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Public-Private Wage Differentials

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Contents

Διαφορισμός μισθών στον δημόσιο και ιδιωτικό τομέα

Περίληψη

Στόχος αυτής της διδακτορικής διατριβής είναι να μελετήσει τον διαφορισμό των μισθών μεταξύ του δημοσίου και του ιδιωτικού τομέα. Το κίνητρο στηρίζεται στο γεγονός ότι παραγωγικά ισοδύναμοι εργαζόμενοι συνήθως αμείβονται καλύτερα στον δημόσιο τομέα απ' ό,τι στον ιδιωτικό. Σύμφωνα με την βιβλιογραφική επισκόπηση, αυτό το φαινόμενο παρατηρείται σε πολλές χώρες ανά τον κόσμο. Οι ερευνητές το αποδίδουν στα διαφορετικά προβλήματα αριστοποίησης που αντιμετωπίζουν οι δύο τομείς. Επίσης, αναγνωρίζουν τον σημαντικό ρόλο των εργατικών ενώσεων, στη διαδικασία καθορισμού των μισθών καθώς και την επίδραση που έχει στους μισθούς ο διαφορετικός χαρακτήρας της απασχόλησης, δηλαδή στον δημόσιο τομέα ο υπάλληλος θεωρείται «μόνιμος» ενώ στον ιδιωτικό θεωρείται «προσωρινός». Ωστόσο, οι ερευνητές δεν ενσωματώνουν στα πλαίσια ενός θεωρητικού υποδείγματος τις παραπάνω παρατηρήσεις ώστε να εξηγήσουν θεωρητικά το πώς αυτές επιδρούν στην διαφόρφωση των μισθών.

Στην παρούσα διατριβή κατασκευάζω ένα υπόδειγμα που ενσωματώνει όλες τις παραπάνω παρατηρήσεις και εξηγώ θεωρητικά τον διαφορισμό των μισθών. Στο υπόδειγμα περιλαμβάνονται δύο τομείς, ο δημόσιος και ο ιδιωτικός. Κάθε τομέας εκπροσωπείται από μία επιχείρηση. Κάθε επιχείρηση διαπραγματεύεται τον μισθό με το εργατικό σωματείο που εκπροσωπεί εργαζομένους του ίδιου τομέα. Η διαδικασία της διαπραγμάτευσης λαμβάνει χώρα ταυτόχρονα, αλλά ανεξάρτητα για κάθε επιχείρηση. Κάθε επιχείρηση διαλέγει τον αριθμό των εργαζομένων που επιθυμεί να απασχολήσει, με δεδομένο το αποτέλεσμα της διαπραγμάτευσης των μισθών. Το υπόδειγμα εκτείνεται σε δύο περιόδους και εξετάζω την περίπτωση που οι μισθοί είναι επαναδιαπραγματεύσιμοι και την περίπτωση που δεν είναι.

Το βασικό συμπέρασμα αυτής της έρευνας είναι ότι ο διαφορισμός των μισθών διαμορφώνεται υπέρ του δημοσίου τομέα και αποδίδεται σε δύο συνιστώσες. Η μία αναφέρεται στους αυστηρότερους περιορισμούς απολύσεων που αντιμετωπίζει ο δημόσιος τομέας σε σχέση με τον ιδιωτικό. Η δεύτερη αναφέρεται στην ασυμμετρία ως προς την αντικειμενική συνάρτηση που μεγιστοποιούν οι δύο τομείς, δηλαδή στον μη ανταγωνιστικό χαρακτήρα του δημόσιου τομέα και στον ανταγωνιστικό του ιδιωτικού.

Chapter 1
Introduction

The aim of this thesis is to investigate wage differentials between the public and the private sector. My motivation is based on the fact that equally productive employees are often better paid in the public than in the private sector. According to the empirical literature, this phenomenon exists in the majority of the countries around the world. Authors attribute wage differentials to different maximization problems that these two sectors face. Also, they recognize the crucial role of unions in the wage setting process as well as the impact on wages of the different structures, namely, the temporary character of jobs in the private sector and the permanent in the public sector. However, this literature does not postulate any explicit theoretical hypothesis of how wages are set.

In the present thesis, I explain theoretically wage differentials. I assume that there are two sectors, the public and the private sector. Each sector is represented by one firm, the public and the private firm. My model contains two features. First, I incorporate the role of the unions in the wage setting process. In particular, I assume that each firm bargains the wage with the union that represents employees of the same sector. Each firm continues to choose the number of workers it wishes to employ once wages have been determined by the bargaining process. In case where unions have all the bargaining power and the firm's bargaining power is set to zero, the model is a "monopoly union model," and is a special case of the "right-to-manage model" (Booth 1995). The bargaining process takes place simultaneously and independently in both sectors. Technically, the solution to the bargaining problem is given by Nash (1950), provided that participants have symmetric bargaining power. Roth (1979) proves that the solution to the bargaining problem given by Nash is feasible even in the case where participants have asymmetric bargaining power.

The literature adopts two assumptions about the objective function of the union. The first is that the union cares only about the economic welfare of the union members. The second is that the union's members are identical, or that unions are concerned about the median member. Dunlop (1944) argues that the most appropriate union objective is the maximization of the total wage bill of the membership.[1] In this case, the utility function is assumed to be linear in wages.

[1] For other assumptions about the objective functions of the union, see de Menil (1971).

In this thesis, I adopt Dunlop's view. Because of the law of labor demand, there is an inverse relationship between wages and employment.

The second feature of my model is related to the asymmetric firing restriction regime that the public and the private sectors face and its impact on the wage setting process. In order to express the firing restriction regime, I use the term "adjustment cost." Adjustment cost has been an issue in the literature since the late 1970s. This term is used in order to express the rigidity of markets due to legislation that protects workers from being "unfairly" fired. Such legislations may make employers reluctant to hire, especially during a recession. Consequently, adjustment costs comprise both hiring and firing costs. Traditionally, the adjustment cost is assumed to be quadratic [Solow (1968), Sargent (1978)], i.e is $\frac{c\left(n_{it}-n_{it-1}\right)^2}{2}$, where *n* is the level of employment for firm *i* in period *t* and c is the adjustment cost. Later, Burgess and Dolado (1989), Pfann and Palm (1993) introduce the linear form for the adjustment cost taking into account the asymmetry in the cost of firing and in the cost of hiring. Namely, linear adjustment cost is $(Hiring - Firing)$. The combination of the quadratic and the linear adjustment cost is given by Hamermesh (1996). In chapter 3, I present analytically how I use the adjustment cost in order to express the asymmetric firing restriction regime that the public and the private sectors face.

Taking into account the above features, I develop a unionized mixed oligopoly model with stochastic demand shocks extending in two periods, in order to explain theoretically wage differentials. I examine two cases. In the first, wages are bargained once at the whole game and in the second wages are re-bargained. The novelty of this analysis is that captures the idea that an asymmetric firing restriction regime between the public and the private sector helps the public sector protect employment and "employees rights" during the business cycle. To my knowledge there is no other analysis in the literature that incorporates this feature.

The rest of this thesis is structured as follows. In chapter 2, I describe the methods that are researchers use in order to estimate the public-private wage differential and I summarize the empirical results of the literature on public-private wage differentials in developed, developing, and poor countries. In chapter 3, I develop the model in the case where wages are bargained once at the whole game. In particular, this chapter reveals how the asymmetry in the firing restriction regime between the public and the private sector provides the public sector with an idiosyncratic incentive to set a premium in the public-sector wage, over the private sector wage contract, irrespectively of the realization and the magnitude of the demand shock. In chapter 4, I consider the model in the case where wages are re-bargained. Specifically, chapter 4 shows that the asymmetry in the firing restriction regime between the public and the private sector provides the public-sector wage with a premium but only in the final round of the bargain. In chapter 5, I re-consider the model that I present in chapter 3, under more realistic assumption about public-firm objective function. In this case the results have strengthened. Finally, chapter 6 concludes.

Chapter 2
Survey of the literature

2.1 Introduction

There is an extensive empirical literature on public-private wage differentials. All the researchers argue that public-private wage differentials are attributed to different maximization problems that both sectors face. More specifically, the public sector is subject to political, social, and economic constraints, whereas the private sector solves an unconstrained profit maximization problem. However, empirical studies do not test economic theories of how wages are set, but are typically based on the definition of public-private sector wage differentials. There are three empirical methods that are used in order to estimate public-private wage differentials. The empirical estimates refer to developed, developing, and poor countries. The main conclusion is that in the majority of countries the public sector pays higher wages than the private sector.

The rest of this chapter is organized as follows. Section 2.2 deals with the characteristics of the two sectors. In section 2.3, I describe the methods that researchers use in order to estimate public-private wage differentials. Sections 2.4, 2.5, and 2.6 deal with wage differentials in developed, developing and poor countries. Finally, section 2.7 summarizes.

2.2 Characteristics of the two sectors

In the literature, the widespread use of the term public-private wage differential refers to the difference between the public and the private sector wage. For an individual i, the public-private wage differential (β_i) can be written as

$$\beta_i = \ln w_{1i} - \ln w_{2i}, \qquad (2.1)$$

where w denotes the wage rate and the subscripts 1 and 2 denote the public and the private sector, respectively. However, for any individual, only one of these two wage rates is observed, whereas the other must be estimated taking into account characteristics such as age, sex, educational level, experience, and union status.

The structure of the two sectors is responsible for the wage differential between them, because it gives rise to different maximization problems. In particular, the public sector is subject to political, social, and fiscal constraints. In the private sector, employers pursue maximum profit in order to survive the competition, and employees work hard in order to keep their jobs. On the contrary, in the public sector a political system is in operation according to which the government is under pressure to employ workers permanently and pay them equal wages, irrespective of their skills and willingness to work. The public sector non-pecuniary characteristics (job security, working hours, and fringe benefits) usually make it more attractive, than the private sector.

In many countries, the government actively participates in the wage setting process. For instance, according to Glinskaya and Lokshin (2005), the Indian government, following the recommendations of the 1948 Committee on Fair Wages, started to provide guidelines for the institution of a minimum wage law. Also, van der Gaag and Vijverberg (1988) refer to Cote d'Ivoire, where the minimum wage legislation drives a wedge between marginal productivity and the observed real wage rate. In general, the larger the institutional or regulatory influence on the labor market, the more likely it becomes for the observed wage rate to exceed the worker's marginal productivity.

An example is given by Van Ophem (1993), who refers to the Netherlands, where during the decade of the 1970s public-sector wages were related to private-sector wages. The government, under the pressure to reduce the budget deficit of 1982, abandoned this relationship between sectors wage, and the public-sector wages fell behind. Adamechik and Bedi (2000) give another example related to fiscal restraints. They refer to Poland, where until 1991 there was wage control in both public and private enterprises due to inflationary pressures and budget deficits, but after 1991 only state-owned enterprises were still subject to wage controls. Blanchard (1998) notes that the dominant goal of the Polish government was to maintain the highest level of employment, through wage cuts, taking into account financial constraints. The aim was to minimize social turbulence.

Borjas (1984) indicates that the U.S. Federal government wage rates rise significantly in election years. Blaise (2005) notes that lack of flexibility in the public sector might result in wages that adjust counter-cyclically. Disney and Gosling (1998) observe that the public-sector wage "premium" in Great Britain is counter-cyclical: when the economy is moving towards its peak, as in the mid 1980s and the mid-1990s, the "premium" stays the same or falls, whereas when the economy is moving into a recession, as in the early 1980s and the late 1980s, the "premium" rises sharply.

Researchers observe that the assumption that public-sector wages are set equal to marginal productivity may not be realistic for many reasons. Jackman and Rutkowski (1994) indicate that prior to the 1990s the government of Poland tried to set wages in accordance with marginal productivity, but it was hard to measure it. Panizza and Zhen-Wei Qiang (2005) refer to the controversy in the literature on the role of productivity in the wage setting process in the public and the private sectors. The problem is that there exist differences in productivity between genders and sectors that are not observable by econometricians.

2.3 Methods of estimation of the wage differential

Estimation of the public-private wage differential is sensitive to the choice of method and to the degree of aggregation used in collecting data. In order to find which estimates we can trust, it is important to understand the methods that researchers use. The method of estimation of the wage differential depends, most of the times, on the available data. There exist three methods, which I now describe briefly.

2.3.1 The first method of estimation

There are many studies that rely on aggregate data and on pooled cross-sectional studies across time estimating the public-private wage differential; see for example, Dell' Aringa et al (2007), Disney and Gosling (1998), Pedersen et al (1990), Nielsen and Rosholm (2001), and Dustmann and van Soest (1997). "Aggregate" or "grouped cross-sectional" studies comprise studies where estimating wages relies on individuals that are aggregated by industry, city, state, or occupation. The estimating equation in this case is as follows:

$$\ln w_i = a + \beta D_i + \delta R_i + \varepsilon_i \qquad (2.2)$$

where $\ln w_i$ is the logarithm of the wage of individual i, D_i is a dummy variable that takes on the value of 1 if an employee works in the public sector and the value 0 otherwise, R_i is a vector of variables of several personal characteristics and job attributes, and a is a constant.

The coefficient β measures the log wage differential. To see that β is the log wage differential, write (2.2) for an employee in the public sector and then for an employee in the private sector.

$$\text{Public sector: } \ln w_{1i} = a + \beta + \delta R_i + \varepsilon_i \qquad (2.2a)$$

$$\text{Private sector: } \ln w_{2i} = a + \delta R_i + \varepsilon_i \,. \qquad (2.2b)$$

Subtract (2.2b) from (2.2a) to get $\beta = \ln w_{1i} - \ln w_{2i}$. If β is positive then the wage differential is formed in favor of the public sector, if it is negative then the wage differential is formed in favor of the private sector. Equation (2.2) can be estimated, for example, with OLS or quantile regression.[2] Note, however, that β can be misleading indicator or biased. One reason may be the omitted-variable problem. For example, in the case of aggregate wages, it is generally impossible to control for labour quality, because of lack of data. In this case, we assume that productivity and job attributes to be equal across sectors, ignoring the market differences in the "wage structure" between sectors. Thus, β is the same for workers employed in the public and in the private sector. Second, estimation of (2.2) excludes workers who are unemployed, since the criterion for selecting the sample is the wage rate of employed workers. This criterion is endogenous and may induce a selectivity bias.[3]

Equation (2.2) allows the number of workers in the public sector to differ from those in the private sector. A problem of this method is the limited number of observations typical of studying aggregate data. A way to increase the sample size is to use independently pooled cross sections. By pooling random samples drawn from the same population, but at different points in time, we get more precise estimators and test statistics with more power.

[2] Quantile regression, as introduced by Koenker and Bassett (1978), is an extension of classical least squares estimation conditional on the θ quantile. In other words, conditional quantile functions are estimated by minimizing an asymmetrically weighed sum of absolute errors. The central special case is the median regression estimator, which minimizes the sum of absolute errors, provided that θ=1/2. This method encompasses models that are linear and non-linear, parametric and non-parametric.

[3] Heckman (1979) develops a two stages estimator to solve the problem related to the selection bias. Though, Manski (1989) notes that very small misspecifications in the selection equation might generate very large biases in the estimates. As well, Blau and Kahn (1996) suggest that when one uses weak instruments or when errors are not normally distributed then the cost of using Heckman's method is higher than the benefit.

2.3.2 The second method of estimation

Blinder (1973) and Oaxaca (1973) suggest a method (often referred to as Oaxaca decomposition) to decompose wage differences by estimating separate equations for the groups that need to be compared.[4] In this case, the earnings of the public and the private sector workers are estimated separately. Thus, it is assumed that the wages in the public and in the private sector are determined by the following equations:

$$\ln w_{1i} = X_{1i}'\beta_1 + \mu_{1i} \qquad (2.3)$$

$$\ln w_{2i} = X_{2i}'\beta_2 + \mu_{2i}\,, \qquad (2.4)$$

where w_{ji} is the earnings of individual *i* who works in sector *j*, $j = 1,2$. X_{ji} is a vector of explanatory variables or individual characteristics. The estimated coefficients in equations (2.3) and (2.4) are used to decompose the raw differences in average earnings between the two sectors. The difference in the mean log wage between the public and the private sector is

$$\ln \bar{w}_1 - \ln \bar{w}_2 = \left[\left(\bar{X}_1 - \bar{X}_2\right)\hat{\beta}^*\right] + \left[\bar{X}_1\left(\hat{\beta}_1 - \hat{\beta}^*\right) - \bar{X}_2\left(\hat{\beta}_2 - \hat{\beta}^*\right)\right], \qquad (2.5)$$

where $\ln \bar{w}_j$ and $\bar{X}_j$ are mean logarithmic wages and mean characteristics, respectively, of workers in the sector *j*. The vector $\hat{\beta}_j$ refers to the returns of workers' characteristics obtained from the estimations of (2.3) and (2.4). $\hat{\beta}^*$ measures the returns to workers' characteristics that would exist in the absence of unequal rates of return to employees in the public and in the private sectors. It is possible to carry out the decomposition under different assumptions for $\hat{\beta}^*$. If $\hat{\beta}^* = \hat{\beta}_1$, equation (2.5) becomes

$$\ln \bar{w}_1 - \ln \bar{w}_2 = \left[\left(\bar{X}_1 - \bar{X}_2\right)\hat{\beta}_1\right] + \bar{X}_2\left(\hat{\beta}_1 - \hat{\beta}_2\right) \qquad (2.6)$$

and if $\beta^* = \hat{\beta}_2$, equation (2.5) becomes

$$\ln \bar{w}_1 - \ln \bar{w}_2 = \left[\left(\bar{X}_1 - \bar{X}_2\right)\hat{\beta}_2\right] + \bar{X}_1\left(\hat{\beta}_1 - \hat{\beta}_2\right). \qquad (2.7)$$

[4] Machato and Mata (2005) propose another decomposition procedure which combines quantile regression and a bootstrap approach.

The first term of the right hand side of equation (2.5) measures log wage differentials due to inter-sector differences in average worker characteristics. The second term of the right hand side of equation (2.5) measures the effect of inter-sector differences in returns to worker characteristics or the public-private wage differential.

Clearly, $\hat{\beta}*$ does not affect the raw wage differential, but it changes the relative size of the composition and the returns effects. This methodology, originally suggested by Cotton (1988),[5] corresponds to choosing a weighted average of $\hat{\beta}_1$ and $\hat{\beta}_2$. Indicatively, some of the studies using this method include Lindauer and Sabot (1983), Mueller (1998), and Blaise (2005).

Oaxaca decompositions have the advantage of not imposing the same vector β on public and private sector workers, but a possible selection bias remains, because a worker's choice between working in the public or in the private sector could be non-random. Moreover, in practice, wages are not exactly related to all observable characteristics, such as productivity, especially in the public sector, where wages are set by the political process.

2.3.3 The third method of estimation

Finally, there are estimates taking into account the possible endogeneity of sector membership, assuming that individuals choose sectors by maximizing utility. This approach helps us consider the theoretical background of an individual's decision to join the public or the private sector. In this case, one must take into account the problems that arise from misspecification and omitted variable bias.

Suppose that individual's preferences (V_i) depend on the wage and non-wage benefits (K_i) of each sector, namely

$$V_i = \ln w_i + D_i K_i . \qquad (2.8)$$

An individual will choose the sector that generates higher utility. Let I_i be individuals i's differential defined as

$$I_i = V_{1i} - V_{2i} = \ln w_{1i} - \ln w_{2i} + K_i . \qquad (2.9)$$

[5] An equivalent method suggested by Neumark (1988).

If $I_i > 0$ $(I_i < 0)$, then public sector jobs are more (less) attractive to individual i. Assume now that each individual has a public and a private wage function given by (2.3) and (2.4). The net non-wage benefits can be written as

$$K_i = \gamma Z_i + u_i , \qquad (2.10)$$

where Z_i is a vector of explanatory variables that affect the non-wage benefits. Equation (2.10) is also called as "switching equation." Substituting equations (2.3), (2.4), and (2.10) into (2.9) yields (2.11):

$$I_i = X_{1i}\beta_1 - X_{2i}\beta_2 + \gamma Z_i + \eta_i = \beta Y_i + \eta_i , \qquad (2.11)$$

where $\beta Y_{\mathrm{i}} = \beta_1 X_{1i} - \beta_2 X_{2i} + \gamma Z_i$ and $\eta_i = \mu_{1i} - \mu_{2i} + u_i$. Assuming that η_i is a standard normal random variable, then an individual *i* will choose the public sector with the following probability

$$\Pr\left(D_i = 1\right) = \Pr\left(I_i > 0\right) = \Pr\left(\beta Y_i > -\eta_i\right) = F\left(\beta Y_i\right). \qquad (2.12)$$

The probability that the individual chooses the private sector is

$$\Pr\left(D_i = 0\right) = \Pr\left(I_i < 0\right) = \Pr\left(\beta Y_i < -\eta_i\right) = 1 - F\left(\beta Y_i\right). \qquad (2.13)$$

Theoretically, it is possible to obtain unbiased estimates of β by running full information maximum likelihood estimations of (2.11).[6] The log-likelihood function can be written as

$$L = \sum_i D_i \ln F\left(\beta Y_i\right) + \sum_i \left(1 - D_i\right) \ln\left(1 - F\left(\beta Y_i\right)\right). \qquad (2.14)$$

See, e.g., Postel-Vinay and Turon (2007), van Ophem (1993), Hartog and Oosterbeek (1993), Dustmann and van Soest (1998), Terrell (1993), Hou (1993), Adamechik and Bedi (2000), Van

[6] In general ML produces consistent estimators. In some cases, they may be biased in small samples, however.

der Gaag and Vijverberg (1988), Christofides and Pashardes (2002), Glinskaya and Lokshin (2005), Venti (1985), Heitmueller (2004), and Casero & Seshan (2006). [7]

2.4 Wage differentials in developed countries

In developed countries, we observe a diversity of results regarding wage differentials. Mueller (1998) refers to Canada for the year 1990 using hourly wages from the Labor Market Activity Survey (LMAS). He finds that the wage differential is formed in favor of the public sector. Indicatively, the public-sector wage premium for males is 2.7% and for females is 8%, respectively. Mueller attributes the wage premium to three factors. First, unions in Canada are more pervasive in the public sector, and this could put upward pressure on wages. Second, the competition between firms in the private sector pushes wages downward. Third, the inelastic demand for government services implies inelastic labor demand, hence the increase in wage can be passed on to consumers.

Venti (1985) uses hourly wages from Current Population Survey (CPS) of the United States (U.S.) in 1982. He indicates that males have a slight wage advantage up to 4.2% and females up to 22.1% in the public sector compared to the private sector. A more recent research is conducted by Borjas (2002), who uses data from the U.S. decennial Censuses and from CPS, for the period 1960-2000. According to his empirical analysis, high skilled private sector workers became less likely to quit their jobs in order to enter the public sector, and high skilled public sector workers are increasingly more likely to switch sector. He attributes this phenomenon to the relative downward pressure on public-sector wages since 1970.

Disney and Gosling (1998) use weekly data for Great Britain for two different periods. Those that referred to 1983 are from the General Household Survey (GHS), and those that referred to the period 1991-1995 are from the British Household Survey (BHPS). Controlling for individual-specific differences (education and age) the public average wage gain for females is approximately 11% and for males is 5% in 1983. In the early 1990s the rates are 14% and 1%, respectively. Trade unions maintain almost complete coverage of the public sector work-force, achieving better working conditions for their members, in contrast to their free-fall decline in the private sector.

[7] Note that researchers may use more than one of the methods of estimation that I describe in this section. From the method that they use I refer to the most "complete" one.

Postel-Vinay and Turon (2007) detect, also, positive public wage premium, both in the present income flows and in the discounted future income flows for the next period 1996-2003 in Great Britain. They use monthly data only for males from the BHPS. Mean public wages tend to be higher than the mean private wages for low and high income classes about 13% and 1%, respectively. For the medium income class, the opposite is true (4% in favor of the private sector) in the first half of their careers, and becomes virtually equal to zero afterwards. Looking at the whole sample, the average public premium is 3.4%.

Heitmueller (2004) conducts independent analysis for Scotland and Wales. He uses data from the BHPS of the year 2000. According to his empirical results, the wage gap between the public and the private sector is 10% for males and 24% for females. These estimates are similar to those of Rees and Shah (1995) for UK.

Dell' Aringa, et al. (2007) investigate public-private wage differentials in Italy by geographical location. They use data (hourly wages) from the Bank of Italy for the decade 1991-2002. According to their empirical estimates, in most of the northern regions public-private wage differentials come are 10% and are above 15% in almost all southern regions reaching 20-25%. In terms of climate and cultural background, northern Italy is often considered more similar to continental Europe (rich north), while southern regions share more features with other Mediterranean countries (poor south). Thus, southern Italy is characterized by high unemployment rates and larger shares of public employment. These differences imply different types of private occupation in both regions. Wage bargaining takes place in national and firm specific level since 1993 in both sectors, though the degree of coordination in the public sector is much stronger.

An alternative case is given by Pederson et al. (1990) that concerns Denmark, where wage differentials are in favour of the private sector. They use annual wage income for the period 1976-1985, divided by the working hours. The data set is described in detail in Westergard-Nielsen (1984). According to their empirical results, for both sexes, there are approximately no wage differentials in 1976. After 1976, private sector wages increased more than those in the public sector for both sexes. Hence, wage differentials are formed in favour of the private sector and are about 11.03% for males and 4.34% for females. This result reflects government's attempt to moderate the simultaneous problems with balance of payments deficits and high unemployment, by cutting public sector budget deficits and expanding private sector employment. As a result, public-sector wages decreased. Moreover, public sector pay scale is more rigid than that in the private sector, and, consequently, less flexible in rounds of collective bargaining or other types of wage adjustments.

Hartog and Oosterbeek (1993) analyze the wage structure of both sectors of the Netherlands in 1983. They include in their investigation hourly wages. The data bases is constructed and updated through the services of municipal administrations of population. According to their findings, wages in the public sector are 3.1% higher than those in the private sector. Although wage differentials are the same for both sexes, females are worse off than males in both sectors. In the Netherlands, until 1982 public and private sector wages were linked. In 1982, due to the necessity of diminishing the government deficit, this linkage was abandoned. Since then public-sector wages have fallen behind. Hans van Ophem (1993) uses data (hourly wages) from the Netherlands in 1986 that were obtained from the Dutch OSA-labor market Survey. His general conclusion is that wages for males are 6.6% higher in the private sector than in the public sector. For females wage differentials are 4% in favor of the public sector.

Dustmann and Van Soest (1997) use data (hourly wages) from German Socio – Economic Panel for the years 1984-1993 to analyze public-private wage differentials for males and females. According to their empirical results, for males, conditional wages on age, marital status, and education are 6.5% lower in the public sector than in the private sector. On the other hand, for females, the wages are 10.6% higher in the public sector than that in the private sector. Later estimation of Dustmann and Van Soest (1998) indicates that wage differentials are in favor of the private sector for male employees. Wage differentials are largest for lower education levels and smallest for the higher levels. The rates are from 4.34% to 38.10%.

A recent research for Germany has been undertaken by Blaise (2005). She uses data (hourly wages) from the GSOEP for the years 1984-2001. According to her empirical estimates, wages in the public sector are about 6.9% lower than wages in the private sector for males. Female public-sector employees earn 8.2% more than private sector employees. The public sector in Germany plays a significant role as an employer, as one out of five individuals in dependent employment work in the public sector. Wage bargaining in Germany is highly centralized, with wage negotiations covering the whole scope of public-sector employment. The rights of private-sector workers are also regulated by negotiated agreements between unions and employers' organizations. In another study, Blanchflower (1996) presents evidence of a public sector premium in 12 OECD countries, such as Germany, Australia, Austria, Canada, Ireland, Italy, Japan, Netherlands, New Zealand, Spain, United Kingdom, and United States.

For developed countries we note four concluding remarks. First, in many developed countries, where the public sector is the dominant employer, wage differentials are formed in favor of the public sector. In countries where the public sector is restricted due to privatizations and the private sector is the dominant employer, wage differentials are formed in favor of the private sector. Second, in many countries, the public sector pushes downward wages, due to fiscal constraints in order to decrease unemployment. Third, unions are characterized as "upcoming force" in wage setting process, defending employees' rights of both sectors. Fourth, in most developed countries females do best in the public sector, despite the decreasing tendency of public-sector wages. The point is that in the private sector, females suffer from gender discrimination, as they cannot retain their job during maternity, in sharp contrast to the public sector. Furthermore, in the private sector, females are crowded in lower ranks of the job hierarchy. In the public sector, there is no overrepresentation of females at the bottom of the job hierarchy, but females suffer from career stop at middle management positions. A representative example, which is referred to gender wage differentials are given by Zweimuller and Winter-Ember (1994). Their research refers to Austria in 1983. Table 1 summarizes all the results for developed countries.

2.5 Wage differentials in developing countries

With respect to developing countries, there is also a great effort to explore public-private wage differentials. Van der Gaag and Vijverberg (1988) compare public-private wages of Cote d'Ivoire. The data come from the Cote d' Ivoire Living Standards Survey (CILSS) for the year 1985 and concern hourly wage rates. According to their empirical results, the need for workers with higher education is higher in the public than in the private sector. Private sector offers wages that are higher for low educational levels, but lower for high educational levels than in the public sector. For an average Ivorian employee, choosing to work in the public sector would earn 12% more than the private sector counterpart. In Cote d'Ivoire 64.2% of public sector workers are union workers. For the private sector this rate is 39.5%.

Terrell (1993) uses data from Haiti for the year 1987. According to her empirical results, the mean wage per hour in the public sector is 37% above the average wage in the private sector. There are no gender wage differentials and no effective labor unions in either sector. Government's policy toward hiring and wage setting in the public sector is influenced by political goals and government's aim as far as the level of employment is concerned. In the private sector government's intervention is confined to the setting of the minimum wage, which is far below the public-sector wage.

Table 1. Developed countries				
Author	**Country/ earnings**	**Method of estimation**	**Year**	**Public – private wage differentials** $(w_1 - w_2)$
Mueller (1998)	**Canada**/ hourly	2nd method	1990	Males: 2.7% Females: 8%
Venti (1985)	**USA**/ hourly	3rd method	1982	Males: 4.2% Females: 22.1%
Disney & Gosling (1998)	**Great Britain**/ weekly	1st method	1991-95	Males: 1% Females: 14%
Postel-Vinay & Turon (2007)	**Great Britain**/ monthly	3rd method	1996- 2003	Males: 3.4%
Heitmueller (2004)	**Scotland**/ hourly	3rd method	2000	Males: 10% Females: 24%
Dell' Aringa et al. (2007)	**Italy**/ hourly	1st method	1991-2002	North: 10% South: 15%-25%
Pedersen et al. (1990)	**Denmark**/ hourly	1st method	1976-1985	Males: -11.03% Females: -4.34%
Hartog & Oosterbeek (1993)	**Netherlands**/ hourly	3rd method	1983	Males: 3.1% Females: 3.1%
Hans van Ophem (1993)	**Netherlands**/ hourly	3rd method	1986	Males: -6.6% Females: 4%
Dustmann & Van Soest (1997)	**Germany**/ monthly/ working hours	1st method	1984 -1993	Males: -6.5% Females: 10.6%
Dustmann & Van Soest (1998)	**Germany**/ hourly	3rd method	1984 -1993	Males: -4.34%-(-38.10%)
Blaise (2005)	**Germany**/ hourly	2nd method	1984 - 2001	Males: -6.9% Females: 8.2%

Hou (1993) derives data from the Human Resource Utilization Survey (HRUS) in Taiwan, in order to examine public-private wage differentials for the period 1978-1985. Wages are hourly. According to his empirical results, there is a wage advantage for both genders in the public sector. Public-sector male employees enjoy roughly a 9.7% and a 16.4% wage premium over their private-sector counterparts. In Taiwan due to industrialization, unions are not effective.

Christofides and Pashardes (2002) use data (weekly earnings) from the Cyprus Household Expenditure and Income Survey (CHEIS) for 1990-1991 to deal with public-private wage differentials in Cyprus. According to their empirical findings wage differentials are in favor of the public sector, the pure rent advantage being 7%. Note that for Cypriot females there is a strong tendency to withdraw from the labor market for family purposes. Since 1977, wages and terms of employment are determined through collective bargaining between unions and specific employers or industries, and the government negotiates wages with the union of public-sector employees.

Adamchik and Bedi (2000) use data on net monthly earnings for Poland of the year 1996 published by the Polish Central Statistical Office. They find a 7% earnings advantage for an average male private-sector worker, compared to the public-sector workers. For females, this difference is 10%, in favor of the private sector. In Poland, after the 1990s, a rapid emergence and growth of the private sector has been taking place. Wages are higher in the private sector for two reasons. First, the public sector, in order to minimize unemployment and consequently social turbulence push wages down, given budget deficit restraint. Second, private enterprises pay efficient wages to discourage workers from shirking or to compensate for lower job security and non-wage benefits.

In another study, Panizza and Zhen-Wei Qiang (2005) use panel data of household surveys to investigate the wage differential between the public and the private sector in 13 Latin American countries. They obtained the data from the Inter-American Development Bank, where are hourly. According to their empirical results, in the majority of Latin American countries, there is a premium associated with work in the public sector. They implement all the methods described in section 2.3. Table 2 summarizes the results for Latin America. Notice the first and the second method give the same sign for the wage differential, but based on the second method the magnitude is smaller. Based on the 3rd method, there are some countries for which public-private wage differentials depend on the choice of sector as well as on the choice of the wage equation used.

Summarizing the results for developing countries, note three things. First the flimsy presence of unions in the wage setting process, contrary to developed countries. Second the tenuous participation of females in the labour force in most countries, where females do not constitute a different category. And finally, wage differentials are formed in favour of the public sector in the majority of developing countries. Table 3 summarizes the results for developing countries.

Table 2. Panel data from 13 Latin American countries
Panizza & Zhen-Wei Qiang (2005)
Public-Private wage differentials

Country	Year	1st Method		2nd Method		3rd Method	
						Does controlling for selection alter the result?	
		Males	Females	Males	Females	Males	Females
Bolivia	1997	-18%	-20%	-7%	-9%	Yes	No
Brazil	1995	28%	19%	6%	5%	Yes	Yes
Colombia	1997	21%	25%	14%	16%	No	No
Costa Rica	1995	13%	14%	6%	4%	No	Yes
Ecuador	1995	11%	0.8%	6%	-2%	No	Yes
Honduras	1996	1.1%	28%	1%	12%	yes	Yes
Mexico	1994	-0.7%	11%	6%	-2%	No	Yes
Nicaragua	1993	-9%	-13%	-6%	-10%	No	No
Panama	1995	-24%	2.2%	-9%	1%	No	Yes
Paraguay	1995	4.5%	14%	1%	5%	No	Yes
Peru	1997	-0.7%	-0.6%	-5%	-5%	No	Yes
El Salvador	1995	18%	27%	8%	12%	No	No
Venezuela	1995	-4%	-0.3%	-3%	0%	No	yes

2.6 Wage differentials in poor countries

Poor countries are characterized by feeble market forces that result in inefficiencies in allocation of human resources, which has a negative impact on the pace of economic growth. Another striking feature is the relative scarcity of educated workers and human capital. Lindauer and Sabot (1983) focus on public-private wage differentials in urban Tanzania in 1971. They use data (monthly earnings) from the National Urban Mobility Employment and Income Survey of Tanzania (NUMEIST). After accounting for the differences in characteristics such as education, experience, and age, they find that the representative government worker earns 14% more than its private-sector counterpart. In Tanzania multinational companies are active but not labor unions.

Table 3. Developing Countries				
Author	**Country/ earnings**	**Method of estimation**	**Year**	**Public-private wage differentials** $(w_1 - w_2)$
Van der Gaag & Vijverberg (1988)	**Cote d'Ivoire**/ hourly	3rd method	1985	12%
Terrell (1993)	**Haiti**/ hourly	3rd method	1987	37%
Hou (1993)	**Taiwan**/ hourly	3rd method	1978-1985	Males: 9.7% Females: 16.4%
Christofides & Pashardes (2002)	**Cyprus**/ weekly	3rd method	1990/1991	7%
Adamchik & Bedi (2000)	**Poland**/ monthly	3rd method	1996	Males: -7% Females: -10%

Nielsen and Rosholm (2001) analyze public-private wage differentials in Zambia, for the period 1991-1996 using monthly earnings data. According to their empirical results wages, are generally higher in the public sector than in the private sector. The public sector premium is close to 15% on average although the returns to education are higher in the private than in the public sector. The public sector in Zambia is characterized as a non-competitive market, contrary to the private sector that is characterized as a competitive market. Thus, in the public sector, skills are not rewarded according to the market price. Zambia is a very poor country, where 84% of the population lives below the official poverty line. In addition, it is a severely indebted country, with an external debt that equaled to 204% of GDP in 1996. Zambia is characterized as a transforming economy and the main target is the large and inefficient public sector. The most noticeable change during the sample period is that the public sector shrinked on account of privatization, from 68% in 1991 to 50% in 1996.

The review of Glinskaya and Lokshin (2005) about public-private wage differentials shows that India has one of the largest differentials in favor of the public sector. On average, the public sector premium ranges from 62% to 102% over the private sector wage. The wage differential in India tends to be higher in rural than in urban areas and is higher among females than among males. They obtained the data from the National Sampling Survey Organization (NSSO). They use weekly wages. India is predominantly a rural country and the public sector accounts for almost 70% of organized employment in 2000. Since 1947, in both sectors workers' unions and the management of each organization settle on wages through a collective bargaining process. Glinskaya and Lokshin note that in India there is also an informal private sector, where wages are even lower and unions are non-existent.

Casero and Seshan (2006) refer to Djibouti, using data (monthly earnings) for the year 1996. They find that public-sector employees earn a 21% wage premium over their private sector counterparts. Public-sector employees are more likely to be males and have parents in the public sector. The positive rent earned in the public sector is likely to be due to the legacy of the French colonial rule, which paid high wages to the public sector rewarding the loyalty of the elite class. With the term "elite class" the authors refer to the educated labor force that lack viable job opportunities in the private sector. Djibouti is characterized as a low income country, where the public sector employs a disproportionate amount of the formal labor force, compared to the private sector. The rates are 56.4% and 17.6%, respectively. Table 4 summarizes the results for poor countries. The main concluding remark is that the public sector pays higher wages than does the private sector in all poor countries.

Table 4. Poor Countries

Author	**Country/ earnings**	**Method of estimation**	**Year**	**Public-private wage differentials** $(w_1 - w_2)$
Lindauer & Sabot (1983)	**Tanzania**/ monthly	2^{nd} method	1971	14%
Nielsen & Rosholm (2001)	**Zambia**/ monthly	1^{st} method	1991-1996	15%
Glinskaya & Lokshin (2005)	**India**/ weekly	3^{rd} method	1993-1994 & 1999-2000	62-102%
Casero & Seshan (2006)	**Djibouti**/ monthly	3^{rd} method	1996	21%

2.7 Summary

This survey of the literature leads to the conclusion that wage differentials between the public and the private sector are a global phenomenon in that in the majority of countries the public sector pays higher wages than does the private sector. The wage differential is attributed to different maximization problems that the two sectors face. Unions play a significant role in the wage setting process in developed and in many developing countries. In developed countries, wage differentials are more intent for females, as they suffer gender discrimination in the private sector. In developing and poor countries, wage differentials are not tested separately for genders, because females are not a representative sample of the workforce.

Chapter 3

A unionized mixed oligopoly model with stochastic demand shocks: public-private wage differentials and "eurosclerosis" reconsidered

3.1 Introduction

The aim of this chapter is to explain, theoretically, the wage differential, which is often evident among equally productive employees, in the public versus the private sector. Thus, I develop a unionized mixed oligopoly model with stochastic demand shocks, proposing that such public-private wage differentials may endogenously emerge whenever the public and the private firms face an asymmetric firing restrictions regime.[8] In particular, I suggest that such wage differentials are always in favor of the public firm's employees and against private firm's employees in equilibrium. Moreover, I show that the public-private wage differential increases as the asymmetry in the firing restriction regime between the two sectors increases. Each alternative firing restrictions regime is institutionalized provided that it receives the approval of the majority of the participants of the game.

The rest of this chapter is organized as follows. In section 3.2, I address my structural model and the game arising in this context. In section 3.3, I investigate public-private wage differentials under the scope of alternative firing restriction regimes. In section 3.4, I conclude.

3.2 The model

I consider a two period mixed oligopoly model with stochastic demand shocks, where two firms producing differentiated goods compete in quantities. One firm, the "public firm" (f_1), represents the public sector, while the other, referred to as the "private firm" (f_2) represents the private sector. Each firm produces with constant returns to scale

[8] Regarding private firms, firing restrictions in the form of state-mandated redundancy payments and/or maximum permissible layoffs (as a percentage of firm-specific total employment), were introduced from the late 50s to the early 70s, and are still present in many European countries [see, e.g., Bentolila and Bertola (1990), Lazear (1990), Layard, Nickell and Jackman (1991), Cook (1997), Nickell (1998), Nickell, Nunziata and Quintini (2001)]. As regards public firms, of course, firing restrictions are strongly evident through the various employee tenure schemes which are prevalent in the public sector.

($C.R.S.$) in only the labor input, given that the deployed capital input is always sufficient to produce the good; effectively, each firm possesses a *Leontief* technology. Thus, the production function of firm $j = 1, 2,$ is $q_j = kL_j$, where q_j denotes output, L_j is the number of employees of firm j, and $k > 0$ represents the labor productivity, assumed to be symmetric across firms. For simplicity, I normalize setting $k \equiv 1$.

Competition in the product market takes place *a la Cournot*, during two consecutive periods $t \in [0,1]$, with firms independently choosing their own outputs in each period. Consumer preferences in each period are represented by a Dixit (1979) quasi-linear specification, hence, the first period (t=0) the public and the private-firm inverse demand functions respectively are,

$$p_{10} = 1 - q_{10} - gq_{20} \quad (3.1a)$$

$$p_{20} = 1 - q_{20} - gq_{10}, \quad (3.1b)$$

where g, $0 \le g \le 1$, is a measure of product differentiation. As $g \to 1 (\to 0)$, the public and the private firm's products become more (less) close substitutes, while if $g = 1(0)$ these products are homogenous (independent). In the first period (t=0) demand is observed, but there is uncertainty about future (e.g., second period) demand. Let, hence, the public and the private firm's expected demand function(s), in the second period (t=1), respectively be,

$$p_{11} = v - q_{11} - gq_{21}, \quad (3.2a)$$

$$p_{21} = v - q_{21} - gq_{11}, \quad (3.2b)$$

where $v = \rho(1+\theta) + (1-\rho)(1-\theta)$. The term v means that a random shock X takes on the following two possible values: θ, is a positive demand shock, and $-\theta$, which is a negative demand shock, with probabilities ρ and $(1-\rho)$ respectively. Thus, the expected value of X is $E(X) = \theta(2\rho - 1)$ and the variance of X is $Var(X) = 4(1-\rho)\rho\theta^2$. I moreover assume a discount factor $\delta^t = 1$. The structure and the timing of my postulated game is then as follows.

At stage one, first period (t=0), the policy maker (*PM*) evaluates the performance of the alternative firing restrictions regimes (frr_x), deciding to activate or not frr_x. The regime frr_x is activated, provided that the majority of the participants of the game approve it.

At stage two, first period (t=0), the public firm and the public firm's labor union, $f_1 | U_1$, as well as the private firm and the private firm's labor union, $f_1 | U_1$, bargain independently and simultaneously about the firm-specific wage, w_{10} and w_{20}, respectively, leaving the firm-specific output/employment decision(s), for both the first and the second period, to each firm's discretion at the subsequent stages of the game. I assume that each union possesses all the bargaining power over the wage, e.g. it behaves as a firm-specific monopoly union which unilaterally sets the firm-specific wage.[9] Moreover, I suppose that unions are identical and are endowed with the same bargaining power, *b*, during negotiations with their firms. Furthermore, I assume that union workers are identical and the individual utility is linear in wages. Therefore, a utilitarian objective function is $U = Lu(w)$, where L is the number of employees and $u(w)$ is utility of income, consequently, I assume that $U_j = (q_{j0} + q_{j1}) w_{j0}$, see Rosen (1969) and Dunlop (1944).[10]

At stage three, first period (t=0), given the firm-specific wage, each firm decides upon (and carries out) its optimal level of employment and output, in the current period, for any (current) employment/output level of its rival firm. At stage four, second period (t=1), firms compete *a la* Cournot, like in the previous period; at this stage, of course, the "current" period is the second period. **Figure 3.1** illustrates the game tree. I solve the game by backward induction. The first aim of this thesis is to investigate whether public-private wage differentials emerge, in the subgame perfect equilibrium, under the alternative firing restrictions regimes.[11]

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[9] That is, for analytical convenience, we undertake the *monopoly union* variant of the *right-to-manage* hypothesis. This is a regular restriction in the union-oligopoly literature, and it is not expected to qualitatively affect our analysis [see Petrakis and Vlassis (2004)].

[10] Dertouzos and Pencavel (1981) examine whether the particular purposive model that is common in the literature provides a viable framework for analyzing union behavior. See also Oswald (1982) for an examination of the structure and predictions of a simple microeconomic model of the trade union.

[11] Note that the first period's actions are taken with certainty (uncertainty) about the product demand in the first (second) period; while, the second period's actions are taken with certainty (uncertainty is resolved at t=1). Nonetheless, subgame perfection ensures that at each stage of the game each agent takes into account the consequences of his actions on the subsequent stages of the game. Hence, at stages two and three, and given the *frr*, both firms and unions "discount" (e.g., bring to the present, t=0) the uncertain demand of stage four (t=1), with $\delta^{t(=1)}$=1.

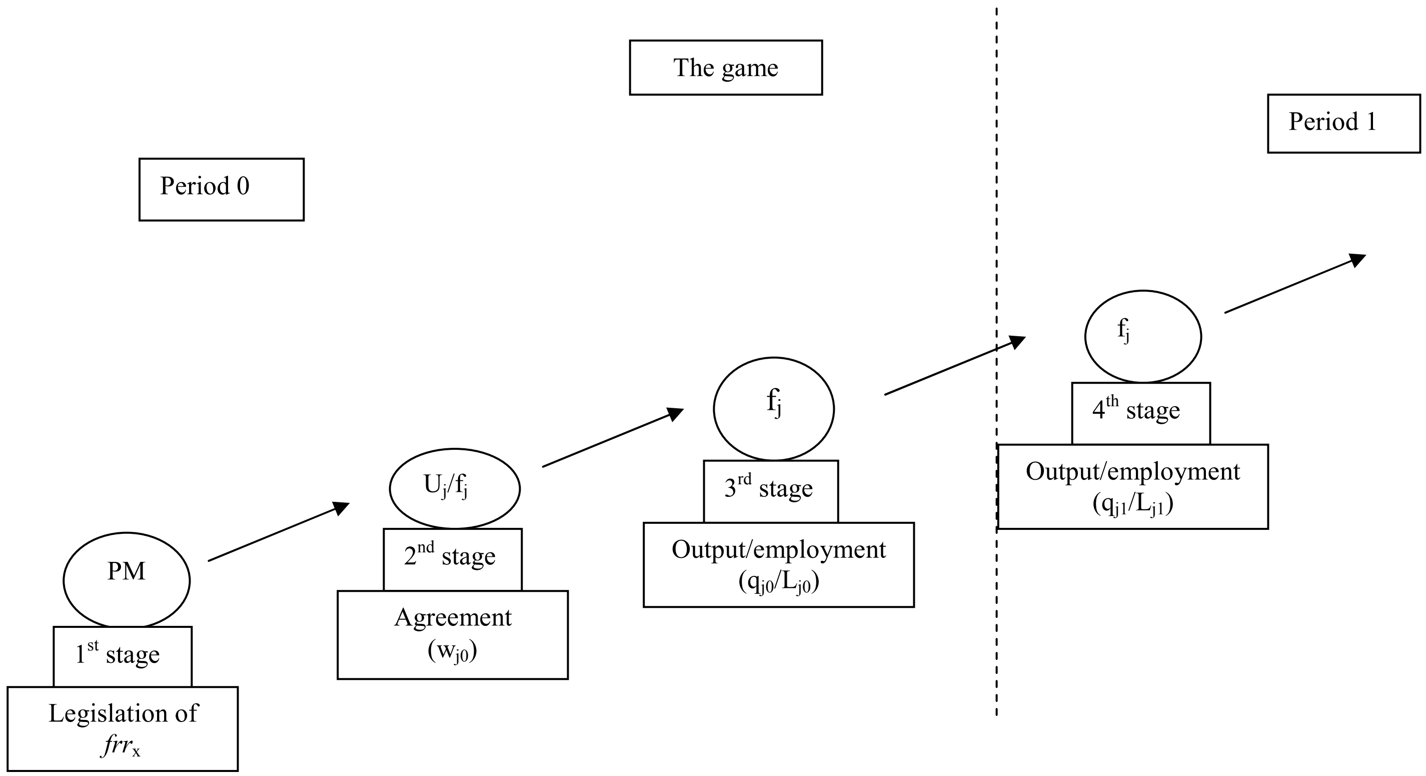

Figure 3.1: The game tree of the model

3.3 Endogenous public-private wage differentials

In this section, I investigate the conditions under which alternative firing restriction regimes may lead to public-private wage differentials of various magnitudes in equilibrium. For that, assuming a particular frr_{X} to be the (equilibrium) outcome at stage one, I solve stages two to four to get the firm-specific wages as well as the employment/output levels, for both the public and the private firm, in equilibrium. I consider three alternative frr_{X}. This section is organized in three paragraphs as follows. Firstly, in subsection 3.3.1, I consider the case frr_1, according to which a strict firing restrictions regime is imposed on the public sector and lenient on the private sector. Secondly, in subsection 3.3.2, I consider the case frr_2, according to which a strict firing restrictions regime is imposed only on the public sector. Finally, in subsection 3.3.3, I consider the case frr_3, according to which a strict firing restriction regime is imposed on both sectors.

3.3.1 Strict firing restrictions in the public sector/ lenient firing restrictions in the private sector (frr_1)

Assume that the *PM*'s institutional choice frr_1 is defined by the following- inter-temporal (e.g., period 2-period 1) employment adjustment cost schedule $\left(AdjC_j\right)$ for the public and the private firm, respectively:

$$AdjC_1 = c\left[\frac{\left(q_{10}-q_{11}\right)^2}{2} - \left(q_{11}-q_{10}\right)\right], \tag{3.3}$$

$$AdjC_2 = c\left[\frac{\left(q_{20}-q_{21}\right)^2}{2}\right], \tag{3.4}$$

where the $c > 0$ parameter reflects the state-mandated cost of adjusting employment, assumed to be symmetric across firms, yet, given the firm-specific *AdjC* rule dictated by the particular *frr* (here, frr_1). Then, the public and the private firm's total cost schedule(s) over the two-period game respectively are

$$TC_1 = w_{10}\left(q_{10}+q_{11}\right)+c\left[\frac{\left(q_{10}-q_{11}\right)^2}{2}-\left(q_{11}-q_{10}\right)\right], \tag{3.5}$$

$$TC_2 = w_{20}\left(q_{20}+q_{21}\right)+c\left[\frac{\left(q_{20}-q_{21}\right)^2}{2}\right], \tag{3.6}$$

where, w_{j0} is the firm-specific wage, bargained once (at stage two, first period) for the entire (two-period) game.[12] Under frr_1, the public firm faces stricter firing restrictions relative to the private firm. To grasp it, suppose that in the second period $(t=1)$ both firms' employment/output, q_{j1}, is lower than that of the first period $(t=0)$, q_{j0}. Then, through the quadratic term, $c\frac{(q_{j0}-q_{j1})^2}{2}$, both firms suffer extra -above unit costs of production- employment adjustment (firing) costs.[13] However, under the present firing restrictions regime, the public firm suffers additional, over its rival private firm, employment adjustment costs, which are moreover increasing with $[q_{11}-q_{10}]$; the opposite is true if q_{11} is higher than q_{10}.[14] This is the novelty of my analysis, capturing the idea of an asymmetric firing restrictions regime accrediting to public firms the role of protecting employment during the business cycle. The public and the private firm's profit(s) over the two period game are subsequently defined by

$$\Pi_1\left(q_{10},q_{20},q_{11},q_{21},w_{10}\right)=TR_{10}+TR_{11}-TC_1, \tag{3.7}$$

$$\Pi_2\left(q_{10},q_{20},q_{11},q_{21},w_{20}\right)=TR_{20}+TR_{21}-TC_2, \tag{3.8}$$

where $TR_{10}=p_{10}q_{10}$ $\left(TR_{11}=p_{11}q_{11}\right)$ are the public firm's first (second) period total revenues, and $TR_{20}=p_{20}q_{20}$ $\left(TR_{21}=p_{21}q_{21}\right)$ respectively are the private firm's first

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[12] Throughout the present analysis I maintain the assumption that wages are not re-bargained after demand uncertainty is resolved (in the second period).

[13] Note that, under specifications (3.3) and (3.4), and as regards the quadratic term, the same happens in the reverse case, $q_{j1}>q_{j0}$, the reasoning being that firms also face state-mandated hiring costs. Yet, (3.4) is a standard in the literature specification for the employment adjustment costs [see e.g., Hamermesh (1996)].

[14] That is, as $q_{j1}>q_{j0}$, the public firm's employment adjustment costs, hence, its total costs, become (increasingly) lower than the counterparts of its (private) rival firm. Of course, if $q_{j1}=q_{j0}$, then the firing/hiring restrictions are not operative, thus, no extra (above labor) costs occur for either firm.

(second) period total revenues. Analytically, profits take the form given by equations (3.9) and (3.10) below. The public and the private firm maximize their profits by chosing q_{11}, q_{21}, respectively, in the second period/fourth stage

$$\underset{q_{11}}{Max}\Pi_1\left(q_{10},q_{20},q_{11},q_{21},w_{10}\right)=-c\left(q_{10}-q_{11}+\frac{1}{2}\left(q_{10}-q_{11}\right)^2\right)+q_{10}\left(1-q_{10}-gq_{20}\right)-\left(q_{10}+q_{11}\right)w_{10}+q_{11}\left(v-q_{11}-gq_{21}\right), \quad (3.9)$$

$$\underset{q_{21}}{Max}\Pi_2\left(q_{10},q_{20},q_{11},q_{21},w_{20}\right)=-\frac{1}{2}c\left(q_{20}-q_{21}\right)^2+q_{20}\left(1-gq_{10}-q_{20}\right)-\left(q_{20}+q_{21}\right)w_{20}+q_{21}\left(v-gq_{11}-q_{21}\right). \quad (3.10)$$

Recall that, at any stage of the game, both firms act independently so as to maximize their own profits for the entire game (see, e.g., footnote 4). Therefore, inducting backwards, from the *foc_s* of (3.9) and (3.10) *w.r.t.* q_{11}, q_{21} respectively, I first get the following second period/fourth stage reaction functions for the public and the private firm:

$$RF_{11}\left(q_{21}\right)=\frac{\left(v+c\right)+cq_{10}-gq_{21}-w_{10}}{2+c}, \quad (3.11)$$

$$RF_{21}\left(q_{11}\right)=\frac{v+cq_{20}-gq_{11}-w_{20}}{2+c}. \quad (3.12)$$

Note that, regarding their employment/output level(s) in the second period $\left(q_{j1}\right)$, the public and the private firm are equally responsive to changes in the rival's current (second period) output; $-g/(2+c)$, as well as to changes in their own past (first period) output; $c/(2+c)$. Quite interestingly, nonetheless, whatever is the realization of the demand shock (hence v), the public firm's maximum employment/output in the second period $\left(q_{11}|q_{21}=0\right)$ is, *other things being equal*, higher than the private firm's counterpart $\left(q_{21}|q_{11}=0\right)$, by $c/(2+c)$. Hence, it seems that the asymmetric firing restrictions regime considered (*frr*$_1$) renders a premium to the wage set by public firm's labor union over the private firm's wage contract. To explicitly check for that, however, let us first move to stage three. To get the first-period optimal employment/output rules, for the public and the private firm $(q_{10},\ q_{20})$ in the (sub-game perfect) equilibrium, I work as follows. Firstly, I solve the system of the second period reaction functions, (3.11) and (3.12), above, to get the optimal q_{11}^* and q_{21}^* - rules in the second period:

$$q_{11}^* = \frac{c^2(1+q_{10})-(g-2)v+c(2+2q_{10}-gq_{20}+v-w_{10})-2w_{10}+gw_{20}}{(2+c-g)(2+c+g)}, \tag{3.13}$$

$$q_{21}^* = \frac{c^2q_{20}-(g-2)v-c(g+gq_{10}-2q_{20}-v+w_{20})+gw_{10}-2w_{20}}{(2+c-g)(2+c+g)}. \tag{3.14}$$

Then, substituting the latter into (3.9) and (3.10), from the *foc$_s$ w.r.t.* q_{10}, q_{20}, of the derived $\Pi_1(q_{10},q_{20},w_{10},w_{20})$, $\Pi_2(q_{10},q_{20},w_{10},w_{20})$, which are given, respectively, by equations (3.A1.1) and (3.A1.2) in the appendix 3.A1, the first period reaction functions for the public and the private firm are derived, and are given by equations (3.15) and (3.16), respectively:

$$RF_{10}(q_{20}) = A + Bq_{20} + Cw_{10} + Dw_{20}, \tag{3.15}$$

$$RF_{20}(q_{10}) = E + Bq_{10} + Cw_{20} + Dw_{10}, \tag{3.16}$$

where, $A = -1 + \frac{3}{2+c} + \frac{3c^2(2+c)}{F} + \frac{c(2+c)(2+c-g)}{F}v > 0$;

$E = \frac{1}{2+c} - \frac{c^2(2+c)(g-1)}{F} + \frac{c(2+c)(2+c-g)}{F}v > 0$, and $E > A$;

$B = \frac{dRF_{10}}{dq_{20}} = \frac{dRF_{20}}{dq_{10}} = -g\left(\frac{1}{2+c} - c\left(\frac{1}{4+2c(3+c)-g^2} - \frac{1}{4+2c-g^2}\right)\right) < 0$;

$C = \frac{dRF_{10}}{dw_{10}} = \frac{dRF_{20}}{dw_{20}} = -\left(\frac{1}{2+c} - (1+c)\left(\frac{1}{4+2c(3+c)-g^2} - \frac{1}{4+2c-g^2}\right)\right) < 0$;

$D = \frac{dRF_{10}}{dw_{20}} = \frac{dRF_{20}}{dw_{10}} = \left(\frac{cg(2+c)}{(2+c)^2[4(1+c)-2g^2]+g^4}\right) > 0$;

$F = (2+c)^2(4+4c-2g^2)+g^4 > 0$.

Solving the system of the first period reaction functions (3.15) and (3.16), above, to get the optimal q_{10}^{*}, q_{20}^{*} rules of the first period, which are given by equations (3.A1.3) and (3.A1.4). Subsequently, by means of (3.11)-(3.12), through (3.15)-(3.16), I get the inter-temporal $q_j \to q_{j'}; j \neq j' = 1,2$, effects given by (3.17) below

$$\frac{\partial q_{j'1}}{\partial q_{j0}} = \frac{dRF_{j'1}}{dq_{j'0}} \frac{\partial q_{j'0}}{\partial q_{j0}} = -\frac{\text{cg}}{2+\text{c}} \left(\frac{1}{2+c} + c\left(\frac{1}{4+2c-g^2} - \frac{1}{4+2c(3+c)} \right) \right). \tag{3.17}$$

Equation (3.17) shows that, given $c > 0$, and $g > 0$, as the j firm's output in the first period (q_{j0}) *ceteris paribus* increases, the rival firm's (j') reaction function in the second period $(RF_{j'1})$ shifts to the left, along the (RF_{j1}). The new reaction functions are given by equations (3.18) and (3.19), respectively:

$$RF_{11}(q_{21})_{NEW} = \frac{(v+c) + c(q_{10}^{*} + B) - gq_{21} - w_{10}^{*}}{2+c}, \tag{3.18}$$

$$RF_{21}(q_{11})_{NEW} = \frac{v + c(q_{20}^{*} + B) - gq_{11} - w_{20}^{*}}{2+c}. \tag{3.19}$$

Hence, firm j''s output in the second period $(q_{j'1})$ would *ceteris paribus* decrease, irrespective of the nature and the magnitude of the stochastic demand shock. This in turn implies that, quite interestingly, the asymmetric firing restrictions considered at first instance render both firms a symmetric incentive to allocate their own production/employment level(s) across periods, given the certainty of the demand in the first period and the stochastic demand in the second period. To see this, recall that both firms, through the quadratic term in their $AdjC_j$, face symmetric employment adjustment costs whether they produce more, or less, in the second than in the first period. Hence, each firm's own products are complements across periods. Also, due to Cournot competition, products are strategic substitutes across firms in each period. This combination gives rise to a twofold strategic interdependence among firms over the two-period game.

On the one hand, each firm has a strategic incentive to over-produce (or under-produce) in the first period, so that the rival firm to under-produce (or over produce) in that period, and thus be locked into high (low) production in the second period.[15] Then, the former firm, by producing a low (high) output level in the second period, will minimize its employment adjustment costs.

On the other hand, however, such a strategy will ex-post prove to be optimal only if the emerging demand conditions in the second period are bad (good) enough relative to the first period. Nonetheless, the public firm, via the extra linear term in $AdjC_1$ is subject to strict firing restrictions and, for that reason, has an idiosyncratic incentive to opt for a higher own production mainly in the second than in the first period. Therefore, whatever the demand conditions turn out to be in the second period, the public firm should be expected to possess a stronger than the private firm incentive to under rather than over produce in the first period.

To explicitly check for the above in the equilibrium, let proceed to stage two (first period). Substituting q_{10}^*, q_{20}^* into (3.A1.1) and (3.A1.2), then profits depend on wages only and are given by (3.A1.5) and (3.A1.6), respectively in the appendix 3.A1; $\Pi_1\left(w_{10}, w_{20}\right)$, $\Pi_2\left(w_{10}, w_{20}\right)$. At this stage firm in the public (private) sector bargain with union that represents workers in the public (private) sector, about wages. The solution to the bargaining problem is given by the generalized Nash bargaining, Roth (1979).[16] The maximization problem in this case is $\max_{w_{j0}}\left(u_j\right)^b\left(\Pi_j\right)^{1-b}$. In the context of this survey I assume, for tractability, that union possesses all the bargaining power (b=1). Under decentralize right to manage bargaining, and having assumed firm-specific monopoly unions each union j, unilaterally and independently from the other union $j \neq j' = 1,2$, sets the firm specific wage, w_{j0}, taking into account how this decision will affect the competiveness of its own firm, hence, the union's employment prospects, in the

[15] Here the terms "over-produce" should be taken to compare a firm's own product under the presence of firing restrictions relative to the case when firing restrictions were absent.

[16] Roth (1979) proves that the static bargaining theory also considers asymmetric solutions. The set of asymmetric Nash solutions is obtained when the symmetry axiom is deleted from the axiomatization of the Nash solution. Modelers often use the asymmetric Nash solution in an attempt to capture the asymmetry bargaining power. See also Binmore, Rubinstein and Wolinsky (1986).

subsequent (two-period) product market game.[17] Thus, the wage bargaining expression (3.20) converted to (3.21):

$$w_{j0} = \max_{w_{j0}} B_i = \max_{w_{j0}} \left(\text{b Log}\left[\text{U}_j \right] + (1 \text{ - b}) \text{ Log}\left[\Pi_j \right] \right) = \quad (3.20)$$

$$w_{j0} = \max_{w_{j0}} U_j \left(= w_{j0} \left(q_{j0}^* + q_{j1}^* \right) \right), \ j = 1,2 \,. \quad (3.21)$$

Substituting q_{10}^*, q_{20}^* into (3.22), below, accrues U_j, $j = 1,2$ depending on wages only. Analytically, $U_1\left(w_{10}, w_{20}\right)$ and $U_2\left(w_{10}, w_{20}\right)$ are given by equations (3.A1.7) and (3.A1.8), respectively:

$$U_j = w_{j0} \left(q_{j0}^* + q_{j1}^* \right). \quad (3.22)$$

Solving the system accruing from the *foc_s* of (3.22) *w.r.t.* w_{j0} accrues a unique stable solution for the equilibrium firm-specific wage contracts w_{10}^*, w_{20}^*, respectively. The result about wages is given from the equations (3.A1.9) and (3.A1.10), respectively. Note that these wages are used in equations (3.11), (3.12), (3.18) and (3.19), about reaction functions, in order to construct **figure 3.2**. Note also that w_{j0} is replaced by w_{j0}^* in equations (3.A1.3) and (3.A1.4), respectively, about optimal quantities in the first period. Alternatively, q_{10}^*, q_{20}^* take the form given by (3.A1.11) and (3.A1.12). In other words in equations (3.11), (3.12), (3.18), and (3.19) $q_{11} \in [0,1]$ and $q_{21} \in [0,1]$, *ceteris paribus*. Diagrammatically, **figure 3.2**, below, illustrates the transposition of reactions functions of (3.11) to (3.18), and of (3.12) to (3.19), respectively. For simplicity I assume that products are perfect substitutes and c=0.3, v=1.5. In case where products are imperfect substitutes, then the transposition is even smaller.

[17] For tractability, we moreover assume that union members are risk-neutral, and we normalize to zero the (exogenous) reservation wage.

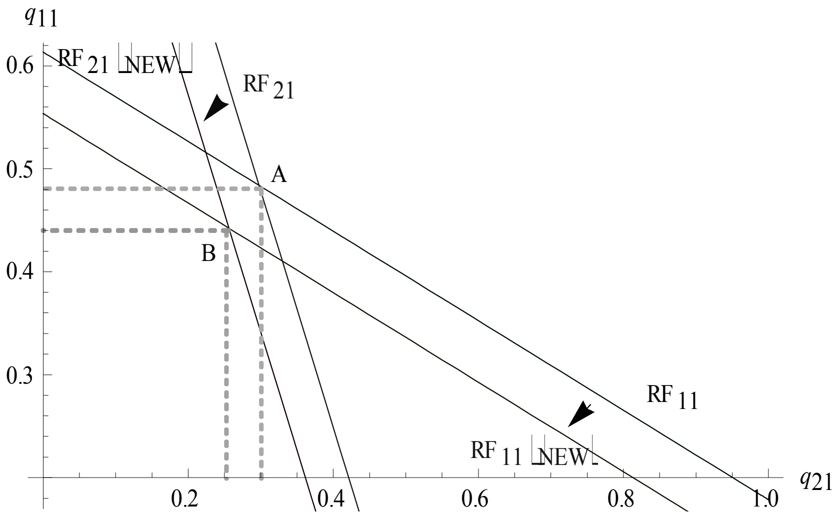

Figure 3.2: Transposition of RF_{11} and RF_{21} due to an increase in rival's product in the first period under frr_1

At the initial equilibrium point A, q_{11}^* and q_{21}^* are given by equations (3.A1.13) and (3.A1.14), which accrues replacing q_{10}^*, q_{20}^*, w_{10}^*, w_{20}^* into (3.13) and (3.14). The optimal values depend on c, g, and v. In the optimal point A, the maximum profits for firms are given by equations (3.A1.15) and (3.A1.16) respectively. The maximum utilities for unions are given by equations (3.A1.17) and (3.A1.18), respectively. The maximum consumer surplus is given by equation (3.A1.19). In this game it turns out (see equation (3.23) below) that the wage differential $\left(w_{10}^* - w_{20}^*\right)$, endogenously emerges in favor of the public firm:

$$w_{10}^* - w_{20}^* = \frac{c^2 g^2 F_1(g,c)}{F_2(g,c) + F_3(g,c)}, \qquad (3.23)$$

where, $F_1(g,c) = 4(1+c)(2+c)^2 + g\left(2\left((2+c)^3 - g^2(2+c)\right) - g^3\right) > 0$;

$F_2(g,c) = (1+c)^2(2+c)^4(32+8g) + 4g^4(2+c)\left(2(2+c)(3+c) + (3+2c)g\right) > 0$;

$F_3(g,c) = -2g^2\Big((2+c)^2\left(4\left(12+c\left(19+c(9+c)\right)\right) + \left(12+c(18+7c)\right)g\right) +$
$g^4\left((4+c)+g\right)\Big) < 0$;

note that $F_2(g,c) > F_3(g,c)$.

