0.049	0.239 [0.081]
6	-0.176	1.398	0.380	0.180	0.183 [0.181]
7	0.171	1.150	0.889	0.107	-0.068 [0.616]
8	0.289	1.043	1.821	0.239	0.121 [0.378]
9	0.525	1.390	10.588	2.296	0.167 [0.223]
10	0.475	1.325	2.978	0.694	0.214 [0.117]
11	-0.106	0.884	0.853	-0.558	-0.028 [0.834]
12	0.241	0.794	0.796	0.265	0.055 [0.686]
13	0.330	0.807	0.993	0.369	0.059 [0.664]
14	0.330	0.807	0.993	0.369	0.059 [0.664]
15	0.332	1.287	0.433	0.345	0.179 [0.190]
16	-0.017	0.703	2.116	0.407	0.248 [0.070]
17	0.063	0.660	2.077	0.545	0.153 [0.264]
18	0.063	0.661	2.075	0.545	0.152 [0.265]
19	0.287	1.058	6.245	1.636	0.099 [0.470]
20	0.144	1.219	4.017	-1.074	0.086 [0.528]
21	-0.232	0.686	0.742	-0.288	0.569 [0.000]**
22	0.004	0.421	1.172	0.484	0.207 [0.129]
23	-0.020	1.078	1.203	-0.451	0.178 [0.193]
24	0.040	0.748	4.097	0.367	-0.248 [0.070]
25	0.112	3.149	4.211	0.439	-0.471 [0.000]**
Risk premium (IV)	0.887	1.061	4.284	-0.445	0.004 [0.973]

Note : In the brackets the corresponding p values are reported : *(**) denotes rejection at 5% (1%) significance level. GPH reports the slope coefficient of the Log periodogram regression. The number of periodogram points is 31 and the bandwidth parameter is set to 0.50.

Table (K) Correlation matrixes estimated for a) the risk premium series and b) the time varying beta estimates using the full sample.

	Risk premium I	Risk premium II	Risk premium III	Risk premium IV
Risk premium I	1			
Risk premium II	-0.055	1		
Risk premium III	0.004	-0.018	1	
Risk premium IV	-0.031	-0.020	0.0006	1
Portfolio-1				
Beta1	1			
Beta2	0.276	1		
Beta3	0.501	0.209	1	
Beta4	0.241	-0.064	0.159	1
Portfolio-2				
Beta1	1			
Beta2	0.181	1		
Beta3	0.377	0.250	1	
Beta4	0.125	0.161	0.134	1
Portfolio-3				
Beta1	1			
Beta2	0.214	1		
Beta3	0.703	0.193	1	
Beta4	0.045	0.081	0.005	1
Portfolio-4				
Beta1	1			
Beta2	0.234	1		
Beta3	0.590	0.236	1	
Beta4	-0.016	0.151	-0.044	1
Portfolio-5				
Beta1	-0.067	1		
Beta2	0.603	-0.098	1	
Beta3	0.139	0.076	0.111	1
Beta4				
Portfolio-6				
Beta1	1			
Beta2	0.373	1		
Beta3	0.646	0.283	1	
	0.012	-0.011	0.070	1
Portfolio-7				
Beta1	1			
Beta2	0.167	1		
Beta3	0.591	0.155	1	
Beta4	0.227	0.168	0.143	1
Portfolio-8				
Beta1	1			
Beta2	0.158			
Beta3	0.644	0.238	1	
Beta4	0.155	0.226	0.092	1
Portfolio-9				
Beta1	1			
Beta2	0.070	1		
Beta3	0.330	0.294	1	
Beta4	-0.040	-0.153	0.033	1
Portfolio-10				
Beta1	1			
Beta2	0.286	1		
Beta3	0.704	0.327	1	
Beta4	0.051	0.142	0.023	1
Portfolio-11				
Beta1	1			

Beta2	0.160	1		
Beta3	0.712	0.245	1	
Beta4	-0.042	0.093	0.087	1
Portfolio-12				
Beta1	1			
Beta2	0.007			
Beta3	0.613	0.004	1	
Beta4	0.074	-0.115	0.176	1
Portfolio-13				
Beta1	1			
Beta2	0.199	1		
Beta3	0.687	0.239	1	
Beta4	0.155	-0.005	0.050	1
Portfolio-14				
Beta1	1			
Beta2	0.197			
Beta3	0.682	0.239	1	
Beta4	0.163	-0.005	0.050	1
Portfolio-15				
Beta1	1			
Beta2	0.481	1		
Beta3	0.727	0.439	1	
Beta4	0.013	-0.122	-0.039	1
Portfolio-16				
Beta1	1			
Beta2	0.444	1		
Beta3	0.760	0.498	1	
Beta4	-0.052	0.018	0.017	1
Portfolio-17				
Beta1	1			
Beta2	0.133	1		
Beta3	0.692	0.135	1	
Beta4	-0.083	0.110	-0.162	1
Portfolio-18				
Beta1	1			
Beta2	0.153	1		
Beta3	0.564	0.260	1	
Beta4	0.001	0.216	-0.035	1
Portfolio-19				
Beta1	1			
Beta2	0.329			
Beta3	0.716	0.344	1	
Beta4	0.158	-0.113	0.079	1
Portfolio-20				
Beta1	1			
Beta2	0.409	1		
Beta3	0.760	0.456	1	
Beta4	-0.107	-0.086	-0.143	1
Portfolio-21				
Beta1	1			
Beta2	0.237	1		
Beta3	0.540	0.266	1	
Beta4	0.141	0.294	0.183	1
Portfolio-22				
Beta1	1			
Beta2	0.248	1		
Beta3	0.593	0.105	1	
Beta4	-0.146	0.123	0.006	1

Portfolio-23				
Beta1	1			
Beta2	0.244	1		
Beta3	0.572	0.193	1	
Beta4	-0.170	0.183	-0.089	1
yy				
Portfolio-24				
Beta1	1			
Beta2	0.491	1		
Beta3	0.711	0.495	1	
Beta4	0.077	0.135	0.101	1
Portfolio-25				
Beta1	1			
Beta2	0.417	1		
Beta3	0.344	0.339	1	
Beta4	-0.037	-0.023	0.160	1

Note : The table reports a) the correlation matrixes of the four estimated risk premiums and b) the correlation matrixes corresponding each Fama-French portfolio using on each occasion the time varying betas that are estimated after applying four different methodologies. Specifically "Beta1" and "Beta2"correspond respectively to the volatility and recursively estimated betas, while "Beta3", "Beta 4" correspond to the Kalman Filter betas assuming respectively the random walk and the AR(1) for the formation of the state equations.

Table (L) Month by month estimates of conditional and Joint conditional CAPM models.

Month	γ_1	γ_{1+}	γ_{1-}
Volatility Approach			
January	0.034 (0.057)	0.121 (0.342)	-0.337 (-0.012)
February	0.003 (0.044)	0.023 (0.356)	-0.378 (-0.239)
March	0.036 (0.056)	0.124 (0.463)	-0.402 (-0.321)
April	0.023 (0.046)	0.234 (0.237)	-0.023 (-0.564)
May	0.086 (0.036)	0.120 (0.461)	-0.109 (-0.289)
June	-0.074 (-0.035)	0.023 (0.023)	-0.237 (-0.102)
July	0.023 (0.083)	0.036 (0.129)	-0.204 (-0.345)
August	0.056 (0.047)	0.147 (0.728)	-0.367 (-0.291)
September	0.025 (0.057)	0.245 (0.269)	-0.102 (-0.241)
October	0.093 (0.023)	0.700 (0.236)	-0.238 (-0.309)
November	-0.056 (-0.034)	0.461 (0.234)	-0.346 (-0.328)
December	0.046 (0.036)	0.209 (0.312)	-0.023 (-0.320)
Recursive OLS			
January	0.032 (0.049)	0.129 (0.445)	-0.411 (-0.367)
February	0.023 (0.032)	0.046 (0.854)	-0.561 (-0.231)
March	0.057 (0.023)	0.123 (0.341)	-0.245 (-0.459)
April	0.078 (0.012)	0.178 (0.201)	-0.201 (-0.031)
May	0.089 (0.238)	0.123 (0.309)	-0.139 (-0.342)
June	-0.032 (-0.230)	0.230 (0.210)	-0.234 (-0.438)
July	0.037 (0.034)	0.093 (0.029)	-0.561 (-0.398)
August	0.047 (0.092)	0.459 (0.034)	-0.301 (-0.467)
September	0.023 (0.003)	0.267 (0.045)	-0.662 (-0.304)
October	0.038 (0.027)	0.039 (0.038)	-0.201 (-0.345)
November	-0.032 (-0.026)	0.036 (0.342)	-0.348 (-0.256)
December	0.012 (0.023)	0.278 (0.467)	-0.561 (-0.372)
Kalman Filter- Random walk assumption			
January	0.340 (0.920)	0.109 (0.331)	-0.142 (-0.287)
February	0.020 (0.321)	0.122 (0.290)	-0.331 (-0.201)
March	0.038 (0.342)	0.311 (0.221)	-0.225 (-0.348)
April	0.093 (0.026)	0.281 (0.301)	-0.109 (-0.441)
May	0.045 (0.309)	0.002 (0.237)	-0.301 (-0.331)
June	0.063 (0.662)	0.267 (0.221)	--0.661 (-0.211)
July	0.012 (0.783)	0.201 (0.256)	-0.301 (-0.101)
August	0.014 (0.012)	0.672 (0.099)	-0.234 (-0.381)
September	0.029 (0.035)	0.320 (0.102)	-0.221 (-0.362)
October	0.010 (0.003)	0.208 (0.180)	-0.561 (-0.209)
November	0.027 (0.023)	0.291 (0.371)	-0.463 (-0.110)
December	0.122 (0.221)	0.041 (0.209)	-0.021 (-0.463)

Kalman Filter-AR(1) assumption			
January	0.020 (0.301)	0.261 (0.267)	-0.431 (-0.320)
February	0.340 (0.012)	0.781 (0.269)	-0.301 (-0.462)
March	0.002 (0.123)	0.119 (0.301)	-0.018 (-0.387)
April	0.568 (0.203)	0.278 (0.021(	-0.202 (-0.190)
May	0.021 (0.039)	0.103 (0.331)	-0.267 (-0.621)
June	0.287 (0.021)	0.108 (0.321)	-0.101 (-0.302)
July	0.107 (0.209)	0.465 (0.129)	-0.467 (-0.101)
August	0.003 (0.661)	0.209 (0.374)	-0.332 (-0.107)
September	0.022 (0.398)	0.311 (0.301)	-0.281 (-0.356)
October	0.078 (0.451)	0.219 (0.363)	-0.374 (-0.763)
November	0.125 (0.289)	0.019 (0.321)	-0.020 (-0.632)
December	0.245 (0.332)	0.077 (0.231)	-0.023 (-0.333)

Note : The table reports the monthly estimated risk premiums and the corresponding t-statistics, the last are reported in parenthesis, for the conditional and the joint conditional CAPM versions using the full sample. *(**) denotes rejection at 5% (1%).

Chapter 2

Fractional Cointegration and the Term Structure Theory of Interest Rates

Evidence from the European Interbank Money Market

Fractional Cointegration and the Term Structure Theory of Interest Rates.

Evidence from the European Interbank Money Market.

A B S T R A C T

The present paper examines the term structure theory of interest rates using daily data from the European interbank money market. The expectations hypothesis suggests the existence of long run equilibrium relations among interest rates of different maturities. The theory implies the stationary nature of spreads while traditionally is verified through cointegration analysis. However, the restrictiveness of I(0)/I(1) dichotomy and the possibility that the time series in question may be fractionally integrated, forces the present application to examine cointegration rank through fractionally integrated systems, and indeed the paper applies such a fractional analysis by following the non parametric variance ratio test of Nielsen (2010) that does not require the specification of a particular data generating process and is invariant to short run dynamics. For the period under consideration and for comparative purposes the present work also estimates the parametric tests of Johansen's (1988,1991) and the fractional alternative of Breitung and Hassler (2002). Results on the cointegration rank among non-parametric and parametric tests differ significantly, even though there seems to be no consensus on the parametric results when different lag augmentations are applied. Finally, the paper proceeds with an informal comparison between the estimated and hypothesized cointegrating space, given that a consistent estimator of the last is easily obtained through the variance ratio test.

Keywords: Fractional integration and cointegration, Interest rates, Cointegration rank, Cointegration space, Long Memory, Unit root processes, Non-parametric, Term structure, Variance ratio

1. Introduction

Over the years cointegration analysis has been one of the milestones of empirical economic research, proven extremely useful when testing the validity of term structure theory. According to expectations hypothesis the yield spread between long and short-term interest rates is an excellent predictor of future changes of short rates over the long run, and if true, the empirical research must provide strong evidence in support of the stationary nature of yield spreads series. Put it in other words, if expectations hypothesis holds then the term premium of interest rates by default is equal to zero, and this in terms of cointegration implies that short and long term interest rates constitute cointegrated series.[56]

Interestingly the empirical validity of the theory provide little evidence in support of the pure expectations hypothesis,[57] and a possible explanation for the observed deviations can be attributed to the restrictiveness of I(1)-I(0) dichotomy that characterizes all traditional methods in cointegration analysis. Indeed the standard cointegration approach allows solely integer values for the memory parameters of the system and this explains why relevant tests for the existence of cointegrating relations rely heavily on unit root tests.

However, many economic and financial series display fractional memory properties in their integration order and this naturally leads next to fractional cointegration, a method that was initially introduced by Granger (1986) and was later analyzed for properties and

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[56]This is the purest form of expectation hypothesis advanced mainly by Irving Fisher. The theory postulates that short term bonds yield the same expected returns as long term ones. This implies that forward interest rates are unbiased estimates of expected future spot rates. Note that a major stream of criticism for the pure form of expectation hypothesis stems from the fact that this ignores several related issues such as a) the risk of capital loss, and b) the unexpected inflation.

[57] See for example Cook and Hahn (1990), Dua (1991), Fama (1990), Friedman (1979), Kane (1983), McCulloch (1975), Mankiw and Miron (1986), Van Horne (1965).

characteristics by Cheung and Lai (1993), Jagannathan (1999), Marinucci and Robinson (2001) and Tsay (2000).[58]

The fractional cointergration method allows both the integration order of the observed series and the integration order of equilibrium errors to take on real values in the (0,1) area, and this justifies why the process is considered an adaptable "tool" in acknowledging and detecting possible cointegrating relations.[59]

The necessity of applying fractional instead of standard cointegration techniques emerges due to a number of reasons. One motivation arises as a result of mounting evidence supporting the existence of long run relations among long memory processes,[60] while another reason mainly demonstrated by Gonzalo and Lee (1998, 2000), states that the null hypothesis of no cointegration will be rejected more often than the nominal's level suggestion, given of course the fractional nature of the observed series.

In the past the identification and modeling of long run relations in fractional cointegrated systems has followed many approaches. Most of them built on the null hypothesis of no cointegration versus the alternative of fractional cointegration. In the semi-parametric frequency domain for example, Marinucci and Robinson (2001) apply a cointegration procedure that compares the estimates in the integration order of the observed series,[61] while Robinson and Yajima (2002) place weight on the eigenvalues problem generated by the estimation of the spectral density matrix. Furthermore an interesting view has recently

[58] Empirical applications can also be found in Booth and Tse (1995), Masih (1995, 1998), Baillie and Bollerslev (1994) , Dueker and Startz (1998).

[59] A process is integrated of order d if it's kth difference has a spectral density $f(\lambda)\sim C|\lambda|^{-2(d-k)}$, such that λ is tending to zero, C>0 and k is nonnegative integer such that d-k<0.5 (Chen and Hurvich, 2003).Consider next two processes, X_t, Y_t that are I(d) processes. We say that those series are fractionally cointegrated if there exists a linear combination $U_t=Y_t-\beta X_t$ such that U_t is $I(d_u)$ with $d_u<d$. Obviously standard cointegration is a generalization of fractional cointegration, with d_u and d set respectively to 0 and 1.

[60] See for example Cheung and Lai (1993), Diebold et al. (1994), Baillie and Bollerslev (1994).

[61] Robinson (2008) provides rigorous theoretical support of this idea.

demonstrated by Marmol and Velasco (2004) and proposes a Wald test of the null of spurious relations against the alternative of a single cointegrating relation, with a similar idea also applied in Hualde and Velasco (2008).

In the time domain the parametric trace test of Breitung and Hassler (2002) stands out and solves a generalized eigenvalue problem of the type proposed by Johansen. However, the approach as any other parametric procedure depends heavily on the correct specification of short run dynamics and therefore the usual dilemmas about the correct number of lags are inserting the fractional analysis, as is common for the Johansen parametric test.

The present paper in order to detect possible fractional cointegrating relations in a system of four interest rates drawn from the European interbank money market applies the nonparametric variance ratio test of Nielsen (2010). The test due to a number of virtues is completely separated from other alternatives that are usual seen in the fractional cointegration literature.

First, the statistic does not depend on the integration order of the observed series, while neither the statistic nor its asymptotic distribution depend on b, that is the strength of the cointegrating relation.[62] Second, inferences on the cointegration rank do not presuppose the estimation of cointegrating vectors, as contrary is true with other methods that depend heavily on some kind of regression analysis.[63] Finally, the most important advantage in applying Nielsen's variance ratio approach can be seen in the non parametric nature of the

[62]Nielsen (2010) states that this is a major advantage since b by default is unobserved and it's estimation requires the estimation of cointegration relations first. The last demands the determination of cointegration rank.

[63]See for example Marmol and Velasco (2004). Their analysis focus on the comparison of OLS and GLS estimates of cointegrating vectors. Hualde and Velasco (2008) on the other hand proceed to inferences on the cointegration rank using the GLS estimates introduced in Robinson and Hualde (2003).

test.[64] In contrast to Johansen (1998, 1991) and Breitung and Hassler (2002) fully parametric trace tests, the analysis does not depend on the misspecification of short run dynamics and therefore avoids possible misspecifications in the lag augmentation, a state that is very often responsible for the inconsistent and the erroneous estimate of cointegrating rank.[65]

Section 2 presents the fundamental mathematic equations of the variance ratio test, while section 3 uses time and frequency domain maximum likelihood methods to answer the question of whether the observed interest rates series should enter Nielsen's approach after adjustments made on the interest rates series for a non zero mean or a deterministic time trend or both. Sections 4 and 5 concentrate on the empirical results of the chapter and moreover present estimations and results over the variance ratio test and the parametric procedures of Johansen (1998, 1991) and Breitung and Hassler (2002) respectively. Finally section 6 proceeds with an informal comparison between the estimated and hypothesized cointegration space and section 7 concludes.

2. The variance ratio test

The nx1 vector Z_t is said being fractionally integrated of order d,[66] that is $Z_t \in I(d)$, if

$$z_t(1-L)^d = u_t \Leftrightarrow z_t = (1-L)^{-d} u_t = \Delta_+^{-d} u_t \tag{1}$$

[64]Even though the variance ratio test is characterized by Nielsen (2010) as non parametric the author remains cautious about accepting the term. This is because the test actually depends on a user chosen parameter, that is d_1, which induces a hall family of tests. The parameter appears in the asymptotic distribution of the test and defines decisively it's shape.

[65] For example Lutkepohl and Saikkonen (1999) note that whenever the number of lags (k) is too small relative to the true size, there can be severe size distortions, while on the other, hand significant power losses may arise if k is too large. In this case too few cointegrating relations are going to be acknowledged.

[66]The definition applies both to univariate and vector cases. Note that a univariate stationary series is one that is characterized by a continuous spectral density function bounded at the zero frequency. A vector defined as an I(d) process is one that it's d-th difference has a continuous spectral density matrix bounded, positive, semi-definite and bounded away from the zero matrix. In terms of the eigenvalues of the spectral density matrix this implies that these are non-negative, while there exists at least one eigenvalue that is bounded away from zero.

where $u_t \in I(0)$ and $(1\text{-}L)^d$ is defined by the following binomial expansion

$$(1-L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)} L^j \quad (2)$$

where $$\Gamma(j) = \int_0^{\infty} t^{z-1} e^{-t} dt \quad (3)$$

The parameter d determines the memory of the process. Specifically, for d>-1/2 vector $\mathbf{Z}_t$ is invertible, for d=0 is stationary with spectral density function bounded at the origin, for d<1/2 is covariance stationary, while for d>1/2 is long memory with spectral density unbounded at low frequencies.[67]

Fractional cointegration although replicates the basic notions of standard cointegration, however allows the observed series to be fractionally integrated. Moreover a vector of time series variables is characterized as fractionally cointegrated if all variables are integrated of order d>0.5 and at the same time exists a linear combination of the same variables with a smaller degree of integration (d-b).

In technical terms the above statement can be recast into the following definition. The nx1 vector Z_t is cointegrated, if $Z_t \in I(d)$[68] and at the same time there exists a full rank nxr matrix β, such that $\beta' Z_t \in I$ (d-b) for b>0,[69] where d and b are real numbers. The r

[67]Long memory time series are stationary processes that display a statistically significant dependence between very distant observations in time. This dependence is formalized in terms of persistence in the observed autocorellations. Specifically for a long memory process it's autocorellation function will decay at a hyperbolical rate, while the for a pure stationary process the same function will die out exponentially. Note that the hyperbolic decay of the sample autocorrelation function does not necessarily stem from a long range dependent process. In fact very often switching regime and change-point processes, display the same empirical characteristics.
[68]The integration order of a vector Z_t is determined by the highest integrated order observed among it's components. This possibly implies the existence of over-differenced elements.
[69]Obviously b can differ in each cointegrating vector and so indexing b implies the following statement $\beta_\kappa' z_t \in I(d-b_k)$ for k=1,…,r

independent columns of $\beta' Z_t$ product define the cointegration rank, while the space spanned by the columns of β constitute the cointegrating space. [70]

Using daily data from the European interbank money market the paper targets the estimation of the non parametric variance ratio test presented in Nielsen (2010). The interest rates under consideration are the Eonia and the Euribor interest rates of 7, 10 and 12 months, denoted respectively as i_{eonia}, i_7,i_{10} and i_{10}.

As was stated in the introduction, the variance ratio test contrary to conventional I(1)/I(0) cointegration analysis,[71] does not depend on the integration order of the observed series or the strength (b) of the cointegrating relations,[72] while another advantage stems from the nonparametric nature of the test that excludes the existence of possible misspecifications in the short run dynamics. Indeed the statistic contrary to most parametric cointegration rank tests, that are sensitive both to different specifications assumed by the underlying model and to the lag-augmentations employed, performs adequately well in the simulations performed in Nielsen (2010). Specifically the ratio test exhibits good size and power properties to different specifications of the simulated models, although the sample sizes, T=100 and T=250, are considered rather small for non parametric tests.

The statistic using a) the sample variances of the observed series and b) their fractional partial sum is constructed as their ratio. In the univariate case of Z_t the following notations are applied

$$z_t = \Delta_+^{-d} u_t \text{, d>1/2, t=1,2,…} \tag{4}$$

[70] Granger (1981,1983) provides an error correction formula for fractionally cointegrated systems. In particular, if y_t~I(d) is a k-dimensional vector and z_t is a set of cointegrating vectors such that z_t=a'y_t~I(d-b) then Granger proves that the appropriate error correction is $H(L)(1-L)^d y_t = -\gamma[1-(1-L)^b](1-L)^{d-b} z_t + C(L)\varepsilon_t$

[71] Stock and Watson (1988), Johansen (1988, 1991).

[72] Note that variance ratio distribution does not depend on b.

$$\bar{z}_t = \Delta_+^{-d_1} u_t \text{, } d_1>0\text{, } t=1,2 \quad (5)$$

The fractional central limit theorem that is applied for Z_t[73] assuming d>1/2 provides

equations (6) and (7). These report the sample second moments of Z_t and $\bar{z}_t$[74] and provide their corresponding convergences in distribution assuming T tends to infinity.

$$T^{-2d} \sum_{t=1}^{T} z_t^2 \xrightarrow{D} \sigma_z^2 \to \int_0^1 W_d(s)^2 ds \quad (6)$$

$$T^{-2(d+d_1)} \sum_{t=1}^{T} \bar{z}_t^2 \xrightarrow{D} \sigma_z^2 \to \int_0^1 W_{d+d_1}(s)^2 ds \quad (7)$$

The univariate variance ratio statistic is then defined as the ratio of 6 to 7

$$p(d_1) = T^{2d_1} \frac{\sum_{t=1}^{T} z_t^2}{\sum_{t=1}^{T} \bar{z}_t^2} \xrightarrow{D} \frac{\int_0^1 W_d(s)^2 ds}{\int_0^1 W_{d+d_1}(s)^2 ds} \quad (8)$$

Note that d_1 is a parameter chosen by the econometrician and is submitted solely to his judgment. However, the fact that d_1 appears in the asymptotic distribution of the ratio

[73]Nielsen (2010) states that for d>1/2 and regularity conditions for u_t the following central limit theorem can be applied. $T^{1/2-d}z \Rightarrow \sigma_Z W_d(s)$, 0< S≤1 as $T \longrightarrow \infty$. Note that double arrow means weak convergence of a process in D[0,1]. The same is valid for $\bar{z}_t$.

[74]Note that $\xrightarrow{D}$ denotes convergence in distribution whereas W_d denotes a special case of fractional standard Brownian motion of order d>1/2 defined $W_d(r)=0$, r=0 and $W_d(r) = \frac{1}{\Gamma(d)} \int_0^r (r-s)^{d-1} dW_1(s)$, r>0. Note that W_1 denotes a standard Brownian motion.

makes this choice less autonomous in the sense that the econometrician will pick up those values of d_1 that maximize the power of the test.

Indeed Nielsen (2010) underlines the important fact that certain values of d_1 tend to maximize the power of the test. Moreover a local power analysis in Nielsen (2010) indicates that the power of the ratio test appears being monotonically decreasing in d_1 and that $d_1=0.1$ probably sets the best choice.[75]This means that higher values of d_1 do not generate higher power for the test, while gains from using lower values to 0.1 are minor or at least equal.

In the vector case the variance ratio test will be based upon the following statement

$$R_T(d_1) = A_T B_T^{-1} \tag{9}$$

where $$A_T = \sum_{T=1}^{T} z_t z_t^{'} \text{ and } B_T = \sum_{T=1}^{T} \vec{z}_t \vec{z}_t^{'},\ \vec{z}_t = (1-L)^{-d_1} z_t \tag{9i}$$

Let now $\lambda_1 \leq \ldots \leq \lambda_n$ be the ordered eigenvalues of R_T (d_1), with η_j, $j=1,\ldots,n$ denote the corresponding eigenvectors. Nielsen (2010) shows that the decision of whether an eigenvector constitutes a cointegrating vector is solely based upon the rate at which the associated eigenvalue converges to zero.[76]

Note at this point the adjustment potentials of the variance ratio to deterministic time trends, nonzero means or both, using the following regression equation

[75]Nielsen (2010) comments that a small value of d_1 may distort the size properties of the test, while another typical choice is that of $d_1= 1$.
[76]Nielsen (2010) acknowledges three cases: a) if λ_j converges to zero at rate $O_P(T^{-2d1})$ then eigenvector η_j is not a cointegrating vector, b) if the rate is $O_P(T^{1-max(2d-2b+2d,1)})$ then η_j is a cointegrating vector and d-b<1/2 and c) if the rate is equal to $O_P(T^{-2d1})$ η_j is again a cointegrating vector but d-b>1/2.

$$Y_t = a'\delta_t + z_t \tag{10}$$

where $\delta_t = [1,t]'$, Z_t denotes the residuals, while Y_t stands for the observed series vector.[77]

In this last case the de-trended variance ratio test is constructed upon the least squares residuals of equation (10) and hence[78]

$$\hat{z}_t = Y_t - \hat{a}'\delta_t \tag{11}$$

Once the variance ratio is estimated and the possibly corrected for the presence of deterministic time trends, the analysis next uses the eigenvalues λ_j, j=1,..,n to estimate the trace statistic[79] that is presented in equation (12)

$$\Lambda_{n,r}(d_1) = T^{2d_1} \sum_{j=1}^{n-r} \lambda_j \tag{12}$$

where $$\Lambda_{n,r}(d_1) \xrightarrow{D} U_{n-r}(\hat{d}, d_1) \tag{13}$$

Note that the asymptotic distribution depends on a) the integration order of the observed series, d, b) the parameter d_1 indexing the family of tests and finally c) the dimensionality of the problem, or state it differently the number of common stochastic trends, that is n-r. On the other hand note that the distribution is independent to the degree of cointegration, that is b.

The null hypothesis and the alternative are given in the following relation[80]

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[77]If δ_t=[1,t]' this stands for the presence of a deterministic linear time trend and a non zero mean. Obviously other definitions of δ_t correspond to different specifications of equation (10). For example, for δ_t=0 there are no deterministic trends, while for δ_t=1 there is a non zero mean.

[78]If a de-trending procedure is under consideration then the asymptotic distribution should adjust through appropriate simulations. This process in order to yield good size properties requires the smooth change of critical values whenever the integration order of the observed series is changing. Actually in order to obtain the right quantiles a consistent estimate of the integration order must be used. So assuming d* is such an estimate then quantiles must be simulated in order to obtain the following asymptotic distribution $U_{n-r}(d^*,d_1)$.

[79]The trace statistic $\Lambda_{n,r}$ (d_1) is asymptotically invariant to short run dynamics. Therefore as Nielsen (2010) quotes any hypothesis test based on this statistic will eventually share this invariant property.

[80]The procedure compares the statistic $\Lambda_{n,0}$ (d_1) with the corresponding critical values $CV_{\xi,n}(d^*, d_1)$. So for example, if the first value is smaller than the second then the null hypothesis H_0 : r=0 is not rejected and the

$$H_0 : r = r_0 \quad H_1 : r > r_0 \tag{13i}$$

3. Interest rates : trend stationary or difference stationary processes.

As was stated in section 2 the non parametric variance ratio test is estimated after corrections made in the observed time series for deterministic terms. Nielsen (2010) decides the de-trending form of his data set solely upon the graph of the interest rates series under consideration, while contrary the present analysis, uses the ARFIMA models, in order to yield a decision based on evidence rather than intuition.[81]

An important debate of the trend behavior of many macroeconomic and financial series often implicates the discussion of whether such trends are best described as deterministic time trends or unit root with drift models.[82]

ARFIMA frameworks that often have been used in this conflict area of research, constitute a conducive tool in determining the stationary nature of the interest rates series under consideration and hopefully will decide in the present analysis the residuals upon the variance ratio test is going to be estimated.

Although many researchers such as Chambers (1996) have used in the past exclusively frequency domain methods, the present analysis applies time and frequency domain maximum likelihood techniques.

Consider for example the following ARFIMA model

$$(1 + a_1 L + ... + a_p L^P)(1 - L)^d x_t = (1 + \theta_1 L + ... + \theta_q L^q)\varepsilon_t \tag{14}$$

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analysis concludes that the cointegration rank is r=0 . However, if it is rejected the process moves on and compares $\Lambda_{n,1}(d_1)$ with the corresponding critical value $CV_{\xi,n-1}(d^*, d_1)$. The same procedure is repeated until at some point there is acceptance of the null hypothesis.

[81] The decision is obviously important since determines the data set upon which the ratio statistic is going to be estimated.

[82] See Perron (1989), Andrews and Zivot (1992)

In equation (14) the short run behavior of the series is captured severally by the ARMA parameters, while the long memory property is modeled by parameter d[83]whose effect on the short run dynamics is limited.[84]

Now equation (15) presents the deterministic model

$$x_t = \mu t + e_t \tag{15}$$

where e_t is a stationary ARMA process[85]

$$a(L)e_t = b(L)u_t \tag{16}$$

Differencing (15) provides equation (17)

$$\Delta x_t = \mu + \Delta e_t \tag{17}$$

Taking first differences in (16) and solving for Δe_t turns over

$$\Delta e_t = a(L)^{-1} B(L) \Delta u_t \tag{18}$$

Finally substituting the last equation in (17) turns over

$$a(L)\Delta x_t = \mu_1 + b(L)\Delta u_t \tag{19}$$

Chamber (1996) notes that Δx_t although being stationary is in fact an over-differenced process. This stems from the fact that first differencing a deterministic trend series adds on a unit root in the moving average representation. Therefore the first difference of Δx_t is integrated of order minus one, and therefore expressing Δx_t through the context of an ARFIMA (p,d,q) should estimate $\hat{d} = -1$.

Consider now the random walk model with a drift as in equation (20)

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[83]The model is non stationary whenever $d \geq 0.5$. On the other hand long range dependence occurs for any value of d greater than zero.

[84]In terms of the spectral density function this means that the long range dependence is easily estimated without imposing any prior restrictions on the higher frequencies components.

[85]u_t is white noise process.

$$x_t = \mu + x_{t-1} + e_t \tag{20}$$

Differencing (20) turns over equation (21)

$$a(L)\Delta x_t = \mu_1 + b(L)u_t \tag{21}$$

where Δx_t is expressed as an ARFIMA (p,d,q) with $\hat{d} = 0$

3.1 Time domain analysis.

The previous analysis states that ARFIMA (p,d,q) nests both the trend and difference stationary versions[86] and corresponds at each a specific estimation of d. In fact this statement introduces a formal test for determining the residuals upon which the variance ratio test will be based. However, a drawback of this process focuses on the sensitivity of the estimated coefficients when different classes of ARFIMA are estimated.

Indeed Schmidt and Tschernig (1993) underlie the decisiveness of determining erroneous AR and MA orders, and particularly state that possible misspecifications in either of the p and q orders, may cause substantial biases in the estimation of the long memory parameter. Specifically, for under-specification or over-specification of either p and q orders, Schmidt and Tschernig (1993) worn for inconsistent estimates of not only AR and MA coefficients, but also of d.

On the other hand, Sowell (1992) does not embrace this opinion at all and in contrast states that ARFIMA is a parameterization where short and long run behavior is captured separately by the ARMA components and the fractional differencing operator respectively.

[86] The first difference of a deterministic trend series is integrated of order -1, while the first difference of a unit root model is 0. A sufficient and necessary condition for the latter statement to be valid is that the corresponding spectral density at zero frequency must not be zero, while for the former statement to exist the spectral density must be zero at zero frequency. The conclusion that occurs is that testing the spectral density at zero frequency is equivalent as testing the integration order of the observed series.

Therefore erroneous estimates on the short memory part do not affect the estimation of the long memory parameter,[87]while the inverse according to Sowell (1992) is also true.

Actually Sowell (1992) suggests the selection of p and q orders according to Akaike and Schwarz information criteria. His procedure starts by setting a-priori max orders for the lagged autoregressive variables (p) and the lagged moving average components (q), while next, using maximum likelihood functions, he estimates all ARFIMA models that are generated from different combinations in the q and p orders.[88] The above information criteria are then helpful in ascertaining the best ARFIMA.

In order to determine the appropriate ARFIMA class, the analysis follows the general scheme of Sowell (1992). However, limiting a-priori as Sowell does the p and q orders seems rather restrictive and therefore the establishment of the appropriate ARFIMA class is completely differentiated at this point from the latter procedure.

Actually the hall process evolves in five steps. Specifically : The first step generates the first differenced interest rates series of i_{eonia},i_7,i_{10} and i_{12} denoted respectively as Δ_{eonia}, Δ_{i7}, Δ_{i10} and Δ_{i12}. The second step uses Δ_{eonia}, Δ_{i7}, Δ_{i10}, Δ_{i12} and applies on each series four long memory methods in order to obtain for each series equal number of long memory estimates, while the third step applies on each series fractional differencing in order to produce the corresponding stationary processes. The fourth step uses the Box-Jenkins

[87]The long range dependence occurs for any positive values of d. As Sowell (1992) claims this dependence in terms of spectral density function occurs even if placing less restrictions on the higher frequency components and this is seen as the flat segment of the spectral density graph.
[88] Sowell (1992) notes that if $d \leq 0.5$ the process consistently estimates all ARFIMA coefficients.

methodology[89] to determine on the stationary series the appropriate AR and MA orders, while finally the fifth step corresponds at each ARFIMA the estimated maximum likelihood value and the Akaike and Schwarz information criteria.[90]

Table (1) reports the estimated long memory coefficients of the first differenced interest rates series of Δ_{eonia}, Δ_{i7}, Δ_{i10} and Δ_{i12} implementing on each occasion four long memory tests. For example, estimating the GPH test for the first differenced series of Eonia using a bandwidth parameter equal to 0.4[91] turns over a long memory coefficient that is equal to 0.331.

Using next the appropriate fractional differencing operator, that is $(1-L)^{0.331}$ the analysis obtains the corresponding stationary process, while applying next the Box-Jenkins methodology to the latter fractionally differenced series determines the AR and MA orders of the ARFIMA.

In this particular case these orders were set equal to one. So synthesizing all the above outcomes the analysis corresponds for Δ_{eonia} and for the GPH test under the bandwidth parameter of 0.4 the ARFIMA (1,0.331,1) framework.

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[89] The method proposed by Box and Jenkins is customarily partitioned in three stages : identification, estimation and diagnostic checking. At the identification stage an ARIMA is specified on the basis of autocorrelations and partial autocorrelations. As it is well known the true autocorrelations of a pure MA process present a cutoff point at the MA order, whereas the partial autocorrelations taper off. In contrast, the autocorrelations of a pure AR processes taper off, whereas it's partial autocorrelations present a cutoff point at the AR order.

[90] A disjoint ARFIMA estimation in a space time context is carried out in Haslett and Raftory (1989). The researchers apply a two step algorithm in order to estimate separately the short and long memory coefficients. In the frequency domain an analogous procedure is delivered by Coli et al (2005). Their full parametric approach is realized combining the orthogonal decomposition of a stochastic process with the Whittle likelihood estimation.

[91] As bandwidth parameter the analysis here defines the value of the function T^m that proclaims the number of low frequency ordinates.

There are many approaches in estimating the fractional differencing parameter. In the present analysis there are four. Those are : a) the rescaled–range method of Hurst (1951) which in the present analysis will be denoted as R/S, b) the modified R/S method of Lo (1991) that will be refer to as modified R/S, c) the GPH test suggested by Geweke and Porter-Hudak (1983) and finally d) the semi-parametric GSP test of Robinson and Henry (1998).

Sowell (1992a) argues that the estimations of the fractional differencing operator can be quite misleading, and using different long memory tests to control the accuracy predominantly as a necessity, especially when acknowledging the drawbacks following every estimating procedure.

For instance, a problem with the R/S statistic is that it's distribution is not well defined, while the long memory estimate appears sensitive to potential short term dependence or heterogeneities occurred in the data generating process.[92] On the other hand the long memory estimate of the modified R/S statistic is invariant to a general class of short memory processes, while the limiting distribution is known.

Furthermore, the semiparametric GPH estimate appears to be sensitive to the choice of the bandwidth parameter and to the presence of short range dependence.[93] As has been stated by Geweke and Porter-Hudak (1983) the number of low frequency periodogram ordinates, that is values of the function T^m, introduces definitely judgment and commonly, large

[92]Robinson states "although the statistic behaves well with respect to long tailed distributions, its limit distribution is not standard and its difficult to use it in statistical inference, while it has no known optimal efficiency properties with respect to any known family of distributions."
[93] Chen and Wohar (1992).

bandwidth parameters will bias the estimate of d due to the use of medium and high frequency components, while on the other hand, small values will tend to generate imprecise estimates due to the limited degrees of freedom. Finally, in simulations made in Dittmann (2000) the modified R/S appeared less powerful to GPH,[94]while on the other hand the size distortions of the former were actually smaller than those presented for the GPH.

Graph (1) displays the graphical output of Δ_{eonia}, Δ_{i7}, Δ_{i10} and $_{\Delta i12}$ while tables (1) to (3) report respectively : a) the estimated long memory coefficients of Δ_{eonia}, $\Delta i7$, Δ_{i10}, Δ_{i12}, using a range of bandwidth parameters for the specification of the sample size functions of both GPH and GSP estimates, b) the orders of AR and MA polynomials after applying appropriate fractional differencing [95]on Δ_{eonia}, $\Delta i7$, Δ_{i10} and $_{\Delta i12}$, and finally c) the log-likelihood value and the Akaike and Schwarz information criteria on every estimated ARFIMA model.

Table (1) Estimation of the long memory coefficient for the first differenced interest rates series Δ_{eonia}, Δ_{i7}, $\Delta_{i10,}$ Δ_{i12} using GPH, GSP, R/S and modified R/S long memory estimates.

	GPH			GSP			Modified R/S			R/S
	$T^{0.4}$	$T^{0.6}$	$T^{0.8}$	$T^{0.4}$	$T^{0.6}$	$T^{0.8}$	Q=5*	Q=10*	Q=15*	-
Δ_{eonia}	0.331 [0.039]*	0.130 [0.040]*	-0.279 [0.000]**	0.476 [0.000]**	0.114 [0.013]*	-0.183 [0.000]**	-0.482	-0.457	-0.444	-0.527
Δ_{i7}	0.335 [0.037]*	0.361 [0.000]**	0.247 [0.000]**	0.251 [0.013]*	0.404 [0.000]**	0.243 [0.000]**	-0.368	-0.389	-0.403	-0.324
Δ_{i10}	0.252 [0.117]	0.288 [0.000]**	0.207 [0.000]**	0.210 [0.038]*	0.363 [0.000]**	0.202 [0.000]**	-0.382	-0.400	-0.412	-0.348
Δ_{i12}	0.252 [0.117]	0.288 [0.000]**	0.207 [0.000]**	0.210 [0.038]*	0.363 [0.000]**	0.202 [0.000]**	-0.382	-0.400	-0.412	-0.348

Number of observations: 2812; number of periodogram points corresponding each bandwidth parameter: 24, 117, 574; Critical Values for Hurst-Mandelbrot and R/S tests 90%: [0.861, 1.747] 95%: [0.809, 1.862] 99%: [0.721, 2.098], Null hypothesis stated for Hurst-Mandelbrot and R/S tests are respectively H_0: no autocorrelation and H_0 : no long-term dependence; The following form is applied for transforming the R/S statistic into the fractional differencing coefficient d=[(log(R/S)/logT]-1/2; p-values are reported in the brackets; * denotes rejection at 5% significance level, ** denotes rejection at 1%.

[94] Particularly if d<0.3 the power of the modified R/S test declines as d decreases.
[95]Applying fractional differencing on a series and then considering the ARMA model that best captures the remaining short run dynamics is a method also applied in Diebold and Rudebusch (1989).

Except of two blocks[96] in table (1) the results indicate that the GPH and GPS estimates of d are statistically significant at 5% significant level, while contrary the implied long memory coefficients in Lo's and Mandelbrot's R/S statistics are non statistically significant at the same level of significance, since both accept the corresponding null hypothesis[97]for the existence of short memory and the presence of no autocorrelation respectively.

Furthermore, it can be stated that the estimated long memory coefficients of both GPH and GSP tests appear quite robust to changes in the bandwidth parameter, although this is not true for Δ_{eonia} under the bandwidth value of 0.8. In this case both tests agree on the definition of the series as anti-persistence.[98]

Table (3) reports the values of the log likelihood functions and the corresponding Akaike-Schwartz information criteria corresponding each ARFIMA model presented in table (2). According to the information criteria the ARFIMA models that best fit Δ_{eonia},Δ_{i7}, $\Delta_{i10,}$ Δ_{i12} are respectively the ARFIMA (1,0.331,1), (1,0.361,1),(1,0.363,1)

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[96] Those are the values reported for the GPH estimates of Δi_{10} and Δi_{12} under the bandwidth of 0.4

[97]The null hypothesis of the R/S statistic of Mandelbrot (1972) is that of an uncorrelated process, while the modified R/S statistic of Lo (1991) focuses on the null of a short memory process against the alternative of a long term dependence. The long memory coefficients of both statistics are estimated using the form presented in Mandelbrot and Wallis (1969). This is d=[(log(R/S)/logT]-1/2. Mandelbrot (1972,1975), Mandelbrot and Taqqu (1979) analyze the properties of this procedure.

[98] McLeod and Hipel (1978) use the autocorrelations p_j to define the long memory. Specifically a discrete time process y_t has long memory if the following quantity is non finite

$$\lim_{n\to\infty} \sum_{j=-n}^{n} \left| p_j \right|$$

For 0<d<0.5 the process is long memory in the sense that it' s autocorrelations are all positive and decay at a hyperbolic rate. For -0.5<d<0 the sum of absolute values of p_j tends to a constant and so it has short memory. The autocorrelations decay hyperbolically to zero. In this case process is referred to as anti-persistence.

Graph (1) Graphical representation of Δi_{eonia}, Δi_7, Δi_{10}, Δ_{12}

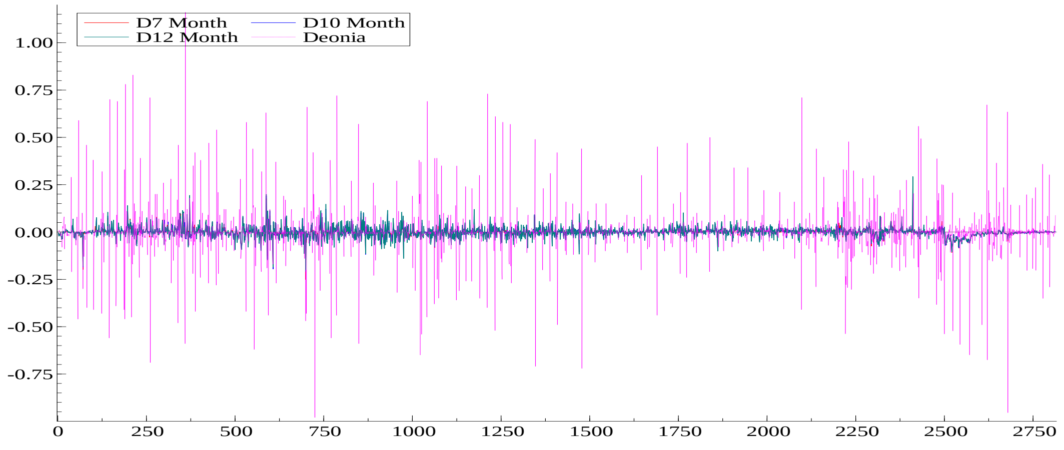

Table (2) The proposed ARFIMA (p,d,q) models for the first differenced interest rates series Δ_{eonia}, Δ_{i7}, $\Delta_{i10,}$ Δ_{i12}

	Δ_{eonia}	Δ_{i7}	Δ_{i10}	Δ_{i12}
$T^{0.4}$ GPH	(1,0.331,1)	(1,0.335,1)	(1,0.252,1)	(1,0.252,0)
$T^{0.6}$ GPH	(1,0.130,1)	(1,0.361,1)	(1,0.288,1)	(2,0.288,1)
$T^{0.8}$ GPH	(0,-0.279,0)	(1,0.247,0)	(1,0.207,0)	(1,0.207,1)
$T^{0.4}$ GSP	(1,0.476,1)	(1,0.251,0)	(1,0.210,0)	(1,0.210,1)
$T^{0.6}$ GSP	(1,0.114,1)	(1,0.404,1)	(1,0.363,1)	(1,0.363,1)
$T^{0.8}$ GSP	(0,-0.183,0)	(1,0.243,1)	(1,0.202,0)	(1,0.202,1)

Note : The orders of AR and MA components are decided upon the graphical representations of ACF and PACF. All estimated models do not include constant terms in the corresponding.

Table (3) Log-likelihood values, Akaike and Schwartz information criteria for the maximum likelihood estimated ARFIMA.

	ARFIMA (p,d,q)					
	$T^{0.4}$ GPH	$T^{0.6}$ GPH	$T^{0.8}$ GPH	$T^{0.4}$ GSP	$T^{0.6}$ GSP	$T^{0.8}$ GSP
Δ_{eonia}	2088.65 (-1.483)* [-1.484]*	2084.33 (-1.480) [-1.481]	2025.41 (-1.439) [-1.440]	2087.64 (-1.482) [-1.484]	2083.69 (-1.479) [-1.481]	2039 (-1.449) [-1.449]
Δ_{i7}	6919.87 (-4.919) [4.919]	6920.01 (-4.919)* [-4.919]*	6917.80 (-4.918) [-4.918]	6917.87 (-4.918) [-4.918]	6919.71 (-4.919) [-4.919]	6917.69 (-4.918) [-4.918]
Δ_{i10}	6379.86 (-4.535) [-4.535]	6380.83 (-4.536) [-4.536]	6376.87 (-4.534) [-4.533]	6376.86 (-4.534) [-4.533]	6381.15 (-4.536)* [-4.536]*	6376.83 (-4.534) [-4.533]
Δ_{i12}	6094.80 (-4.333) [-4.333]	6106.42 (-4.340) [-4.341]	6103.74 (-4.339) [-4.339]	6103.91 (-4.339) [-4.333]	6106.71 (-4.349)* [-4.341]*	6103.46 (-4.338) [-4.339]

Note : In parenthesis and brackets the Akaike and Schwartz information criteria are reported respectively; K denotes the number of estimated coefficients and (1,0.363,1) respectively.[99]Note the congruence of both information criteria in every occasion.

As was stated in the previous section the fractional differencing operator contains critical information over the nature of trend appearing in the series. A brief review of these statements is that the ARFIMA model that best fits the first differenced process of a series will estimate a value of d equal to zero, if the trend in the original series is stemming from a unit root process and will estimate a value of d equal to -1 if the trend occurs from a deterministic trend model. Therefore testing the null hypothesis of d=0 or d=-1 decides the de-trending procedure in the present analysis.

Specifically, for the ARFIMA model that corresponds the first differenced process, if it estimates a value of d equal to zero, then the trend of the series is due to a unit root process, while if d is equal to -1, the trend occurs from a deterministic time trend model. Therefore testing the null hypothesis of d=0 or d=-1 decides the de-trending strategy in the present analysis.

However, before conducting such a testing analysis two things must be stated. First, the consistency and asymptotic normality of the maximum likelihood estimates of the Gaussian fractional ARIMA are presented in Dahlhaus (1989) and they are valid when $0<d<1/2$. This case obviously is in line with the ARFIMA models presented in table (3). Second, it will prove useful to remember that first differencing a deterministic trend series

[99] The present selection of ARFIMA is supported inversely by the following procedure. Using the first differenced interest rates series, the orders of AR and MA polynomials are defined according to the optimal orders found in table (3). The procedure next estimates the maximum likelihood the value of d. Results are presented in the appendix.

imposes a unit root in it's ARMA moving average representation. This statement will prove helpful when testing the null hypothesis

d=-1.

Table (4) presents results on the likelihood ratio test when examining either of the above restrictions. For example, imposing the null hypothesis d=0 on the ARFIMA (1,0.331,1)[100] that corresponds the first differenced series of Eonia, Δ_{eonia}, turns over the restricted ARMA(1,1) framework. The likelihood ratio in the last case turns over a value equal to 0.011 and for a chi-squared distribution with one degree of freedom the null hypothesis d=0 is accepted.

On the other hand, testing the restriction d= -1on the same ARFIMA, implies that the restricted framework is the ARMA (1,2).[101] In this case the estimated likelihood ratio statistic is 23.19 and for a chi-square distribution with one degree of freedom the result indicates the rejection of the corresponding null hypothesis. Therefore the results conduce that the presence of stochastic trend is the one characterize best the Eonia interest rates series.

Table (4) Results of the likelihood ratio test for deciding the stochastic or deterministic trend in the first differenced interest rate series.

	Δ_{eonia}	Δ_{i7}	Δ_{i10}	Δ_{i12}
d=0	0.011	19.6**	7**	7.1**
d=1	23.19**	2.76	12.06**	11**

Note : *(**) denote rejection at 5(1)% significant level

As can be seen in the results presented in table (4) neither of the hypothesis dominates the other. Specifically, although Δ_{eonia} and Δ_{i7} clearly provide support over the unit root and

[100]This ARFIMA is estimated for the first differenced series of Eonia, that is Δ_{eonia}, and fopr the GPH estimator that uses $T^{0.4}$ as a sample function $T^{0.4}$.

[101]Estimating the ARFIMA (1,-1,1) with PcGive turns over no outcome since the estimating process deos not reach any convergence.

the time trend models respectively, however, the other two processes, Δ_{i10} and Δ_{i12}, clearly reject at 5% and 1% significant levels both null hypothesis.

The fact that there is no homogeneity in the above results may attributed to possible misspecifications existing in the AR, MA orders. If that is the case then the author assesses that these errors are caused more by the segregate specification of the long memory part and the short dynamics and less by a non-successful Box-Jenkins methodology. Either way the conclusion remains the same and this is that time domain analysis provides little help in deciding a common de-trending strategy in all interest rates series under examination. Therefore the latter procedure is considered impractical. However the paper continues upon the same quest in section 3.2 although in this case estimates ARFIMA frameworks under the frequency domain analysis.

3.2 Frequency domain analysis.

Chambers (1996) states that the frequency domain analysis[102] is generally invariant to specifications applied on the short and long memory modeling, and so using this process to estimate the coefficients of an ARFIMA it is expected to deliver reliable outcomes, even if possible misspecifications exist in the AR and MA orders. The fundamental equations of Chambers (1996) are given below.

Specifically, let θ denote the vector of ARFIMA's (p,d,q) coefficients

$$\theta = (d, a_1, ..., a_p, b_1, ..., b_q)$$

[102]As Chambers (1996) notes the Wald statistic in this case can be seen as the t ratio of the estimated differencing parameter.

Using a sample of T observations the estimation of θ vector is obtained through minimizing the negative of the frequency domain log-likelihood function. The last is defined as in the following equation

$$LnL(\theta) = 0.5\sum_{s=1}^{T-1}[\ln h(\lambda_s;\theta) + I_t(\lambda_s)/h(\lambda_s;\theta)] \quad (21)$$

where $\lambda_s=2\pi\varsigma/T$, s=1,…,T-1 denotes the set of Fourier frequencies and $I_T(\lambda)$ stands for the periodogram function defined as in equation (22)

$$I(\lambda) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} x_t e^{it\lambda}\right|^2 \quad (22)$$

Function (21) is stressed in Hannan (1973) and depends on θ through the spectral density function of the assumed ARFIMA (p,d,q) class. The latter sets up a discrete version of the Whittle (1951) frequency domain method.