Note, also, that, if $g = 0$ and/ or $c = 0$, namely, if the firms' products are independent and/ or firing restrictions does not exist, then the differential is null. Moreover, as shown in **figure 3.3**, given $c(g)$, the higher the $g(c)$, the more intense the public- private wage differential. Note that *g* increases both wages decreases but w_{20} decreases more than w_{10}. Diagrammatically, there is a move to a higher contour line.

Figure 3.3:
Contour Plot
$(w_{10}^* - w_{20}^*)frr_1$,

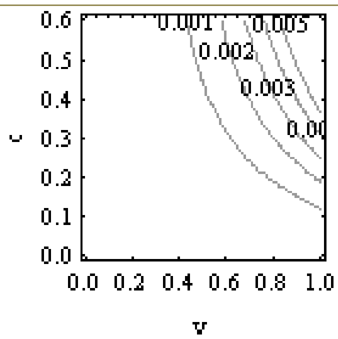

Figure 3.3: Wage differentials increase with c|g|; g|c| under *frr₁*

It is a necessity to make assumptions about *v* and *c*, in order to ensure interior solution, namely both firms to produce in each period. Technically, this means that products, prices, wages, and total costs must be positive. I consider the case where $v \in [0.01, 1.99]$, which means that there is high uncertainty about the demand shock, If *g*=1 then interior solution is ensured provided that $v > 0.32$ and $c \in (0, 0.4]$ and If *g*=0.5 then $c \in (0, 0.58]$. The TC_1 imposes the main restriction related to *c*. The restriction related to *v* is imposed by the private sector. I assume that the positive and the negative demand shock is of equal magnitude thus I restrict symmetrically the interval for v. A representative case for v is $0.5 \le v \le 1.5$, see in the appendix 3.B1 the **figures 3.B1.1-6**. Under these restrictions of v and c profits, utilities and consumer surplus are positive see **figures 3.B1.7-8**.

Private sector is benefited from supple character of its commitment under negative demand shock (Π2>Π1), producing more in the first period rather than in the second ($q_{20}>q_{21}$). In general, producing more than public sector in the first period and less in the second ($q_{20}>q_{10}$ and $q_{11}>q_{21}$). The total production of both firms is higher in the second period than in the first under positive demand shock $(tq_B > tq_A)$[18]. Under positive demand

[18] Where $tq_A=q_{10}+q_{20}$ and $tq_B=q_{11}+q_{21}$.

shock public sector is benefited from the stricter firing restriction regime that face ($\Pi1>\Pi2$), in this case produce higher quantity in the second period than in the first ($q_{11}>q_{10}$). In general, public sector conquer the market ($tq_1>tq_2$)[19]. Consequently, there are two influences in favor of utility of public sector employees against private sector, the first is on account on wage $\left(w_{10} > w_{20}\right)$, the second is due to products, thus U1>U2, see **figure**s **3.B1.9-10**. The results are similar, even the products are perfect substitutes or imperfect substitutes, see **figures 3.B1.11-12**.

In order to investigate the conditions under which firing restrictions regime are beneficial, I compare the proposal firing restrictions regime (*frr₁*) with the case where no firing restrictions regime are imposed (*frr₀*). Under *frr₀*, neither the public nor the private sector is engaged with firing restriction regime, thus $c = 0$. The game, then, is symmetric ($q_{10} = q_{20}$, $q_{11} = q_{21}$, $P_{10} = P_{20}$, $P_{11} = P_{21}$, $w_{10} = w_{20}$, $\Pi1 = \Pi2$, and $U1 = U2$). Note that the comparison between frr_1 and frr_0 is meaningful provided that interior solution is ensured in both cases. According to **figures 3.B1.13-14** interior solution is ensured under frr_0.

Consequently, the possible abolition of the adjustment cost results in increase in the public sector's production and in reduction in the private sector's production in the first period, since the products are complements across firms. In the second period, private sector's production is enhanced by the abolition of *c* and public sector's production is declined, since its firm own products are substitutes across periods, see **figures 3.B1.15-16**. Between periods, production increases more under frr_1 for the public sector and under frr_0 for the private sector, see **figures 3.B1.17.** The total production of each firm as well as the total production of the game is higher under frr_1 than frr_0. In the first period, both firms together produce more under frr_0 regime than frr_1, but in the second period the opposite happens, see figures **3.B1.18-19**.

Consumer surplus and U1 are generally higher under frr_1 than frr_0. The other yields are higher under frr_1 under restrictions. Firstly, profits for the public sector are higher under frr_1, provided that $v > 1$. For these values of the adjustment cost, private sector profits are higher under frr_0 than frr_1 and U2 is higher under frr_1 than frr_0, if *g*=1. If g=0.5 then U2 is always higher under frr_1 than frr_0, see **figures 3.B1.20-21**.

At the first stage of the game policy maker has to make a decision about the institutionalizion of the adjustment cost. This will happen provided that the adjustment cost

[19] Where $tq_1=q_{10}+q_{11}$ and $tq_2=q_{20}+q_{21}$.

(c) receives the approval of at least 3 of the participants of this game (firms and unions). The participants of this model will vote in favor of the proposal regime (*frr_1*) as long as its profits are increasing with *c*. Technically, if g=1, under *frr_1*, public sector's profits are increasing with *c*, provided that $c \in [0.15, 0.4]$, irrespective of *v*. For these values of *c* private sector's profits are also increasing with c provided that $v \leq 1.3$. U1 is also increasing with c, but U2 is decreasing with *c*, thus U2 does not support *frr_1*. U2's behavior is due to high level of the wage differential in favor of public sector thus there is a "leakage" of employees out of the private sector out towards the public sector, thus its utility is formed at extremely low levels. Consequently, there is a tacit agreement between f1, f2 and U1, and policy maker can institutionalize it. If g=0.5, then $c \in [0.2, 0.6]$ under these combinations of *v* and *c*, Π1, U1 and U2 are increasing simultaneously with *c*. Note that union that represent employees of the private sector has changed its attitude, because wage differentials are less intense and the massive leakage of employees is restrained. Π2 is also increasing with *c* provided that $v \leq 1$. Consequently, there is a tacit agreement between f1, U1 and U2. For these combinations of *v* and *c* consumer surplus is increasing also with *c* either g=1 or g=0.5, see **figures 3.B1.22-23**. Proposition 1 summarizes.

***Proposition 1**: Under frr_1 regime wage differentials are in favor of the public sector provided that both firms produce in both periods. Wage differentials are more intense in case where products are perfect substitutes. According to the reaction functions, products across firms are strategic substitutes and each firm own products are complements across periods. The frr_1 regime receives the approval of the majority of the participants of the game and this permits its institutionalization by the policy maker.*

In order to assess the impact of varX on variables and yields I separate two cases. The first one is under negative demand shock $\left(\rho = \rho_1 = \dfrac{\theta^2 - \sqrt{-(\operatorname{var} X)\theta^2 + \theta^4}}{2\theta^2} \right)$ and the second one is under positive demand shock $\left(\rho = \rho_2 = \dfrac{\theta^2 + \sqrt{-(\operatorname{var} X)\theta^2 + \theta^4}}{2\theta^2} \right)$. I make, also, the assumptions that θ=0.5 and that *c* takes the maximum permissible value under *frr_1*

as a representative cases. Thus under *frr_1*, I assume that if *g*=1 then *c*=0.4 and if *g*=0.5 then *c*=0.5.

We assume that $\rho \in [0,1]$. For $\rho \in [0.1, 0.5)$ the negative demand shock is more likely to occur, then $v \in [0.5, 1)$ and $\text{var}\, X \in [0.09, 0.25)$. The relationship between ρ and $\text{var}\, X$ is proportional. For $\rho \in (0.5, 1]$ the positive demand shock is more likely to occur, then $v \in (1, 1.5]$ and $\text{var}\, X \in [0.09, 0.25)$. The relationship between ρ and $\text{var}\, X$ is inversely proportional.

Figure 3.B1.24 illustrates that under ρ_1 (negative demand shock), w_{10} and w_{20} takes the maximum value provided that $\text{var}\, X$ takes also the maximum value that means that the demand shock takes a value near the median of v (v=1), which means that the negative demand shock is less likely to occur. Under ρ_2 (positive demand shock), w_{10} and w_{20} takes the maximum value provided that $\text{var}\, X$ takes the minimum value that means that the positive demand shock is more likely to occur (v=1.5) either g=1 or g=0.5. Wage differentials though do not affected by *v*, and consequently, by $\text{var}\, X$, under *frr_1*.

In the first period each firm production increases as the probability a negative demand shock is more likely to occur satisfying the demand of the current period. Of course, this phenomenon is more intense for the private sector that face lenient firing restriction regime. In the second period, q_{11} q_{21}, and therefore $tq_B=q_{11}+q_{21}$, increases as the probability a positive demand shock to occur increases irrespective of g, satisfying the demand of the second period. This phenomenon is more intent for the public sector that face stricter firing restriction regime. On the other hand, under *frr_0* employment is not protected thus product of the first period increases even when a negative demand shock is more likely to occur, covering the demand of the first period. In the second period, though, production increases only if the probability a positive demand shock is more likely to occur, see **figures 3.B1.23-25**.

An increase in the production and employment across period in the public sector is benefited when the probability a positive demand shock is more likely to occur. Private sector fills the gap in the market in both periods as products are substitutes across firms, see **figure 3.B1.26**. Public-sector total production increase as the probability a positive demand shock is more likely to occur. If g=1, private-sector total production increases as the probability a positive demand shock is more likely to occur, if g=0.5 the opposite happens, see **figure 3.B1.27**. The total production of the game as well as the yields Π_j, U_j, CS, EV1, EV2, EV3 increase as the probability a positive demand shock is more likely to occur see **figures 3.B1.28-3.B1.32**.

An interesting question to answer is what changes to this model if strict firing restriction regime imposed in the public sector only. In the next paragraph I deal with this issue.

3.3.2 Strict firing restrictions in the public sector only (frr_2)

Assume now that PM's institutional choice is the frr_2. In this case, the private sector does not face any firing restriction regime that is mean that does not face adjustment cost:

$$AdjC_2 = 0. \tag{3.24}$$

On the other hand, public sector continues to face strict firing restriction regime. Thus, the state-mandated cost of adjusting employment (c) is assumed to be asymmetric across firms. Then the public and the private firm's total cost scheduled(s) over the two-period game, respectively, are given by equations (3.5) and (3.25) for the public and the private sector respectively:

$$TC_2 = w_{20}\left(q_{20} + q_{21}\right). \tag{3.25}$$

The game is deployed in two periods/ four stages as in the previous case and is solved step by step in the appendix (3.A2). Thus, in this paragraph I am going to emphasize the difference between frr_1 and frr_2 regime. Firstly, private sector's reaction function of the second period is differentiated now. Reaction Functions are given in the appendix (3.A2) and I re-write them below:

$$RF_{11}\left(q_{21}\right) = \frac{\left(v+c\right) + cq_{10} - gq_{21} - w_{10}}{2+c}, \tag{3.26}$$

$$RF_{21}\left(q_{11}\right) = \frac{v - gq_{11} - w_{20}}{2}. \tag{3.27}$$

Under frr_2 regime, the public and the private sector are unequally responsive to changes in the rival's current (second period) output, as only public sector's decision is restricted by the adjustment cost c; $\frac{dRF_{11}}{dq_{21}} = -\frac{g}{2+c} < \frac{dRF_{21}}{dq_{11}} = -\frac{g}{2}$. Furthermore, only public sector is affected positively by changes in its own past output, due to the adjustment cost that faces; $\frac{dRF_{11}}{dq_{10}} = \frac{c}{2+c} > \frac{dRF_{21}}{dq_{20}} = 0$. Note also that whatever is the realization of the demand shock (v), the public firm's maximum employment/ output in the second period is, *other things being equal*, higher than the private's firm counterpart; $\frac{v+c}{2+c} > \frac{v}{2}$. This premium is higher under frr_2 than under frr_1 regime; $\frac{v+c}{2+c} > \frac{v}{2} > \frac{v}{2+c}$, thus it is expected wage differentials increase under frr_2 comparing to frr_1. According to the RFs of the first period, which are given in the appendix (3.A2) and I display them below, it is clear that private sector's production of the first period is not affected by public sector's wage:

$$RF_{10}(q_{20}) = L + Mq_{20} + Nw_{10} + Ow_{20}, \tag{3.28}$$

$$RF_{20}(q_{10}) = \frac{1 - gq_{10} - w_{20}}{2}, \tag{3.29}$$

where,

$$\text{L}=\frac{\left(\text{g}^2\text{-4}\right)^2 \text{ - c}\left(\text{g-2}\right)\left(\text{g}^2\left(\text{2+g}\right)\text{+4v}\right)\text{+ c}^2\left(\text{-4 + 4g}^2\text{+ 4v -2gv}\right)}{\text{P}}>0, \qquad \text{M}=\frac{\text{g}\left(\text{4 + 2c - g}^2\right)^2}{\text{P}}<0,$$

$$\text{N}=\frac{\left(\text{2+c}\right)\left(\text{4g}^2\text{-8}\left(\text{1+c}\right)\right)\text{+g}^4}{\text{P}}<0, \quad \text{O}=\frac{\text{-2cg}}{\left(\text{4+4c-g}^2\right)\left(\text{g}^2\text{-4}\right)}>0,$$

$\text{P}=\left(\text{2+c}\right)\left(\text{g}^2\text{-4}\right)\left(\text{g}^2\text{-4c-4}\right)$. By means of (3.26)-(3.27), through (3.28)-(3.29), accrues the inter-temporal $q_j \rightarrow q_{j'}$, $j \neq j' = 1,2$ effects given by (3.30), (3.31) below:

$$\frac{dq_{11}}{dq_{20}} = \frac{dq_{11}}{dq_{10}}\frac{dq_{10}}{dq_{20}} = \frac{c}{2+c}M < 0, \tag{3.30}$$

$$\frac{dq_{21}}{dq_{10}} = \frac{dq_{21}}{dq_{20}}\frac{dq_{20}}{dq_{10}} = 0. \tag{3.31}$$

Alternatively, reaction function of second period moves downward, but only for the public sector, due to rival's product increase, and this transposition is equal to (3.30). The new reaction function for the public sector is given by equation (3.32):

$$RF_{11}\left(q_{21}\right)_{NEW} = \frac{\left(v+c\right)+c\left(q_{10}^{*}+M\right)-gq_{21}-w_{10}^{*}}{2+c}. \tag{3.32}$$

Hence, the public firm's output in the second period would *ceteris paribus* decrease, due to an increase in rival's product in the first period, irrespective of the nature and the magnitude of the stochastic demand shock. This in turn implies that the asymmetric firing restrictions render only the public firm an incentive to allocate its own production across periods, given the deterministic demand of the first period and the stochastic demand of the second period. Due to Cournot competition products are strategic substitutes across firms in each period. Consequently, given that public sector own products are complementary across periods, thus private sector own product are complementary too. This combination gives rise to the twofold strategic interdependence among firms over the two-period game. **Figure 3.4** illustrates the transposition of the reaction function (3.26) to (3.32). Private sector's reaction function does not move. For simplicity we assume that products are perfect substitutes, c=0.3, and v=1.5. In case where products are imperfect substitutes, the transposition is even smaller.

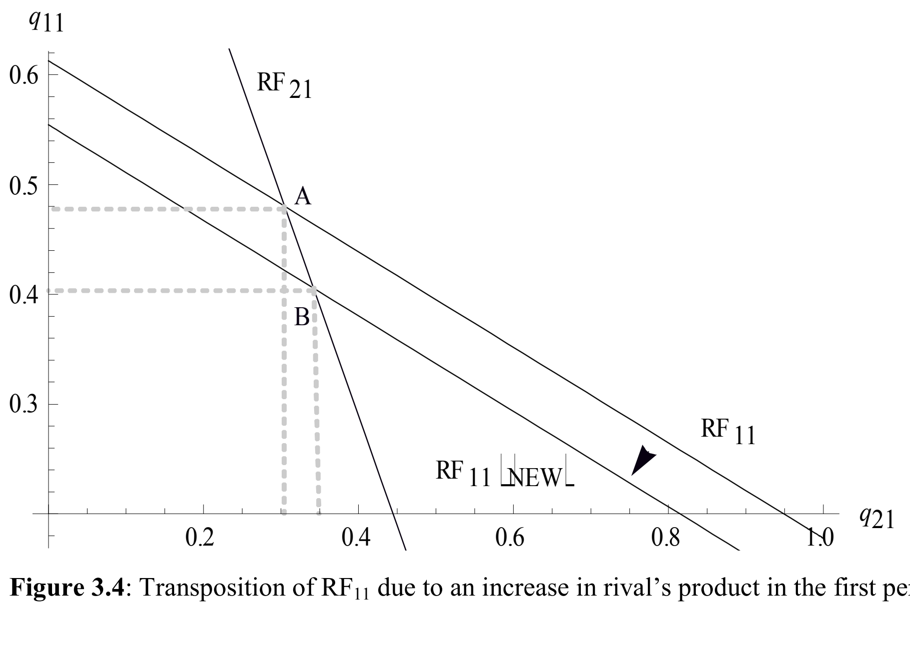

Figure 3.4: Transposition of RF_{11} due to an increase in rival's product in the first period under frr_2

In this game wage differentials are, also, in favor of the public sector and are given by (3.33). Note that, if $g = 0$ and/ or $c = 0$, namely, if the products are independent and/ or firing restrictions does not exist, then the differential is null. **Figure 3.5** illustrates diagrammatically the wage differential.

$$w_{10}^{*} - w_{20}^{*} = \frac{cg^2\left(H_1 + H_2\right)}{H_3} > 0\,, \tag{3.33}$$

note that $H_1\left(c, g, v\right) < H_2\left(c, g, v\right)$, where,

$$\begin{aligned}\mathrm{H}_1\left(\mathrm{g,c,v}\right)=&\,\mathrm{g}^2\Big(\Big(\big(128+\mathrm{c}\left(64+120\mathrm{c}+32\mathrm{c}^2\right)\big)+\mathrm{g}\left(48+96\mathrm{c}+42\mathrm{c}^2\right)+\mathrm{g}^2\left(24+2\mathrm{cg}^2+\mathrm{g}^3\right)\Big)+\\ &\mathrm{v}\Big(\left(96+96\mathrm{c}+24\mathrm{c}^2\right)+6\mathrm{gc}\left(2+\mathrm{c}\right)+2\mathrm{g}^4\Big)\Big)>0;\end{aligned}$$

$$\begin{aligned}\mathrm{H}_2\left(\mathrm{g,c,v}\right)=&-\Big(64\mathrm{c}\left(1+5\mathrm{c}+2\mathrm{c}^2\right)+\mathrm{g}\Big(16\Big(4+6\mathrm{g}+\mathrm{c}\left(12+11\mathrm{c}+3\mathrm{c}^2\right)\Big)+\mathrm{g}^3\big(2\mathrm{c}\left(10+7\mathrm{c}\right)+\\ &2\mathrm{g}\left(6+6\mathrm{c}+\mathrm{g}\right)\big)\Big)+\mathrm{v}\Big(16\Big(8+12\mathrm{c}+4\mathrm{c}^2+\mathrm{cg}\left(2+3\mathrm{c}+\mathrm{c}^2\right)\Big)+\mathrm{g}^4\left(24+12\mathrm{c}+\mathrm{cg}\right)\Big)\Big)<0;\end{aligned}$$

$$\begin{aligned}\mathrm{H}_3\left(\mathrm{c,g}\right)=&\,2\Big(8\left(1+\mathrm{c}\right)\left(2+\mathrm{c}\right)-\left(8+5\mathrm{c}\right)\mathrm{g}^2+\mathrm{g}^4\Big)\Big(8\left(2+\mathrm{c}\right)\left(-16\left(1+\mathrm{c}\right)+3\left(3+\mathrm{c}\right)\mathrm{g}^2\right)-\\ &\mathrm{g}^4\left(3\left(8+3\mathrm{c}\right)-\mathrm{g}^2\right)\Big)<0.\end{aligned}$$

Figure 3.5a: g=1
Contour Plot w_{10}*-w_{20}*

Figure 3.5b: g=0.5
Contour Plot w_{10}*-w_{20}*

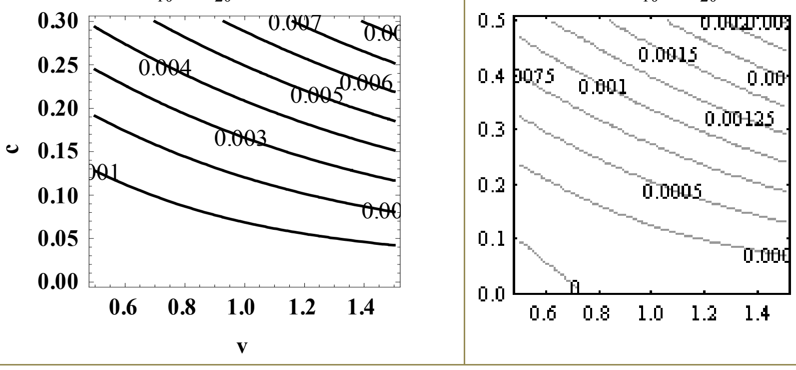

Figure 3.5: Wage differentials w_{10}*-w_{20}*, exist if θ=0.5 under *frr*$_2$

The interesting point is that demand shock affects wage differentials, contrary to *frr*$_1$ case, due to the fact that private sector is not engaged with the adjustment cost, which mainly affects the allocation of the production between periods in the public sector. Given c, as v increases wage differentials increase too, diagrammatically we move up to a higher contour line. Moreover, as it is shown in **figure 3.6** wage differentials are more intense in case where products are perfect substitutes.

Figure 3.6:
Contour Plot
$(w_{10}^{*}-w_{20}^{*})(frr_{2\ \text{Perfect Substitutes}})-$
$(w_{10}^{*}-w_{20}^{*})(frr_{2\ \text{Imperfect Substitutes}})-$

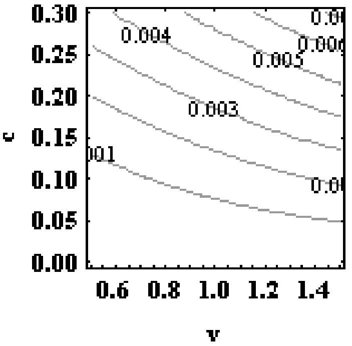

Figure 3.6: Wage differentials increase as g increases under frr_2

Note also that wage differentials are more intense under frr_2 than under frr_1, due to the fact that the private sector does not face adjustment cost now, see **figure 3.7**. This happens because public sector's wages decreases, as the adjustment cost disperses on the private sector. Similarly, public sector's wages increases as the adjustment cost is confined to the public sector. Private sector's wages are higher under frr_2 regime and under negative demand shock and higher under frr_1 in case of positive demand shock, in case where products are perfect or imperfect substitutes see **figure 3.8**.

Figure 3.7a: g=1
Contour Plot
$(w_{10}^{*}-w_{20}^{*})frr_1-(w_{10}^{*}-w_{20}^{*})frr_2$

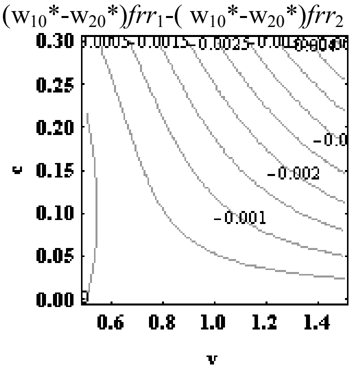

Figure 3.7b: g=0.5
Contour Plot
$(w_{10}^{*}-w_{20}^{*})frr_1-(w_{10}^{*}-w_{20}^{*})frr_2$

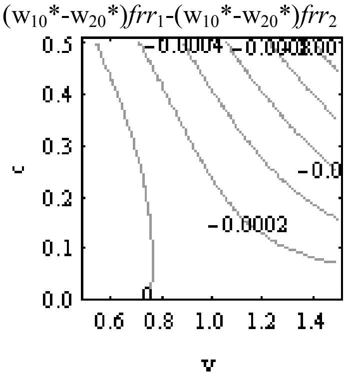

Figure 3.7: Wage differentials are more intense under frr_2

Figure 3.8a: g=1
Contour Plot
[w_{10}* (frr_1)- w_{10}* (frr_2), {v, 0.5, 1.5}, {c, 0, 0.3}]

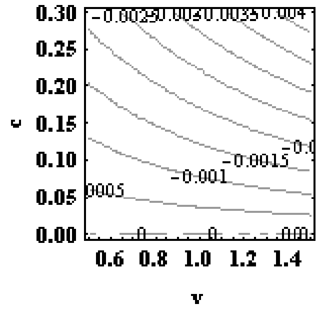

Figure 3.8b: g=1
Contour Plot
[w_{20}* (frr_1)- w_{20}* (frr_2), {v, 0.5, 1.5}, {c ,0, 0.3}]

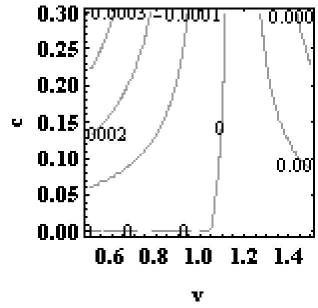

Figure 3.8a: g=0.5
Contour Plot
w_{10}* (frr_1)- w_{10}* (frr_2)

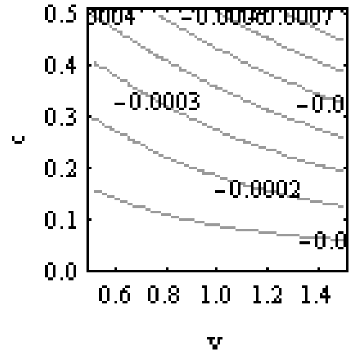

Figure 3.8b: g=0.5
Contour Plot
w_{20}* (frr_1)- w_{20}* (frr_2)

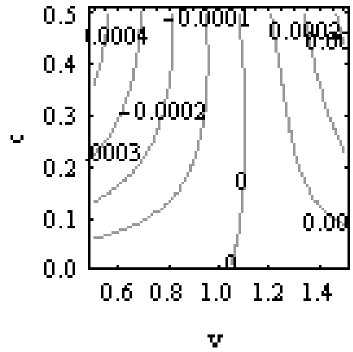

Figure 3.8: w_{10}*(frr_1)<w_{10}*(frr_2) and w_{20}*(frr_1)<w_{20}*(frr_2) v<1, but w_{20}*(frr_1)>w_{20}*(frr_2) v>1

Interior solution is ensured provided that $c \leq 0.3$ and $v \geq 0.5$ in case where *g*=1, the main restriction, related to *c* and *v*, comes from the private sector, in order to produce in the second period. If *g*=0.5 then $c \leq 0.5$ and $v \geq 0.5$, on account of public-sector total cost, see **figure 3.B2.1-6**.Under these restrictions for *v* and *c* profits, utilities, and consumer surplus are positive, see **figure 3.B2.7-8**.

Similar to the frr_1 case, private sector is benefited from supple character of its commitment under negative demand shock (Π2>Π1), producing more in the first period rather than in the second (q_{20}>q_{21}). Under positive demand shock public sector is benefited from strict firing restriction regime that face (Π1>Π2), in this case produce higher quantity in the second period than in the first (q_{11}>q_{10}). In general, the private sector produces more than the public sector in the first period and less in the second (q_{20}>q_{10} and q_{11}>q_{21}). Moreover the public sector conquers the market (tq_1>tq_2)[20]. Consequently, there are two influences in favor of the utility of the public sector employees against private sector

[20] Where tq_1=q_{10}+q_{11} and tq_2=q_{20}+q_{21}.

employees. The first one is on account of wages. The second one is due to the production, thus U1>U2, see **figure**s **3.B2.9-10**. The results are similar, even the products are perfect substitutes or imperfect substitutes, see **figures 3.B2.11-12**.

Comparing frr_1 regime with frr_2 regime, notice two things. The first one is that the stricter the firing restriction regime is the more suspending policy for the private sector in the first period, in case of negative demand shock and more expansive policy in case of positive demand shock. In the second period the opposite is true. Recall that products are strategic substitutes across firms and its firm own products are complements across periods, see **figure 3.B2.13-14**.

Irrespective of the firing restrictions regime, frr_2 or frr_1, the abolition of the adjustment cost has the same impact on firm's strategic decisions as far as the allocation of the production in both periods is concerned, see **figures 3.B2.15-16**. Between periods, public-sector production increases more under frr_2 (the same with frr_1) and private sector's production increases more under frr_0, see **figures 3.B2.17.** The total production of the game is higher under frr_2 than frr_0, as it happens with the total production of the public sector (the same with frr_1). Though, the total production of the private sector is higher under frr_0 than frr_2 regime (unlike frr_1). In the first period, both firms, together, produce more under frr_0 regime than frr_2, but in the second period the opposite happens (the same with frr_1), see **figures 3.B2.18-19**. Consequently, consumer surplus and U1 are generally higher under frr_2 than frr_0. U2 is always lower under frr_2 than frr_0. The proposal regime (frr_2) is proved to be more beneficial (compared to frr_0) for the public sector under positive demand shock and for the private sector under negative demand shock, see **figures 3.B2.20-21**.

As for the institutionalization of the frr_2 regime there are some differences with the frr_1 case. The main difference is that U2 is always decreasing with c either the products are perfect substitutes or imperfect substitutes. If g=1 then Π1 is increasing with c provided that $c \in [0.14, 0.3]$. For that c, Π2 is increasing with c, provided that $v \le 1.3$. U1 and CS are always increasing with c either g=1 or g=0.5. If g=0.5, then $c \in [0.2, 0.5]$ in combination with proper $(v \le 1.2)$, Π1, Π2 and U1 are increasing with c. Restrictions related to c are imposed by the f1 and restrictions related to v are imposed by the f2. Thus there is a tacit agreement between f1, f2 and U1, see **figures 3.B2.22-23**.

U2 does not approve the proposal regime frr_2, because as we saw, under frr_2 regime w_{20} is even lower than under frr_1 also employment is figured at very low levels since $tq_1>tq_2$, thus U1>U2. As well as f2 does not approve the asymmetry in the firing restriction regime in favor of the public sector, in case of "economic boom" (v>1.3), because its profits are lower than Π1, but n case of the economic boom, there are plenty of jobs thus employees do not "hurt" essentially by the abolition of *c*. Note though, that in case of economic recession, private sector votes in favor of frr_2 that is meant to be in favor of the public sector employees, through the institutionalization of the adjustment cost. Proposition 2 summarizes.

Proposition 2: *Under frr_2 regime wage differentials are in favor of the public sector provided that both firms produce in both periods and are more intense comparing to the frr_1 regime. Wage differentials are more intense in case where products are perfect substitutes. According to the reaction functions, changes in public firm's product of the first period affect positively its product of the next period due to the adjustment cost that only the public firm faces. Strict firing restriction regime is proved to be more beneficial for the public sector under positive demand shock. Adjustment cost receives the approval of the majority of the participants of the game and this permits its institutionalization by the policy maker.*

In order to assess the impact of varX on variables and yields I assume, similar to the case frr_1, that *c* takes the maximum permissible value under frr_2, consequently, if g=1 then c=0.3 and if g=0.5 then c=0.5. The results are similar to the case frr_1 except for the following differences. The first one is that wage differentials increase, under frr_2, as the probability a positive demand shock to occur increases. This happens due to the stochastic term *v* that affects wage differentials. The second one is that public-private product differentials affected by $\text{var}\, X$ under frr_2 unlike frr_1 case. Specifically, the public-private product differentials in the first period, which are in favor of the private sector, increase as the probability a negative demand shock to occur increase. Consequently, in the second period, public-private wage differentials, which are in favor of the public sector, increase too. The public-private total product differentials, which are in favor of the public sector increase as the probability a negative demand shock to occur increases, see **figures 3.B2.24-3.B2.36**.

The question now is what changes to our model if both sectors face the same strict firing restriction regime. The next paragraph deals with this issue.

3.3.3 Strict firing restrictions in both sectors (frr_3)

Assume now that PM's institutional choice is the *frr_3*. In this case both sectors face strict firing restrictions regime, thus PM's institutional choice is defined by (3.3) and (3.34) for the public and the private sector respectively:

$$AdjC_2 = c\left[\frac{\left(q_{20}-q_{21}\right)^2}{2}-\left(q_{21}-q_{20}\right)\right]. \tag{3.34}$$

Then the public and the private firm's total cost schedule(s) over the two-period game respectively are given by (3.5) and (3.35):

$$TC_2 = w_{20}\left(q_{20}+q_{21}\right)+c\left[\frac{\left(q_{20}-q_{21}\right)^2}{2}-\left(q_{21}-q_{20}\right)\right]. \tag{3.35}$$

The game is deployed in two periods/ four stages as in the cases *frr_1* and *frr_2* and is solved step by step in the appendix 3.A3. Thus, in this paragraph I emphasize the differences with the previous cases. First of all, under *frr_3* regime the game is symmetric, consequently reaction functions are symmetric, which are given by the equations (3.A3.3) and (3.A3.4) in the appendix A3 and I re-write them below;

$$RF_{11}\left(q_{21}\right)=\frac{\left(v+c\right)+cq_{10}-gq_{21}-w_{10}}{2+c}, \tag{3.36}$$

$$RF_{21}\left(q_{11}\right)=\frac{\left(v+c\right)+cq_{20}-gq_{11}-w_{20}}{2+c}. \tag{3.37}$$

As it is expected both firms are equally responsive to changes in the rival's current (second period) output; $-\frac{g}{2+c}$, as well as to changes in their own past (first) period output; $\frac{c}{2+c}$. The symmetric firing restriction regime that both firms face does not render a premium to the wage set by sectors, thus I expect equality in wages between sectors. Indeed, wage differentials are eliminated in this game ($w_{10}^{*} = w_{20}^{*}$). As the adjustment cost disperses to private sector, public sector's wages decreases and private sector increases see **figure 3.B3.1**. Reaction functions of the second period are also symmetric, they are given in the appendix and I re-write them below:

$$RF_{10}(q_{20}) = Y + Bq_{20} + Cw_{10} + Dw_{20}, \tag{3.38}$$

$$RF_{20}(q_{10}) = Y + Bq_{10} + Cw_{20} + Dw_{10}, \tag{3.39}$$

$$\text{where } Y = -\frac{(2+c-g)\left((g-2)(2+g)^2 + c^3(1+g-v) - c^2\left(4v+(g-3)g-2\right) - c\left(4+g^2+g^3+4v\right)\right)}{4(1+c)(2+c)^3 - 2(2+c)^3 g^2 + (2+c)g^4} > 0,$$

note that *B*, *C*, and *D* are given in paragraph *3.3.1*. Recall that $B < 0$, $C < 0$, and $D > 0$. Subsequently, by means of (3.36)-(3.37), through (3.38)-(3.39), accrues the inter-temporal $q_j \rightarrow q_{j'}$; $j \neq j' = 1,2$ effects in (3.40) below:

$$\frac{dq_{11}}{dq_{20}} = \frac{dq_{11}}{dq_{10}}\frac{dq_{10}}{dq_{20}} = \frac{dq_{21}}{dq_{10}} = \frac{dq_{21}}{dq_{20}}\frac{dq_{20}}{dq_{10}} = \frac{c}{2+c}B < 0, \tag{3.40}$$

$$\text{where, } B = -g\left(\frac{1}{2+c} - c\left(\frac{1}{4+2c(3+c)-g^2} - \frac{1}{4+2c-g^2}\right)\right) < 0.$$

Alternatively, reaction function of second period moves downward, for both sectors, due to rival's product increase in the first period, and this transposition is equal to (3.40). The new reaction functions for the public and the private sector are given by

$$RF_{11}(q_{21})_{NEW} = \frac{(v+c) + c\left(q_{10}^{*} + B\right) - gq_{21} - w_{10}^{*}}{2+c}, \tag{3.41}$$

$$RF_{21}(q_{11})_{NEW} = \frac{(v+c) + c\left(q_{20}^{*} + B\right) - gq_{11} - w_{20}^{*}}{2+c}. \tag{3.42}$$

Diagrammatically, in **figure 3.9** below, we can see the transposition of reaction function of (3.36) to (3.41) and of (3.37) to (3.42). I assume that $q_{11} \in [0,1]$ and $q_{21} \in [0,1]$, *ceteris paribus*. For simplicity I also assume that products are perfect substitutes, *c*=0.3 and *v*=1.5. In case where products are imperfect substitutes, the transposition is even smaller.

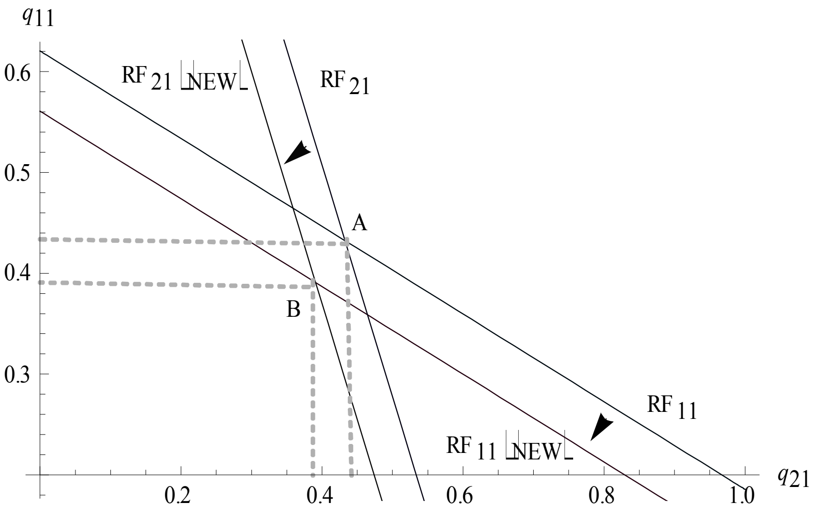

Figure 3.9: Transposition of RF_{11} and RF_{21} due to an increase in rival's product in the first period under *frr*$_3$

In this case interior solution is ensured under *frr*$_3$ provided that $v \in [0.4, 1.6]$. Either products are perfect substitutes or imperfect substitutes $c \in (0.01, 0.6]$. Ensuring interior solution both firms enjoy positive profits, unions positive utilities, and consumer's positive surplus, see **figures 3.B3.4-7**.

As strict firing restriction regime disperses in both sectors public sector's product increases and private sector's product decreases in the first period. The opposite is true as for the products of the second period, in order to reach the symmetric solution $q_{10}=q_{20}$ and $q_{11}=q_{21}$, see **figures 3.B3.2-3**. It is self-evident, given the game is symmetric, that $\Pi_1^* = \Pi_2^*$ and $U_1^* = U_2^*$. The total output for the public sector is equal to the total output for the private sector. **Figure 3.B3.6** illustrates that under positive demand shock or under negative demand shock provided that $c \in (0.3, 0.6]$, then $q_{j1} > q_{j0}$. If the demand shock is negative, then $q_{j1} < q_{j0}$, provided that $c \in (0, 0.3]$. The symmetric firing restriction regime (*frr*$_3$) receives the approval of all the participants of the game. The results are similar in case where products are imperfect substitutes. Proposition 3 summarizes.

***Proposition 3**: Wage differentials are eliminated under frr_3 regime. According to reaction functions changes in the product of the first period affects positively the product of the next period due to the adjustment cost that each firm face. Provided that interior solution is ensured the public and the private sector share equally the market and enjoy equal profits. Thus unions for public sector employees enjoy the same utility as unions that represent workers in the private sector.*

3.4 Conclusions

In order to explain wage differentials between the public and the private sector I construct a mixed duopoly model, with stochastic demand shocks, extended in two periods, examining if firing restrictions are responsible for public-private wage differentials. The main conclusion is that wage differentials are in favor of the public sector, provided that the public sector faces stricter firing restriction regime compared to the private firm. In this case the private sector has the higher share in the market. If the two firms face the same firing restriction regime, then there are no wage differentials.

In order to ensure interior solution, is important to test under which interval of c each firm produce in each period. We ensure interior solution under the representative case $\theta=0.5$ or $v \in [0.5,1.5]$, where less uncertainty lies in the market and encourages both firm's production. As adjustment cost disperses in both sectors the monopolistic character of firms is lost and both firms are more tolerant with *c*. Moreover, adjustment cost receives the approval of the majority of economic agents and this permits its institutionalization by the policy maker. The market seems to accept frr_1 more than frr_2. In this point is remarkable the essential role of private sector that supporting its own benefits, indirectly, supports employees benefit especially under economic recession. Finally, yields under frr_1 and frr_2 take the maximum value provided that an extremely positive demand shock is more likely to occur.

Chapter 4

A unionized mixed oligopoly model with stochastic demand shocks: public-private wage differentials under re-bargaining

4.1 Introduction

In chapter 4, I investigate the public-private wage differentials in case where wages are re-bargained, after the uncertainty is resolved. The main finding is that wage differentials are formed in favor of the private sector in the first period, but in favor of the public sector in the second, provided that the public sector faces stricter firing restriction regime than the private sector. In the first period wage differentials are in favor of the private sector for two reasons. The first one is that the private sector compensates more, probably, for lower job security and lower non-wage benefits. The second one is that the public sector faces less flexibility during collective bargaining that reflects to lower wages, in the first period. According to the empirical literature, these happen in Denmark [Pederson *et al*. (1990)], Poland [Adamchik and Bedi, (2000)] and Netherlands [Hartog and Oosterbeek, (1993), van Ophen (1993)]. The rest of this chapter is organized as follows. In section 4.2, I investigate public-private wage differentials under re-bargaining and the scope of alternative firing restriction regimes. In section 4.3, I conclude.

4.2 Endogenous public-private wage differentials under re-bargaining

Under the assumption that wages are re-bargained the game is deployed in two periods/five stages that are described below.

- At **1st stage/ 1st period,** (t=0), the policy maker (*PM*) evaluates the performance of alternative firing restrictions regimes (frr_x), deciding to activate or not frr_x. The particular frr_x has been activated, provided that the majority of the participants of the game approve it.
- At **2nd stage/ 1st period (t=0)**, the public firm and the public firm's labor union as well as the private firm and the private firm's labor union; $\left(f_1|U_1\right)$, $\left(f_2|U_2\right)$, bargain independently and simultaneously about the firm-specific current wage; w_{10}, $\left(w_{20}\right)$, leaving the firm-specific output/employment decision(s), to each firm's

discretion at the subsequent stages of the game. I assume that each union possesses all the bargaining power over the wage.

- At **3rd stage/ 1st period** (t=0), given the firm-specific wage, each firm decides upon (and carries out) its optimal level of employment and output, in the current period, for any (current) employment/output level of its rival firm.
- At **4th stage/ 2nd period** (t=1), under universal RTM the f1/U1 and f2/U2 bargaining units, bargain independently and simultaneously about the current - second period- firm-specific wage; w_{11} $\left(w_{21}\right)$, leaving the output/employment decision(s), of the second period, to each firm's discretion at the subsequent stage of the game. Each union possesses all the bargaining power over the wage.
- At **5th stage/ 2nd period** (t=1), firms compete *a la* Cournot deciding about the level of employment/output of the current period, like in the previous period; at this stage, of course, the "current" period is the second period. I solve the game by backward induction. **Figure 4.1** illustrates the game tree under re-bargaining.

This section is organized in two paragraphs as follows. Firstly, subsection 4.2.1 deals with frr_1 under re-bargaining. Secondly, subsection 4.2.2 deals with frr_2 under re-bargaining. I do not investigate the case where firms face the same firing restriction regime (frr_3), because does not result in wage differentials.

4.2.1 Strict firing restrictions in the public sector/ lenient firing restrictions in the private sector (frr_1) under re-bargaining

In case frr_1, under re-bargaining, the total cost schedule(s) over the two-period game respectively are

$$TC_1 = w_{10}q_{10} + w_{11}q_{11} + c\left(\frac{\left(q_{10}-q_{11}\right)^2}{2} - \left(q_{11}-q_{10}\right)\right), \tag{4.1}$$

$$TC_2 = w_{20}q_{20} + w_{21}q_{21} + c\frac{\left(q_{20}-q_{21}\right)^2}{2}, \tag{4.2}$$

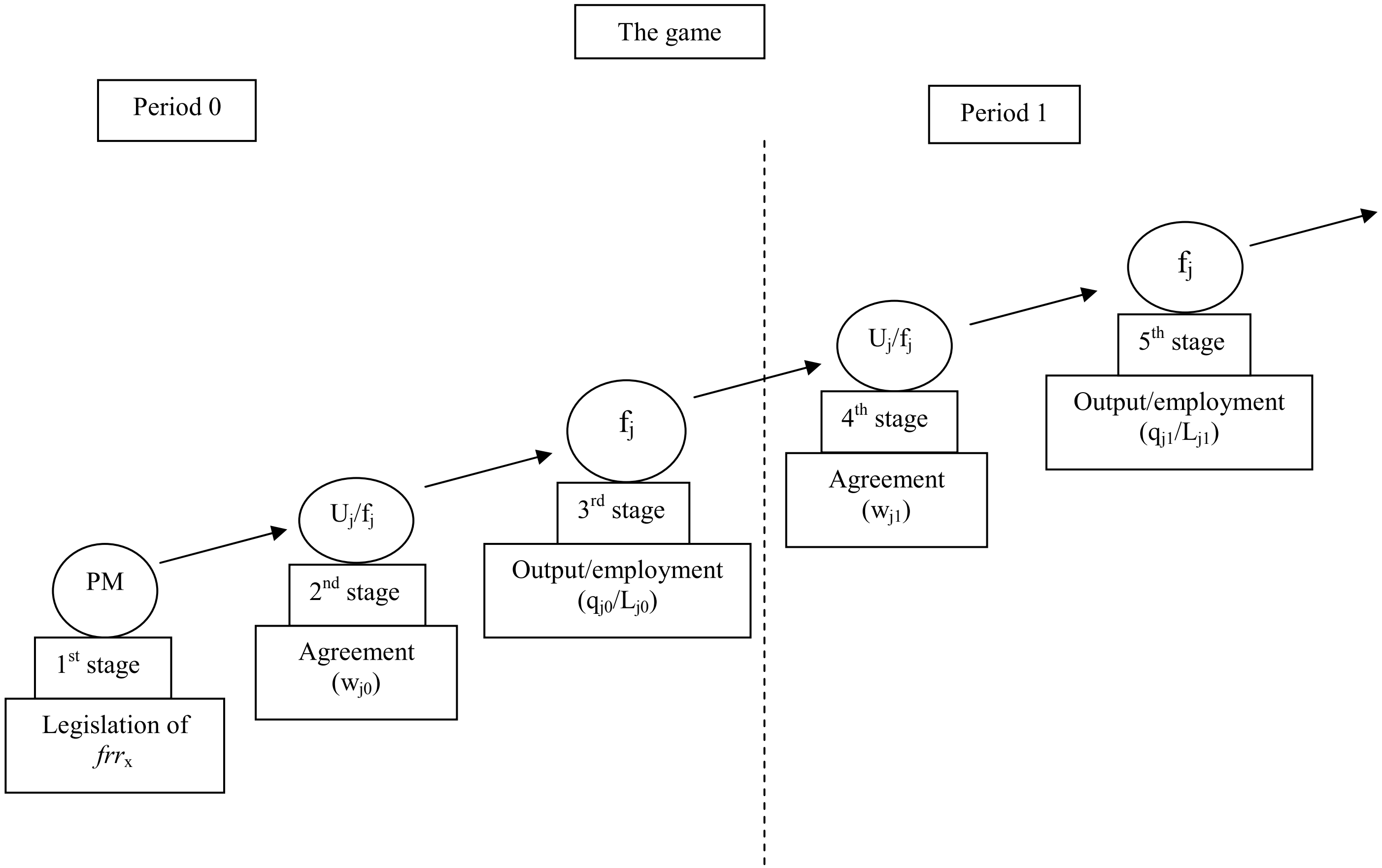

Figure 4.1: The game tree of the model under re-bargaining

where w_{j0}; $j=1,2$ is the firm specific wage bargained at the first period, stage two, and w_{j1}; $j=1,2$ is the firm specific wage bargained at the second period, stage four. Thus, profits that the public and the private firm maximizes *w.r.t.* q_{11}, q_{21} respectively, in the second period/ fifth stage are

$$\underset{q_{11}}{\text{Max}}\,\Pi_1\left(q_{10},q_{20},q_{11},q_{21},w_{10},w_{11}\right)=\text{-c}\left(\text{q}_{10}\text{-q}_{11}+\frac{1}{2}\left(\text{q}_{10}\text{-q}_{11}\right)^2\right)+\left(1\text{-q}_{10}\text{-gq}_{20}\right)\text{q}_{10}+ \\ \left(\text{v-q}_{11}\text{-gq}_{21}\right)\text{q}_{11}\text{-q}_{10}\text{w}_{10}\text{-q}_{11}\text{w}_{11}, \tag{4.3}$$

$$\underset{q_{21}}{\text{Max}}\,\Pi_2\left(\text{q}_{10},\text{q}_{20},\text{q}_{21},\text{q}_{22},\text{w}_{20},\text{w}_{21}\right)=\text{-}\frac{1}{2}\text{c}\left(\text{q}_{20}\text{-q}_{21}\right)^2+\left(1\text{-gq}_{10}\text{-q}_{20}\right)\text{q}_{20}+ \\ \left(\text{v-gq}_{11}\text{-q}_{21}\right)\text{q}_{21}\text{-q}_{20}\text{w}_{20}\text{-q}_{21}\text{w}_{21}. \tag{4.4}$$

Recall that at any stage of the game, both firms act independently so as to maximize their own profits for the entire game. Therefore, inducting backwards, from the *foc*$_s$ of (4.3) and (4.4) *w.r.t* q_{11}, q_{21}, respectively, the following-second period/ fifth stage reaction functions accrue for the public and the private firm:

$$RF_{11}\left(q_{21}\right)=\frac{\left(v+c\right)+cq_{10}-gq_{21}-w_{11}}{2+c} \tag{4.5}$$

$$RF_{21}\left(q_{11}\right)=\frac{v+cq_{20}-gq_{11}-w_{21}}{2+c}. \tag{4.6}$$

Comparing (3.11) and (3.12) to (4.5) and (4.6), it is clear that the only difference concerns the role of wages. While in equations (3.11) and (3.12) w_{10} and w_{20} affects negatively RF_{11} and RF_{21} respectively, now, in case of re-bargaining, wages of the first period does not affect, directly, the product of the second period. But product of the second period affected by the wage of the second period. The asymmetric firing restrictions regime considered frr_1 under re-bargaining, renders a premium to the wage, of the second period, set by the public firm's labor union over the private firm's wage contract. Solving the system (4.5) and (4.6) accrues the optimal q_{11}^*, q_{21}^* - rules of the second period,

$$q_{11}^{*}=\frac{c^{2}\left(1+q_{10}\right)-\left(g-2\right)v+c\left(2+2q_{10}-gq_{20}+v-w_{11}\right)-2w_{11}+gw_{21}}{(2+c-g)(2+c+g)}, \tag{4.7}$$

$$q_{21}^{*}=\frac{c^{2}q_{20}-\left(g-2\right)v-c\left(g+gq_{10}-2q_{20}-v+w_{21}\right)+gw_{11}-2w_{21}}{\left(2+c-g\right)\left(2+c+g\right)}. \tag{4.8}$$

At stage four/second period, given the product of the second period/ fifth stage, under universal RTM the f1/U1 and f2/U2 bargaining units independently and simultaneously bargain about w_{11} and w_{21} respectively. For tractability, I more over assume that each union possesses all the bargaining power over the wage. The wage bargaining expression (4.9) converted to (4.10):

$$w_{j1}=\max_{w_{j1}} B_i=\max_{w_{j1}}\left(bLog\left[U_j\right]+\left(1-b\right)Log\left[\Pi_j\right]\right)= \tag{4.9}$$

$$w_{j1}^{*}=\max_{w_{j1}} U_j\left(=\left(w_{j0}q_{j0}+w_{j1}q_{j1}^{*}\right)\right),\ j=1,2. \tag{4.10}$$

Solving the system accrues from foc_s of (4.10)[21], optimal wages of the second period are given by (4.A1.3) and (4.A1.4) in the appendix 4.A1. To get the first period- optimal employment/ output rules for the public and the private sector (q_{10}^{*},q_{20}^{*}) in the (sub-game perfect) equilibrium, I substitute the optimal q_{j1}^{*} and w_{j1}^{*}, where j=1,2 into (4.3) and (4.4). In the first period, third stage, maximizing the derived profits, $\Pi_1\left(q_{10},q_{20},w_{10},w_{20}\right)$, $\Pi_2\left(q_{10},q_{20},w_{10},w_{20}\right)$[22], as for q_{10} and q_{20} respectively, the first period- reaction functions for the public and the private firm are derived, and are given by equations (4.11) and (4.12), respectively:

$$RF_{10}\left(q_{20}\right)=A+Bq_{20}+Cw_{10}, \tag{4.11}$$

$$RF_{20}\left(q_{10}\right)=D+Bq_{10}+Cw_{20}, \tag{4.12}$$

where,

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[21] The analytical forms of $U1\left(q_{10},q_{20},w_{10},w_{11},w_{21}\right)$ and $U2\left(q_{10},q_{20},w_{20},w_{11},w_{21}\right)$ are given by (4.A1.1) and (4.A1.2), respectively, in the appendix 4.A1.

[22] The analytical forms of $\Pi_1\left(q_{10},q_{20},w_{10},w_{20}\right)$ and $\Pi_2\left(q_{10},q_{20},w_{10},w_{20}\right)$, are given by (4.A1.5) and (4.A1.6), respectively, in the appendix 4.A1.

$$\mathrm{B}=\frac{dRF_{10}}{q_{20}}=\frac{dRF_{20}}{q_{10}}=$$

$$=\frac{\mathrm{g}\left(-2(2+\mathrm{c})^{6}(32+\mathrm{c}(32+9\mathrm{c}))+(2+\mathrm{c})^{4}(160+\mathrm{c}(160+41\mathrm{c}))\mathrm{g}^{2}-33(2+\mathrm{c})^{4}\mathrm{g}^{4}+10(2+\mathrm{c})^{2}\mathrm{g}^{6}-\mathrm{g}^{8}\right)}{E}<0;$$

$$\mathrm{C}=\frac{\mathrm{dRF}_{10}}{\mathrm{w}_{10}}=\frac{\mathrm{dRF}_{20}}{\mathrm{w}_{20}}=-\frac{\left(4(2+\mathrm{c})^{4}-5(2+\mathrm{c})^{2}\mathrm{g}^{2}+\mathrm{g}^{4}\right)^{2}}{\mathrm{E}}<0\ ;$$

and E, A, and D are given in the appendix 4.A1 by (4.A1.6a-6c). Note that D > A, thus frr_1 under re-bargaining, renders a premium to the private sector's wage, over the public firm's wage contract. Observing reaction functions of the first period, contrary to (3.15) and (3.16), rival setting wage does not affect the choice about quantity of each firm, namely $\frac{dRF_{10}}{w_{20}}=\frac{dRF_{20}}{w_{10}}=0$. Solving the system of (4.11) and (4.12), above, accrues the optimal q_{10}^{*}, q_{20}^{*} rules of the first period, which are given by (4.A1.7) and (4.A1.8). Subsequently, by means of (4.5)-(4.6), through (4.11)-(4.12), the inter-temporal $q_j \to q_{j'}$; $j \neq j' = 1,2$, effects are

$$\frac{\partial q_{j'1}}{\partial q_{j0}}=\frac{dRF_{j'1}}{dq_{j'0}}\frac{\partial q_{j'0}}{\partial q_{j0}}=\frac{c}{2+c}B<0\,. \tag{4.13}$$

Equation (4.13) shows that, given *c*>0 and *g*>0, as the j firm's output in the first period (q_{j0}) *ceteris paribus* increases, the rival firm's (j') reaction function in the second period $(RF_{j'1})$ shifts to the left, along (RF_{j1}). The new reaction functions are

$$RF_{11}(q_{21})_{NEW}=\frac{(v+c)+c\left(q_{10}^{*}+B\right)-gq_{21}-w_{11}^{*}}{2+c}, \tag{4.14}$$

$$RF_{21}(q_{11})_{NEW}=\frac{v+c\left(q_{20}^{*}+B\right)-gq_{11}-w_{21}^{*}}{2+c}. \tag{4.15}$$

Similar to the case where wages bargained once at the entire game, under re-bargaining each firm's own products are complements across periods. Also, due to Cournot competition, products are strategic substitutes across firms in each period. The transposition of the reaction functions of (4.5) and (4.6) into (4.14) and (4.15) respectively is depicted in **figure 4.2**. In order to construct reaction functions I assume that $q_{11}^{*} \in [0,1]$, and $q_{21}^{*} \in [0,1]$, *ceteris paribus*, namely q_{10}^{*}, q_{20}^{*}, w_{11}^{*}, w_{21}^{*} are the optimal and are given by (4.A1.15) and (4.A1.16), (4.A1.17), and (4.A1.18) respectively, note that the optimal values accrues provided the game is entirely solved. For simplicity I assume that products are perfect substitutes and *c*=0.5 and *v*=1.5, in case where products are imperfect substitutes the transposition is smaller.

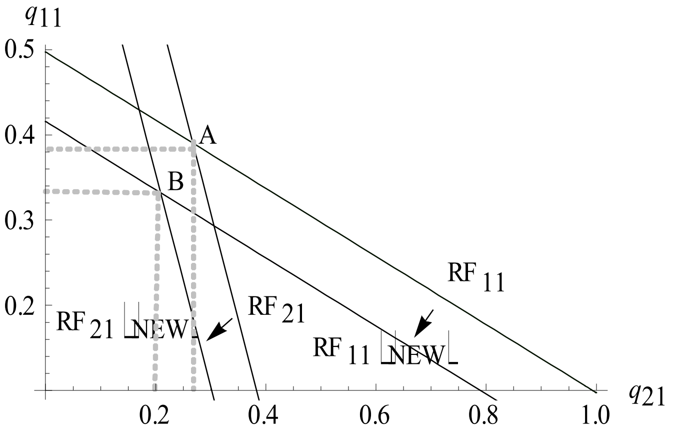

Figure 4.2: Transposition of RF_{11} and RF_{21} due to an increase in rival's product in the first period under frr_1 and re-bargaining

Similar to stage 4/ second period, at stage 2/ first period, under universal RTM the f1/U1 and f2/U2 bargaining units, independently and simultaneously bargain about w_{10} and w_{20} respectively leaving output and employment decisions to each firm's discretion in the subsequent stages of the game. For tractability I retain the assumption that each union possesses all the bargaining power over the wage. Thus, (4.16) converted to (4.17):

$$w_{j0} = \max_{w_{j0}} B_j = \max_{w_{j0}} \left(\text{b Log}\left[\text{U}_j\right] + \text{(1 - b) Log}\left[\Pi_j\right] \right) = \tag{4.16}$$

$$w_{j0} = \max_{w_{j0}} U_i \left(= w_{j0} q_{j0}^{*} + w_{j1} q_{j1}^{*} \right),\ j = 1,2\,. \tag{4.17}$$

Substituting q_{10}^{*}, q_{20}^{*} into (4.18), below, then U_j, $j = 1,2$ depend on wages only;[23]

[23] Analytically, $U_1(w_{10}, w_{20})$ and $U_2(w_{10}, w_{20})$ are given by equations (4.A1.9) and (4.A1.10), respectively.

$$U_j = w_{j0}q^*_{j0} + w_{j1}q^*_{j1},\ j = 1,2\,. \qquad (4.18)$$

Solving the system accruing from the *foc_s* of (4.18) *w.r.t.* $w_{j0} : j = 1,2,$ accrues a unique stable solution for the equilibrium firm-specific wage contracts $w^*_{10},\ w^*_{20}$ respectively. The results about wages are given by equations (4.A1.11) and (4.A1.12). In **figure 4.2**, in the initial equilibrium point A, q^*_{11} and q^*_{21} are given by equations (4.A1.13) and (4.A1.14), which accrue replacing q^*_{10}, q^*_{20}, w^*_{11}, w^*_{21}, which are given by (4.A1.15), (4.A1.16), (4.A1.17) and (4.A1.18) respectively, into (4.7) and (4.8). The optimal values depend on *c*, *g* and *v*. In the optimal point A, the maximum profits for firms are given by equations (4.19) and (4.20), respectively:

$$\Pi^*_1(v, c, g) = -c\left(q^*_{10} - q^*_{11} + \frac{\left(q^*_{10} - q^*_{11}\right)^2}{2} \right) + q^*_{10}\left(1 - q^*_{10} - gq^*_{20}\right) + q^*_{11}\left(v - q^*_{11} - gq^*_{21}\right) - q^*_{10}w^*_{10} - q^*_{11}w^*_{11}, \qquad (4.19)$$

$$\Pi^*_2(v, c, g) = \left(1 - gq^*_{10} - q^*_{20}\right)q^*_{20} - \frac{1}{2}c\left(q^*_{20} - q^*_{21}\right)^2 + q^*_{21}\left(v - gq^*_{11} - q^*_{21}\right) - q^*_{20}w^*_{20} - q^*_{21}w^*_{21}\,. \qquad (4.20)$$

The maximum utilities for unions are given by equation (4.21);

$$U^*_j = w^*_{j0}q^*_{j0} + w^*_{j1}q^*_{j1},\ j = 1,2\,. \qquad (4.21)$$

Indeed, in this game the wage differential endogenously emerges in favor of the private sector in the first period, but in favor of the public sector in the second, see equations (4.22) and (4.23) below:

$$w^*_{10} - w^*_{20} = \frac{c\left[A1(c,g) + A2(c,g)\right]}{B1(c,g) + B2(c,g)} < 0\,, \qquad (4.22)$$

where, (*A*1 < 0) > (*A*2 > 0), and (*B*1 < 0) < (*B*2 > 0) are given in the appendix 4.A1 by (4.A1.12a), (4.A1.12b), (4.A1,12c), and (4.A1.12d),

$$w^*_{11} - w^*_{21} = \frac{c\left(2 + c + g\right)\left(1 + q^*_{10} - q^*_{20}\right)}{4 + 2c + g} > 0\,. \qquad (4.23)$$

Products of the first period exert equal but opposite influence on wage differentials of the second period. In equilibrium, though, the private sector produces more than the public sector in the first period, but the difference $\left|q_{10}^{*}-q_{20}^{*}\right|<1$, also this difference does not affected by *v*, see equation (4.A1.19) and **figure 4.B1.7a**, thus wage differentials are formed in favor of the public sector. **Figure 4.3** illustrates combinations of *g* and *c* that yields contour lines about wage differentials in the first and the second period, respectively. Note, also, that, if $c=0$, namely, if firing restrictions do not exist, then this differential is null. Under re-bargaining, if *g*=0 then in both periods wage differentials do not eliminated contrary to chapter 3. Moreover, as shown in **figure 4.3**, given g the higher the c the more intense is the public-private wage differential.

Figure 4.3a:
Contour Plot[$w_{10}^{*}-w_{20}^{*}$, {g, 0, 1}, {c, 0, 0.6}]

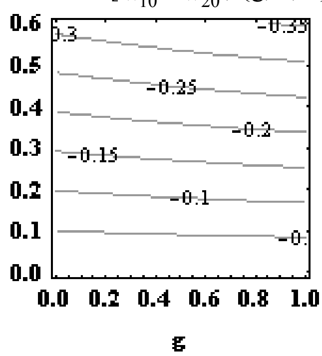

Figure 4.3b:
Contour Plot [$w_{11}^{*}-w_{21}^{*}$, {g, 0, 1}, {c, 0, 0.6}]

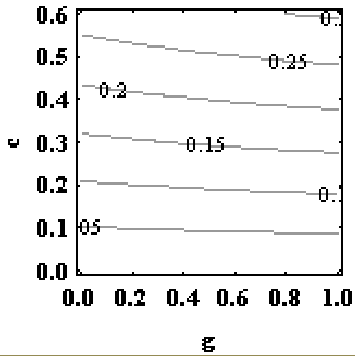

Figure 4.3: Wage differentials increase with *c*, given *g* under frr_1 and re-bargaining

Under frr_1, interior solution ensured provided that $v\in[0.03,1.97]$ and $c\leq 0.5$, in case where *g*=1. The restriction related to v comes from private sector's product in the second period (q_{21}) and the restriction related to c comes from the public sector in order to produce in the first period(q_{10}) and to pay positive wages (w_{10}). If g=0.5 then $v\in[0.01,1.99]$ and $c\leq 0.6$. Prices and total costs are also positive, see **figure 4.B1.1-6** in the appendix 4.B1. Under these restrictions for *v* and *c* profits, utilities, consumer surplus are positive, see **figure 4.B1.7-8**.

The private sector is benefited from the supple character of its commitment under negative demand shock (*Π*2>*Π*1), producing more in the first period rather than in the second (q_{20}>q_{21}). In general, producing more than the public sector in the first period and less in the second (q_{20}>q_{10} and q_{11}>q_{21}). Under positive demand shock the public sector is benefited from the strict firing restriction regime that face ($\Pi1$>$\Pi2$), in this case produce higher quantity in the second period than in the first (q_{11}>q_{10}). In general, the public sector conquer the market (tq_1>tq_2)[24]. Consequently, there are two influences on utility of each sector. The first one is due to wages, the second one is due to products, thus under positive demand shock $U1$>$U2$, the influence of wage and product of the second period are stronger related to first. Under negative demand shock, though, the stimulus is reversed thus $U1$<$U2$. The results are similar, even the products are perfect substitutes or imperfect substitutes, see **figure 3.B1.9-10**.

Comparing each alternative regime with the case where no firing restriction regime (frr_0) is imposed accrues that under re-bargaining the results are similar to the case where wages bargained once at the whole game (see Ch. 3). Thus, briefly, I will comment the behavior of firms. Firstly, a possible abolition of the adjustment cost (frr_0) stimulates public sector's production in the first period and private sector's production in the second. Note that firms are strategic substitutes across firms, thus the allocation of the total production is re-adjusted so as firms to possess equal share in the market under frr_0, see **figures 4.B1.9-11**.[25]

Between periods, public sector's production increases more under frr_1. For the private sector the difference in production between periods is more intense under frr_0, see **figures 4.B1.13**. Surprisingly, there is a strike difference with chapter 3, where wages bargained once at the whole game. The total production of the game under re- bargaining is higher, in case where no one engaged with any restriction regime (frr_0). The re-bargaining of wages inserts uncertainty and makes both sectors reserved as far as the production is concerned. Both sectors together produce higher quantity in the first period under frr_0 regime. On the contrary, in the second period, they produce higher quantity under frr_1, see **figures 4.B1.14-17**.

$\Pi2$ and CS are higher under frr_0 rather than frr_1. $\Pi1$ as well as $U1$ are higher under frr_1 rather frr_0 under positive demand shock and U2 is higher under frr_1 rather frr_0 under negative demand shock, see **figures 4.B1.18-19**. Technically, in case where g=1 public sector's profits and utility $U1$ are increasing with c, provided that $c \in [0.4, 0.5]$. $U2$ is increasing with c provided that $v \leq 0.7$ and $\Pi2$ is always increasing with c. If g=0.5 only

[24] Where tq_1=q_{10}+q_{11} and tq_2=q_{20}+q_{21}.

[25] Under frr_0 inrerior solution is ensured too, see **figure 4.B1.12**.

$\Pi 1$ and $U1$ are increasing with c provided that $c \in [0.47, 0.6]$. Thus, there is a tacit agreement between f1, U1 and U2 only if products are perfect substitutes and v<0.7, see **figures 4.B1.20-21**. Consumer surplus is decreasing with *c*. In sharp contrast to the case where wages bargained once at the whole game (chapter 3), f2 and U2 does not support frr_1 under re-bargaining thus there is no any stable institutionalization of adjustment cost *c*. Proposition 4 summarizes.