However, Coli et al (2005) contrary to Chambers (1996) state that misspecifications in the AR and MA coefficients definitely affect the estimation of the long memory parameter and moreover correspond in particular combinations of AR and MA components specific long memory features. For example, for the ARFIMA (0,d,0) they state that it's spectral density function is given by the following equation

$$g(w,d) = |1 - e^{-iw}|^{-2d} \quad (23)$$

which entirely is concentrated at low frequencies. In fact g(w,d) irrespective of the value in the long memory parameter, appears being a decreasing function of w. In this case as w tends to zero the function g (w,d) tends to infinity.

In the general ARFIMA(p,d,q) framework the spectral density function is given by

$$g(w,d) = |1-e^{-iw}|^{-2d} \frac{|\theta(e^{-iw})|^2}{|\varphi(e^{-iw})|^2} \tag{24}$$

The last is strongly affected by the presence of short run dynamics.

Some typical examples are the ARFIMA (1,d,0) and ARFIMA (0,d,1). If these frameworks estimate a positive AR and MA coefficients respectively, then their spectral density function will exhibit an increased intensity at low frequencies, whereas if these coefficients are negative this fact alone will shift the concentration of the function to higher frequencies, while if a MA of opposite sign is added to ARFIMA (1,d,0) it will cause a reduction of it's memory value.Another typical example is the ARFIMA (1,d,1). Coli et. al. (2005) argue that for positive AR and MA coefficients the relative spectral density function will be concentrated to low frequencies, while on the other hand if these coefficients are both negative then the density function will be centered to higher frequencies. Furthermore if the estimated AR and MA coefficients exhibit opposite signs the behavior of the spectral will be determined by the larger absolute coefficient.

Finally, in the general ARFIMA (p,d,q) case the large positive AR parameters will increase the long memory value, while the large negatives will restricted it. Note that the large and negative AR and MA coefficients will tend to eliminate the long memory of the process. In this case these models will exhibit similar properties to (1,0,1) ARFIMA.

Setting by default the max AR and MA orders equal to three as in Sowell (1992) the present work, uses appropriate spectral density functions to estimate all ARFIMA frameworks that are generated from different combinations in the AR and MA orders.[103]The fact that the paper applies multiple combinations in these orders stems exactly from the statements of Coli et al (2005) that imply a straightforward link among the sensitivity of long memory estimates and the existence of possible misspecifications in the AR and MA orders.

Furthermore, in an attempt to reduce the parameter space region the analysis applies in all estimated ARFIMA frameworks the standard parameterization choice, an option provided by the afm tools package of the R statistical program.[104]/[105]

Table (5) presents the ARFIMA that estimate the smallest Schwarz information criteria for every differenced interest rate series, while table (6) tests the trend and differenced stationary nature of the observed series using the results of the wald and likelihood ratio tests.

The results in table (5) underline certain characteristics. First of all under this approach non of the differenced interest rates series of the analysis is defined as anti-persistent.[106]

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[103]Estimations are carried out with the R statistical program using the afmtools-package. All estimated ARFIMA are presented extensively in the appendix.

[104]Standard parameterization requires certain premises. First, the parametric spectral density function must have the following form $f(\lambda;\theta;\sigma^2) = (\sigma^2/2\pi)h(\lambda;\theta)$, where θ is an r dimensional vector and σ^2 is regarded as varying freely from θ. Second, the following relation must be true

$$\int_{-\pi}^{\pi} \log h(\lambda;\theta)d\lambda = 0$$

Practically standard parameterization means that the residual variance σ^2 is located out of the likelihood function in order to reduce the dimensions of the parameter space.

[105]Although the Whittle estimates are asymptotically efficient only when x_t is Gaussian, however limiting distributions in the case of standard parameterizations are steady under many departures from Gaussianity. This property is initially established for short memory series. The justification of using Whittle estimates in long memory models is provided in Fox and Taqqu (1986).Their objective function is a continuous version of (21) but their insight may well be applied to the discrete case. Fox and Taqqu (1986) and Dahlhaus (1989) under assumptions made for the long memory parameter and the correct specification of short run dynamics, have shown that θ estimated vector is consistent and asymptotically normally distributed. Dalhaus (1989) also establishes asymptotic efficiency.

[106] This outcome strengthens the previous choice of the analysis to consider only the GPH and GSP estimates of table (1).

Second, all estimated long memory coefficients are statistically significant at both conventional levels of significance and range from 0.348 to 0.405, and finally third, all chosen ARFIMA frameworks are estimated using one AR and one MA order.

Table (6) reports the results on the Wald and likelihood ratio statistics. Both tests reject the null hypothesis of the trend and differenced stationary nature at 1% and 5% significant levels, and so the analysis is left at this point solely with an intuitive selection strategy for the nature of data that are finally going to be used. Following Nielsen (2010) the analysis decides to adjust the observed interest rates series for a deterministic time trend and a non zero-mean.

Table (5) Whittle estimated ARFIMA models that minimize the Schwarz information criteria

	Δ_{eonia}	Δ_{i7}	Δ_{i10}	Δ_{i12}
α_1	0.290 [14.095]**	0.460 [19.242]**	0.448 [19.002]**	0.472 [19.456]
θ_1	-0.945 [-5.315]**	-0.654 [-19.839]**	-0.669 [-19.596]**	-0.699 [19.456]
d	0.405 [13.105]**	0.375 [12.129]**	0.351 [11.350]**	0.348 [11.249]

Note : t estimates are reported in the brackets//*(**) denotes rejection at 5%(1%) significant level

Table (6) Wald and likelihood ratio statistics for testing the null hypothesis of differenced or trend stationary series.

	Δ_{eonia}	Δ_{i7}	Δ_{i10}	Δ_{i12}
Null hypothesis d=0 (difference stationarity)				
Wald	13.5**	12.5**	11.7**	11.6**
Likelihood ratio	43.07**	76.09**	55.05**	46.07**
Null hypothesis d=-1 (trend stationarity)				
Wald	46.83**	45.83**	45.03**	44.93**
Likelihood ratio	76.03**	46.07**	56.02**	42.01**

Note : *(**) denote rejection at 5%(1%) significant level

4. Estimating the variance ratio test.

Table (7) uses the de-trended interest rates series to estimate the GPH and GSP estimates of d. The applied bandwidth choices follow Nielsen (2010) and are consistent with the previous parts in the analysis. Furthermore, table (8) uses those latter estimates to test whether those series are in fact unit root processes. One and two asterisks denote respectively rejection of the null at 5% and 1% significant levels.[107]Note, that the unit root hypothesis is against the two sided alternative.

In table (7) all estimated long memory coefficients are statistically significant at 5% and 1% significant level. Interestingly some of the GPH estimates are below the threshold value of one,[108] although table (8) indicates that these series are generally no different to I(1) processes. Specifically, table (7) indicates that six GPH estimates are below one, while table (8) shows that only one of them is actually different to a unit root process.

Furthermore, the results in table (8) show that the unit root hypothesis is rejected 15 out of 24 times.[109] The outcome is consistent with the majority of evidence existing in many related empirical applications,[110]and implies the introduction of two variance ratio tests : a) one that uses the hypothesis d=1, and b) a second that sets d equal to the average value of those GSP estimations that correspond to a particular bandwidth choice. In the present analysis this is 0.8.[111]

However, the analysis does not rest upon this double assumption for the integration order of the de-trended interest rates series, and applies additionally two choices over the index

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[107]Nielsen (2010) underlines the invariant nature of the GSP estimates and states that adjustments made in the interest rates series for a non zero mean and a deterministic trend do not alter the good properties of the GSP estimators.

[108]Those estimated values are between (0.812-0.963).

[109]Geweke and Porter-Hudak (1983) shows that GPH test provides consistent estimate of 1-d. This is the trend coefficient in their linear regression equation, where periodogam points are evaluated at Fourier frequencies $(2\pi j)/T$, j=1,.., n, where n is the number of low frequency periodogram points used in estimation. In the present analysis this is refer to as bandwidth. Note that any hypothesis test of d is based upon a t-statistic.

[110] See Chen and Hurvich (2003), Nielsen (2010)

[111]Nielsen (2010) set d equal to 1.0025. This is the average value of those GSP estimates that correspond to bandwidth of 0.4. This average is very close to the one estimated in table (6) when m is set equal to 0.8. In this case the average value of d is equal to 1.003.

appearing in the asymptotic distribution of the variance ratio test.[112] These are d_1=0.1 and d_1=1.

Since $\Lambda_{n,r}(d_1)$ statistic in the present analysis is estimated under all the above combinations of d and d_1, eventually the paper conducts four fractional cointegration variance ratio tests. Note that in the variance ratio methodology d parameter refers to the integration order of the observed data series, while d_1 indexes a particular family of tests and is a parameter chosen exclusively by the econometrician.

Table (7) GPH and GSP long memory tests for the de-trended univariate interest rates series.

ARFIMA (p,d,q)						
	$T^{0.4}$ GPH	$T^{0.6}$ GPH	$T^{0.8}$ GPH	$T^{0.4}$ GSP	$T^{0.6}$ GSP	$T^{0.8}$ GSP
eonia	0.812 [0.000]**	0.941 [0.000]**	0.753 [0.000]**	1.247 [0.000]**	1.080 [0.000]**	0.69 [0.000]**
I_7	0.953 [0.000]8*	1.171 [0.000]**	1.073 [0.000]**	1.110 [0.000]**	1.242 [0.000]**	1.100 [0.000]**
I_{10}	0.963 [0.000]**	1.173 [0.000]**	1.077 [0.000]**	1.082 [0.000]**	1.234 [0.000]**	1.114 [0.000]**
I_{12}	0.960 [0.000]**	1.166 [0.000]**	1.085 [0.000]**	1.064 [0.000]**	1.230 [0.000]**	1.108 [0.000]**

Note : number in parenthesis reporte p-values, *(**) denotes rejection of unit root hypothesis at 5%(1%)

Table (8) Testing the null hypothesis of d=1 for the GPH and GSP estimates of long memory in the de-trended interest rates series.

	$T^{0.4}$ GPH	$T^{0.6}$ GPH	$T^{0.8}$ GPH	$T^{0.4}$ GSP	$T^{0.6}$ GSP	$T^{0.8}$ GSP
eonia	-1.167	-0.936	-9.148**	2.421*	1.739	-8.6**
I_7	-0.291	2.714**	2.703**	1.078	5.260**	6.11**
I_{10}	-0.229	2.746**	2.851**	0.803	5.086**	5.7**
I_{12}	-0.248	2.634**	3.148**	0.627	5**	5.4**

Note : *(**) denotes rejection of unit root hypothesis at 5%(1%)

Table (9) uses the de-trended interest rates series and reports : a) the univariate variance ratio test $p(d_1)$, b) the ordered eigenvalues of $R_T(d_1)$ ratio for both values of d_1, and c) the variance ratio trace statistic. Table (10) reports the 1%, 5% and 10 % simulated critical

[112] d_1 appears in the asymptotic distribution of the ratio. Nielsen (2010) states that certain values of d_1 maximize the power of the test.

values of all four cointegration variance ratio trace tests. These critical values are generated after appropriate simulations and are reported in Nielsen (2010).

Table (9) Univariate variance ratio, ordered eigenvalues and the variance ratio cointegration rank test.

Univariate Variance Ratio Test				
d_1	eonia	I_7	I_{10}	I_{12}
0.1	5.1111e+003	5.7334e+003	5.7156e+003	5.7137e+003
1	1.0559e+005	1.4324e+005	1.4628e+005	1.4738e+005m
Eigenvalues				
d_1	λ_1	λ_2	λ_3	λ_4
0.1	0.349	0.399	0.472	0.484
1	0.00001	0.00002	0.00005	0.0001
Variance Ratio Trace Statistic				
d_1	n-r=1or r=3	n-r=2 or r=2	n-r=3 or r=1	n-r=4 or r=0
0.1	1.708	3.662	5.973	8.342
1	79.12	237.38	633.03	1424.3

Note : The table reports the eigenvalues of $R_T(d_1)$ ratio, the univariate variance ratio test and the cointegration rank test for the de-trended interest rate series.

Table (10) Critical values of the four variance ratio cointegration rank tests using de-trended interest rate series.

Table (10)-A		d=1			
Null Hypothesis		n-r=1 or r=3	n-r=2or r=2	n-r=3 or r=1	n-r=4 or r=0
d_1=0.1	α=0.10	[1.93]	[3.81]	[5.75]*	[7.74]*
	α=0.05	[1.98]	[3.88]	[5.82]*	[7.82]*
	α=0.01	[2.08]	[4.01]	[5.97]*	[7.97]*
d_1=1	α=0.10	[228.18]	[586.52]	[1157.34]	[1970.49]
	α=0.05	[291.93]	[697.41]	[1325.41]	[2202.48]
	α=0.01	[457.46]	[971.59]	[1706.81]	[2.709.98]
Table (10)-B		d=average GSP estimates when bandwidth is set to 0.8-(1.003)			
d_1=0.1	α=0.10	[1.93]	[3.81]	[5.75]*	[7.74]*
	α=0.05	[1.98]	[3.87]	[5.83]*	[7.82]*
	α=0.01	[2.08]	[4.00]	[5.97]*	[7.97]*
d_1=1	α=0.10	[228.81]	[586.32]	[1159.92]	[1960.74]
	α=0.05	[293.45]	[691.22]	[1330.46]	[2198.69]
	α=0.01	[447.33]	[950.59]	[1691.45]	[2.695.75]

Note : number in brackets report the simulated critical values reported in Nielsen (2010). * denotes rejection of the null hypothesis at the corresponding significant level.

Comparison of the simulated critical values in tables 10-A and 10-B reveal many similarities among them. Specifically, the critical values correspond the choice of d_1=0.1 in 9 out of 12 cases are identical, while the rest critical values of either d_1 parameter although not matching perfectly each other are indeed very close. The evident similarity stems from the fact that the average GSP estimate of d is indeed very close to one and therefore the simulated critical values in both tables are not expected to be much different.

In general lines it can be stated that results of table (10) depend significantly on the value of d_1 parameter, since changing the assumption of the integration order for the observed series, that is d, does not really alter any of the cointegrating results. For example, under the choice of d=1 and d_1=0.1 the variance ratio test acknowledges two cointegrating relations, whereas if the combination of d=1 and d_1=1 is applied, the ratio concludes that no cointegrating relations exist, with the same results also repeat when the average GSP estimate of d is employed. The apparent dilemma on the cointegration rank is resolved when remembering that the power of the variance ratio test is maximized under the choice of d_1=0.1. The cointegration rank corresponding this choice must generally considered more reliable.

So, the results in table (10) support the presence of two common stochastic trends, or in other words indicate the presence of two cointegrating relations. Given that expectations hypothesis implies the existence of a unique common stochastic term among n interest rates of different maturities, the variance ratio clearly rejects the validity of term structure theory.

The result is in line with the outcomes presented in Chen and Hurvich (2003) where it is found that two common stochastic trends exist among eight interest rates of different maturities.

Although comparison is not straightforward, however, both outcomes speculate the existence of one common trend driving the short maturity interest rates and another driving the long term ones. Note that an alternative explanation for the finding of two cointegrating relations can be seen in the argument that expectations hypothesis holds better in the shorter end of the yield curve. Of course in this case every interest rate that corresponds to any duration between the first and eleventh month may considered as a potential shorter end part of the yield curve shorter end shorter end is considered a period of less than six months.

5. Estimating the cointegration rank using Johansen's (1991, 1998) and Breitung and Hassler (2002) tests.

This section extends the previous analysis by applying two different cointegration methodologies. Those are a) the Johansen (1988, 1991) cointegration approach and b) the Breitung and Hassler (2002) fractional trace test.

5.1 Applying Johansen's test (1991,1998).

Since Johansen's test by its nature assumes the existence[113] of I(1) and I(0) variables, in the appendix the interested reader may find results over the ADF, Phillips-Perron and KPSS unit root tests. All of these tests indicate the strong presence of unit roots in the observed interest rate series and indeed, both the ADF and Phillips-Peron accept the corresponding null hypothesis in all interest rates series at both conventional levels of significance, while KPSS fails to reject the null of stationarity at 1% significant level, although this is not true for Eonia.

[113] If I(2) variables are existing then it will be necessary to use the approach developed by Johansen (1995b).

Even though Diebold and Rudebusch (1991) and Sowell (1990a) argue that standard unit root tests may lack high power against fractional alternatives, their argument is probably moderated by the results presented in the first column of table (8). In this case although all the estimated fractional differencing operators are below 1, however the null hypothesis of d=1 is accepted in all four blocks, and so the interest rates series are considered being unit root processes in this particular case.

Table (11) isolates the long memory tests and the bandwidth parameters under which the hypothesis d=1 is not rejected. The table sets a useful transformation of the results reported in table (8). Obviously the unit root assumption is supported by both long memory tests, GPH and GSP, under mainly the 0.4 value for the bandwidth parameter.[114]

Table (11) : Long memory tests accepting the hypothesis d=1.

eonia	GPH-$T^{0.4}$& GPH-$T^{0.6}$	GSP-$T^{0.6}$
I_7	GPH-$T^{0.4}$	GSP-$T^{0.4}$
I_{10}	GPH-$T^{0.4}$	GSP-$T^{0.4}$
I_{12}	GPH-$T^{0.4}$	GSP-$T^{0.4}$

Before proceeding with the actual cointegration analysis the VAR order, k, must first be decided. This is a very critical point in Johansen's analysis, since possible

[114] The log-periodogram regression estimator of Geweke and Porter-Hudak (1983), that was latter formalized by Robinson (1995) and Hurvich et al. (1998) develops the idea that if an interest rate series presents long memory characteristics then the spectrum of the process should be a linear function of the frequencies close to zero. Let $I(w_J)$ denote the sample periodogram at the jth Fourier frequency. The estimate is obtained from the least square regression $\log[I(w_J)]=b_0+b_1\log(w_J)+u_J$, where j=1,..,m and $d=(-1/2)b_1$. The asymptotic standard error for d depends only on m. As has been stated by Geweke and Porter-Hudak (1983) the choice of the number of low frequency ordinates, that is T^m, necessarily involves judgment. Specifically Geweke and Porter-Hudak (1983) note " Although a too large value of n will cause contamination of the estimate of the d estimate due to medium or high frequency components, a too small value will lead to imprecise estimates due to limited degrees of freedom in the estimation process.

misspecifications in the lag augmentation, may cause phenomena of autocorrelation, non normality and conditional heteroskedasticity on the residual series.

On the other hand specifying the value of k is frequently implicated with issues of omitted variables biases. In a situation where autocorrelations in the residual series are due to omitted variables, these absences will likely become part of the error term and increasing the lag length to restore any such phenomena usually is not the correct answer. [115]

On the other hand over-parameterization which is associated in many monte carlo experiments with a reduction in the power of cointegrating tests, can also create potential problems, since very often implies that too few cointegrating relations will be acknowledged.[116]Avoiding over-parameterization is also underlined in Johansen (1995b) who states that to many lags will cause the number of parameters to grow very fast and as a consequence the information criteria that strike a compromise between the number of lags and parameters will tend to reject the alternatives most of the times.

In the present analysis the max number of lag augmentations is set equal to 25, representing the business days of one month period. At the same time following Lee and Siklos (1997) the analysis does not apply seasonal adjustments, since evidence in many

[115]Residual misspecification very often arise as a consequence of omitting important conditioning variables. In the last case increasing the lag length may result as reported in Harris and Sollis (2003) in a harmful parameterization, that affects the estimation of cointegration rank, and makes hard the economical interpretation of the present cointegrating relations. The same is stated also by Johansen (1995b) who encourages researcher to increase the information set instead of automatically increasing the lag length.

[116]This opinion is mainly supported by Lutkepohl and Saikkonen (1999) who report the existence of size distortions and power losses when the number of lags (k) is too small and too large respectively. Their recommendation is to choose the lag-length using information criteria such as Akaike. This criterium tends to create a balance between a good approximation of the data generating process and an efficient use of the sample information. However, Cheung and Lai (1993) state that cointegration tests are rather robust to over-parameterization, while additionally argue that the size distortions, when k is to small, are probably minor.

monte carlo experiments suggest that relevant attempts usually end up in less or spurious cointegration.

Table (12) reports the values of Akaike, Hanna-Quin and Schwartz information criteria for VAR (25) and VAR(1) models. Although the majority of outcomes support the parsimonious version, however, the suggested in Johansen (1995) likelihood ratio test clearly rejects the model reduction.

Table (12) : Information criteria and Likelihood ratio test for the VAR order selection

Model	T	N.P	Logl	SC	HQ	AIC
Var(25)	2788	404	31033.054	-21.112	-21.662	-21.972
Var(1)	2788	20	30364.043	-21.725	-21.752	-21.768
Likelihood Ratio~x^2(384)	1338.0 [0.000]**					

Note : T stands for the number of observations, Logl is the loglikelihood value, p-values are reported in the brackets, *(**) denotes rejection of unit root hypothesis at 5%(1%)

The latter contradiction forces the econometrician to use both VAR models simultaneously, while another challenge focus on the nature of deterministic variables that eventually will eventually enter the cointegrating space. In the present analysis following Johansen (1992b) the deterministic components are examined alongside the number of cointegrating relations using the pantula principle (Johansen 1992, 1995).

This is a tool for deciding simultaneously a) the correct rank order and b) the deterministic components that will enter the VECM. The strategy starts with the VECM framework presented in equation (25). For notation simplicity the number of lags, k, is set equal to two.

$$\Delta z_t = \Gamma_1 \Delta z_{t-1} + a \begin{bmatrix} \beta \\ \mu_1 \\ \delta_1 \end{bmatrix} \tilde{z}_{t-k} + a_1 \mu_2 + \alpha_1 \delta_2 \tau + u_t \tag{25}$$

Imposing specific restrictions in equation (25) generates three models. Those are models 1,2 and 3, each introducing a different set of deterministic components.[117]The applied principle moving on from the most restrictive model to the least restrictive one, compares at each stage of the process the trace statistic to its critical value and stops the first time the null hypothesis is not rejected.[118]

Tables (13) and (14) report respectively the trace and max eigenvalues statistics when the number of lag augmentations in all restricted models is set equal to one. Starting with table (13) and the most restrictive model, that is model1, the trace rank statistic is estimated at a value of 432.541, exceeding its 95% and 99% critical values. Proceeding with the next most restrictive model and keeping r=0 the null hypothesis is rejected again.

Keeping this track, hence moving from left to right in every row of table (13), the first time the null hypothesis is not rejected, belongs to the block that corresponds model 1 with cointegration rank equal to three.

Even though the monte carlo experiments in Cheung and Lai (1993) suggest the superiority of the trace rank test to the maximum eigenvalue statistics mainly due to the

[117]Model1 is generated after setting δ_1,δ_2,μ_2 equal to zero. This restricts the intercept solely to the cointegrating space. This model is suitable if there are no linear trends in the levels of the data, such that the first differenced series have a zero mean. The critical values for this model are available in Osterwald – Lenum (1992), although Doornik and Ooms (1999) has also produced critical values using the Gamma distribution. Note that these values are the default option in the econometric package Pcgive 10.1. If there are linear trends in the levels of the data then the analysis should specify model 2, that allows the non stationary relationships in the model to drift. This model is generated when δ_1 and δ_2 parameters are set equal to zero. The critical values for this model are found in Pesaran et al (2000). Finally, model 3 sets δ_2=0 and as a result the cointegration space includes solely time as a trend stationary variable. This model is proper whenever there is some long run linear growth that the model cannot account for.
[118] For other applications of Pantula principle see Love and Chandra (2005) and Dawson (2006).

robustness of the former test in phenomena of skewness and excess kurtosis, however in this case the maximum eigenvalues statistics reported in table (14) estimate identical results and so these results strengthen the overall credibility of Johansen analysis.

Table (13) : Eigenvalues and trace rank statistics for VAR (1) and models 1,2 and 3.

r	n-r	Eigenvalue1	Model1	Eigenvalue2	Model2	Eigenvalue3	Model3
0	4	0.093	432.541 [0.000]**	0.092	427.030 [0.000]**	0.101	494.468 [0.000]**
1	3	0.035	156.859 [0.000]**	0.035	155.477 [0.000]**	0.035	192.628 [0.000]**
2	2	0.018	54.986 [0.000]**	0.018	53.966 [0.000]**	0.030	91.088 [0.000}**
3	1	0.000	2.585 [0.660]	0.000	2.494 [0.114]	0.001	4.355 [0.234]

Note : MacKinnon-Haug-Michelis (1999) p-values are reported in the brackets.*(**) denote respectively rejection at 5%(1%) significance level.

Table (14) : Eigenvalues and Maximum eigenvalues statistics for VAR (1)- models 1,2 and 3.

r	n-r	Eigenvalue1	Model1	Eigenvalue2	Model2	Eigenvaue3	Model3
0	4	0.093	275.681 [0.000]**	0.092	271.553 [0.000]**	0.101	301.840 [0.000]**
1	3	0.035	101.873 [0.000]**	0.035	101.510 [0.000]**	0.035	101.539 [0.000]**
2	2	0.018	52.400 [0.000]**	0.018	51.472 [0.000]**	0.030	86.733 [0.000]**
3	1	0.000	2.585 [0.660]	0.000	2.494 [0.114]	0.001	4.355 [0.690]

Note : MacKinnon-Haug-Michelis (1999) p-values are reported in the brackets.*(**) denote respectively rejection at 5%(1%) significance level.

Furthermore tables (15) and (16) estimate correspondingly the trace and maximum eigenvalues statistics when the number of lag augmentations is set equal to 25. Both statistics support the choice of model 1, while both suggest the presence of one cointegrating relation. The outcome clearly rejects the validity of term structure theory and obviously contrasts the previous result for the presence of three cointegrating vectors or the presence of only one common stochastic trend.

Table (15) : Eigenvalues and trace rank statistics for VAR (25) - models 1,2 and 3.

r	n-r	Eigenvalue1	Model1	Eigenvalue2	Model2	Eigenvalue3	Model3
0	4	0.014	72.989 [0.000]**	0.014	72.091 [0.000]**	0.015	83.520 [0.000]**
1	3	0.007	32.266* [0.100]	0.007	31.662 [0.030]	0.008	40.695 [0.082]
2	2	0.003	11.676 [0.478]	0.003	11.188 [0.200]	0.004	18.308 [0.323]
3	1	0.000	1.943 [0.788]	0.000	1.566 [0.210]	0.001	4.757 [0.631]

Note : MacKinnon-Haug-Michelis (1999) p-values are reported in the brackets.*(**) denote respectively rejection at 5%(1%) significant level

Table (16) Eigenvalues and Max-eigenvalue statistics for VAR (25)- models 1,2 and 3.

r	n-r	Eigenvalue1	Model1	Model2	Eigenvalue2	Eigenvalue3	Model3
0	4	0.014	40.723 [0.000]**	0.014	40.429 [0.000]**	0.015	42.825 [0.001]**
1	3	0.007	20.590 [0.085]	0.007	20.473 [0.061]	0.008	22.387 [0.133]
2	2	0.003	9.732 [0.359]	0.003	9.622 [0.237]	0.004	13.550 [0.285]
3	1	0.000	1.943 [0.788]	0.000	1.566 [0.210]	0.001	4.757 [0.631]

Note : MacKinnon-Haug-Michelis (1999) p-values are reported in the brackets.*(**) denote respectively rejection at 5%(1%) significant level

5.2 Applying Breitung and Hassler (2002) test.

The analysis in this section is separated out in two parts. The first makes a brief presentation of the fundamental mathematical relations of Breitung and Hassler's (2002) fractional parametric trace test, and the second proceeds with an application of the statistic to the observed interest rates series.

Breitung and Hassler's (2002) parametric fractional trace test is based upon a generalized eigenvalue problem analogous to Johansen (1988). The statistic sets up a multivariate version of the regression based score test that results in a chi-squared distribution, with degrees of freedom depending on the cointegration rank under the null. Note that the distribution under consideration does not rely on the integration order of the observed series or the integration order of deviations corresponding the long run relationships.

Suppose x_t is a nx1 vector of Gaussian components, originally generated from y_t vector after applying the fractional differencing operator of equation (1). Note that Gaussianity assumption is only important for the set up of the log-likelihood function which is presented in equation (26) and is not necessary for further asymptotic analysis.

$$L(\theta,\Sigma) = -\mathrm{T}/2(2\pi|\Sigma|) - 0.5\sum_{t=1}^{T}[(1-L)^{d+\theta} y_t^{'}\Sigma^{-1}(1-L)^{d+\theta} y_t] \tag{26}$$

Defining now

$$S_{10} = \sum_{t=2}^{T} x_{t-1}^{*} x_t^{'},\ S_{11} = \sum_{t=2}^{T} x_{t-1}^{*} x_{t-1}^{*}{}',\ x_{t-1}^{*} = \sum_{j=1}^{t-1} j^{-1} x_{t-1} \tag{27}$$

and replacing Σ with a consistent estimate as in the following equation

$$\hat{\Sigma} = T^{-1}\sum_{t=1}^{T} x_t x_t^{'} \tag{28}$$

Breitung and Hassler (2002) set up their multivariate score statistic as in the next equation

$$\Lambda_0(d) = tr[\hat{\Sigma}^{-1} S_{10}^{'} S_{11}^{-1} S_{10}] \tag{29}$$

Since the trace of a matrix is the sum of all eigenvalues, Breitung and Hassler (2002) follow at this point Johansen (1995) and estimate their cointegration rank statistic through the sum of the n-r smallest eigenvalues. So actually based their test on the following problem

$$\left| \lambda \hat{\Sigma} - S_{10}^{'} S_{11}^{-1} S_{10} \right| = 0 \tag{29i}$$

Under the null hypothesis H_0: $r = r_0$ the trace statistic is introduced as

$$\Lambda_{r_0}(d) = \sum_{j=1}^{n-r_0} \lambda_j \tag{30}$$

with $\lambda_1 \leq \ldots \leq \lambda_n$ denote the ordered eigenvalues. As T tends to infinity the trace has an asymptotic X^2 distribution with $(n-r_0)^2$ degrees of freedom.

Finally two issues must be analyzed: a) the use of de-trended series and b) the existence of possible misspecifications in d. For the first it has to be stated that if a possible de-trending is of interest, then regressing x_t on a vector containing different deterministic terms, such as constants, time trends and possibly seasonal dummy variables must first be realized. In this case the trace statistic remains practically invariable and the only difference centers on the substitution of the fractionally differenced series in equations (27) and (28) with the above estimated residuals.

The second possibility explores the use of short run dynamics in the analysis, due to possible misspecifications existing in the long memory parameter. These short run dynamics are modeled through estimating appropriate VAR (X) forms. In the latter case the trace statistic is re-estimated using the residuals of a well suited autoregressive process.

As has been pointed by Breitung and Hassler (2002) assuming a known long memory parameter places severe restrictions on the testing procedure, and a possible misspecification may be responsible for the presence of serially correlated errors. If that is the case then Breitung and Hassler (2002) suggest that the effects of a possible misspecification of d can likely be reduced when accounting for short run dynamics, and this strategy is well explored in the estimations presented in table (17). However, Breitung and Hassler (2002) point that this process is rather trivial, since a large lag number will tend on one hand to eliminate the size distortions, while on the other will weaken drastically the power of the test.

Table (17) which considers this last notation induces three alternatives for modeling short run dynamics. These are a) the zero lag-augmentations choice, b) the one lag option and c) the twenty five lags alternative. Obviously the last two cases follow the specifications applied in Johansen parametric test.[119]The results in table (17) depend heavily on the modeling of short run dynamics and specifically options a and c identify significantly three cointegrating relations, whereas the b option clearly rejects all null hypothesis.

Table (17) : Breitung and Hassler (2002) fractional trace test.

r	n-r	$\Lambda_{ro\,-0}$ (d)	Λ_{ro-1} (d)	Λ_{ro-25} (d)	X^2~(n-ro)^2
0	4	360.128*	145.944*	40.253*	26.296
1	3	85.516*	36.219*	14.048*	16.919
2	2	9.578*	21.921*	3.730*	9.488

[119]The fractionally differenced series are generated with the R program and particularly with the Fracdiff package. For estimation of the long memory parameter the analysis uses the GPH test. Those estimates are produced after setting the number of low -frequency ordinates equal to 575 and this implies that the m parameter in the sample function is set equal to 0.8. If the lag augmentation strategy is not, then Breitung and Hassler (2002) suggest the computation of the statistic using an estimated d value. In this last case the analysis is better to use the bootstrap critical values presented in Davidson (2002).

3	1	2.837	8.665*	0.1418	3.841

Note : $\Lambda_{ro-0}(d)$, $\Lambda_{ro-1}(d)$, $\Lambda_{ro-25}(d)$ denote respectively the estimated trace statistics estimated under using a) no lags , b) one lag and c) twentyfive lags. The critical values are reported in the last column. * denotes rejection of the null.

Finally, table (18) concentrates results on the Johansen (1998,1991) and Breitung and Hassle (2002) cointegration tests. Two things must be stated here. First, that the applied pantula principle picks up model1 in all four cases examined, and second that results on the cointegration rank appear to be sensitive arise to different lag augmentations applied.[120] Note that the last statement underlines the advantage of the non parametric variance ratio test, that does not depend on the specification of short run dynamics or any other tuning parameter.

Table (18) : Results of Johansen's (1998,1991) and Breitung and Hassler's (2002) tests

Lag length	Cointegration rank	Specifications	term structure theory
Panel A: Johansen (1998,1991) trace test			
1	r=3	Model 1	acceptance
25	r=1	Model1	rejection
Panel B: Johansen (1998,1991) max eigenvalue			
1	r=3	Model1	acceptance
25	r=1	Model 1	rejection
Panel C: Breitung and Hassler (2002) fractional trace test			
0	r=3	De-trended interest rate	acceptance
1	Rejection of the null	De-trended interest rate	rejection
25	r=3	De-trended interest rate	acceptance

Note Model 1 is generated from equation (23) after applying $\delta_1= \delta_2=\mu_2=0$. This determination restricts the intercept solely to the cointegration space. This model is suitable if there are no linear trends in the levels of the data such that the first differenced series have a zero mean.

6. An informal test of the variance ratio using the estimated cointegration space.

The space spanned by all linearly independent cointegration vectors is the cointegration space and Johansen (1988,1991) states that the reduced rank regression will indicates the number of unique cointegration vectors spanning it. However, any linear combination of stationary vectors will generate another stationary vector, and therefore estimates of the cointegrating vectors will not necessarily be unique. Therefore without imposing certain

[120] This is true for all panels A,B and C.

restrictions motivated by economic arguments it will be possible only to estimate a basis[121]of the space spanned by the cointegrating vectors.[122]

This last statement is obviously true for the variance ratio test, since the statistic does not identify all the restrictions imposed by expectation hypothesis theory. Note that the last in terms of cointegration imposes two restrictions. The first implies that among n interest rates of different maturities there should be n-1 cointegrating relations, while, the second states that if expectations hypothesis holds, then any linear combination of such n interest rates must normally sum its coefficients to zero. Put it differently, the last restriction implies that coefficients in each of the cointegrating vectors must add up to zero if expectations hypothesis holds.

In the present analysis the first of these two implications was extensively tested in sections 4 and 5 through the use of different cointegration techniques, while the second, as is reported in Hall et al (1992), is testable using the following relation

$$B = DF, D = [I_{n-1},...,-i]' \qquad (31)$$

where i=[1,…,1], D is a n x (n-1) matrix and F is a (n-1) x (n-1) matrix of free parameters. Based on this last equation the estimated cointegration space in Nielsen (2010) is informally compared to D.[123]

[121] Basis is a set of linearly independent vectors that in a linear combination can represent every vector in a given vector space. Put it simple, a basis is a linearly independent spanning set. In mathematical terms, a basis B of a vector space V over a field F is a linearly independent subset of V that spans V. So B satisfies the following conditions : a) the linear independent property and b) the spanning property. In more details, suppose that B=(v_1,...,v_n) is a finite subset of a vector space V over a field F(with real or complex numbers R or C). The linear independence property implies that for all a_1,..,a_n of F the following statement is true $a_1v_1+\ldots+a_nv_n=0$. The spanning property implies that for every x in V it is possible to choose $a_1,\ldots,a_n$ such that $x=a_1v_1+\ldots+a_nv_n$.h.

[122] See Johansen (1988,1991), and Chen and Hurvich (2003, 2006)

Although the variance ratio statistic is purely a strategy for testing the cointegration rank and is not associated formally with a distribution theory for the estimated cointegration space, inferences on the validity of expectations theory may produced indirectly, through comparing the estimated and the hypothesized cointegration space. The former is the space spanned by a subset of eigenvectors generated from the following eigenproblem

$$\left|\lambda B_T - A_T\right| = 0 \tag{32}$$

where equation (32) uses the eigenvalues of equation (9).

Since, the estimated variance ratio test rejects the term structure theory by acknowledging two cointegrating relations instead of three, the rest part of the analysis retains a confirmatory task, in the sense that if expectation hypothesis does not hold then the estimated and hypothesized cointegration space must deviate.

Although defying particular eigenvectors of the equation (32) as estimates of the real cointegrating vectors is not accurate, since the variance ratio does not deliver a distribution theory for the estimated cointegrating space, however these vectors are proven useful for the rest part of section 6.

Panels (19i) and (19ii) of table (19) present the eigenvectors of $R_T(d_1)$ matrix when d_1 parameter is set equal to 0.1 and 1 respectively. The eigenvectors correspond to the eigenvalues reported in table (9), and specifically eigenvector η_j is linked to the eigenvalue

[123] See Johansen (1998,1991) for a simultaneous test on both the zero sum and the rank restrictions.

λ_j for j=1,..,4, while these vectors are simultaneously sorted in the same order as the corresponding eignevalues. Note that the last row of both panels reports the sum of the elements of each eigenvector.

According to expectation hypothesis the elements of all the columns of any basis of the cointegration space must sum up to zero and this does not seem to be the case in either panel of table (19), although it could be said that the sum corresponding panel 19i is more close to zero the other one.

Specifically in panels (19i) and (19-ii) the sums are -0.122 and 1,818 respectively. However these vectors cannot be regarded as straightforward estimates of particular cointegrating vectors, and so as in Nielsen (2010) the analysis concentrates in one matrix on the matrix $\eta(3)$, which is comprised by the eigenvectors corresponding the three largest eigenvalues of $R_T(d_1)$ matrix. The analysis continues next with the rotation of this subspace, since this makes easier the economical interpretation of it's vector.[124]The rotation is achieved by estimating the following product

$$\eta(3)([I_3,0_{3X1}]\eta(3))^{-1} \qquad (33)$$

which applies a normalization on each cointegrating vector, so that each can be considered an estimated spread between two interest rates series.

Table (20) that presents the estimated cointegration space assuming r=3, verifies that the estimated and hypothesized space are not relevant to each other and so the comparison casts doubts on the validity of expectations hypothesis.

[124]In order to proceed with the analysis we assume r=3.

Table (19) Eigenvectors of the de-trended data after setting d_1=0.1 and d_1=1

	Panel (19)-i d_1=0.1				Table (19)-ii d_1=1			
	η_1	η_2	η_3	η_4	η_1	η_2	η_3	η_4
eonia	-0.4434	-0.8741	0.8841	0.3319	0.4147	-0.9052	0.0930	0.0005
I_7	-0.5165	-0.3617	0.1649	0.4602	0.5230	0.1573	-0.7998	-0.2493
I_{10}	-0.5180	-0.2546	0.2841	0.5380	0.5264	0.2566	0.1461	0.7973
I_{12}	-0.5179	-0.2546	0.3325	0.6234	0.5267	0.3001	0.5748	-0.5496
sum	-1.9958	**-1.745**	**1.6656**	**1.9535**	1.9908	-0.1912	0.0136	0.0006

Note : The eigenvectors of $R_t(d_1)$ matrix are reported for the detrended data. The eigenvectors are sorted in the same order as the eigenvalues in table 7(i). The final row reports the sum of all the elements of the corresponding eigenvector.

Table (20) Estimated cointegration space assuming r=3

	Table (15)-i d_1=0.1			Table (15)-ii d_1=1		
	η_2	η_3	η_4	η_2	η_3	η_4
eonia	1	0	0	1	0	0
I_7	0	1	0	0	1	0
I_{10}	0	0	1	0	0	1
I_{12}	-0.017	-0.193	1.334	-0.7862	-0.9923	-0.998
sum	**0.983**	**0.807**	**2.334**	0.213	0.008	0.002

Note : The table reports the estimated and rotated cointegration space assuming r=3

7. Conclusions.

The paper using four European interest rates of different maturities. applies the non parametric variance ratio test of Nielsen (2010) in order to produce inferences on the expectation hypothesis. The procedure contrary to the parametric tests of Johansen (1988,1991) and the fractional version of Breitung and Hassler (2002), which both are estimated in the paper, is invariant to short run dynamics and naturally avoids the usual misspecification errors that rise when the other two methods are applied.

Furthermore Nielsen's (2010) strategy beside the latter advantage does not require the estimation of cointegrating vectors or inferences made on the integration order of the series under consideration, although the test does depend on a parameter appearing in the asymptotic distribution of the ratio, that is d_1, and whose values correspond to significant differences in the power of the ratio test.

Following Nielsen who suggests the use of $d_1=0.1$ since this maximizes the power of the test, the analysis proceeds with this choice and with an alternative of setting $d_1=1$. In the first case the sequential test of the variance ratio comes up with two cointegrating relations and hence rejects the expectations hypothesis, while the same result occurs when considering the alternative choice of d_1, although in this case no cointegrating relations exist.

The paper proceeds with the application of some known parametric tests in order to underline the drawbacks which are present when different lag-augmentations are applied.

As far as the Johansen methodology is concerned the VAR (1) and VAR (25) models which both are estimated after controlling for the relevant short run dynamics, they produce not surprisingly two different outcomes. Specifically VAR (1) concludes the presence of three and therefore accepts the expectations hypothesis , while VAR (25) acknowledges the presence of only one cointegrating relation. Note that these results are consistent either when estimating the trace rank or the maximum eigenvalue statistics.

The same lag-augmentations are applied when the analysis estimates the fractional cointegration rank of Breitung and Hassler (2002) although in this particular case there is also the choice of setting the lag-length equal to zero. The results show that when lag augmentations are equal to 25 the expectation hypothesis holds, while the remaining two options reject the term structure theory. Finally, the paper with an informal testing comparison of the estimated and hypothesized cointegration space. This require a rotation of the estimated cointegration space so that every column represent the spread of two interest rates of different maturities. The results show that the estimated and hypothesized space are not relative to each other and so this informal testing comparison of the estimated

Rejection is also valid when considering the alternative choice for d_1, although no cointegrating relations are acknowledged in this case. The paper proceeds with the application of some known parametric tests in order to prove the drawbacks which are attached with the decision over the lag-augmenations. As far as the Johansen methodology is concerned, VAR(1) and VAR(25) models which are selected after controlling the short run dynamics, they produce not surprisingly two different outcomes. On one hand VAR(1) framework stands for the presence of three cointegrating relations and hence accepts the expectations hypothesis, whereas VAR(25) acknowledges only one. The same lag-augmentations are chosen when alternatively we apply the fractional cointegration rank test of Breitung and Hassler (2002), although in this particular testing there is also the choice of setting the lag-length equal to zero. For this last case and when lags are set to 25 the expectations hypothesis holds, whereas for the remaining case all null hypothesis are being rejected. Finally, the paper proceeds with an informal testing of the cointegration space. This require a rotation of the estimated cointegration space so that every columns will stand for the spread of two interest rates. The results show that the estimated and the hypothesized space are not relative to each other and so this informal testing procedure constitutes another indication that in the present data set the expectation hypothesis does not hold.

Appendix

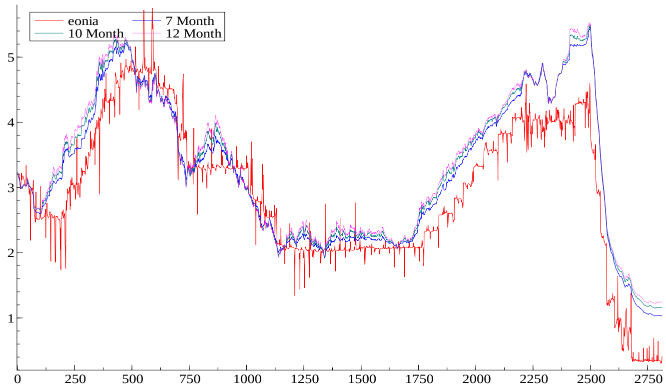
1. Plots and descriptive statistics of interest rates series under examination.

Table (A) Descriptive statistics of eonia, i_7, i_{10} and i_{12}

	mean	Std.dev	skewness	Kurtosis	Jarque/Bera	Q(50)	Q(100)	Q(150)	Q(200)
i_{eonia}	2.932	1.125	-0.294	-0.285	50.315 [1.1864e-011]	125729. [0.000]**	219097. [0.000]**	275483. [0.000]**	302072. [0.000]**
i_7	3.2486	1.1246	0.060076	-0.99523	117.78 [2.6508e-026]	129620. [0.000]**	224797. [0.000]**	282689. [0.000]**	311964. [0.000]**
i_{10}	3.305	1.121	0.075	-1.0355	128.38 [1.3243e-028]	129430. [0.000]**	223829. [0.000]**	280708. [0.000]**	309565. [0.000]**
i_{12}	3.346	1.1218	0.082569	-1.0444	131.04 [3.5100e-029]	129197. [0.000]**	222729. [0.000]**	278440. [0.000]**	306562. [0.000]**

Note : p-values are reported in the brackets/*(**) denotes rejection at 5% and 1 significant level/ Q(50),Q(100),Q(150),Q(200) denotes the Ljung Box statistic for

Graph (A) Plot of Eonia, i_7, i_{10}, i_{12}.



2. Modeling fractional ARIMA models for the first differenced interest rates series of Δ_{eonia}, Δ_{i7}, Δ_{i10} and Δ_{i12}.

Table (3) uses the Akaike and Schwartz information criteria and corresponds to Δ_{eonia},Δ_{i7}, $\Delta_{i10,}$ Δ_{i12} the ARFIMA (1,0.331,1), (1,0.361,1),(1,0.363,1) and (1,0.363,1) respectively. The procedure starts by estimating initially the fractional differencing operator of Δ_{eonia},Δ_{i7}, $\Delta_{i10,}$ Δ_{i12}, while in a next stage applies fractional differencing to the above series, and finally uses ACF and PACF plots, hence applies the Box-Jenkins methodology to decide the appropriate orders of AR and MA polynomials.

Table (B) using the Pc.Give10, which provides the option of estimating the ARFIMA using a fixed long memory parameter, presents the estimated parameters of the above selected fractional ARIMA models when the fractional differencing operator is fixed. Table (C) which immediately follows repeats those estimates but allows this time the fractional parameter to be freely estimated through the maximum likelihood process.

Table (B) The estimated coefficients and the portmanteau statistics of the selected ARFIMA models using a fixed long memory parameters.

	Δ_{eonia}	Δ_{i7}	Δ_{i10}	Δ_{i12}
d fixed	0.331	0.361	0.363	0.363
α_1	0.354 [0.000]**	0.463 [0.000]**	0.448 [0.000]**	0.469 [0.000]**
θ_1	-0.932 [0.000]**	-0.644 [0.000]**	-0.680 [0.000]**	-0.711 [0.000]**
Q (50)	62.568 [0.077]	84.087 [0.001]**	77.541 [0.004]**	70.349 [0.019}*
Q(100)	118.45 [0.078]	132.95 [0.010]*	133.69 [0.009]**	130.37 [0.016]*
Q(150)	180.50 [0.035]**	219.92 [0.000]**	214.70 [0.000]**	204.39 [0.001]**
Q(200)	232.56 [0.046]*	313.90 [0.000]**	295.96 [0.000]**	288.16 [0.000]**

Note :P(50),P(100),P(150),P(200) denote the portmanteau statistics for residual autocorrelation. p-values are reported in the brackets. *(**) rejection at 5% and 1% significant level

Table (C) The estimated coefficients and the portmanteau statistics of the selected ARFIMA models when long memory parameter is freely estimated using the maximum likelihood process.

	Δ_{eonia}	Δ_{i7}	Δ_{i10}	Δ_{i12}
d fixed	0.376 [0.000]**	0.254 [0.000]**	0.325 [0.000]**	0.320 [0.000]**
α_1	0.323 [0.000]**	-0.083 [0.919]	0.452 [0.000]**	0.480 [0.000]**
θ_1	-0.239 [0.000]**	-0.001 [0.999]	-0.649 [0.000]**	-0.682 [0.000]**
P(50)	61.803 [0.072]	91.584 [0.001]**	77.837 [0.003]**	70.734 [0.014]*
P(100)	171.55 [0.076]	142.18 [0.001]**	134.16 [0.007]**	130.95 [0.012]*
P(150)	179.54 [0.035]*	228.88 [0.000]**	214.8 [0.000]**	204.68 [0.001]**
P(200)	231.56 [0.046]*	331.92 [0.000]**	297.00 [0.000]**	289.53 [0.000]**

Note :P(50),P(100),P(150),P(200) denote the portmanteau statistics for residual autocorrelation. p-values are reported in the brackets. *(**) rejection at 5% and 1% significant level

Tables D, E, F and G report the Whittled estimated coefficients of the ARFIMA models corresponding Δ_{i7}, Δ_{i10}, Δ_{i12} and Δ_{Eonia} . Setting the max AR and MA order equal to 3 and using all possible combinations in modeling the short memory dynamics as in Sowell (1992), each of the above tables estimates the same 16 ARFIMA frameworks. In the brackets below the estimated coefficients the t-statistic values are reported, while *(**) denote respectively the rejection of the null at 5% and % significant level.

The last column in every table reports the Schwarz (Bayesian) information criteria. These estimations are generated using the R statistical program and specifically the afmtools-package. The above information criterium decides the ARFIMA frameworks upon which the rest part of the analysis will actually evolve and as already has stated the selected ARFIMA models are all employing common AR and MA specifications which sets the AR and MA order equal to one.Using these ARFIMA models images (B) to (E) below each table generate graphically outputs on a) the inverse AR and MA roots, b) the theoretical and empirical spectrum and finally c) the correlogram of the estimated residuals.

The fact that all inverse AR and MA roots are well inside the unit circle indicates a good approximation of the long memory segments of the ARFIMA, since otherwise the roots of the autoregressive and moving average polynomials would normally approach the unit circle. In fact for a positive long range dependence the root of the autoregressive polynomial would approach the unit circle, while for a negative long run dependence the root of the moving average polynomial would do the same

Furthermore, plots of the implied spectral densities over the corresponding periodograms indicate the appropriateness of the selected ARFIMA, while the same fit is underlined by the autocorrelation functions of the estimated residuals.

Table (D) : Whittled estimated ARFIMA models corresponding Δ_{i7} interest rate.

Δ_{i7}							
ARFIMA	d	α_1	α_2	α_3	Θ_1	Θ_2	Θ_3
(0,d,0)	0.210 [6.806]**	-	-	-	-	-	-
(0,d,1)	0.265 [8.560]**	-	-	-	-0.094 [-4.978]**	-	-
(0,d,2)	0.267 [8.647]**	-	-	-	-0.097 [-5.1203]**	-0.003 [-0.177]**	-
(0,d,3)	0.348 [11.253]**	-	-	-	-0.183 [-9.274]**	-0.036 [-1.828]	-0.064 [-3.271]**
(1,d,0)	0.259 [8.363]**	-0.088 [-4.638]**	-	-	-	-	-
(2,d,0)	0.259 [8.373]**	-0.088 [-4.655]**	-0.0004 [-0.024]	-	-	-	-
(3,d,0)	0.2855 [9.221**	-0.114 [-5.991]**	-0.018 [-0.989]	-0.041 [-2.170]**	-	-	-
(1,d,1)	0.375 [12.129]**	0.460 [19.242]**	-	-	-0.654 [-19.839]**	-	-
(1,d,2)	0.258 [8.342]**	-0.822 [-14.085]**	-	-	0.735 [15.219]**	-0.057 [-1.192]	-
(1,d,3)	0.453 [14.647]**	0.492 [19.785]**	-	-	-0.780 [-18.306]**	0.083 [1.965]*	-0.051 [-1.210]
(2,d,1)	0.253 [8.175]**	-0.886 [-13.656]**	-0.053 [-0.821]	-	0.805 [14.991]**	-	-
(2,d,2)	0.462 [14.928]**	0.136 [6.541]**	0.250 [12.049]**	-	-0.431 [-13.688]**	-0.274 [-8.698}**	-
(2,d,3)	-0.085 [-2.762]**	1.254 [1.091]	-0.265 [-0.231]	-	-1.003 [-8.393]**	0.152 [1.272]	-0.063 [-0.530]
(3,d,1)	0.446 [14.422]**	0.471 [18.718]**	0.073 [2.933]**	-0.034 [-1.387]	-0.751 [-17.360]**	-	-
(3,d,2)	0.474 [15.324]**	-0.267 [-9.091]**	0.454 [15.413]**	0.040 [1.389]	-0.039 [-1.315]	-0.604 [-20.139]**	-
(3,d,3)	0.399 [12.911]**	0.946 [11.421]**	-1.066 [-12.872]**	-1.175 [-11.723]**	-1.175 [-11.723]**	1.223 [12.204]**	-0.584 [-5.829]**

Graph (B) : Plots of the estimated ARFIMA (1,d,1) for Δi_7 a) inverse AR and MA roots, b) theoretical and empirical spectrum, c) ACF-residuals.