Proposition 4*: Under* frr_1 *regime wage differentials are in favor of the private sector in the first period but in favor of the public sector in the second, provided that interior solution is ensured. Wage differentials are not affected by the degree of homogeneity between products. According to reaction functions changes in the product of the first period affects positively the product of the next period due to the adjustment cost that both firms face. Strict firing restrictions regime is proved to be beneficial for the public sector but only under positive demand shock. Adjustment cost receives the approval of the majority of the participants of the game only for limited values of v.*

In order to assess the impact of varX I make the assumption that θ=0.5, as a representative case, therefore the range of *v* restricted, $v \in [0.5, 1.5]$. Also, I assume that *c* takes the maximum permissible, namely if *g*=1 then *c*=0.5 and if *g*=0.5 then *c*=0.6. **Figure 4.B1.22** illustrates that w_{10} and w_{20} takes the maximum value provided that the negative demand shock is more likely to occur.[26] Notice that in the first period both firms reduce their production in case where a negative demand shock is more likely to occur. Thus firms are in position to increase wages, though wages remain at low levels. More over private firm produces more than the private sector in the first period and faces more lenient firing restriction regime thus provokes the competition between the two sectors. This result is in contrast to the case where wages bargaining once at the whole game (chapter 3). In the second period though, after the second round of bargaining, where uncertainty is eliminated, w_{11}, w_{21} take the maximum value provided that the probability a positive demand shock is more likely to occur, but *v* still does not affect wage differentials. Each firm product in each period as well as profits and utilities increase as the positive demand shock is more likely to occur, see **figures 4.B1.23 – 4.B1.32**.

[26] Recall that wage differentials are not affected by *v* in the first period.

4.2.2 Strict firing restrictions in the public sector only (frr_2) under re-bargaining

In case frr_2 under re-bargaining, public employment cost schedule is given by equation (4.1) for the public sector and by equation (4.24) for the private sector:

$$TC_2 = w_{20}q_{20} + w_{21}q_{21}\,. \tag{4.24}$$

The game is deployed in two periods/ five stages as in the previous case and is solved step by step in the appendix (4.A2). Thus, in this paragraph I am going to emphasize some crucial points. First of all, reaction functions of the second period are

$$RF_{11}(q_{21}) \;=\; \frac{(v+c)+cq_{10} - \;gq_{21} - \;w_{11}}{2+c}\,, \tag{4.25}$$

$$RF_{21}(q_{11}) = \frac{v-gq_{11}-w_{21}}{2}\,. \tag{4.26}$$

Comparing (3.26) and (3.27) to (4.25) and (4.26), the only difference concerns the role of wages. While in equations (3.26) and (3.27) w_{10} and w_{20} affects negatively RF_{11} and RF_{21} respectively, in case of re-bargaining, product of the second period affected by the wage of the second period. The asymmetric firing restriction regime considered frr_2 under re-bargaining, renders a premium to the public firm's wage, of the second period, over the private firm's wage contract, wage differentials tends to be more intense under frr_2 compared to frr_1 case. Reaction functions of the first period are

$$RF_{10}(q_{20}) = K + Lq_{20} + Mw_{10} \tag{4.27}$$

$$RF_{20}(q_{10}) = \frac{1-gq_{10}-w_{20}}{2}\,, \tag{4.28}$$

where, $M = \dfrac{dRF_{10}}{w_{10}} = gL < 0$;

$$L = \frac{dRF_{10}}{q_{20}} = -\frac{g\left(16(2+c)^2-10(2+c)g^2+g^4\right)^2}{(2+c)\left(8(2+c)(4+c)-4(5+2c)g^2+g^4\right)\left(8(2+c)(4+3c)-4(5+3c)g^2+g^4\right)} < 0\,;$$

$$K=\left(-192c^5+\left(64-20g^2+g^4\right)^2+16c^4\left(-88+18g^2+4v-gv\right)+4c^3\left(-16\left(52-23g^2+2g^4\right)+\left(4+g\right)\left(24+\left(-12+g\right)g\right)v\right)+4c^2\left(-384+448g^2-97g^4+5g^6+\left(-4+g\right)\left(-48+g^2\left(8+g\right)\right)v\right)-c\left(-2+g\right)\left(4+g\right)\left(g^2-8\right)\left(-16g-2g^3+g^4-8\left(8+v\right)\right)\right)/\left(\left(2+c\right)\left(8\left(2+c\right)\left(4+c\right)-4\left(5+2c\right)g^2+g^4\right)\left(8\left(2+c\right)\left(4+3c\right)-4\left(5+3c\right)g^2+g^4\right)\right)>0.$$

Observing reaction functions of the first period, contrary to (3.28) and (3.29), under re-bargaining rival setting wage does not affect the choice about quantity of each firm, namely $\frac{dRF_{10}}{w_{20}}=\frac{dRF_{20}}{w_{10}}=0$. By means of (4.25)-(4.26), through (4.27)-(4.28), the inter-temporal $q_j \to q_{j'}$; $j \neq j' = 1,2$, effects given by (4.29)-(4.30):

$$\frac{\partial q_{11}}{\partial q_{20}}=\frac{dRF_{11}}{dq_{10}}\frac{\partial q_{10}}{\partial q_{20}}=\frac{c}{2+c}L<0\,, \tag{4.29}$$

$$\frac{\partial q_{21}}{\partial q_{10}}=\frac{dRF_{21}}{dq_{20}}\frac{\partial q_{20}}{\partial q_{10}}=0\left(-\frac{g}{2}\right)=0\,. \tag{4.30}$$

Equations (4.29) and (4.30) show that, only public sector's product decreases in the second period, due to an increase in the private firm's output *ceteris paribus* in the first period, given that c>0. The new reaction function is given by equation (4.31):

$$RF_{11}\left(q_{21}\right) \;=\; \frac{\left(v+c\right)+c\left(q_{10}^{*}+L\right)-\,gq_{21}-\,w_{11}^{*}}{2+c}\,. \tag{4.31}$$

Similarly to the case where wages bargained once in the entire game, under re-bargaining public sector retains the incentive to allocate its own production across periods, given the deterministic demand in the first period and the stochastic demand in the second. The transposition of reaction functions of (4.25) into (4.31) is depicted in **figure 4.4**. In order to construct reaction functions I assume that $q_{11}^{*}\in\left[0,1\right]$, and $q_{21}^{*}\in\left[0,1\right]$, *ceteris paribus*, namely q_{10}^{*}, q_{20}^{*}, w_{11}^{*}, w_{21}^{*} are the optimal. For simplicity I assume that products are perfect substitutes, c=0.5 and v=1.3, in case where products are imperfect substitutes the transposition is smaller.

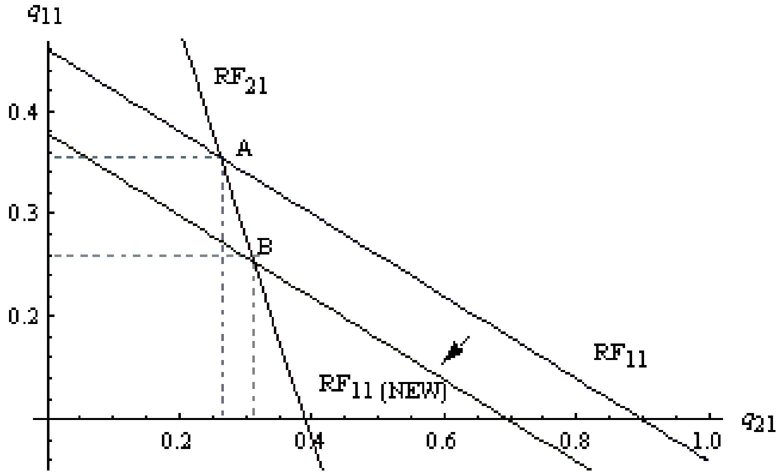

Figure 4.4: Transposition of RF_{11} due to an increase in rival's product in the first period under frr_2 and re-bargaining; RF_{21} does not move

In this game the wage differential endogenously emerges in favor of the private sector in the first period, but in favor of the public sector in the second:

$$\mathrm{w}_{10}^{*}\text{-}\mathrm{w}_{20}^{*}=\mathrm{F}(c,g,v)<0, \tag{4.32}$$

$$w_{11}^{*}-w_{21}^{*}=\frac{c\left(4c-(g-4)(g+2)\right)\left(1+q_{10}^{*}\right)-cgv}{16+8c-g^{2}}>0. \tag{4.33}$$

Note that I suppose that $g\in[0,1]$ and $0.03\leq v\leq 1.97$; also, in **figure 4.B2.1c** and **4.B2.2c** it is clear that $0<q_{10}<1$, provided that $c<1$ in order to ensure interior solution. Only q_{10}^{*} exerts positive influence on the wage differential of the second period. q_{20}^{*} does not affect wage differentials of the second period, despite the fact that the private sector produces more than the public sector in equilibrium, in the first period; see **figure (4.B2.9a)** and **(4.B2.11a)**. In this case the difference $\left(q_{10}^{*}-q_{20}^{*}\right)<0$ affected by *v*, contrary to previous case (frr_1, under re-bargaining). **Figure 4.4** illustrates combinations of *g* and *c* that yields contour lines about wage differentials in the first and the second period respectively. Note, also, that, if $c=0$, namely, if firing restrictions do not exist, then there are no wage differentials neither in the first nor in the second period. Under re-bargaining, if *g*=0 then in both periods wage differentials do not eliminated contrary to chapter 3.

Figure 4.5a: g=1
Contour Plot
$w^*_{10} - w^*_{20}$

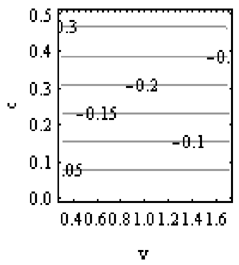

Figure 4.5b: g=0.5
Contour Plot
$w^*_{10} - w^*_{20}$

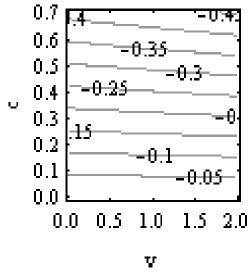

Figure 4.4c: g=1
Contour Plot
$w^*_{11} - w^*_{21}$

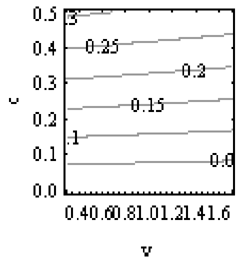

Figure 4.5d: g=0.5
Contour Plot
$w^*_{11} - w^*_{21}$

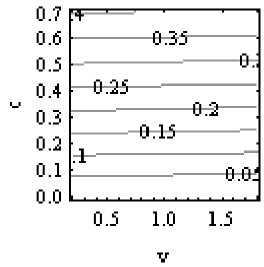

Figure 4.5: Wage differentials increase with c, given v under frr_2 and re-bargaining

Moreover, **figure 4.6** illustrates that public-private wage differentials in the first period is more intense provided that g=1. In the second period though, wage differentials are more intense if g=1 under negative demand shock. Under positive demand shock is higher provided that g=0.5, contrary to chapter 3, where wage differentials are more intent in case where g=1.

In the first period wage differentials are higher under frr_1 rather than frr_2, this is reasonable as public sector's wage increases and private sector's wage decreases as the adjustment cost spreads to the private sector. In the second period, though, wage differentials are more intense under frr_2 rather than frr_1. This happens because as the adjustment cost spreads to the private sector its wages increase more than public sector's wage do, see **figures 4.7- 4.8**.

Figure 4.6a:
Contour Plot
$\{(w^*_{10} - w^*_{20})(g{=}1)\}$-$\{(w^*_{10} - w^*_{20})(g{=}0.5)\}$

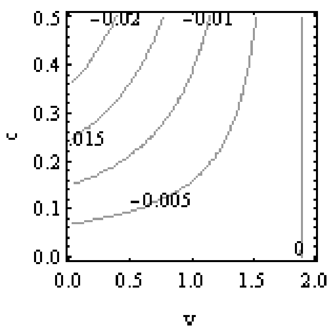

Figure 4.6b:
Contour Plot
$\{(w^*_{11} - w^*_{21})(g{=}1)\}$-$\{(w^*_{11} - w^*_{21})(g{=}0.5)\}$

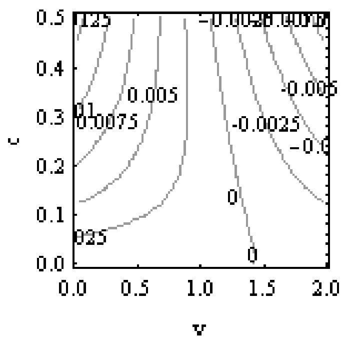

Figure 4.6: Comparing wage differentials in case where g=1 and g=0.5 under frr_2 and re-bargaining

Figure 4.7a: g=1
Contour Plot
$(w^*_{10} - w^*_{20})frr_1$-
$(w^*_{10} - w^*_{20})frr_2$,

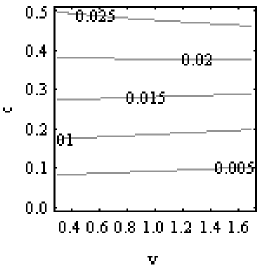

Figure 4.7b: g=1
Contour Plot
$(w^*_{11} - w^*_{21})frr_1$-
$(w^*_{11} - w^*_{21})frr_2$,

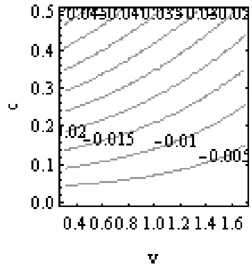

Figure 4.7c: g=0.5
Contour Plot
$(w^*_{10} - w^*_{20})frr_1$-
$(w^*_{10} - w^*_{20})frr_2$,

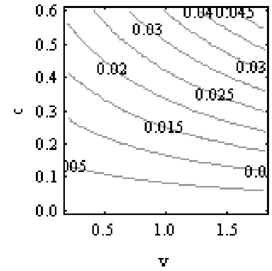

Figure 4.7d: g=0.5
Contour Plot
$(w^*_{11} - w^*_{21})frr_1$-
$(w^*_{11} - w^*_{21})frr_2$,

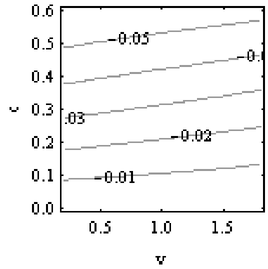

Figure 4.7: Wage differentials are more intense under frr_1 in the first period but under frr_2 in the second, under re-bargaining

Figure 4.8a: g=1
Contour Plot
$w^*_{10}(frr_1)$- $w^*_{10}(frr_2)$,

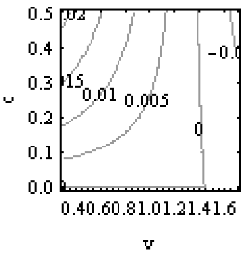

Figure 4.8b: g=1
Contour Plot
$w^*_{20}(frr_1)$- $w^*_{20}(frr_2)$,

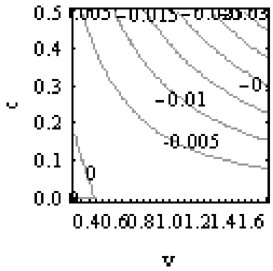

Figure 4.8c: g=1
Contour Plot
$w^*_{11}(frr_1)$- $w^*_{11}(frr_2)$,

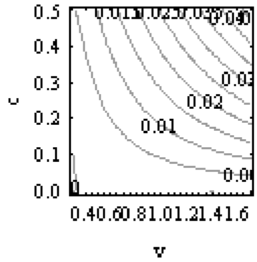

Figure 4.8d: g=1
Contour Plot
$w^*_{21}(frr_1)$- $w^*_{21}(frr_2)$,

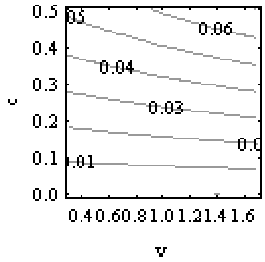

Figure 4.8e: g=0.5
Contour Plot
$w^*_{10}(frr_1)$- $w^*_{10}(frr_2)$,

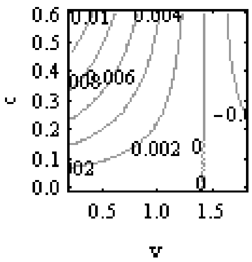

Figure 4.8f: g=0.5
Contour Plot
$w^*_{20}(frr_1)$- $w^*_{20}(frr_2)$,

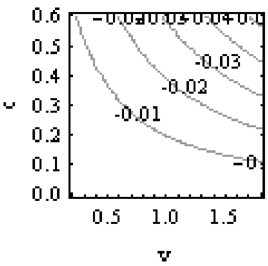

Figure 4.8g: g=0.5
Contour Plot
$w^*_{11}(frr_1)$- $w^*_{11}(frr_2)$,

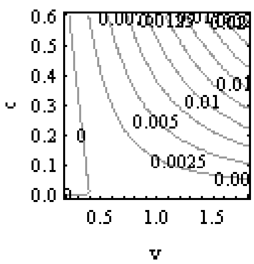

Figure 4.8h: g=0.5
Contour Plot
$w^*_{21}(frr_1)$- $w^*_{21}(frr_2)$,

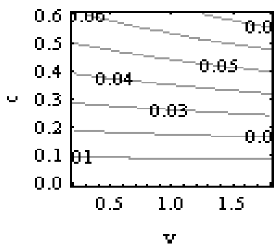

Figure 4.8: w^*_{10}, w^*_{11}, w^*_{21} are higher under frr_1; but w^*_{20} is higher under frr_2 under re-bargaining

In case frr_2 under re-bargaining, interior solution ensured provided that $v \in [0.03, 1.97]$ and $c \leq 0.5$ in case where g=1; If g=0.5 then $v \in [0.02, 1.98]$ and $c \leq 0.7$. The restriction related to v comes from q_{21}, and q_{10} imposes restriction related to c, namely, further restriction related to c comes from w_{10}, which is positive provided that $c \leq 0.5$ ($c \leq 0.7$), then total costs are positive, see **figure 4.B2.1-6** in the appendix 4.B2. Under these restrictions for v and c profits, utilities, and consumer surplus are positive, see **figure 4.B2.7-8**.

Private sector is benefited from supple character of its commitment under negative demand shock ($\Pi 2 > \Pi 1$), producing more in the first period rather than in the second ($q_{20} > q_{21}$). Under positive demand shock public sector is benefited from strict firing restriction regime that face ($\Pi 1 > \Pi 2$), producing higher quantity in the second period than in the first ($q_{11} > q_{10}$). Although the private sector produces more under positive demand, in the second period too, does not produce more than the public sector in the second period; ($q_{20} > q_{10}$ and $q_{11} > q_{21}$), irrespective of g. Surprisingly, in this case, contrary to all the other cases, private sector conquer the market ($tq_2 > tq_1$)[27]. Consequently, there are two influences on utility of each sector. The first one is due to wages; the second one is due to products. Thus, under positive demand shock $U1 > U2$, the influence of wage and product of the second period are stronger related to first. Under negative demand shock, though, the stimulus is reversed, thus $U1 < U2$. The results are similar, even the products are perfect or imperfect substitutes, see **figure 3.B2.9-12**.

Comparing each alternative regime with the case where no firing restriction regime (frr_0) is imposed accrues that under re-bargaining the results are similar to the case where wages bargained once at the whole game (see Ch. 3). Thus, briefly, I will comment the behavior of firms. Firstly, a possible abolition of the adjustment cost (frr_0) stimulates public sector's production in the first period and private sector's production in the second. Note that firms are strategic substitutes across firms, thus the allocation of the total production is re-adjusted in order to firms to possess equal share in the market under frr_0, see **figures 4.B2.13-14**.

Private sector's product of the first period as well as its total production, and consequently $U2$ are higher under frr_2 than under frr_0. Moreover, in the second period both firms together produce more under frr_1. As for wages w_{21} does not affected essentially by c and w_{20} is affected positively by c under frr_2 regime, see **figures 4.B2.15** and **4.B2.19**. Thus, contrary to previous case, (frr_1 under re-bargaining) and to the case where wages bargained once at the whole game, $U2$ is always increasing with c. If g=1, then $\Pi 1$ and $U1$ are increasing with c, provided that $c \in [0.28, 0.5]$ and if g=0.5, then $\Pi 1$, $\Pi 2$, and $U1$ are increasing with c provided that $c \in [0.38, 0.7]$, see **figures 4.B2.20-21**. Therefore, I conclude that there is a tacit agreement, between f1, f2, U1, and U2 provided that g=1 and there is a tacit agreement between f1, U1, and U2 if g=0.5. Proposition 5 summarizes.

[27] Where $tq_1 = q_{10} + q_{11}$ and $tq_2 = q_{20} + q_{21}$.

Proposition 5: *Under frr_2 regime wage differentials are in favor of the private sector in the first period but in favor of the public sector in the second, provided that interior solution is ensured. Wage differentials are affected by the degree of homogeneity between products, contrary to frr_1 under re-bargaining. According to reaction functions we observe that only for the public sector holds that changes in the product of the first period affects positively the product of the next period, due to the adjustment cost that only this sector face. Strict firing restriction regime is proved to be beneficial for the public sector but only under positive demand shock. Adjustment cost receives the approval of the majority of economic agents.*

In order to assess the impact of varX on variables and yields I assume that if g=1 then c=0.5 and if g=0.5 then c=0.7. The results are similar to the previous case except from the following differences. First, in both periods, under frr_2 wage differentials affected by v, therefore affected by $\text{var}\, X$. Wage differentials in the first period increase as the probability a positive demand shock to occur increases. In the second period wage differentials increase as the negative demand shock is more likely to occur. Second, q_{20} increases as the negative demand shock is more likely to occur. Note that the private sector does not face any firing restriction regime under frr_2, thus conquer higher share of the market in the first period satisfying the demand of the first period. And thirdly, the differences (q_{10}-q_{20}), (q_{11}-q_{21}) and (tq_1-tq_2) under frr_1 do not affected by $\text{var}\, X$ but under frr_2 affected, see **figures 4.B2.22 – 3.B2.34**.

4.3 Conclusions

In chapter 4, I expand the model that I construct in chapter 3 assuming that wages are re-bargained. Thus, the game deployed in two periods/ five stages. Under the new assumption wage differentials are figured in favor of the private sector in the first period, but in favor of the public sector in the second, provided that sectors face an asymmetric firing restriction regime. Contrary to chapter 3, under re-bargaining interior solution is ensured even if v lies in a wider interval, namely v $\in$ [0.3, 1.7].[28] This happens because

[28] Recall that in case where wages bargained once at the whole game v $\in$ [0.5, 1.5].

firms are in position to re-adjust their strategy concerning wages, under re-bargaining. On the other hand, firms are less tolerant to the adjustment cost, especially under frr_1.

Finally, the triple alliance is succeeded under frr_2, where both unions and f1 accept the institutionalization of *c*. Under frr_1, though, adjustment cost does not receive the approval of the majority of economic against, except from the case where g=1, $c \geq 0.4$ and $v < 0.7$, where there is alliance between f1 and both unions, contrary to the case where wages bargained once at the whole game, where the market seems to accept frr_1 more than frr_2. Analytically, the main difference under frr_1 regime in case where wages bargained once at the whole game and in case frr_1 under re-bargaining is behavior of f2. In the first case, f2 accepts the institutionalization of *c*, but in the second do not. *U*2 accepts the institutionalization of *c* in both cases under restrictions. On the other hand, under frr_2 regime, the strike difference is due to *U*2. Thus, in case where wages bargained once at the whole game *U*2 does not support frr_2, but under re-bargaining, supports it. In any case, seems that under re-bargaining unions, especially *U*2, has more power to influence wages positively, as a "compensation" for the lower level of employment in the private sector. In contrast, in case where wages bargained once at the whole game seems that f2 has more power to influence wages.

Chapter 5

A unionized mixed oligopoly model with stochastic demand shocks: public-private wage differentials under an alternative prism

5.1 Introduction

The main aim of this chapter is to reconsider the model that I described in chapter 3 under an alternative assumption about the social welfare-objective function of the public firm. In particular, I assume that public-firm objective function depends on public-firm profits and consumer surplus,[29] namely $EV_1 = \Pi_1 + CS$. Under this alternative assumption the model becomes more complicated but also more realistic. Thus, similar to chapter 3, the game is extended in two periods/four stages and wages are bargained once at the whole game. Given that the methodology is the same I am going to focus, only, on the crucial points of this analysis, however in the appendices 5.A1-A3 I solve the game, step by step, under frr_1,[30] frr_2,[31] and frr_0.[32] Notice that, frr_3 and frr_0 cases give the same result concerning public-private wage differentials, because they are symmetrical cases as for the adjustment cost. The main difference in that the solution of frr_3 is more complicated, thus it is omitted. The main conclusion is that the results concerning public-private wage differentials have strengthened.

5.2 Endogenous public-private wage differentials under the alternative prism

Under the alternative assumption about the public-firm objective function there is a public-private objective function differential that leads to public-private wage differentials in favor of the public sector, even if firms face the same firing restriction regime.[33]

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[29] In the context of a unionized mixed duopoly, public-firm objective function depends on the welfare of the participants of the game, see Kangsik (2009). In the context of my model, though, public-firm objective function depends only on public-firm profits and *CS*, given the assumptions about the total cost and union's objective function.
[30] See appendix 5.A1.
[31] See appendix 5.A2.
[32] See appendix 5.A3.
[33] In the appendix 5.A3 compare reaction functions of the first period; $\frac{H1}{I1} < \frac{H2}{I2}$, (eq. 5.A3.9-5.A3.10).

Consequently, under frr_1 of the alternative prism, in the first period, public-private wage differentials are formed in favor of the public sector, due to the public-private objective function differential and in the second due to the public-private adjustment cost differential.[34] Moreover, its firm own product are complements across periods, however, according to the reaction functions it is clear the non-competitive character of the public firm and the competitive character of the private firm. Public sector is the dominant employer and under this alternative prism produces more that the private sector in both periods.

Interior solution is ensured provided that products are imperfect substitutes, $g \in (0, 0.2]$. Both firms have strong incentive to differentiate their products. Restrictions related to v, namely $v \in [0.7, 1.3]$ are imposed by both firms also. The public sector imposes the restriction related to c, namely $c \leq 0.37$. Yields Π_2, U_1, U_2, CS, and EV_1 are positive provided that interior solution is ensured. See **figures 5.B1.1-5.B1.12c**, in the appendix 5.B. In this game, public-sector profits, consumer surplus, and union's utility are increasing simultaneously with c, provided that $0.27 < c < 0.37$. Note also that private firm votes in favor of the proposal regime under negative demand shock. See **figures 5.B1.12d-5.B1.13**, in the appendix 5.B.

In case frr_2-under the alternative prism wage differentials are also in favor of the public sector. In this case interior solution is ensured provided that $g \in (0, 0.2]$ and $c \leq 0.4$. In this case the public firm and private-firm labor union votes in favor of the proposal regime provided that $0.1 \leq c \leq 0.4$. Private firm votes in favor of the proposal regime for $v < 1$. The frr_3 case under the alternative prism does not receive the approval of the majority of economic agents. Wage differentials are also in favor of the public sector, see figures in the appendices 5.B2 and 5.B3.

[34] Note that his observation implies that public-private wage differentials are formed in favor of the public sector in both periods even in case where wages are re-bargaining.

Chapter 6
General conclusions

The main aim of this thesis is to investigate public-private wage differentials under alternative assumptions about the rigidities of markets that the public and the private sector face, due to legislation that protects workers from being unfairly fired. For this purpose, I construct a unionized mixed oligopoly model with stochastic demand shocks, extended in two periods.

Provided that the wages are bargained once at the whole game, the main finding is that public-private wage differentials emerge whenever the public firm faces stricter firing restriction regime than the private firm, provided that both firms maximizes their profits. In this case, the public sector is the dominant employer and the total production of the public sector is higher than that of the private sector. If firms face the same firing restriction regime, then there are no wage differentials and firms conquer equal share in the labor market. The proposal regimes frr_1 and frr_2 receive the approval of the majority of the participants of this game.

Wage differentials are also formed in favor of the public sector even in case where public sector maximizes a social welfare, instead of maximizing its profits. In this case, the game becomes more complicated but also more realistic. The different maximization problem that the public and the private sectors face leads the wage differential in favor of the public sector even if firms face the same firing restriction regime. Under this view, the public sector is the dominant employer. Public sector produces more than the private sector in both periods of the game. The proposal regimes frr_1, frr_2, and frr_3 receive the approval of the majority of the participants of this game.

On the other hand, provided that firms maximize their profits the assumption that the wages are re-bargained lead the wage differential in favor of the private sector in the first period but in favor of the public sector in the second, after the uncertainty is resolved, given that the public sector faces stricter firing restriction regime. Under re-bargain more uncertainty lies in the market and the public firm is reluctant to pay high enough wages in the first round of bargaining. The public sector is the dominant employer under frr_1 but the private sector is the dominant employer under frr_2. Note that if the public firm maximizes the social welfare then the wage differential would form in favor of the public sector in the first period. Under re-bargaining the proposal regime that receives the approval of the majority of the participants of this game is frr_2.

For further research maybe it is interesting to study the impact of different polices such as taxes or subsidizes on wages and employment, Fullerton and Henderson (1985). Finally, it seems worthy to assess wage differentials constructing a macroeconomic model, McDonald and Solows (1981).

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APPENDIX 3.A1

In order to get first-period reaction functions I maximize the profits Π_1 and Π_2 as for q_{10} and q_{20}, respectively[35]

$$\Pi_1 (q_{10}, q_{20}, w_{10}, w_{20})= -q_{10} (-1 + q_{10} + g\, q_{20}) + (c\, (c^2 + (-4 + g^2)\, q_{10} + 2\, v - g\, v - 2\, w_{10} - c\, (-2 + 2\, q_{10} + g\, q_{20} - v + w_{10}) + g\, w_{20})\, (c^2 + 2\, (4 + 2\, q_{10} - v + w_{10}) + c\, (6 + 2\, q_{10} + g\, q_{20} - v + w_{10}) - g\, (g\, (2 + q_{10}) - v + w_{20})))/2K^2 - w_{10}\, (q_{10} + (c^2\, (1 + q_{10}) - (-2 + g)\, v + c\, (2 + 2\, q_{10} - g\, q_{20} + v - w_{10}) - 2\, w_{10} + g\, w_{20})/K) - 1/(K^2)\, (c^2\, (1 + q_{10}) - (-2 + g)\, v + c\, (2 + 2\, q_{10} - g\, q_{20} + v - w_{10}) - 2\, w_{10} + g\, w_{20})\, (c^2\, (1 + q_{10} + g\, q_{20} - v) - 2\, (v + w_{10}) + g\, (v + g\, w_{10} - w_{20}) - c\, (-2 - 2\, q_{10} + 3\, v + w_{10} + g\, (g + g\, q_{10} - q_{20} - v + w_{20}))) \qquad (3.A1.1)$$

$$\Pi_2 (q_{10}, q_{20}, w_{10}, w_{20})= -q_{20} (-1 + g\, q_{10} + q_{20}) - (c\, (-(-4 + g2)\, q_{20} + (-2 + g)\, v - g\, w_{10} + 2\, w_{20} + c\, (g + g\, q_{10} + 2\, q_{20} - v + w_{20}))2)/2\, K^2 - ((c2\, q_{20} - (-2 + g)\, v + g\, w_{10} - 2\, w_{20} - c\, (g + g\, q_{10} - 2\, q_{20} - v + w_{20}))\, (c^2\, (g + g\, q_{10} + q_{20} - v) + c\, (2\, q_{20} - 3\, v + g\, (1 + q_{10} - g\, q_{20} + v - w_{10}) - w_{20}) - 2\, (v + w_{20}) + g\, (v - w_{10} + g\, w_{20})))/(K^2) - w_{20}\, (q_{20} + (c^2\, q_{20} - (-2 + g)\, v + g\, w_{10} - 2\, w_{20} - c\, (g + g\, q_{10} - 2\, q_{20} - v + w_{20}))/K) \qquad (3.A1.2)$$

Where, $K = (2 + c - g)\, (2 + c + g)$

Equilibrium point of the first period is

$$q_{10}{}^* = -(c3\, (-32\, (1 + 7\, v - 22\, w_{10}) + g4\, (19 + v + 2\, w_{10}) - 4\, g2\, (31 + 27\, w_{10}) + 32\, g\, (6 + 5\, v - 11\, w_{20}) - 4\, g^3\, (7 + 3\, v - 10\, w_{20})) + c5\, (28 - 36\, v + 80\, w_{10} - 2\, g^2\, (11 + w_{10}) + 20\, g\, (1 + v - 2\, w_{20})) + 2\, c^6\, (2 - 2\, v + 4\, w_{10} + g\, (1 - g + v - 2\, w_{20})) + (-4 + g^2)^3\, (2 - 2\, w_{10} + g\, (-1 + w_{20})) + 2\, c^4\, (28 - 64\, v + 164\, w_{10} + g\, (42 + 40\, v - 82\, w_{20} + g\, (-6\, (7 + 2\, w_{10}) + g\, (-2 + 2\, g - v + 3\, w_{20})))) - c\, (-4 + g2)\, (-16\, (5 + v - 8\, w_{10}) + g\, (16\, (3 + v - 4\, w_{20}) + g\, (-4\, (-4 + v + 7\, w_{10}) + g\, (-12 + g + g\, w_{10} + 12\, w_{20})))) - c^2\, (64\, (4 + 3\, v - 13\, w_{10}) + g\, (-32\, (8 + 5\, v - 13\, w_{20}) + g\, (8 + 232\, w_{10} + g\, (4\, (19 + 6\, v - 25\, w_{20}) + g\, (-2\, (11 + 2\, v + 7\, w_{10}) + g\, (-3 + g + 3\, w_{20})))))))/Z \qquad (3.A1.3)$$

$$q_{20}{}^*= -(2\, c^4\, (g\, (26 + 40\, v - 82\, w_{10} + g^2\, (6 - v + 3\, w_{10}) - 12\, g\, (-1 + w_{20})) - 4\, (25 + 16\, v - 41\, w_{20})) + (-4 + g^2)^3\, (2 + g\, (-1 + w_{10}) - 2\, w_{20}) + 2\, c^6\, (-2 + g - 2\, v + g\, v - 2\, g\, w_{10} + 4\, w_{20}) + 2\, c^5\, (-22 - 18\, v + 40\, w_{20} + g\, (8 + g + g^2 + 10\, v - 20\, w_{10} - g\, w_{20})) + c\, (-4 + g^2)\, (16\, (7 + v - 8\, w_{20}) + g\, (-16\, (2 + v - 4\, w_{10}) + g\, (4\, (-8 + v + 7\, w_{20}) + g\, (4 + g + g^2 - 12\, w_{10} - g\, w_{20})))) + c^3\, (-32\, (15 + 7\, v - 22\, w_{20}) + g\, (32\, (3 + 5\, v - 11\, w_{10}) + g\, (-108\, (-1 + w_{20}) + g\, (24 - 12\, v + 40\, w_{10} + g\, (-3 - 3\, g + v + 2\, w_{20}))))) + c^2\, (-64\, (10 + 3\, v - 13\, w_{20}) + g\, (32\, (4 + 5\, v - 13\, w_{10}) + g\, (-232\, (-1 + w_{20}) + g\, (4 - 24\, v + 100\, w_{10} - 3\, g^2\, (3 + w_{10}) + 2\, g\, (-9 + 2\, v + 7\, w_{20}))))))/Z \qquad (3.A1.4)$$

$$Z = (16\, (1 + c)^2\, (2 + c)^4 - 4\, (2 + c)^6\, g^2 + 8\, (2 + c)^2\, (3 + c\, (3 + c))\, g^4 - 4\, (2 + c)^2\, g^6 + g^8)$$

Given the optimal level of employment of the first and the second period, profits depend on wages only

$$\Pi_1 (w_{10}, w_{20})= ((2 + c)\, (16\, c^{12}\, (-4 + g^2)^2 + 16\, c^{11}\, (g^4\, (20 + v) + 4\, (65 + 6\, v - 4\, w_{10}) + 4\, (v - 2\, w_{10})^2 - 4\, g\, (1 + v - 2\, w_{10})\, (1 + v - 2\, w_{20}) + g^2\, (-143 - 6\, v + v^2 - 4\, (1 + v)\, w_{20} + 4\, w_{20}{}^2)) + (-4 + g^2)^6\, (4\, (1 + v^2 - 2\, v\, w_{10} + 2\, (-1 + w_{10})\, w_{10}) + g^2\, (1 + v^2 - 2\, v\, w_{20} + 2\, (-1 + w_{20})\, w_{20}) + 4\, g\, (-1 - v^2 + w_{10} + w_{20} - 2\, w_{10}\, w_{20} + v\, (w_{10} + w_{20}))) - 4\, c^{10}\, (14\, g^6 - 16\, (467 + 19\, v^2 + v\, (102 - 72\, w_{10}) + 72\, (-1 + w_{10})\, w_{10}) + 4\, g^5\, (v - w_{20}) + 32\, g\, (8 + 19\, v + 9\, v^2 - 18\, w_{10} - 18\, v\, w_{10} - 18\, (1 + v - 2\, w_{10})\, w_{20}) + 4\, g3\, (1 - 10\, v - 3\, v^2 + 6\, w_{10} + 6\, v\, w_{10} + 6\, (1 + v - 2\, w_{10})\, w_{20}) - 16\, g^2\, (-281 + 3\, v^2 - 4\, (-1 + w_{10})\, w_{10} + 2\, v\, (-15 + 2\, w_{10} - 9\, w_{20}) + 18\, (-1 + w_{20})\, w_{20}) + g^4\, (-763 +$$

[35] I use the program "mathematica" in order to solve the game

$v^2 + 8 w_{10} + 8 (-1 + w_{20}) w_{20} - 2 v (41 + 4 w_{20}))) - 2 c (-4 + g^2)^4 (-32 (5 + 3 v + 7 v^2 - 15 (1$
$+ v) w_{10} + 15 w_{10}^2) + g^5 (-1 + v + v^2 + w_{10} (-1 + w_{20}) - v w_{20}) + 16 g (11 + 6 v + 13 v^2 - 15$
$w_{10} - 15 v w_{10} - 15 (1 + v - 2 w_{10}) w_{20}) + 2 g^4 (7 + 5 v^2 + w_{10} + v w_{10} - w_{10}^2 - 13 (1 + v) w_{20}$
$+ 13 w_{20}^2) + 4 g^3 (-10 - 14 v^2 + 14 w_{10} + 13 w_{20} - 27 w_{10} w_{20} + v (-3 + 13 w_{10} + 14 w_{20})) + 8$
$g^2 (-2 + 2 v^2 + 14 (-1 + w_{10}) w_{10} - 15 (-1 + w_{20}) w_{20} + v (-1 - 14 w_{10} + 15 w_{20}))) - 8 c^9 (g^6$
$(101 + 7 v) + 8 (-1947 - 163 v^2 - 584 (-1 + w_{10}) w_{10} + v (-774 + 584 w_{10})) + g^3 (22 - 2 v$
$(182 + 57 v - 110 w_{10}) + 228 w_{10} + 232 w_{20} + 224 (v - 2 w_{10}) w_{20}) + 8 g (115 + 322 v + 147$
$v^2 - 292 w_{10} - 292 v w_{10} - 292 (1 + v - 2 w_{10}) w_{20}) + g^5 (-5 + 2 v^2 - 2 w_{10} + v (35 - 2 w_{10} - 3$
$w_{20}) - 29 w_{20} + 4 w_{10} w_{20}) - 4 g^2 (-2541 + 19 v^2 - 140 (-1 + w_{10}) w_{10} + 2 v (-265 + 70 w_{10} -$
$146 w_{20}) + 292 (-1 + w_{20}) w_{20}) + 2 g^4 (-1072 + 3 v^2 + (28 - 5 w_{10}) w_{10} + v (-194 + 6 w_{10} - 41$
$w_{20}) + w20 (-43 + 42 w_{20}))) + 2 c^3 (-4 + g^2) (g^{10} (3 + 2 v + w_{10}) - 512 (66 + 164 v^2 + v (225$
$- 403 w_{10}) + 403 (-1 + w_{10}) w_{10}) + 256 g (169 + 362 v + 275 v^2 - 403 w_{10} - 403 v w_{10} - 403$
$(1 + v - 2 w_{10}) w_{20}) - 64 g^3 (286 + 661 v^2 - 892 w_{10} + v (803 - 840 w_{10} - 883 w_{20}) - 867 w_{20}$
$+ 1732 w_{10} w_{20}) + 16 g^5 (36 + v (667 + 543 v - 554 w_{10}) - 626 w_{10} - 597 w_{20} + 59 (-11 v +$
$20 w_{10}) w_{20}) + 2 g^8 (18 + 20 v^2 + 2 (4 - 5 w_{10}) w_{10} + v (-65 + 6 w_{10} - 85 w_{20}) + 85 (-1 + w_{20})$
$w_{20}) + g^9 (-29 + 6 v^2 - 5 w_{10} - 22 w_{20} + 6 w_{10} w_{20} - v (-51 + w_{10} + 6 w_{20})) + g^7 (440 - 588 v^2$
$+ 568 w_{10} + 620 w_{20} - 976 w_{10} w_{20} + 4 v (-277 + 102 w_{10} + 159 w_{20})) + 128 g^2 (-30 + 316 v^2$
$+ 903 (-1 + w_{10}) w_{10} - 403 (-1 + w_{20}) w_{20} + v (442 - 903 w_{10} + 403 w_{20})) - 8 g^6 (216 + 36 v^2$
$+ (169 - 167 w_{10}) w_{10} + v (-206 + 167 w_{10} - 511 w_{20}) + w_{20} (-527 + 519 w_{20})) - 32 g^4 (-288 +$
$152 v^2 + w_{10} (-659 + 665 w_{10}) + (845 - 829 w_{20}) w_{20} + v (362 - 677 w_{10} + 813 w_{20}))) + 4 c^7$
$(g^8 (151 + 15 v + 2 w_{10}) + 64 (2314 + 704 v^2 + 3 v (833 - 747 w_{10}) + 2241 (-1 + w_{10}) w_{10}) +$
$32 g (-715 - 2590 v - 1177 v^2 + 2241 w_{10} + 2241 v w_{10} + 2241 (1 + v - 2 w_{10}) w_{20}) + 8 g^3$
$(19 + 1224 v^2 - 2295 w_{10} + v (3323 - 2149 w_{10} - 2234 w_{20}) + 44 (-53 + 101 w_{10}) w_{20}) +g^7 (-$
$45 + 9 v^2 - 8 w_{10} + 2 (-55 + 7 w_{10}) w_{20} - 2 v (-79 + 3 w_{10} + 6 w_{20})) + 2 g^6 (-2127 + 3 v^2 + 2$
$(29 - 12 w_{10}) w_{10} + 2 v (-319 + 13 w_{10} - 73 w_{20}) + 2 w_{20} (-81 + 77 w_{20})) + 4 g^5 (228 - 191 v^2$
$+ 268 w_{10} + 544 w_{20} - 509 w_{10} w_{20} + v (-880 + 241 w_{10} + 299 w_{20})) + 8 g^4 (4187 + 43 v^2 + 4$
$w_{10} (-131 + 94 w_{10}) + (995 - 959 w_{20}) w_{20} + v (1996 - 407 w_{10} + 923 w_{20})) - 16 g^2 (6656 +$
$468 v^2 + 2526 (-1 + w_{10}) w_{10} - 2241 (-1 + w_{20}) w_{20} + v (4529 - 2526 w_{10} + 2241 w_{20}))) + 4$
$c^8 (16 g^8 + 16 (5197 + 833 v^2 + v (3450 - 2816 w_{10}) + 2816 (-1 + w_{10}) w_{10}) + 2 g^7 (-1 + 6 v -$
$5 w_{20}) + 32 g (-247 - 801 v - 360 v^2 + 704 w_{10} + 704 v w_{10} + 704 (1 + v - 2 w_{10}) w_{20}) + g^6 (-$
$1254 + v^2 - 2 (-5 + w_{10}) w_{10} + 2 v (-102 + w_{10} - 10 w_{20}) + w_{20} (-22 + 21 w_{20})) + 4 g^5 (37 - 21$
$v^2 + 27 w_{10} + 108 w_{20} - 52 w_{10} w_{20} + v (-157 + 25 w_{10} + 33 w_{20})) + 8 g^3 (243 v^2 + v (725 -$
$449 w_{10} - 463 w_{20}) + 924 w_{10} w_{20} - 5 (4 + 95 w_{10} + 97 w_{20})) - 4 g^4 (-3772 + 6 v^2 + (205 - 93$
$w_{10}) w_{10} + 3 v (-364 + 35 w_{10} - 121 w_{20}) + w_{20} (-389 + 376 w_{20})) - 4 g^2 (14477 + 165 v^2 +$
$2192 (-1 + w_{10}) w_{10} - 2816 (-1 + w_{20}) w_{20} + v (5454 - 2192 w_{10} + 2816 w_{20}))) - 4 c^5 (2 g^{10}$
$(19 + 3 v + w_{10}) - 256 (592 + 686 v^2 + 7 v (237 - 275 w_{10}) + 1925 (-1 + w_{10}) w_{10}) + 896 g$
$(85 + 306 v + 159 v^2 - 275 w_{10} - 275 v w_{10} - 275 (1 + v - 2 w10) w_{20}) + 8 g^5 (-575 + 1615 v^2$
$- 2255 w_{10} + v (3853 - 2014 w_{10} - 2335 w_{20}) - 2558 w_{20} + 4269 w_{10} w_{20}) + g^9 (-47 + v (110$
$+ 8 v - 3 w_{10}) - 6 w_{10} - 62 w_{20} + 9 (-v + w_{10}) w_{20}) + g^8 (-977 + 21 v^2 + 2 (5 - 16 w_{10}) w_{10} + v$
$(-570 + 26 w_{10} - 226 w_{20}) - 242 w_{20} + 234 w_{20}^2) + 4 g^7 (288 - 200 v^2 + 214 w_{10} + 421 w_{20} -$
$387 w_{10} w_{20} + v (-763 + 173 w_{10} + 254 w_{20})) + 64 g^2 (1268 + 1176 v^2 + 3878 (-1 + w_{10}) w_{10}$
$- 1925 (-1 + w_{20}) w_{20} + 7 v (555 - 554 w_{10} + 275 w_{20})) - 32 g^3 (502 + 2268 v2 - 3740 w_{10} -$
$3725 w_{20} + 7255 w_{10} w_{20} - 5 v (-925 + 703 w_{10} + 734 w_{20})) + 8 g^6 (945 + 29 v^2 + 2 w_{10} (-$
$167 + 147 w_{10}) + 9 (101 - 97 w_{20}) w_{20} + v (1175 - 310 w_{10} + 837 w_{20})) - 16 g^4 (1741 + 645$
$v^2 + w_{10} (-2643 + 2557 w_{10}) + (3477 - 3377 w_{20}) w_{20} + v (4094 - 2635 w_{10} + 3277 w_{20}))) +$
$c^4 (5 g^{12} + 2048 (197 + 403 v^2 + v (759 - 1059 w_{10}) + 1059 (-1 + w_{10}) w_{10}) + 2 g^{11} (-5 + 11$
$v - 6 w_{20}) + 128 g^7 (-58 + 193 v + 85 v^2 - 94 w_{10} - 78 v w_{10} - 4 (29 + 26 v - 43 w_{10}) w_{20}) +$
$1024 g (-365 - 1094 v - 659 v^2 + 1059 w^{10} + 1059 v w^{10} + 1059 (1 + v - 2 w^{10}) w_{20}) - 8 g^9 (-$
$93 + 39 v^2 - 34 w_{10} + v (189 - 21 w_{10} - 43 w_{20}) - 92 w_{20} + 55 w_{10} w_{20}) + 512 g^3 (304 + 882$
$v^2 - 1339 w_{10} + v (1457 - 1270 w_{10} - 1322 w_{20}) - 1321 w_{20} + 2609 w_{10} w_{20}) + g^{10} (-241 + 11$
$v^2 - 4 w_{10} (6 + w_{10}) + 48 (-1 + w_{20}) w_{20} - 2 v (71 + 24 w_{20})) + 4 g^8 (461 - 93 v^2 + 6 w_{10} (-32$

$+ 35 w_{10}) + 24 (44 - 43 w_{20}) w_{20} + 2 v (557 - 98 w_{10} + 504 w_{20})) - 64 g^{5} (-45 + 1723 v^{2} + v$
$(3008 - 2079 w_{10} - 2347 w_{20}) - 2339 w_{20} + 7 w_{10} (-329 + 626 w_{20})) - 512 g^{2} (187 + 927 v^{2} +$
$2730 (-1 + w_{10}) w_{10} - 1059 (-1 + w_{20}) w_{20} + v (2005 - 2730 w_{10} + 1059 w_{20})) - 16 g^{6} (-135 +$
$229 v^{2} + 8 w_{10} (-228 + 223 w_{10}) + 3974 w_{20} - 3858 w_{20}^{2} + v (3032 - 1840 w_{10} + 3742 w_{20}))$
$+ 64 g^{4} (-395 + 1391 v^{2} - 4950 w_{10} + 4936 w_{10}^{2} + 5084 w_{20} - 4976 w_{20}^{2} + v (4552 - 5018$
$w_{10} + 4868 w_{20}))) - 4 c^{6} (7 g^{10} - 896 (200 + 118 v^{2} + v (351 - 353 w_{10}) + 353 (-1 + w_{10}) w_{10})$
$+ g^{9} (-4 + 13 v - 9 w_{20}) + 448 g (107 + 406 v + 193 v2 - 353 w_{10} - 353 v w_{10} - 353 (1 + v - 2$
$w_{10}) w_{20}) + 8 g^{5} (-365 + 495 v^{2} - 706 w_{10} + v (1606 - 629 w_{10} - 751 w_{20}) - 985 w_{20} + 1335$
$w_{10} w_{20}) - 16 g^{3} (229 + 2016 v^{2} - 3569 w_{10} + v (4832 - 3338 w_{10} - 3484 w_{20}) - 3593 w_{20} +$
$6907 w_{10} w_{20}) + g^{8} (-566 + 2 v^{2} - w_{10} (9 + 2 w_{10}) + v (-146 + w_{10} - 21 w_{20}) + w_{20} (-23 + 22$
$w_{20})) + 2 g^{7} (167 - 65 v^{2} + 65 w_{10} + 273 w_{20} - 116 w_{10} w_{20} + v (-460 + 51 w_{10} + 85 w_{20})) +$
$224 g^{2} (550 + 138 v^{2} + 542 (-1 + w_{10}) w_{10} - 353 (-1 + w_{20}) w_{20} + v (725 - 542 w_{10} + 353$
$w_{20})) + 2 g^{6} (4077 + 7 v^{2} + 10 w_{10} (-34 + 23 w_{10}) + 16 (64 - 61 w_{20}) w_{20} + v (2226 - 248 w_{10}$
$+ 928 w_{20})) - 4 g^{4} (11134 + 705 v^{2} + 5 w_{10} (-786 + 697 w_{10}) + 22 (293 - 283 w_{20}) w_{20} + v$
$(9820 - 3664 w_{10} + 6006 w_{20}))) - c^{2} (-4 + g^{2})^{2} (-512 (22 + 44 v^{2} + v (39 - 101 w_{10}) + 101 (-1$
$+ w_{10}) w_{10}) + 16 g^{5} (31 + 82 v + 115 v^{2} - 119 w_{10} - 103 v w_{10} - 3 (35 + 41 v - 74 w_{10}) w_{20}) +$
$256 g (55 + 70 v + 77 v^{2} - 101 w_{10} - 101 v w_{10} - 101 (1 + v - 2 w_{10}) w_{20}) - 64 g^{3} (99 + 2 v$ (3.A1.5)
$(64 + 85 v - 96 w_{10}) - 203 w_{10} - 195 w_{20} - 202 v w_{20} + 395 w_{10\ w20}) - 8 g^{6} (54 + 23 v^{2} + (20 -$
$19 w_{10}) w_{10} + v (-13 + 18 w_{10} - 99 w_{20}) + 99 (-1 + w_{20}) w_{20}) + g^{9} (4 v - 2 (1 + w_{20})) + 4 g^{7}$
$(15 - 18 v^{2} + 17 w_{10} + 13 w_{20} - 25 w_{10} w_{20} + 2 v (-14 + 4 w_{10} + 9 w_{20})) + 128 g^{2} (-2 + 70 v^{2}$
$+ 204 (-1 + w_{10}) w_{10} - 101 (-1 + w_{20}) w_{20} + v (51 - 204 w_{10} + 101 w_{20})) - 16 g^{4} (-148 + 19 v^{2}$
$+ w_{10} (-244 + 247 w_{10}) + 386 w_{20} - 382 w_{20}^{2} + v (42 - 250 w_{10} + 378 w_{20})) + g^{8} (5 v^{2} - (-2 +$
$w_{10}) w_{10} - 2 v (4 + 7 w_{20}) + 2 (4 + 7 (-1 + w_{20}) w_{20})))))/2I^{2}$

Where, $I = (16 (1 + c)^{2} (2 + c)^{4} - 4 (2 + c)^{6} g^{2} + 8 (2 + c)^{2} (3 + c (3 + c)) g^{4} - 4 (2 + c)^{2} g^{6} + g^{8})$

$\Pi_{2} = ((2 + c) (16 c^{11} (-2 + g - 2 v + g v - 2 g w_{10} + 4 w_{20})^{2} + (-4 + g^{2})^{6} (g ((-4 + g) (1 + v^{2}) -$
$2 (-2 + g) (1 + v) w_{10} + 2 g w_{10}^{2}) + 4 (1 + v^{2} - 2 w_{20}) + 4 (-2 v + g (1 + v - 2 w_{10})) w_{20} + 8$
$w_{20}^{2}) + 8 c^{9} (g^{6} (9 + w_{10}) + g^{5} (21 - 19 v - 2 v^{2} + w_{10} + 3 v w_{10} + 2 (-1 + v - 2 w_{10}) w_{20}) + 8 g$
$(-115 - 322 v - 147 v^{2} + 292 w_{10} + 292 v w_{10} + 292 (1 + v - 2 w_{10}) w_{20}) + 8 (163 + 163 v^{2} +$
$v (258 - 584 w_{20}) + 584 (-1 + w_{20}) w_{20}) - 2 g^{4} (24 + 3 v^{2} + w_{10} (-17 + 42 w_{10}) + (4 - 5 w_{20})$
$w_{20} + v (18 - 41 w_{10} + 6 w_{20})) + 4 g^{2} (47 + 19 v^{2} + 292 (-1 + w_{10}) w_{10} - 140 (-1 + w_{20}) w_{20} +$
$4 v (29 - 73 w_{10} + 35 w_{20})) + 2 g^{3} (-39 + 57 v^{2} - 62 w_{20} + 112 w_{10} (-1 + 2 w_{20}) - 2 v (-77 +$
$56 w_{10} + 55 w_{20}))) - 4 c^{7} (g^{8} (49 + 3 v + 2 w_{10}) + 32 g (715 + 2590 v + 1177 v^{2} - 2241 w_{10} -$
$2241 v w_{10} - 2241 (1 + v - 2 w_{10}) w_{20}) + 4 g^{5} (-472 + 191 v^{2} - 210 w_{10} + v (636 - 299 w_{10} -$
$241 w_{20}) - 114 w_{20} + 509 w_{10} w_{20}) - 64 (704 + 704 v^{2} + v (833 - 2241 w_{20}) + 2241 (-1 + w_{20})$
$w_{20}) + 16 g^{2} (202 + 468 v^{2} - 2241 (-1 + w_{10}) w_{10} + v (-749 + 2241 w_{10} - 2526 w_{20}) + 2526 (-$
$1 + w_{20}) w_{20}) + g^{7} (113 - 9 v^{2} + 2 w_{10} - 2 (10 + 7 w_{10}) w_{20} + 6 v (-15 + 2 w_{10} + w_{20})) - 2 g^{6}$
$(365 + 3 v^{2} + 22 w_{10} (-3 + 7 w_{10}) + 22 w_{20} - 24 w_{20}^{2} + 2 v (79 - 73 w_{10} + 13 w_{20})) - 8 g^{3}$
$(1224 v^{2} + v (3031 - 2234 w_{10} - 2149 w_{20}) + 4444 w_{10} w_{20} - 13 (21 + 170 w_{10} + 141 w_{20})) -$
$8 g^{4} (-329 + 43 v^{2} + (775 - 959 w_{10}) w_{10} + v (-619 + 923 w_{10} - 407 w_{20}) + w20 (-345 + 376$
$w_{20}))) + c^{4} (-10 g^{12} + 4 g^{11} (-5 + 3 v + 2 w_{20}) + 1024 g (-365 - 1094 v - 659 v^{2} + 1059 w_{10} +$
$1059 v w_{10} + 1059 (1 + v - 2 w_{10}) w_{20}) - 8 g^{9} (-155 + 39 v^{2} - 12 w_{10} + v (127 - 43 w_{10} - 21$
$w_{20}) + 10 w_{20} + 55 w_{10} w_{20}) + 128 g^{7} (-96 + 85 v^{2} - 68 w_{10} + v (155 - 104 w_{10} - 78 w_{20}) - 66$
$w_{20} + 172 w_{10} w_{20}) - 64 g^{5} (-321 + v (2732 + 1723 v - 2347 w_{10}) - 2035 w_{10} - 2055 w_{20} + 7$
$(-297 v + 626 w_{10}) w_{20}) + g^{10} (403 + 11 v^{2} + 6 v (33 - 8 w_{10}) + 48 (-1 + w_{10}) w_{10} - 4 (-2 +$
$w_{20}) w_{20}) + 2048 (403 + 403 v^{2} + v (253 - 1059 w_{20}) + 1059 (-1 + w_{20}) w_{20}) - 512 g^{2} (941 +$
$927 v^{2} - 1059 (-1 + w_{10}) w_{10} + v (-337 + 1059 w_{10} - 2730 w_{20}) + 2730 (-1 + w_{20}) w_{20}) + 64$
$g^{4} (1209 + 1391 v^{2} + 4956 w_{10} - 4976 w_{10}^{2} + v (-3784 + 4868 w_{10} - 5018 w_{20}) - 4854 w_{20} +$
$4936 w_{20}^{2}) - 4 g^{8} (823 + 93 v^{2} + 8 w_{10} (-124 + 129 w_{10}) + 14 (16 - 15 w_{20}) w_{20} + 2 v (785 -$
$504 w_{10} + 98 w_{20})) - 16 g^{6} (-327 + 229 v^{2} + 2 (1891 - 1929 w_{10}) w_{10} + 2 v (-1988 + 1871$

$$
\begin{gathered}
w_{10} - 920 w_{20}) + 8 w_{20} (-216 + 223 w_{20})) + 512 g^3 (261 + 882 v^2 - 1287 w_{10} - 1287 w_{20} + \\
2609 w_{10} w_{20} - 2 v (-707 + 661 w_{10} + 635 w_{20}))) - 2 c^3 (-4 + g^2) (g^{10} (17 + 5 v) + 256 g (-169 \\
- 362 v - 275 v^2 + 403 w_{10} + 403 v w_{10} + 403 (1 + v - 2 w_{10}) w_{20}) + 64 g^3 (266 + 661 v^2 - \\
849 w_{10} + v (783 - 883 w_{10} - 840 w_{20}) - 870 w_{20} + 1732 w_{10} w_{20}) + 4 g^7 (147 v 2 - 85 (2 + \\
w_{10}) + v (217 - 159 w_{10} - 102 w_{20}) + 4 (-23 + 61 w_{10}) w_{20}) + 512 (164 + 164 v 2 + v (75 - \\
403 w_{20}) + 403 (-1 + w_{20}) w_{20}) - 128 g^2 (350 + 316 v^2 - 403 (-1 + w_{10}) w_{10} + v (-202 + 403 \\
w_{10} - 903 w_{20}) + 903 (-1 + w_{20}) w_{20}) + g^9 (49 - 6 v^2 - 13 w_{20} - 6 w_{10} w_{20} + v (-31 + 6 w_{10} + \\
w_{20})) - 2 g^8 (52 + 20 v^2 + 85 (-1 + w_{10}) w_{10} + 2 (7 - 5 w_{20}) w_{20} + v (171 - 85 w_{10} + 6 w_{20})) - \\
16 g^5 (543 v^2 + v (607 - 649 w_{10} - 554 w_{20}) + 59 w_{10} (-9 + 20 w_{20}) - 4 (6 + 143 w_{20})) + 8 g^6 \\
(-20 + 36 v^2 + 519 (-1 + w_{10}) w_{10} - 167 (-1 + w_{20}) w_{20} + v (568 - 511 w_{10} + 167 w_{20})) + 32 \\
g^4 (224 + 152 v^2 + (837 - 829 w_{10}) w_{10} + v (-684 + 813 w_{10} - 677 w_{20}) + w_{20} (-653 + 665 \\
w_{20}))) + 4 c^5 (g^{10} (38 + 5 v) + 896 g (-85 - 306 v - 159 v^2 + 275 w_{10} + 275 v w_{10} + 275 (1 + \\
v - 2 w_{10}) w_{20}) - 8 g^5 (-1111 + 1615 v^2 - 1934 w1_0 + v (3317 - 2335 w_{10} - 2014 w_{20}) - 1807 \\
w_{20} + 4269 w_{10} w_{20}) + 32 g^3 (294 + 2268 v^2 - 3585 w_{10} + v (4417 - 3670 w_{10} - 3515 w_{20}) - \\
3464 w_{20} + 7255 w_{10} w_{20}) + 4 g^7 (-496 + 200 v^2 - 133 w_{10} + v (555 - 254 w_{10} - 173 w_{20}) + \\
43 (-2 + 9 w_{10}) w_{20}) + 1792 (98 + 98 v^2 + v (79 - 275 w_{20}) + 275 (-1 + w_{20}) w_{20}) - 448 g^2 \\
(154 + 168 v^2 - 275 (-1 + w_{10}) w_{10} + v (-87 + 275 w_{10} - 554 w_{20}) + 554 (-1 + w_{20}) w_{20}) - 8 g^6 \\
(-412 + 29 v^2 + (793 - 873 w_{10}) w_{10} + v (-936 + 837 w_{10} - 310 w_{20}) - 278 w_{20} + 294 w_{20}^2) \\
+g^9 (91 - 8 v^2 - 20 w_{20} - 9 w_{10} w_{20} + 3 v (-22 + 3 w_{10} + w_{20})) - g^8 (663 + 21 v^2 + 2 w_{10} (-97 + \\
117 w_{10}) + 38 w_{20} - 32 w_{20}^2 + v (446 - 226 w_{10} + 26 w_{20})) + 16 g^4 (79 + 645 v^2 + (3261 - \\
3377 w_{10}) w_{10} + v (-2544 + 3277 w_{10} - 2635 w_{20}) + w_{20} (-2479 + 2557 w_{20}))) + 4 c^6 (5 g^{10} + \\
g^9 (10 - 7 v - 3 w_{20}) + 448 g (-107 - 406 v - 193 v^2 + 353 w_{10} + 353 v w_{10} + 353 (1 + v - 2 \\
w_{10}) w_{20}) + 2 g^7 (-329 + 65 v^2 - 31 w_{10} + v (298 - 85 w_{10} - 51 w_{20}) + 17 w_{20} + 116 w_{10} w_{20}) - \\
8 g^5 (-686 + 495 v^2 - 584 w_{10} + v (1285 - 751 w_{10} - 629 w_{20}) - 465 w_{20} + 1335 w_{10} w_{20}) + 16 \\
g^3 (-81 + 2016 v^2 - 3423 w_{10} + v (4522 - 3484 w10 - 3338 w_{20}) - 3119 w_{20} + 6907 w_{10} w_{20}) \\
+ 896 (118 + 118 v^2 + v (117 - 353 w_{20}) + 353 (-1 + w_{20}) w_{20}) - 224 g^2 (104 + 138 v^2 - 353 \\
(-1 + w_{10}) w_{10} + v (-113 + 353 w_{10} - 542 w_{20}) + 542 (-1 + w_{20}) w_{20}) - g^8 (252 + 2 v^2 + w_{10} (- \\
7 + 22 w_{10}) + (3 - 2 w_{20}) w_{20} + v (62 - 21 w_{10} + w_{20})) + 2 g^6 (1059 - 7 v^2 + 16 w_{10} (-46 + 61 \\
w_{10}) + 212 w_{20} - 230 w_{20}^2 + v (1042 - 928 w_{10} + 248 w_{20})) + 4 g^4 (-1090 + 705 v^2 + 2 (2823 \\
- 3113 w_{10}) w_{10} + v (-4436 + 6006 w_{10} - 3664 w_{20}) + w_{20} (-3306 + 3485 w_{20}))) + 4 c^{10} (g^6 - 2 \\
g^5 (-1 + v) - g^4 (5 + v (2 + v) - 8 v w_{10} + 8 w_{10}^2) + 4 g^3 (-3 + v (8 + 3 v - 6 w_{10}) - 6 w_{10} - 2 \\
w_{20} - 6 (v - 2 w_{10}) w_{20}) + 32 g (-8 - 19 v - 9 v^2 + 18 w_{10} + 18 v w_{10} + 18 (1 + v - 2 w_{10}) w_{20}) \\
+ 16 (19 + 19 v^2 + v (34 - 72 w_{20}) + 72 (-1 + w_{20}) w_{20}) + 16 g^2 (3 v^2 + v (8 - 18 w_{10} + 4 \\
w_{20}) + 2 (2 + 9 (-1 + w_{10}) w_{10} - 2 (-1 + w_{20}) w_{20}))) + 2 c (-4 + g^2)^4 (g^6 (-1 + v) + 16 g (-11 - \\
6 v - 13 v^2 + 15 w_{10} + 15 v w_{10} + 15 (1 + v - 2 w_{10}) w_{20}) - g^5 (-1 + v + v^2 - v w_{10} + (-1 + \\
w_{10}) w_{20}) + 4 g^3 (10 + 14 v^2 - 13 w_{10} + v (3 - 14 w_{10} - 13 w_{20}) - 14 w_{20} + 27 w_{10} w_{20}) + 32 \\
(7 + v + 7 v^2 - 15 v w_{20} + 15 (-1 + w_{20}) w_{20}) - 2 g^4 (1 + 5 v^2 + 13 (-1 + w_{10}) w_{10} + w_{20} - w_{20} \\
{}^2 + v (6 - 13 w_{10} + w_{20})) - 8 g^2 (2 v^2 - 15 (-1 + w_{10}) w_{10} + v (-7 + 15 w_{10} - 14 w_{20}) + 2 (2 + \\
7 (-1 + w_{20}) w_{20}))) + c^2 (-4 + g^2)^2 (2 g^{10} - 2 g^9 (-2 + v + w_{20}) + 256 g (-55 - 70 v - 77 v^2 + \\
101 w_{10} + 101 v w_{10} + 101 (1 + v - 2 w_{10}) w_{20}) - 16 g^5 (25 + 115 v^2 - 99 w_{10} + v (76 - 123 \\
w_{10} - 103 w_{20}) - 113 w_{20} + 222 w_{10} w_{20}) + g^8 (8 - 5 v^2 - 14 (-1 + w_{10}) w_{10} + 2 v (-24 + 7 \\
w_{10}) + (-2 + w_{20}) w_{20}) + 512 (44 + 44 v^2 + v (13 - 101 w_{20}) + 101 (-1 + w_{20}) w_{20}) + 4 g^7 (18 \\
v^2 - 7 (3 + w_{10}) + (-11 + 25 w_{10}) w_{20} - 2 v (-11 + 9 w_{10} + 4 w_{20})) + 8 g^6 (-16 + 23 v^2 + 99 (- \\
1 + w_{10}) w_{10} + (20 - 19 w_{20}) w_{20} +v (97 - 99 w10 + 18 w_{20})) + 64 g^3 (97 + 170 v^2 - 193 w_{10} \\
- 201 w_{20} + 395 w_{10} w_{20} - 2 v (-63 + 101 w_{10} + 96 w_{20})) + 16 g^4 (96 + 19 v^2 + 2 (193 - 191 \\
w_{10}) w_{10} + v (-282 + 378 w_{10} - 250 w_{20}) + w20 (-244 + 247 w_{20})) - 128 g^2 (70 v^2 - 101 (-1 \\
+ w_{10}) w_{10} + v (-55 + 101 w_{10} - 204 w_{20}) + 12 (7 + 17 (-1 + w_{20}) w_{20}))) - 4 c^8 (4 g^8 - 2 g^7 (- \\
4 + 3 v + w_{20}) + 32 g (247 + 801 v + 360 v^2 - 704 w_{10} - 704 v w_{10} - 704 (1 + v - 2 w_{10}) w_{20}) \\
+ 4 g^5 (21 v^2 - 19 (5 + w_{10}) + v (99 - 33 w_{10} - 25 w_{20}) + 52 w_{10} w_{20}) - 8 g^3 (-105 + 243 v^2 - \\
461 w_{10} + v (640 - 463 w_{10} - 449 w_{20}) - 329 w_{20} + 924 w_{10} w_{20}) + 4 g^2 (-167 + 165 v^2 - \\
2816 (-1 + w_{10}) w_{10} + 2 v (-507 + 1408 w_{10} - 1096 w_{20}) + 2192 (-1 + w_{20}) w_{20}) + 4 g^4 (179 +
\end{gathered}
\tag{3.A1.6}
$$

$$6 v^{2} + w_{10} (-245 + 376 w_{10}) + 81 w_{20} - 93 w_{20}^{2} + v (208 - 363 w_{10} + 105 w_{20})) + 16 (-833 - 833 v^2 - 2816 (-1 + w_{20}) w_{20} + 2 v (-575 + 1408 w_{20})) - g^{6} (v^{2} + w_{10} (10 + 21 w_{10}) + 2 v (10 - 10 w_{10} + w_{20}) + 2 (75 + w_{20} - w_{20}^{2})))))/2I^2.$$