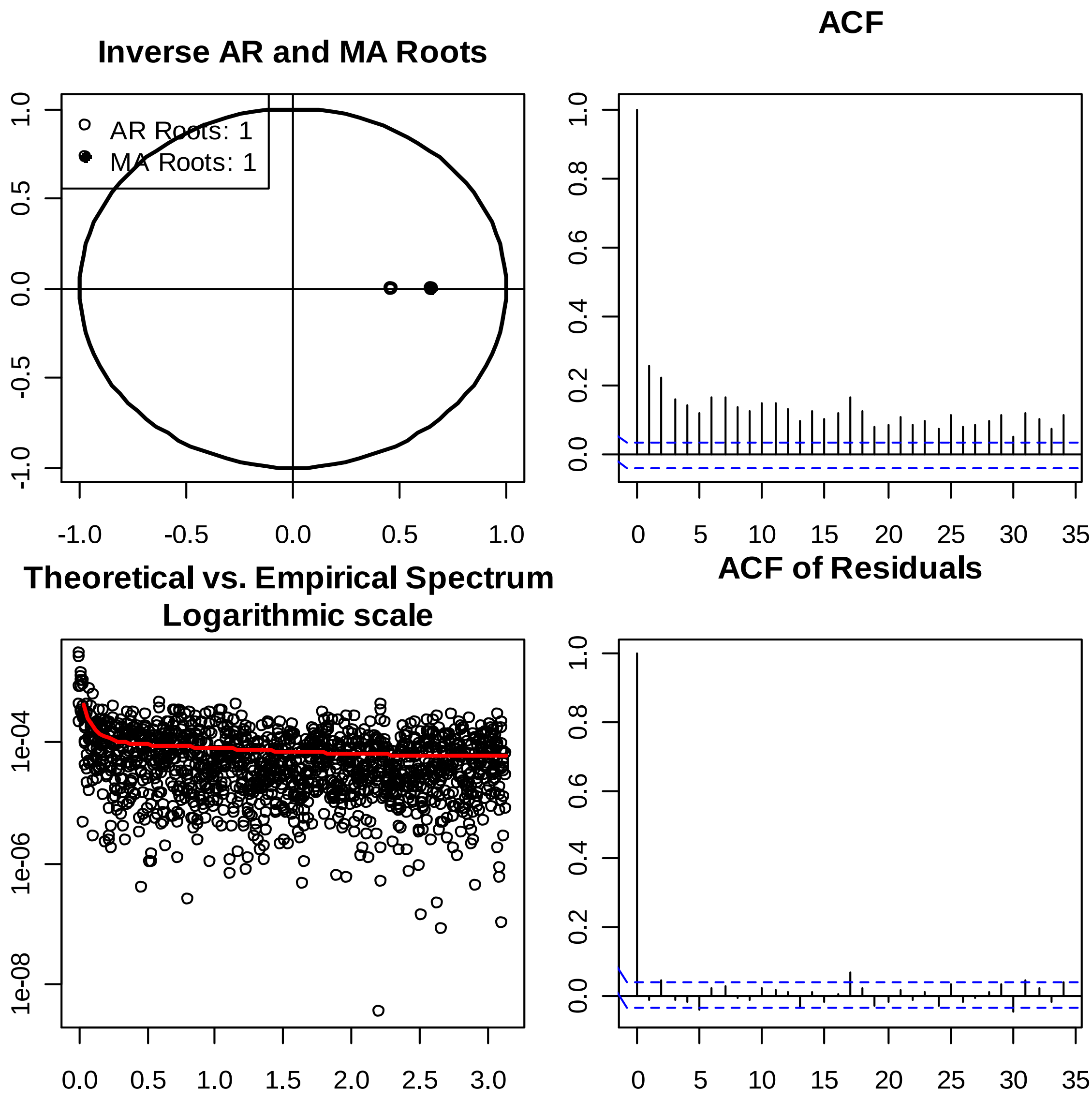


Table (E) : Whittled estimated ARFIMA models corresponding Δ_{i10} interest rate.

Δ_{i10}								
ARFIMA	d	α_1	α_2	α_3	Θ_1	Θ_2	Θ_3	Sic
(0,d,0)	0.164 [5.314]**	-	-	-	-	-	-	
(0,d,1)	0.219 [7.077]**	-	-	-	-0.097 [-5.105]**	-	-	
(0,d,2)	0.243 [7.864]**	-	-	-	-0.122 [-6.378]**	-0.029 [-1.543]	-	
(0,d,3)	0.323 [10.447]**	-	-	-	-0.204 [-10.220]**	-0.058 [-2.906]**	-0.063 [-3.187]**	
(1,d,0)	0.209 [6.773]**	-0.084 [-4.456]**	-	-	-	-	-	
(2,d,0)	0.222 [7.196]**	-0.098 [-5.190]**	-0.020 [-1.102]*	-	-	-	-	
(3,d,0)	0.252 [8.148]	-0.128 [-6.711]**	-0.042 [-2.195]*	-0.045 [-2.391]*	-	-	-	
(1,d,1)	0.351 [11.350]**	0.448 [19.002]**	-	-	-0.669 [-19.596]**	-	-	
(1,d,2)	0.440 [14.234]**	0.612 [20.294]**	-	-	-0.932 [-13.950]**	0.103 [1.542]	-	
(1,d,3)	0.419 [13.552]**	0.504 [19.941]**	-	-	-0.804 [-17.566]**	0.075 [1.639]	-0.033 [-0.731]	
(2,d,1)	0.446 [14.426]**	0.475 [18.537]**	0.069 [2.714]**	-	-0.803 [-15.127]**	-	-	
(2,d,2)	0.410 [13.262]**	0.015 [0.761]	0.272 [13.379]**	-	-0.305 [-11.174]**	-0.344 [-12.610]**	-	
(2,d,3)	0.053 [1.743]	0.238 [1.180]	0.732 [3.626]**	-	-0.174 [-2.500]*	-0.691] [-9.940]**	-0.043 [-0.622]	
(3,d,1)	0.418 [13.506]**	0.464 [18.676]**	0.062 [2.510]*	-0.019 [-0.796]	-0.763 [-16.882]**	-	-	
(3,d,2)	0.040 [1.314]	0.320 [1.366]	0.707 [3.015]**	-0.054 [-0.233]	-0.242 [-3.304]**	-0.668 [-9.119]**	-	
(3,d,3)	0.410 [13.262]**	0.584 [15.869]**	0.263 [7.156]**	-0.155 [-4.210]**	-0.874 [-10.540]	-0.170 [-2.056]*	0.195 [2.362]*	

Note : In the brackets t-statistics are reported. *(**) denote respectively rejection of the null at 5% and 1% significant level

Graph (D) : Plots of the estimated ARFIMA (1,d,1) for Δi_{10} a) inverse AR and MA roots, b) theoretical and empirical spectrum, c)ACF-residuals.

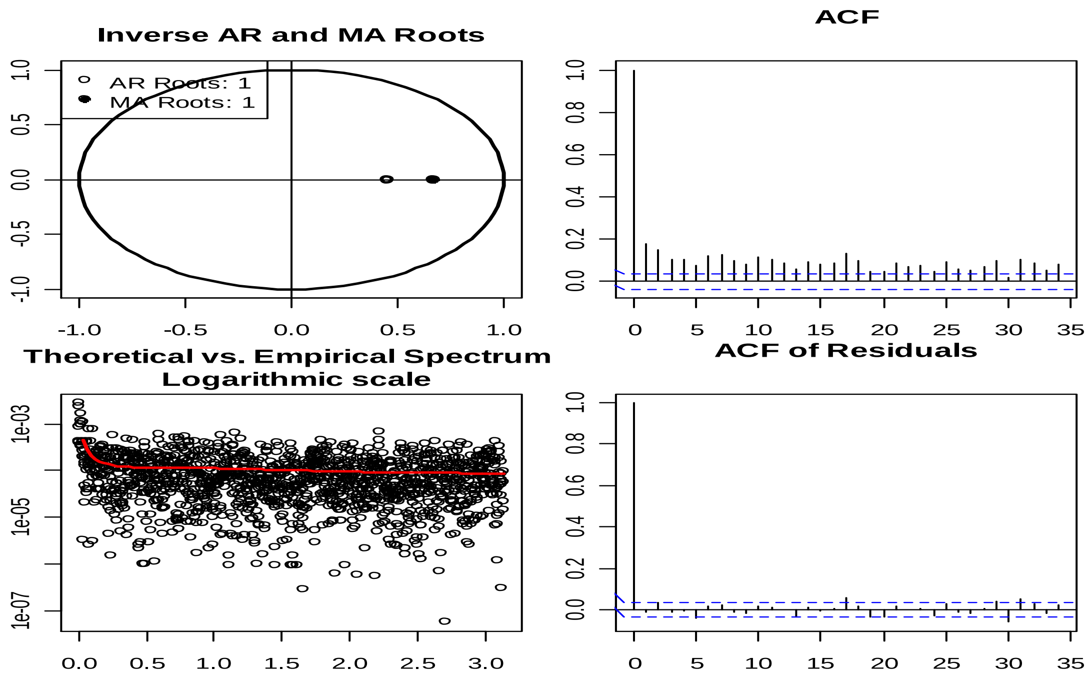


Table (F) : Whittled estimated ARFIMA models corresponding Δ_{i10} interest rate.

Δ_{i12}								
ARFIMA	d	α_1	α_2	α_3	Θ_1	Θ_2	Θ_3	Sic
(0,d,0)	0.148 [4.800]**	-	-	-	-	-	-	
(0,d,1)	0.191 [6.182]**	-	-	-	-0.077 [-4.060]**	-	-	
(0,d,2)	0.228 [7.374]**	-	-	-	-0.115 [-6.031]**	-0.044 [-2.324]*	-	
(0,d,3)	0.307 [9.913]**	-	-	-	-0.196 [-9.778]**	-0.073 [-3.658]**	-0.062 [-3.139]**	
(1,d,0)	0.183 [5.930]**	-0.065 [-3.465]**	-	-	-	-	-	
(2,d,0)	0.203 [6.562]**	-0.086 [-4.551]**	-0.031 [-1.673]	-	-	-	-	
(3,d,0)	0.233 [7.527]**	-0.117 [-6.122]**	-0.052 [-2.732]**	-0.046 [-2.423]**	-	-	-	
(1,d,1)	0.348 [11.249]**	0.472 [19.456]**	-	-	-0.699 [-18.938]**	-	-	
(1,d,2)	0.416 [13.456]**	0.586 [20.404]**	-	-	-0.890 [-14.799]**	0.076 [1.267]	-	
(1,d,3)	0.400 [12.926]**	0.509 [20.009]**	-	-	-0.798 [-17.281]**	0.055 [1.199]	-0.022 [-0.490]	
(2,d,1)	0.418 [13.510]**	0.485 [18.972]**	0.050 [1.967]*	-	-0.792 [-15.627]**	-	-	
(2,d,2)	0.387 [12.509]**	-0.083 [-3.947]**	0.309 [14.606]**	-	-0.190 [-7.388]**	-0.422 [-16.413]**	-	
(2,d,3)	0.407 [13.144]**	-0.220 [-8.051]**	0.434 [15.843]**	-	-0.074 [-2.505]*	-0.593 [-20.075]**	0.033 [1.142]	
(3,d,1)	0.399 [12.895]**	0.476 [19.044]**	0.045 [1.814]	-0.013 [-0.547]	-0.764 [-16.854]**	-	-	
(3,d,2)	0.406 [13.109]**	-0.285 [-9.751]**	0.420 [14.374}**	0.023 [0.793]	-0.008 [-0.282]	-0.598 [-20.364]**	-	
(3,d,3)	0.406 [13.124]**	0.686 [9.426]**	0.410 [5.642]**	-0.245 [-3.372]**	-0.979 [-4.289]**	-0.305 [-1.337]	0.353 [1.547]	

Note : In the brackets t-statistics are reported. *(**) denote respectively rejection of the null at 5% and 1% significant level

Graph (E) : Plots of the estimated ARFIMA (1,d,1) for Δi_{12} a) inverse AR and MA roots, b) theoretical and empirical spectrum, c)ACF-residuals.

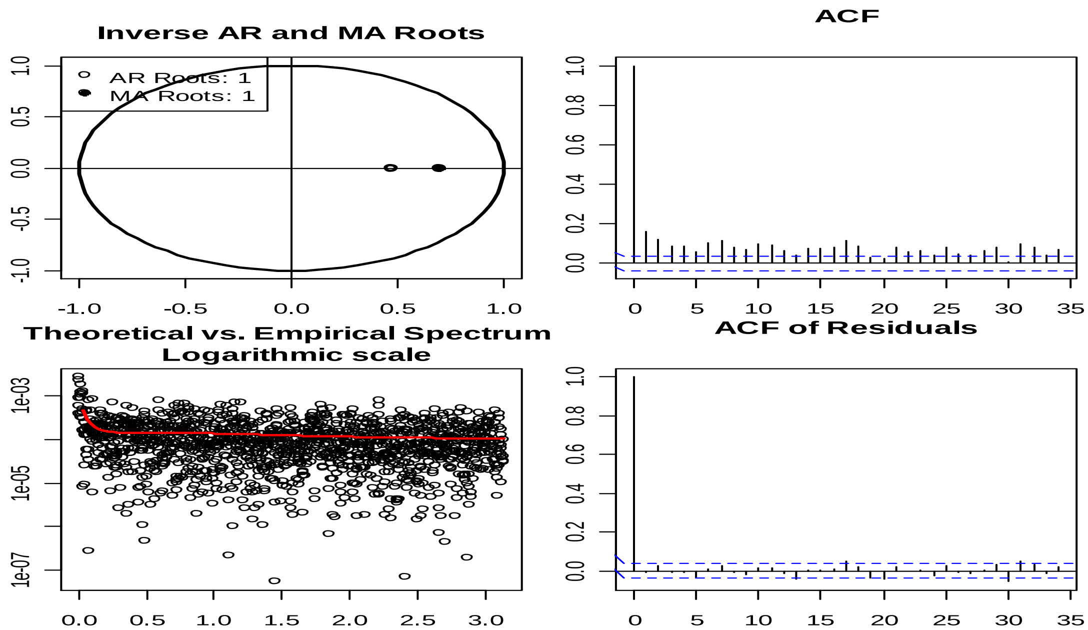
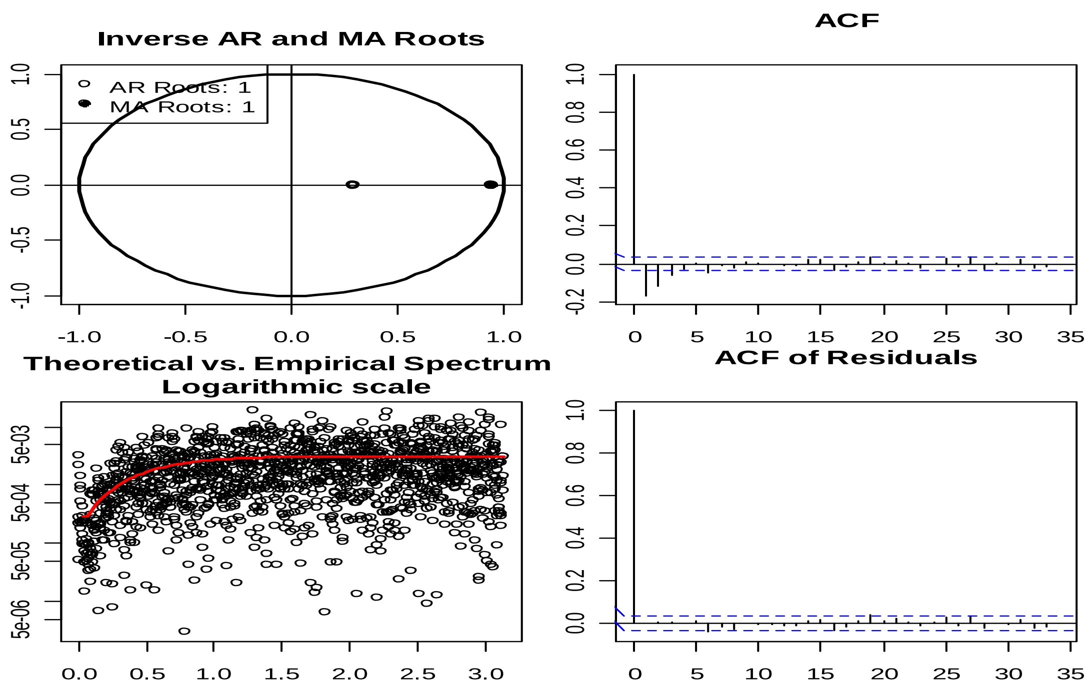


Table (Civ) : Whittled estimated ARFIMA models corresponding Δ_{Eonia}

Δ_{eonia}								
ARFIMA	d	α_1	α_2	α_3	Θ_1	Θ_2	Θ_3	Sic
(0,d,0)	-0.214 [-6.940]**	-	-	-	-	-	-	
(0,d,1)	-0.208 [-6.731]**	-	-	-	-0.014 [-0.744]	-	-	
(0,d,2)	-0.117 [-3.778]**	-	-	-	-0.121 [-6.193]**	-0.126 [-6.498]	-	
(0,d,3)	0.450 [14.547]**	-	-	-	-0.701 [-5.828]**	-0.179 [-1.487]	-0.055 [-0.460]	
(1,d,0)	-0.210 [-6.783]**	-0.010 [-0.563]	-	-	-	-	-	
(2,d,0)	-0.173 [-5.591]**	-0.048 [-2.531]*	-0.083 [-4.363]**	-	-	-	-	
(3,d,0)	-0.142 [-4.596]**	-0.083 [-4.336]**	-0.102 [-5.313]**	-0.065 [-3.431]**	-	-	-	
(1,d,1)	0.405 [13.105]**	0.290 [14.095]**	-	-	-0.945 [-5.315]**	-	-	
(1,d,2)	0.395 [12.756]**	0.322 [15.314]**	-	-	-0.968 [-5.311]**	0.022 [0.124]	-	
(1,d,3)	0.390 [12.611]**	0.359 [16.588]**	-	-	-1.000 [-5.079]**	0.043 [0.218]	0.009 [0.050]	
(2,d,1)	0.395 [12.772]**	0.297 [14.374]**	0.006 [0.335]	-	-0.944 [-5.418]**	-	-	
(2,d,2)	0.393 [12.690]**	0.481 [20.082]**	-0.047 [-1.962]*	-	-1.125 [-4.439]**	0.170 [0.674]	-	
(2,d,3)	0.386 [12.477]**	-0.577 [-6.545]**	0.336 [3.813]**	-	-0.058 [-0.599]	-0.897 [-9.250]**	0.058 [0.602]	
(3,d,1)	0.389 [12.588]**	0.302 [14.567]**	0.006 [0.322]	0.004 [0.206]	-0.943 [-5.465]**	-	-	
(3,d,2)	0.386 [12.484]**	-0.643 [-6.573]**	0.295 [3.018]**	0.019 [0.198]	0.006 [0.067]	-0.896 [-9.318]**	-	
(3,d,3)	0.395 [12.751]**	-1.185 [-3.194]**	-0.060 [-0.163]	0.166 [0.448]	0.540 [2.466]*	-0.891 [-4.071]**	-0.481 [-2.201}*	

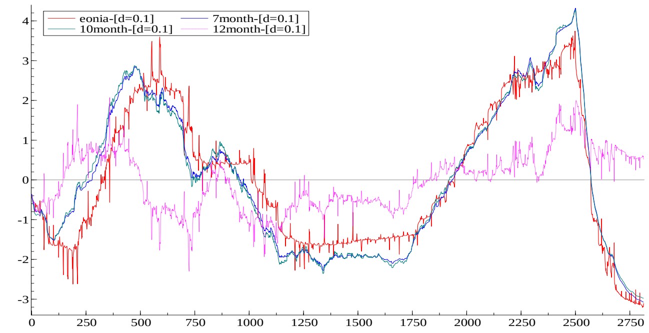
Note : In the brackets t-statistics are reported. *(**) denote respectively rejection of the null at 5% and 1% significant level

Graph (F) : Plots of the estimated ARFIMA (1,d,1) for Δ_{Eonia} a) inverse AR and MA roots, b) theoretical and empirical spectrum, c)ACF-residuals.

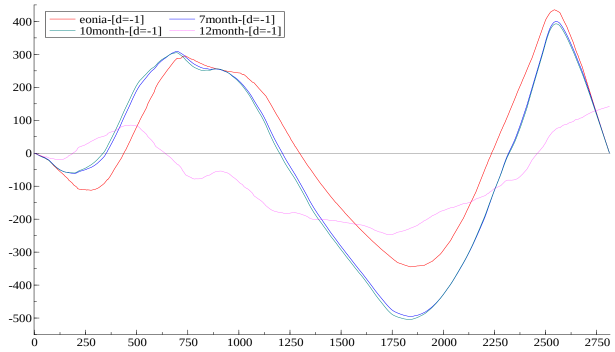


3. Graphical representation of interest rates series after applying fractional differencing.

Graph (G) : Fractionally differences interest rate series when d=0.1



Graph (H) : Fractionally differences interest rate series when d=1



4. Testing the unit root hypothesis using ADF, Phillips-Peron and KPSS tests

Table (H) The ADF, Phillips-Peron and KPSS test

Interest rates	ADF	Phillips-Peron	KPSS	Lags
eonia	-0.329 [0.918]	-1.598 [0.483]	1.108**	8
I_7	-0.346 [0.915]	-0.454 [0.897]	0.628*	7
I_{10}	-0.384 [0.909]	-0.522 [0.884]	0.624*	7
I_{12}	-0.438 [0.900]	-0.580 [0.872]	0.626*	7

Note : For the ADF tests the Schwartz information criteria was used for lag selection, while maxlag was set equal to 27 and the MacKinnon (1996) one-sided p-values were used for all cases. The Phillips-Peron spectral estimations were based uppon Barlett Kernel, while for the KPSS test statistic the asymptotic critical values are 0.739 and 0.463 for 1 % and 5% significance levels respectively. On all test equations a constant but no trend was included, *(**) denotes rejection at 5% and 1% significant level.

Chapter 3

Forecasting volatility and the risk return trade off .

An application on the Fama-French Benchmark market return

Forecasting volatility and the risk return trade off :

An application on the Fama-French Benchmark market return

A B S T R A C T

The paper presents an application of the exponential fractional GARCH-M (FIEGARCH-M) model to the daily stock market index returns of Fama-French. The model extends the basic long memory volatility framework of Bollerslev and Mikkelsen (1996) by introducing a possible volatility in mean effect. However, as has been stated by Christensen and Nielsen (2007) the introduction of volatility in the return equation may not be empirically warranted since often generates long memory in returns. Avoiding this spillover effect could be crucial and in order to achieve the co-existence of long memory in volatility and short memory in returns, the paper follows Ang et all. (2006) and Christensen, Nielsen and Zhu (2010) and estimates their filtering volatility frameworks (FIEGARCH-MG and FIEGARCH-MH). However, there is no reason to assume as Christensen, Nielsen and Zhu (2010) that innovations in the return equations necessarily will follow the normal distribution and therefore the present work enriches the estimation by introducing various distributional assumptions settings on the corresponding maximum likelihood functions. The results indicate the existence of a statistically significant in mean effect when both filtered models are estimated under the assumption of t-student. However both cases cannot outperform in terms of forecasting criteria the parsimonious FIEGARCH version which dominate filtered and non filtered volatility models in various forecasting horizons.

Keywords : FIEGARCH, Financial leverage, GARCH, Long memory, Risk-return trade off, Stock returns, Volatility feedback.

1. Introduction.

The main goal of the paper is to compare different volatility models in terms of their volatility forecasting potentials. However, before dealing with its final goal the paper addresses first the issue of specifying properly the competing volatility frameworks, and naturally this pre-requests answers upon questions such as : a) the ideal number of ARCH and GARCH terms that should be employed in every volatility model, b) the distributional assumptions applied and upon which the corresponding likelihood functions will be formed, c) the incorporation or not in the estimated frameworks of volatility-return relations, and finally d) issues related to long memory features and spillover effects when both phenomena are present.

Especially when dealing with the question of whether the estimated volatility frameworks identify possible volatility-return relations,[125]the paper answers the question of whether there exist statistically significant risk-return trade offs. So, the analysis although initially sets its eye on volatility forecasting, simultaneously addresses the issue of whether particular volatility specifications imply straightforward risk and return relations.

There are three theoretical approaches that justify the existence of such a relation : (a) the risk-return tradeoff, (b) the financial leverage effect and (c) the volatility feedback effect mechanism. The first assuming rational investors that will take on additional risk whenever they expect higher returns, underlines a positive relation between volatility and returns.[126] The second approach is attributed to Black (1976) and is broadly known as the leverage effect. The theory states that bad news decrease the price of a stock and increase at the same time the financial leverage or the debt/equity ratio of the corresponding firm. This

[125] These volatility models are denoted with the M designation. For example GARCH-M, EGARCH-M, FIEGARCH-M.

[126]This idea triggered Engle (1982) to introduce GARCH-M, that sets volatility as explanatory variable in the conditional mean equation.

situation sets stocks riskier after the price drop and hence increase future expected volatility. So, this leverage effect obviously introduces a negative relation between volatility and returns. Finally, the last approach is attributed to Cambell and Hentsel (1992) and their volatility feedback mechanism. The last resumes the following chain of events. An increase in volatility increases further the risk premium and the discount rate in the economy, and given an unchanged stream of dividends, this lowers the price of a stock,[127] and produces naturally a negative volatility return-relation.

Although the daily stock market returns are not characterized by the long memory feature and seem to be rather unpredictable, volatility in returns is highly predictable and can be modeled as a long memory process. These findings that are common in a number of studies,[128]set naturally the FIEGARCH model of Bollerslev and Mikkelsen (1996) at the core of the present analysis. The last not only addresses the issue of the asymmetric volatility reactions to positive and negative innovations as the exponential EGARCH model of Nelson (1991) does, but also accounts for the long memory features in volatility as those have been modeled by the FIGARCH approach of Baillie et al. (1996).

Furthermore the paper focuses on FIEGARCH-M models. These frameworks beside modeling volatility as a combination of asymmetrical and long memory features moreover estimate the in mean relation among the latter volatility specification and the examined return series.[129]

An interesting view of this risk-return relation that is also explored in the present analysis is explored in Christensen, Nielsen and Zhu (2010) and states that the introduction of

[127]Christensen,Nielsen and Zhu (2010) state that the volatility feedback effect mechanism possibly is strongest at the market level, while the leverage effect mainly affects individual stocks.
[128] See Crato and de Lima (1994), Baillie et al (1996), Robinson (1991), Baillie and Morana (2007).
[129]These are the basic operations of any volatility-Mmodel.

volatility in the return equation may generate long memory in returns. Avoiding this spillover effect may prove crucial for both, the forecasting properties and the statistical acknowledgment of the risk-return trade off, and so modeling the co-existence of long memory in volatility and short memory in returns as in Ang et all. (2006) and Christensen, Nielsen and Zhu (2010) sets an interesting empirical application of the present analysis. The procedure introduces two FIEGARCH-M models, the FIEGARCH-MG and FIEGARCH-MH, which both consider the option that it is changes in volatility entering the conditional in mean equation rather than volatility levels themselves.

In terms of forecasting FIEGARCH is initially compared to GARCH, IGARCH, FIGARCH, EGARCH and GJR. All these models are estimated for the daily stock market index returns of Fama-French for a period of 37 years, from 01.07.1963 to 31.06.2010.[130]The applied specifications are decided upon standard information criteria that reward both the goodness of fit and parsimony as well as the out of sample forecasting.[131]For example, although all the above models are estimated initially under the normality assumption, however, a trialing process of applying alternative distributions such as the t-student, general error and the skewed t-student, indicates that the fit corresponding all the above models is significantly enhanced whenever the skewed t-student distribution is applied. So the majority of the above models are estimated under this distributional assumption.

At this initial stage and controlling for autocorrelation in data returns, comparisons reveal that FIEGARCH is indeed the best forecasting model, while the volatility in mean effect

[130]The data are available on http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

[131]The "specification" term here refers to certain characteristics of the estimated volatility frameworks. Such characteristics are (a) the hypothesized distribution of innovations , (b) the number of GARCH-ARCH terms, (c) the inclusion or not in the estimated frameworks of possible M features and (d) the number of statistically significant autoregressive terms when controlling the conditional in mean equation for the existence of possible autocorrelations in the daily stock market returns.

irrespective of the estimated volatility framework, turns out being not statistically significant, contrary to results presented in Christensen, Nielsen and Zhu (2010). However this conclusion is generally sensitive to the distributional assumptions applied, since the choice of normal distribution generally acknowledges the statistical presence of the latter relation, although in this case the forecasting and fitting properties of all the competing volatility models are rather poor and non-competitive.

As far as the filtered long memory volatility models is concerned, there is no obvious reason why their innovations must necessarily follow the standard normal distribution as in Christensen, Nielsen and Zhu (2010) and therefore the paper changes the estimation settings by introducing various possibilities over the distributional assumption. The t-student distribution that is finally chosen acknowledges a statistically significant in mean effect, although both filtered models cannot outperform in terms of forecasting the parsimonious FIEGARCH model according to standard forecasting criteria.

The rest of the paper is organized as follows. Section 2 presents the mathematical equations that correspond the M volatility models and the quasi maximum likelihood method. Section 3 estimates all the competing volatility frameworks and particularly section 3.1 estimates the non filtered EGARCH, FIEGARCH, FIGARCH, GARCH, GJR, IGRACH volatility models, while section 3.2 estimates the filtered FIEGARCH models, FIEGARCH-MH and FIEGARCH-MG. Section 4 presents a comparative forecasting analysis of all estimated volatility frameworks and finally section 5 concludes.

2. The volatility–M models and the quasi maximum likelihood estimation.

The basic idea of a volatility-M model is the introduction of volatility in the return equation. Denoting r_t as the daily return of a stock or stock market index at time t, F_{t-1} as the available set information up to t-1moment, z_t as a white noise process at time t, the general representation of any volatility-M model is given by equation system (1), where (1.a), (1.b) and (1.c) equations state respectively the conditional in mean specification, the conditional variance of the residuals, and the general ARCH representation :

$$r_t = \mu + \lambda\sigma_t^2 + \varepsilon_t \quad (1.a)$$

$$\sigma_t^2 = E(\varepsilon_t^2 / F_{t-1}) \quad (1.b)$$

$$\varepsilon_t = z_t (v_t a)^{1/2} \quad (1.c)$$

$$v_t = (1, \varepsilon_{t-1}^2, ..., \varepsilon_{t-q}^2, \sigma_{t-1}^2, ..., \sigma_{t-p}^2) \quad (1.d)$$

Equation system 1 is estimated by quasi maximum likelihood (QML).[132]Although ordinary least squares (OLS) deliver consistent estimates, however, the maximum likelihood method is more efficient, in the sense that the estimated parameters converge to their population counterparts at a much faster rate.[133]Obviously the log-likelihood function depends upon the assumed distribution of innovations and specifically upon the assumed conditional distributions of ε_t and r_t.[134]

Although Engle (1982) notes that applying conditional normality may not be as restrictive as it initially appears,[135]however it is common strategy to let alternative assumptions enter

[132]When normality is assumed but the true conditional distribution is not normal, the maximum likelihood estimations are known to be quasi maximum likelihood. Weiss (1986) and Bollerslev and Wooldridge (1992) shows that these (estimations) are consistent whenever the equations of the conditional mean and the conditional variance are correctly specified.

[133]Engle (1982) shows that estimating the ARCH (1) model using the maximum likelihood provides gains in efficiency that are quite large.

[134]Note that Z_t term in (1.c) equation is usually named as innovation process. The reader must keep in mind that the term innovation in the present context refers either to the previous definition or the one stated by Cristensen, Nielsen and Zhu (2010). The last will be clarified as soon as there is presentation of the filtered volatility models.

[135]Engle (1982) states that if conditional distribution of returns is normal, the unconditional will not be normal since it's shape strongly displays a leptokurtic shape.

the conditional distributions. Indeed Palm (1996), Pagan (1996) and Bollerslev, Chou and Kroner (1992) underline the widespread use of fat-tailed distributions in the volatility literature,[136] an particularly state that the symmetric t-student, the generalized error (GED), and the skewed student distribution of Fernandez and Steel (1998) are in fact the most popular alternatives. Since the distribution of asset returns is most of the times negatively skewed,[137]the latter distribution by incorporating and adjusting for phenomena of kurtosis and skewness, turns out being extremely useful.

Equation system (1) describes the general framework of a volatility-M model. However, Christensen, Nielsen and Zhu (2010) propose an alternative presentation which introduces volatility changes instead of volatility levels as the explanatory variable entering the conditional in mean equation. The approach in the present analysis is applied solely for the FIEGARCH-M model, and further details are displayed as soon as there is presentation of the basic FIEGARCH framework.

FIEGARCH, nests both the FIGARCH model of Baillie.et al (1996) and the asymmetrical EGARCH model of Nelson (1991). This model is specified in the next equation

$$\varphi(L)(1-L)^d(\ln\sigma_t^2-\omega)=\psi(L)g(z_{t-1}) \tag{2}$$

where ω is the mean of the logarithmic conditional variance, $\varphi(L)$ and $\psi(L)$ are polynomials in the lag operator, $(1\text{-}L)^d$ stands for the fractional difference operator presented in equation (3), z_t is the normalized innovation at time t, that is $z_t=\varepsilon_t/\sigma_t$, while

[136]Bollerslev (1987), Hsieh (1989), Baillie and Bollerslev (1989), Palm and Vlaar (1997) among others show that these distributions perform better in order to capture the higher observed Kurtosis.
[137]Note that this is the case in our data set. For more information on the use of non normal distributions when estimating GARCH models, see Laurent and Peters (2002).

finally g (z_{t-1}) introduces the news impact function that is defined as in equation (4) that follows

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(d+1)}{\Gamma(k+1)\Gamma(d-k+1)} L^K \quad (3)$$

$$g(z_t) = \theta z_t + \gamma(|z_t| - E|z_t|) \quad (4)$$

The fractional difference operator engages a key roll in the FIEGARCH model, allowing a persistence of shocks to volatility which is stronger than the one designating the short memory volatility models of GARCH, ARCH, EGARCH etc. Furthermore, the incorporation of asymmetries or leverage effects in the estimated conditional variance is ensured by the presence of the news impact function g (z_{t-1}), which manages the way in which past shocks affect the current levels in volatility.[138]

As Nelson (1991) states the leverage effect should be modeled as both a function of magnitude and sign, and coefficients θ and γ presented in equation (4) manage exactly that. Specifically, γ coefficient displays the rate at which innovations enter volatility, while θ manages the way the sign of the normalized innovations affect the current levels in volatility. Obviously for a value θ that is below zero, the negative innovations will induce higher volatility levels than the positive innovations of the same magnitude.

Christensen, Nielsen and Zhu (2010) introduce the following stationary input in the conditional in mean equation

$$h_t = (1-L)^d (\ln \sigma_t^2 - \omega) \quad (5)$$

[138]Using in FIEGARCH the natural log of conditional variance as the reliant variable implies that the latter will always positive, even in the presence of negative estimated coefficients. This transformation eliminates the necessity of imposing non negativity parameter restrictions, a strategy most common in the GARCH and ARCH models.

The equation operates as a filter that abstracts the long memory feature of volatility, while an analogous stationary expression is presented in Ang et.al (2006) and uses the news impact function of the previous period, that is $g(z_{t-1})$ function. The last resumes the most recent innovation to volatility.

Both stationary products, h_t and $g(z_{t-1})$, when substitute the independent variable of equation (1.a) generate the following two expressions

$$r_t = \mu + \lambda g(z_{t-1}) + \varepsilon_t \qquad (6)$$

$$r_t = \mu + \lambda h_t + \varepsilon_t \qquad (7)$$

These conditional in mean equations alongside the FIEGARCH modeling of conditional variance, premise the filtered long memory volatility models presented in Christensen, Nielsen and Zhu (2010). Particularly, equation (6) corresponds to the FIEGARCH-MG model, while equation (7) belongs to the FIEGARCH-MH case.

3. Application to the Fama-Frech stock market index, 1963-2010.

3.1 Estimating volatility models with out filtering : GARCH, EGARCH, FIGARCH, GJR, FIEGARCH, IGARCH.

The first part of the analysis uses the daily returns of the Fama-French stock market for a period of 37 years from 01.07.1963 to 31.06.2010, to estimate different non filtered volatility models. Those models are GARCH-M, EGARCH-M, FIGARCH-M, FIEGARCH-M, IGARCH-M and GJR-M, while table (1) concentrates the relative mathematical expressions.

Following at this point Bollerslev and Mikkelsen (1996) the above volatility frameworks add in the conditional mean equations the autoregressive terms of an AR(3) process, and so these models beside including in the return equation the corresponding volatility estimates[139] account also for autocorrelations in stock market returns.

This adjustment which theoretically is based on the arguments of Scholes and Williams (1977) and Lo and MacKinlay (1990) underline the important fact that potential discontinuous trading of stocks that make up the market index may result in significant serial dependence over the index returns. Obviously the structure of autocorrelation will depend on the specific feature that defines the exact nature of non synchroneity.

The above comments imply two things. First, that all the above volatility models use equation system (1) as their general mathematical expression, and second, that only equation (1.a) is changing into the following expression

$$r_t = \mu + \mu_1 r_{t-1} + \mu_2 r_{t-2} + \mu_3 r_{t-3} + \lambda \sigma_t^2 + \varepsilon_t \quad (8)$$

Table (2) concentrates the estimations on the above volatility-M models. All cases estimate equation (8) as the fundamental conditional in mean equation. Furthermore following Christensen, Nielsen and Zhu (2010) all models of table (2) use one ARCH and one GARCH term, while all models are estimated under the assumption of standard normal distribution. Hence the estimated volatility models of table (2) are the GARCH(1,1)-M-[1,2,3], EGARCH-(1,1)-M-[1,2,3], FIGARCH-(1,1)-M-[1,2,3], FIEGARCH-(1,1)-M-[1.2.3], IGARCH-(1,1)-M-[1,2,3] and GJR (1,1)-M-[1,2,3].[140]

[139] The presence of the conditional in mean equation is denoted by the M designate.

[140] Estimation of the above models is carried out with the G@rch 6 program. For the calculation of standard errors the option of sandwich formula is consistently used. Elements x and y in parenthesis (x,y) denote

Table (1) Mathematical expressions of the conditional variances for the estimated volatility models.

GARCH	$\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_t \sigma_{t-i}^2 = a_0 + a(L)\varepsilon_t^2 + B(L)\sigma_t^2$	α(L)=a_1L+…+$a_q L^q$ B(L)=β_1L+…+$\beta_p L^P$
IGARCH	$\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_t \sigma_{t-i}^2 = a_0 + a(L)\varepsilon_t^2 + B(L)\sigma_t^2$	$\sum_{i=1}^{q} a_i + \sum_{i=1}^{p} \beta_i = 1$
EGARCH	$\log(\sigma_t^2) = \omega + [1-\beta(L)]^{-1}[1+a(L)]g(z_{t-1})$	$g(z_t)=\gamma_1 z_t+\gamma_2(\lvert z_t\rvert-E(\lvert z_t\rvert))$
GJR	$\sigma_t^2 = \omega + \sum_{i=1}^{q}(a_i \varepsilon_{t-i}^2 + \gamma_i S_{t-i}^{-} \varepsilon_{t-i}^2) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$	If $\varepsilon_{t-i}>0$ then $S^{-1}=1$ If $\varepsilon_{t-i}<0$ then $S^{-1}=0$
FIGARCH	$\sigma_t^2 = \omega\{1-\beta(L)\}^{-1} + \{1-\Phi(L)(1-L)^d[1-\beta(L)]^{-1}\}\varepsilon_t^2$	Φ(L)=(1-φ_1L-…-$\varphi_p L^P$) Θ(L)=(1+θ_1L+…+$\Theta_q L^q$)

The statistical significance of λ coefficient in results presented in table (2) indicates that in four out of six models[141]the return-volatility relation is present and favors a positive risk-return trade off. On the other hand the estimated θ and γ parameters in both news impact functions of EGARCH-M and FIEGARCH-M models are statistically significant at conventional levels and have the expected signs.[142] Furthermore the fractionally differenced parameter, d, although positive and strongly significant in both long memory volatility models differ significantly between the two and create speculations about the

respectively the number of GARCH (x) and ARCH (x) terms, while number in brackets indicate the autoregressive variables enriching the conditional in mean equation. For example FIEGARCH (1,1)-M-[1] denotes an M volatility model that uses one GARCH and ARCH term for the volatility specification, while adds in the conditional in mean equation beside the logσ^2 the first lagged variable r_{t-1} as an additional explanatory variable. If the model under consideration does not belong to the M family of volatility models then the letter M is missing from the above notation. For example FIEGARCH (2,1)-[1,3] states that the model under consideration is the FIEGARCH with two GARCH and one ARCH terms specifying the conditional variance equation, and the first and third lagged return variables, those are r_{t-1} and r_{t-3}, added in the conditional mean equation. Note that the same notation is consistently used throughout the analysis.

[141] These models are IGARCH, FIGARCH, FIEGARCH and GARCH.

[142] The news impact function, as clearly can be seen in the mathematical expressions presented in table (1) and equation (2) is used solely in EGARCH and FIEGARH models. The successive incorporation of both asymmetries induced by the sign and size of innovations implies that the estimated coefficients θ and γ fulfill certain characteristics. Those features are mainly two. First, the statistical significance of both estimated coefficients at conventional levels, and second the verification of their expected signs. Specifically, θ is expected to be negative and γ positive. By default GJR incorporates only the asymmetries induced by the sign of innovations and in this case by default the expected sign of θ is positive. Therefore, the model does not incorporate the news impact function presented in equation (4).

realized goodness of fit. Moreover, for FIGARCH and FIEGARCH models he estimated long memory parameters are 0.525 and 0.173 respectively.

As for the serial correlation of residuals in all models the estimated non statistically significant Ljung–Box Q statistics[143] at both choices of lags, those are 100 and 200, indicate clearly the absence of misspecifications in the corresponding conditional in mean equations, an outcome mainly attributed to the presence of autoregressive terms[144]introduced in equation (8).

Furthermore the Ljung–Box Q* statistics that are estimated on the squared residuals, turn over non statistically significant outcomes except in one case. The exception irrespective of the lag choices applied concerns the FIEGARCH model. The results clearly suggest a misspecification of the FIEGARCH volatility equation although Harris and Sollis (2003) argue that in GARCH type models the p-values drawn from X^2 distributions cannot generally considered reliable. However these results since retain useful information for further model comparison they are reported constantly in the present analysis.

Beyond this clearly FIEGARCH-M reports the best Akaike and log-likelihood value among all estimated volatility models, while EGARCH-M practically matches those standings and in the case of Schwarz information criteria outperforms FIEGARCH-M.

Finally the last row of table (2) presents results on the mean square error (MSE). The last is estimated over the last 100 days of the sample period, using one-day ahead out of sample volatility forecasts. Oddly, FIEGARCH-M turns over the highest value and hence the

[143] The level of significance is 5%

[144]Estimating the same conditional in mean equations without the autoregressive terms of equation (8) turns over everywhere statistically significant Ljung-Box Q* statistics at both lag-choices.

worst forecasting property among all estimated models, while at same time the best forecasting performance is delivered by FIGARCH-M.

Since the models in table (2) are estimated under a common set of applied specifications that concern mainly a) the number of ARCH and GARCH terms, b) the assumption of conditional distribution and c) the orders of the autoregressive variables introduced in the corresponding conditional in mean equations, estimates in table (2) may embed biases and therefore mislead the quest for the best forecasting volatility model. So, the specifications applied in table (2) should not by any means considered final and contrary wise they should be regarded as the initial milestones for evolving further the fit of the presented volatility models.

A versatile and perhaps a more realistic approach than the previous one would let the volatility specifications change every time a different volatility model is estimated. Such an analysis finds auspicious ground in many empirical applications[145]and is carried over in the estimations presented in table (3). The specifications and settings chosen for every volatility model are decided upon standard information criteria that reward both the goodness of fit and parsimony as well as the out of sample forecasting.[146]

For example all models presented in table (3) except EGARCH which incorporates the generalized error distribution (GED) are estimated presuming the skewed student

[145]Nelson (1991) for example suggests that EGARCH model should be estimated using the generalized error distribution (GED) while French, Schwert and Stambaugh[145](1987) propose the GARCH (2,1) specification every time a GARCH volatility model is estimated.

[146]Retaining the volatility frameworks presented in table (2) the analysis re-estimates all models assuming however non-normal distributions, such as a) the t-student b) the generalized error and c) the skewed t-student distribution. The estimates are reported in the appendix, while table (3) resumes the final specification of each volatility model. This process consists of two parts. The first decides the distributional assumptions upon which the volatility models are estimated. These decisions are taken after comparing the log-likelihood values and the Akaike/Schwarz information criteria corresponding every distributional alternative. The second locates in those latter estimates the non statistically significant coefficients, and uses the likelihood ratio test to decide the restricted forms. The results are reported in the appendix.

distribution for the corresponding innovations, since the last significantly not only improves the fit but in many cases enhances the forecasting properties of the corresponding volatility models. Indeed comparison of the log-likelihood values and the Akaike-Schwarz information criteria in tables (2) and (3), unveil that all models of table (3) achieve considerably higher and smaller values respectively.

Table (2) Estimations of GARCH(1,1)-M-[1,2,3], EGARCH(1,1)-M-[1,2,3], IGARCH(1,1)-M-[1,2,3], GJR(1,1)-M-[1,2,3], FIGARCH(1,1)-M-[1,2,3], FIEGARCH(1,1)-M-[1,2,3] assuming standard normal distribution**.**

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
μ	0.022 [0.005]**	0.009 [0.341]	0.015 [0.048]*	0.021 [0.012]*	-0.004 [0.527]	0.022 [0.008]**
μ_1	0.153 [0.000]**	0.157 [0.000]**	0.161 [0.000]**	0.155 [0.000]**	0.1614 [0.000]**	0.153 [0.000]**
μ_2	-0.015 [0.122]	-0.005 [0.535]	-0.008 [0.390]	-0.016 [0.125]	-0.003 [0.706]	-0.016 [0.126]
μ_3	0.011 [0.249]	0.021 [0.012]	0.018 [0.072]	0.009 [0.372]	0.022 [0.028]	0.010 [0.271]
ω	0.005 [0.000]**	0.000 [1.000]	0.007 [0.000]**	0.015 [0.000]**	-6.265 [0.000]**	0.006 [0.000]**
λ	0.028 [0.004]**	0.002 [0.874]	0.005 [0.633]	0.033 [0.004]**	0.042 [0.001]**	0.030 [0.006]**
α_1	0.090 [0.000]**	-0.363 [0.000]**	0.024 [0.000]**	0.225 [0.000]**	-0.517 [0.000]**	0.087 [0.000]**
β_1	0.909 [0.000]**	0.989 [0.000]**	0.915 [0.000]**	0.651 [0.000]**	0.955 [0.000]**	0.908 [0.000]**
θ	-	-0.125 [0.000]**	0.104 [0.000]**	-	-0.142 [0.000]**	-
γ	-	0.194 [0.000]**	-	-	0.193 [0.000]**	-
d	-	-	-	0.525 [0.000]**	0.173 [0.000]**	-
Logl	-13729.9	-13565.7	-13610.8	-13710.2	-13543.6	-13727.707
Akaike	2.342	2.315	2.322	2.339	2.311	2.342
Schwarz	2.346	2.315	2.322	2.339	2.318	2.342
Q(100)	101.689 [0.434]	101.123 [0.449]	97.181 [0.561]	106.808 [0.302]	106.494 [0.309]	101.954 [0.426]
Q(200)	219.196 [0.167]	218.316 [0.178]	217.383 [0.189]	224.067 [0.116]	220.992 [0.147]	219.744 [0.161]
Q*(100)	95.9477 [0.539]	101.123 [0.449]	99.173 [0.447]	91.327 [0.670]	161.449 [0.000]**	94.659 [0.576]
Q*(200)	186.545 [0.710]	222.698 [0.110]	210.752 [0.254]	173.752 [0.892]	282.624 [0.000]**	189.056 [0.663]
MSE	12.76	13.57	13.45	12.96	16.39	13.1

Note: Quasi Maximum Likelihood estimates are reported. Also reported are the Logl, this is the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung-Box portmanteau statistic for testing the up to m'th order serial dependence of standardized and absolute standardized residuals denoted respectively as Q(m) and Q*(m). Finally, the table reports the mean square error (MSE) for 100 one-day-ahead out of sample forecasts. The values in parenthesis stand for p-values. *denotes rejection at 5% significance, while ** denotes rejection at 1% significant level.

Table (3) Estimations of IGARCH (1,1)-[1], GARCH(1,1)-[1], FIGARCH(1,1-)[1], FIEGARCH(1,1)-[1], GJR(1,1)-[1] under the skewed distribution. Estimation EGARCH (2,1)-M-[1] using the GED distribution.

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
μ	0.034 [0.000]**	0.040 [0.000]**	0.021 [0.000]**	0.034 [0.000]**	0.019 [0.000]**	0.034 [0.000]**
μ_1	0.137 [0.000]**	0.138 [0.000]**	0.144 [0.000]**	0.137 [0.000]**	0.145 [0.000]**	0.137 [0.000]**
ω	0.003 [0.000]**	2.257 [0.000]**	0.005 [0.000]**	0.009 [0.000]**	-2.294 [0.000]**	0.004 [0.000]**
λ	-	-0.026 [0.003]**	-	-	-	-
α_1	0.079 [0.000]**	-0.980 [0.000]**	0.026 [0.000]**	0.196 [0.000]**	-0.468 [0.000]**	0.078 [0.000]**
β_1	0.920 [0.000]**	1.928 [0.000]**	0.920 [0.000]**	0.697 [0.000]**	0.759 [0.000]**	0.919 [0.000]**
μ_2	-	-	-	-	-	-0.928 [0.000]**
μ_3	-	-	-	-	-	-
θ	-	-0.114 [0.000]**	0.097 [0.000]**	-	-0.121 [0.000]**	-
γ	-	0.156 [0.000]**	-	-	0.158 [0.000]**	-
d	-	-	-	0.573 [0.000]	0.565 0.000	-
Logl	-13466.2	-13348.2	-13381.1	-13453.9	-13296.2	-13465.8
Akaike	2.297	2.277	2.282	2.295	2.268	2.297
Schwarz	2.300	2.283	2.287	2.300	2.275	2.301
Q(100)	110.010 [0.231]	112.954 [0.177]	102.255 [0.418]	114.201 [0.157]	108.880 [0.255]	109.821 [0.235]
Q(200)	228.858 [0.079]	230.429 [0.068]	222.613 [0.130]	173.619 [0.893]	226.944 [0.077]	228.809 [0.079]
Q*(100)	97.545 [0.493]	123.147 [0.037]	94.132 [0.591]	95.1898 [0.561]	127.563 [0.024]	96.846 [0.513]
Q*(200)	228.858 [0.079]	230.429 [0.068]	196.698 [0.512]	173.619 [0.893]	226.944 [0.077]	184.429 [0.746]
MSE	12.8	13.25	13.04	12.93	13.97	12.98
Asymmetry	-0.062 [0.000]**	-	-0.064 [0.000]**	-0.061 [0.000]**	-0.062 [0.000]**	-0.062 [0.000]**

Note : Quasi Maximum Likelihood estimates are reported. Also reported are the Logl, the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung-Box portmanteau statistic for testing the up to m'th order serial dependence in the standardized and absolute standardized residuals denoted respectively as Q(m) and Q*(m). Finally, the table reports the mean square error (MSE) for 100 one-day-ahead out of sample forecasts and the estimated asymmetry coefficient of skewed student distribution. The values in parenthesis stand for p-values.*denotes rejection at 5% significance level, while ** denotes rejection at 1 significance.

Take for example the results reported for the IGARCH model in tables (2) and (3). The estimated log-likelihood values are -13729.9 and -13466.2 respectively, while the Akaike criterium is significantly improved, and from the initial value of 2.342 finally reaches in table (3) the value of 2.297. However, implementing a non normal distribution not only improves the fit of the presented volatility models, but in many cases alters the statistical significance of certain coefficients.

Take for example the GARCH model and watch out how coefficients λ and μ_3 are missing from the estimations presented in table (3). Precluding the corresponding variables is a decision made in a two stage process. The first stage estimates the unrestricted GARCH version, proclaiming the variables that are individually non significant and therefore may be dropped, while the second stage estimates the likelihood ratio test and draws conclusions on the reception of the restricted forms.

For example the estimation of the unrestricted GARCH[147]model indicates λ_1, μ_2 and μ_3 as individually not statistically significant at 5% significance level, while the likelihood ratio statistic that tests the joint exclusion of these coefficients accepts the corresponding null hypothesis and so GARCH (1,1)-[1,2] is the restricted framework presented in table (3). Obviously applying this two stage process in all models of table (2) delivers the estimations presented in table (3).

Except EGARCH that acknowledges a negative statistically significant volatility-return relation, all other models in table (3) reject the validity of the risk-return trade off.[148]Note that although all models in table (3) are estimated using one ARCH and one GARCH term, however the EGARCH model of Nelson (1991) does not follow this pattern and applies a different combination that constitutes of two GARCH and one ARCH terms. Finally even though all models strengthen the interpretive properties of the corresponding conditional in

[147] This is GARCH(1,1)-M-[1,2,3] estimated under the skewed student distribution.

[148]Using t-student, GED and skewed student distribution for the formation of the corresponding likelihood functions ends up in non statistically significant risk-return coefficients. However applying the normality assumption results in the acknowledgement of the risk–return trade off in four at of six volatility models. Therefore the analysis concludes that the statistical significance of the risk-return relation is sensitive to the distribution applied.

mean equations by introducing r_{t-1} in the relative equations, the GARCH model does the same and moreover adds r_{t-2}.

Again the majority of the estimated Ljung–Box Q and Q*statistics are not statistically significant at 5% significant level,[149] while coefficients θ and γ of both news impact functions acknowledge the presence of a leverage sign and size effect and justify the competitive fit succeeded by EGARCH and FIEGARCH.

Further evidence on the superiority of FIEGARCH are reported in results of table (4) where the Engle and Ng (1993) sign and size bias misspecification tests are presented.[150] As expected IGARCH, FIGARCH and GARCH reject the null hypothesis in all Engle and Ng (1993) tests, while the rest models although account for asymmetries, do not always manage successfully the leverage and size effects.

Specifically the results in table (4) indicate that only FIEGARCH incorporates efficiently all three asymmetries, since EGARCH and GJR models although` both accept null hypothesis in the sign bias (S.B) and negative size bias test (N.S.B), however turn over statistically significant statistics when the positive size bias tests (P.S.B) are estimated.

The results of table (4) are considered somewhat unexpected. The acceptance of null hypothesis in the N.S.B test for the GJR model is completely unjustified since the model by default accounts only for the sign asymmetries. On the other hand FIEGARCH although deals efficiently with the asymmetries induced by the sign and size of innovations, however unexpectedly rejects the null in the joint Engle and Ng (1993) test at 5%

[149] Only Q*(100) statistics of EGARCH and FIGARCH are statistically significant at 5%.