Given the optimal level of employment of the first and the second period, unions utility depend on wages only

$$U_1(w_{10}, w_{20}) = -(w_{10} (-2 c^3 (352 (1 + v - 2 w_{10}) + g (g (-48 - 54 v + 118 w_{10} + g (26 + 20 v + g (1 + v - 2 w_{10}) - 46 w_{20})) - 176 (1 + v - 2 w_{20}))) + c (-4 + g^2) (128 (1 + v - 2 w_{10}) + g (g (-4 (8 + 7 v - 15 w_{10}) + g (16 + (12 + g) v - g w_{10} - 28 w_{20})) - 64 (1 + v - 2 w_{20}))) + c^2 (-832 (1 + v - 2 w_{10}) + g (g (8 (31 + 29 v - 62 w_{10}) + g (g (-14 (1 + v - 2 w_{10}) + g (5 + g + 3 v - 8 w_{20})) - 4 (31 + 25 v - 56 w_{20}))) + 416 (1 + v - 2 w_{20}))) + 2 c^5 (g^2 (-1 + v - 2 w_{10}) - 40 (1 + v - 2 w_{10}) + 20 g (1 + v - 2 w_{20})) + 4 c^6 (-2 + g - 2 v + g v + 4 w_{10} - 2 g w_{20}) - (-4 + g^2)^3 (-2 + g - 2 v + g v + 4 w_{10} - 2 g w_{20}) + 2 c^4 (-164 (1 + v - 2 w_{10}) + g (82 (1 + v - 2 w_{20}) + g (4 + 12 v - 26 w_{10} + g (-4 - 3 v + 7 w_{20}))))))/I \qquad (3.A1.7)$$

$$U_2(w_{10}, w_{20}) = -(w_{20} (2 c^4 (82 g (1 + v - 2 w_{10}) + g^3 (2 - 3 v + 7 w_{10}) + 2 g^2 (7 + 6 v - 13 w_{20}) - 164 (1 + v - 2 w_{20})) + 2 c^5 (g (20 (1 + v - 2 w_{10}) + g (1 + g + v - 2 w_{20})) - 40 (1 + v - 2 w_{20})) + c^2 (-4 g^3 (27 + 25 v - 56 w_{10}) + g^5 (1 + 3 v - 8 w_{10}) + 416 g (1 + v - 2 w_{10}) + 8 g^2 (33 + 29 v - 62 w_{20}) - 832 (1 + v - 2 w_{20}) - 14 g^4 (1 + v - 2 w_{20})) + 4 c^6 (-2 + g - 2 v + g v - 2 g w_{10} + 4 w_{20}) - (-4 + g^2)^3 (-2 + g - 2 v + g v - 2 g w_{10} + 4 w_{20}) - 2 c^3 (352 (1 + v - 2 w_{20}) + g (-176 (1 + v - 2 w_{10}) + g (-64 - 54 v + g (14 + 20 v - 46 w_{10} + g (1 + g + v - 2 w_{20})) + 118 w_{20}))) + c (-4 + g^2) (128 (1 + v - 2 w_{20}) + g (-64 (1 + v - 2 w_{10}) + g (-4 (8 + 7 v - 15 w_{20}) + g (16 + (12 + g) v - 28 w_{10} - g w_{20}))))))/I \qquad (3.A1.8)$$

The results of the bargaining are the following wages

$$w_{10}^* = (16 c^{12} (-2 + g) (4 + g) (1 + v) + (-2 + g)^7 (2 + g)^6 (4 + g) (1 + v) + 8 c^{11} (g^4 - 320 (1 + v) + 80 g (1 + v) - g^3 (1 + v) + 8 g^2 (5 + 6 v)) - 4 c^{10} (5824 (1 + v) + g (-1456 (1 + v) + g (-864 + 62 g - 19 g^2 + (32 + 3 g) (-34 + 5 g) v))) + c (-2 + g)^5 (2 + g)^4 (512 (1 + v) + g (128 (1 + v) + g (-124 - 30 g + g^2 - 4 (29 + 8 g) v))) + c^2 (-2 + g)4 (2 + g)^2 (-14848 (1 + v) + g (-11136 (1 + v) + g (64 (32 + 27 v) + g (2888 + 348 g - 78 g^2 - 9 g^3 - 8 (-353 + g (-56 + g (6 + g))) v)))) - 2 c^9 (63744 (1 + v) + g (-15936 (1 + v) + g (-16 (775 + 943 v) + g (4 (363 + 329 v) + g (40 + g (-13 + 11 g - 9 v) + 564 v))))) + c^5 (-3055616 (1 + v) + g (763904 (1 + v) + g (128 (14317 + 14351 v) + g (-2 g (186272 + g (-28600 + g (-13880 + g (1414 + g (201 + g (-11 + 3 g)))))) + g (-384832 + g (42624 + g (31536 + 5 g (-312 + (-150 + g) g)))) v - 64 (5830 + 5157 v))))) + 2 c^8 (-233536 (1 + v) + g (58384 (1 + v) + g (8 (7679 + 8791 v) + g (-9284 - 8268 v + g (-2522 - 4830 v + g (260 - 82 g + 182 v + 43 g v)))))) + 2 c^7 (-603136 (1 + v) + g (150784 (1 + v) + g (213232 + 230352 v + g (-8 (4660 + 4113 v) + g (-4 (4645 + 6177 v) + g (2160 + 1534 v + g (42 + g (-17 + 9 g - 7 v) + 638 v))))))) + c^3 (-2 + g)^2 (2 + g) (-258048 (1 + v) + g (-64512 (1 + v) + g (64 (2257 + 2179 v) + g (35360 + 37632 v + g (-112 (221 + 198 v) + g (-8 (709 + 778 v) + g (1220 + g^3 + 872 v - g^2 (5 + v) + 8 g (27 + 29 v)))))))) + c^6 (-2250752 (1 + v) + g (562688 (1 + v) + g (11520 (91 + 94 v) + g (-544 (369 + 325 v) + g (-16 (9233 + 10408 v) + g (24 (832 + 603 v) + g (5376 + 8360 v - g (482 + 234 v + g (-63 + 67 v))))))))) + c^4 (-2 + g) (1497088 (1 + v) + g (374272 (1 + v) + g (-256 (3689 + 3625 v) + g (-64 (3617 + 3887 v) + g (64 (3063 + 2866 v) + g (45040 + 50768 v + g (-32 (449 + 375 v) + g (-4 (677 + 810 v) + g (232 + 142 v + g (9 + 31 v)))))))))))/J \qquad (3.A1.9)$$

Where, $J=(2 (-256 (1 + c)^4 (2 + c)^8 + 16 (1 + c)^2 (2 + c)^6 (100 + c (164 + c (85 + c (14 + c)))) g^2 - 8 (2 + c)^4 (528 + c (1768 + c (2370 + c (1618 + c (593 + c (112 + 9 c)))))) g^4 + (2 +c)^4 (1520 + c (3232 + c (2508 + c$

$$(844 + 113\ c)))) g^6 - 4 (2 + c)^2 (320 + c (632 + c (476 + c (163 + 22\ c)))) g^8 + 2 (2 + c)^2 (78 + c (70 + 19\ c)) g10 - (40 + 9\ c (4 + c)) g^{12} + g^{14}))$$

$$w_{20}^* = -(2\ g (4 (1 + c)^2 (2 + c)^4 - (2 + c)^2 (12 + c (18 + 7\ c)) g^2 + 2 (2 + c) (3 + 2\ c) g^4 - g^6) (-4\ c^6 (-2 + g) (1 + v) + (-2 + g)^4 (2 + g)^3 (1 + v) -c (-2 + g)^3 (2 + g) (16 (2 + g) + (32 + g (16 + g)) v) + 2\ c^5 (40 (1 + v) + g (-20 + g - (20 + g) v)) + 2\ c^4 (164 (1 + v) + g (4 (-1 + g) g + 3 (-4 + g) g v - 82 (1 + v))) - c^2 (-2 + g) (416 (1 + v) + g2 (-4 (31 + 29\ v) + g (-8\ v + g (7 + g + 3\ v)))) + 2\ c^3 (352 (1 + v) + g (-176 (1 + v) + g (-6 (8 + 9\ v) + g (26 + g + (20 + g) v))))) - 2 (16 (1 + c)^2 (2 + c)^4 - 4 (2 + c)^2 (12 + c (19 + c (9 + c))) g^2 + 4 (2 + c)^2 (3 + c) g^4 - (4 + c) g^6) (4\ c^6 (-2 + g) (1 + v) - (-2 + g)^4 (2 + g)^3 (1 + v) + c (-2 + g)^3 (2 + g) (16 (2 + g) + (32 + g (16 + g)) v) + 2\ c^5 (-40 (1 + v) + g (20 + g + g^2 + (20 + g) v)) +2\ c^4 (-164 (1 + v) + g (2\ g (7 + g) - 3 (-4 + g) g v + 82 (1 + v))) + c^2 (-2 + g) (416 (1 + v) + g^2 (-4 (33 + 29\ v) + g (-12 + g - 8\ v + 3\ g\ v))) - 2\ c^3 (352 (1 + v) + g (-176 (1 + v) + g (-64 - 54\ v + g (14 + g + g^2 + (20 + g) v))))))/H \qquad (3.A1.10)$$

Where, $H=(-4 (-16 (1 + c)^2 (2 + c)^4 +4 (2 + c)^2 (12 + c (19 + c (9 + c))) g^2 - 4 (2 + c)^2 (3 + c) g^4 + (4 + c) g^6)^2 + 4 (-4 (1 + c)^2 (2 + c)^4 g + (2 + c)^2 (12 + c (18 + 7\ c)) g3 - 2 (2 + c) (3 + 2\ c) g^5 + g^7)^2)$

Given the optimal value of wages, the optimal value of products in the first period is

$$q_{10}^* = -(-(-2 + g)^{10} (2 + g)^9 (4 + g) (6 + g (-1 + v) - 2\ v) +64\ c^{18} (-2 + g) (4 + g) (-6 - 2\ g + g^2 + 2\ v) +c (-2 + g)^8 (2 +g)^7 (2\ g (-128 + g (532 + g (22 + (-22 + g) g))) +1280 (-3 + v) +g (-256 + g (-520 + g (64 + g (48 + g)))) v) -16\ c^{16} (23360 (-3 + v) +g (-6832 + 33808\ g + 312\ g^2 - 3028\ g^3 - 2\ g^4 + 27\ g^5 +8 (-838 + 3 (-8 + g) g (19 + g)) v)) +64\ c^{17} (-448 (-3 + v) +g (120 (1 + v) + g (-570 + 39\ g^2 + 62\ v - 2\ g (1 + v)))) +c^5 (-2 + g)^2 (2 + g) (132120576 (-3 + v) +g (49152 (2877 + 29\ v) +g (-8192 (-38737 + 24328\ v) +g (4\ g (-22863872 +g (19506944 +g (2674080 +g (-4417072 +g (-69968 +g (502336 +g (-6378 + g (-25697 + g (227 + 403\ g))))))))) +g (105385984 +g (1216000 +g (-25848576 +g (-703872 +g (3056992 +g (126512 +g (-154976 +g (-7064 + g (2234 + 59\ g))))))))) v -2048 (82364 + 135\ v))))) +2\ c^{11} (-232837120 (-3 + v) +g (3072 (54059 + 31171\ v) +g (128 (-5343853 + 484613\ v) +g (-64 (1008406 + 311519\ v) +g (207995968 + 8\ g^5 (629 - 381\ v) - 972576\ v -3\ g^6 (5271 + 71\ v) + 16\ g (517082 + 6959\ v) +16\ g^3 (-24490 + 7387\ v) +4\ g^4 (306001 + 8916\ v) -8\ g^2 (3197761 + 83938\ v)))))) -16\ c^{15} (187904 (-3 + v) +g (-448 (137 + 129\ v) +g (310624 - 33408\ v +g (5376 + 3912\ v +g (-36692 + 560\ v +g (-96 + 853\ g - 14\ v + 4\ g\ v)))))) +8\ c^{14} (-2083200 (-3 + v) +g (96 (8133 + 7157\ v) +g (-3946832 + 421520\ v +g (-160 (697 + 433\ v) +g (606216 - 11648\ v +g (3946 + 634\ v +g (g (-21 + 141\ g + 8\ v) -3 (8437 + 63\ v)))))))) +c^2 (-2 + g)^6 (2 + g)^5 (93184 (-3 + v) +g (-256 (39 + 55\ v) +g (512 (289 - 131\ v) +g (64 (23 + 117\ v) +g (16 (-1483 + 980\ v) +g (808 - 856\ v +g (1056 - 1244\ v +g (g (-5 + 3\ g + 11\ v) - 2 (77 + 13\ v))))))))) +8\ c^{13} (-8409600 (-3 + v) +g (2496 (1497 + 1193\ v) +g (-18325584 + 1918192\ v +g (-16 (49401 + 25579\ v) +g (3590416 - 68792\ v +g (47020 + 5996\ v +g (-230839 - 2503\ v +g (-701 + 226\ v + g (3617 + 33\ v))))))))) +4\ c^{12} (-50865920 (-3 + v) +g (64 (435869 + 302741\ v) +g (64 (-2002100 + 200083\ v) +g (-96 (85309 + 35311\ v) +g (31472752 - 461856\ v +g (740348 + 55380\ v +g (-8 (358677 + 5944\ v) +g (-21358 + 6424\ v +g (g (96 - 381\ g - 62\ v) +92 (929 + 17\ v)))))))))) -c^3 (-2 + g)^5 (2 + g)^3 (2023424 (-3 + v) +g (2048 (-1423 + 393\ v) +g (-768 (-4373 + 2301\ v) +g (1424384 - 707712\ v +g (64 (-10608 + 7907\ v) +g (800 (-248 + 261\ v) +g (73632 - 49984\ v +g (64 (79 - 353\ v) +g (-5592 + 516\ v +g (168 + 508\ v + 11\ g (15 + v))))))))))) +2\ c^{10} (-399872000 (-3 + v) +g (11264 (35453 + 15877\ v) +g (512 (-2723111 + 199769\ v) +g (-2560 (77402 + 15749\ v) +g (64 (8155697 + 109060\ v) +g (34248640 - 1769120\ v +g (-16 (5247129 + 220610\ v) +g (8 (-302356 + 95563\ v) +g (32 (184047 + 8018\ v) +g (60336 - 34852\ v +g (g (-245 + 603\ g + 198\ v) -2 (74261 + 2108\ v)))))))))))) -c^4 (-2 + g)^3 (2 + g)^2 (28631040 (-3 + v) +g (-6144 (-$$

1747 + 237 v) +g (1024 (67893 - 34441 v) +g (512 (-26055 + 2827 v) +g (256 (-80967 + 61973 v) +g (6121856 - 322944 v +g (64 (42448 - 51437 v) +g (-32 (40901 + 507 v) +g (16 (-8786 + 19545 v) +g (32 (4087 + 278 v) +g (1508 - 10800 v +g (g (9 + 34 g + 58 v) -4 (1262 + 93 v)))))))))))) +2 c9 (-494714880 (-3 + v) +g (360448 (2166 + 679 v) +g (512 (-4166491 + 165895 v) +g (-1280 (375626 + 37159 v) +g (128 (7729540 + 349763 v) +g (-128 (-842499 + 99425 v) +g (-32 (6396437 + 434322 v) +g (352 (-30259 + 10252 v) +g (8 (2494282 + 160761 v) +g (435584 - 244056 v +g (-20 (41419 + 1936 v) +g (-5120 + 3940 v + g (9949 + 182 v))))))))))))) +c^{8} (-764690432 (-3 + v) +g (135168 (18379 + 3283 v) +g (-4096 (1134744 + 39107 v) +g (2048 (-906449 + 2824 v) +g (1024 (2697083 + 285296 v) +g (524443392 - 96537344 v +g (-128 (5746321 + 646368 v) +g (64 (-1080773 + 396354 v) +g (96810848 + 9401280 v +g (48 (87684 - 48391 v) +g (-16 (377752 + 27155 v) +g (-96800 + 71184 v +g (g (381 - 595 g - 343 v) +8 (18089 + 723 v)))))))))))))) +c^{7} (-91488256 (-3 + v) +g (65536 (48881 + 2329 v) +g (-8192 (367165 + 101067 v) +g (36864 (-77382 + 7825 v) +g (2048 (1287842 + 315967 v) +g (1024 (968098 - 240851 v) +g (-1280 (734233 + 146412 v) +g (256 (-658707 + 262081 v) +g (448 (371794 + 57151 v) +g (14328640 - 7889728 v +g -64 (233173 + 25766 v) +g (-542928 + 384072 v +g (602872 + 41428 v -g (-6252 + 5386 v +g (7477 + 186 v))))))))))))))) +c^{6} (-2 + g) (-336855040 (-3 + v) +g (-262144 (4400 + 161 v) +g (32768 (-19269 + 23020 v) +g (16384 (86607 + 2500 v) +g (-6144 (-10527 + 81500 v) +g (-1024 (679035 + 23239 v) +g (256 (135149 + 585266 v) +g (256 (674582 + 31053 v) +g (-64 (159469 + 346986 v) +g (-160 (142075 + 8241 v) +g (16 (61623 + 97217 v) +g (16 (93185 + 5818 v) +g (-12 (2741 + 3428 v) +g (g (182 + 185 g + 179 v) -2 (19563 + 947 v)))))))))))))))))/K (3.A1.11)

q_{20}* = ((-2 + g)^{10} (2 + g)^{9} (4 + g) (6 + g (-1 + v) - 2 v) -128 c^{18} (-2 + g) (4 + g) (1 + v) +c (-2 + g)^{8} (2 +g)^{7} (2 (2176 +g (384 + g (-548 + g (-102 + g (6 + g (3 + g)))))) - (-2 +g) (-640 + g (-192 + g (164 + g (50 + g)))) v) +128 c^{16} (8 (481 + 365 v) +g (-406 - 838 v +g (-782 - 144 g + 23 g^{2} + 10 g^{3} +3 (-8 + g) (19 + g) v))) +32 c^{17} (128 (8 + 7 v) +g (-16 (11 + 15 v) +g (g (g (2 + g) + 4 (-5 + v)) - 4 (41 + 31 v)))) +8 c^{15} (1024 (564 + 367 v) +g (-128 (227 + 903 v) +g (-256 (574 + 261 v) +g (48 (-656 + 163 v) +g (8 (943 + 140 v) +g (3120 - 28 v + g (-46 - 29 g + 8 v))))))) -8 c^{14} (-1920 (1973 + 1085 v) +g (96 (229 + 7157 v) +g (16 (74357 + 26345 v) +g (269632 - 69280 v +g (-32 (2893 + 364 v) +g (-37078 + 634 v +g (1563 - 189 v + g (906 + 8 v)))))))) -c^{2} (-2 + g)^{6} (2 + g)^{5} (1024 (-361 + 91 v) +g (-1280 (75 + 11 v) +g (256 (703 - 262 v) +g (64 (796 + 117 v) +g (80 (-271 + 196 v) +g (-8 (997 + 107 v) +g (-4 (100 + 311 v) +g (306 - 26 v + g (83 + 11 v))))))))) +8 c^{13} (7680 (2402 + 1095 v) +g (523584 - 2977728 v +g (-16 (440921 + 119887 v) +g (-1609488 + 409264 v +g (8 (95948 + 8599 v) +g (295464 - 5996 v +g (-24371 + 2503 v +g (g (108 + 79 g - 33 v) -2 (6595 + 113 v))))))))) +8 c^{12} (640 (107983 + 39739 v) +g (3630176 - 9687712 v +g (-32 (993218 + 200083 v) +g (-7055360 + 1694928 v +g (4588928 + 230928 v +g (1670318 - 27690 v +g (-231434 + 23776 v +g (-116829 - 3212 v +g (2943 - 782 v + g (1972 + 31 v)))))))))) -c^{6} (-2 + g) (-262144 (-24669 + 1285 v) +g (-262144 (-12175 + 161 v) +g (32768 (-177207 + 23020 v) +g (40960 (-80073 + 1000 v) +g (-6144 (-296027 + 81500 v) +g (1024 (1278685 - 23239 v) +g (256 (-738189 + 585266 v) +g (256 (-997165 + 31053 v) +g (-192 (72719 + 115662 v) +g (2 g (2119288 +g (-537912 + g (-132518 + g (7061 + 2122 g)))) +32 (775997 - 41205 v) +g (1555472 +g (93088 +g (-41136 + g (-1894 + 179 g)))) v))))))))))) -c^{3} (-2 + g)^{5} (2 + g)^{3} (-8192 (-1193 + 247 v) +g (6144 (1325 - 131 v) +g (256 (-12551 + 6903 v) +g (128 (-33248 + 5529 v) +g (-64 (5198 + 7907 v) +g (96 (6406 - 2175 v) +g (32 (5633 + 1562 v) +g (32 (-243 + 706 v) +g (-4 (3172 + 129 v) +g (g (111 + 22 g - 11 v) -4 (471 + 127 v))))))))))) +2 c^{11} (20480 (39769 + 11369 v) +g (1024 (53159 - 93513 v) +g (-128 (3479931 + 484613 v) +g (64 (-1457714 + 311519 v) +g (82273792 + 972576 v +g (27744448 - 111344 v +g (8 (-749133 + 83938 v) +g (-8 (348939 + 14774 v) +g (147044 - 35664 v +g (90156 + 3048 v +g (-549 - 443 g + 213 v))))))))))) +2 c^{10} (225280 (8491 +

$$1775 v) +g (-11264 (-12131 + 15877 v) +g (-512 (2405715 + 199769 v) +g (1280 (-183663 + 31498 v) +g (320 (885281 - 21812 v) +g (32 (2697426 + 55285 v) +g (-28045392 + 3529760 v -g (104 (113856 + 7351 v) +g (8 (-139309 + 32072 v) +g (-4 (154826 + 8713 v) +g (12146 - 4216 v + g (8815 + 198 v)))))))))))) +2 c^9 (901120 (4003 + 549 v) +g (-180224 (-1329 + 1358 v) +g (-512 (5294721 + 165895 v) +g (256 (-1758962 + 185795 v) +g (760484864 - 44769664 v +g (128 (1583207 + 99425 v) +g (864 (-113543 + 16086 v) +g (-64 (572139 + 56386 v) +g (5717888 - 1286088 v +g (8 (353777 + 30507 v) +g (4 (-30487 + 9680 v) +g (-78872 + 421 g + 362 g^2 -2 (1970 + 91 g) v)))))))))))) +c^4 (-2 + g)3 (2 + g)2 (122880 (-1467 + 233 v) +g (-6144 (11629 + 237 v) +g (1024 (136561 - 34441 v) +g (512 (121003 + 2827 v) +g (256 (-141283 + 61973 v) +g (-384 (51905 + 841 v) +g (64 (41794 - 51437 v) +g (96 (29937 - 169 v) +g (244384 + 312720 v +g (-173712 + 8896 v +g (-12 (3071 + 900 v) +g (3044 - 372 v + g (897 + 58 v))))))))))))) +c^8 (180224 (60743 + 4243 v) +g (-45056 (-12575 + 9849 v) +g (4096 (-2326196 + 39107 v) +g (-2048 (627277 + 2824 v) +g (1024 (3132411 - 285296 v) +g (256 (2787189 + 377099 v) +g (128 (-4084681 + 646368 v) +g (-64 (2602379 + 396354 v) +g (41943008 - 9401280 v +g (18009536 + 2322768 v +g (16 (-91636 + 27155 v) +g (-8 (103775 + 8898 v) +g (14956 - 5784 v +g (11257 + 343 v)))))))))))))) +c^5 (-2 + g)2 (2 + g) (-393216 (-3133 + 336 v) +g (-49152 (-11275 + 29 v) +g (16384 (-62185 + 12164 v) +g (10240 (-50806 + 27 v) +g (-4096 (-69882 + 25729 v) +g (-512 (-362324 + 2375 v) +g (384 (-63277 + 67314 v) +g (192 (-161779 + 3666 v) +g (-32 (70634 + 95531 v) +g (2411648 - 126512 v +g (8 (57697 + 19372 v) +g (-71036 + 7064 v +g (g (360 + 117 g - 59 v) -2 (9546 + 1117 v)))))))))))))) +c^7 (262144 (50861 + 349 v) +g (-65536 (-5591 + 2329 v) +g (483328 (-27617 + 1713 v) +g (-184320 (6994 + 1565 v) +g (2048 (2598780 - 315967 v) +g (1024 (895656 + 240851 v) +g (256 (-4183441 + 732060 v) +g (-256 (1078468 + 262081 v) +g (64 (1767972 - 400057 v) +g (64 (632802 + 123277 v) +g (64 (-91581 + 25766 v) +g (-8 (352790 + 48009 v) +g (118240 - 41428 v +g (76456 + 5386 v +g (-409 - 367 g +186 v))))))))))))))))/K$$

(3.A1.12)

Given the optimal value of wages and product of the first period, the optimal value of products in the first period is

$$q_{11}^* = ((2 + c - g) (2 + c +g) ((-2 + g)^9 (2 + g)^8 (4 + g) (2 + g (-1 + v) - 6 v) +64 c^{15} (2 (992 + g (-52 + g (-305 + g + 17 g^2))) + (-8 +g) (-56 + g (6 + g (10 + g))) v) +c^4 (-4 +g^2)^2 ((-2 + g) (-13127680 +g (-8690688 +g (8070656 +g (5852160 +g (-1689856 +g (-1375072 +g (137904 +g (132272 +g (-3880 +3 g (-1550 + g (7 + 10 g)))))))))) + (2 +g) (29429760 +g (-23180288 +g (-10278400 +g (10740736 +g (606656 +g (-1608320 +g (79472 +g (85328 +g (-6268 + g (-1136 + 49 g)))))))))) v) +64 c16 (-2 + g) (4 + g) ((-2 + g) g - 2 (5 + v)) +2 c^8 (2 g (-15577344 +g (-289303552 +g (4594112 +g (89033600 +g (-41168 +g (-12917072 +g (-72668 +g (872752 +g (4514 + g (-22516 + g (-34 + 101 g))))))))))) +g (-102707712 +g (-326406656 +g (56858752 +g (81120832 +g (-11056416 +g (-8788496 +g (886904 +g (387968 +g (-25436 + g (-4782 + 121 g)))))))))) v +92160 (7799 + 5147 v)) -16 c^{14} (-320 (285 + 73 v) +g (4976 + 5104 v +g (31744 + 4976 v +g (-232 - 2356 g + 21 g^3 +2 (-132 + (-80 + g) g) v)))) -c^2 (-2 + g)5 (2 + g)4 (2560 (3 + 119 v) +g (128 (401 + 369 v) +g (-832 (-3 + 199 v) +g (3 g (-848 +g (1592 + g (124 + g (-78 + (-1 + g) g)))) +2 g (13872 +g (1728 + g (-686 + g (-54 + 5 g)))) v -32 (887 + 745 v))))) -c (-2 + g)^7 (2 + g)^6 (128 (-5 + 31 v) +g (480 (1 + v) +g (280 - 1112 v +g (-4 (32 + 29 v) + g (-32 + 3 g + 2 (24 + g) v))))) -16 c^{13} (-1280 (503 + 147 v) +g (128 (283 + 307 v) +g (32 (7996 + 1581 v) +g (-8 (371 + 484 v) +g (-4 (6362 + 705 v) +g (18 + 80 v + 19 g (30 + v))))))) +8 c^{12} (640 (9751 + 3275 v) +g (-32 (11171 + 13299 v) +g (-16 (177931 + 43653 v) +g (128 (348 + 533 v) +g (8 (46421 + 7348 v) +g (-738 - 2762 v +g (-14629 - 1091 v +g (-4 + 84 g + 17 v)))))))) +8 c^{11} (2560 (8583 + 3343 v) +g (-704 (1787 + 2443 v) +g (-16 (721257 + 217493 v) +g (144 (1515 + 2833 v)$$

$$+g (8 (240423 + 51088 v) +g (-4 (1625 + 6897 v) +g (-115247 - 14215 v +g (-68 + 473 v + 2 g (908 + 35 v))))))))) -c^5 (-4 + g2) (32768 (7329 + 10733 v) +g (-8192 (-1471 + 11353 v) +g (-2048 (115881 + 142787 v) +g (3072 (-4713 + 22360 v) +g (256 (367915 + 360993 v) +g (256 (21673 - 72934 v) +g (-320 (58613 + 42821 v) +g (2 g (955256 +g (29304 + g (-44626 + g (-561 + 665 g)))) +g (939520 +g (-114736 + g (-24236 + g (1712 + 105 g)))) v +96 (-9287 + 23538 v))))))))) +c^3 (-2 + g)^4 (2 + g)^3 (163840 (5 + 22 v) +g (1024 (797 + 653 v) +g (-256 (1617 + 8161 v) +g (-128 (3859 + 2833 v) +g (60128 + 395040 v +g (98544 + 60464 v +g (-16 (119 + 1639 v) +g (-8 (895 + 372 v) +3 g (2 + 150 v + g (53 + 7 v))))))))))) +4 c^{10} (1280 (90607 + 41395 v) +g (-64 (100801 + 167689 v) +g (-192 (365735 + 135142 v) +g (32 (45461 + 110285 v) +g (16 (913057 + 252566 v) -g (62052 + 361100 v +g (4 (307501 + 55713 v) +g (1396 - 11914 v +g (g (-26 + 171 g + 62 v) -8 (4526 + 393 v)))))))))) +2 c^9 (10240 (45551 + 24771 v) +g (-512 (46911 + 102919 v) +g (-384 (848121 + 385267 v) +g (576 (11409 + 39754 v) +g (32 (2593797 + 921454 v) -g (308336 + 3311296 v +g (72 (129660 + 32611 v) +g (23072 - 178608 v +g (-24 (18152 + 2663 v) +g (-984 + 2644 v + g (5849 + 269 v))))))))))) +4 c^7 (81920 (5159 + 4243 v) +g (-32768 (367 + 2428 v) +g (-2048 (191502 + 136681 v) +g (512 (4123 + 106522 v) +g (64 (2253469 + 1329007 v) +g (-320 (-3472 + 42923 v) +g (-32 (822522 + 376073 v) +g (16 (-20393 + 96182 v) +g (2432164 + 781188 v +g (26044 - 72328 v +g (-2 (50867 + 9458 v) +g (-528 + 996 v + g (1321 + 76 v))))))))))))) +c6 (32768 (45595 + 48559 v) +g (-8192 (257 + 47561 v) +g (-40960 (39045 + 36362 v) +g (24576 (-1043 + 13163 v) +g (512 (1362950 + 1064269 v) +g (-256 (-67771 + 400425 v) +g (-128 (1229498 + 763595 v) +g (64 (-64276 + 239669 v) +g (1152 (16615 + 7607 v) +g (48 (8536 - 22381 v) +g (-16 (73828 + 21833 v) +g (16 (-937 + 1814 v) +g (29940 + g (95 - 145 g - 139 v) +4176 v)))))))))))))))/K \qquad (3.A1.13)$$

$$q_{21}{}^* = -((2 + c - g) (2 + c +g) (-(-2 + g)^9 (2 + g)^8 (4 + g) (2 + g (-1 + v) - 6 v) +128 c^{16} (-2 + g) (4 + g) (1 + v) +32 c^{15} (-768 + 272 g + 108 g^2 - 20 g^3 + g^5 -2 (-8 + g) (-56 + g (6 + g (10 + g))) v) +c^4 (-4 +g^2)^2 (-(-2 + g) (4075520 +g (6071296 +g (-512512 +g (-3855360 +g (-838912 +g (841504 +g (273520 +g (-70832 +g (-26760 + g (1694 + 681 g)))))))))) - (2 +g) (29429760 +g (-23180288 +g (-10278400 +g (10740736 +g (606656 +g (-1608320 +g (79472 +g (85328 +g (-6268 + g (-1136 + 49 g)))))))))) v) +2 c8 (-4 g (-41556096 +g (-4353536 +g (28511200 +g (-259648 +g (-7443784 +g (169048 +g (896266 +g (-14304 +g (-46929 + 5 g (57 + 149 g)))))))))) +g (102707712 +g (326406656 +g (-56858752 +g (-81120832 +g (11056416 +g (8788496 +g (-886904 +g (-387968 +g (25436 + (4782 - 121 g) g))))))))) v -92160 (663 + 5147 v)) +8 c^{10} (2 g (5197328 +g (1148208 +g (-2404528 +g (-71672 +g (390883 +g (-2144 + 3 g (-8287 + g (43 + 149 g)))))))) +g (5366048 +g (12973632 +g (-1764560 +g (-2020528 +g (180550 +g (111426 +g (-5957 + g (-1572 + 31 g)))))))) v -640 (12303 + 41395 v)) -c (-2 + g)^7 (2 + g)^6 (128 (9 - 31 v) +g (32 - 480 v +g (-312 - 32 g + 5 g^3 + 2 g^4 -2 (-556 + g (-58 + g (24 + g))) v))) +32 c^{14} (-160 (53 + 73 v) +g (8 (503 + 319 v) +g (8 (172 + 311 v) +g (-4 (142 + 33 v) +g (-12 + 31 g + (-80 + g) v))))) +8 c^{13} (-2560 (89 + 147 v) +g (256 (549 + 307 v) +g (192 (225 + 527 v) -g (29616 + 7744 v +g (976 + 5640 v +g (g (4 + 21 g - 38 v) - 20 (103 + 8 v))))))) +8 c^{11} (-2560 (1310 + 3343 v) +g (64 (53101 + 26873 v) +g (862896 + 3479888 v +g (-48 (26153 + 8499 v) +g (-8 (5775 + 51088 v) +g (132 (1182 + 209 v) +g (-467 +g (-6587 + 2 g (7 + 22 g - 35 v) - 473 v) +14215 v))))))) +c^2 (-2 + g)^5 (2 + g)^4 (2560 (-29 + 119 v) +g (1152 (-23 + 41 v) +g (64 (491 - 2587 v) +g (160 (99 - 149 v) +g (-944 + 27744 v +g (-3048 + 3456 v +g (-860 + 186 g + 73 g^2 +2 (-686 + g (-54 + 5 g)) v))))))) -8 c12 (640 (1619 + 3275 v) +g (-32 (25603 + 13299 v) +g (-16 (14369 + 43653 v) +g (32 (7323 + 2132 v) +g (8864 + 58784 v +g (-21826 - 2762 v +g (53 - 1091 v + g (537 + 17 v)))))))) -c^5 (-4 + g^2) (-32768 (-851 + 10733 v) +g (8192 (8873 + 11353 v) +g (2048 (-16123 + 142787 v) +g (-5120 (12901 + 13416 v) +g (768 (17153 - 120331 v) +g (256 (93631 + 72934 v) +g (2 g (-2142896 +g (96872 +g$$

(187000 +g (-3166 + g (-6595 + 4 g (5 + 11 g)))))) -g (2259648 +g (939520 +g (-114736 + g (-24236 + g (1712 + 105 g))))) v +64 (-36731 + 214105 v)))))))) +c^3 (-2 + g)^4 (2 + g)^3 (-40960 (-17 + 88 v) +g (1024 (539 - 653 v) +g (256 (-845 + 8161 v) +g (128 (-2563 + 2833 v) +g (-32 (1349 + 12345 v) +g (64368 - 60464 v +g (19776 + 26224 v +g (8 (-545 + 372 v) +g (-1666 + 47 g + 19 g^2 -3 (150 + 7 g) v))))))))) +2 c9 (-10240 (5195 + 24771 v) +g (512 (186663 + 102919 v) +g (384 (43453 + 385267 v) +g (-192 (282511 + 119262 v) +g (-32 (26315 + 921454 v) +g (16 (705151 + 206956 v) +g (72 (-1794 + 32611 v) +g (-16 (63073 + 11163 v) +g (9536 - 63912 v +g (g (-73 - 189 g + 269 v) +4 (8370 + 661 v))))))))))) +2 c^7 (-163840 (229 + 4243 v) +g (32768 (6647 + 4856 v) +g (4096 (-329 + 136681 v) +g (-1024 (174595 + 106522 v) +g (8992896 - 170112896 v +g (128 (450836 + 214615 v) +g (64 (-38151 + 376073 v) +g (-64 (141776 + 48091 v) +g (240344 - 1562376 v +g (16 (43120 + 9041 v) +g (-8340 + 37832 v +g (g (50 + 117 g - 152 v) -24 (873 + 83 v))))))))))))) +c^6 (-32768 (-741 + 48559 v) +g (8192 (51553 + 47561 v) +g (8192 (-10021 + 181810 v) +g (-4096 (100609 + 78978 v) +g (512 (97594 - 1064269 v) +g (256 (635533 + 400425 v) +g (128 (-97078 + 763595 v) +g (-64 (511102 + 239669 v) +g (1446080 - 8763264 v +g (16 (214678 + 67143 v) +g (-74288 + 349328 v +g (-8 (21211 + 3628 v) +g (1208 - 4176 v +g (2739 + 139 v))))))))))))))))/K (3.A1.14)

Where, K = (2 (4096 (1 + c)^6 (2 + c)^{12} - 256 (1 + c)^4 (2 + c)^{10} (164 + c (292 + c (181 + c (46 + 5 c)))) g^2 + 64 (1 + c)^2 (2 + c)^8 (3040 + c (10896 + c (16276 + c (13124 + c (6190 + c (1724 + c (271 + c (22 + c))))))) g^4 - 16 (2 + c)^8 (8400 + c (37056 + c (69124 + c (70924 + c (43593 + c (16362 + c (3651 + 448 c + 26 c^2))))))) g^6 + 4 (2 + c)^6 (61440 + c (261888 + c (480416 + c (494896 + c (313056 + c (124608 + c (30596 + c (4292 + 273 c)))))))) g^8 - 8 (2 + c)^6 (9744 + c (30576 + c (39698 + c (27264 + c (10487 + 5 c (433 + 39 c)))))) g^{10} + 4 (2 + c)^4 (17472 + c (53088 + c (67688 + c (46308 + c (17994 + c (3796 + 345 c)))))) g^{12} - (2 +c)^4 (11040 + c (21696 + c (16540 + c (5772 + 805 c)))) g^{14} + 4 (2 + c)^2 (1200 + c (2304 + c (1718 + c (589 + 79 c)))) g^{16} - 2 (2 + c)^2 (170 + c (154 + 41 c)) g^{18} + (56 + 13 c (4 + c)) g^{20} - g^{22}))

The optimal yields are also depending on *c*, *g*, and *v*;

$$\Pi_1^*(c,g,v) = -c\left(q_{10}^* - q_{11}^* + \frac{1}{2}\left(q_{10}^* - q_{11}^*\right)^2\right) + q_{10}\left(1 - q_{10}^* - g q_{20}^*\right) - \left(q_{10}^* + q_{11}^*\right) w_{10}^* + q_{11}^*\left(v - q_{11}^* - g q_{21}^*\right) \tag{3.A1.15}$$

$$\Pi_2^*(c,g,v) = -\frac{1}{2}c\left(q_{20}^* - q_{21}^*\right)^2 + q_{20}^*\left(1 - g q_{10}^* - q_{20}^*\right) - \left(q_{20}^* + q_{21}^*\right) w_{20}^* + q_{21}^*\left(v - g q_{11}^* - q_{21}^*\right) \tag{3.A1.16}$$

$$U_1^*(c,g,v) = w_{10}^*\left(q_{10}^* + q_{11}^*\right) \tag{3.A1.17}$$

$$U_2^*(c,g,v) = w_{20}^*\left(q_{20}^* + q_{21}^*\right) \tag{3.A1.18}$$

$$CS^*(c,g,v) = \frac{1}{4}(1+g)\left(q_{10}^* + q_{11}^* + q_{20}^* + q_{21}^*\right) \tag{3.A1.19}$$

APPENDIX 3.A2

Second period/ four stage each firm maximazizes its profits as for q_{11} and q_{21} respectively;

$$\underset{q_{11}}{Max}\Pi_1\left(q_{10},q_{20},q_{11},q_{21},w_{10}\right)=-c\left(q_{10}-q_{11}+\frac{1}{2}\left(q_{10}-q_{11}\right)^2\right)+q_{10}\left(1-q_{10}-gq_{20}\right)-\left(q_{10}+q_{11}\right)w_{10}+ q_{11}\left(v-q_{11}-gq_{21}\right) \quad (3.A2.1)$$

$$\underset{q_{21}}{Max}\Pi_2\left(q_{10},q_{20},q_{11},q_{21},w_{10},w_{20}\right)=q_{20}\left(1-gq_{10}-q_{20}\right)-\left(q_{20}+q_{21}\right)w_{20}+q_{21}\left(v-gq_{11}-q_{21}\right). \quad (3.A2.2)$$

From the *foc* accrues reaction functions of the second period;

$$RF_{11}\left(q_{21}\right)=\frac{\left(v+c\right)+cq_{10}-gq_{21}-w_{10}}{2+c} \quad (3.A2.3)$$

$$RF_{21}\left(q_{11}\right)=\frac{v-gq_{11}-w_{20}}{2}. \quad (3.A2.4)$$

We solve the system of the second period RFs to get the optimal q_{11}^* and q_{21}^* -rules in the second period;

$$q_{11}^*=\frac{-\left(g-2\right)v+2c\left(1+q_{10}\right)-2w_{10}+gw_{20}}{4+2c-g^2} \quad (3.A2.5)$$

$$q_{21}^*=\frac{-\left(g-2\right)v-c\left(g+gq_{10}-v+w_{20}\right)+gw_{10}-2w_{20}}{4+2c-g^2}. \quad (3.A2.6)$$

Substituting the later into (3.A2.1) and (3.A2.2) accrues profits that depend on products of the first period and wages;

$$\Pi1(q_{10}, q_{20}, w_{10}, w_{20}) = 1/(2 (-4 - 2 c + g^2)^2) (4 c^3 +c (-g^4 q_{10} (2 + q_{10}) + 8 g^3 q_{10} q_{20} +4 (-12 q_{10}^2 + 4 q_{10} (v - 3 w_{10}) + (v - w_{10}) (4 + v - w_{10})) +g^2 (8 q_{10} (1 + 2 q_{10} + w_{10}) + (v - w_{20})^2) +4 g (-(2 + v - w_{10}) (v - w_{20}) - 2 q_{10} (4 q_{20} + v - w_{20}))) +2 (-(-4 + g^2)^2 q_{10}^2 - (-4 + g^2)^2 q_{10} (-1 + g q_{20} +w_{10}) + ((-2 + g) v + 2 w_{10} - g w_{20})^2) +4 c^2 (2 + (-4 + g^2) q_{10}^2 - (-2 + g) v - 2 w_{10} + g w_{20} +q_{10} (2 (-1 + v - 2 w_{10}) + g (2 g - 2 q_{20} - v + w_{20})))) \quad (3.A2.7)$$

$$\Pi2(q_{10}, q_{20}, w_{10}, w_{20})= (1 - g q_{10} - q_{20}) q_{20} -w_{20} (q_{20} - ((-2 + g) v - g w_{10} + 2 w_{20} + c (g + g q_{10} - v + w_{20}))/(4 + 2 c - g^2)) - (((-2 + g) v - g w_{10} + 2 w_{20} +c (g + g q_{10} - v + w_{20})) (v - (g (2 c (1 + q_{10}) - (-2 + g) v - 2 w_{10} + g w_{20}))/(4 + 2 c - g^2) + ((-2 + g) v - g w_{10} + 2 w_{20} +c (g + g q_{10} - v + w_{20}))/(4 + 2 c - g^2)))/(4 + 2 c - g^2). \quad (3.A2.8)$$

First period/ third stage each firm maximizes its profits as for q_{10} and q_{20} respectively. Reaction functions of the first period are given below;

$$RF_{10}(q_{20})=L+Mq_{20}+Nw_{10}+Ow_{20} \quad (3.A2.9)$$

$$RF_{20}(q_{10})=\frac{1 - gq_{10} - w_{20}}{2}, \quad (3.A2.10)$$

where, $L=\frac{\left(g^2-4\right)^2 - c\left(g-2\right)\left(g^2\left(2+g\right)+4v\right)+c^2\left(-4+4g^2+4v-2gv\right)}{P}>0$, $M=\frac{g\left(4+2c-g^2\right)^2}{P}<0$,

$N=\frac{\left(2+c\right)\left(4g^2-8\left(1+c\right)\right)+g^4}{P}<0$, $O=\frac{-2cg}{\left(4+4c-g^2\right)\left(g^2-4\right)}>0$, $P=\left(2+c\right)\left(g^2-4\right)\left(g^2-4c-4\right)$.

We solve the system of the first period RFs to get the optimal q_{10}^{*} and q_{20}^{*} -rules in the first period;

$q_{10}^* = (4 c^2 (2 - 2 v + 4 w_{10} + g (1 - 2 g + v - 2 w_{20})) - (-4 +g^2)^2 (2 - 2 w_{10} + g (-1 + w_{20})) +2 c (-8 (v - 3 w_{10}) +g (4 (2 + v - 3 w_{20}) +g (-4 (1 + w_{10}) + g (-2 + g + 2 w_{20})))))/Q$ (3.A2.11)

$q_{20}^*= -(1/(Q) ((-4 + g^2)^2 (2 + g (-1 + w_{10}) - 2 w_{20}) +2 c2 (8 - 8 w_{20} + g (2 - 2 v + 4 w_{10} + g (-2 - 2 g + v + w_{20}))) +c (-48 (-1 + w_{20}) +g (-8 (v - 3 w_{10}) +g (4 (-4 + v + 3 w_{20}) +g (g + g^2 - 4 (1 + w_{10}) - g w_{20}))))).$ (3.A2.12)

Substituting the later into (3.A2.7) and (3.A2.8) accrues profits that depend on wages only;

$\Pi1(w_{10}, w_{20})= (2 (4 + 2 c -g^2) (2 c (-2 (3 + v - 2 w_{10}) + g (1 + g + v - 2 w_{20})) - (-4 +g^2) ((-2 + g) v + 2 w_{10} - g w_{20})) (-c (-4 +g^2) (-4 (-3 + 4 v + 3 w_{10}) +g (-2 + 8 v + g (-8 + g + g^2 + 2 v - g v + 6 w_{10}) -6 w_{20})) + (-4 + g^2)^2 ((-2 + g) v + (-2 + g2) w_{10} -g w_{20}) +2 c^2 (-4 (-3 + 3 v + 2 w_{10}) +g (-2 + 6 v - 4 w_{20} +g (4 (-2 + v + w_{10}) + g (1 + g - 2 v + w_{20}))))) +2 (-(-4 + g^2)^2 (g + g^2 w_{10} - 2 (1 + w_{10}) - g w_{20}) +c (-(-2 + g) (g (8 + g (g (-6 + g (3 + g)) + 4 (-6 + v))) -8 (-6 + v)) + 4 (12 - 8 g^2 + g4) w_{10} +g (24 - 8 g^2 + g^4) w_{20}) +2 c^2 (20 + 2 g^4 - 4 v + 2 g^2 (-6 + v - 2 w_{10}) + 8 w_{10} -g^3 (-2 + v + w_{20}) + 2 g (-3 + v + 2 w_{20}))) (-4 c^2 (2 -2 v + 4 w_{10} + g (1 - 2 g + v - 2 w_{20})) + (-4 + g^2)^2 (2 -2 w1_0 + g (-1 + w_{20})) -2 c (-8 (v - 3 w_{10}) +g (4 (2 + v - 3 w_{20}) +g (-4 (1 + w_{10}) + g (-2 + g + 2 w_{20}))))) -2 (-6 c (-4 + g^2)^2 + (-4 + g^2)^3 +4 c2 (-8 + 3 g^2)) w_{10} ((-4 + g^2)^2 (-2 + g - 2 v + g v +4 w_{10} - 2 g w_{20}) -4 c^2 (4 (1 + v - 2 w_{10}) + g (-2 + g - 2 v + 4 w_{20})) + 2 c (-24 (1 + v - 2 w_{10}) +g (12 (1 + v - 2 w_{20}) +g (6 + 4 v - 10 w_{10} + g (-3 - 2 v + 5 w_{20}))))) - c (-2 (4 + 2 c - g^2) (-6 c (-4 + g^2)^2 + (-4 + g^2)^3 +4 c^2 (-8 + 3 g^2)) (2 c (-2 (3 + v - 2 w_{10}) +g (1 + g + v - 2 w_{20})) - (-4 + g^2) ((-2 + g) v + 2 w_{10} -g w_{20})) + (4 c^2 (-8 + 3 g^2) + (-2 + g)3 (2 +g)^2 (-1 + v) +2 c (-8 (3 + v) +g (4 (-1 + v) +g (14 + 4 v - 2 w_{10} + g (1 - 2 g - 2 v + w_{20})))))^2 +2 (-6 c (-4 + g^2)^2 + (-4 + g2)^3 +4 c^2 (-8 + 3 g^2)) (4 c^2 (2 - 2 v + 4 w_{10} +g (1 - 2 g + v - 2 w_{20})) - (-4 + g^2)^2 (2 - 2 w_{10} +g (-1 + w_{20})) +2 c (-8 (v - 3 w_{10}) +g (4 (2 + v - 3 w_{20}) +g (-4 (1 + w_{10}) +g (-2 + g + 2 w_{20})))))))/2Q^2$ (3.A2.13)

$\Pi2(w_{10}, w_{20}) =(((-(-4 + g^2)^2 ((-2 + g) v -g w_{10} + (-2 + g^2) w_{20}) -c (-4 + g^2) (g (-6 - 4 v + 6 w_{10} + g (1 + g - v - 6 w_{20})) +12 (v + w_{20})) +2 c^2 (8 (v + w_{20}) +g (-6 + g + g2 - 2 v - 2 g v + 4 w_{10} - 5 g w_{20}))) (-c (-4 +g^2) (g (-6 + g + g^2 - 4 v - g v + 6 w_{10}) +12 (v - w_{20})) - (-4 + g^2)^2 ((-2 + g) v - g w_{10} + 2 w_{20}) +2 c^2 (8 (v - w_{20}) +g (-2 (3 + v - 2 w_{10}) +g (1 + g - 2 v + w_{20})))) - (-6 c (-4 + g^2)^2 + (-4 +g^2)^3 +4 c^2 (-8 +3 g^2)) w_{20} (2 c^2 (g (4 (1 + v - 2 w_{10}) +g (1 + g + v - 2 w_{20})) - 8 (1 + v - 2 w_{20})) + (-4 +g^2)^2 (-2 + g - 2 v + g v - 2 g w_{10} + 4 w_{20}) +c (-48 (1 + v - 2 w_{20}) +g (24 (1 + v - 2 w_{10}) +g (12 (1 + v - 2 w_{20}) +g (-6 - (4 + g) v + 10 w_{10} + g w_{20}))))) + (-(-4 +g^2)^2 (2 + g (-1 + w_{10}) - 2 w_{20}) +2 c^2 (2 g^3 + 2 g (-1 + v - 2 w_{10}) + 8 (-1 + w_{20}) -g^2 (-2 + v + w_{20})) +c (-g^5 + 8 g (v - 3 w_{10}) + 4 g3 (1 + w_{10}) + 48 (-1 + w_{20}) +g^4 (-1 + w_{20}) - 4 g2 (-4 + v + 3 w_{20}))) ((-4 +g^2)^2 (-2 + g - g w_{10} + (-2 + g^2) w_{20}) +2 c^2 (-8 (1 + w_{20}) +g (2 (-1 + v - 2 w_{10}) + g (2 + 2 g - v + 5 w_{20}))) -c (48 (1 + w_{20}) +g (-8 (v - 3 w_{10}) +g (4 (-4 + v - 9 w_{20}) +g (g + g2 - 4 (1 + w_{10}) + 5 g w_{20})))))))/Q^2.$ (3.A2.14)

Union's utility is also depending on wages only. Union's utility accrues substituting optimal quantities in the equation below;

$$U_j\left(= w_{i0}\left(q^*_{j0}+q^*_{j1}\right)\right),\ j=1,2, \qquad (3.A2.15)$$

thus

$$U1(w_{10}, w_{20}) = w_{10} ((-4 + g^2)^2 (-2 + g - 2 v + g v + 4 w_{10} - 2 g w_{20}) -4 c^2 (4 (1 + v - 2 w_{10}) + g (-2 + g - 2 v + 4 w_{20})) +2 c (-24 (1 + v - 2 w_{10}) +g (12 (1 + v - 2 w_{20}) +g (6 + 4 v - 10 w_{10} + g (-3 - 2 v + 5 w_{20})))))/Q \qquad (3.A2.16)$$

$$U2 (w_{10}, w_{20}) = w_{20} (2 c^2 (g (4 (1 + v - 2 w_{10}) + g (1 + g + v - 2 w_{20})) -8 (1 + v - 2 w_{20})) + (-4 + g^2)^2 (-2 + g - 2 v + g v -2 g w_{10} + 4 w_{20}) +c (-48 (1 + v - 2 w_{20}) +g (24 (1 + v - 2 w_{10}) +g (12 (1 + v - 2 w_{20}) +g (-6 - (4 + g) v + 10 w10 + g w_{20})))))/Q, \qquad (3.A2.17)$$

Where, $Q = (-6 c (-4 + g^2)^2 + (-4 + g^2)^3 + 4 c^2 (-8 + 3 g^2))$.

In the first period/ second stage, each firm bargains with the union that represents employees of the same sector the wage. I make the assumption that unions possess all the bargaining power (b=1). The bargaining problem is set as follow;

$$w_{j0} = \max_{w_{j0}} B_j = \max_{w_{j0}}\left(\mathrm{b}\ \mathrm{Log}\left[\mathrm{U}_j\right] + (1-\mathrm{b})\ \mathrm{Log}\left[\Pi_j\right]\right).$$

Solving the system that accrues from the first order conditions of the above problem as for w_{10} and w_{20} accrues a unique stable solution for the equilibrium firm-specific wage contracts w^*_{10} and w^*_{20}, respectively;

$$w_{10}^* = ((-2 + g)^5 (2 + g)^4 (4 + g) (1 + v) +16 c^4 (2 g^4 + 8 g^2 v - 32 (1 + v) + 8 g (1 + v) -g^3 (1 + v)) +c (-2 + g)^3 (2 + g)^2 (g^4 + 192 (1 + v) + 48 g (1 + v) -11 g^3 (1 + v) - 8 g^2 (6 + 5 v)) -2 c^3 (1536 (1 + v) +g (-384 (1 + v) +g (-32 (15 + 19 v) +g (32 (4 + 3 v) +g (4 + g (-11 + 7 g - 3 v) + 60 v))))) +c^2 (-2 + g) (3328 (1 + v) +g (832 (1 + v) +g (-64 (25 + 24 v) +g (-8 (43 + 51 v) +g (8 (26 + 23 v) +g (26 + 2 g^2 + 50 v - g (4 + v))))))))/R, \qquad (3.A2.18)$$

where $R = (2 (8 (1 +c) (2 + c) - (8 + 5 c) g^2 + g^4) (-128 (1 + c) (2 + c) +24 (2 + c) (3 + c) g^2 - 3 (8 + 3 c) g^4 + g^6))$

$$w_{20}^*= (-6 c (-2 + g)^2 (2 + g) (4 + g) -8 c (24 + g (-6 + g (-9 + g + g^2))) v + (-2 + g)3 (2 + g)^2 (4 +g) (1 + v) +4 c^2 (-16 (1 + v) + g ((2 + g)^2 + 4 (1 + g) v)))/(-256 (1 +c) (2 + c) + 48 (2 + c) (3 + c) g^2 - 6 (8 + 3 c) g^4 + 2 g^6). \qquad (3.A2.19)$$

Substituting the wages with the optimal in the equations (3.A2.11) and (3.A2.12) accrues the optimal quantities of the first period, depending only on *c*, *g*, and *v*;

$$q_{10}^*(c,g,v) = (-(-2 + g)^7 (2 + g)^6 (4 + g) (6 + g (-1 + v) - 2 v) -2 c^2 (-2 + g)^3 (2 +g)^2 (2 g (-2528 +g (-720 + g (1584 + g (64 - 309 g + 18 g^3)))) -1280 (-3 + v) +g (2112 +g (2208 + g (-928 + g (-688 + g (102 + 59 g))))) v) +4 c^3 ((-4 + g2)^2 (-4992 +g (-5216 +g (6528 +g (1756 + g (-1780 + g (-139 + 131 g)))))) + (26624 +g (18944 +g (-11264 +g (-24896 +g (960 +g (10320 +g (88 + g (-1736 + g (-8 + 103 g))))))))) v) -256 c^6 (11 g^4 + 8 g2 (-9 + v) - 32 (-3 + v) + 8 g (1 + v) -g^3 (1 + v)) +32 c^5 (1792 (-3 + v) +g (-64 (13 + 5 v) +g (4992 + 114 g^4 + 11 g^3 (-3 + v) - 640 v +8 g (41 + v) + 8 g^2 (-169 + 7 v)))) +c (-2 + g)^5 (2 + g)^4 (320 (-3 + v) +g (16 (7 - 9 v) +g (248 - 168 v + g (-50 + 30 v + g (-11 + 4 g + 17 v))))) -8 c^4 (-17408 (-3 + v) \qquad (3.A2.20)$$

$$+g (256 (65 + v) +g (256 (-241 + 31 v) +g (32 (-323 + 69 v) +g (32 (770 - 37 v) +g (2108 - 884 v +g (-4090 + 58 v +g (-141 + 242 g + 87 v)))))))))/RQ$$

$$q_{20}^{*}(c,g,v) = -(-64 c^{6} (g (g (g (g (22 + 19 g - 14 v) + 4 (-37 + v)) +96 (-2 + v)) - 32 (-7 + v)) - 128 (-3 + v)) + (-2 +g)^{7} (2 + g)^{6} (4 + g) (6 + g (-1 + v) - 2 v) +4 c^{3} ((-4 + g^{2})^{2} (-24192 +g (-4512 + g (11024 +g (4172 +g (-1544 + g (-968 + 63 g + 60 g^{2})))))) - (-2 +g) (64512 +g (22784 +g (-57856 +g (-16480 +g (19568 +g (4624 +g (-2968 + g (-616 + g (171 + 34 g))))))))) v) +8 c^{5} (9216 (-3 + v) +g (256 (-51 + 5 v) +g (17536 - 7808 v +g (3 g (-1168 + g (-892 + g (74 + 67 g))) -32 (-341 + v) - 2 g (-920 + g (22 + 67 g)) v)))) +c (-2 + g)^{5} (2 + g)^{4} (-576 (-3 + v) +g (16 (33 + v) +g (-352 + 208 v +g (-10 (15 + v) + g (g (6 + 2 g - v) - 19 (1 + v)))))) -c^{2} (-2 + g)^{3} (2 + g)^{2} (16896 (-3 + v) +g (128 (-201 + 23 v) +g (64 (311 - 163 v) +g (128 (110 - 9 v) +g (64 (-5 + 34 v) +g (4 (-582 + 49 v) +g (-512 + 103 g + 34 g^{2} -2 (77 + 9 g) v))))))) +4 c^{4} (67584 (-3 + v) +g (512 (-135 + v) +g (512 (311 - 126 v) +g (192 (381 + 23 v) +g (64 (-706 + 323 v) +g (-8 (3347 + 221 v) +g (5456 - 2760 v +g (4072 + 174 v +g (-233 - 217 g + 133 v))))))))))/RQ. \tag{3.A2.21}$$

Substituting now the above optimal quantities of the first period as well as the optimal wages in equations (3.A2.5) and (3.A2.6) accrues the optimal quantities of the second period depending on *c*, *g*, and *v*;

$$q_{11}^{*}(c,g,v) = ((4 + 2 c -g^{2}) (-(-2 + g)^{6} (2 + g)^{5} (4 + g) (2 + g (-1 + v) - 6 v) +2 c^{2} (-2 + g)^{2} (2 +g) (-(-2 + g) (2 + g) (3008 +g (1936 + g (-760 + g (-558 + g (23 + 28 g))))) -2 (-6528 +g (-800 +g (3472 + g (388 + g (-554 + g (-47 + 23 g)))))) v) +128 c^{5} (32 (5 + v) +g (-8 (1 + v) + g (-8 (9 + v) + g (1 + 7 g + v)))) +c (-2 + g)^{4} (2 + g)^{3} (256 (1 + 5 v) +g (128 (3 + v) +g (-16 (3 + 22 v) +g (-110 - 5 g + 4 g^{2} + 15 (-2 + g) v)))) -16 c^{4} (-256 (29 + 9 v) +g (64 (3 + 11 v) +g (128 (35 + 8 v) +g (-8 (1 + 33 v) +g (-16 (54 + 7 v) + g (-5 + 52 g + 23 v)))))) +4 c^{3} (-2 + g) (-512 (59 + 31 v) +g (-128 (123 + 19 v) +g (64 (255 + 139 v) +g (16 (557 + 77 v) +g (-8 (337 + 187 v) +g (-22 (71 + 7 v) +g (122 + 79 g + 68 v)))))))))/RQ \tag{3.A2.22}$$

$$q_{21}^{*} (c,g,v)= ((-2 + g)^{7} (2 + g)^{6} (4 + g) (2 + g (-1 + v) - 6 v) +c^{2} (-2 (-4 + g2)4 (1056 +g (552 + g (-216 + g (-230 + g (10 + 17 g))))) + (-2 + g)^{3} (2 + g)2 (-50688 +g (-11392 +g (27072 +g (5760 +g (-4480 + g (-876 + g (214 + 35 g))))))) v) -64 c^{6} (128 - 384 v +g (32 (9 + v) +g (96 (-1 + 2 v) +g (g (14 + 17 g - 22 v) - 4 (35 + v))))) +c (-2 + g)^{5} (2 + g)^{4} (576 (1 - 3 v) +g (240 - 272 v +g (80 (-1 + 6 v) +g (-82 + 66 v + g (g (5 + 2 g - 2 v) - 23 (1 + v)))))) +8 c^{5} (9216 (-1 + 3 v) +g (-256 (69 + 13 v) +g (7808 - 17536 v +g (32 (363 + 41 v) +g (-1904 + 3504 v +g (-2556 + 142 g + 187 g^{2} -6 (22 + 37 g) v)))))) +2 c^{3} (-2 + g)^{2} (2 + g) (32256 (-1 + 3 v) +g (128 (-393 + 215 v) +g (1408 - 54528 v +g (29168 - 15024 v +g (6904 + 9688 v +g (-5416 + 2532 v +g (-1784 + 313 g + 118 g^{2} -2 (265 + 63 g) v))))))) +4 c^{4} (67584 (-1 + 3 v) +g (-512 (201 + 65 v) +g (512 (126 - 311 v) +g (64 (1369 + 323 v) +g (64 (-335 + 706 v) +g (-8 (3505 + 527 v) +g (2984 - 5456 v +g (3968 + 282 v +g (-147 - 208 g + 233 v))))))))))/RQ. \tag{3.A2.23}$$

The optimal yields are also depending on *c*, *g*, and *v*;

$$\Pi_1^*(c,g,v) = -c\left(q_{10}^* - q_{11}^* + \frac{1}{2}\left(q_{10}^* - q_{11}^*\right)^2\right) + q_{10}\left(1 - q_{10}^* - g q_{20}^*\right) - \left(q_{10}^* + q_{11}^*\right) w_{10}^* + q_{11}^*\left(v - q_{11}^* - g q_{21}^*\right) \tag{3.A1.24}$$

$$\Pi_2^*(c,g,v) = q_{20}^*\left(1 - g q_{10}^* - q_{20}^*\right) - \left(q_{20}^* + q_{21}^*\right) w_{20}^* + q_{21}^*\left(v - g q_{11}^* - q_{21}^*\right) \tag{3.A1.25}$$

$$U_1^*(c,g,v) = w_{10}^*\left(q_{10}^* + q_{11}^*\right) \tag{3.A1.26}$$

$$U_2^*(c,g,v) = w_{20}^*\left(q_{20}^* + q_{21}^*\right) \tag{3.A1.27}$$

$$CS^*(c,g,v) = \frac{1}{4}(1+g)\left(q_{10}^* + q_{11}^* + q_{20}^* + q_{21}^*\right). \tag{3.A1.28}$$

APPENDIX 3.A3

Second period/ four stage each firm maximazizes its profits as for q_{11} and q_{21} respectively;

$$\underset{q_{11}}{Max}\Pi_1\left(q_{10},q_{20},q_{11},q_{21},w_{10}\right)=-c\left(q_{10}-q_{11}+\frac{1}{2}\left(q_{10}-q_{11}\right)^2\right)+q_{10}\left(1-q_{10}-gq_{20}\right)-\left(q_{10}+q_{11}\right)w_{10}+ q_{11}\left(v-q_{11}-gq_{21}\right) \quad (3.A3.1)$$

$$\underset{q_{21}}{Max}\Pi_2\left(q_{10},q_{20},q_{11},q_{21},w_{20}\right)=-c\left(q_{20}-q_{21}+\frac{1}{2}\left(q_{20}-q_{21}\right)^2\right)+q_{20}\left(1-gq_{10}-q_{20}\right)-\left(q_{20}+q_{21}\right)w_{20}+ q_{21}\left(v-gq_{11}-q_{21}\right). \quad (3.A3.2)$$

From the *foc* accrues reaction functions of the second period;

$$RF_{11}\left(q_{21}\right)=\frac{\left(v+c\right)+cq_{10}-gq_{21}-w_{10}}{2+c} \quad (3.A3.3)$$

$$RF_{21}\left(q_{11}\right)=\frac{\left(v+c\right)+cq_{20}-gq_{11}-w_{20}}{2+c}. \quad (3.A3.4)$$

We solve the system of the second period RFs to get the optimal q_{11}^* and q_{21}^* -rules in the second period;

$$q_{11}^*=\frac{c^2\left(1+q_{10}\right)-\left(g-2\right)v+c\left(2-g+2q_{10}-gq_{20}+v-w_{10}\right)-2w_{10}+gw_{20}}{\left(2+\mathrm{c}-\mathrm{g}\right)\left(2+c+g\right)} \quad (3.A3.5)$$

$$q_{21}^*=\frac{c^2\left(1+q_{20}\right)-\left(g-2\right)v+c\left(2-g+2q_{20}-gq_{10}+v-w_{20}\right)-2w_{20}+gw_{10}}{\left(2+\mathrm{c}-\mathrm{g}\right)\left(2+c+g\right)}. \quad (3.A3.6)$$

Substituting the later into (3.A3.1) and (3.A3.2) accrues profits that depend on products of the first period and wages;

$\Pi1(q_{10}, q_{20}, w_{10}, w_{20}) = q_{10} (1 - q_{10} - g q_{20}) -w_{10} (q_{10} + (c^2 (1 + q_{10}) - (-2 + g) v - 2 w_{10} -c (-2 + g - 2 q_{10} + g q_{20} - v + w_{10}) +g w_{20})/((2 + c - g) (2 + c + g))) + ((c^2 (1 + q_{10}) - (-2 +g) v - 2 w_{10} - c (-2 + g - 2 q_{10} + g q_{20} - v + w_{10}) +g w_{20}) (v - (c^2 (1 + q_{10}) - (-2 + g) v - 2 w_{10} -c (-2 + g - 2 q_{10} + g q_{20} - v + w_{10}) +g w_{20})/((2 + c - g) (2 + c + g)) - (g (c^2 (1 + q_{20}) - (-2 + g) v + g w_{10} - 2 w_{20} -c (-2 + g + g q_{10} - 2 q_{20} - v + w_{20})))/((2 + c - g) (2 + c +g))))/((2 + c - g) (2 + c + g)) -c (q_{10} - (c^2 (1 + q_{10}) - (-2 + g) v - 2 w_{10} -c (-2 + g - 2 q_{10} + g q_{20} - v + w_{10}) +g w_{20})/((2 + c - g) (2 + c + g)) +1/2 (q_{10} - (c^2 (1 + q_{10}) - (-2 + g) v - 2 w_{10} -c (-2 + g - 2 q_{10} + g q_{20} - v + w_{10}) +g w_{20})/((2 + c - g) (2 + c + g)))^2)$ (3.A3.7)

$\Pi2 = (1 - g q_{10} - q_{20}) q_{20} + ((c^2 (1 + q_{20}) - (-2 + g) v + g w_{10} -2 w_{20} - c (-2 + g + g q_{10} - 2 q_{20} - v + w_{20})) (v - (g (c^2 (1 + q_{10}) - (-2 + g) v - 2 w_{10} -c (-2 + g - 2 q_{10} + g q_{20} - v + w_{10}) + g w_{20}))/((2 + c -g) (2 + c + g)) - (c^2 (1 + q_{20}) - (-2 + g) v + g w_{10} - 2 w_{20} -c (-2 + g + g q_{10} - 2 q_{20} - v + w_{20}))/((2 + c - g) (2 + c +g))))/((2 + c - g) (2 + c + g)) -w_{20} (q_{20} + (c^2 (1 + q_{20}) - (-2 + g) v + g w_{10} - 2 w_{20} -c (-2 + g + g q_{10} - 2 q_{20} - v + w_{20}))/((2 + c - g) (2 + c +g))) - c (q_{20} -$ (3.A3.8)

$(c^2 (1 + q_{20}) - (-2 + g) v + g w_{10} - 2 w_{20} -c (-2 + g + g q_{10} - 2 q_{20} - v + w_{20}))/((2 + c - g) (2 + c +g)) + 1/2 (q_{20} - (c^2 (1 + q_{20}) - (-2 + g) v + g w_{10} - 2 w_{20} -c (-2 + g + g q_{10} - 2 q_{20} - v + w_{20}))/((2 + c - g) (2 + c +g)))^2)$.

First period/ third stage each firm maximizes its profits as for q_{10} and q_{20} respectively. Reaction functions of the first period are given below;

$$RF_{10}(q_{20}) = Y + Bq_{20} + Cw_{10} + Dw_{20} \tag{3.A3.9}$$

$$RF_{20}(q_{10}) = Y + Bq_{10} + Cw_{20} + Dw_{10}, \tag{3.A3.10}$$

where, $Y = -\frac{(2+c-g)\left((g-2)(2+g)^2 + c^3(1+g-v) - c^2\left(4v+(g-3)g-2\right) - c\left(4+g^2+g^3+4v\right)\right)}{4(1+c)(2+c)^3 - 2(2+c)^3 g^2 + (2+c)g^4} > 0$,

note that *B*, *C*, and *D* are given in paragraph *3.3.1*. Recall that $B < 0$, $C < 0$, and $D > 0$.

We solve the system of the first period RFs to get the optimal q_{10}^* and q_{20}^* -rules in the first period;

$q_{10}^* = -(c (-4 + g^2) (16 (5 + v - 8 w_{10}) +g (g (4 (-4 + v + 7 w_{10}) +g (4 + g^2 - g (1 + w_{10}) - 12 w_{20})) -16 (2 + v - 4 w_{20}))) +2 c^5 (14 - 18 v + 40 w_{10} +g (8 + g^2 + 10 v - g (11 + w_{10}) - 20 w_{20})) +2 c^6 (2 - 2 v + 4 w_{10} + g (1 - g + v - 2 w_{20})) + (-4 +g^2)^3 (2 - 2 w_{10} + g (-1 + w_{20})) +2 c^4 (28 - 64 v + 164 w_{10} +g (26 + 40 v - 82 w_{20} +g (-6 (7 + 2 w_{10}) + g (6 + 2 g - v + 3 w_{20})))) +c^3 (-32 (1 + 7 v - 22 w_{10}) +g (32 (3 + 5 v - 11 w_{20}) +g (-4 (31 + 27 w_{10}) +g (24 - 12 v + g (19 - 3 g + v + 2 w_{10}) + 40 w_{20})))) -c2 (64 (4 + 3 v - 13 w10) +g (-32 (4 + 5 v - 13 w_{20}) +g (8 + 232 w_{10} +g (4 (-1 + 6 v - 25 w_{20}) +g (-2 (11 + 2 v + 7 w_{10}) +g (9 + g + 3 w_{20}))))))))/V$ (3.A3.11)

$q_{20}^* = -((-4 + g^2)^3 (2 + g (-1 + w_{10}) - 2 w_{20}) +2 c^6 (2 - 2 v + g (1 - g + v - 2 w_{10}) + 4 w_{20}) +2 c^5 (14 - 18 v + 40 w_{20} +g (8 + g^2 + 10 v - 20 w_{10} - g (11 + w_{20}))) +2 c^4 (28 - 64 v + 164 w_{20} +g (26 + 40 v - 82 w_{10} +g (g (6 + 2 g - v + 3 w_{10}) - 6 (7 + 2 w_{20})))) +c (-4 + g^2) (16 (5 + v - 8 w_{20}) +g (-16 (2 + v - 4 w_{10}) +g (4 (-4 + v + 7 w_{20}) +g (4 + g2 - 12 w_{10} - g (1 + w_{20}))))) +c^3 (-32 (1 + 7 v - 22 w_{20}) +g (32 (3 + 5 v - 11 w_{10}) +g (-4 (31 + 27 w_{20}) +g (24 - 12 v + 40 w_{10} + g (19 - 3 g + v + 2 w_{20}))))) -c^2 (64 (4 + 3 v - 13 w_{20}) +g (-32 (4 + 5 v - 13 w_{10}) +g (8 + 232 w_{20} +g (4 (-1 + 6 v - 25 w_{10}) +g (g (9 + g + 3 w_{10}) -2 (11 + 2 v + 7 w_{20})))))))/V$. (3.A3.12)

Substituting the later into (3.A3.7) and (3.A3.8) accrues profits that depend on wages only

$\Pi1(w_{10}, w_{20}) = -(-2 (16 (1 + c)^2 (2 + c)^4 - 4 (2 + c)^6 g^2 +8 (2 + c)^2 (3 + c (3 + c)) g^4 - 4 (2 + c)2 g^6 +g^8) w_{10} (-2 c^3 (352 (1 + v - 2 w_{10}) +g (g (-48 - 54 v + 118 w_{10} +g (14 + 20 v + g (1 + g + v - 2 w_{10}) - 46 w_{20})) -176 (1 + v - 2 w_{20}))) +c (-4 + g^2) (128 (1 + v - 2 w_{10}) +g (g (-4 (8 + 7 v - 15 w_{10}) +g (16 + (12 + g) v - g w_{10} - 28 w_{20})) -64 (1 + v - 2 w_{20}))) +2 c^5 (-40 (1 + v - 2 w_{10}) +g (g (-1 + g + v - 2 w_{10}) + 20 (1 + v - 2 w_{20}))) +c^2 (-832 (1 + v - 2 w_{10}) +g (g (8 (31 + 29 v - 62 w_{10}) +g (g (-14 (1 + v - 2 w_{10}) +g (1 + g + 3 v - 8 w_{20})) -4 (27 + 25 v - 56 w_{20}))) + 416 (1 + v - 2 w_{20}))) +4 c^6 (-2 + g - 2 v + g v + 4 w_{10} - 2 g w_{20}) - (-4 +g^2)^3 (-2 + g - 2 v + g v + 4 w_{10} - 2 g w_{20}) +2 c^4 (-164 (1 + v - 2 w_{10}) +g (82 (1 + v - 2 w_{20}) +g (4 + 12 v - 26 w_{10} + g (2 - 3 v + 7 w_{20}))))) +2 (2 + c - g) (2 + c + g) V (2 c^3 (-30 - 14 v + 24 w_{10} +g (8 + 6 v + g (6 + v - w_{10}) - 12 w_{20})) +2 c^4 (-2 (3 + v - 2 w_{10}) + g (1 + g + v - 2 w_{20})) + (-4 +g^2)^2 ((-2 + g) v + 2 w_{10} - g w_{20}) +c (-4 + g^2) (4 (3 + 5 v - 6 w_{10}) +g (-8 + g^2 - 8 v - g (1 + v) + 12 w_{20})) -2 c^2 (48 + 36 v - 52 w_{10} +g (-20 - 14 v +g (-6 (2 + v - w_{10}) + g (3 + g + v - 2 w_{20})) +26 w_{20}))) (v + ((2 + c - g) (2 + c +g) (c (-4 + g^2) (4 (3 + 5 v - 6 w_{10}) +g (g (-9 + g^2 - 9 v - g v + 12 w_{10})$

$$
\begin{gathered}
+4 (1 + 3 v - 3 w_{20}))) + (-4 +g^2)^2 ((-2 + g) (1 + g) v + 2 w_{10} +g (-g w_{10} + w_{20})) +2 c^4 (-2 (3 + \\
v - 2 w_{10}) +g (-5 - v + g (2 + g + v - 2 w_{10}) + 2 w_{20})) +2 c^3 (-30 - 14 v + 24 w_{10} +g (-22 - 8 v \\
+g (14 + 7 v - 13 w_{10} + g (6 + v - w_{20})) +12 w_{20})) -2 c^2 (48 + 36 v - 52 w_{10} +g (28 + 22 v - \\
26 w_{20} +g (-4 (8 + 5 v - 8 w_{10}) +g (-9 - 5 v + g (4 + g + v - 2 w_{10}) +4 w_{20}))))))/(V)) +2 (2 c^6 \\
(2 (-5 + v - 2 w_{10}) +g (-3 + v + g (2 + g - v + 2 w_{10}) - 2 w_{20})) - (-4 +g^2)^3 (g + g^2 w_{10} - 2 (1 + \\
w_{10}) - g w_20) -2 c^4 (356 - 64 v + 164 w_{10} +g (54 - 24 v +g (-136 + 40 v - 94 w_{10} +g (-36 - v \\
+ g (12 + 2 g - v + 3 w_{10}) - 9 w_{20})) +82 w_{20})) -2 c^5 (94 - 18 v + 40 w_{10} +g (22 - 8 v + 20 w_{20} \\
+g (-27 + g^2 + 10 v - 21 w_{10} - g (10 + w_{20})))) -c (-4 + g^2) (16 (-11 + v - 8 w_{10}) +g (48 - 64 \\
w_{20} +g (80 + g^4 - 12 v + 92 w_{10} - 13 g^2 (1 + w_{10}) -g^3 w_{20} + 4 g (-3 + v + 4 w_{20})))) +c^2 (64 (- \\
22 + 3 v - 13 w_{10}) +g (32 (4 + v - 13 w_{20}) +g (840 - 160 v + 648 w_{10} +g (g (2 (-89 + 10 v - \\
57 w_{10}) +g (-13 - 4 v + g (14 + g + 3 w_{10}) - 11 w_{20})) +4 (1 + 6 v + 33 w_{20}))))) +c^3 (32 (-43 + \\
7 v - 22 w_{10}) +g (32 (-2 + 2 v - 11 w_{20}) +g (668 - 160 v + 460 w_{10} +g (4 (25 + 3 v + 17 w_{20}) \\
+g (-99 + 3 g^2 + 11 v - 42 w_{10} -g (16 + v + 2 w_{20}))))))) (-c (-4 +g^2) (16 (5 + v - 8 w_{10}) +g (g \\
(4 (-4 + v + 7 w_{10}) +g (4 + g^2 - g (1 + w_{10}) - 12 w_{20})) -16 (2 + v - 4 w_{20}))) -2 c^5 (14 - 18 v + \\
40 w_{10} +g (8 + g^2 + 10 v - g (11 + w_{10}) - 20 w_{20})) +c^3 (3 g^5 + 32 (1 + 7 v - 22 w_{10}) - g^4 (19 + \\
v + 2 w_{10}) +4 g^2 (31 + 27 w_{10}) - 32 g (3 + 5 v - 11 w20) +4 g^3 (-6 + 3 v - 10 w_{20})) - (-4 + \\
g^2)^3 (2 - 2 w_{10} +g (-1 + w_{20})) +2 c^6 (2 (-1 + v - 2 w_{10}) + g (-1 + g - v + 2 w_{20})) -2 c^4 (28 - 64 \\
v + 164 w_{10} +g (26 + 40 v - 82 w_{20} +g (-6 (7 + 2 w_{10}) + g (6 + 2 g - v + 3 w_{20})))) +c^2 (64 (4 \\
+ 3 v - 13 w_{10}) +g (-32 (4 + 5 v - 13 w_{20}) +g (8 + 232 w_{10} +g (4 (-1 + 6 v - 25 w_{20}) +g (-2 \\
(11 + 2 v + 7 w_{10}) +g (9 + g + 3 w_{20}))))))) +c (2 (2 + c - g) (2 + c + g) V (2 c^3 (-30 - 14 v + \\
24 w_{10} + g (8 + 6 v + g (6 + v - w_{10}) - 12 w_{20})) + 2 c^4 (-2 (3 + v - 2 w10) + g (1 + g + v - 2 \\
w_{20})) + (-4 +g^2)^2 ((-2 + g) v + 2 w_{10} - g w_{20}) +c (-4 + g^2) (4 (3 + 5 v - 6 w_{10}) +g (-8 + g^2 - 8 v \\
- g (1 + v) + 12 w_{20})) -2 c^2 (48 + 36 v - 52 w_{10} +g (-20 - 14 v +g (-6 (2 + v - w_{10}) + g (3 + g \\
+ v - 2 w_{20})) +26 w_{20}))) + (-4 c^6 (-4 + g^2) + (-2 + g)^4 (2 +g)^3 (-1 + v) +2 c^5 (4 (17 + v) + g (- \\
4 + g^2 - g (21 + v))) +c^2 (64 (5 + 7 v) -g (32 (5 + 3 v) +g (8 (33 + 29 v - 4 w_{10}) +g (g (-58 - \\
22 v + g (19 + 3 g + 3 v - 2 w_{20})) -4 (29 + 13 v - 6 w_{20}))))) +2 c^4 (4 (55 + 9 v) +g (-2 (15 + \\
v) +g (2 (-44 - 6 v + w_{10}) + g (10 + 4 g + v - w_{20})))) +c (-4 + g^2) (32 - 96 v +g (32 v +g (2 g^3 \\
+ 36 v - 4 w_{10} - g^2 (2 + v + w_{10}) +4 g (-2 - 3 v + w_{20})))) -4 c^3 (-32 (5 + 2 v) +g (8 (5 + v) +g \\
(86 + 27 v - 5 w_{10} +g (-19 + g^2 - 4 v - g (10 + v) + 3 w_{20})))))^2 -2 V (c (-4 + g^2) (16 (5 + v - 8 \\
w_{10}) +g (g (4 (-4 + v + 7 w_{10}) +g (4 + g^2 - g (1 + w_{10}) - 12 w_{20})) -16 (2 + v - 4 w_{20}))) +2 c^5 \\
(14 - 18 v + 40 w_{10} +g (8 + g^2 + 10 v - g (11 + w_{10}) - 20 w_{20})) +2 c^6 (2 - 2 v + 4 w_{10} + g (1 - \\
g + v - 2 w_{20})) + (-4 +g^2)^3 (2 - 2 w_{10} + g (-1 + w_{20})) +2 c^4 (28 - 64 v + 164 w_{10} +g (26 + 40 v \\
- 82 w_{20} +g (-6 (7 + 2 w_{10}) + g (6 + 2 g - v + 3 w_{20})))) +c^3 (-32 (1 + 7 v - 22 w_{10}) +g (32 (3 + \\
5 v - 11 w_{20}) +g (-4 (31 + 27 w_{10}) +g (24 - 12 v + g (19 - 3 g + v + 2 w_{10}) +40 w_{20})))) -c^2 \\
(64 (4 + 3 v - 13 w_{10}) +g (-32 (4 + 5 v - 13 w_{20}) +g (8 + 232 w10 +g (4 (-1 + 6 v - 25 w_{20}) \\
+g (-2 (11 + 2 v + 7 w_{10}) +g (9 + g + 3 w_{20}))))))))))/(2V^2)
\end{gathered}
\tag{3.A3.13}
$$

$$
\begin{gathered}
\Pi 2(w_{10}, w_{20}) =-(2 (2 + c - g) (2 + c + g) (16 (1 + c)^2 (2 + c)^4 -4 (2 + c)^6 g^2 + 8 (2 + c)^2 (3 + c \\
(3 + c)) g^4 -4 (2 + c)^2 g^6 +g^8) (c (-4 + g^2) (g (-8 + g^2 - 8 v - g (1 + v) + 12 w_{10}) +4 (3 + 5 v - \\
6 w_{20})) +2 c^4 (g (1 + g + v - 2 w_{10}) - 2 (3 + v - 2 w_{20})) -2 c^2 (48 + 36 v +g (-20 - 14 v + 26 \\
w_{10} +g (g (3 + g + v - 2 w_{10}) - 6 (2 + v - w_{20}))) -52 w_{20}) + (-4 + g^2)^2 ((-2 + g) v - g w_{10} + 2 \\
w_{20}) +2 c^3 (-30 - 14 v + g (8 + 6 v - 12 w_{10} + g (6 + v - w_{20})) +24 w_{20})) (v + ((2 + c - g) (2 + \\
c +g) (2 c^4 (g (-5 - v + 2 w_{10} + g (2 + g + v - 2 w_{20})) -2 (3 + v - 2 w_{20})) -2 c^2 (48 + 36 v +g \\
(28 + 22 v - 26 w_{10} +g (g (-9 - 5 v + 4 w_{10} + g (4 + g + v - 2 w_{20})) -4 (8 + 5 v - 8 w_{20}))) - 52 \\
w_{20}) +2 c^3 (-30 - 14 v +g (-22 - 8 v + 12 w_{10} +g (14 + 7 v + g (6 + v - w_{10}) - 13 w_{20})) +24 \\
w_{20}) + (-4 + g^2)^2 ((-2 + g) (1 + g) v + 2 w_{20} +g (w_{10} - g w_{20})) +c (-4 + g^2) (4 (3 + 5 v - 6 w_{20}) \\
+g (4 (1 + 3 v - 3 w_{10}) +g (-9 + g^2 - 9 v - g v + 12 w_{20})))))/V) -2 (V) w_{20} (c^2 (g (416 (1 + v - \\
2 w_{10}) +g (g (-4 (27 + 25 v - 56 w_{10}) +g (g (1 + g + 3 v - 8 w_{10}) -14 (1 + v - 2 w_{20}))) + 8 (31 \\
+ 29 v - 62 w_{20}))) -832 (1 + v - 2 w_{20})) +2 c^4 (g (82 (1 + v - 2 w_{10}) +g (4 + 12 v + g (2 - 3 v \\
+ 7 w_{10}) - 26 w_{20})) -164 (1 + v - 2 w_{20})) +2 c^5 (g (20 (1 + v - 2 w_{10}) + g (-1 + g + v - 2 w_{20})) \\
-40 (1 + v - 2 w_{20})) +4 c^6 (-2 + g - 2 v + g v - 2 g w_{10} + 4 w_{20}) - (-4 +g^2)^3 (-2 + g - 2 v + g v -
\end{gathered}
$$

$2 g w_{10} + 4 w_{20}) -2 c^3 (352 (1 + v - 2 w_{20}) +g (-176 (1 + v - 2 w_{10}) +g (-48 - 54 v +g (14 +$
$20 v - 46 w_{10} + g (1 + g + v - 2 w_{20})) +118 w_{20}))) +c (-4 + g^2) (128 (1 + v - 2 w_{20}) +g (-64 (1$
$+ v - 2 w_{10}) +g (-4 (8 + 7 v - 15 w_{20}) +g (16 + (12 + g) v - 28 w_{10} - g w_{20}))))) +2 (2 c^6 (g (-1$
$+ g - v + 2 w_{10}) + 2 (-1 + v - 2 w_{20})) - (-4 +g^2)^3 (2 + g (-1 + w_{10}) - 2 w_{20}) +c^3 (3 g^5 - 32 g (3$
$+ 5 v - 11 w_{10}) +4 g^3 (-6 + 3 v - 10 w_{10}) + 32 (1 + 7 v - 22 w_{20}) -g^4 (19 + v + 2 w_{20}) + 4 g^2$
$(31 + 27 w_{20})) -2 c^5 (14 - 18 v + 40 w_{20} +g (8 + g^2 + 10 v - 20 w_{10} - g (11 + w_{20}))) -2 c^4 (28 -$
$64 v + 164 w_{20} +g (26 + 40 v - 82 w_{10} +g (g (6 + 2 g - v + 3 w_{10}) - 6 (7 + 2 w_{20})))) -c (-4 +$
$g^2) (16 (5 + v - 8 w_{20}) +g (-16 (2 + v - 4 w_{10}) +g (4 (-4 + v + 7 w_{20}) +g (4 + g^2 - 12 w_{10} - g$
$(1 + w_{20}))))) +c^2 (64 (4 + 3 v - 13 w_{20}) +g (-32 (4 + 5 v - 13 w_{10}) +g (8 + 232 w_{20} +g (4 (-1$
$+ 6 v - 25 w_{10}) +g (g (9 + g + 3 w_{10}) -2 (11 + 2 v + 7 w_{20}))))))) (-2 c^5 (94 - 18 v +g (22 - 8 v$
$+ 20 w_{10} +g (-27 + g^2 + 10 v - g (10 + w_{10}) - 21 w_{20})) +40 w_{20}) - (-4 + g^2)^3 (-2 + g - g w_{10} +$
$(-2 + g^2) w_{20}) +2 c^6 (2 (-5 + v - 2 w_{20}) +g (-3 + v - 2 w_{10} + g (2 + g - v + 2 w_{20}))) +c^3 (32 (-$
$43 + 7 v - 22 w_{20}) +g (32 (-2 + 2 v - 11 w_{10}) +g (668 - 160 v +g (4 (25 + 3 v + 17 w_{10}) +g (-$
$99 + 3 g^2 + 11 v - g (16 + v + 2 w_{10}) -42 w_{20})) + 460 w_{20}))) -c (-4 + g^2) (16 (-11 + v - 8 w_{20})$
$+g (48 - 64 w_{10} +g (80 + g^4 - 12 v - g^3 w_{10} + 4 g (-3 + v + 4 w_{10}) +92 w_{20} - 13 g^2 (1 + w_{20}))))$
$-2 c4 (356 - 64 v + 164 w_{20} +g (54 - 24 v + 82 w_{10} +g (-136 + 40 v - 94 w_{20} +g (-36 - v - 9$
$w_{10} + g (12 + 2 g - v + 3 w_{20}))))) +c^2 (64 (-22 + 3 v - 13 w_{20}) +g (32 (4 + v - 13 w_{10}) +g$
$(840 - 160 v + 648 w_{20} +g (4 (1 + 6 v + 33 w_{10}) +g (2 (-89 + 10 v - 57 w_{20}) +g (-13 - 4 v -$
$11 w_{10} +g (14 + g + 3 w_{20})))))))) +c (2 (2 + c - g) (2 + c + g) V (c (-4 +g^2) (g (-8 + g^2 - 8 v -$
$g (1 + v) + 12 w_{10}) +4 (3 + 5 v - 6 w_{20})) +2 c^4 (g (1 + g + v - 2 w_{10}) - 2 (3 + v - 2 w_{20})) -2 c^2$
$(48 + 36 v +g (-20 - 14 v + 26 w_{10} +g (g (3 + g + v - 2 w_{10}) - 6 (2 + v - w_{20}))) -52 w_{20}) + (-4$
$+ g2)^2 ((-2 + g) v - g w_{10} + 2 w_{20}) +2 c^3 (-30 - 14 v +g (8 + 6 v - 12 w_{10} + g (6 + v - w_{20}))$ (3.A3.14)
$+24 w_{20})) + (-4 c^6 (-4 + g^2) + (-2 + g)^4 (2 +g)^3 (-1 + v) +2 c^5 (4 (17 + v) + g (-4 + g^2 - g (21$
$+ v))) +c^2 (64 (5 + 7 v) -g (32 (5 + 3 v) +g (g (g (-58 - 22 v + g (19 + 3 g + 3 v - 2 w_{10})) -4$
$(29 + 13 v - 6 w_{10})) +8 (33 + 29 v - 4 w_{20})))) -4 c^3 (-32 (5 + 2 v) +g (8 (5 + v) +g (86 + 27 v$
$+g (-19 + g^2 - 4 v - g (10 + v) + 3 w_{10}) -5 w_{20}))) +2 c^4 (4 (55 + 9 v) +g (-2 (15 + v) +g (g (10$
$+ 4 g + v - w_{10}) + 2 (-44 - 6 v + w_{20})))) +c (-4 + g^2) (32 - 96 v +g (32 v +g (2 g^3 + 36 v + 4 g$
$(-2 - 3 v + w_{10}) - 4 w_{20} -g^2 (2 + v + w_{20})))))^2 -2 (V) ((-4 + g^2)^3 (2 + g (-1 + w_{10}) - 2 w_{20}) +2$
$c^6 (2 - 2 v + g (1 - g + v - 2 w_{10}) + 4 w_{20}) +2 c^5 (14 - 18 v + 40 w_{20} +g (8 + g^2 + 10 v - 20$
$w_{10} - g (11 + w_{20}))) +2 c^4 (28 - 64 v + 164 w_{20} +g (26 + 40 v - 82 w_{10} +g (g (6 + 2 g - v + 3$
$w_{10}) - 6 (7 + 2 w_{20})))) +c (-4 + g^2) (16 (5 + v - 8 w_{20}) +g (-16 (2 + v - 4 w_{10}) +g (4 (-4 + v +$
$7 w_{20}) +g (4 + g^2 - 12 w_{10} - g (1 + w_{20}))))) +c^3 (-32 (1 + 7 v - 22 w_{20}) +g (32 (3 + 5 v - 11$
$w_{10}) +g (-4 (31 + 27 w_{20}) +g (24 - 12 v + 40 w_{10} +g (19 - 3 g + v + 2 w_{20}))))) -c^2 (64 (4 + 3$
$v - 13 w_{20}) +g (-32 (4 + 5 v - 13 w_{10}) +g (8 + 232 w_{20} +g (4 (-1 + 6 v - 25 w_{10}) +g (g (9 + g$
$+ 3 w_{10}) -2 (11 + 2 v + 7 w_{20})))))))))/2V^2.$

Union's utility is also depending on wages only. Union's utility accrues substituting optimal quantities in the equation below;

$$U_j \left(= w_{i0} \left(q_{j0}^* + q_{j1}^* \right) \right), \; j = 1, 2, \qquad (3.A3.15)$$

thus

$U1(w_{10}, w_{20}) = -(w_{10} (-2 c^3 (352 (1 + v - 2 w_{10}) +g (g (-48 - 54 v + 118 w_{10} +g (14 + 20 v +$
$g (1 + g + v - 2 w_{10}) - 46 w_{20})) -176 (1 + v - 2 w_{20}))) +c (-4 + g^2) (128 (1 + v - 2 w_{10}) +g (g$
$(-4 (8 + 7 v - 15 w_{10}) +g (16 + (12 + g) v - g w_{10} - 28 w_{20})) -64 (1 + v - 2 w_{20}))) +2 c^5 (-40$
$(1 + v - 2 w_{10}) +g (g (-1 + g + v - 2 w_{10}) + 20 (1 + v - 2 w_{20}))) +c^2 (-832 (1 + v - 2 w_{10}) +g$ (3.A3.16)
$(g (8 (31 + 29 v - 62 w_{10}) +g (g (-14 (1 + v - 2 w_{10}) +g (1 + g + 3 v - 8 w_{20})) -4 (27 + 25 v -$
$56 w_{20}))) + 416 (1 + v - 2 w_{20}))) +4 c^6 (-2 + g - 2 v + g v + 4 w_{10} - 2 g w_{20}) - (-4 +g^2)^3 (-2 +$

$g - 2 v + g v + 4 w10 - 2 g w_{20}) +2 c^4 (-164 (1 + v - 2 w_{10}) +g (82 (1 + v - 2 w_{20}) +g (4 + 12 v - 26 w_{10} + g (2 - 3 v + 7 w_{20}))))))/V$

$$U2 (w_{10}, w_{20}) = -(w_{20} (c^2 (g (416 (1 + v - 2 w_{10}) +g (g (-4 (27 + 25 v - 56 w_{10}) +g (g (1 + g + 3 v - 8 w_{10}) -14 (1 + v - 2 w_{20}))) + 8 (31 + 29 v - 62 w_{20}))) -832 (1 + v - 2 w_{20})) +2 c^4 (g (82 (1 + v - 2 w_{10}) +g (4 + 12 v + g (2 - 3 v + 7 w_{10}) - 26 w_{20})) -164 (1 + v - 2 w_{20})) +2 c^5 (g (20 (1 + v - 2 w_{10}) + g (-1 + g + v - 2 w_{20})) -40 (1 + v - 2 w_{20})) +4 c^6 (-2 + g - 2 v + g v - 2 g w10 + 4 w_{20}) - (-4 +g^2)^3 (-2 + g - 2 v + g v - 2 g w_{10} + 4 w_{20}) -2 c^3 (352 (1 + v - 2 w_{20}) +g (-176 (1 + v - 2 w_{10}) +g (-48 - 54 v +g (14 + 20 v - 46 w_{10} + g (1 + g + v - 2 w_{20})) +118 w_{20}))) +c (-4 + g^2) (128 (1 + v - 2 w_{20}) +g (-64 (1 + v - 2 w_{10}) +g (-4 (8 + 7 v - 15 w_{20}) +g (16 + (12 + g) v - 28 w_{10} - g w_{20}))))))/V, \quad (3.A3.17)$$

Where, $V = (16 (1 + c)^2 (2 +c)^4 - 4 (2 + c)^6 g^2 + 8 (2 + c)^2 (3 + c (3 + c)) g^4 -4 (2 + c)^2 g^6 + g^8)$.

In the first period/ second stage, each firm bargains with the union that represents employees of the same sector the wage. I make the assumption that unions possess all the bargaining power (b=1). The bargaining problem is set as follow;

$$w_{j0} = \max_{w_{j0}} B_j = \max_{w_{j0}} \left(\text{b Log}\left[\text{U}_j\right] + (1-\text{b}) \text{ Log}\left[\Pi_j\right]\right)$$

Solving the system that accrues from the first order conditions of the above problem as for w_{10} and w_{20} accrues a unique stable solution for the equilibrium firm-specific wage contracts w_{10}^* and w_{20}^*, respectively;

$$w_{10}^*= w_{20}^*= ((-4 (1 + c) (2 + c)^2 + 2 (2 + c)^3 g - 2 (2 + c) g^3 +g^4) (2 c^3 (1 + v) - (-2 + g) (2 + g)^2 (1 + v) +c^2 (10 + g + g^2 + (10 + g) v) +c (16 (1 + v) + 4 g (1 + v) - g^2 (2 + v))))/(-32 (1 + c)^2 (2 +c)^4 + 8 (1 + c)^2 (2 + c)^4 g +8 (2 + c)^2 (12 + c (19 + c (9 + c))) g^2 -2 (2 + c)^2 (12 + c (18 + 7 c)) g3 - 8 (2 + c)^2 (3 + c) g^4 +4 (2 + c) (3 + 2 c) g^5 + 2 (4 + c) g^6 - 2 g^7). \quad (3.A3.18)$$

Substituting the wages with the optimal in the equations (3.A3.11) and (3.A3.12) accrues the optimal quantities of the first and the second period, depending only on *c*, *g*, and *v*;

$$q_{10}^*= q_{20}^*= -(c (-2 + g)^2 (2 + g)^3 (384 +g (-64 + g (-60 + g (8 + g (-7 + 2 g))))) +4 c^8 (g (-52 + g (36 + g)) + 52 (-3 + v)) + (-2 + g)^4 (2 +g)^5 (6 + g (-1 + v) - 2 v) -4 c (-4 + g2)^3 (-16 + 5 g^2) v +8 c^9 (-6 - 2 g + g2 + 2 v) -4 c^7 (-284 (-3 + v) + g (284 + g (-275 + g (-20 + 7 g) + v))) +4 c^5 (1312 (-3 + v) +g (-1312 +g (g (421 + g (-369 + g (-22 + 7 g) - 23 v)) +38 (71 + 2 v)))) -4 c^6 (-828 (-3 + v) +g (828 + g (-1137 - v + 2 g (-67 + g (40 + g + v))))) -c^2 (-4 + g^2) (-1472 (-3 + v) +g (1472 +g (64 (-7 + 18 v) +g (-144 +g (-4 (73 + 52 v) +g (-44 + g2 + 5 g (8 + v)))))) +c^4 (3392 (-3 + v) +g (-3392 +g (16 (869 + 107 v) +g (2528 +g (-4 (853 + 114 v) +g (-332 + g (197 + 5 g + 11 v))))))) +c^3 (-2304 (-3 + v) +g (2304 +g (64 (98 + 69 v) +g (992 +g (-8 (469 + 156 v) +g (-484 +g (498 + (31 - 11 g) g + 80 v))))))))/X, \quad (3.A3.19)$$

Substituting now the above optimal quantities of the first period as well as the optimal wages in equations (3.A3.5) and (3.A3.6) accrue the optimal quantities of the second period depending on *c*, *g*, and *v*;

$$q_{11}^*= q_{21}^*= ((2 + c - g) (2 + c +g) (-2 c (-2 + (-2 + g) g) (-4 + g^2)^3 - (-2 + g)^3 (2 +g)^4 (2 + g (-1 + v) - 6 v) +c (-2 + g) (2 + g)^2 (416 +g (-80 + g (-100 + g (20 + g)))) v +8 c^7 ((-2 + g) g - 2 (5 + v)) +4 c^6 (-4 (49 + 13 v) + g (-4 (10 + v) + g (26 + g + 2 v))) -4 c^5 (780 + 284 v +g (164 + 4 g^3 + 40 v - g^2 (13 + v) - 3 g (49 + 9 v))) -4 c^4 (1596 + 844 v +g (4 (88 + 41 v) +g (- \quad (3.A3.20)$$

437 - 145 v + g (-59 - 13 v + g (29 + g + 3 v))))) +4 c^3 (-64 (27 + 23 v) +g (-32 (13 + 11 v) +g (692 + 402 v +g (124 + 59 v +g (-83 + 2 g^2 - 23 v - g (7 + v)))))) +c^2 (2 + g) (-64 (27 + 47 v) +g (352 + 672 v +g (856 + 888 v +g (-4 (45 + 49 v) +g (g (29 + g + 5 v) -2 (61 + 19 v))))))))/X,

where, X = (2 (-64 $(1 +c)^3$ $(2 + c)^6$ - 16 $(1 + c)^2$ $(2 + c)^6$ (3 + c) g + 8 (1 + c) $(2 + c)^4$ (32 + c (58 + c (36 + c (9 + c)))) g^2 + 4 $(2 + c)^4$ (52 + c (110 + c (81 + c (23 + 2 c)))) g^3 - 2 $(2 + c)^4$ (48 + c (84 + c (48 + 11 c))) g^4 - 4 $(2 + c)^3$ (44 + c (65 + c (32 + 5 c))) g^5 + 2 $(2 + c)^2$ (32 + c (60 + c (38 + 9 c))) g^6 + $(2 + c)^2$ (72 + c (66 + 17 c)) g^7 - 2 $(2 + c)^2$ (2 + 3 c) g^8 - 2 (2 + c) (7 + 3 c) g^9 + c g^{10} + g^{11})).

The optimal yields are also depending on *c*, *g*, and *v*;

$$\Pi_j^*\left(c,g,v\right)=-c\left(q_{j0}^*-q_{j1}^*+\frac{1}{2}\left(q_{j0}^*-q_{j1}^*\right)^2\right)+q_{j0}\left(1-q_{j0}^*-gq_{j0}^*\right)-\left(q_{j0}^*+q_{j1}^*\right)w_{j0}^*+q_{j1}^*\left(v-q_{j1}^*-gq_{j1}^*\right) \quad (3.A3.21)$$

$$U_j^*\left(c,g,v\right)=w_{j0}^*\left(q_{j0}^*+q_{j1}^*\right),\ \text{where}\ \ j=1,2 \quad (3.A3.22)$$

$$CS^*(c,g,v)=\frac{1}{4}\left(1+g\right)\left(q_{10}^*+q_{11}^*+q_{20}^*+q_{21}^*\right) \quad (3.A3.23)$$

APPENDIX 3.B1

***frr*₁**: Note that all the figures concern the optimal point.

Figure 3.B1.1a |g=1
Contour Plot $q_{11}(frr_1)$

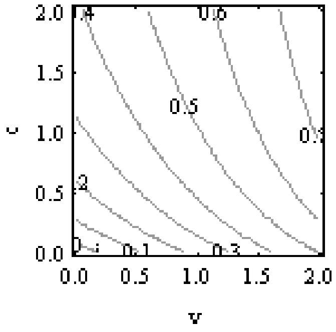

Figure 3.B1.1b | g=1
Contour Plot $q_{21}(frr_1)$

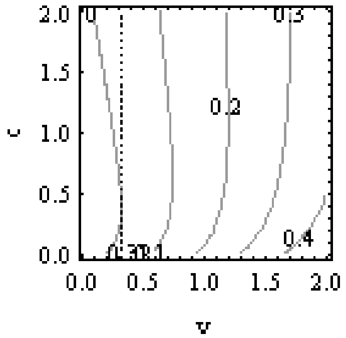

Figure 3.B1.1c | g=1
Contour Plot $q_{10}(frr_1)$

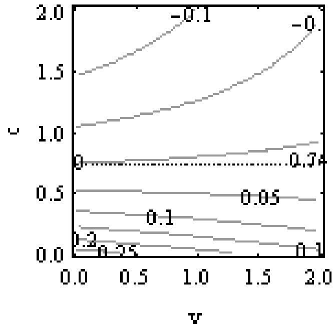

Figure 3.B1.1d | g=1
Contour Plot $q_{20}(frr_1)$

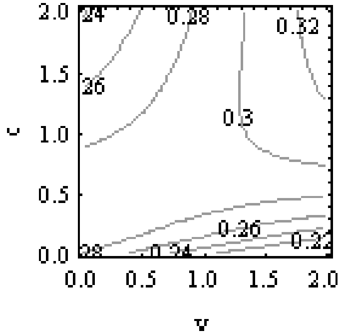

Figure 3.B1.1: Positive isoquants in each period simultaneously, provided that $v > 0.32$ and $c \leq 0.7$

Figure 3.B1.2a | g=0.5
Contour Plot $q_{11}(frr_1)$

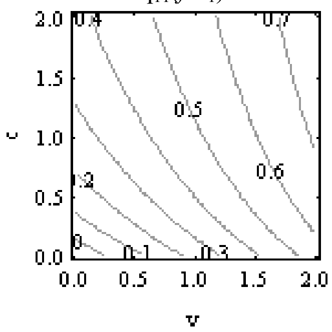

Figure 3.B1.2b | g=0.5
Contour Plot $q_{21}(frr_1)$

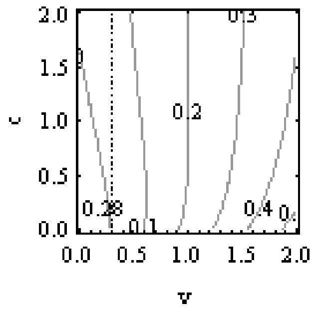

Figure 3.B1.2c | g=0.5
Contour Plot $q_{10}(frr_1)$

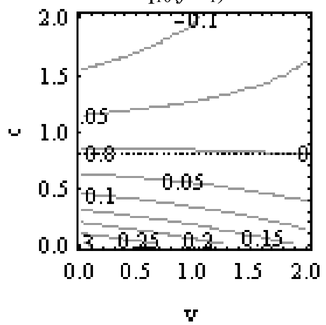

Figure 3.B1.2d | g=0.5
Contour Plot $q_{20}(frr_1)$

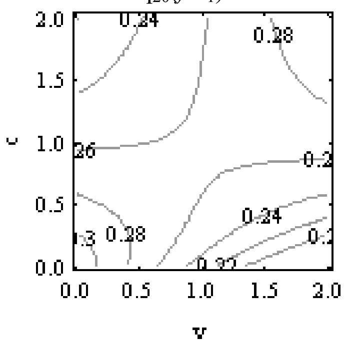

Figure 3.B1.2: Positive isoquants in each period, simultaneously, provided that $v > 0.28$ [36] and $c \leq 0.75$

[36] Note that v is restricted even for positive demand shock, because we assume that θ is positive or negative demand shock of equal magnitude.

Figure 3.B1.3a | g=1
Contour Plot w_{10} (frr_1)

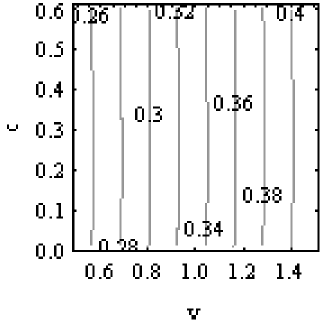

Figure 3.B1.3b | g=1
Contour Plot w_{20} (frr_1)

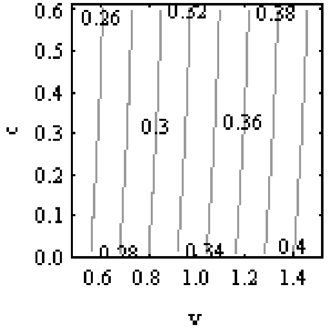

Figure 3.B1.3c | g=1
Contour Plot TC_1 (frr_1)

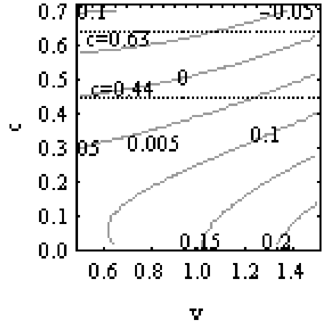

Figure 3.B1.3d | g=1
Contour Plot TC_2(frr_1)

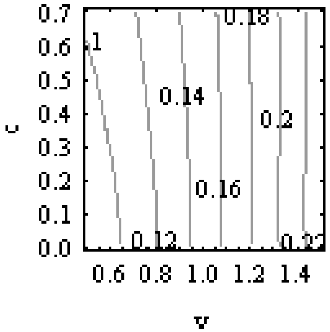

Figure 3.B1.3: Positive wages and total costs simultaneously, provided that $c \leq 0.44$

Figure 3.B1.4a | g=0.5
Contour Plot w_{10} (frr_1)

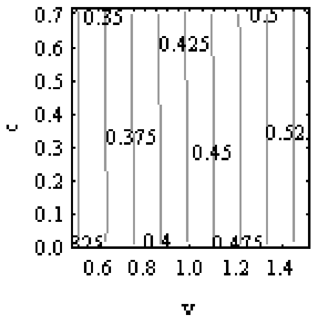

Figure 3.B1.4b | g=0.5
Contour Plot w_{20}(frr_1)

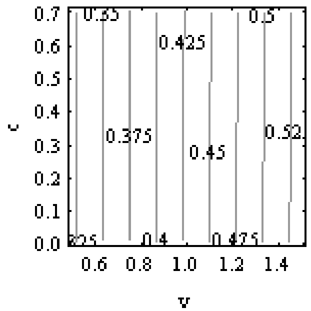

Figure 3.B1.4c | g=0.5
Contour Plot TC_1 (frr_1)

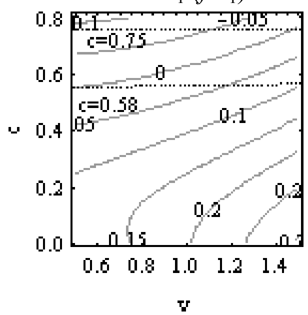

Figure 3.B1.4d | g=0.5
Contour Plot TC_2 (frr_1)

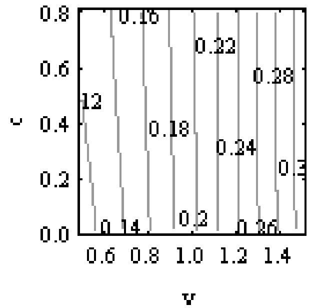

Figure 3.B1.4: Positive wages and total costs, provided that $c \leq 0.58$

Figure 3.B1.5a | g=1
Contour Plot P_{10} (frr_1)

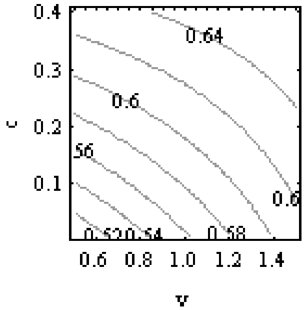

Figure 3.B1.5b | g=1
Contour Plot P_{20}, (frr_1)

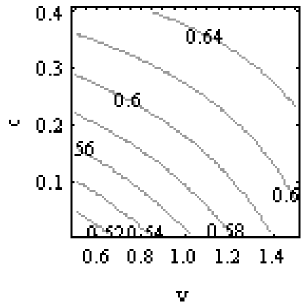

Figure 3.B1.5c | g=1
Contour Plot P_{11}, (frr_1)

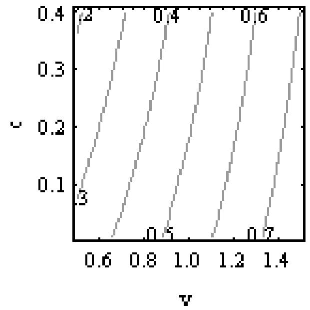

Figure 3.B1.5d | g=1
Contour Plot P_{21}, (frr_1)

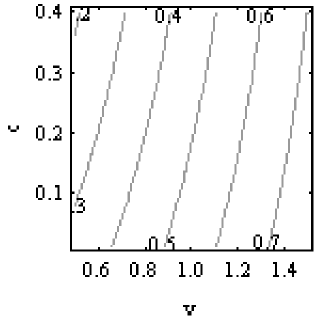

Figure 3.B1.5: Positive prices in each period

Figure 3.B1.6a | g=0.5
Contour Plot P_{10} (frr_1)

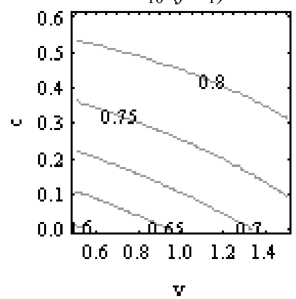

Figure 3.B1.6b | g=0.5
Contour Plot P_{20} (frr_1)

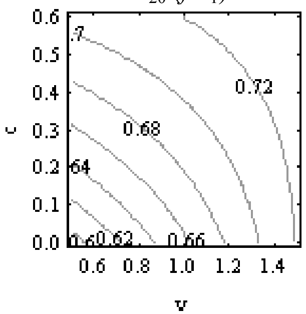

Figure 3.B1.6c | g=0.5
Contour Plot P_{11} (frr_1)

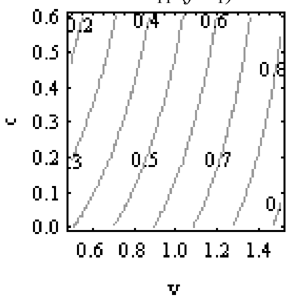

Figure 3.B1.6d | g=0.5
Contour Plot P_{21} (frr_1)

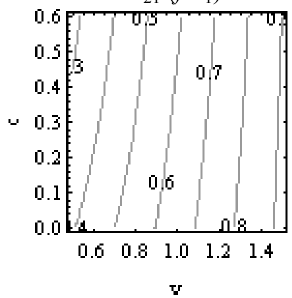

Figure 3.B1.6: Positive prices in each period

Figure 3.B1.7a | g=1
Contour Plot Π1 (*frr*$_1$)

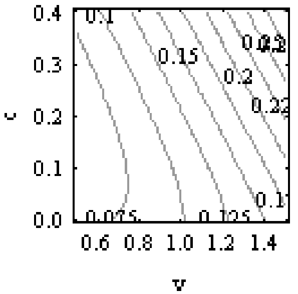

Figure 3.B1.7b | g=1
Contour Plot Π2 (*frr*$_1$)

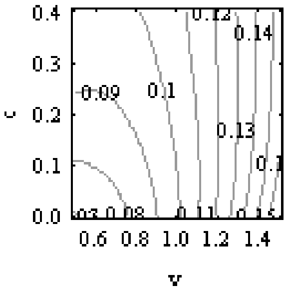

Figure 3.B1.7c | g=1
Contour Plot U1 (*frr*$_1$)

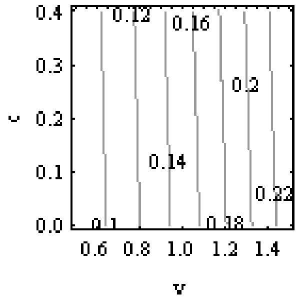

Figure 3.B1.7d | g=1
Contour Plot U2 (*frr*$_1$)

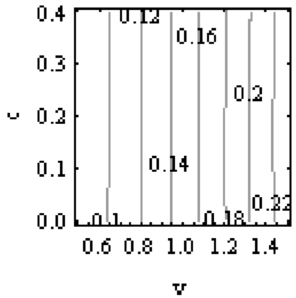

Figure 3.B1.7e | g=1
Contour Plot CS (*frr*$_1$)

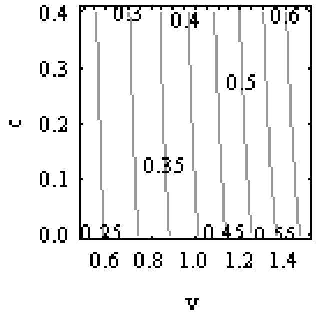

Figure 3.B1.7: Π_1, Π_2 U_1, U_2 and CS are positive, provided that interior solution is ensured

Figure 3.B1.8a | g=0.5
Contour Plot Π1 (*frr*$_1$)

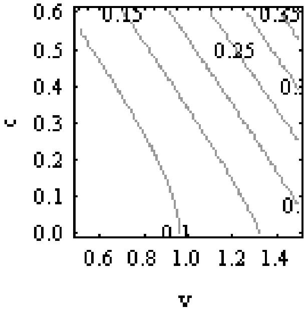

Figure 3.B1.8b | g=0.5
Contour Plot Π2 (*frr*$_1$)

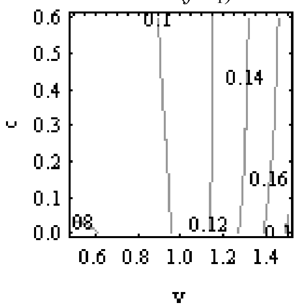

Figure 3.B1.8c | g=0.5
Contour Plot U1 (*frr*$_1$)

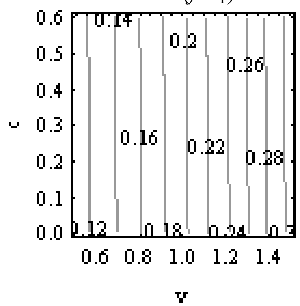

Figure 3.B1.8d | g=0.5
Contour Plot U2 (*frr*$_1$)

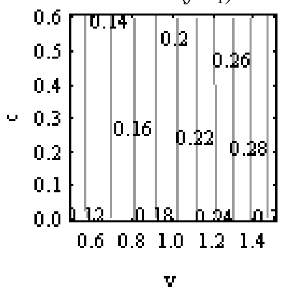

Figure 3.B1.8e | g=0.5
Contour Plot CS (*frr*$_1$)

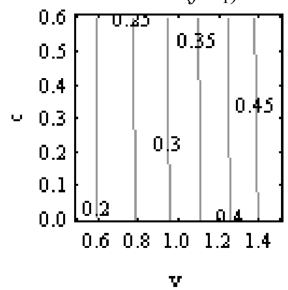

Figure 3.B1.8: Π_1, Π_2 U_1, U_2 and CS are positive, provided that interior solution is ensured

Figure 3.B1.9a | g=1
Contour Plot (q_{10}-q_{20}) (*frr*$_1$)

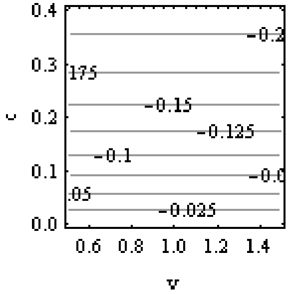

Figure 3.B1.9b | g=1
Contour Plot (q_{11}-q_{21}) (*frr*$_1$)

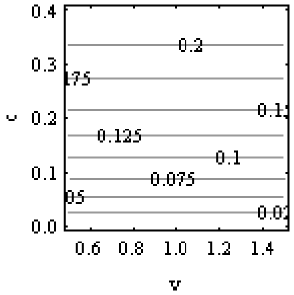

Figure 3.B1.9c | g=1
Contour Plot (q_{11}-q_{10}) (*frr*$_1$)

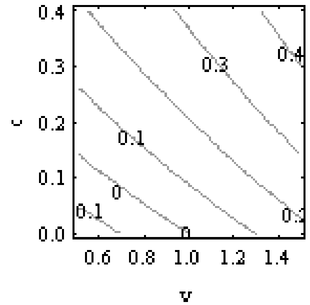

Figure 3.B1.9d | g=1
Contour Plot (q_{21}-q_{20}) (*frr*$_1$)

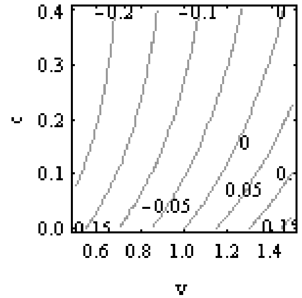

Figure 3.B1.9: $q_{10}<q_{20}$ and $q_{11}>q_{21}$, irrespective of v and $q_{11}>q_{10}$ if v>1 and $q_{20}>q_{21}$ if v<1

Figure 3.B1.10a | g=1
Contour Plot (tq_1-tq_2)(*frr*$_1$)

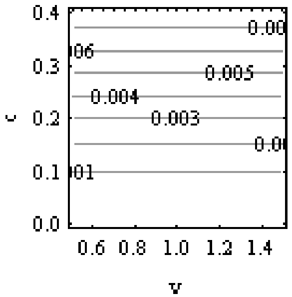

Figure 3.B1.10b | g=1
Contour Plot (tq_A-tq_B)(*frr*$_1$)

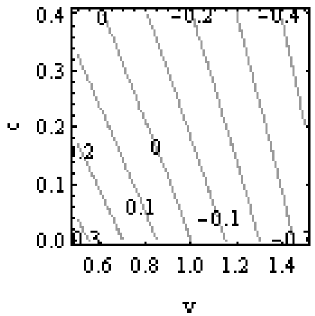

Figure 3.B1.10c | g=1
Contour Plot (Π1-Π2) (*frr*$_1$)

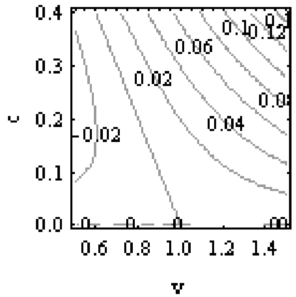

Figure 3.B1.10d | g=1
Contour Plot (U1-U2) (*frr*$_1$)

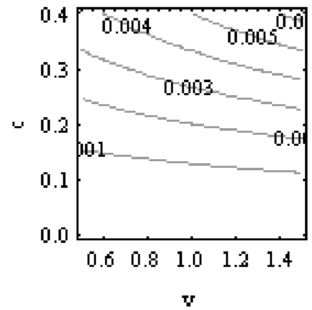

Figure 3.B1.10: $tq_1>tq_2$; $tq_A<tq_B$, v>1; $\Pi_1>\Pi_2$ if v>1 and U1>U2

Figure 3.B1.11a | g=0.5
Contour Plot (q_{10}-q_{20}) (*frr*$_1$)

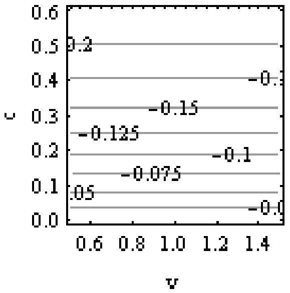

Figure 3.B1.11b | g=0.5
Contour Plot (q_{11}-q_{21}) (*frr*$_1$)

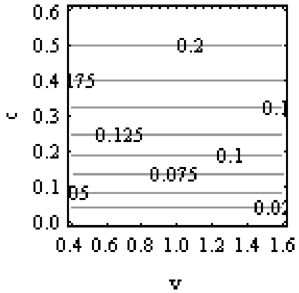

Figure 3.B1.11c | g=0.5
Contour Plot (q_{11}-q_{10}) (*frr*$_1$)

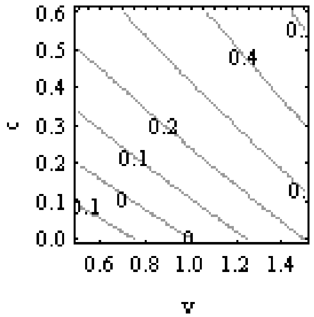

Figure 3.B1.11d | g=0.5
Contour Plot (q_{21}-q_{20}) (*frr*$_1$)

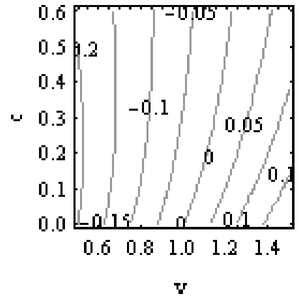

Figure 3.B1.11: $q_{10}<q_{20}$ and $q_{11}>q_{21}$; $q_{11}>q_{10}$ if v>1 and $q_{20}>q_{21}$ if v<1

Figure 3.B1.12a | g=0.5
Contour Plot (tq_1-tq_2) (*frr*$_1$)

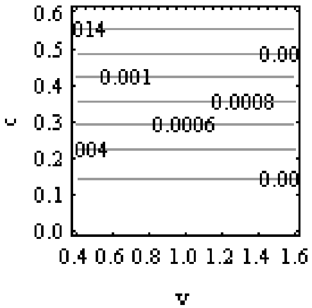

Figure 3.B1.12b | g=0.5
Contour Plot (tq_A-tq_B) (*frr*$_1$)

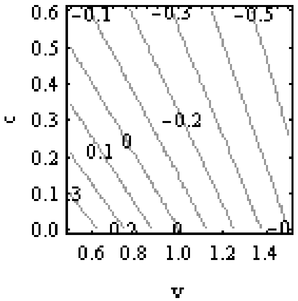

Figure 3.B1.12c | g=0.5
Contour Plot (Π1-Π2) (*frr*$_1$)

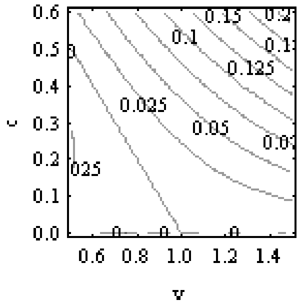

Figure 3.B1.12d | g=0.5
Contour Plot (U1-U2) (*frr*$_1$)

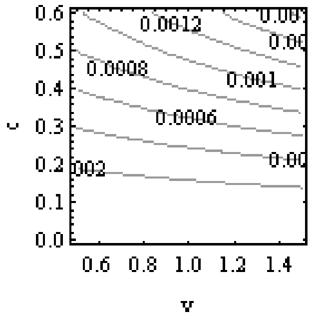

Figure 3.B1.12: $tq_1>tq_2$; $tq_A<tq_B$, v>1; $\Pi_1>\Pi_2$ if v>1 and U1>U2

Figure 3.B1.13a |g=1
Contour Plot $q_{10}(frr_0)$

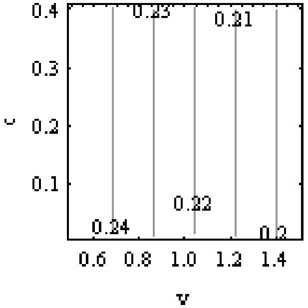

Figure 3.B1.13b |g=1
Contour Plot $q_{11}(frr_0)$

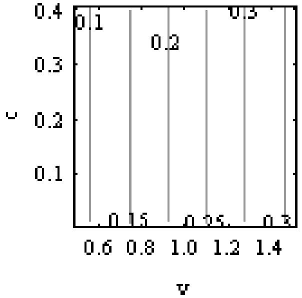

Figure 3.B1.13c |g=1
Contour Plot $P_{10}(frr_0)$

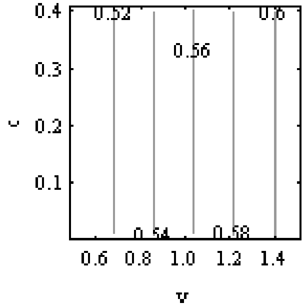

Figure 3.B1.13d |g=1
Contour Plot $P_{11}(frr_0)$

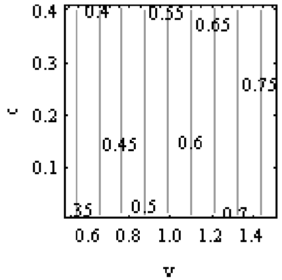

Figure 3.B1.13e |g=1
Contour Plot $w_{10}(frr_0)$

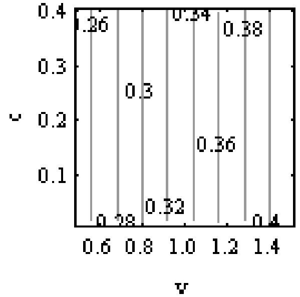

Figure 3.B1.13: Note that is important to ensure interior solution under frr_0 for {v, 0.5, 1.5} and {c, 0.01, 0.4} as is the case under frr_1, frr_2, and frr_3

Figure 3.B1.14a |g=0.5
Contour Plot $q_{10}(frr_0)$

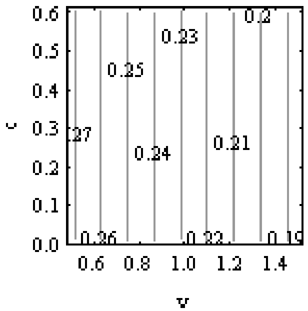

Figure 3.B1.14b |g=0.5
Contour Plot $q_{11}(frr_0)$

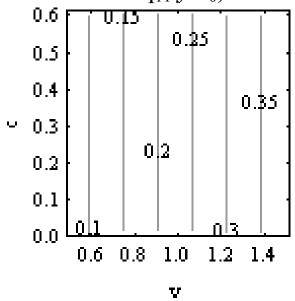

Figure 3.B1.14c |g=0.5
Contour Plot $P_{10}(frr_0)$

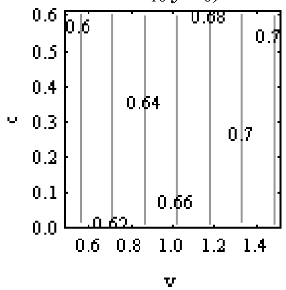

Figure 3.B1.14d |g=0.5
Contour Plot $P_{20}(frr_0)$

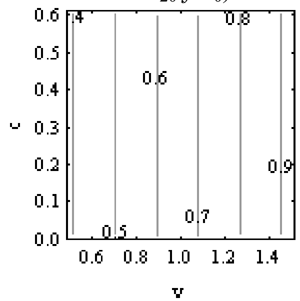

Figure 3.B1.14e |g=0.5
Contour Plot $w_{10}(frr_0)$

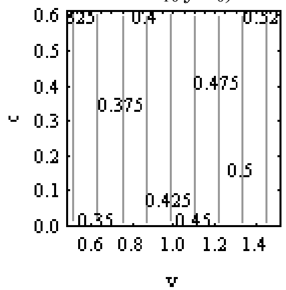

Figure 3.B1.14: Note that is important to ensure interior solution under frr_0 for {v, 0.5, 1.5} and {c, 0.01, 0.6} as is the case under frr_1, frr_2, and frr_3

Figure 3.B1.15a |g=1
Contour Plot $q_{10}(frr_1)$-$q_{10}(frr_0)$

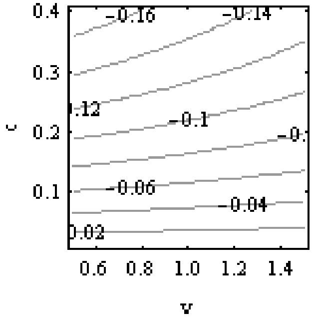

Figure 3.B1.15b |g=1
Contour Plot $q_{20}(frr_1)$-$q_{20}(frr_0)$

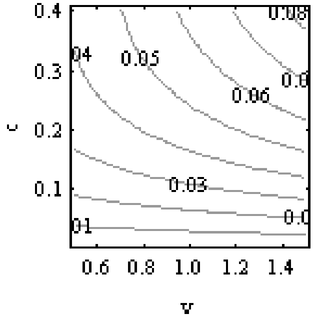

Figure 3.B1.15c |g=1
Contour Plot $q_{11}(frr_1)$-$q_{11}(frr_0)$

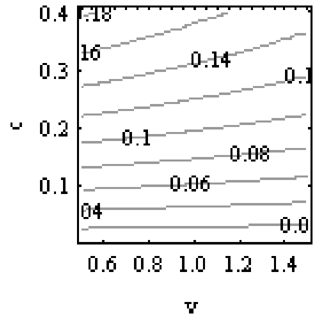

Figure 3.B1.15d |g=1
Contour Plot $q_{21}(frr_1)$-$q_{21}(frr_0)$

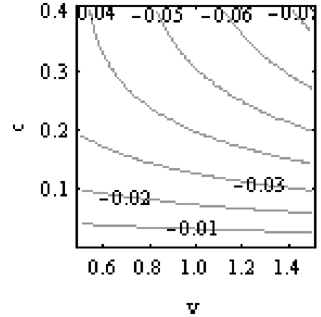

Figure 3.B1.15: $q_{10}(frr_1)<q_{10}(frr_0)$ $q_{20}(frr_1)>q_{20}(frr_0)$ $q_{11}(frr_1)>q_{11}(frr_0)$ and $q_{21}(frr_1)<q_{21}(frr_0)$

Figure 3.B1.16a |g=0.5
Contour Plot $q_{10}(frr_1)$-$q_{10}(frr_0)$

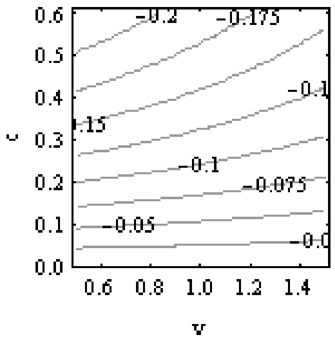

Figure 3.B1.16b |g=0.5
Contour Plot $q_{20}(frr_1)$-$q_{20}(frr_0)$

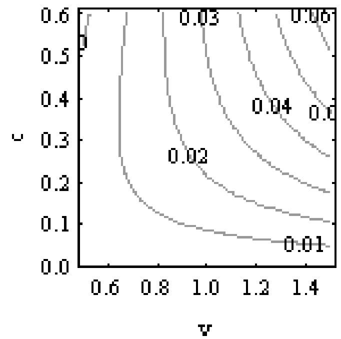

Figure 3.B1.16c |g=0.5
Contour Plot $q_{11}(frr_1)$-$q_{11}(frr_0)$

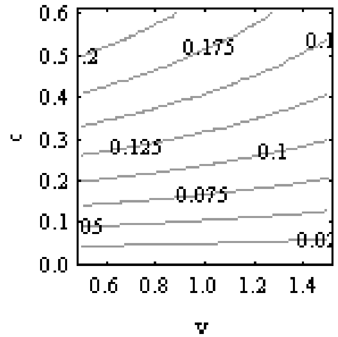

Figure 3.B1.16d |g=0.5
Contour Plot $q_{21}(frr_1)$-$q_{21}(frr_0)$

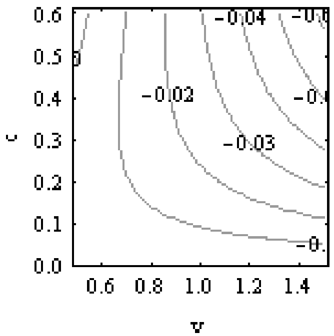

Figure 3.B1.16: $q_{10}(frr_1)<q_{10}(frr_0)$ $q_{20}(frr_1)>q_{20}(frr_0)$ $q_{11}(frr_1)>q_{11}(frr_0)$ and $q_{21}(frr_1)<q_{21}(frr_0)$

Figure 3.B1.17a |g=1
Contour Plot
$(q_{11}-q_{10})$ (*frr*$_1$)-$(q_{11}-q_{10})$ (*frr*$_0$)