[150] Engle and Ng (1993) proposed three tests: a) the sign bias test (SBT), b) the negative sign bias test (NSBT) and c) the positive sign bias test (PSBT). The logic of these tests is to see whether having estimated a particular GARCH model, an asymmetry dummy variable is significant in predicting the squared residuals. The tests are of the null hypothesis that the null model is correctly specified (i.e there is on remaining asymmetry).

significant level. This contradiction naturally surprises, but similar outcomes have reported before in other empirical studies.[151]

Table (4) Engle and Ng (1993) Sign and Size bias test.

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
S.B	2.187 [0.028]*	1.242 [0.213]	2.882 [0.003]**	2.006 [0.044]*	1.558 [0.119]	2.148 [0.031]*
NSB	2.452 [0.014]*	0.120 [0.904]	0.303 [0.761]	3.091 [0.001]**	0.100 [0.920]	2.631 [0.008]**
PSB	3.000 [0.002]**	2.417 [0.015]*	2.287 [0.022]*	2.798 [0.005]**	1.716 [0.085]	2.912 [0.003]**
Joint	55.730 [0.000]**	16.404 [0.000]**	31.505 [0.000]**	58.505 [0.000]**	12.648 [0.049]**	56.176 [0.000]**

Note : (SB) stands for sign bias, (NSB) for negative size bias test, (PSB) for positive size bias test, (Joint) for the joint statistic. Number in parenthesis report p-values.* (**) denotes rejection at 5% and 1% significant levels respectively.

The previous analysis indicated FIEGARCH (1,1)-[1] as the best volatility model, although the outcomes on the mean square error in table (3) make this vantage less obvious, since FIEGARCH actually delivers the worst forecast error measurement.

However, the results in tables (B) and (C) in the appendix, where different error measurements on one day ahead out of sample forecasts are reported, for 100 and 200 days of forecasts respectively, reverse this impression.[152] The outcomes affirm that although FIEGARCH(1,1)-[1] in terms of MSE does not stamp it's forecasting superiority, however the model does provide competitive forecasts in the majority of forecast error measurements. See for example the reported values on the median squared error, the mean absolute error, the mean absolute percentage error and the logarithmic loss function.

Furthermore, since no model in tables (B) and (C) provides consistently best forecasts naturally no model is considered dominant in terms of forecasting. However, the results

[151] See for example Harris and Solis (2003).

[152] The forecasts error measurements are (MSE) mean squared error, (MedSE) median squared error, (MAE) mean absolute error, (RMSE) root mean squared error, (MAPE) mean absolute percentage error, (AMAPE) Adjusted mean absolute percentage error,(TIC) Theil inequality coefficient, (LL) Logarithmic loss function.

in table (5) where the 300 one day ahead out of sample forecasts are reported turn around this impression and consolidate the forecasting superiority of FIEGARCH (1,1)-[1]. Impressively the model exceeds in terms of forecasting any other model in 7 out of 8 forecasting error measurements.[153]

These outcomes provide even greater endorsement for the results presented in table (3) and further justify the belief that FIEGARCH (1,1)-[1] is indeed the best volatility model among all non filtered alternatives presented so far.

Table (5) Forecasts errors for 300 one day ahead out of sample forecasts

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
MSE	46.21	10.52	23.41	25.96	9.858*	26.99
MedSE	52.61	2.706	25.43	26.68	1.693*	27.28
MAE	6.508	2.320	4.526	4.781	2.052*	4.882
RMSE	6.797	3.243	4.838	5.095	3.14*	5.196
MAPE	1498	400.2	1023	1066	318.3*	1081
AMAPE	0.748	0.608	0.705	0.712	0.594*	0.714
TIC	0.593	0.504*	0.529	0.537	0.534	0.540
LL	15.16	8.956	13.06	13.34	8.162*	13.44

Note (MSE) mean squared error, (MedSE) median squared error, (MAE) mean absolute error, (RMSE) root mean squared error, (MAPE) mean absolute percentage error, (AMAPE) Adjusted mean absolute percentage error,(TIC) Theil inequality coefficient, (LL) Logarithmic loss function * denotes the best forecasting model

3.2 Estimating volatility models with filtering: FIEGARCH-MH and FIEGARCH-MG.

The previous analysis indicated FIEGARCH (1,1)-[1] as the best volatility model according to information criteria rewarding the goodness of fit and parsimony as well as the out of sample forecasting. Furthermore, results revealed that the inclusion of the long memory feature alone as in the FIGARCH model does not imperatively improve the fit, while the statistical significance of the risk-return relation, although sensitive to the distributional assumptions applied, is mainly conceded under the assumption of normality.

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[153] The only exception concerns Theil's Inequality coefficient.

This section follows the approach of Christensen, Nielsen and Zhu (2010) and estimates their filtered long memory volatility models. These models are FIEGARCH-MH and FIEGARCH-MG.

Furthermore the analysis beside including in the corresponding conditional mean equations the autoregressive terms of an AR(3) process as in the previous section, accounts also for potential lagged volatility in mean effects and so, after appropriate adjustments made, the conditional in mean equation of FIEGARCH-MH is defined as in equation (10)

$$r_t = \mu + \mu_1 r_{t-1} + \mu_2 r_{t-2} + \mu_3 r_{t-3} + \lambda_1 h_t + \lambda_2 h_{t-1} + \lambda_3 h_{t-2} \quad (10)$$

For FIEGARCH-MG the following specification is applied

$$r_t = \mu + \mu_1 r_{t-1} + \mu_2 r_{t-2} + \mu_3 r_{t-3} + \lambda_1 g(z_{t-1}) + \lambda_2 g(z_{t-2}) + \lambda_3 g(z_{t-3}) \quad (11)$$

Assuming innovations follow the normal distribution, tables (6) to (10) concentrate estimations on both filtered long memory volatility models.[154]

Although both cases aim the filtering of conditional variance, however the models turn over quite different results as far as the risk–return relation is concerned. Particularly, the statistical significance of λ_1 coefficient at 5% significant level in FIEGARCH-MH underlines the strong presence of a positive risk-return trade off, while at the same time the acceptance of null hypothesis in the t-statistics in all lagged volatility variables in the FIEGARCH-MG model indicates a complete absence of such relations.

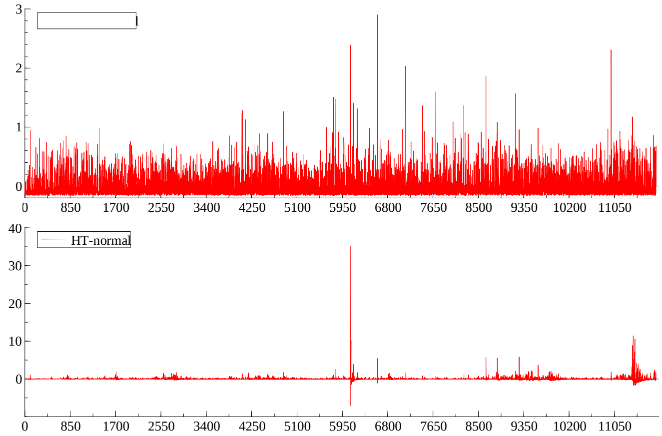
[154] The visual representations of both regressors appears in Graph (1).

However, beside these differences the models present a number of common characteristics. First, both filtered models acknowledge the statistical significance of the first autoregressive component, while both incorporate well specified conditional in mean equations.[155] On the other hand another common feature concerns the Ljung Box Q* statistics, and specifically the rejection of the null hypothesis irrespective of the lag selection. The outcomes clearly suggest the existence of possible misspecifications in the corresponding volatility equations, although these inferences are not affirmed when the LM statistics are computed. These results which are reported in table (6) indicate that conditional variances are probably well specified.[156]

Table (9) estimates different error measurements for 100 one day ahead out of sample forecasts. Comparing these results with the ones reported in table (B) in the appendix where different forecasts error measurements are reported for the same forecasting horizon but for various non filtered volatility models, reveals the poor forecasting potentials of FIEGARCH-MG and FIEGARCH-MH. Both models deliver forecasts that never challenge the best values of each forecast error measurement.

[155] This is seen in the results reported on the Ljung box Q statistic.
[156] Except of ARCH (1-100) test that rejects LM's null hypothesis at 5% significant level all other statistics accept null hypothesis.

Graph (1) Graphical representation of h_t and $g(z_{t-1})$ variables assuming innovations follow the standard normal distribution.



The analysis so far aimed the filter of long memory in volatility. However, the a-priori use of normality as in Christensen, Nielsen and Zhu (2010) is not entirely warranted by the present data set, since the last is characterized by a strong leptokurtic, negative skewed shape.

The possible misuse of normality can also be seen in table (10) where the outcomes on the adjusted Pearson goodness of fit test are reported. The test compares the empirical distribution of innovations with it's theoretical shape. The rejection of the null in all sections of cells indicates a possible mismatch and this obviously seems to be the case for both filtered long memory volatility models.

Table (6) Estimated coefficients FIEGARCH-MH, FIEGARCH-MG assuming standard normal distribution.

FIEGARCH-MH				FIEGARCH-MG			
μ	0.022 [0.000]**	d	0.549 [0.000]**	μ	0.020 [0.000]	d	0.534 [0.000]**
μ_1	0.187 [0.000]**	β_1	0.777 [0.000]**	μ_1	0.154 [0.000]	β_1	0.818 [0.000]**
μ_2	0.008 [0.304]	Logl	-13518.3	μ_2	0.001 [0.922]	Logl	-13533.4
μ_3	0.026 [0.098]	Akaike	2.307	μ_3	0.019 [0.114]	Akaike	2.310
λ_1	0.262 [0.000]**	Schwarz	2.315	λ_1	-0.039 [0.437]	Schwarz	2.318

λ_2	0.107 [0.069]	Q(100)	106.832 [0.301]	λ_2	0.051 [0.310]	Q(100)	104.485 [0.359]
λ_3	0.014 [0.837]	Q(200)	223.073 [0.126]	λ_3	0.018 [0.721]	Q(200)	222.575 [0.130]
ω	0.000 [1.000]	Q*(100)	133.828 [0.009]**	ω	0.000 [1.000]	Q*(100)	134.152 [0.008]**
α_1	-0.535 [0.000]**	Q*(200)	240.772 [0.020]*	α_1	-0.588 [0.000]**	Q*(200)	238.715 [0.025]*
θ	-0.131 [0.000]**	γ	0.176 [0.000]**	θ	-0.134 [0.000]**	γ	0.186 [0.000]**
Table (7) Engle and Ng tests(1993)							
SBT	1.352 [0.176	NSB	0.791 [0.428]	SBT	0.987 [0.323]	NSB	0.638 [0.522]
PSB	1.593 [0.110]	Joint	8.761 [0.032]	PSB	1.953 [0.050]	Joint	8.876 [0.030]
Table (8) Engle's LM ARCH test (1982) for FIEGARCH-MG, FIEGARCH-MH							
ARCH 1-100 test	1.2939 [0.026]*	ARCH 1-200 test	1.1231 [0.114]	ARCH 1-100 test	1.2845 [0.029]	ARCH 1-200 test	1.1052 [0.149]
ARCH 1-300 test	1.0573 [0.240]	-	-	ARCH 1-300 test	1.0370 [0.3202]	-	-

Note : Quasi Maximum Likelihood estimates are reported. Also reported are the Logl, the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung-Box portmanteau statistic for testing the up to m'th order serial dependence of the standardized and absolute standardized residuals denoted respectively as Q(m) and Q*(m). (SBT) stands for the sign bias test, (NSBT) for the negative size bias test, (PSBT) for the positive size bias test and (Joint) for the Joint statistic. The table also reports Engle's (1982) LM ARCH test. The values in parenthesis stand for p-values.*denotes rejection at 5% significance level, while ** denotes rejection at 1 significance.

Table (9) Error measurements for 100 one day ahead out of sample forecasts.

	MSE	MedSE	MAE	RMSE	MAPE	AMAPE	TIC	LL
FIEGARCH MG	13.21	2.851	2.242	3.635	388.8	0.640	0.611	11.13
FIEGARCH MH	13.18	2.648	2.201	3.63	375.7	0.637	0.617	10.97

Note (MSE) mean squared error, (MedSE) median squared error, (MAE) mean absolute error, (RMSE) root mean squared error, (MAPE) mean absolute percentage error, (AMAPE) Adjusted mean absolute percentage error,(TIC) Theil inequality coefficient, (LL) Logarithmic loss function.

Table (10) The adjusted Pearson goodness of fit test for FIEGARCH-MH and FIEGARCH-MG under normality.

cells	FIEGARCH-MH			FIEGARCH-MG		
	statistic	P-Value (g-1)	P-Value (g-k-1)	statistic	P-Value (g-1)	P-Value (g-k-1)
300	436.6828	[0.000]**	[0.000]**	481.1917	[0.000]**	[0.000]**
400	561.2224	[0.000]**	[0.000]**	589.8035	[0.000]**	[0.000]**
600	742.4638	[0.000]**	[0.000]**	800.5812	[0.000]**	[0.000]**

*denotes rejection at 5% significance level, while ** denotes rejection at 1 significance

The next part of the analysis uses different distributional assumptions to estimate FIEGARCH-MH and FIEGARCH-MG.[157] Obviously introducing assumptions beside

[157]The appropriateness of the applied distributions is judged according to standard criteria used in the volatility literature. These criteria are parsimony, goodness of fit and out of sample forecasting.

normality requires re-estimating the news impact function $g(z_t)$ and the stationary variable h_t. Once this stage is completed the estimates of both filtered long memory volatility models are concentrated in table (11) and then models are tested for the joint exclusion of the non statistically significant variables. The estimated restricted models that come out of these process and are denoted as FIEGARCH-MH*and FIEGARCH MG*, are both presented in table (12).The objective remains the estimation of the best filtered volatility models considering the options of the analysis as far as the distributional assumptions are concerned.

Specifically for FIEGARCH-MH the choice results among estimating the filtered MH model either under the t-student or GED distributions, since the estimated asymmetry coefficient of the skewed student distribution in table (11) is non statistically significant and naturally excludes the corresponding model from the rest part of the analysis.[158]

However, making a choice between the remaining two restricted[159]options is no easy task, since both models achieve equal fit and identical estimations in almost all implicated coefficients.

Specifically, both models acknowledge the same set[160] of coefficients as statistically significant at conventional levels, while both estimate a positive λ_1 coefficient and therefore conclude the presence of a positive return-risk trade off.

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[158]Note that G@RCH does not estimate ξ but log ξ facilitating inferences about the null hypothesis of symmetry. Note also that skewed–student equals the symmetric student distribution when ξ=1 or in this case when log(ξ)=0. The estimated value of log (ξ) is reported in the output under the label asymmetry. See Lambert and Laurent (2001) and Bauwens and Laurent (2005) for more details on this outcome.

[159]The full models are estimated in table (11) while the restricted versions are reported in table (12). The latter are generated after the joint test for the significance of all or part of the individually non statistically significant variables in the models presented in table (11) is accepted. Take for example the FIEGARCH-MH under the assumption of t-student. Looking at t-statistics and the corresponding p-values in table (11) the individually non statistical significant variables are the second and third order autoregressive term and the innovation h_{t-2}. Those variables correspond respectively to the coefficients μ_2, μ_3 and λ_2 and λ_3. The likelihood ratio statistic that tests the joint exclusion of these variables accepts the null hypothesis and the restricted model is presented in table (2).

Table (11) Estimation of the unrestricted FIEGARCH-MH-MG models assuming t- student, GED, and skewed student innovations.

	FIEGARCH-MH			FIGARCH-MG		
	t-student	GED	skewed	t-student	GED	skewed
μ	0.014 [0.005]**	0.016 [0.005]**	0.015 [0.015]*	-0.011 [0.561]	0.264 [0.000]**	0.024 [0.551]
μ_1	0.176 [0.000]**	0.167 [0.000]**	0.170 [0.000]**	0.138 [0.000]**	-0.023 [0.009]**	0.137 [0.000]**
μ_2	-0.005 [0.635]	-0.008 [0.400]	-0.011 [0.212]	-0.010 [0.151]	-0.001 [0.899]	-0.011 [0.587]
μ_3	0.013 [0.235]	0.012 [0.289]	0.007 [0.458]	0.009 [0.252]	-0.015 [0.057]	0.008 [0.608]
λ_1	0.267 [0.000]**	0.244 [0.001]**	0.232 [0.001]**	-0.105 [0.002]**	1.819 [0.000]**	-0.115 [0.319]
λ_2	0.117 [0.100]	0.085 [0.234]	0.079 [0.226]	0.061 [0.018]*	-0.214 [0.000]**	0.061 [0.583]
λ_3	-0.011 [0.861]	-0.022 [0.736]	-0.055 [0.369]	0.009 [0.252]	0.2621 [0.000]**	0.007 [0.961]
α_1	-0.439 [0.002]**	-0.524 [0.000]**	-0.467 [0.000]**	-0.494 [0.000]**	-0.552 [0.000]**	-0.496 [0.000]**
β_1	0.722 [0.000]**	0.760 [0.000]**	0.743 [0.000]**	0.753 [0.000]**	0.779 [0.000]**	0.755 [0.000]**
ω	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]
θ	-0.116 [0.000]**	-0.124 [0.000]**	-0.113 [0.000]**	-0.120 [0.000]**	-0.127 [0.000]**	-0.119 [0.000]**
γ	0.161 [0.000]**	0.177 [0.000]**	0.164 [0.000]**	0.172 [0.000]**	0.179 [0.000]**	0.171 [0.000]**
d	0.605 [0.000]**	0.589 [0.000]**	0.602 [0.000]**	0.599 [0.000]**	0.590 [0.000]**	0.598 [0.000]**
Logl	-13316.1	-13355.7	-13319.4	-13324.3	-13337.2	-13323.6
Akaike	2.273	2.279	2.273	2.274	2.276	2.274
Schwarz	2.282	2.288	2.283	2.283	2.285	2.284
Q(100)	107.189 [0.293]	109.357 [0.245]	106.202 [0.316]	104.503 [0.359]	101.155 [0.448]	104.343 [0.363]
Q(200)	225.352 [0.105]	227.740 [0.086]	224.904 [0.109]	224.171 [0.115]	219.525 [0.163]	224.218 [0.115]
Q*(100)	117.399 [0.088]	120.253 [0.063]	117.803 [0.084]	117.288 [0.089]	121.926 [0.051]	118.119 [0.081]
Q*(200)	219.033 [0.145]	222.383 [0.112]	219.338 [0.142]	218.073 [0.156]	224.006 [0.099]	219.245 [0.143]
MSE	13.21	13.21	13.20	13.24	13.24	13.23
Asymmetry (log)	-	-	0.006 [0.712]	-	-	0.006 [0.702]
SB	1.339 [0.180]	1.271 [0.203]	1.530 [0.125]	1.500 [0.133]	1.458 [0.144]	1.331 [0.182]
NSB	0.397 [0.690]	0.708 [0.478]	0.436 [0.662]	0.547 [0.584]	0.945 [0.344]	0.415 [0.677]
PSB	2.141 [0.032]*	2.250 [0.024]*	2.139 [0.032]*	2.215 [0.026]*	2.408 [0.016]*	2.253 [0.024]*

[160] The statistical significant coefficients are μ, μ_1, λ_1, d, θ, γ.

Joint	13.380 [0.003]**	12.951 [0.004]**	14.711 [0.002]**	14.614 [0.002]**	15.163 [0.001]**	13.944 [0.002]**

Note : Quasi Maximum Likelihood estimates are reported. Also reported are the Logl, the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung-Box portmanteau statistic for testing the up to m'th order serial dependence of the standardized and absolute standardized residuals, denoted respectively by Q(m) and Q*(m), (SB) stands for sign bias, (NSB) for negative size bias test, (PSB) for positive size bias test, (Joint) for the joint statistic. Number in parenthesis report p-values. Finally, the table reports the mean square error (MSE) for 100 one-day-ahead out of sample forecast and the asymmetry coefficient estimated when the distribution skewed student is assumed. The values in parenthesis stand for p-values.*denotes rejection at 5% significance level, while ** denotes rejection at 1% significance.

Table (12) Estimation of restricted FIEGARCH-MH and FIEGARCH-MG assuming t- student and GED distributions

	FIEGARCH-MH*		FIEGARCH-MG*	
	t-student	GED	t-student	GED
μ	0.014 [0.000]**	0.015 [0.001]**	-	0.232 [0.000]**
μ_1	0.175 [0.000]**	0.165 [0.000]**	0.136 [0.000]**	-0.025 [0.019]*
μ_2	-	-	-	-
μ_3	-	-	-	-
λ_1	0.274 [0.000]**	-	-0.106 [0.004]**	1.834 [0.000]**
λ_2	-	-	0.088 [0.016]*	-0.211 [0.000]**
λ_3	-	-	-	-
α_1	-0.428 [0.002]**	-0.512 [0.000]**	-0.491 [0.000]**	-0.552 [0.000]**
β_1	0.721 [0.000]**	0.757 [0.000]**	0.750 [0.000]**	0.778 [0.000]**
ω	-	-	-	-
θ	-0.117 [0.000]**	-0.123 [0.000]**	-0.120 [0.000]**	-0.127 [0.000]**
γ	0.161 [0.000]**	0.176 [0.000]**	0.172 [0.000]**	-0.127 [0.000]**
d	0.603 [0.000]**	0.588 [0.000]**	0.600 [0.000]**	0.589 [0.000]**
Logl	-13335.3	-13359.4	-13325.1	-13338.3
Akaike	2.273	2.279	2.273	2.276
Schwarz	2.278	2.285	2.279	2.282
Q(100)	111.326 [0.206]	113.526 [0.167]	107.000 [0.297]	103.981 [0.372]
Q(200)	228.332 [0.082]	230.501 [0.068]	226.332 [0.097]	222.389 [0.132]
Q*(100)	117.338 [0.089]	119.949 [0.065]	117.088 [0.091]	121.093 [0.056]
Q*(200)	217.060 [0.168]	220.242 [0.133]	217.800 [0.159]	222.389 [0.132]
MSE	14.29	13.24	13.25	13.24
SB	1.641 [0.100]	1.370 [0.170]	1.548 [0.121]	1.473 [0.140]
NSB	0.454 [0.649]	0.656 [0.511]	0.565 [0.571]	0.953 [0.340]

PSB	1.975 [0.048]*	2.143 [0.032]*	2.184 [0.028]*	2.437 [0.014]*
Joint	14.22 [0.002]**	12.828 [0.005]**	14.698 [0.002]**	15.490 [0.001]**

Note Quasi Maximum Likelihood estimates are reported. Also reported are the Logl, the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung-Box portmanteau statistic for testing the up to m'th order serial dependence of the standardized and absolute standardized residuals, denoted respectively by Q(m) and Q*(m), (SB) stands for sign bias, (NSB) for negative size bias test, (PSB) for positive size bias test, (Joint) for the joint statistic. Number in parenthesis report p-values. Finally, the table reports the mean square error (MSE) for 100 one-day-ahead out of sample forecast and the asymmetry coefficient estimated when the distribution skewed student is assumed. The values in parenthesis stand for p-values.*denotes rejection at 5% significance level, while ** denotes rejection at 1% significance.

Furthermore, the models estimate almost identical values on the Akaike and Schwarz information criteria, while both estimate Ljung Box Q and Q* statistics at 100 and 200 lags that are not statistically significant. As for the sign and size bias tests of Engle and Ng (1993) both filtered models estimate identical results. Specifically the S.B.T and N.S.B.T statistics accept the null, whereas the joint and P.S.B tests both rejects it.

Table (13) reports the log-likelihood values and the Akaike/ Schwarz information criteria for all the competing filtered volatility models. In bold letters the table denotes the best estimates of each criterium. Obviously the best ones belong to the t-student assumption.

Table (13) The log-likelihood values and the Akaike/Schwarz information criteria of all estimated filtered volatility models.

	FIEGARCH-MH			FIEGARCH-MG		
	Lilkelihood value	Akaike	Schwarz	Likelihood value	Akaike	Schwarz
Normal	-13518.3	2.307	2.315	-13533.4	2.1310	2.318
t-student-MH*- MG*	**-13335.3**	**2.273**	**2.278**	**-13325.1**	**2.273**	**2.279**
GED-MH*-MG*	-13359.74	2.279	2.285	-13338.3	2.276	2.282

4. Forecasting volatility : A comparative analysis.

A last criterium for selecting the best volatility model involves inescapably out of sample forecasts. However, using only one forecast error measurement, or estimating various forecasts error measurements under the same forecasting horizon definitely induces biases. For example, the mean square error values (MSE) reported in table (12) according to the forecasting accuracy test of Diebold and Mariano (1995) are statistically equal.[161] However, using a different forecasting horizon than the 100 one step ahead out of sample forecasts of table (12), for example 200, turns over this impression since the MSE values are no longer consider being statistically equal.[162] So the analysis in this case is left with no real power over the selection procedure.

In an attempt to overcome this drawback, it is worth using a criteria that assesses the overall mean square errors of different forecasting horizons and in this direction the present analysis applies the Clements and Hendry (1993) approach which introduces the generalized forecast error second moment statistic (GFESM). The last is given by the determinant of the complete forecast error second matrix which is presented in the following equation

$$\text{GFESM}=\left|E[uu']\right| \qquad (12)$$

where u is the vector of forecast errors.[163]

[161] For more details see the appendix.
[162] Results are reported in the appendix
[163] In order to avoid large numbers the analysis reports results on the log transformation of GFESM. This is denoted as Log-GFESM.

Tables (14) and (15) use one step ahead out of sample forecasts for 20,40,60,80 and 100 days to estimate the log-GFESM statistic for both restricted filtered volatility models, under the t student and GED distributions.

Obviously FIEGARCH-MH* under the t-student assumption is constantly generating the smaller Log-GFESM values. Taking in mind this outcome along with the smaller Akaike and Schwarz values of FIEGARCH-MH* in table (13), the analysis eventually decides to estimate the filter FIEGARCH-MH model under the t-student assumption.

Note that for FIEGARCH-MG* the results reported in tables (11) and (12) are in fact very similar to the ones presented for FIEGARCH-MH*, while the basic conclusions are briefly discussed in the following lines.

Under the assumption of skewed student distribution the FIEGARCH-MG[164] estimates an asymmetry coefficient that is strongly insignificant. The result naturally precludes the model from the rest part of the analysis and so the remain competing volatility frameworks are the FIEGARCH-MG, estimated under either the t-student or the GED distribution. Although the above models acknowledge different statistically significant sets on the estimated coefficients, however, the models appear identical according to standard information criteria.[165]

The selection procedure continues next with the estimation of the restricted versions of the above models. Those are presented in table (12). Note that the outcomes on Q and Q* Ljung Box statistics imply the correct specification of the mean and volatility equations, while the sign and size bias tests reiterate the same pattern as in FIEGARCH-MH*case,

[164] The analysis denotes this model as FIEGARCH-MG-skewed.

[165] As has been stated the selection procedure involves the restricted models. So here identical refers to the latter specification.

with S.B.T and N.S.B.T statistics accept the null hypothesis at 5% significant level, and joint and P.S.B tests rejects it, at both conventional levels.

The similar results of the above models make impossible a sustained choice among them. Moreover, since the comparison of the mean squared errors value (MSE) reported in table (12) does not really contribute any additive information in the selection procedure, the weight of the selection falls entirely on forecasting properties and particular on Log-GFESM statistic.

As already is mentioned the results on the Log-GFESM statistic in table (14) are endorsing the estimation of the FIGARCH-MH*under the t-student distribution, while this option is also supported by the smaller Akaike value.