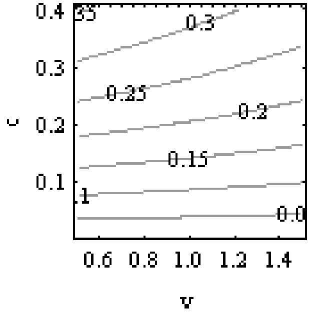

Figure 3.B1.17b |g=1
Contour Plot
$(q_{21}-q_{20})$ (*frr*$_1$)-$(q_{21}-q_{20})$ (*frr*$_0$)

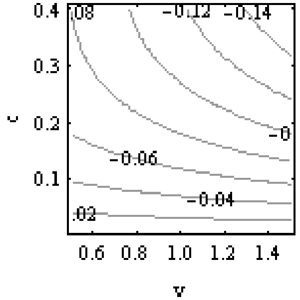

Figure 3.B1.17c |g=0.5
Contour Plot
$(q_{11}-q_{10})$ (*frr*$_1$)-$(q_{11}-q_{10})$ (*frr*$_0$)

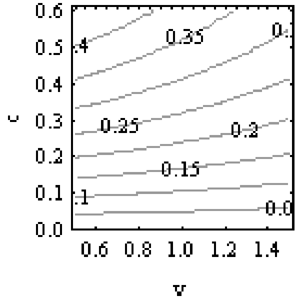

Figure 3.B1.17d |g=0.5
Contour Plot
$(q_{21}-q_{20})$ (*frr*$_1$)-$(q_{21}-q_{20})$ (*frr*$_0$)

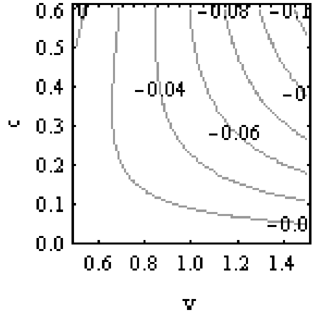

Figure 3.B1.17: $(q_{11}-q_{10})$ (*frr*$_1$)>$(q_{11}-q_{10})$ (*frr*$_0$); $(q_{21}-q_{20})$ (*frr*$_1$)<$(q_{21}-q_{20})$ (*frr*$_0$)

Figure 3.B1.18a |g=1
Contour Plot
tq_1(*frr*$_1$)-tq_1(*frr*$_0$)

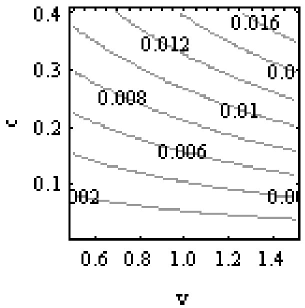

Figure 3.B1.18b |g=1
Contour Plot
tq_2(*frr*$_1$)-tq_2(*frr*$_0$)

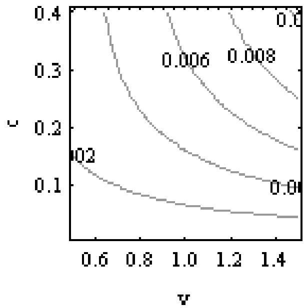

Figure 3.B1.18c |g=1
Contour Plot
Tq_A(*frr*$_1$)-Tq_A(*frr*$_0$)

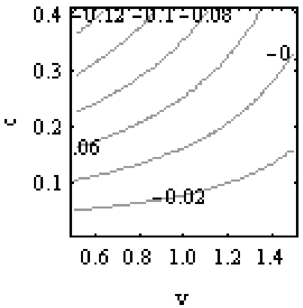

Figure 3.B1.18d |g=1
Contour Plot
Tq_B(*frr*$_1$)-Tq_B(*frr*$_0$)

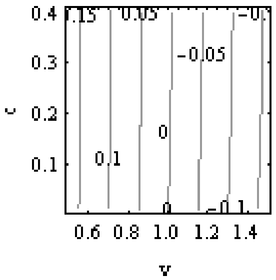

Figure 3.B1.18e |g=1
Contour Plot
Tq(*frr*$_1$)-Tq(*frr*$_0$)

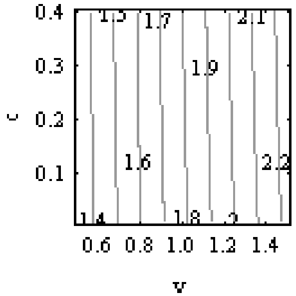

Figure 3.B1.18: $tq_1=q_{10}+q_{11}$ and $tq_2= q_{20}+q_{21}$; tq_1(*frr*$_1$)>tq_1(*frr*$_0$) and tq_2(*frr*$_1$)>tq_2(*frr*$_0$);$Tq_A=q_{10}+q_{20}$ and $Tq_B=q_{11}+q_{21}$; Tq_A(*frr*$_1$)<Tq_A(*frr*$_0$) and Tq_B(*frr*$_1$)>Tq_B(*frr*$_0$) if v<1; Tq(*frr*$_1$)>Tq(*frr*$_0$)

Figure 3.B1.19a |g=0.5
Contour Plot
$tq_1(frr_1)-tq_1(frr_0)$

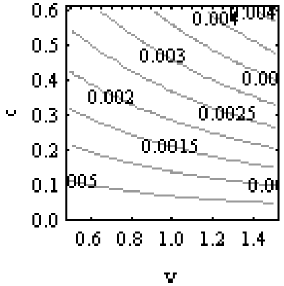

Figure 3.B1.19b |g=0.5
Contour Plot
$tq_2(frr_1)-tq_2(frr_0)$

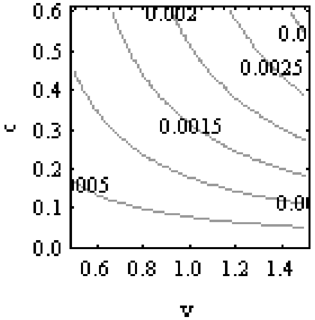

Figure 3.B1.19c |g=0.5
Contour Plot
$Tq_A(frr_1)-Tq_A(frr_0)$

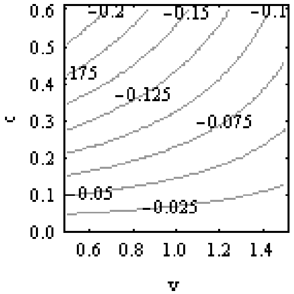

Figure 3.B1.19d |g=0.5
Contour Plot
$Tq_B(frr_1)-Tq_B(frr_0)$

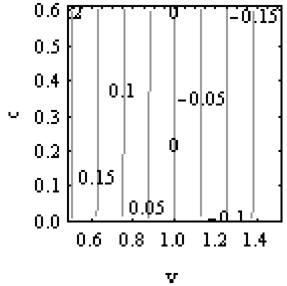

Figure 3.B1.19e |g=0.5
Contour Plot
$Tq(frr_1)-Tq(frr_0)$

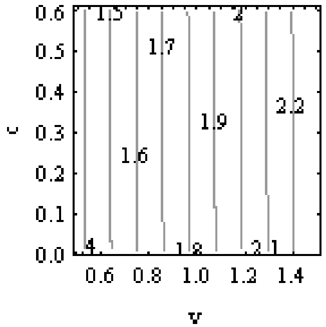

Figure 3.B1.19: $tq_1=q_{10}+q_{11}$ and $tq_2= q_{20}+q_{21}$; $tq_1(frr_1)>tq_1(frr_0)$ and $tq_2(frr_1)>tq_2(frr_0)$;$Tq_A=q_{10}+q_{20}$ and $Tq_B=q_{11}+q_{21}$; $Tq_A(frr_1)<Tq_A(frr_0)$ and $Tq_B(frr_1)>Tq_B(frr_0)$, if $v<1$; $Tq(frr_1)>Tq(frr_0)$

Figure 3.B1.20a |g=1
Contour Plot
$\Pi1(frr_1)-\Pi1(frr_0)$

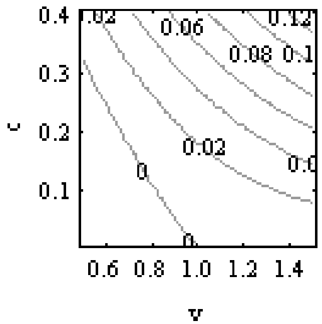

Figure 3.B1.20b |g=1
Contour Plot
$\Pi2(frr_1)-\Pi2(frr_0)$

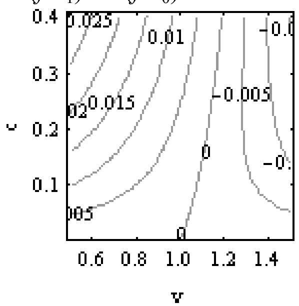

Figure 3.B1.20c |g=1
Contour Plot
$U1(frr_1)-U1(frr_0)$

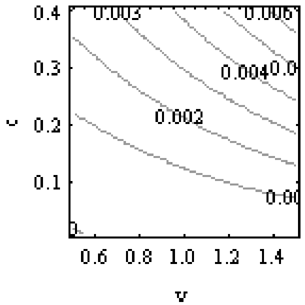

Figure 3.B1.20d |g=1
Contour Plot
$U2(frr_1)-U2(frr_0)$

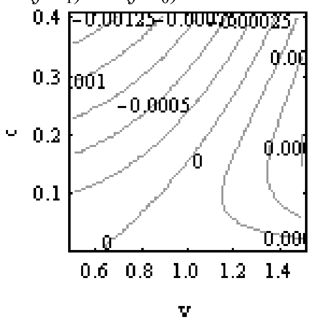

Figure 3.B1.20e |g=1
Contour Plot
$CS(frr_1)-CS(frr_0)$

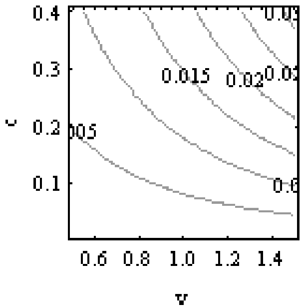

Figure 3.B1.20: $\Pi1(frr_1)>\Pi1(frr_0)$ but $\Pi2(frr_1)<\Pi2(frr_0)$ $v>1$; $U1(frr_1)>U1(frr_0)$; $U2(frr_1)>U2(frr_0)$ under restriction; $CS(frr_1)>CS(frr_0)$

Figure 3.B1.21a |g=0.5
Contour Plot
Π1(frr_1)-Π1(frr_0)

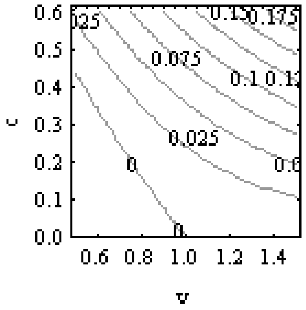

Figure 3.B1.21b |g=0.5
Contour Plot
Π2(frr_1)-Π2(frr_0)

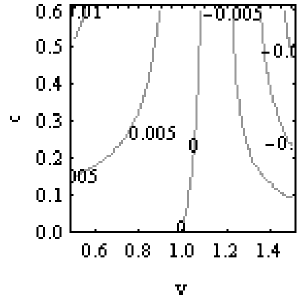

Figure 3.B1.21c |g=0.5
Contour Plot
U1(frr_1)-U1(frr_0)

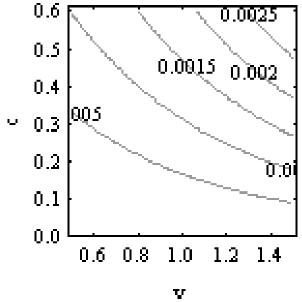

Figure 3.B1.21d |g=0.5
Contour Plot
U2(frr_1)-U2(frr_0)

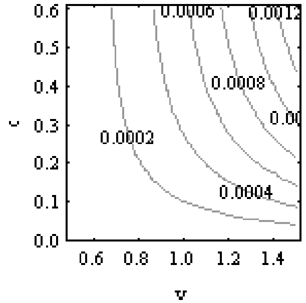

Figure 3.B1.21e |g=0.5
Contour Plot
CS(frr_1)-CS(frr_0)

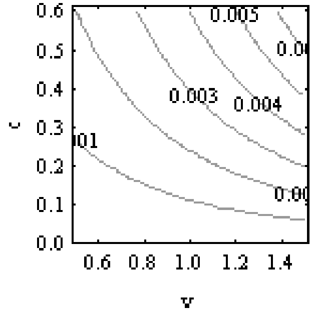

Figure 3.B1.21: Π1(frr_1)>Π1(frr_0) but Π2(frr_1)<Π2(frr_0) if v>1; U1(frr_1)>U1(frr_0); U2(frr_1)>U2(frr_0; CS(frr_1)>CS(frr_0)

Figure 3.B1.22a |g=1
Contour Plot $\frac{d\Pi 1}{dc}(frr_1)$

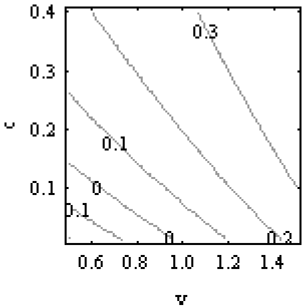

Figure 3.B1.22b |g=1
Contour Plot $\frac{d\Pi 2}{dc}(frr_1)$

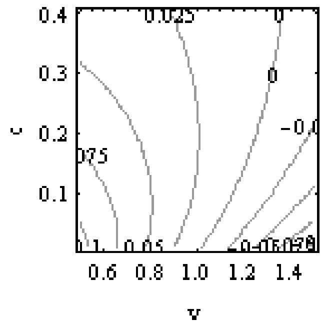

Figure 3.B1.22c |g=1
Contour Plot $\frac{dU1}{dc}(frr_1)$

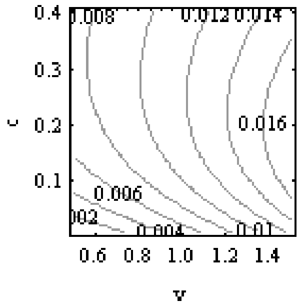

Figure 3.B1.22d |g=1
Contour Plot $\frac{dU2}{dc}(frr_1)$

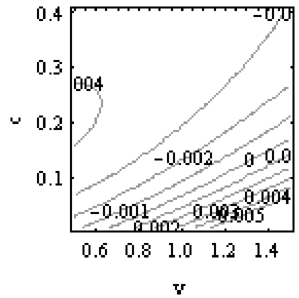

Figure 3.B1.22e |g=1
Contour Plot $\frac{dCS}{dc}(frr_1)$

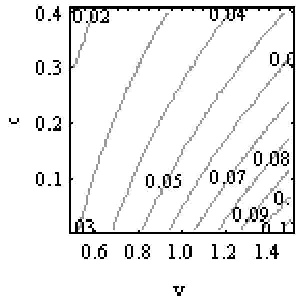

Figure 3.B1.22: Π1, Π2, U1 increasing with c provided that $0.15 \le c \le 0.4$ and $0.5 \le v \le 0.13$

Figure 3.B1.23a |g=0.5

Contour Plot $\frac{d\Pi 1}{dc}$ (frr_1)

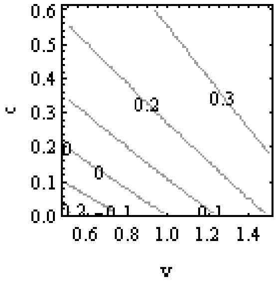

Figure 3.B1.23b |g=0.5

Contour Plot $\frac{d\Pi 2}{dc}$ (frr_1)

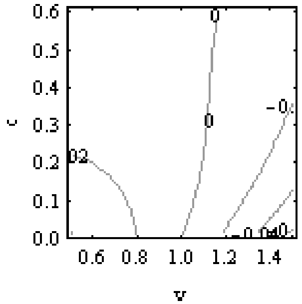

Figure 3.B1.23c |g=0.5

Contour Plot $\frac{dU1}{dc}$ (frr_1)

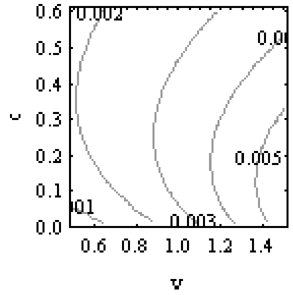

Figure 3.B1.23d |g=0.5

Contour Plot $\frac{dU2}{dc}$ (frr_1)

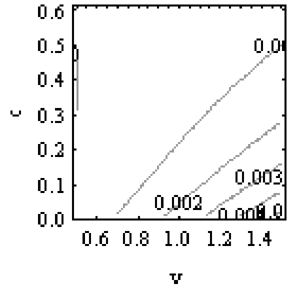

Figure 3.B1.23e |g=0.5

Contour Plot $\frac{dCS}{dc}$ (frr_1)

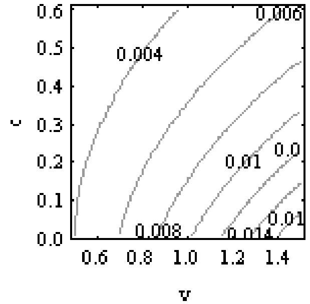

Figure 3.B1.23: Π1, U1 and U2 are increasing with c provided that $0.2 \le c \le 0.6$

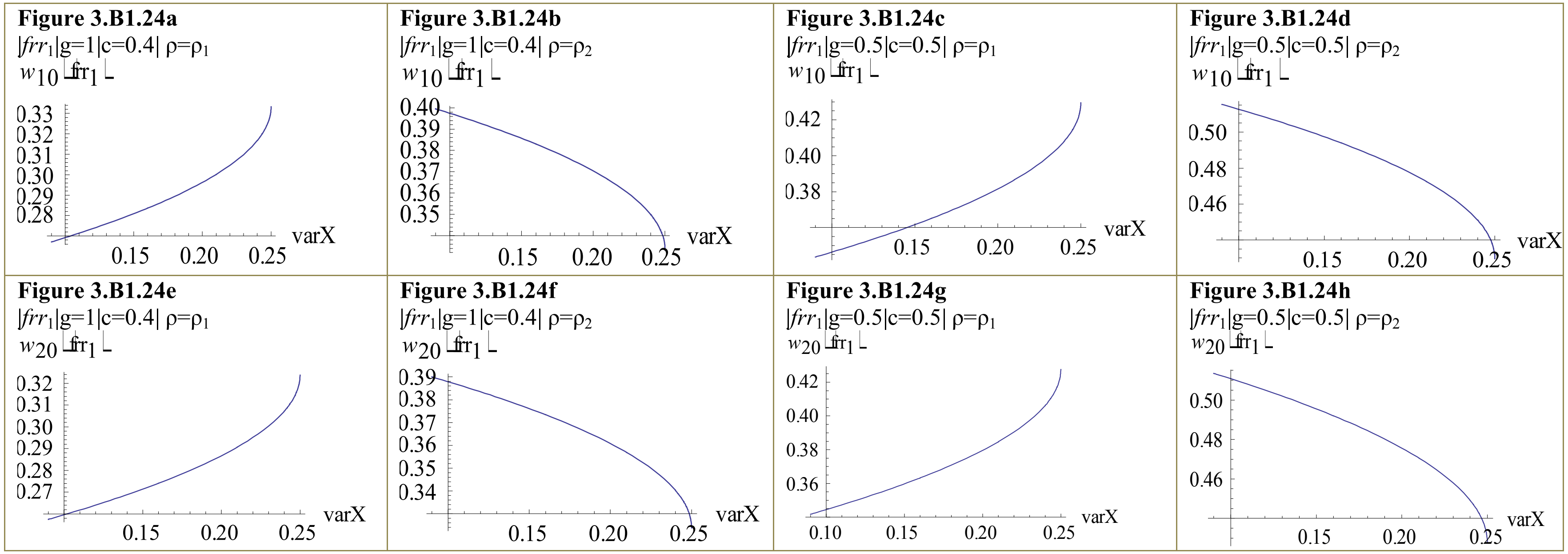

Figure 3.B1.24: w_{10}, w_{20} increasing (decreasing) with varX under ρ_1 (ρ_2)

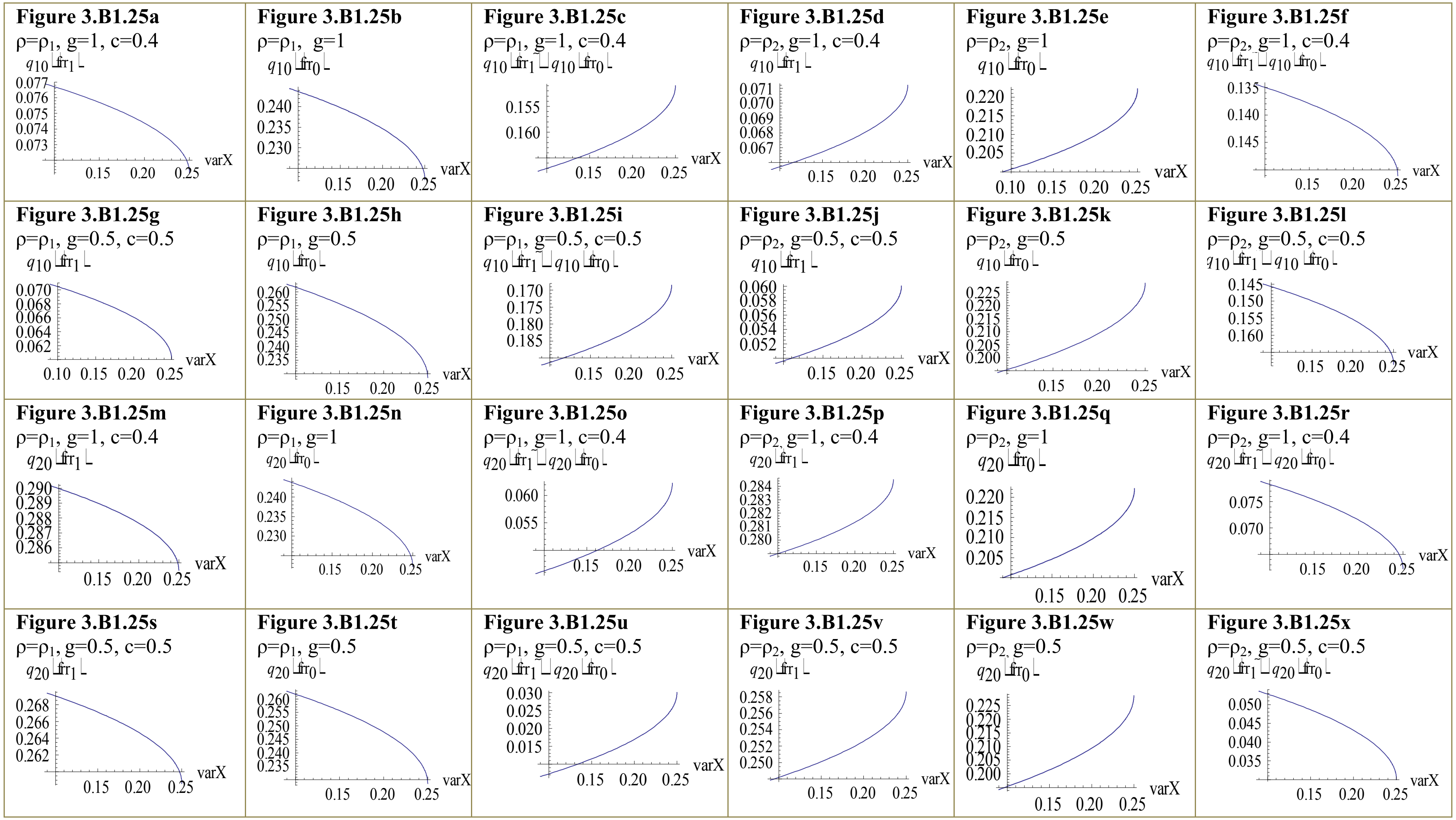

Figure 3.B1.25: How varX affects $q_{10}(frr_1)$ $q_{10}(frr_0)$ $q_{10}(frr_1)$-$q_{10}(frr_0)$ $q_{20}(frr_1)$ $q_{20}(frr_0)$ $q_{20}(frr_1)$-$q_{20}(frr_0)$ under ρ_1 and ρ_2

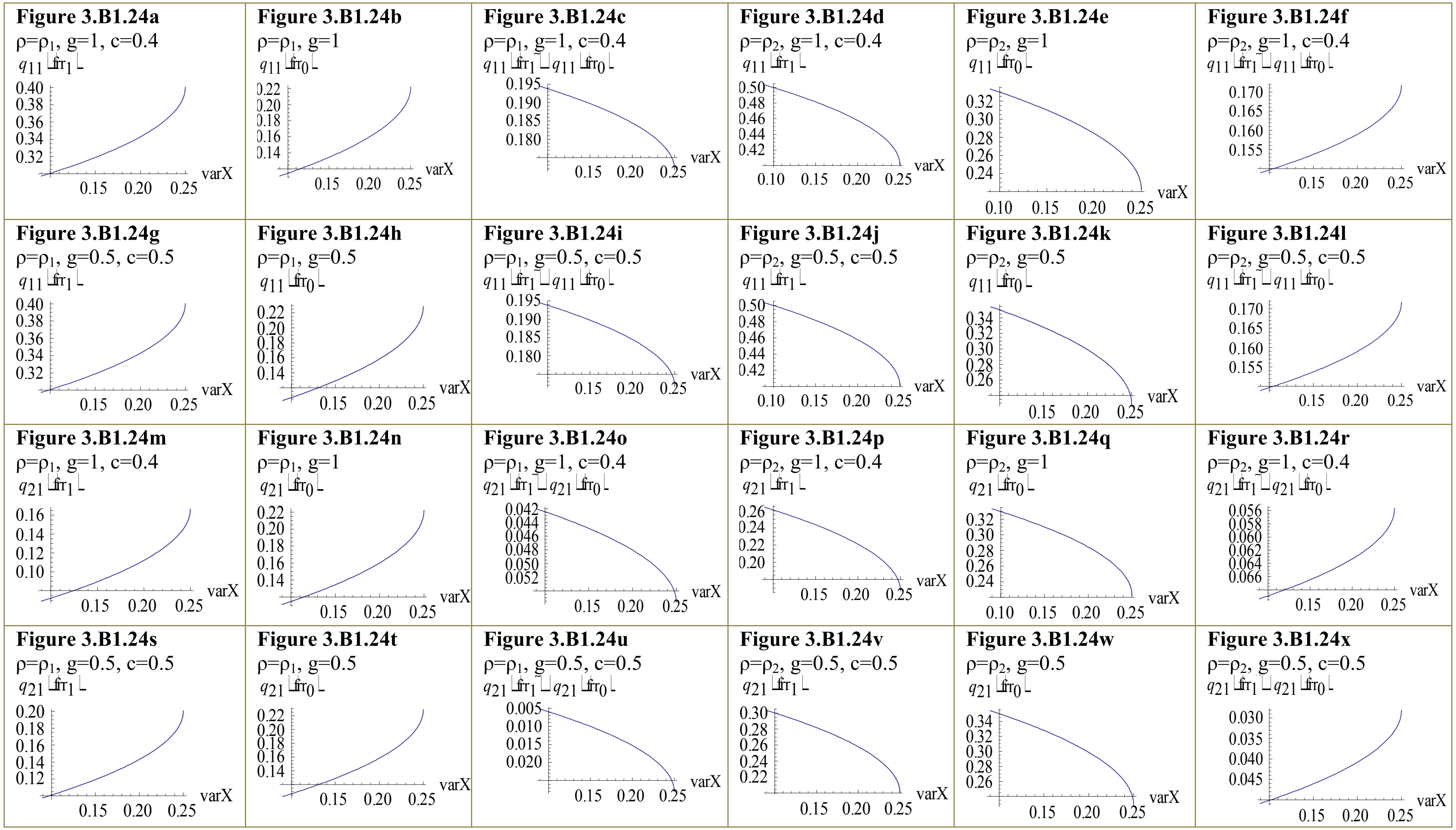

Figure 3.B1.24: How varX affects $q_{11}(frr_1)$ $q_{11}(frr_0)$ $q_{11}(frr_1)$-$q_{11}(frr_0)$ $q_{21}(frr_{21})$ $q_{21}(frr_0)$ $q_{21}(frr_1)$-$q_{21}(frr_0)$ under ρ_1 and ρ_2

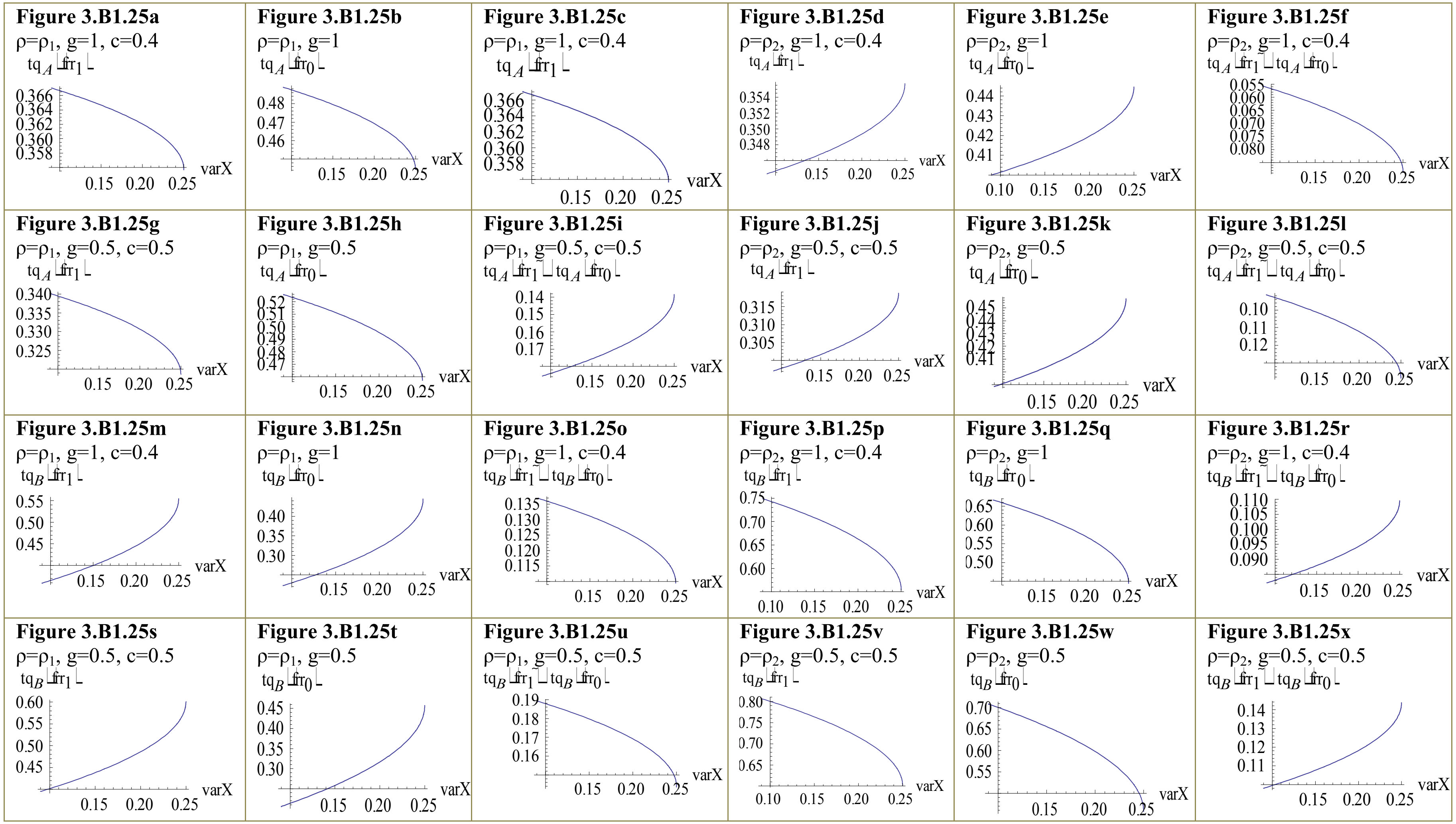

Figure 3.B1.25: How varX affects $tq_A(frr_1)$ $tq_A(frr_0)$ $tq_A(frr_1)-tq_A(frr_0)$ $tq_B(frr_1)$ $tq_B(frr_0)$ $tq_B(frr_1)-tq_B(frr_0)$ under ρ_1 and ρ_2

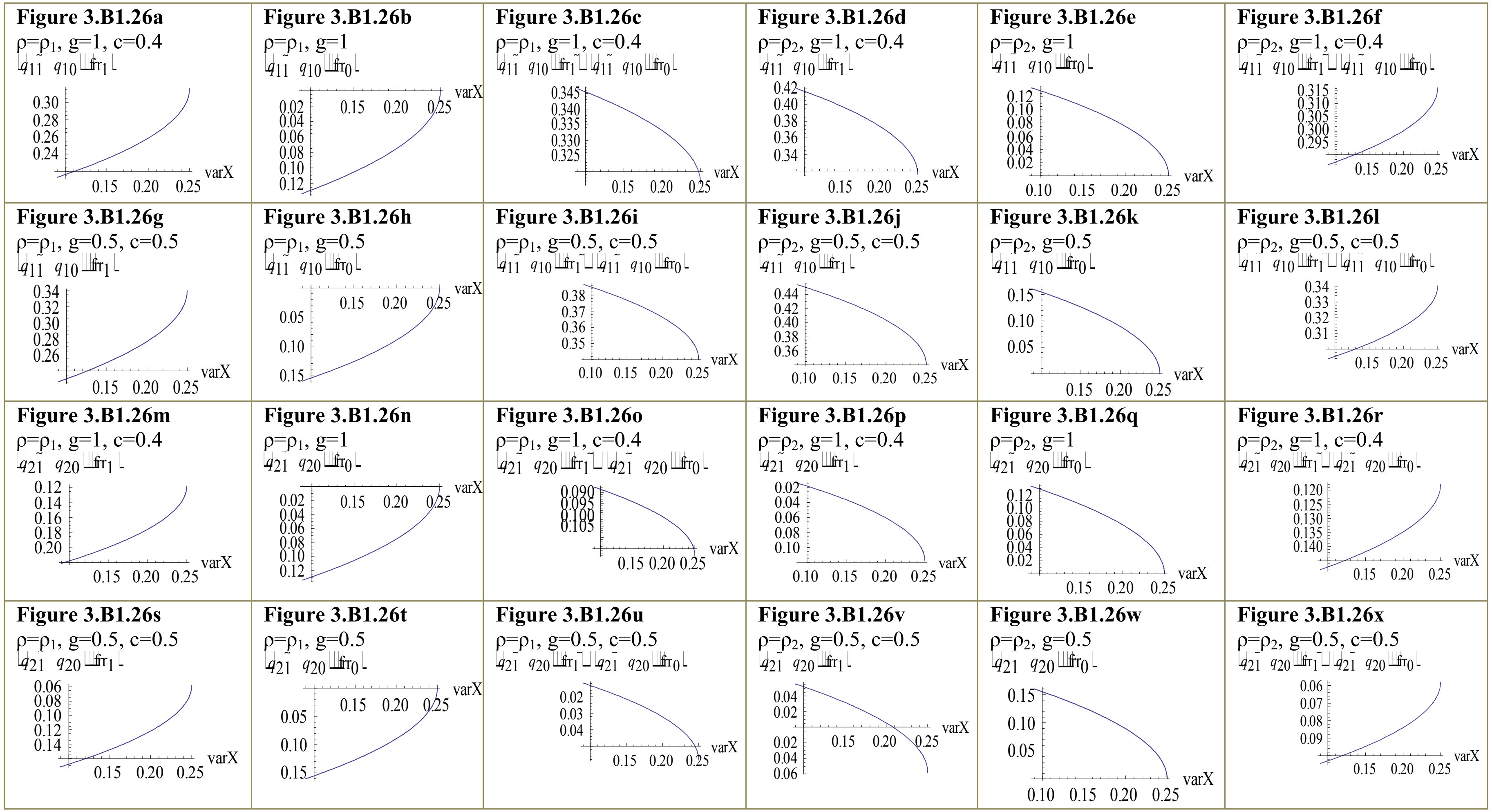

Figure 3.B1.26: How varX affects $(q_{11}-q_{10})(frr_1)$ $(q_{11}-q_{10})(frr_0)$ $(q_{11}-q_{10})(frr_1)-(q_{11}-q_{10})(frr_0)$ $(q_{21}-q_{20})(frr_1)$ $(q_{21}-q_{20})(frr_0)$ $(q_{21}-q_{20})(frr_1)-(q_{21}-q_{20})(frr_0)$ under ρ_1 and ρ_2

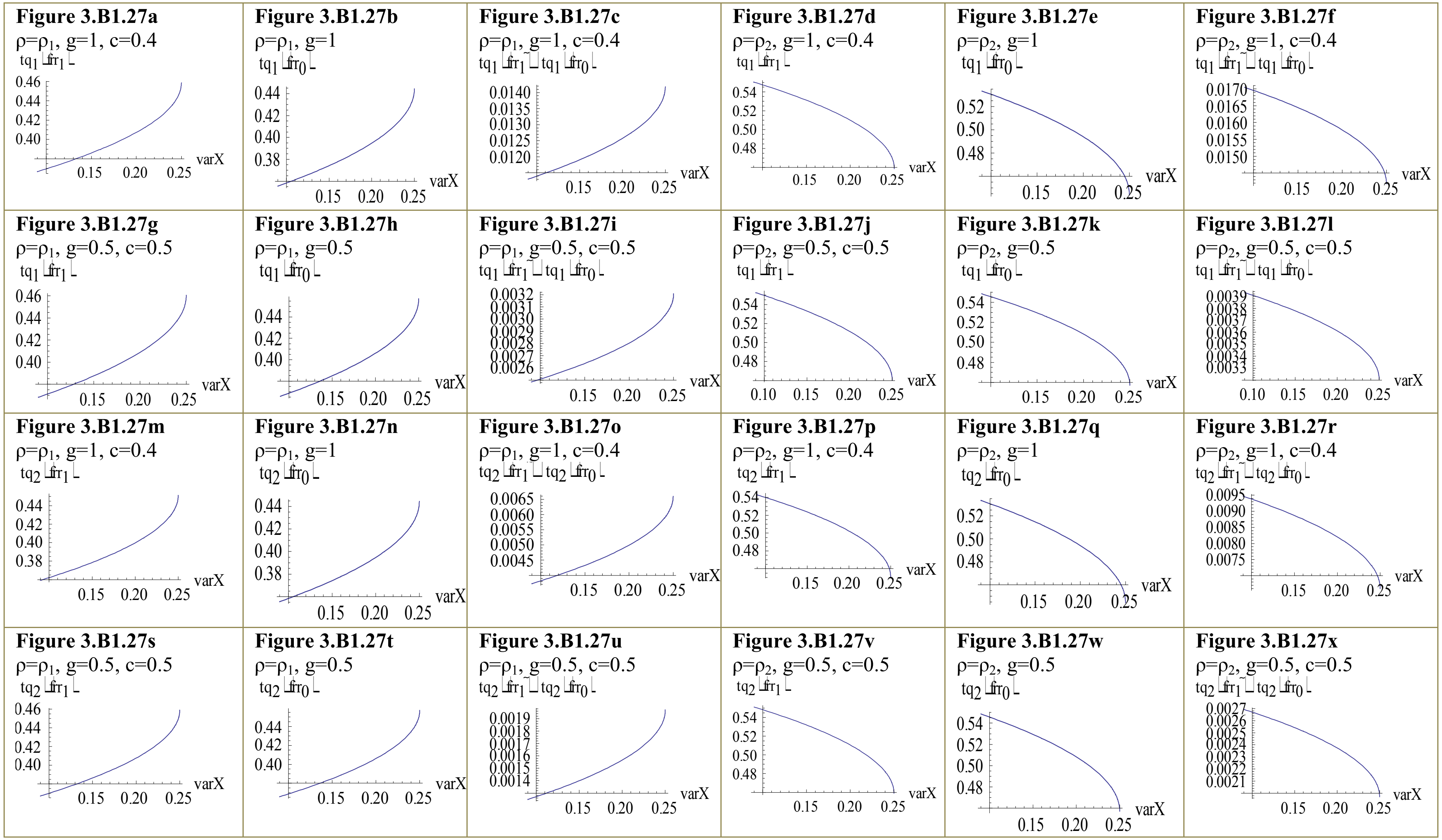

Figure 3.B1.27: How varX affects $tq_1(frr_1)$ $tq_1(frr_0)$ $tq_1(frr_1)-tq_1(frr_0)$ $tq_2(frr_1)$ $tq_2(frr_0)$ $tq_2(frr_1)-tq_2(frr_0)$ under ρ_1 and ρ_2

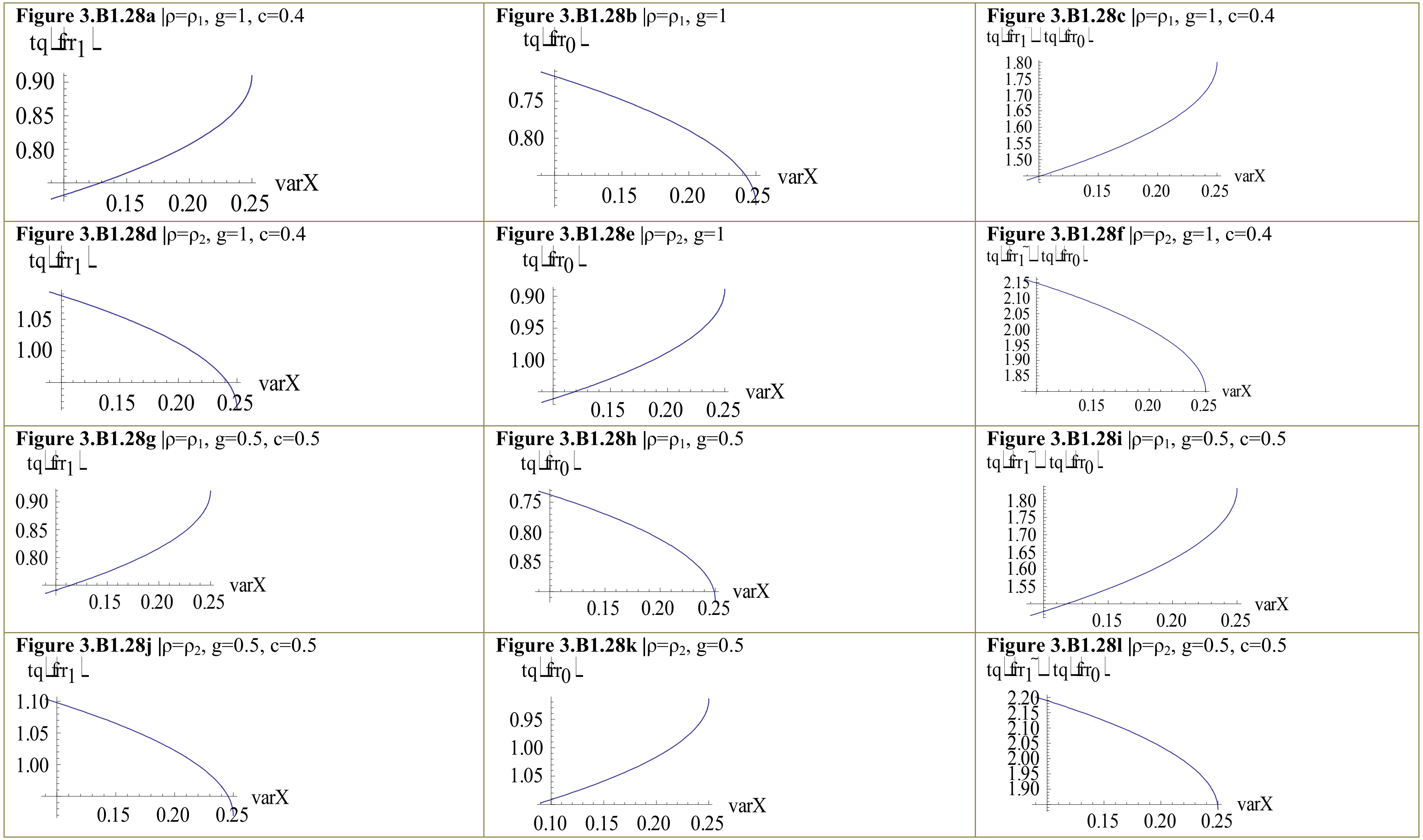

Figure 3.B1.28: How varX affects tq (*frr*$_1$) tq (*frr*$_0$) tq (*frr*$_1$)-tq(*frr*$_0$) under ρ_1 and ρ_2

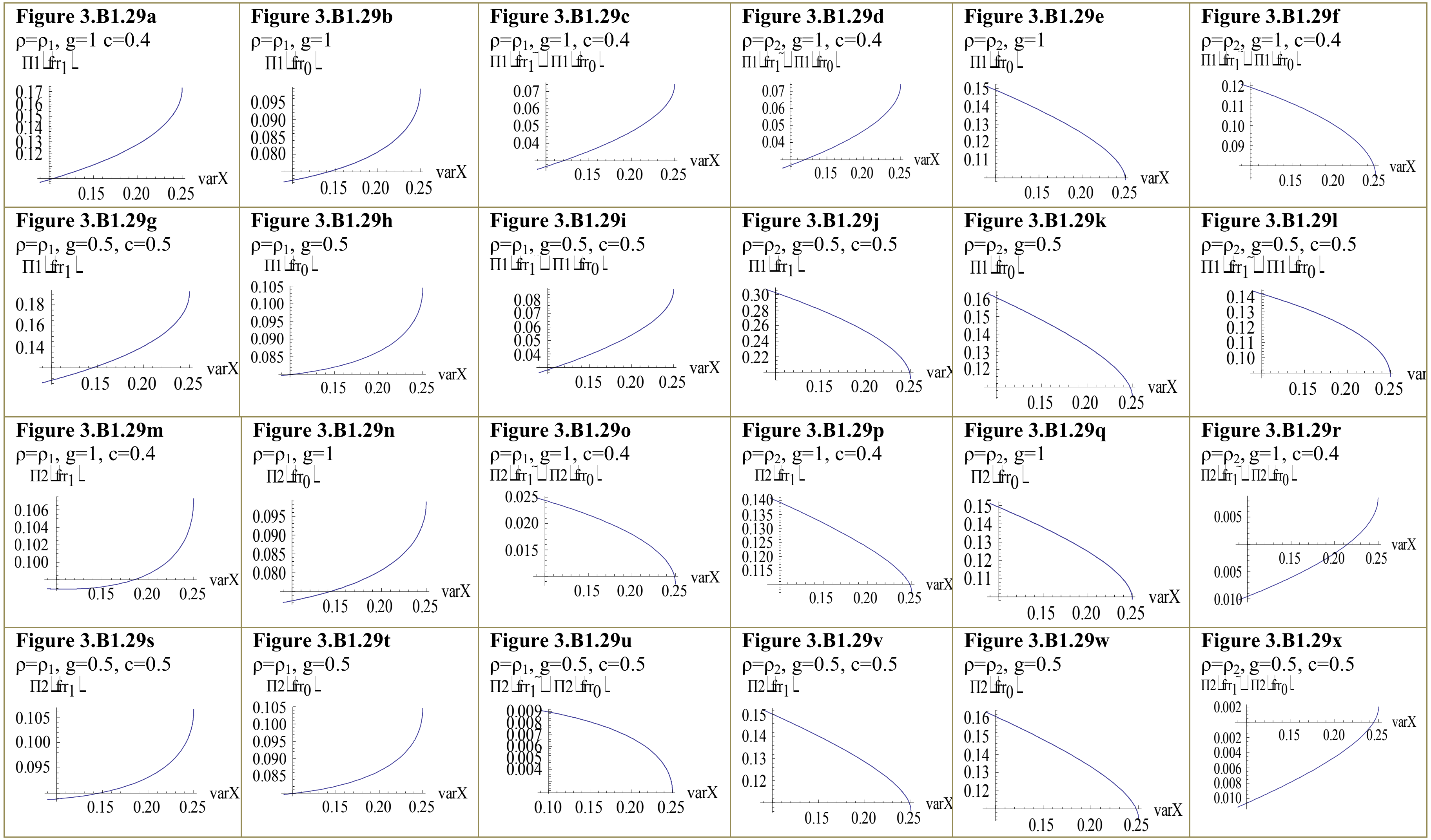

Figure 3.B1.29: How varX affects $\Pi1(frr_1)$ $\Pi1(frr_0)$ $\Pi1(frr_1)$-$\Pi1(frr_0)$ $\Pi2(frr_1)$ $\Pi2(frr_0)$ $\Pi2(frr_1)$-$\Pi2(frr_0)$ under ρ_1 and ρ_2

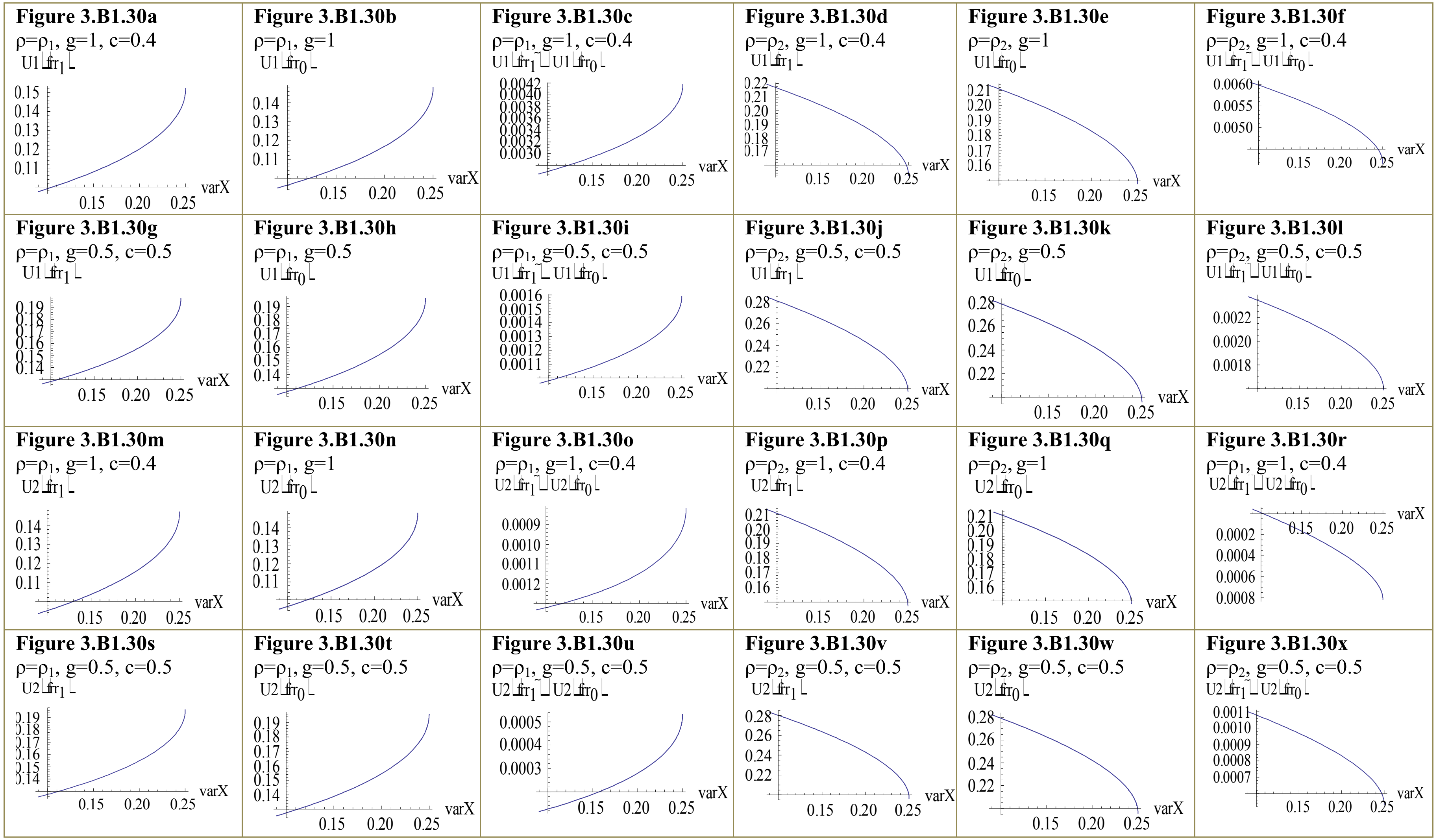

Figure 3.B1.30: How varX affects U1(*frr*$_1$) U1(*frr*$_0$) U1(*frr*$_1$)-U1(*frr*$_0$) U2(*frr*$_1$) U2(*frr*$_0$) U2(*frr*$_1$)-U2(*frr*$_0$) under ρ_1 and ρ_2

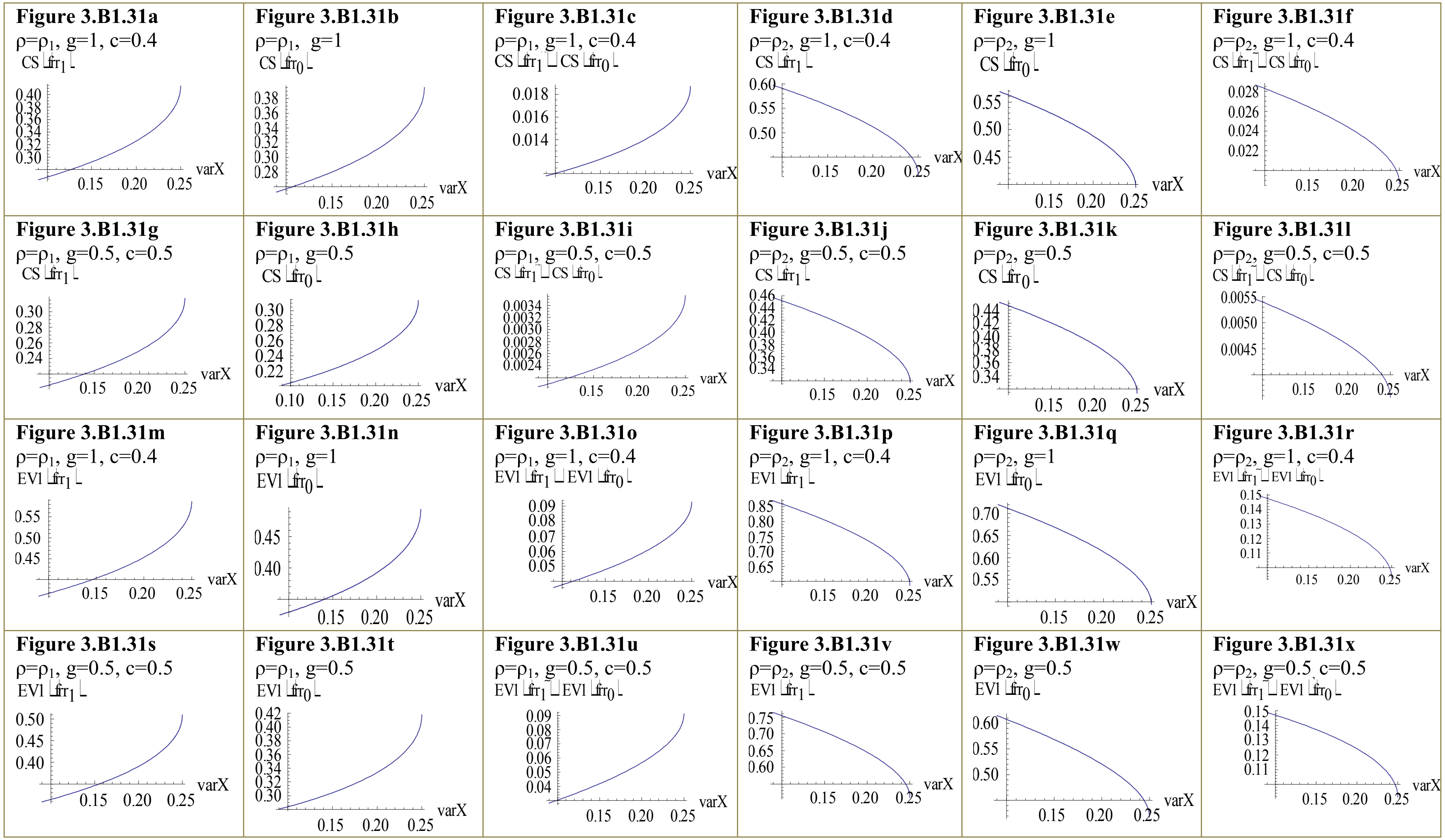

Figure 3.B1.31: How varX affects CS(*frr*$_1$) CS(*frr*$_0$) CS(*frr*$_1$)-CS(*frr*$_0$) EV1(*frr*$_1$) EV1(*frr*$_0$) EV1(*frr*$_1$)-EV1(*frr*$_0$) under ρ_1 and ρ_2

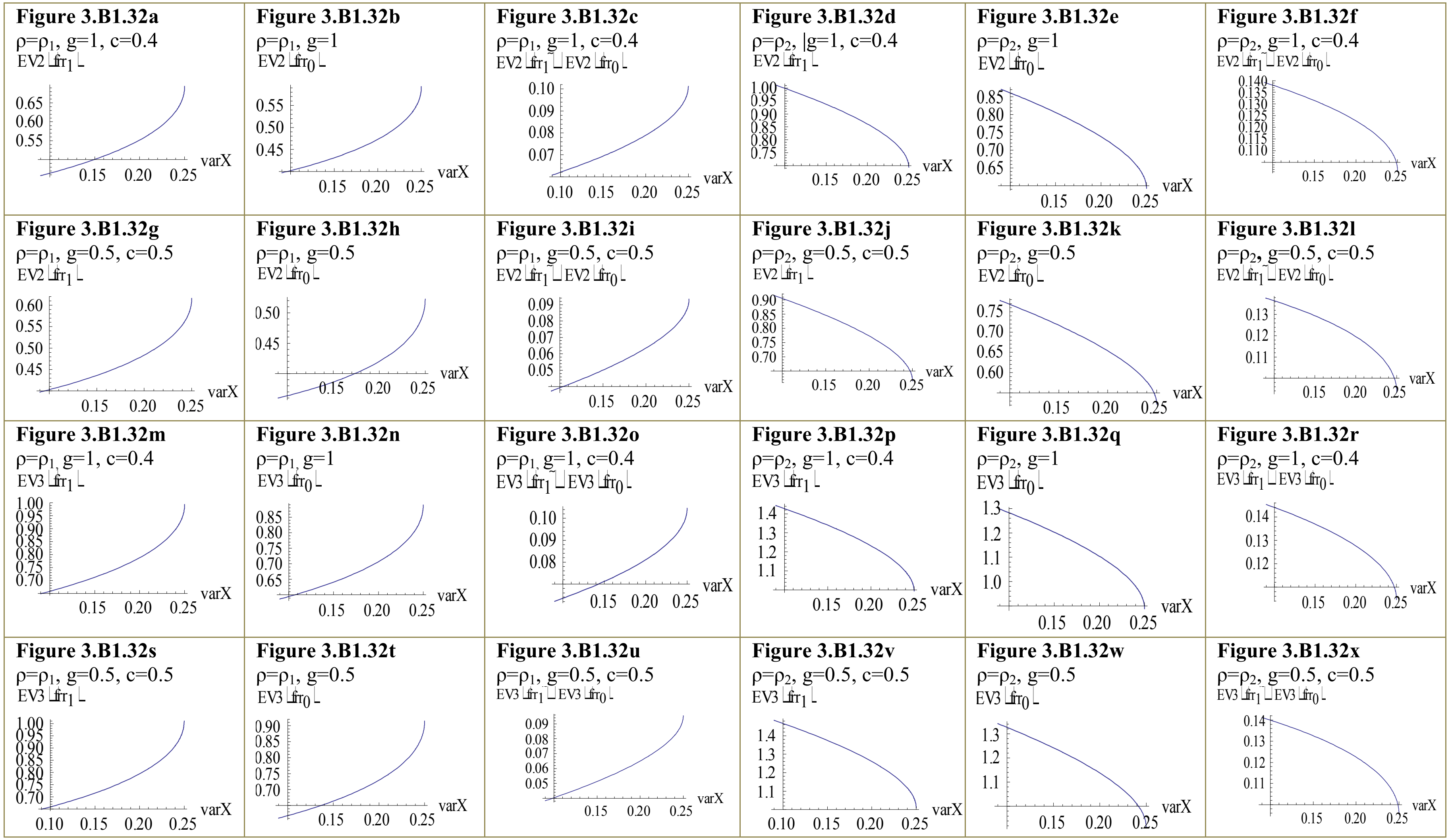

Figure 3.B1.32: How varX affects EV2(*frr*$_1$) EV2(*frr*$_0$) EV2(*frr*$_1$)-EV2(*frr*$_0$) EV3(*frr*$_1$) EV3(*frr*$_0$) EV3(*frr*$_1$)-EV3(*frr*$_0$) under ρ_1 and ρ_2

APPENDIX 3.B2

***frr*$_2$**: All the figures concern the optimal point

Figure 3.B2.1a | g=1
Contour Plot q_{11} (*frr*$_2$)

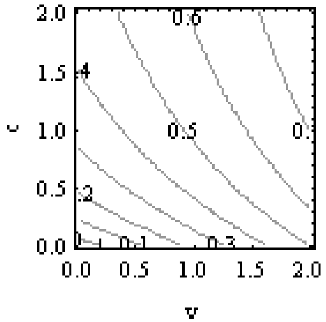

Figure 3.B2.1b | g=1
Contour Plot q_{21} (*frr*$_2$)

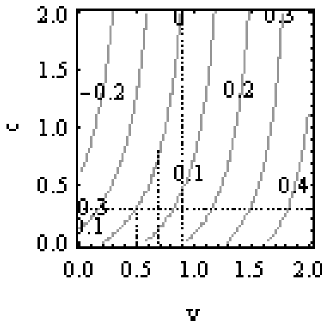

Figure 3.B2.1c | g=1
Contour Plot q_{10} (*frr*$_2$)

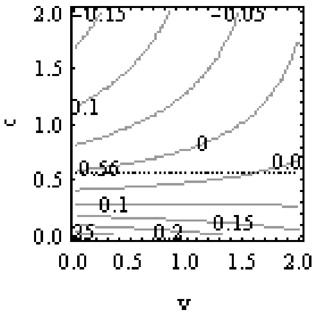

Figure 3.B2.1d | g=1
Contour Plot q_{20} (*frr*$_2$)

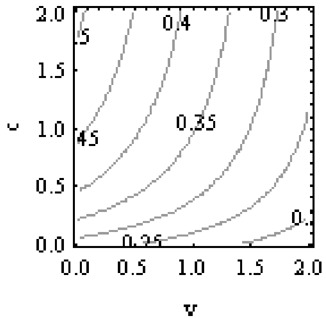

Figure 3.B2.1: Positive isoquants in each period simultaneously, provided that $v > 0.5$ and $c \leq 0.3$

Figure 3.B2.2a | g=0.5
Contour Plot q_{11} (*frr*$_2$)

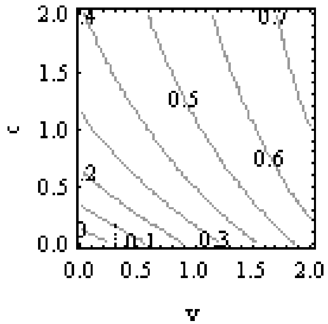

Figure 3.B2.2b | g=0.5
Contour Plot q_{21} (*frr*$_2$)

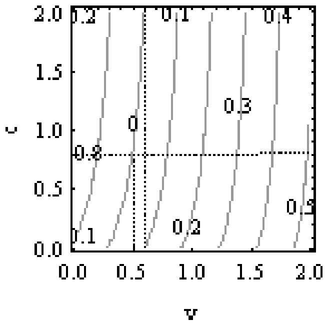

Figure 3.B2.2c | g=0.5
Contour Plot q_{10} (*frr*$_2$)

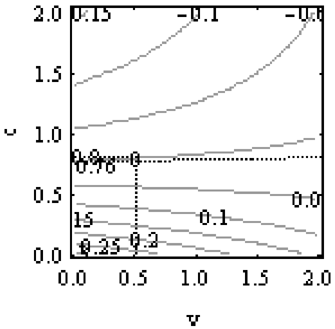

Figure 3.B2.2d | g=0.5
Contour Plot q_{20} (*frr*$_2$)

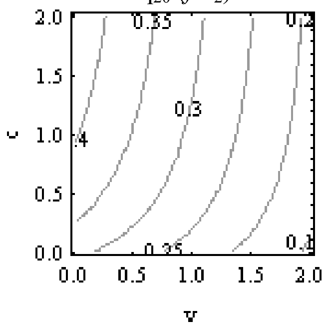

Figure 3.B2.2: Positive isoquants in each period, simultaneously, provided that $v > 0.5$ and $c \leq 0.7$

Figure 3.B2.3a | g=1
Contour Plot w_{10} (*frr*$_2$)

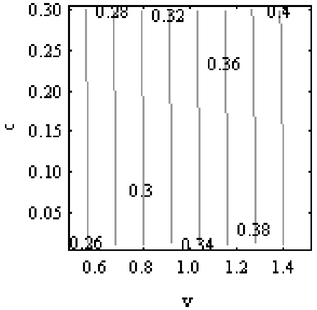

Figure 3.B2.3b | g=1
Contour Plot w_{20} (*frr*$_2$)

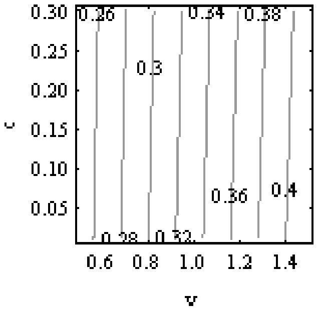

Figure 3.B2.3c | g=1
Contour Plot TC_1 (*frr*$_2$)

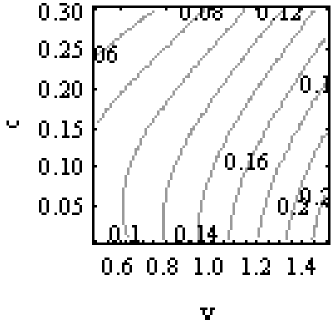

Figure 3.B2.3d | g=1
Contour Plot TC_2 (*frr*$_2$)

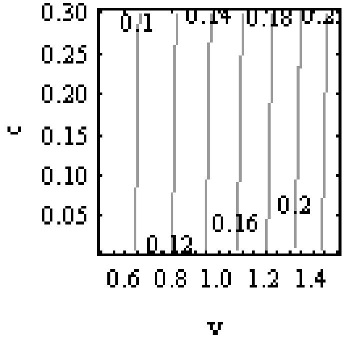

Figure 3.B2.3: Positive wages and total costs

Figure 3.B2.4a | g=0.5
Contour Plot w_{10} (*frr*$_2$)

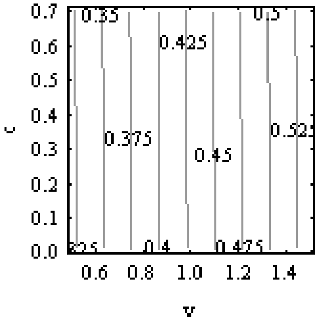

Figure 3.B2.4b | g=0.5
Contour Plot w_{20} (*frr*$_2$)

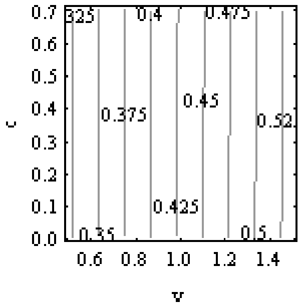

Figure 3.B2.4c | g=0.5
Contour Plot TC_1 (*frr*$_2$)

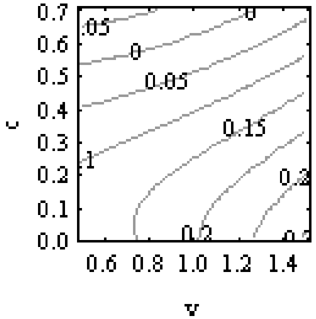

Figure 3.B2.4d | g=0.5
Contour Plot TC_2 (*frr*$_2$)

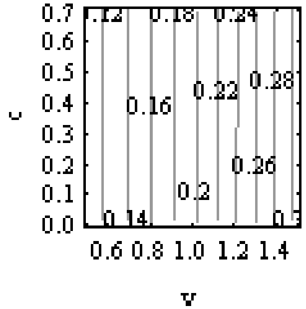

Figure 3.B2.4: Positive wages and total costs provided that $c \leq 0.53$

Figure 3.B2.5a | g=1
Contour Plot P_{10} (frr_2)

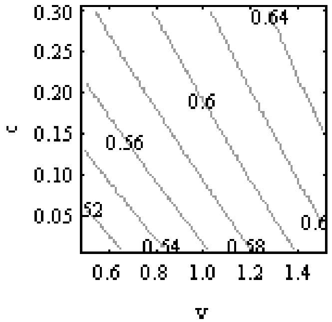

Figure 3.B2.5b | g=1
Contour Plot P_{20} (frr_2)

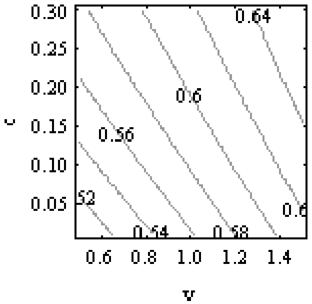

Figure 3.B2.5c | g=1
Contour Plot P_{11} (frr_2)

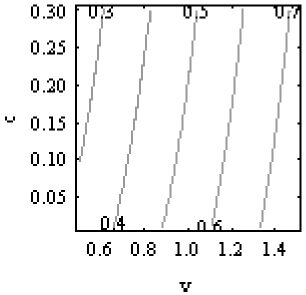

Figure 3.B2.5d | g=1
Contour Plot P_{21} (frr_2)

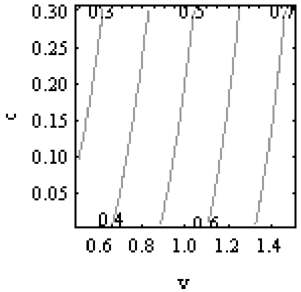

Figure 3.B2.5: Positive prices and total costs

Figure 3.B2.6a | g=0.5
Contour Plot P_{10} (frr_2)

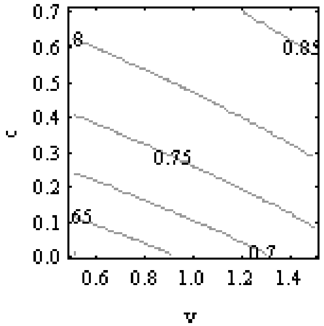

Figure 3.B2.6b | g=0.5
Contour Plot P_{20} (frr_2)

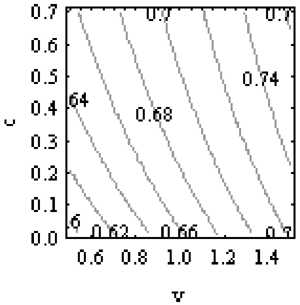

Figure 3.B2.6c | g=0.5
Contour Plot P_{11} (frr_2)

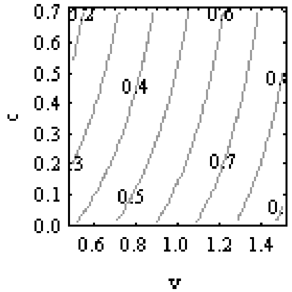

Figure 3.B2.6d | g=0.5
Contour Plot P_{21} (frr_2)

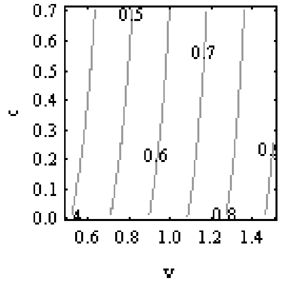

Figure 3.B2.6: Positive prices and total costs

Figure 3.B2.7a | g=1
Contour Plot Π1 (*frr*$_2$)

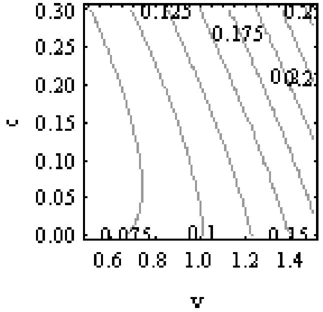

Figure 3.B2.7b | g=1
Contour Plot Π2 (*frr*$_2$)

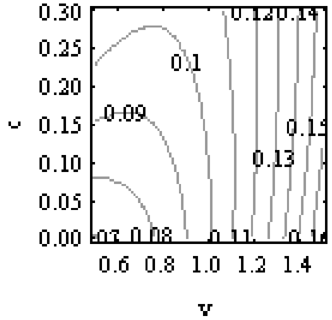

Figure 3.B2.7c | g=1
Contour Plot U1 (*frr*$_2$)

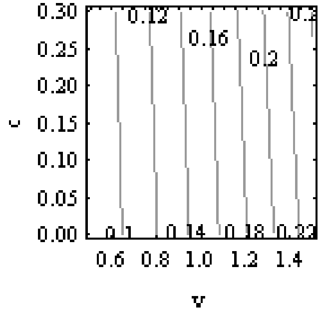

Figure 3.B2.7d | g=1
Contour Plot U2 (*frr*$_2$)

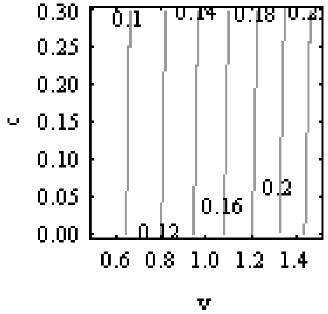

Figure 3.B2.7e | g=1
Contour Plot CS (*frr*$_2$)

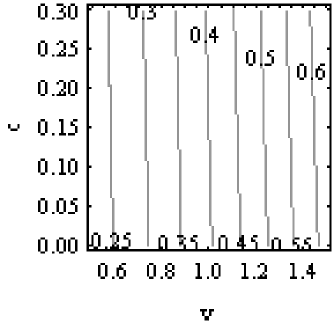

Figure 3.B2.7: Π$_1$, Π$_2$ U$_1$, U$_2$ and CS are positive, provided interior solution is ensured

Figure 3.B2.8a | g=0.5
Contour Plot Π1 (*frr*$_2$)

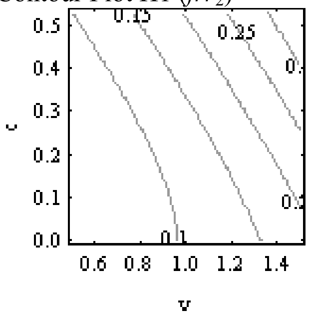

Figure 3.B2.8b | g=0.5
Contour Plot Π2 (*frr*$_2$)

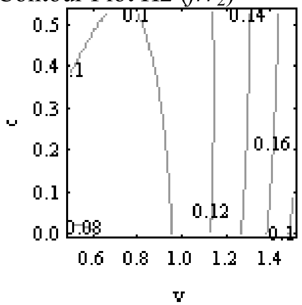

Figure 3.B2.8c | g=0.5
Contour Plot U1 (*frr*$_2$)

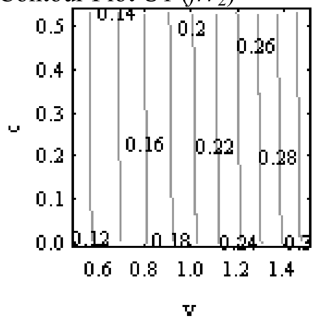

Figure 3.B2.8d | g=0.5
Contour Plot U2 (*frr*$_2$)

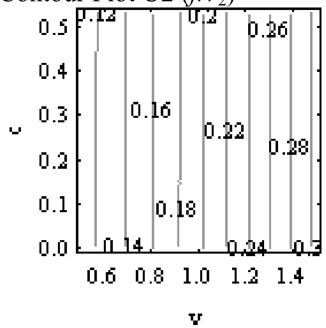

Figure 3.B2.8e | g=0.5
Contour Plot CS (*frr*$_2$)

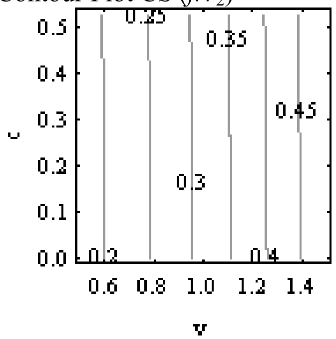

Figure 3.B2. 8: Π$_1$, Π$_2$ U$_1$, U$_2$ and CS are positive, provided interior solution is ensured

Figure 3.B2.9a | g=1
Contour Plot (q_{10}-q_{20})(*frr*$_2$)

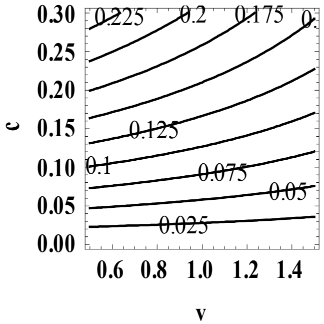

Figure 3.B2.9b | g=1
Contour Plot (q_{11}-q_{21})(*frr*$_2$)

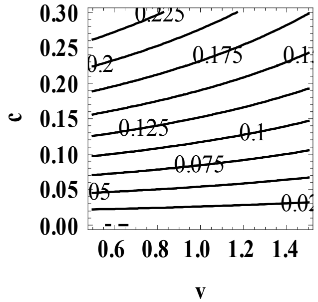

Figure 3.B2.9c | g=1
Contour Plot (q_{11}-q_{10}) (*frr*$_2$)

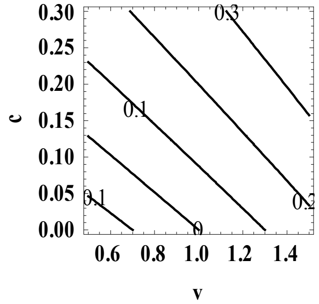

Figure 3.B2.9d | g=1
Contour Plot (q_{21}-q_{20}) (*frr*$_2$)

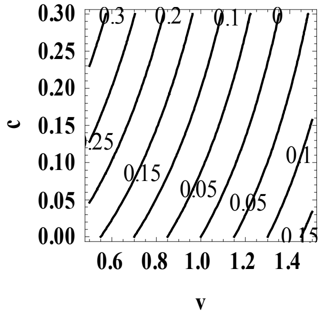

Figure 3.B2. 9: q_{10}<q_{20} and q_{11}>q_{21}, irrespective of v and q_{11}>q_{10} if c>0.15 and q_{20}>q_{21} if v<1

Figure 3.B2.10a | g=1
Contour Plot (tq_1-tq_2) (*frr*$_2$)

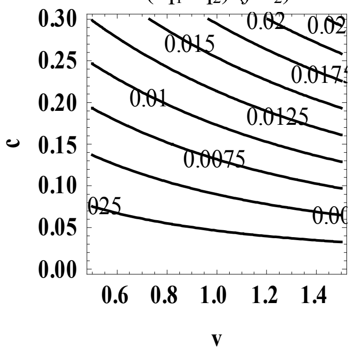

Figure 3.B2.10b | g=1
Contour Plot (Π1-Π2) (*frr*$_2$)

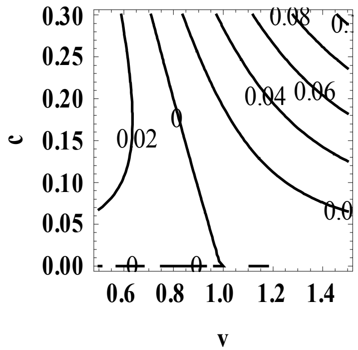

Figure 3.B2.10c | g=1
Contour Plot (U1-U2) (*frr*$_2$)

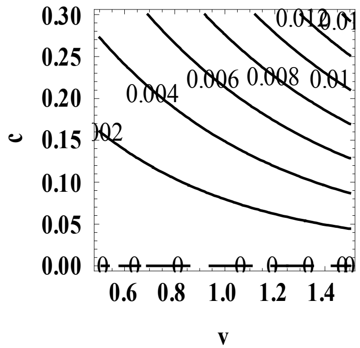

Figure 3.B2.10: tq_1>tq_2, Π_1>Π_2 if v>1 and U1>U2

Figure 3.B2.11a | g=0.5
Contour Plot (q_{10}-q_{20}) (*frr*$_2$)

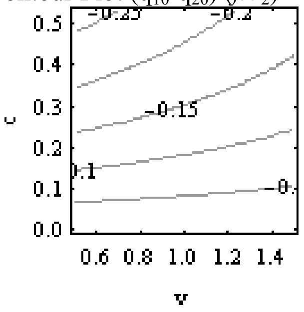

Figure 3.B2.11b | g=0.5
Contour Plot (q_{11}-q_{21}) (*frr*$_2$)

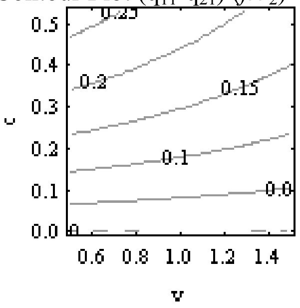

Figure 3.B2.11c | g=0.5
Contour Plot (q_{11}-q_{10}) (*frr*$_2$)

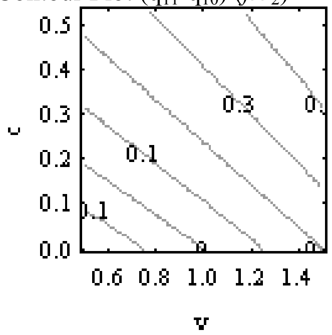

Figure 3.B2.11d | g=0.5
Contour Plot (q_{21}-q_{20}) (*frr*$_2$)

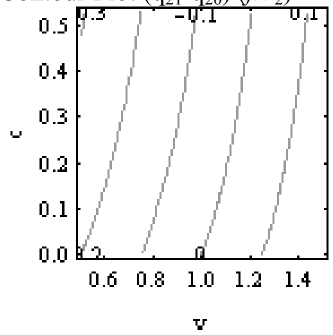

Figure 3.B2.11: q_{10}<q_{20} and q_{11}>q_{21} and q_{11}>q_{10} if c>0.2 and q_{20}>q_{21} if v<1

Figure 3.B2.12a | g=0.5
Contour Plot (tq_1-tq_2) (*frr*$_2$)

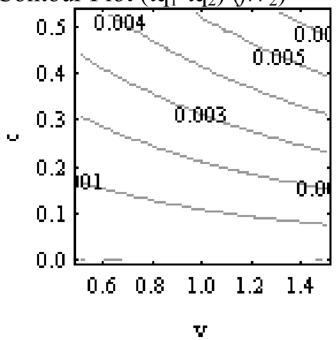

Figure 3.B2.12b| g=0.5
Contour Plot (Π1-Π2) (*frr*$_2$)

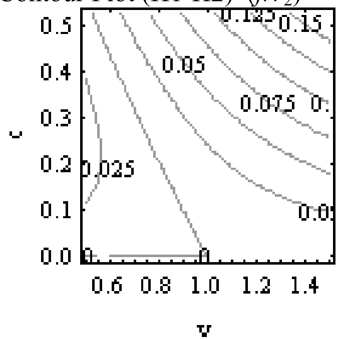

Figure 3.B2.12c | g=0.5
Contour Plot (U1-U2) (*frr*$_2$)

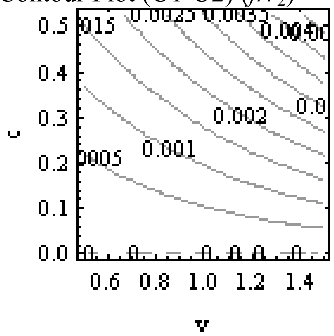

Figure 3.B2.12: tq_1>tq_2 irrespective of v, Π_1>Π_2 if v>1 and U1>U2

Figure 3.B2.13a| g=1
Contour Plot $q_{10}(frr_1)$-$q_{10}(frr_2)$

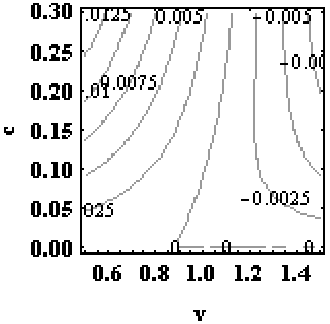

Figure 3.B2.13b| g=1
Contour Plot $q_{20}(frr_1)$-$q_{20}(frr_2)$

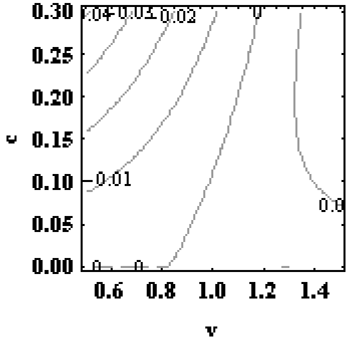

Figure 3.B2.13c| g=1
Contour Plot $q_{11}(frr_1)$-$q_{11}(frr_2)$

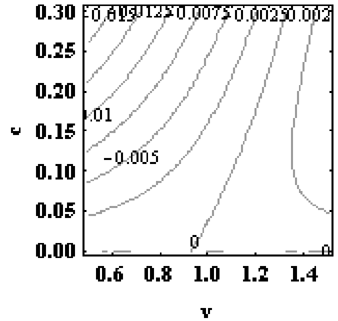

Figure 3.B2.13d| g=1
Contour Plot $q_{21}(frr_1)$-$q_{21}(frr_2)$

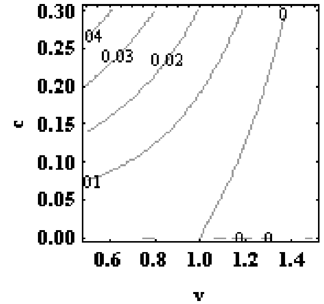

Figure 3.B2.13: $q_{10}(frr_1)>q_{10}(frr_2)$ $q_{20}(frr_1)<q_{20}(frr_2)$ $q_{11}(frr_1)<q_{11}(frr_2)$ and $q_{21}(frr_1)>q_{21}(frr_2)$ if v<1, and the opposite if v<1

Figure 3.B2.14a| g=0.5
Contour Plot $q_{10}(frr_1)$-$q_{10}(frr_2)$

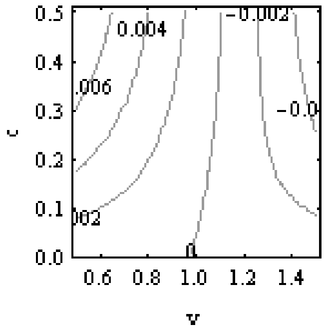

Figure 3.B2.14b| g=0.5
Contour Plot$q_{20}(frr_1)$-$q_{20}(frr_2)$

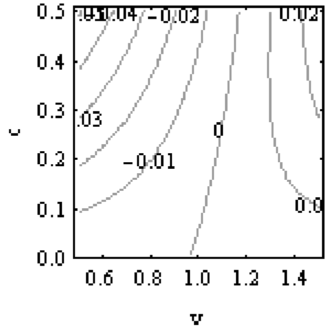

Figure 3.B2.14c| g=0.5
Contour Plot $q_{11}(frr_1)$-$q_{11}(frr_2)$

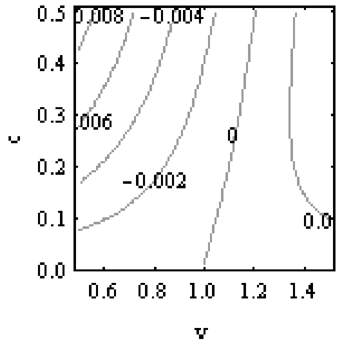

Figure 3.B2.14d| g=0.5
Contour Plot $q_{21}(frr_1)$-$q_{21}(frr_2)$

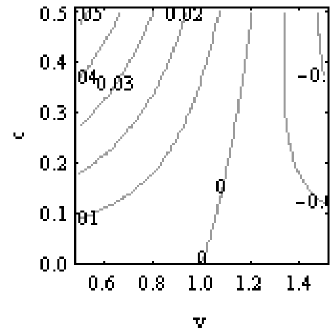

Figure 3.B2.14: $q_{10}(frr_1)>q_{10}(frr_2)$ $q_{20}(frr_1)<q_{20}(frr_2)$ $q_{11}(frr_1)<q_{11}(frr_2)$ and $q_{21}(frr_1)>q_{21}(frr_2)$ if v<1, and the opposite if v<1, provided that g=0.5

Figure 3.B2.15a |g=1

Contour Plot $q_{10}(frr_2)$-$q_{10}(frr_0)$

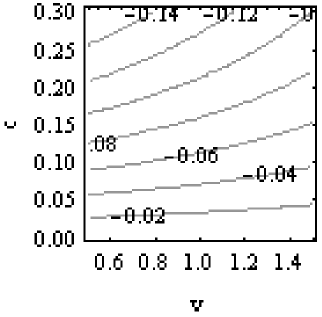

Figure 3.B2.15b |g=1

Contour Plot $q_{20}(frr_2)$-$q_{20}(frr_0)$

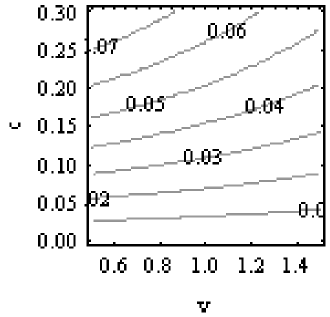

Figure 3.B2.15c |g=1

Contour Plot $q_{11}(frr_2)$-$q_{11}(frr_0)$

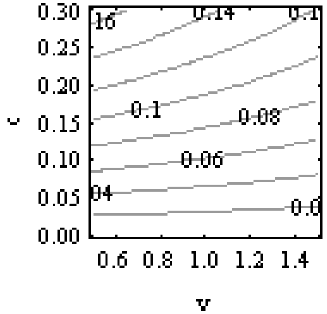

Figure 3.B2.15d |g=1

Contour Plot $q_{21}(frr_2)$-$q_{21}(frr_0)$

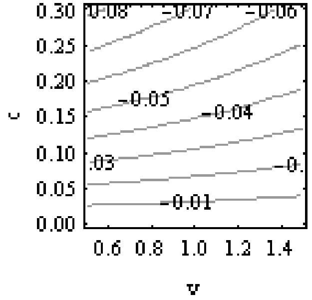

Figure 3.B2.15: $q_{10}(frr_2)<q_{10}(frr_0)$ and $q_{20}(frr_2)>q_{20}(frr_0)$; $q_{11}(frr_2)>q_{11}(frr_0)$ and $q_{21}(frr_2)<q_{21}(frr_0)$

Figure 3.B2.16a |g=0.5

Contour Plot $q_{10}(frr_2)$-$q_{10}(frr_0)$

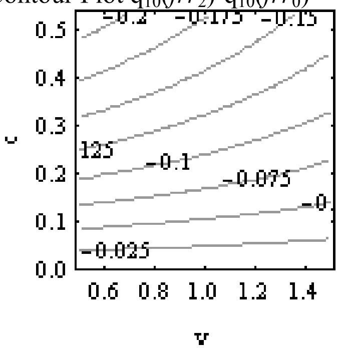

Figure 3.B2.16b |g=0.5

Contour Plot $q_{20}(frr_2)$-$q_{20}(frr_0)$

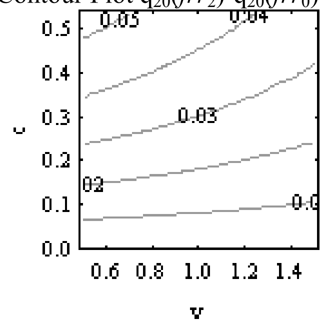

Figure 3.B2.16c |g=0.5

Contour Plot $q_{11}(frr_2)$-$q_{11}(frr_0)$

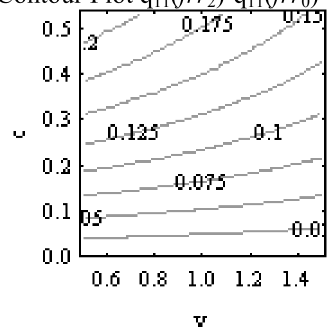

Figure 3.B2.16d |g=0.5

Contour Plot $q_{21}(frr_2)$-$q_{21}(frr_0)$

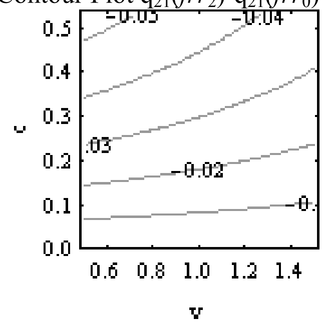

Figure 3.B2.16: $q_{10}(frr_2)<q_{10}(frr_0)$ and $q_{20}(frr_2)>q_{20}(frr_0)$; $q_{11}(frr_2)>q_{11}(frr_0)$ and $q_{21}(frr_2)<q_{21}(frr_0)$

Figure 3.B2.17a |g=1
Contour Plot
$(q_{11}-q_{10})$ $(frr_2)-(q_{11}-q_{10})(frr_0)$

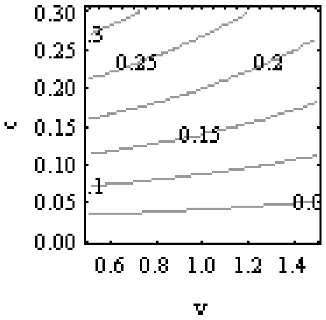

Figure 3.B2.17b |g=1
Contour Plot
$(q_{21}-q_{20})$ $(frr_2)-(q_{21}-q_{20})$ (frr_0)

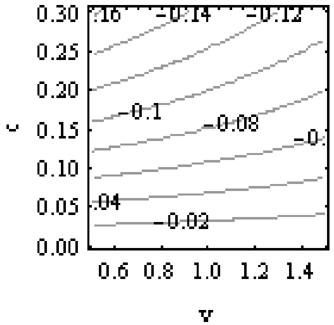

Figure 3.B2.17c |g=0.5
Contour Plot
$(q_{11}-q_{10})$ $(frr_2)-(q_{11}-q_{10})(frr_0)$

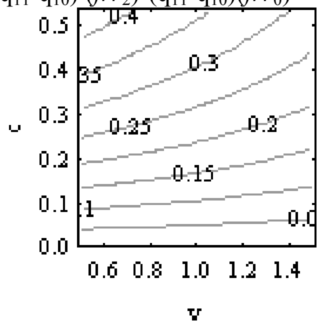

Figure 3.B2.17d |g=0.5
Contour Plot
$(q_{21}-q_{20})$ $(frr_2)-(q_{21}-q_{20})$ (frr_0)

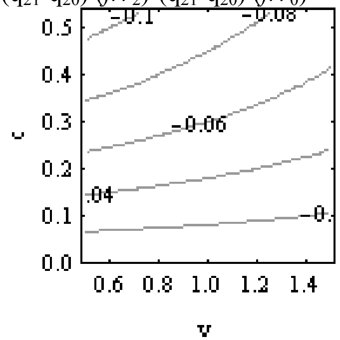

Figure 3.B2.17: $(q_{11}-q_{10})$ $(frr_2)>$ $(q_{11}-q_{10})$ (frr_0) and $(q_{21}-q_{20})$ (frr_2) < $(q_{21}-q_{20})$ (frr_0)

Figure 3.B2.18a |g=1
Contour Plot
$tq_1(frr_2)-tq_1(frr_0)$

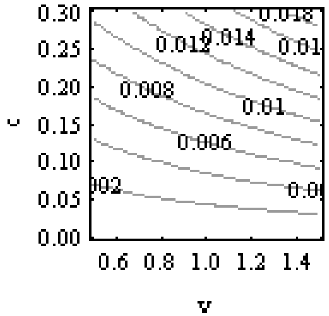

Figure 3.B2.18b |g=1
Contour Plot
$tq_2(frr_2)-tq_2(frr_0)$

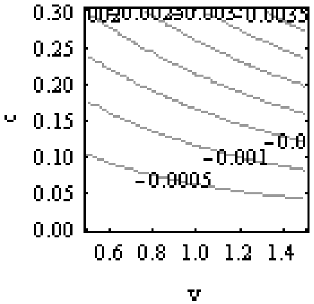

Figure 3.B2.18c |g=1
Contour Plot
$Tq_A(frr_2)-Tq_A(frr_0)$

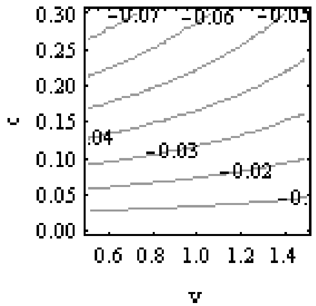

Figure 3.B2.18d |g=1
Contour Plot
$Tq_B(frr_2)-Tq_B(frr_0)$

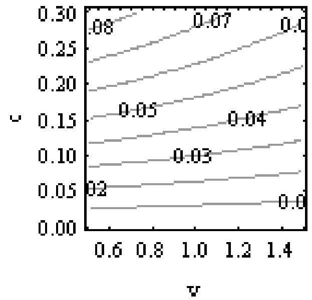

Figure 3.B2.18e |g=1
Contour Plot
$Tq(frr_2)-Tq(frr_0)$

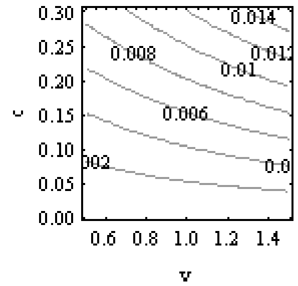

Figure 3.B2.18: $tq_1(frr_2)>tq_1(frr_0)$ and $tq_2(frr_2)<tq_2(frr_0)$; $Tq_A(frr_2)<Tq_A(frr_0)$ and $Tq_B(frr_2)>Tq_B(frr_0)$; $Tq(frr_2)>Tq(frr_0)$

Figure 3.B2.19a |g=0.5
Contour Plot
$tq_1(frr_2)$-$tq_1(frr_0)$

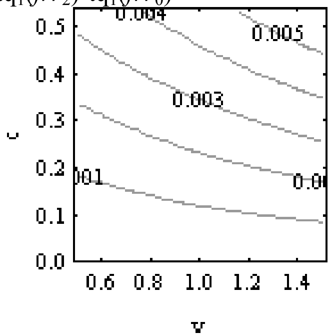

Figure 3.B2.19b |g=0.5
Contour Plot
$tq_2(frr_2)$-$tq_2(frr_0)$

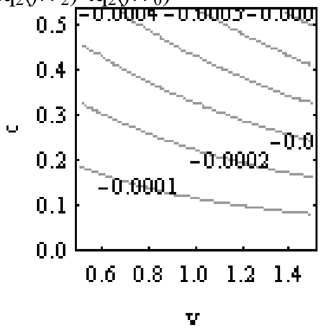

Figure 3.B2.19c |g=0.5
Contour Plot
$Tq_A(frr_2)$-$Tq_A(frr_0)$

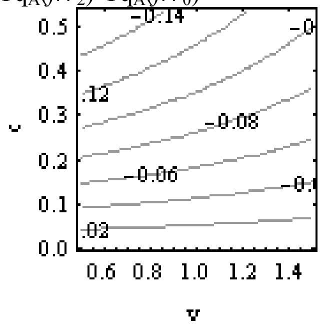

Figure 3.B2.19d |g=0.5
Contour Plot
$Tq_B(frr_2)$-$Tq_B(frr_0)$

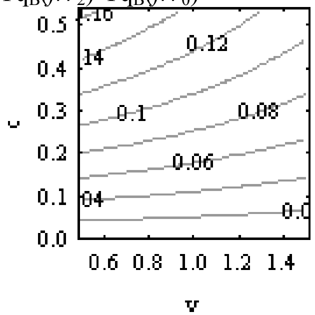

Figure 3.B2.19e |g=0.5
Contour Plot
$Tq(frr_2)$-$Tq(frr_0)$

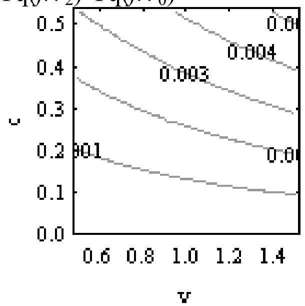

Figure 3.B2.19: $tq_1(frr_2)>tq_1(frr_0)$ and $tq_2(frr_2)<tq_2(frr_0)$; $Tq_A(frr_2)<Tq_A(frr_0)$ and $Tq_B(frr_2)>Tq_B(frr_0)$; $Tq(frr_2)>Tq(frr_0)$

Figure 3.B2.20a |g=1
Contour Plot
Π1(frr_2)-Π1(frr_0)

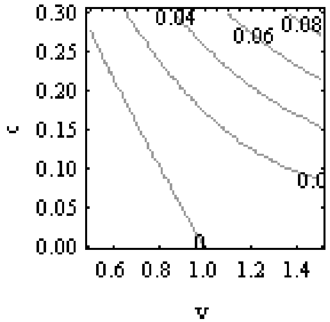

Figure 3.B2.20b |g=1
Contour Plot
Π2(frr_2)-Π2(frr_0)

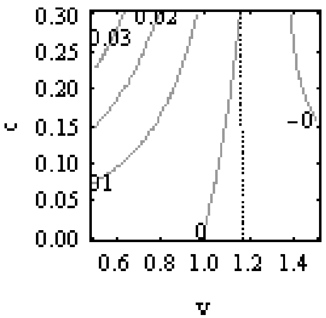

Figure 3.B2.20c |g=1
Contour Plot
U1(frr_2)-U1(frr_0)

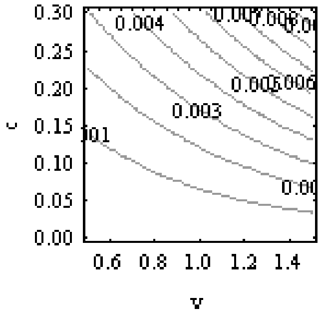

Figure 3.B2.20d |g=1
Contour Plot
U2(frr_2)-U2(frr_0)

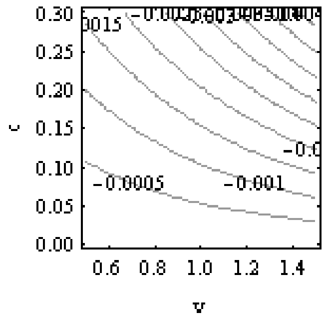

Figure 3.B2.20e |g=1
Contour Plot
CS(frr_2)-CS(frr_0)

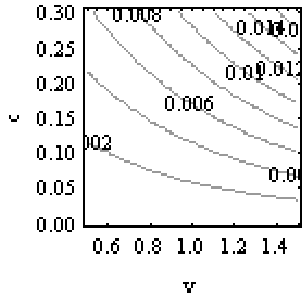

Figure 3.B2.20: Π1(frr_2)>Π1(frr_0) under restriction, Π2(frr_2)>Π2(frr_0) if v<1; U1(frr_2)>U1(frr_0) and U2(frr_2)<U2(frr_0) CS(frr_2)>CS(frr_0)

Figure 3.B2.21a |g=0.5
Contour Plot
$\Pi 1(frr_2)$-$\Pi 1(frr_0)$

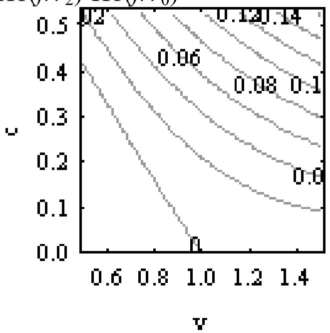

Figure 3.B2.21b |g=0.5
Contour Plot
$\Pi 2(frr_2)$-$\Pi 2(frr_0)$

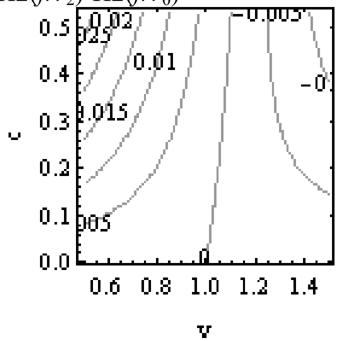

Figure 3.B2.21c |g=0.5
Contour Plot
$U1(frr_2)$-$U1(frr_0)$

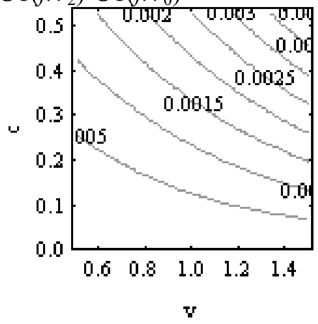

Figure 3.B2.21d |g=0.5
Contour Plot
$U2(frr_2)$-$U2(frr_0)$

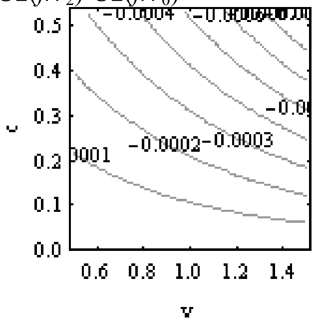

Figure 3.B2.21e |g=0.5
Contour Plot
$CS(frr_2)$-$CS(frr_0)$

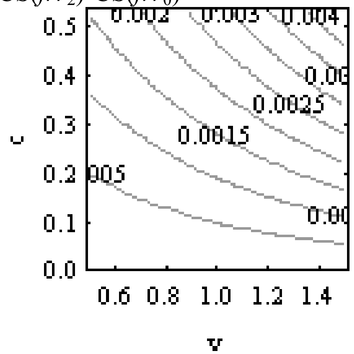

Figure 3.B2.21: $\Pi 1(frr_2)>\Pi 1(frr_0)$ under restriction, $\Pi 2(frr_2)>\Pi 2(frr_0)$ if v<1; $U1(frr_2)>U1(frr_0)$ and $U2(frr_2)<U2(frr_0)$ $CS(frr_2)>CS(frr_0)$

Figure 3.B2.22a |g=1
Contour Plot $\frac{d\Pi 1}{dc}(frr_2)$

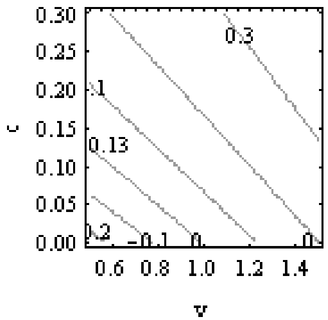

Figure 3.B2.22b |g=1
Contour Plot $\frac{d\Pi 2}{dc}(frr_2)$

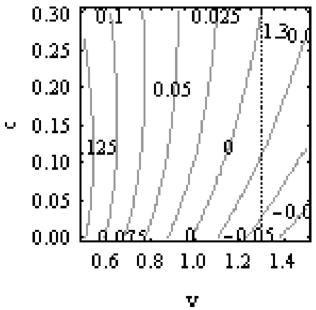

Figure 3.B2.22c |g=1
Contour Plot $\frac{dU1}{dc}(frr_2)$

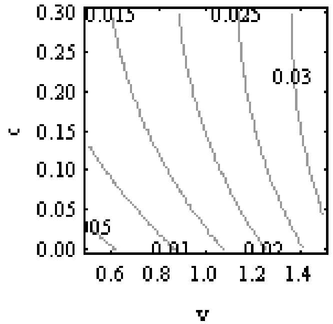

Figure 3.B2.22d |g=1
Contour Plot $\frac{dU2}{dc}(frr_2)$

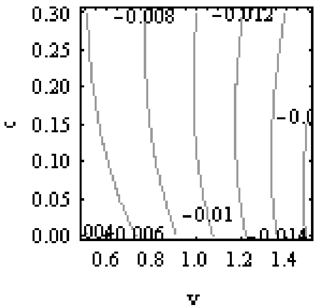

Figure 3.B2.22e |g=1
Contour Plot $\frac{dCS}{dc}(frr_2)$

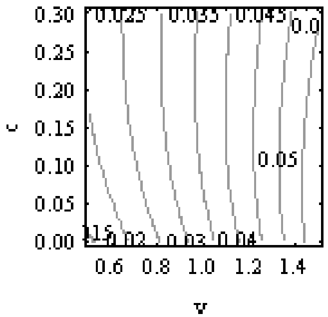

Figure 3.B2.22: $\Pi 1$, $\Pi 2$ and U1 are increasing with c provided that $0.13 \le c \le 0.3$, and v<1.3

Figure 3.B2.23a |g=0.5

Contour Plot $\frac{d\Pi 1}{dc}(frr_2)$

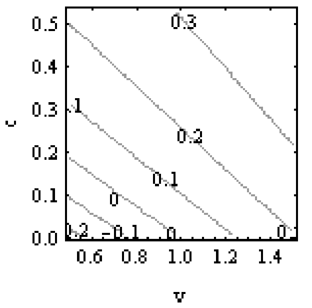

Figure 3.B2.23b |g=0.5

Contour Plot $\frac{d\Pi 2}{dc}(frr_2)$

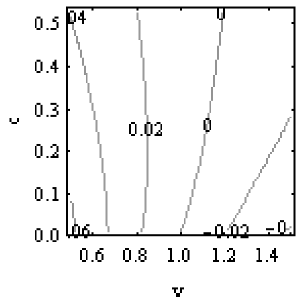

Figure 3.B2.23c |g=0.5

Contour Plot $\frac{dU1}{dc}(frr_2)$

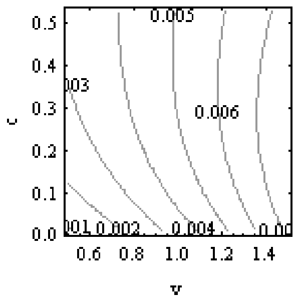

Figure 3.B2.23d |g=0.5

Contour Plot $\frac{dU2}{dc}(frr_2)$

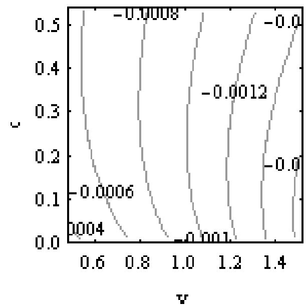

Figure 3.B2.23e |g=0.5

Contour Plot $\frac{dCS}{dc}(frr_2)$

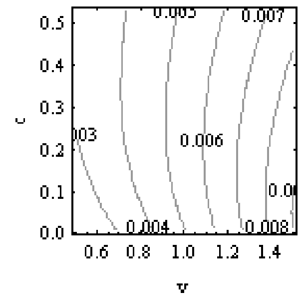

Figure 3.B2.23: Π1, Π2, U1 are increasing with c provided that $0.2 \le c \le 0.53$, and v<1.2

Figure 3.B2.24.a

|g=1| c=0.3| ρ=ρ_1

$\tilde{w}_{10}$ w_{20} frr$_2$

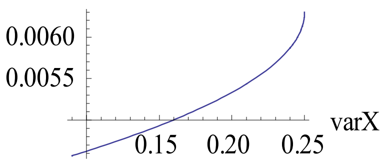

Figure 3.B2.24.b

|g=1| c=0.3| ρ=ρ_2

$\tilde{w}_{10}$ w_{20} frr$_2$

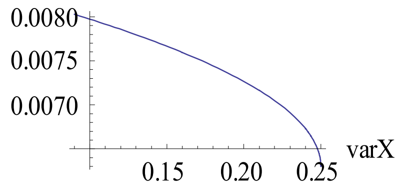

Figure 3.B2.24.c

|g=0.5| c=0.5| ρ=ρ_1

$\tilde{w}_{10}$ w_{20} frr$_2$

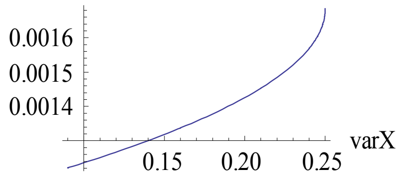

Figure 3.B2.24.d

|g=0.5| c=0.5| ρ=ρ_2

$\tilde{w}_{10}$ w_{20} frr$_2$

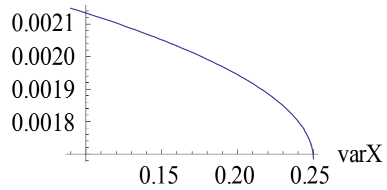

Figure 3.B2.24: (w_{10}-w_{20}) increasing (decreasing) with varX under ρ_1 (ρ_2)

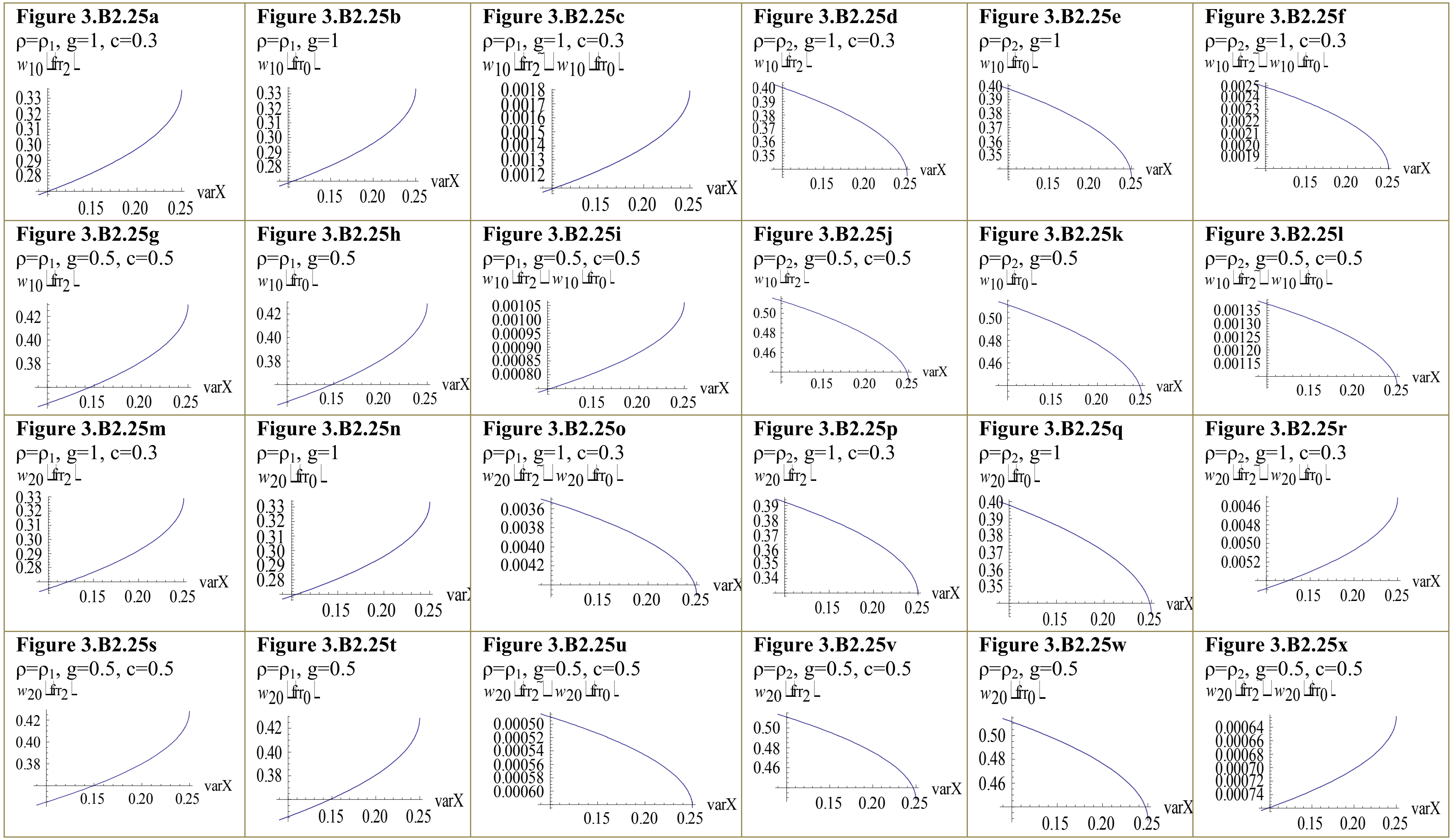

Figure 3.B2.25: w_{10}, w_{20} increasing (decreasing) with varX under ρ_1 (ρ_2)

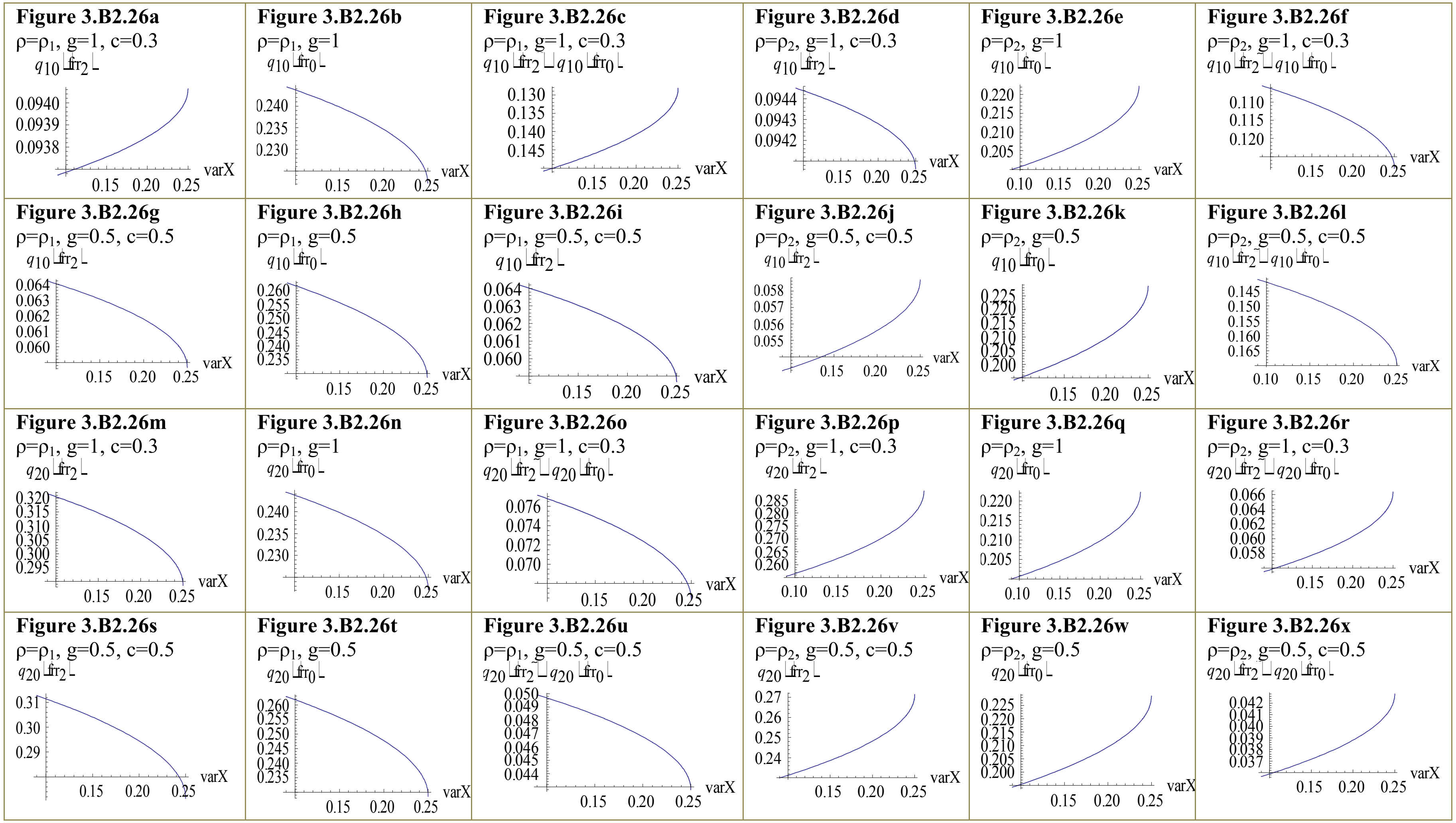

Figure 3.B2.26: How varX affects $q_{10}(frr_2)$ $q_{10}(frr_0)$ $q_{10}(frr_2)$-$q_{10}(frr_0)$ $q_{20}(frr_2)$ $q_{20}(frr_0)$ $q_{20}(frr_2)$-$q_{20}(frr_0)$ under ρ_1 and ρ_2

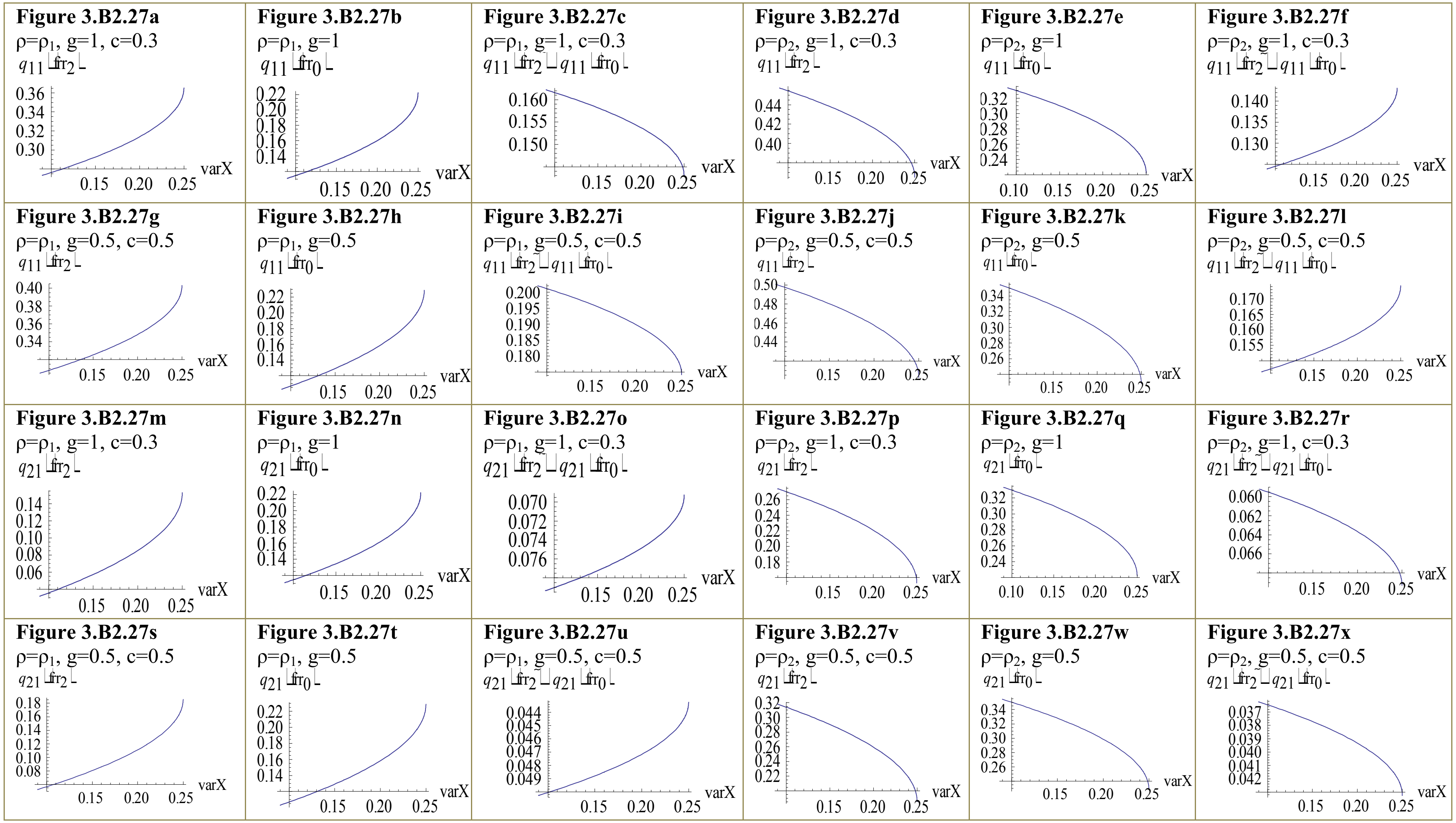

Figure 3.B2.27: How varX affects $q_{11}(frr_2)$ $q_{11}(frr_0)$ $q_{11}(frr_2)$-$q_{11}(frr_0)$ $q_{21}(frr_2)$ $q_{21}(frr_0)$ $q_{21}(frr_2)$-$q_{21}(frr_0)$ under ρ_1 and ρ_2

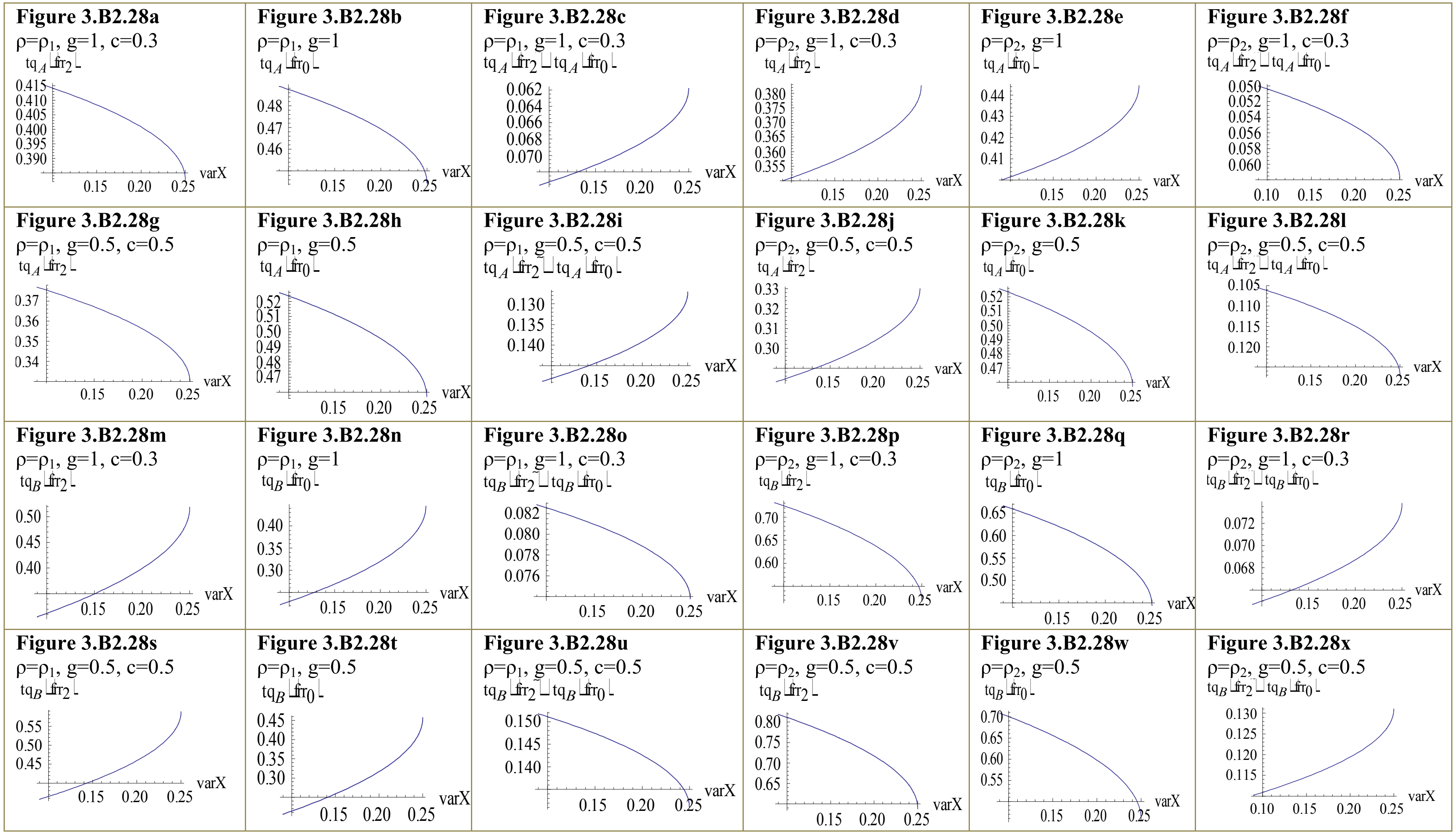

Figure 3.B2.28: How varX affects $tq_A(frr_2)tq_A(frr_0)$ $tq_A(frr_2)-tq_A(frr_0)$ $tq_B(frr_2)$ $tq_B(frr_0)$ $tq_B(frr_2)-tq_B(frr_0)$ under ρ_1 and ρ_2

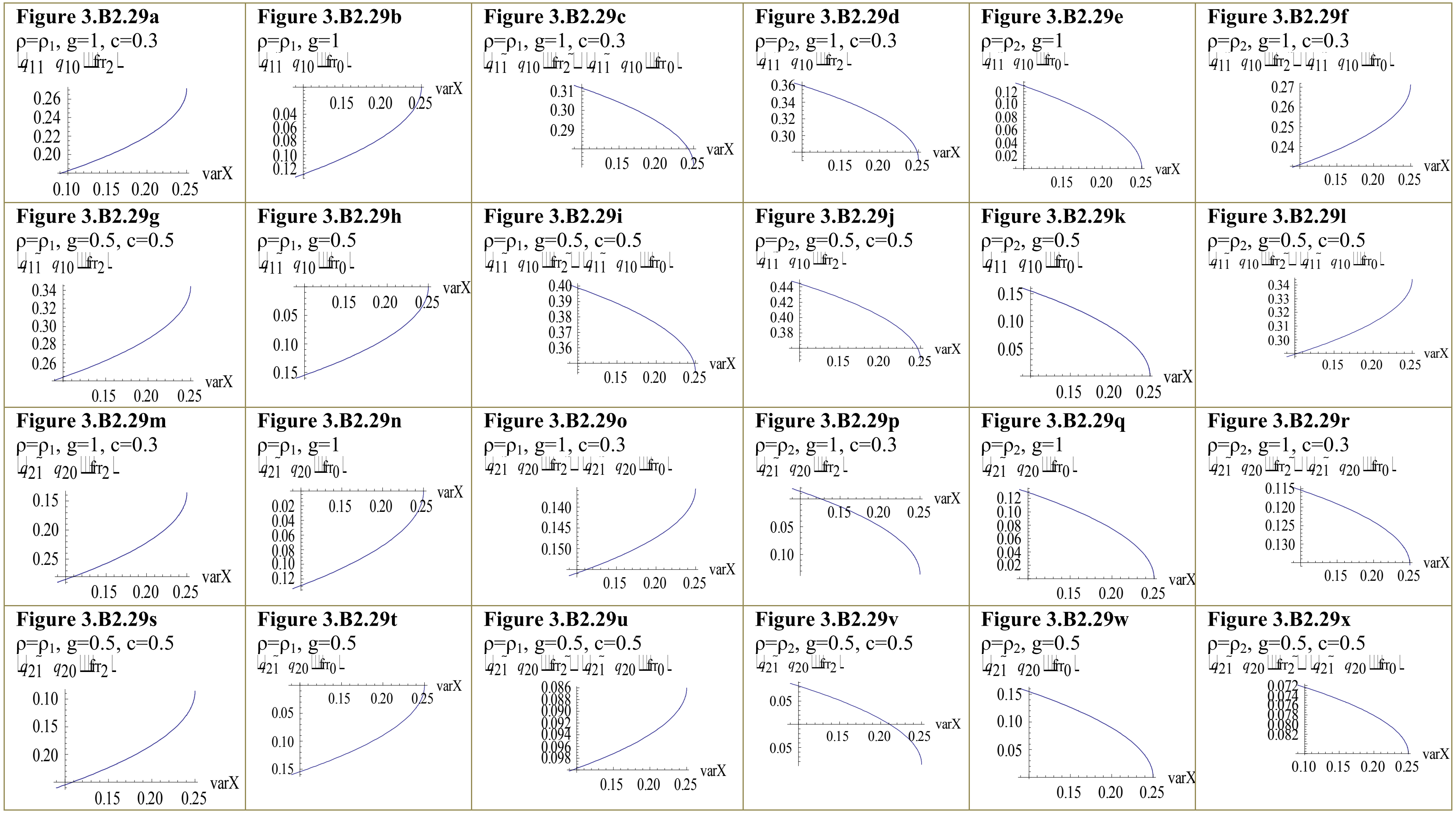

Figure 3.B2.29: How varX affects $(q_{11}-q_{10})(frr_2)$ $(q_{11}-q_{10})(frr_0)$ $(q_{11}-q_{10})(frr_2)-(q_{11}-q_{10})(frr_0)$ $(q_{21}-q_{20})(frr_2)$ $(q_{21}-q_{20})(frr_0)$ $(q_{21}-q_{20})(frr_2)-(q_{21}-q_{20})(frr_0)$ under ρ_1 and ρ_2

Figure 3.B2.30a
g=1, c=0.3, ρ=ρ_1
$\tilde{q}_{10}$ q_{20} frr_2

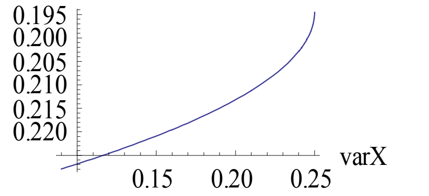

Figure 3.B2.30b
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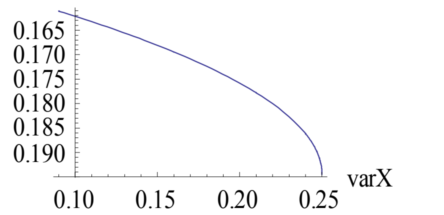

Figure 3.B2.30c
g=0.5, c=0.5, ρ=ρ_1
$\tilde{q}_{10}$ q_{20} frr_2

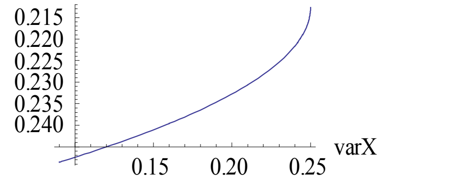

Figure 3.B2.30d
g=0.5, c=0.5, ρ=ρ_2
$\tilde{q}_{10}$ q_{20} frr_2

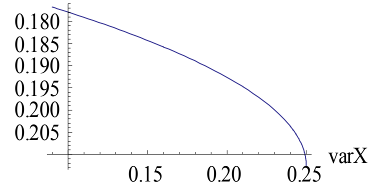

Figure 3.B2.30e
g=1, c=0.3, ρ=ρ_1
$\tilde{q}_{11}$ q_{21} frr_2

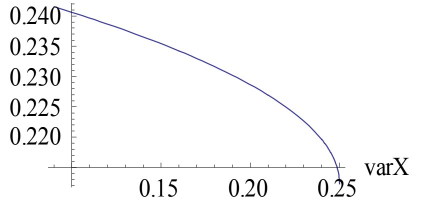

Figure 3.B2.30f
g=1, c=0.3, ρ=ρ_2
$\tilde{q}_{11}$ q_{21} frr_2

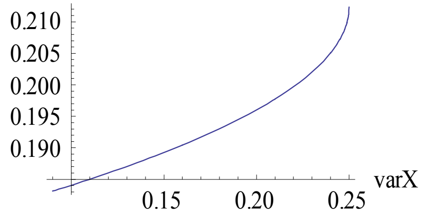

Figure 3.B2.30g
g=0.5, c=0.5, ρ=ρ_1
$\tilde{q}_{11}$ q_{21} frr_2

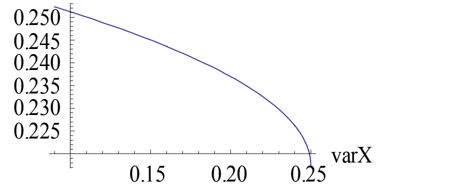

Figure 3.B2.30h
g=0.5, c=0.5, ρ=ρ_2
$\tilde{q}_{11}$ q_{21} frr_2

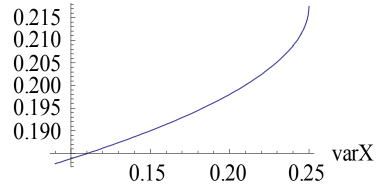

Figure 3.B2.30i
g=1, c=0.3, ρ=ρ_1
$\tilde{tq}_1$ tq_2 frr_2

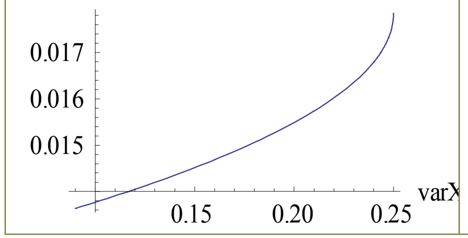

Figure 3.B2.30j
g=1, c=0.3, ρ=ρ_2
$\tilde{tq}_1$ tq_2 frr_2

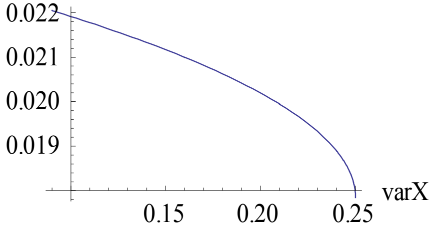

Figure 3.B2.30k
g=0.5, c=0.5, ρ=ρ_1
$\tilde{tq}_1$ tq_2 frr_2

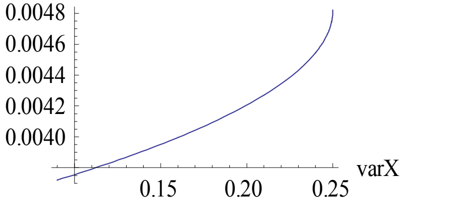

Figure 3.B2.30l
g=0.5, c=0.5, ρ=ρ_2
$\tilde{tq}_1$ tq_2 frr_2

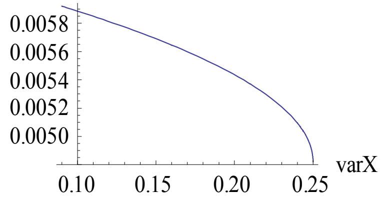

Figure 3.B2.30: How varX affects $(q_{10}$-$q_{20})(frr_2)$ $(q_{10}$-$q_{20})(frr_0)$; $(q_{11}$-$q_{21})(frr_2)$ $(q_{11}$-$q_{21})(frr_0)$; $(tq_1$-$tq_2)(frr_2)$ $(tq_1$-$tq_2)(frr_0)$ under ρ_1 and ρ_2