Table (14)-(15) Estimation of the Log-GFESM statistic for FIEGARCH-MH* and FIEGARCH-MG* under the t –student and GED distributions.

	Table (14)		**Table (15)**	
	FIEGARCH-MH* student	FIEGARCH-MH* GED	FIEGARCH-MG* student	FIEGARCH-MG* GED
Log-GFESM				
20	0,067	0,466	0,476	0,457
40	0,196	0,948	0,969	0,928
60	1,166	1,473	1,503	1,445
80	2,333	2,571	2,601	2,543
100	3,488	3,692	3,723	3,665

As far as the FIEGARCH-MG* is concerned the results in table (15) indicate that the two models appear equally good performances when controlling for different forecasting horizons, even though FIEGARCH-MG* model under the GED distribution is constantly generating smaller Log-GFESM values. However, the comparison of the corresponding MSE of 20, 40, 60, 80, 100 one step ahead out of sample forecasts, indicates that those differences are not statistically significant. [166]

[166] Note that Log-GFSM statistics for FIEGARCH-MG* under the t-student and GED are considered statistically equal according to Diebold and Mariano test (1995).

So the decision of which model to choose falls entirely on Akaike and Schwarz criteria reported in table (13). Considering the slightly smaller values of both, when FIEGARCH-MG* is estimated under t-student, there is finally a decision reached.

Results of tables (16) and (17) estimate the adjusted Pearson goodness of fit test and provide further support on the decision to estimate both filtered restricted models under the assumption of t-student. Specifically the results report that under the t student assumption the null hypothesis is accepted in every section of cells, while the last is always rejected when the GED distribution is chosen. So, results provide solid support over the decision to estimate both filtered models under the assumption of t-student.

Table (16) Adjusted Pearson Chi-square Goodness-of-fit test for restricted FIEGARCH-MH*

cells	FIEGARCH-MH* t-student			FIEGARCH-MH* GED		
	stat	P	P*	stat	P	P*
300	334.514	0.077	0.056	385.932	0.000	0.000
400	432.129	0.121	0.069	473.705	0.005	0.002
600	580.851	0.695	0.598	666.338	0.028	0.015

Note : P-column reports p-values corresponding p-each number of cells. The P*-column reports the corresponding adjusted p-values.

Table (17) Adjusted Pearson Chi-square Goodness-of-fit test for restricted FIEGARCH-MG*

cells	FIEGARCH-MH* t-student			FIEGARCH-MG* GED		
	stat	P	P*	stat	P	P*
300	335.642	0.071	0.053	416.065	0.000	0.000
400	434.346	0.107	0.059	473.023	0.006	0.002
600	597.579	0.508	0.405	695.090	0.003	0.001

Note : P-column reports p-values corresponding p-each number of cells. The P*-column reports the corresponding adjusted p-values.

However, before proceeding with the final forecasting analysis there are two things that must be stated. First, the estimates on the long memory parameter in all non filtered volatility models are quite robust and vary between 0.525 and 0.573. The only exception is the FIEGARCH model in table (2) which surprisingly estimates a value 0.173. On the other hand the same long memory estimates for the restricted filtered volatility models are approximately 0.60 and match the values found in Bollerslev and Mikkelsen (1996).

The second statement concerns the risk-volatility relation and the fact that FIEGARCH-MG*indicates the presence of two opposite in sign statistically significant relations.

Moreover, λ_1 coefficient is found being negative, while λ_2 is positive. The results suggest the dominance of a volatility feedback effect mechanism in the first lagged period and the existence of a positive volatility-return trade off in the second.

The combination of signs reported here is the one appearing in Christensen, Nielsen and Zhu (2010). The researchers justify this finding by stating that the negative sign of the first period is something to be seen first, since the volatility feedback effect mechanism induces an immediate price drop as soon as the discount rate in the economy reacts to an increase in volatility. So, the negative effect is something to be seen first. On the other hand the positive sign of the second period is attributed mainly to adjustments made in expectations, a phenomena that naturally requires time in order to mature and in the present analysis this occurs in the second period.

Two things must be underlined when dealing with such phenomena. First, a possible non linearity in either relation would make difficult the separation of the two results, and second an omitted variable bias would produce a negative risk-return relation when in fact there is not any.

Next, the paper compares the best volatility models of each approach. Competing frameworks are the FIEGARCH (1,1)-[1] estimated under the skewed student assumption and the restricted filtered FIEGARCH-MH* and FIEGARCH-MG* both estimated under the t-student distribution.

Obviously, the estimations reported in tables (3) and (13) indicate that the non filtered FIEGARCH (1,1)-[1] outperforms both filtered FIEGARCH versions according to the log-likelihood values and the Akaike and Schwarz criteria. This may be due to the fact that FIEGARCH (1,1)-[1] accepts all null hypothesis in the relevant sign and size bias tests of Engle and Ng (1993), although this is not true for the joint test at 5% significant level since the p-value in this case is 0.049 and hence rejects on the limit the null hypothesis.

Ignoring this latter rejection and having in mind the statement of Engle and Ng (1993) that the joint test is actually more powerful to the individual ones, then this might be the only framework that efficiently manages the leverage effects. So the above statements leave no doubt about the opportune incorporation of all asymmetrical effects. Note that the estimation of FIEGARCH (1,1)-[1] under the skewed student distribution is endorsed by the results presented in appendix for the adjusted Pearson test.[167]

A last crucial criterium involves out of sample forecasting under different forecasting horizons. Earlier in the analysis the paper used 20, 40, 60, 80 and 100 one step ahead out of sample forecasts, estimating on each occasion the corresponding log-GFESM statistic.

Table (17) re-estimates this statistic for the competing volatility frameworks, while table (18) extends the foresting horizons of the analysis by assuming 100, 150, 200 and 250 one step ahead out of sample forecasts. At the same time the table reports the values of eight forecasts error measurements in order to provide an overall assessment of the forecasting performances.

Both tables verify the complete dominance of FIEGARCH(1,1)-[1] in short and long term forecasting horizons, as clearly can be seen in the nine forecasts error measurements. The only exception is reported in table (17) for the 60 one step ahead out of sample forecasts. In this case the FIEGARCH(1,1)-[1] delivers the worst forecasts error measurement.

[167]Another endorsement for estimating FIEGARCH underthe skewed student distribution can be seen in the statistical significance of the estimated asymmetric coefficient reported in table (3).

Table (17) One step ahead forecasts of FIEGARCH models considering different forecasting horizons for L-GFESM statistic

L-GFESM	FIEGARCH(1,1)-[1] Skewed student	FIEGARCH-MH*(1,1) t-student	FIEGARCH-MG*(1,1) t-student
20	0,480*	0,067	0,457
40	1,147*	0,196	0,928
60	1,686	1,166*	1,445
80	2,140*	2,333	2,543
100	2,673*	3,488	3,665

Table (18) One step ahead forecasts of FIEGARCH models considering different forecasting horizons

	FIEGARCH* Skewed student	FIEGARCH-MH* t-student	FIEGARCH-MG* t-student
L-GFESM			
100	0.708*	0.919	0.942
150	1.361*	1.844	1.886
200	1.926*	2.767	2.825
250	2.840*	3.810	3.878
MSE			
100	5.109*	8.309	8.762
150	4.504*	8.404	8.779
200	3.669*	8.378	8.690
250	8.209*	11.06	11.310
MEDSE			
100	3.003*	8.852	9.349
150	2.336*	9.253	9.813
200	1.776*	9.298	9.717
250	1.714*	8.852	9.135
MAE			
100	1.914*	2.694	2.773
150	1.715*	2.705	2.771
200	1.538*	2.725	2.780
250	1.814*	2.843	2.890
RMSE			
100	2.260*	2.883	2.960
150	2.122*	2.899	2.963
200	1.915*	2.894	2.948
250	2.865*	3.326	3.363
MAPE			
100	432.2*	658.9	676.7
150	296.1*	456.6	468.7
200	348.8*	679.9	690
250	299.9*	603.5	612.3
AMAPE			
100	0.588*	0.639	0.643
150	0.588*	0.659	0.662
200	0.604*	0.689	0.692
250	0.604*	0.668	0.670
TIC			
100	0.466*	0.485	0.489
150	0.476*	0.505	0.508
200	0.477*	0.531	0.534

250	0.578*	0.514	0.515
LL			
100	8.209*	9.847	9.982
150	7.457*	9.582	9.694
200	8.628*	11.600	11.700
250	8.333*	11.190	11.270

Note : * denote the best forecasting model

5. Conclusions.

The present analysis uses the daily returns of the Fama-French stock market index to estimate various volatility models, with the aim to compare their volatility forecasting potentials. The competing volatility frameworks are GARCH, FIGARCH, EGARCH, IGARCH, GJR and FIEGARCH.

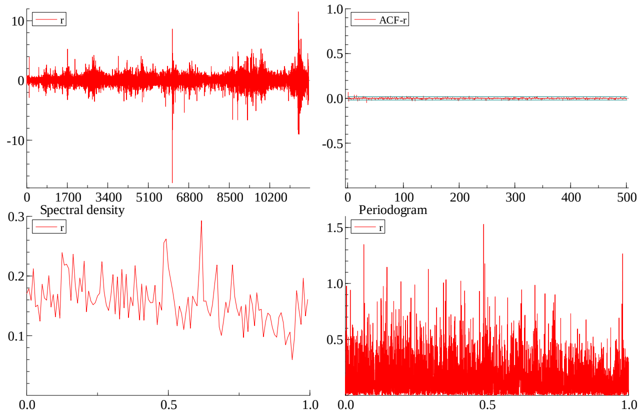
The pursue of the above objective naturally requires the exact specification of the competing volatility frameworks, and this obviously demands decisions upon dilemmas such as, a) the number of ARCH and GARCH terms used in every volatility model, b) the distributional assumptions applied, and finally c) decisions about whether to include in the estimated frameworks volatility-return relations. Especially when dealing with the last issue, the analysis answers the question about the presence of a statistically significant volatility-return relation.

Furthermore the analysis focuses on FIEGARCH model and applies a filter of long memory in volatility in order to prevent possible spill over effects. Specifically, the analysis follows Christensen, Nielsen and Zhu (2010) and estimates their filter volatility models, FIEGARCH-MH and FIEGARCH-MG, that introduce stationary volatility representations in the conditional in mean equations.

As far as the volatility-relation is concerned the results suggest that the acknowledgement of such a relation is generally sensitive to the distributional assumptions applied and to specific estimated models. On the other hand and as far as volatility forecasting is concerned, the Fama-French stock market index provides strong support for models not incorporating such a risk-return relation, with FIEGARCH (1,1)–[1] truly outperforming any other model, either according to standard information criteria that reward the fit, or according to combined forecasting analysis, that examines various forecasting error measurements under different forecasting horizons.

Appendix

1. ACF, Spectral density, long memory tests.



The spectrum at zero frequency is a finite function and therefore suggests that stock market index of Fama-French is a well defined stationary process. The same results are confirmed in table (A) when implementing both the log-periodogram regression method of Geweke and Porter-Hudak (1983) and the Gaussian semi-parametric method of Robinson and Henry (1998). Furthermore, results confirm the negative skewed and leptokyrtic nature of the data set, which justifies the selection of skewed student t distributions in the majority of estimations.

Table (A) Descriptive statistics of Fama-French data set

	mean	std.dev	skewness	Kurtosis	J.B	GPH test	GSP test
r	0.019	0.983	-0.54655 4.5806e-129	17.655 [0.000]	1.5295e+005 [0.000]	0.017 [0.052]	0.011 [0.073]

2. Forecasts Error Measurements for different volatility models.

Table (B) Forecasts Error measurements for 100 one day ahead out of sample forecasts

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
MSE	12.80	14.19	13.04	12.93	13.97	12.98
MedSE	2.943	1.114	4.458	3.785	1.473	2.529
MAE	2.207	1.986	2.430	2.337	2.049	2.142
RMSE	3.577	3.766	3.612	3.596	3.738	3.602
MAPE	396.6	226.4	475.6	442.8	261.7	359.6
AMAPE	0.633	0.635	0.647	0.641	0.637	0.631
TIC	0.597	0.718	0.567	0.578	0.689	0.620

LL	11.120	9.57	11.92	11.61	9.96	10.78

Table (C) Forecasts Error measurements for 200 one day ahead out of sample forecasts

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
MSE	7.679	8.127	7.727	9.207	8.225	7.824
MedSE	1.636	0.783	1.222	5.747	0.981	1.262
MAE	1.658	1.46	1.567	2.362	1.527	1.558
RMSE	2.771	2.851	2.78	3.034	2.868	2.797
MAPE	266.5	168	230.4	430.8	191.7	222.8
AMAPE	0.580	0.570	0.572	0.635	0.579	0.574
TIC	0.583	0.682	0.611	0.515	0.661	0.622
LL	8.172	6.89	7.732	10.28	7.25	7.641

Table (D) Estimations of GARCH(1,1)-M-[1,2,3], EGARCH(1,1)-M-[1,2,3], IGARCH(1,1)-M-[1,2,3], GJR(1,1)-M-[1,2,3], FIGARCH(1,1)-M-[1,2,3], FIEGARCH(1,1)-M-[1,2,3] assuming the t-student distribution**.**

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
μ	0.035 [0.000] **	0.023 [0.002]**	0.028 [0.000]**	0.035 [0.000]**	0.002 [0.775]	0.035 [0.000]**
μ_1	0.147 [0.000]**	0.147 [0.000]**	0.151 [0.000]**	0.147 [0.000]**	0.134 [0.000]**	0.147 [0.000]**
μ_2	-0.027 [0.003]**	-0.018 [0.023]*	-0.021 [0.025]*	-0.028 [0.003]**	-0.023 [0.036]*	-0.027 [0.003]**
μ_3	0.003 [0.692]	0.009 [0.231]	0.009 [0.335]	0.002 [0.775]	0.008 [0.544]	0.003 [0.689]
ω	0.003 [0.000]**	-	0.005 [0.000]**	0.010 [0.000]**	-	0.004 [0.000]**
λ	0.016 [0.118]	-0.016 [0.381]	0.002 [0.777]	0.018 [0.116]	0.007 [0.000]**	0.017 [0.120]
α_1	0.080 [0.000]**	-0.327 [0.000]**	0.026 [0.000]**	0.195 [0.000]**	-0.293 [0.673]	0.078 [0.000]**
β_1	0.919 [0.000]**	0.994 [0.000]**	0.919 [0.000]**	0.693 [0.000]**	0.922 [0.000]**	0.919 [0.000]**
θ	-	-0.109 [0.000]**	0.910 [0.000]**	-	-1.000 [0.151]	-
γ	-	0.181 [0.000]**	-	-	1.000 [0.160]	-
d	-	-	-	0.569 [0.000]**	0.217 [0.008]**	-
Logl	-13.470.7	-13.364.6	-13.389	-13.458	-13345.6	-13.470.3
Akaike	2.298	2.280	2.285	2.296	2.367	2.453
Schwarz	2.303	2.287	2.291	2.303	2.374	2.302
Q(100)	106.903 [0.300]	99.838 [0.485]	98.029 [0.537]	111.305 [0.206]	114.273 [0.155]	112.151 [0.167]
Q(200)	226.916 [0.092]	219.202 [0.167]	219.489 [0.164]	231.070 [0.065]	231.490 [0.062]	223.430 [0.172]
Q*(100)	97.428 [0.497]	100.486 [0.411]	94.036 [0.594]	94.7586 [0.573]	350.714 [0.000]**	264.372 [0.450]
Q*(200)	185.233 [0.733]	203.100 [0.386]	197.540 [0.495]	174.781 [0.881]	507.979 [0.000]**	493.129 [0.000]**
MSE	13.25	13.01	12.98	13.47	12.98	13,90

Note: Quasi Maximum Likelihood estimates are reported. Also reported are the Logl, this is the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung-Box portmanteau statistic for testing the up to m'th order serial dependence of standardized and absolute standardized residuals denoted respectively as Q(m) and Q*(m). Finally, the table reports the mean square error (MSE) for 100 one-day-ahead out of sample forecasts. The values in parenthesis stand for p-values. *denotes rejection at 5% significance, while ** denotes rejection at 1% significant level.

Table (E) Estimations of GARCH(1,1)-M-[1,2,3], EGARCH(1,1)-M-[1,2,3], IGARCH(1,1)-M-[1,2,3], GJR(1,1)-M-[1,2,3], FIGARCH(1,1)-M-[1,2,3], FIEGARCH(1,1)-M-[1,2,3] assuming the GED distribution**.**

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
μ	0.029 [0.001]**	0.024 [0.001]**	0.023 [0.002]**	0.029 [0.000]**	-0.033 [0.000]**	0.029 [0.000]**
μ_1	0.142 [0.000]**	0.148 [0.000]**	0.147 [0.000]**	0.142 [0.000]**	0.186 [0.000]**	0.142 [0.000]**
μ_2	-0.032 [0.000]**	-0.018 [0.006]**	-0.024 [0.010]*	-0.032 [0.000]**	0.024 [0.031]*	-0.032 [0.000]**
μ_3	0.001 [0.884]	0.010 [0.181]	0.008 [0.371]	0.000 [0.975]	0.042 [0.004]**	0.001 [0.881]
ω	0.003 [0.000]**	-	0.005 [0.000]**	0.010 [0.000]**	-	0.004 [0.000]**
λ	0.011 [0.284]		-0.001 [0.862]	0.014 [0.221]	-0.091 [0.000]**	0.012 [0.262]
α_1	0.079 [0.000]**	-0.316 [0.000]**	0.027 [0.000]**	0.196 [0.000]**	0.654 [0.011]*	0.078 [0.000]**
β_1	0.920	0.992 [0.000]**	0.920 [0.000]**	0.693 [0.000]**	-0.247 [0.322]	0.919 [0.000]**
θ	-	-0.107 [0.000]**		-	-0.239 [0.000]**	-
γ	-	0.179 [0.000]**	-	-	0.042 [0.000]**	-
d	-	-	-	0.568 [0.000]**	0.664 [0.000]**	-
Logl	-13458.2	-13.362.7	-13376.3	-13445.4	-13807.3	-13457.7
Akaike	2.296	2.280	2.282	2.294	2.356	2.296
Schwarz	2.302	2.288	2.289	2.301	2.364	2.302
Q(100)	112.028 [0.193]	99.579 [0.493]	99.912 [0.483]	116.905 [0.118]	177.063 [0.000]**	111.872 [0.196]
Q(200)	232.636 [0.056]	218.749 [0.172]	221.664 [0.140]	237.306 [0.036]*	300.369 [0.000]**	232.578 [0.056]
Q*(100)	97.354 [0.499]	101.654 [0.380]	93.479 [0.610]	94.557 [0.579]	2180.330 [0.000]**	96.615 [0.520]
Q*(200)	184.487 [0.745]	206.254 [0.329]	195.929 [0.528]	174.010 [0.889]	2539.56 [0.000]**	184.964 [0.737]
MSE	13.23	12.93	14.56	13.8	13.46	13.58
asymmetry	-0.065 [0.000]**	0.011 [0.513]	-0.066 [0.000]**	-0.065 [0.000]**	-0.145 [0.000]**	-0.065 [0.000]**

Note: Quasi Maximum Likelihood estimates are reported. Also reported are the Logl, this is the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung-Box portmanteau statistic for testing the up to m'th order serial dependence of standardized and absolute standardized residuals denoted respectively as Q(m) and Q*(m). Finally, the table reports the mean square error (MSE) for 100 one-day-ahead out of sample forecasts. The values in parenthesis stand for p-values. *denotes rejection at 5% significance, while ** denotes rejection at 1% significant level.

Table (F) Estimations of GARCH(1,1)-M-[1,2,3], EGARCH(1,1)-M-[1,2,3], IGARCH(1,1)-M-[1,2,3], GJR(1,1)-M-[1,2,3], FIGARCH(1,1)-M-[1,2,3], FIEGARCH(1,1)-M-[1,2,3] assuming the Generalized Error Distribution**.**

	GARCH	EGARCH	GJR	FIGARCH	FIEGARCH	IGARCH
μ	0.038 [0.000]**	0.026 [0.001]**	0.031 [0.000]**	0.038 [0.000]**	-0.025 [0.000]**	0.038 [0.000]**
μ_1	0.137 [0.000]**	0.140 [0.000]**	0.143 [0.000]**	0.139 [0.000]**	0.192 [0.000]**	0.137 [0.000]**
μ_2	-0.028 [0.003]**	-0.019 [0.022]*	-0.022 [0.018]*	-0.029 [0.002]**	0.027 [0.004]**	-0.028 [0.002]**
μ_3	0.002 [0.785]	0.010 [0.259]	0.008 [0.396]	0.001 [0.869]	0.044 [0.000]**	0.002 [0.779]
ω	0.005 [0.000]**	-	0.005 [0.000]**	0.229 [0.000]**	-	0.004 [0.000]**
λ	0.014 [0.214]	-0.021 [0.177]	0.000 [0.929]	0.017 [0.209]	-0.098 [0.000]**	0.013 [0.211]
α_1	0.082 [0.000]**	-0.368 [0.000]**	0.026 [0.000]**	0.243 [0.000]**	0.687 [0.007]**	0.093 [0.230]
β_1	0.915 [0.000]**	0.993 [0.000]**	0.917 [0.000]**	0.571 [0.000]**	-0.334 [0.000]**	0.937 [0.000]**
θ	-	-0.118 [0.000]**	0.645 [0.000]**	-	-0.228 [0.000]**	-
γ	-	0.193 [0.000]**	-	-	0.034 [0.000]**	-
d	-	-	-	0.408 [0.000]**	0.672 [0.000]**	-
Logl	-13498.2	-13398.2	-13472.4	-13472.4	-13921.5	-13498.9
Akaike	2.303	2.286	2.290	2.299	2.376	2.303
Schwarz	2.309	2.293	2.296	2.305	2.383	2.308
Q(100)	111.782 [0.197]	101.269 [0.445]	100.754 [0.460]	119.852 [0.085]	195.649 [0.000]**	112.079 [0.192]
Q(200)	232.448 [0.057]	220.959 [0.147]	222.890 [0.127]	239.744 [0.028]*	320.131 [0.000]**	232.589 [0.056]
Q*(100)	95.182 [0.561]	102.210 [0.365]	95.339 [0.557]	92.4645 [0.638]	2427.05 [0.000]**	96.292 [0.529]
Q*(200)	186.127 [0.717]	205.531 [0.341]	201.811 [0.411]	170.296 [0.923]	320.131 [0.000]**	185.208 [0.056]
MSE	14.00	12.97	14.01	13.47	13.8	13.76

Note: Quasi Maximum Likelihood estimates are reported. Also reported are the Logl, this is the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung-Box portmanteau statistic for testing the up to m'th order serial dependence of standardized and absolute standardized residuals denoted respectively as Q(m) and Q*(m). Finally, the table reports the mean square error (MSE) for 100 one-day-ahead out of sample forecasts. The values in parenthesis stand for p-values. *denotes rejection at 5% significance, while ** denotes rejection at 1% significant level.

3. Forecast Evaluation.

Very often the analysis compares the MSE values of the competing volatility frameworks in order to produce inferences on their forecasting volatility potentials. However, if a volatility model does estimate a lower MSE value than a competing alternative it is probably precarious to talk about forecasting superiority, since the differences of the compared MSE values may actually turn out being not statistically significant. So as

Diebold and Mariano (1995) underline it is very important not only to compare the MSE values but it is of interest also to test whether possible reductions in the MSE values are statistically significant.

Let assume that two different volatility models generate m, h step ahead out of sample forecasts. This implies two sets of forecasts errors, e_{1t} and e_{2t}, where obviously t=1,…,m. If the analysis uses as criterium for the forecasting potential the MSE value then if

$$d = e_{1t}^2 - e_{2t}^2$$

the hypothesis of equal forecast accuracy can be represented as $E[d_t]=0$.The Diebold and Mariano (1995) statistic for testing the null hypothesis of equal forecast accuracy is

$$S_1 = [\hat{V}(\bar{d})]^{-1/2}\bar{d}$$

where

$$V(\bar{d}) \simeq n^{-1}[\hat{\gamma}_0 + 2\sum_{\kappa=1}^{h-1}\hat{\gamma}_k]$$

$$\hat{\gamma}_\kappa = n^{-1}\sum_{t=k+1}^{n}(d_t - \bar{d})(d_{t-k} - \bar{d})$$

Under the null hypothesis S_1 statistic follows asymptotically the standard normal distribution. Note that the monte carlo experiments conducted in Diebold and Mariano (1995) indicate that the performance of the statistic is good either for samples that are small or for forecasts errors that are autocorrelated and have non-normal distributions.

An alternative statistic for testing the equality of forecasts errors is the one offered by Harvey, Leybourne and Newbold (1997) and is denoted in the equation following. This actually modifies Diebold and Mariano's test and claims the improvement of the finite sample performances of the latter statistic.

$$S_2 = [\frac{m+1-2h+m^{-1}h(h-1)}{m}]^{1/2} s_1$$

where S_1 is the initial statistic. Note that S_2 is compared with t-student critical values.

Table (G) Diebold and Mariano (1995) and Harvey, Leybourne and Newbold (1997) statistics.

Null Hypothesis : E[d]=0	100-one step ahead out of sample forecasts	
	FIEGARCH-MH*	FIEGARCH-MG*
	t-student-GED	t-student-GED
$S_{1\text{-normal distribution}}$	3.450**	0.653
$S_{2\text{-tstudent}}$	3.210**	0.610

Note : *(**) denote the rejection of the null at 5% and 1% significant level.

Table (H) Diebold and Mariano (1995) and Harvey, Leybourne and Newbold (1997) statistics.

Null Hypothesis : E[d]=0	200-one step ahead out of sample forecasts	
	FIEGARCH-MH*	FIEGARCH-MG*
	t-student-GED	t-student-GED
$S_{1\text{-normal distribution}}$	3.234**	2.774**
$S_{2\text{-tstudent}}$	3.201**	2.720**

Note : *(**) denote the rejection of the null at 5% and 1% significant level.

Table (I) Diebold and Mariano (1995) and Harvey, Leybourne and Newbold (1997) statistics.

Null Hypothesis : E[d]=0	FIEGARCH-MG*[t-student and GED]				
	Different one step ahead out of sample forecasts				
	20	40	60	80	100
$S_{1\text{-normal distribution}}$	0.541	0.671	0.978	1.231	1.340

$S_{2\text{-t student}}$	0.532	0.598	0.951	1.200	1.278

Note : *(**) denote the rejection of the null at 5% and 1% significant level.

The results in table (G) indicate that the MSE values reported on table (12) for 100 one step ahead out of sample forecasts of FIEGARCH-MH*models of t-student and GED distributions are statistically different to each other, since the null hypothesis in Diebold and Mariano test (1995) is rejected at 5% and 1% significant levels. On the other hand comparison of the MSE values of FIEGARCH-MG* modes of t-student and GED distributions on the same forecasting, horizon turns over the acceptance of null hypothesis in the relevant Diebold and Mariano's test. As for the results reported on table (H), those reject the corresponding null hypothesis at both levels of significance and hence the results conclude that the relevant MSE values are in fact statistically different to each other at all occasions.

Finally, in table (I) the outcomes of S_1 and S_2 statistics strongly suggest the indifference of the corresponding MSE values at all examined forecasting horizons. Furthermore note that the previous outcomes are all verified when the results on the S_2 statistic are reported.

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