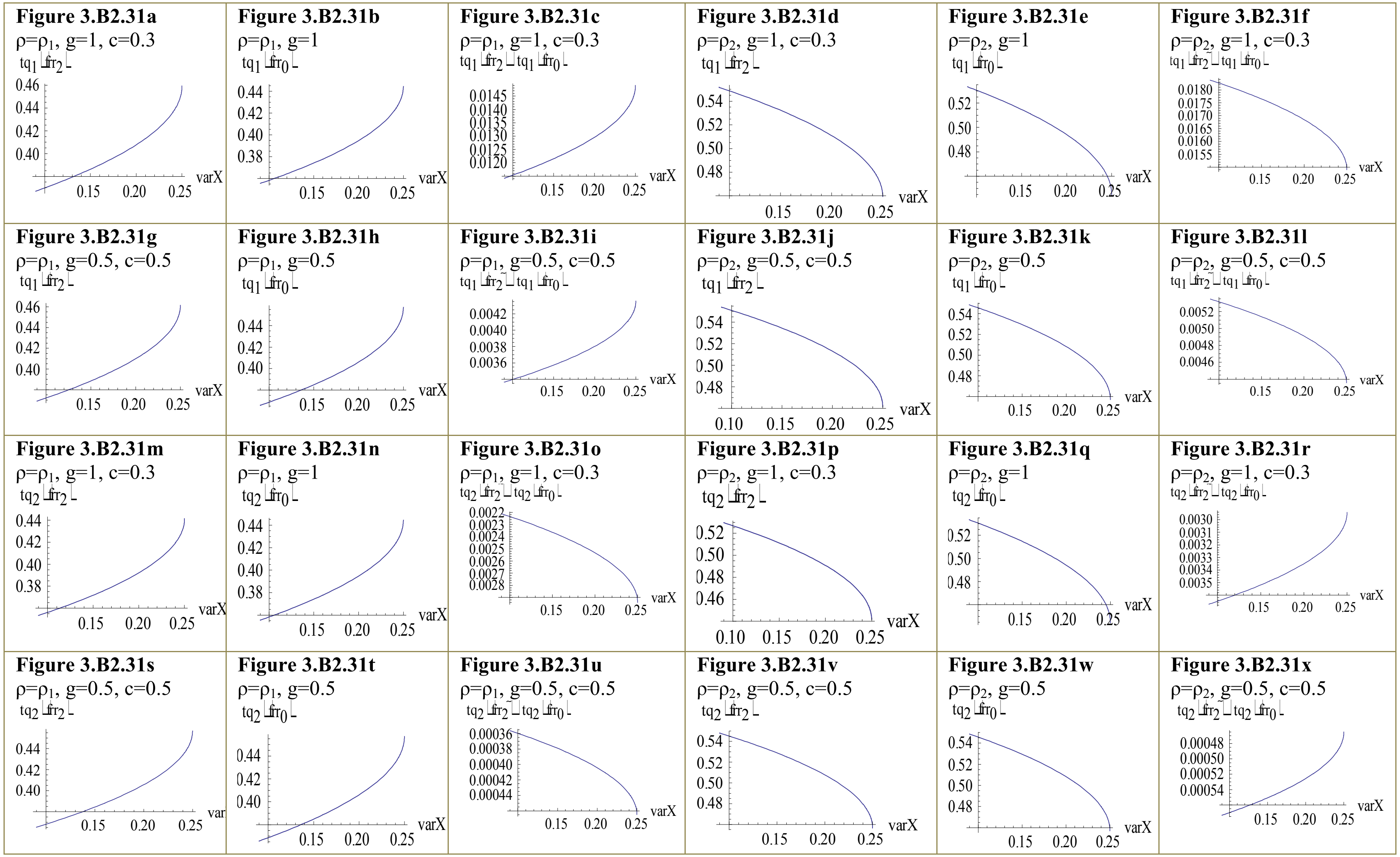

Figure 3.B2.31: How varX affects $tq_1(frr_2) tq_1(frr_0)$ $tq_1(frr_2)-tq_1(frr_0)$ $tq_2(frr_2)$ $tq_2(frr_0)$ $tq_2(frr_2)-tq_2(frr_0)$ under ρ_1 and ρ_2

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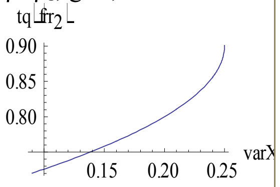</td>
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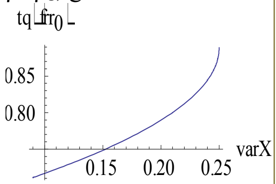</td>
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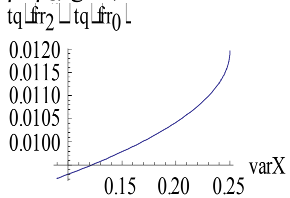</td>
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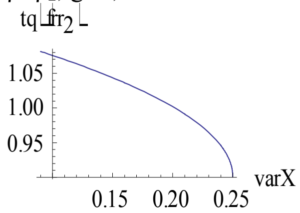</td>
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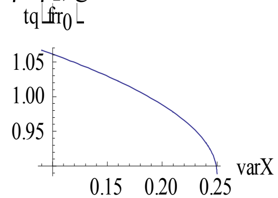</td>
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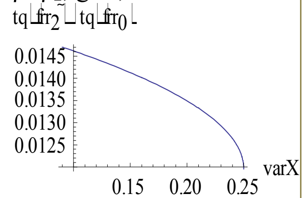</td>
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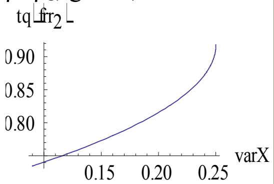</td>
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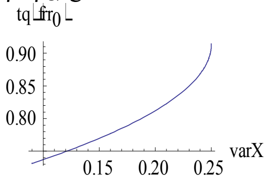</td>
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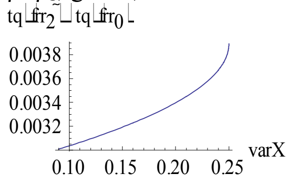
$\rho=\rho_1$, g=0.5, c=0.5
</td>
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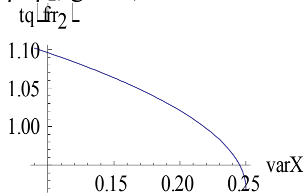</td>
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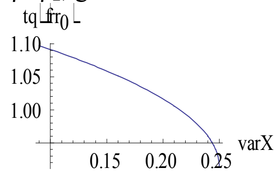</td>
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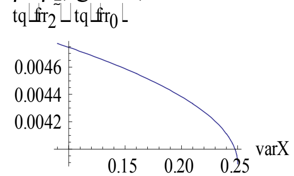</td>
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Figure 3.B2.32: How varX affects tq(*frr*$_2$)tq(*frr*$_0$) tq(*frr*$_2$)-tq(*frr*$_0$) under ρ_1 and ρ_2

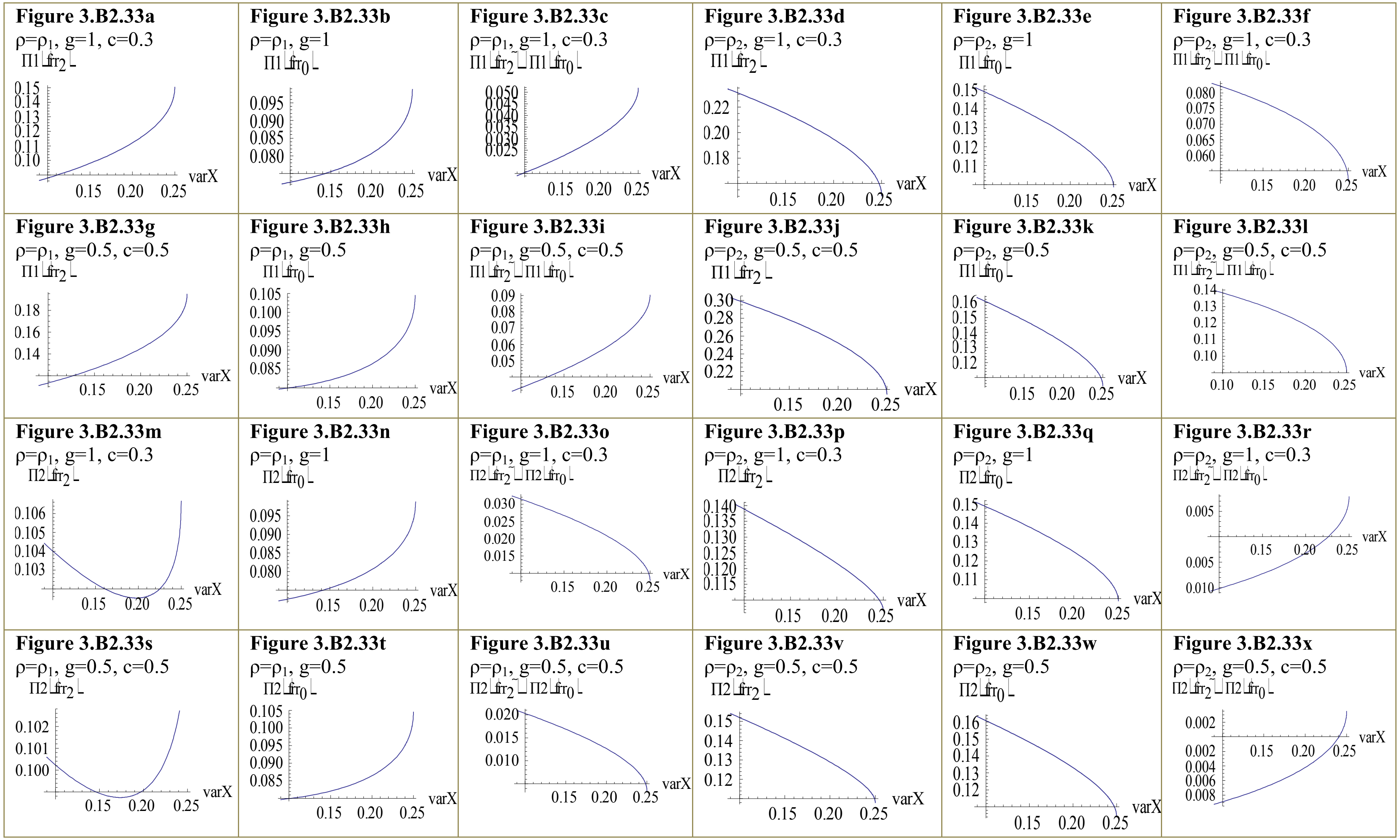

Figure 3.B2.33: How varX affects Π1(*frr*$_2$) Π1(*frr*$_0$) Π1(*frr*$_2$)-Π1(*frr*$_0$) Π2(*frr*$_2$) Π2(*frr*$_0$) Π2(*frr*$_2$)-Π2(*frr*$_0$) under ρ_1 and ρ_2

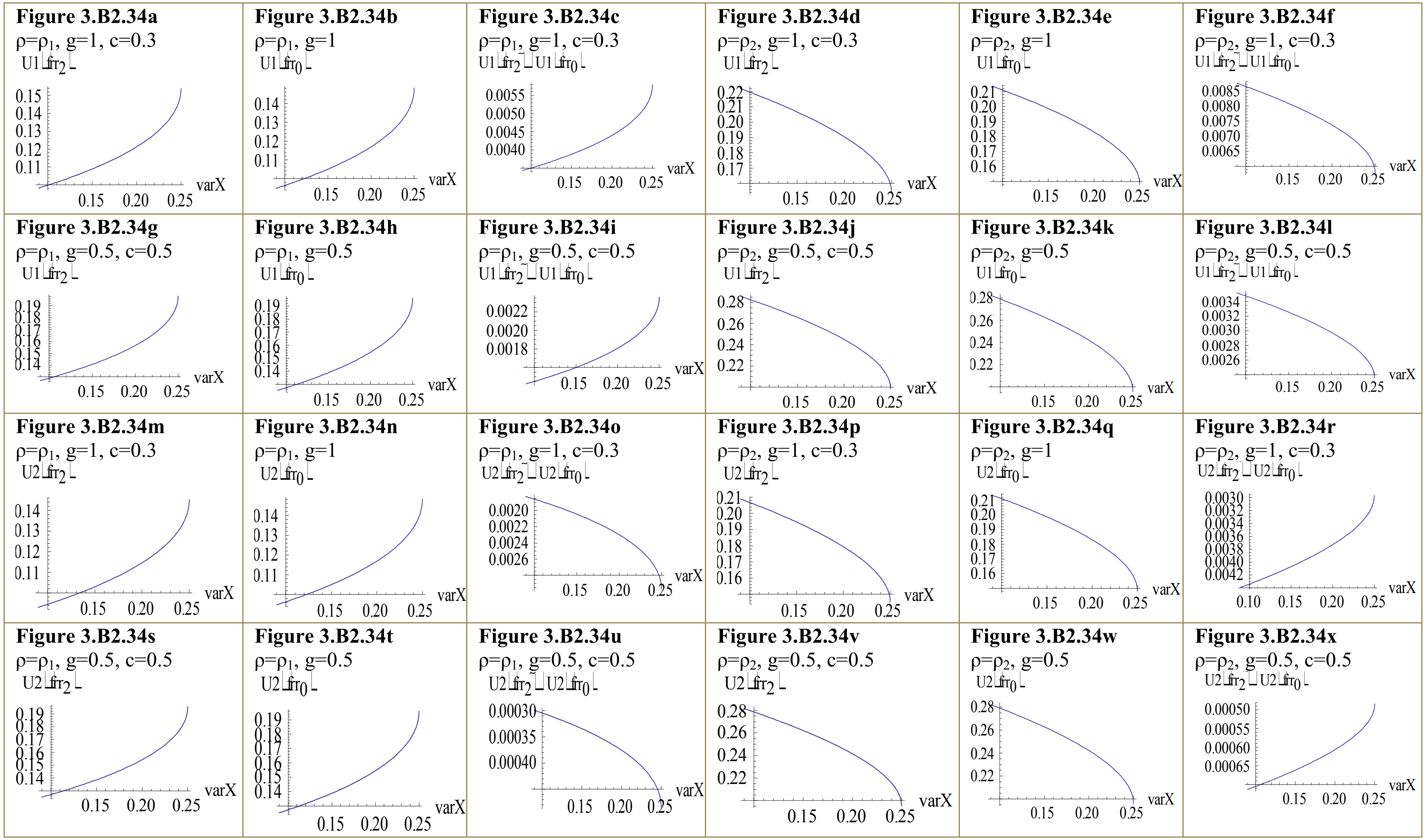

Figure 3.B2.34: How varX affects U1(*frr*$_2$) U1(*frr*$_0$) U1(*frr*$_2$)-U1(*frr*$_0$) U2(*frr*$_2$) U2(*frr*$_0$) U2(*frr*$_2$)-U2(*frr*$_0$) under ρ_1 and ρ_2

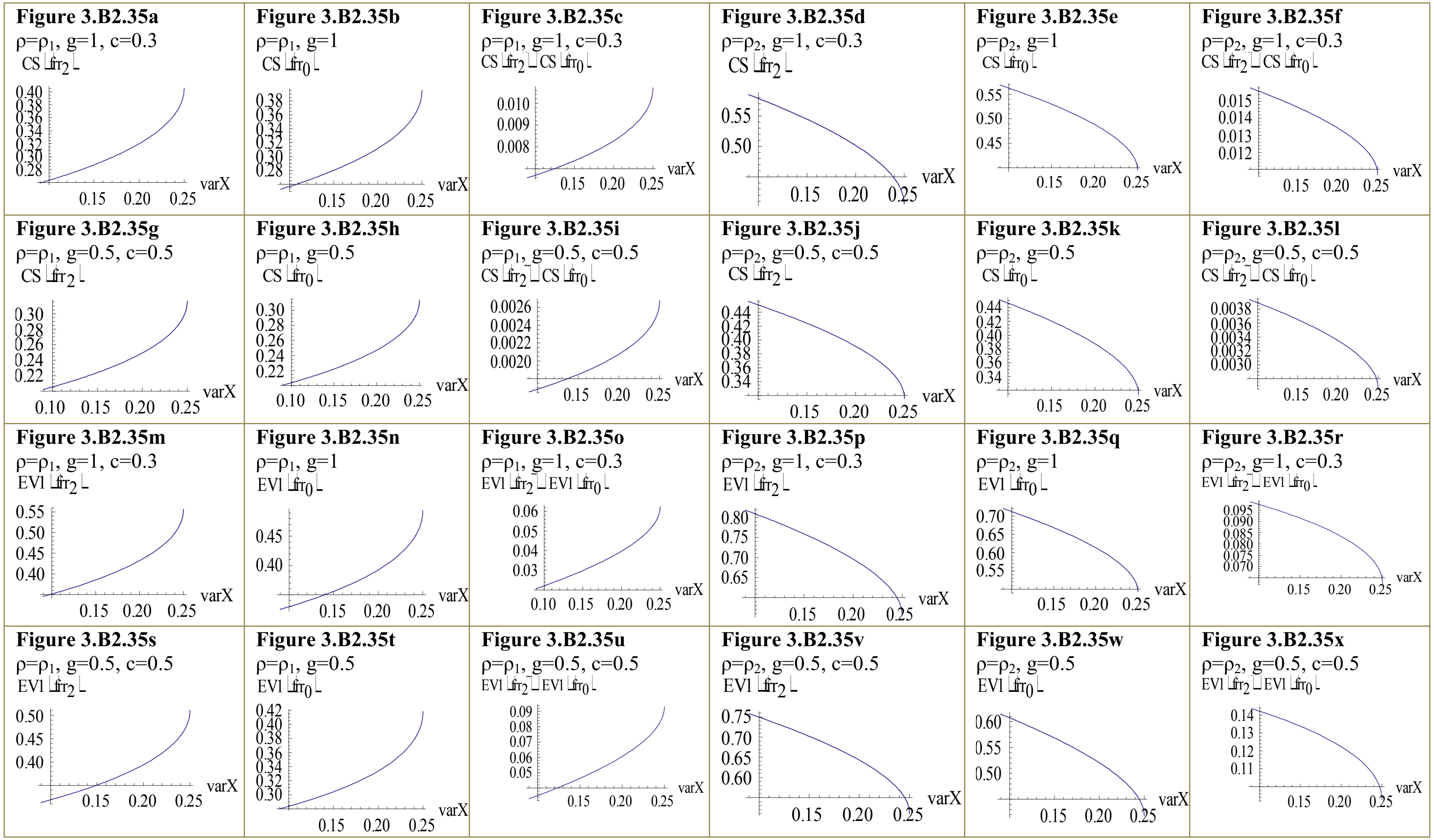

Figure 3.B2.35: How varX affects CS(*frr*$_2$) CS(*frr*$_0$) CS(*frr*$_2$)-CS(*frr*$_0$) EV1(*frr*$_2$) EV1(*frr*$_0$) EV1(*frr*$_2$)-EV1(*frr*$_0$) under ρ_1 and ρ_2

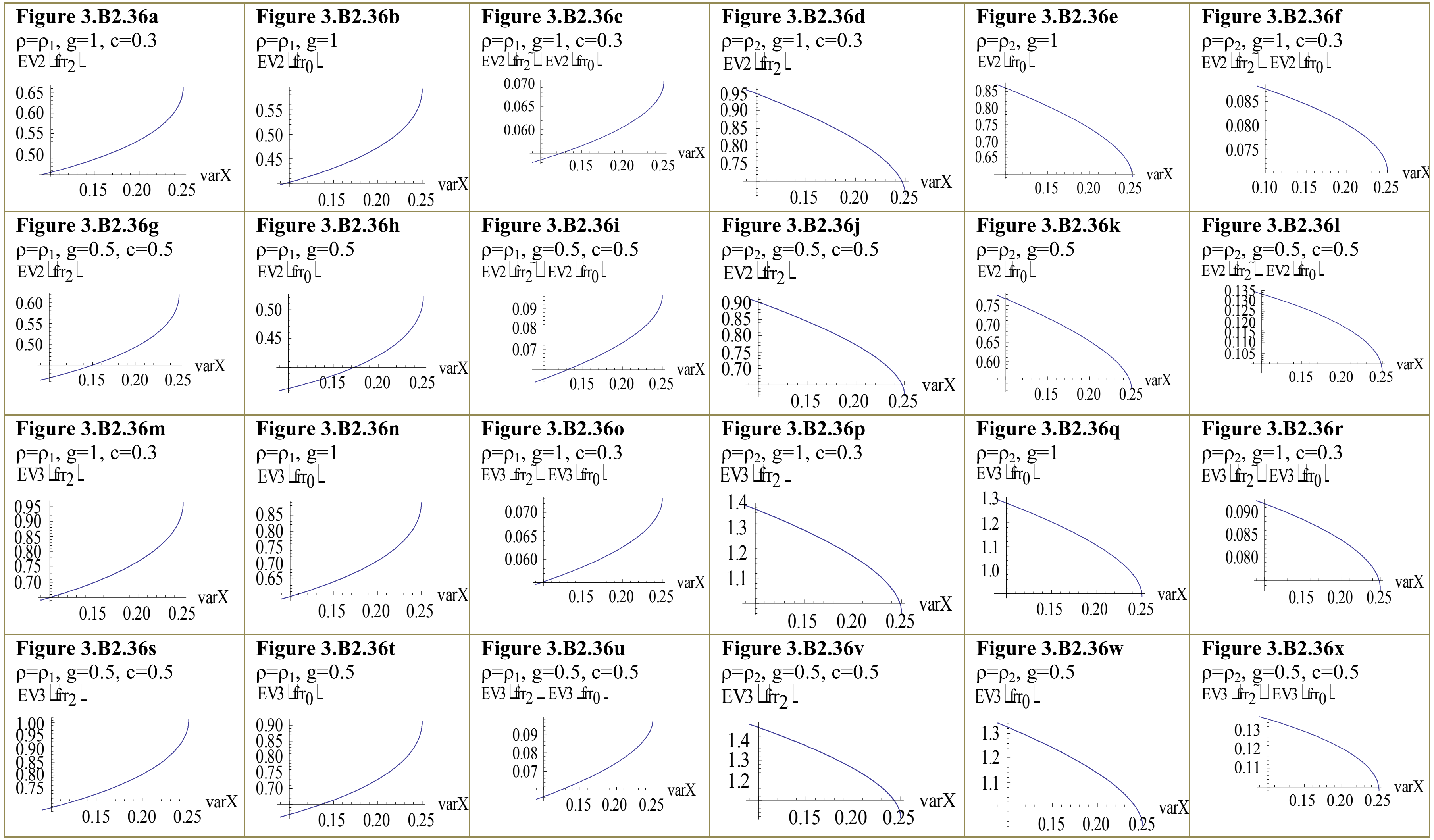

Figure 3.B2.36: How varX affects EV2(frr_2) EV2(frr_0) EV2(frr_2)-EV2(frr_0) EV3(frr_2) EV3(frr_0) EV3(frr_2)-EV3(frr_0) under ρ_1 and ρ_2

APPENDIX 3.B3

frr$_3$: The figures concern the optimal point

Figure 3.B3.1a |g=1
Contour Plot $w_{10}(frr_1)$-$w_{10}(frr_3)$

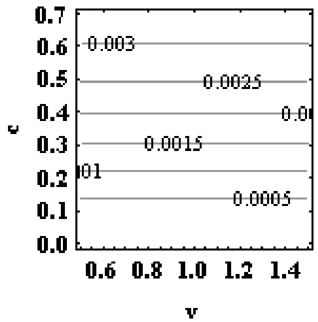

Figure 3.B3.1b |g=1
Contour Plot $w_{20}(frr_1)$-$w_{20}(frr_3)$

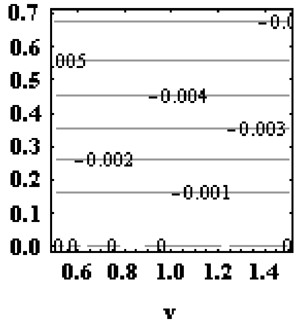

Figure 3.B3.1c |g=0.5
Contour Plot $w_{10}(frr_1)$-$w_{10}(frr_3)$

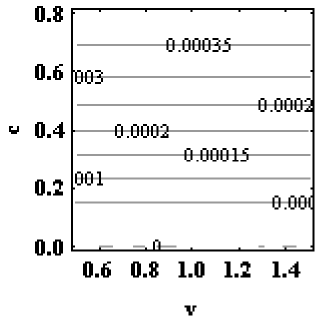

Figure 3.B3.1d |g=0.5
Contour Plot $w_{20}(frr_1)$-$w_{20}(frr_3)$

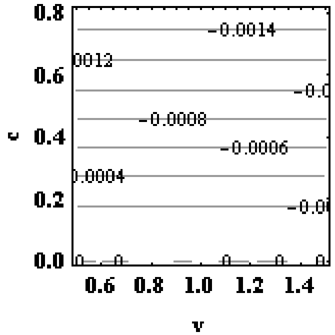

Figure 3.B3.1: $w_{10}(frr_1)>w_{10}(frr_3)$ and $w_{20}(frr_1)<w_{20}(frr_3)$

Figure 3.B3.2a |g=1
Contour Plot $q_{10}(frr_1)$-$q_{10}(frr_3)$

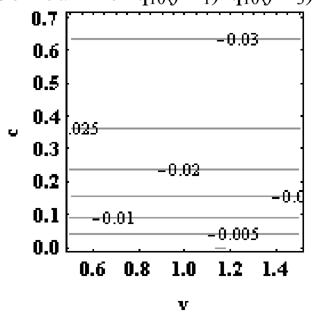

Figure 3.B3.2b |g=1
Contour Plot $q_{20}(frr_1)$-$q_{20}(frr_3)$

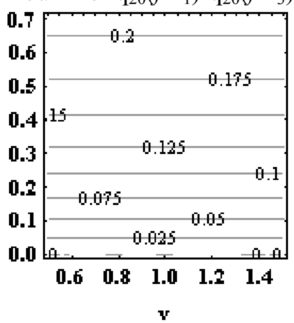

Figure 3.B3.2c |g=1
Contour Plot $q_{11}(frr_1)$-$q_{11}(frr_3)$

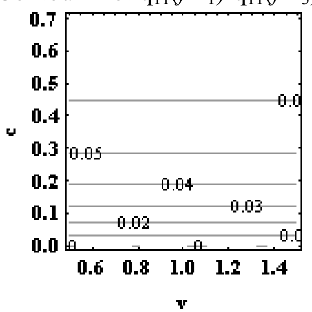

Figure 3.B3.2d |g=1
Contour Plot $q_{21}(frr_1)$-$q_{21}(frr_3)$

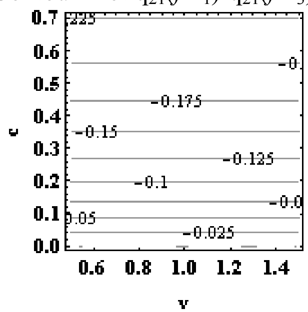

Figure 3.B3.2: $q_{10}(frr_1)<q_{10}(frr_3)$ $q_{20}(frr_1)>q_{20}(frr_3)$ and $q_{11}(frr_1)>q_{11}(frr_3)$ and $q_{21}(frr_1)<q_{21}(frr_3)$

Figure 3.B3.3a |g=0.5
Contour Plot $q_{10}(frr_1)$-$q_{10}(frr_3)$

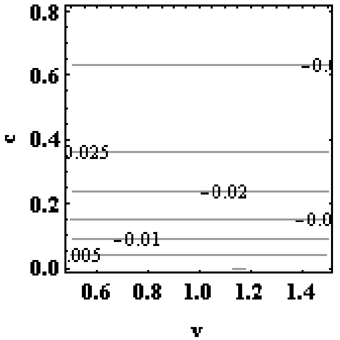

Figure 3.B3.3b |g=0.5
Contour Plot $q_{20}(frr_1)$-$q_{20}(frr_3)$

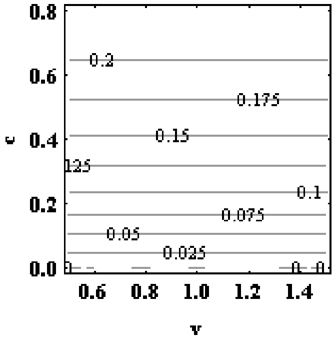

Figure 3.B3.3c |g=0.5
Contour Plot$q_{11}(frr_1)$-$q_{11}(frr_3)$

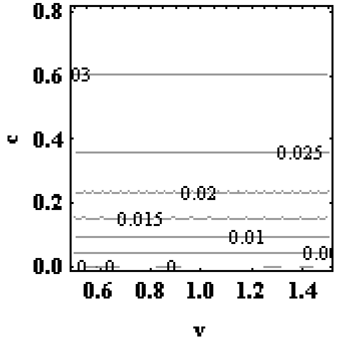

Figure 3.B3.3d |g=0.5
Contour Plot $q_{21}(frr_1)$-$q_{21}(frr_3)$

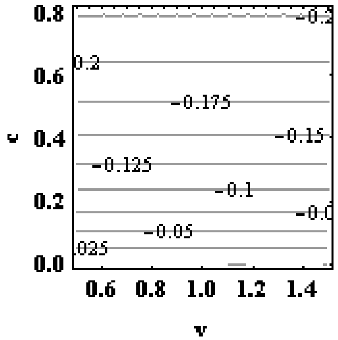

Figure 3.B3.3: $q_{10}(frr_1)<q_{10}(frr_3)$ $q_{20}(frr_1)>q_{20}(frr_3)$ and $q_{11}(frr_1)>q_{11}(frr_3)$ and $q_{21}(frr_1)<q_{21}(frr_3)$

Figure 3.B3.4a |, g=1
Contour Plot $q_{11}(frr_3)=q_{21}$, (frr_3)

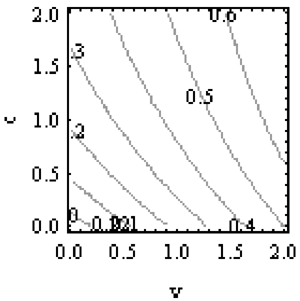

Figure 3.B3.4b |, g=1
Contour Plot $q_{10}(frr_3)=q_{20}(frr_3)$,

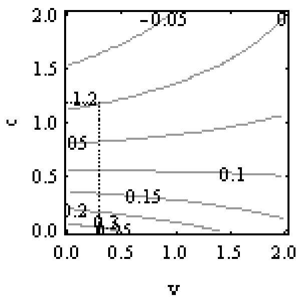

Figure 3.B3.4c |, g=1
Contour Plot $P_{10}(frr_3)=P_{20}(frr_3)$

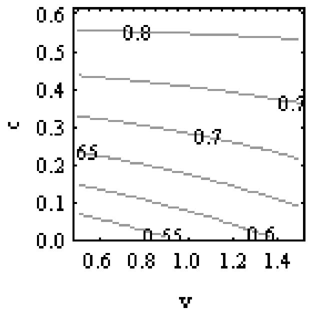

Figure 3.B3.4d |, g=1
Contour Plot $P_{11}(frr_3)=P_{21}(frr_3)$,

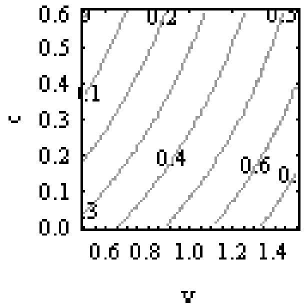

Figure 3.B3.4: Positive isoquants in each period and prices provided that $c \le 0.6$

Figure 3.B3.5a | g=0.5
Contour Plot $q_{11}(frr_3)=q_{21}(frr_3)$,

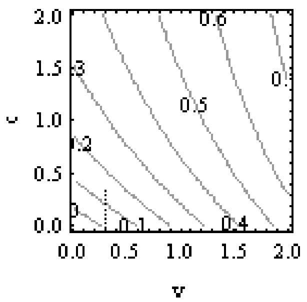

Figure 3.B3.5b | g=0.5
Contour Plot $q_{10}(frr_3)=q_{20}(frr_3)$,

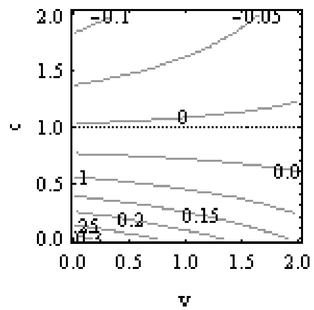

Figure 3.B3.5c | g=0.5
Contour Plot $P_{10}(frr_3)=P_{20}(frr_3)$

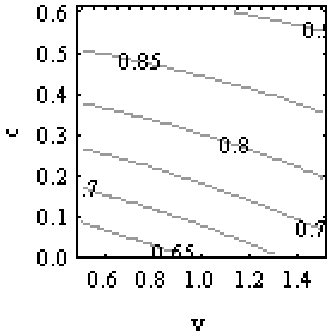

Figure 3.B3.5d | g=0.5
Contour Plot $P_{11}(frr_3)=P_{21}(frr_3)$

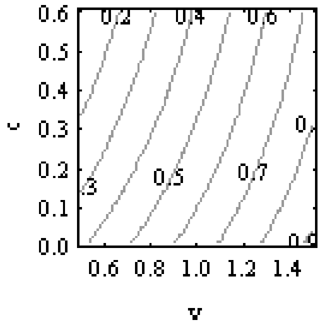

Figure 3.B3.5: Positive isoquants in each period and prices provided that $c \leq 0.6$ and $v \geq 0.4$

Figure 3.B3.6a | g=1
Contour Plot
$w_{10}(frr_3)=w_{20}(frr_3)$

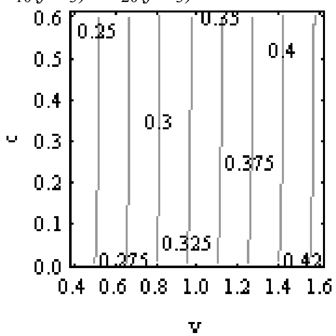

Figure 3.B3.6b | g=1
Contour Plot
$TC_1(frr_3)=TC_2(frr_3)$

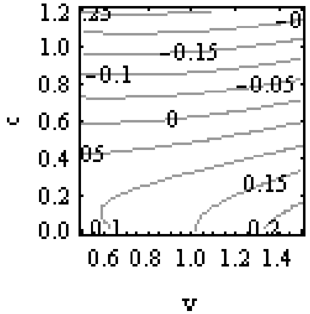

Figure 3.B3.6c | g=1
Contour Plot
$\Pi 1(frr_3)=\Pi 2(frr_3)$

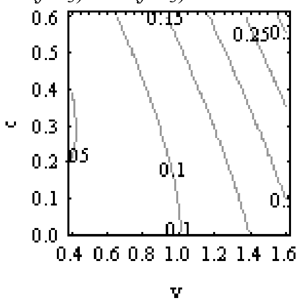

Figure 3.B3.6d | g=1
Contour Plot
$U1(frr_3)=U2(frr_3)$

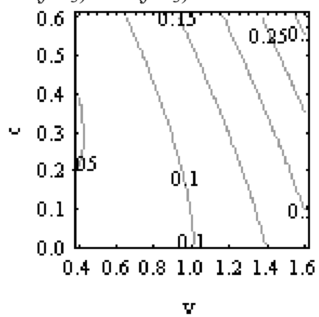

Figure 3.B3.6e | g=1
Contour Plot
$CS(frr_3)$

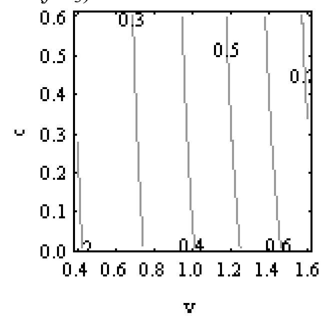

Figure 3.B3.6: Positive wages, total costs and yields provided that $c \leq 0.6$ and $v \geq 0.4$

Figure 3.B3.7a |g=0.5
Contour Plot
$w_{10}(frr_3)=w_{20}(frr_3)$

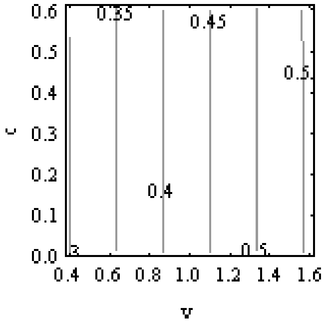

Figure 3.B3.7b | g=0.5
Contour Plot
$TC_1(frr_3)=TC_2\ (frr_3)$

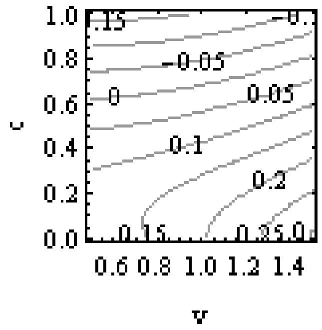

Figure 3.B3.7c | g=0.5
Contour
Plot$\Pi 1(frr_3)=\Pi 2(frr_3)$

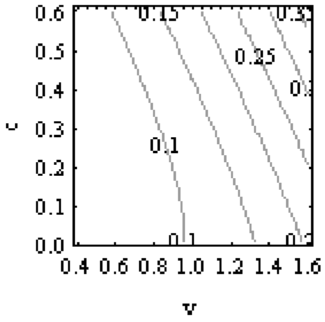

Figure 3.B3.7d | g=0.5
Contour Plot
$U1(frr_3)=U2(frr_3)$

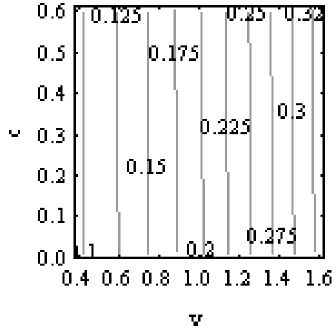

Figure 3.B3.7e | g=0.5
Contour Plot
CS (frr_3)

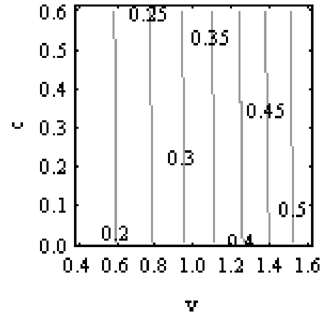

Figure 3.B3.7: Positive wages, total costs and yields provided that $c \le 0.6$

Figure 3.B3.8a| g=1
Contour Plot $(q_{11}-q_{10})\ (frr_3) = (q_{21}-q_{20})\ (frr_3)$

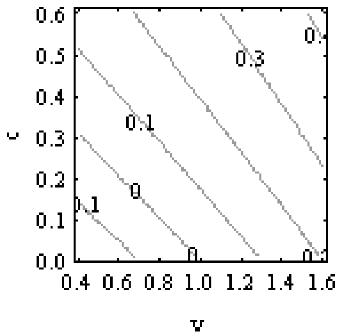

Figure 3.B3.8b | g=0.5
Contour Plot $(q_{11}-q_{10})\ (frr_3)= (q_{21}-q_{20})\ (frr_3)$

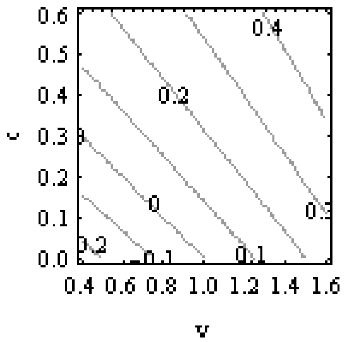

Figure 3.B3.8: $q_{11}>q_{10}$ $(q_{21} > q_{20})$ if v>1

Figure 3.B3.9a |g=1
Contour Plot
$q_{10}(frr_3)-q_{10}(frr_0)$

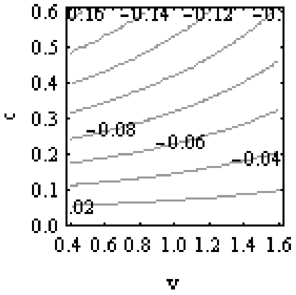

Figure 3.B3.9b |g=1
Contour Plot
$q_{11}(frr_3)-q_{11}(frr_0)$

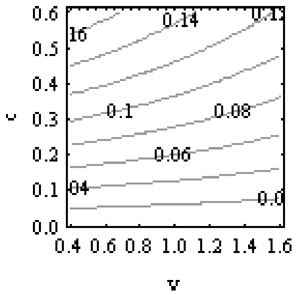

Figure 3.B3.9c |g=1
Contour Plot
$(q_{11}-q_{10})\ (frr_3)-(q_{11}-q_{10})\ (frr_0)$

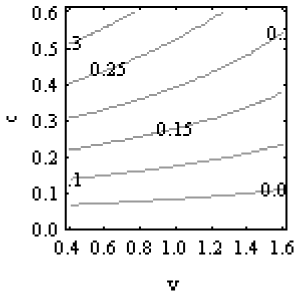

Figure 3.B3.9d |g=1
Contour Plot
$tq_1(frr_3)-tq_1(frr_0)$

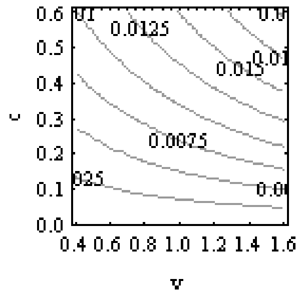

Figure 3.B3.9e |g=1
Contour Plot
$tq(frr_3)-tq(frr_0)$

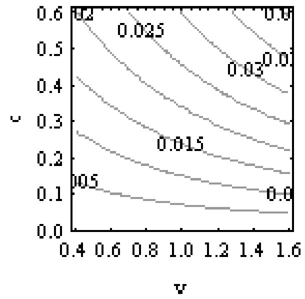

Figure 3.B3.9: $q_{10}=q_{20}$ and $q_{11}=q_{21}$; $q_{10}(frr_3)<q_{10}(frr_0)$ and $q_{11}(frr_3)>q_{11}(frr_0)$; $(q_{11}-q_{10})\ (frr_3)>(q_{11}-q_{10})\ (frr_0)\ (q_{21}-q_{20})$; $tq_1=q_{10}+q_{11}=tq_2$; $tq=tq_1+tq_2$; $tq_1(frr_3)>tq_1(frr_0)$ $tq(frr_3)>tq(frr_0)$

Figure 3.B3.10a |g=0.5
Contour Plot
$_{10}(frr_3)-q_{10}(frr_0)$

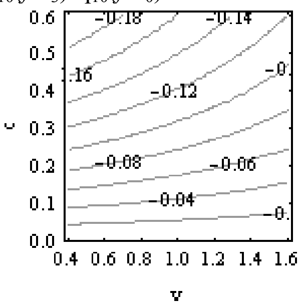

Figure 3.B3.10b |g=0.5
Contour Plot
$q_{11}(frr_3)-q_{11}(frr_0)$

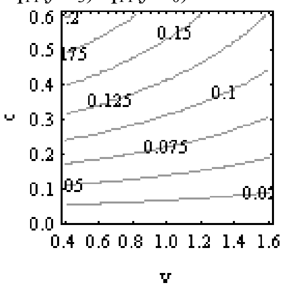

Figure 3.B3.10c |g=0.5
Contour Plot
$(q_{11}-q_{10})(frr_3)-(q_{11}-q_{10})\ (frr_0)$

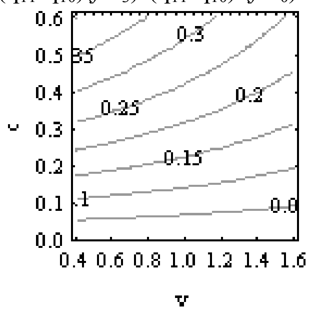

Figure 3.B3.10d |g=0.5
Contour Plot
$tq_1(frr_3)-tq_1(frr_0)$

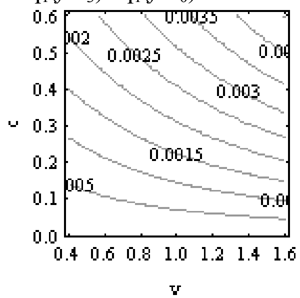

Figure 3.B3.10e |g=0.5
Contour Plot
$tq(frr_3)-tq(frr_0)$

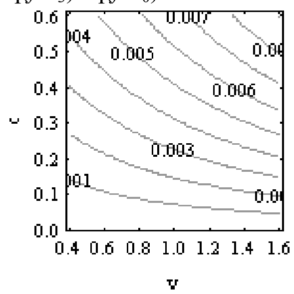

Figure 3.B3.10: $q_{10}=q_{20}$ and $q_{11}=q_{21}$; $q_{10}(frr_3)<q_{10}(frr_0)$ and $q_{11}(frr_3)>q_{11}(frr_0)$; $(q_{11}-q_{10})\ (frr_3)>(q_{11}-q_{10})\ (frr_0)\ (q_{21}-q_{20})$; $tq_1=q_{10}+q_{11}=tq_2$; $tq=tq_1+tq_2$; $tq_1(frr_3)>tq_1(frr_0)$ $tq(frr_3)>tq(frr_0)$

Figure 3.B3.11a |g=1
Contour Plot Π1(*frr*$_3$)-Π1(*frr*$_0$)

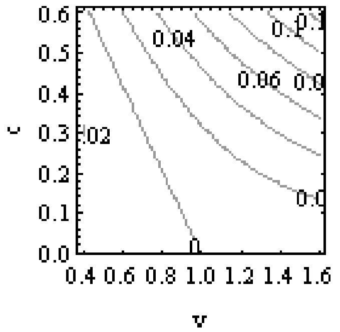

Figure 3.B3.11b |g=1
Contour Plot U1(*frr*$_3$)-U1(*frr*$_0$)

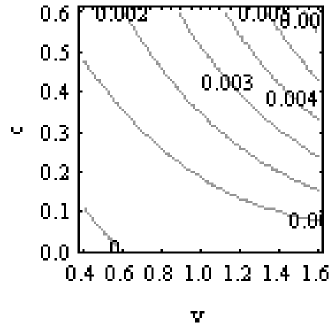

Figure 3.B3.11c |g=1
Contour Plot CS(*frr*$_3$)-CS(*frr*$_0$)

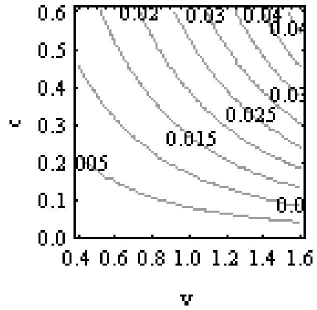

Figure 3.B3.11: Π1=Π2 and U1=U2; Π1(*frr*$_3$)>Π1(*frr*$_0$) if v>1; U1(*frr*$_3$)>U1(*frr*$_0$) and CS(*frr*$_3$)>CS(*frr*$_0$)

Figure 3.B3.12a |g=0.5
Contour Plot Π1(*frr*$_3$)-Π1(*frr*$_0$)

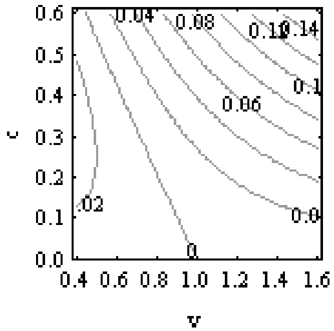

Figure 3.B3.12b |g=0.5
Contour Plot U1(*frr*$_3$)-U1(*frr*$_0$)

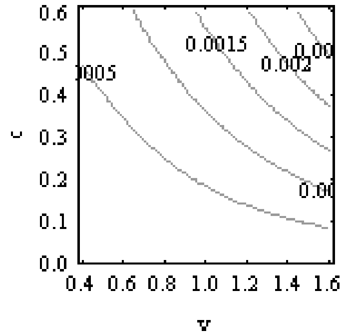

Figure 3.B3.12c |g=0.5
Contour Plot CS(*frr*$_3$)-CS(*frr*$_0$)

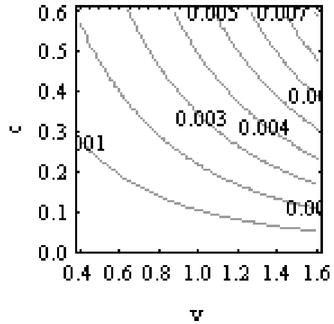

Figure 3.B3.12: Π1=Π2 and U1=U2; Π1(*frr*$_3$)>Π1(*frr*$_0$) if v>1; U1(*frr*$_3$)>U1(*frr*$_0$) and CS(*frr*$_3$)>CS(*frr*$_0$)

Figure 3.B3.13a |g=1
Contour Plot
$$\frac{d\Pi 1(frr_3)}{dc} = \frac{d\Pi 2(frr_3)}{dc}$$

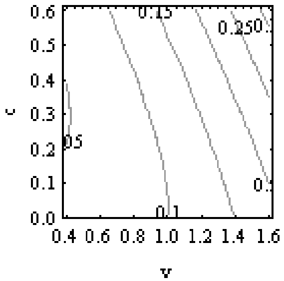

Figure 3.B3.13b |g=1
Contour Plot
$$\frac{dU1(frr_3)}{dc} = \frac{dU2(frr_3)}{dc}$$

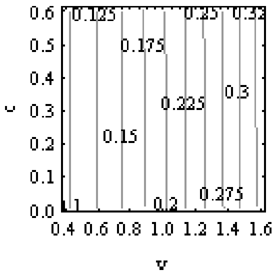

Figure 3.B3.13c |g=0.5
Contour Plot
$$\frac{d\Pi 1(frr_3)}{dc} = \frac{d\Pi 2(frr_3)}{dc}$$

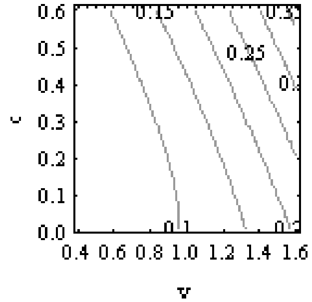

Figure 3.B3.13d |g=0.5
Contour Plot
$$\frac{dU1(frr_3)}{dc} = \frac{dU2(frr_3)}{dc}$$

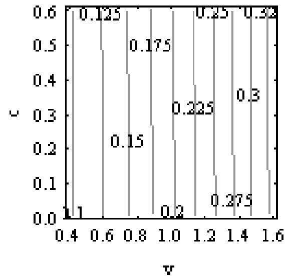

Figure 3.B3.13: The proposed regime receives the approval

APPENDIX 4.A1

Given the optimal product of the second period union-utilities are

$$U1(q_{10}, q_{20}, w_{10}, w_{11}, w_{21}) = q_{10}\, w_{10} + (w_{11}\, (c^2\, (1 + q_{10}) - (-2 + g)\, v + c\, (2 + 2\, q_{10} - g\, q_{20} + v - w_{11}) - 2\, w_{11} + g\, w_{21}))/((2 + c - g)\, (2 + c + g)) \qquad (4.A1.1)$$

$$U2(q_{10}, q_{20}, w_{20}, w_{11}, w_{21}) = q_{20}\, w_{20} + (w_{21}\, (c^2\, q_{20} - (-2 + g)\, v + g\, w_{11} - 2\, w_{21} - c\, (g + g\, q_{10} - 2\, q_{20} - v + w_{21})))/((2 + c - g)\, (2 + c + g)) \qquad (4.A1.2)$$

The result of the bargaining process about wages in the second period is

$$w_{11}^{*} = \frac{c\left(2\left(2+c\right)^2 - g^2\right)\left(1+q_{10}\right) - c\left(2+c\right)g q_{20} + \left(2+c-g\right)\left(4+2c+g\right)v}{4\left(2+c\right)^2 - g^2} \qquad (4.A1.3)$$

$$w_{21}^{*} = \frac{-c\left(2+c\right)g\left(1+q_{10}\right) + c\left(2\left(2+c\right)^2 - g^2\right)q_{20} + \left(2+c-g\right)\left(4+2c+g\right)v}{4\left(2+c\right)^2 - g^2} \qquad (4.A1.4)$$

Given optimal wages and products of the second periods, profits are

$$\Pi_1(q_{10}, q_{20}, w_{10}, w_{20})=1/(2\, (4\, (2 + c)^4 - 5\, (2 + c)^2\, g^2 + g^4)^2)\, (-4\, c^9\, (-1 + 3\, q_{10}\, (2 + q_{10})) - 2\, (g -2)^2\, (4 + g)^2\, ((g-4)^2\, (2 + g)^2\, q_{10}\, (-1 + q_{10} + g\, q_{20}) - 4\, v^2) + c^3\, (10g^6\, q_{10}\, (2 + q_{10}) - 528g^5\, q_{10}\, q_{20} + 64\, g\, (-4\, (3 + 227\, q_{10})\, q_{20} - 15\, (1 + q_{10} + q_{20})\, v - 10v^2) + 16\, g^3\, (v\, (3 + 3\, q_{10} + v) + q_{20}\, (4 + 804\, q10 +3\, v)) + g^4\, (12 -1308q_{10}^2 + v\, (12 + v) + 12q_{10}\, (-86 +v - 44w_{10})) - 448(188\, q_{10}^2 - (2 + v)\, (2 +5v) - 4\, q_{10}\, (2 + 3\, v - 32\, w_{10})) + 40\, g^2\, (552\, q_{10}^2 + 2\, (-4 + q_{20}^2) + 4\, (-4 + q_{20})\, v - 3\, v^2 - 16\, q_{10}\, (-9 + v - 20\, w_{10}))) - 2\, (64 - 20\, g^2 + g^4)^2\, q_{10}\, w_{10} + 2\, c^2\, (-792g^5\, q_{10}\, q_{20} + 10g^7\, q_{10}\, q_{20} + 8\, g^3\, (2\, (1 + 601q_{10})\, q_{20} + 4\, (1 + q_{10} + q_{20})\, v + 3\, v^2) - 32\, g\, (4\, q_{20}\, (1 + 225q_{10} + 3\, v) + 3\, v\, (4 + 4\, q_{10} + 5\, v)) + g^4\, (4 -1316\, q_{10}^2 + 3\, v\, (4 + v) + 4\, q_{10}\, (-64 + 3\, v - 198\, w_{10})) + 8\, g^2\, (-8 + 1672\, q_{10}^2 + 2\, q_{20}^2 + 10\, q_{20}\, v -5v\, (8+3v) - 8q_{10}\, (32+5v-150w_{10})) - 64\, (-4+ 572\, q_{10}^2 - 7v\, (4 + 3v) -4q_{10}\, (50 + 7v - 112\, w_{10})) + 10\, g^6\, q_{10}\, (3 + 3\, q_{10} + w_{10})) - 4\, c^8\, (g\, q_{20} - 2\, (7 + v) + q_{10}\, (92 + 58\, q_{10} + 9\, g\, q_{20} - 2\, v + 8\, w_{10})) + c^7\, (g^2\, (-4 + 36\, q_{10}\, (2 + q_{10}) + q_{20}^2) - 4\, g\, (4\, (3 + 35\, q_{10})\, q_{20} + (1 + q_{10} + q_{20})\, v) - 4\, (-84 + 492\, q_{10}^2 - v\, (28 + v) + 4\, q_{10}\, (150 - 7\, v + 32\, w_{10}))) + c^5\, (g^4\, (1 - 32\, q_{10}\, (2 + q_{10})) + 2\, g^3\, (v + q_{10}\, v + q_{20}\, (8 + 488q_{10} + v)) +16g\, (-3v\, (5 + 5q_{10} +v) - q_{20}\, (40 + 936q_{10} + 15v)) + g^2\, (3200q_{10}^2 + 40(-4 + q_{20}^2) +20\, (-4 + q_{20})\, v - 3\, v^2 - 80\, q_{10}\, (-44 + v - 12\, w_{10})) -112\, (-20 + 268\, q_{10}^2 - v\, (20 + 3v) + 4\, q_{10}\, (38 - 5v +32w_{10}))) - 2c^4\, (33\, g^5\, q_{10}\, q_{20} + 40\, g\, (4\, (3 + 115q_{10})\, q_{20} + 8\, (1 +q_{10} +q_{20})\, v + 3v^2) - g^3\, (v\, (8 +8\, q_{10} + v) + 8\, q_{20}\, (3 + 303\, q_{10} + v)) - 5\, g^2\, (1088\, q_{10}^2 + 8\, (-4 + q_{20}^2) + 8\, (-4 +q_{20})\, v - 3\, v^2 - 32q_{10}\, (-23 + v - 15w_{10})) + g^4\, (-3 - v +q_{10}\, (225 + 162\, q_{10} - v + 33\, w_{10})) + 112\, (-12 + 276q_{10}^2 - 5\, v\, (4 + v) + 4q_{10}\, (18 - 5v + 40w_{10}))) + 2c^6\, (-8\, g\, (15 + 239q_{10})\, q_{20} + g^3\, (q_{20} +41q_{10}\, q_{20}) - 24\, g\, (1 + q_{10} + q_{20})\, v - 2\, g\, v^2 - 28\, (172\, q_{10}^2 - (2 + v)\, (10 + v) + 4\, q_{10}\, (38 - 3\, v + 16\, w_{10})) + g^2\, (260\, q_{10}^2 - 4\, (5 + v) + q_{20}\, (5\, q_{20} + v) + q_{10}\, (400 - 4\, v + 40\, w_{10}))) - c\, (-2 + g)\, (4 + g)\, ((-4 +g)\, (2 +g)\, (576 - 100\, g^2 + g^4)\, q_{10}^2 - 4\, v\, (-8\, (4 + 7\, v) + g\, (8\, q_{20} + 10\, v + g\, (4 + 3\, v))) + 2\, q_{10}\, (64\, (24 + v - 32\, w_{10}) + g\, (-128\, (-3 +16\, q_{20} + 4\, w_{10}) + g\, (-8\, (44 + 64\, q_{20} + v - 72\, w_{10}) + g\, (g\, (12 + g\, (-2 + g - 40\, q_{20}) + 80\, q_{20} - 40\, w_{10}) + 8\, (-5 + 72\, q_{20} + 10\, w_{10}))))))) \qquad (4.A1.5)$$

$$\Pi_2(q_{10}, q_{20}, w_{10}, w_{20})=1/(2(4\, (2 + c)^4 -5\, (2 + c)^2\, g^2 + g^4)^2)\, (-12\, c^9\, q_{20}^2 + c^7\, (g^2\, ((1 + q_{10})^2 + 36q_{20}^2) - 4g\, (4\, (3 + 35\, q_{10})\, q_{20} + (1 + q_{10} + q_{20})\, v) + 4\, (-492\, q_{20}^2 + v^2 + 4\, q_{20}\, (32 + 7\, v - 32\, w_{20}))) +c^3\, (-528g^5\, q_{10}\, q_{20} + 10\, g^6\, q_{20}^2 - 64g\, (4\, (3 + 227\, q_{10})\, q_{20} + 15\, (1 +q_{10} + q_{20})\, v + 10\, v^2) + 16g^3\, (v\, (3 + 3q_{10} + v) + q_{20}\, (4 + 804\, q_{10} + 3\, v)) + g^4\, (-1308\, q_{20}^2 + v^2 + 12q_{20}\, (44 + v - 44\, w_{20})) - 448\, (188\, q_{20}^2 - 5v^2 - 4q_{20}\, (32 + 3\, v - 32w_{20})) + 40g^2\, (2 + 2\, q_{10}^2 + 552\, q_{20}^2 + (4 - 3\, v)\, v + 4\, q_{10}\, (1 + v) - 16\, q_{20}\, (20 + v -20\, w_{20}))) + c^5\, (-32\, g^4\, q_{20}^2 + 2\, g3\, (v + q_{10}\, v + q_{20}\, (8 + 488$$

$$q_{10} + v)) - 16 g (3 v (5 + 5 q_{10} + v) + q_{20} (40 + 936q_{10} + 15v)) - 112(268 q_{20}^2 - 3v^2 - 4 q_{20} (32 +5v - 32w_{20})) + g^2 (40 + 40q_{10}^2 +3200 q_{20}^2 +(20 - 3v) v + 20q_{10} (4 + v) - 80 q_{20} (12 + v - 12w_{20}))) -2 c^4 (33 g^5 q_{10} q_{20} + 40 g (4 (3 +115 q_{10}) q_{20} + 8 (1 + q_{10} + q_{20}) v + 3v^2) - g^3 (v (8 + 8 q_{10} + v) + 8 q_{20} (3 + 303 q_{10} + v)) - 5 g^2 (8 + 8q_{10}^2 +1088q_{20}^2 + (8 -3v) v + 8q_{10} (2 + v) - 32q_{20} (15 + v - 15 w_{20})) + 112 (276 q_{20}^2 - 5v^2 - 20q_{20} (8+v - 8 w_{20})) g^4 q_{20}+(162q_{20} - v + 33 (-1+w_{20}))) - 4c^8 q_{20} (-8 + g + 9gq_{10} + 58q_{20} - 2v + 8w_{20}) - 2 (-2 + g)^2 (4 + g)^2 (-4 v^2 +(g-4)^2 (2 + g)^2 q_{20} (-1+ gq_{10} +q_{20} + w_{20})) + 2c^2 (-792g^5 q_{10} q_{20} + 10g^7 q_{10} q_{20} + 8g^3 (2 (1+ 601 q_{10}) q_{20} +4 (1 +q_{10} + q_{20}) v + 3v^2) - 32g (4 q_{20} (1 + 225q_{10} + 3v) + 3 v (4 + 4 q_{10} + 5v)) + g^4 (-1316 q_{20}^2 + 3v^2 +12q_{20} (66 + v - 66 w_{20})) + 8g^2 (2 + 2 q_{10}^2 + 1672 q_{20}^2 + 5 (2 - 3v) v + 2 q_{10} (2 + 5v) - 40 q_{20} (30 + v - 30 w_{20})) - 64 (572 q_{20}^2 - 21 v^2 - 28 q_{20} (16 + v - 16 w_{20})) + 10 g^6 q_{20} (-1 + 3 q_{20} + w_{20})) + 2c^6 (g^3 (q_{20} + 41 q_{10} q_{20}) - 2 g (4 (15 + 239 q_{10}) q_{20} + 12 (1 + q_{10} + q_{20}) v + v^2) - 28 (172 q_{20}^2 - v^2 - 4 q_{20} (16 + 3 v - 16 w_{20})) + g^2 (5 + 5 q_{10}^2 + v + q_{10} (10 + v) + 4 q_{20} (-10 + 65 q_{20} - v + 10 w_{20}))) - c (-2 + g) (4 + g) ((-4 +g) (2 + g) (576 - 100g^2 + g^4) q_{20}^2 - 4v (8 g (1+ q_{10}) + (-56 + g (10 +3g)) v) - 16q_{20} (-8 (32 + v- 32w_{20}) + g (64 (-1 + 4 q_{10} + w_{20}) + g (72 + 64 q_{10} + v - 72 w_{20} + g (5 g (-1 + (-2 + g) q_{10} + w_{20}) - 2 (-5 + 36 q_{10} +5 w_{20}))))))) \quad (4.A1.6)$$

Maximizing the above profits *w.r.t* q_{10}, q_{20}, respectively accrue the optimal quantities of the first period Reactions functions are give n by (4.11) and (4.12) where

$$E=4(2+c)^7(4+c)(4+3c)-4(2+c)^5(40+c(40+9c))g^2+4(2+c)^3(33+c(33+8c))g^4-10(2+c)^3g^6+(2+c)g^8; \quad (4.A1.6a)$$

$$A=(4096+12288c+12800c^2+1792c^3-8064c^4-8512c^5-4256c^6-1200c^7-184c^8-12c^9-2560g^2-5120cg^2-2048c^2g^2+2880c^3g^2+3680c^4g^2+1760c^5g^2+400c^6g^2+36c^7g^2+528g^4+528cg^4-256c^2g^4-516c^3g^4-225c^4g^4-32c^5g^4-40g^6+30c^2g^6+10c^3g^6+g^8-cg^8+c(2+c)^3(2+c-g)(4+2c+g)(2(2+c)^2-g^2)v)/F>0; \quad (4.A1.6b)$$

$$D=\Big(\big(64-20g^2+g^4\big)^2-2c^8(g-2(4+v))-8c(-2+g)(4+g)(g(64+g(-72+5(-2+g)g-v))+8(32+v))+c^4\big(8g^3(3+v)+2240(8+v)-160g^2(15+v)+g^4(33+v)-160g(3+2v)\big)+c^6\big(g^3-24g(5+v)-4g^2(10+v)+112(16+3v)\big)+c^5\big(g^3(8+v)-40g^2(12+v)-40g(8+3v)+224(32+5v)\big)+c^7(-2g(12+v)+8(32+7v))+2c^3(448(32+3v)+g(-48(4+5v)+\big(g(4g(4+33g)+3g(4+g)v-160(20+v))))+2c^2\big(896(16+v)+g\big(-64(1+3v)+g\big(-160(30+v)+g\big(8-5g^3+16v+6g(66+v)\big)\big)\big)\big)\Big)/F>0; \quad (4.A1.6c)$$

$$q_{10}^*=(-36 c^{14} + 12 c^{13} (v - 4 (20 + w_{10})) + c^9 (128 (-10768 + 885 v - 5060 w_{10}) + g (g (-4592 v + 256 (859 + 186 w_{10}) + g (900 v + g (-5340 + 13 v - 320 w_{10}) - 5024 (-1 + w_{20}))) - 320 (99 v - 376 (-1 + w_{20})))) - 2 c^7 (-768 (-3428 + 581v - 4268w_{10}) + g (g (64 (-15545 + 700v - 9333 w_{10}) +g (g (86072 - 754 v + 23880 w_{10} + g (245 v + g (-1540 + v - 118w_{10}) - 2826 (-1 + w_{20}))) - 32 (525 v - 3698 (-1 + w_{20})))) + 2304 (77 v - 366 (-1 + w_{20})))) + c^{10} (128 (-3196 + 190 v - 979 w_{10}) + g (g (8(4799 - 62v + 579 w_{10}) + g (254 -307g +50v - 254 w_{20})) - 96 (55v - 191(-1+ w_{20})))) - 2 c^6 (-1536 (-1643 +508v -4356w_{10}) +g (g (896 (-1747 +122v -1899w_{10}) +g (g (8 (27777 -382v+14126w_{10}) +g (g (2 (-4190 +6 v - 835 w_{10}) + g (43g - 5v + 123 (-1 +w_{20}))) +294 (67 + 5v - 67w_{20}))) -1120 (45v-367 (w_{20}-1)))) + 10752 (33v-181 (w_{20} -1)))) +c^8 (1536 (-2125+243v-1562w_{10}) +g (g (128 (6347-196v+2286w_{10}) +g (g (-40380 + 234g^2 + 212v - 5872 w_{10} - 5 g (71 + 7 v - 71 w_{20})) + 32 (225 v - 1399 (-1 + w_{20})))) - 3840 (33v-139 (-1+ w_{20})))) + c^3 (4096 (2140+263v-4532w_{10}) + g (64g^3 (4260+391v-29036w_{10}) -$$

$$
\begin{aligned}
&1024g (3377 +314v - 9819w_{10}) + 15g^9 (5+ w_{10}) -4g^7 (871 + 867w_{10}) + 32g^5 (859- \\
&15v+4265w_{10}) + 1024 g^2 (225v - 3614(-1+w_{20})) + 32g^6 (25v - 1226 (-1 + w_{20})) - 3360 g^4 \\
&(186 +7v-186w_{20})- 61440(118 + 11 v - 118 w_{20}) - 6 g^8 (116 + v - 116 w_{20}))) - 4 c^{11} (21136 - \\
&876v +4112w_{10}+g (424+132v + g (-983 + 6 v - 51 w_{10}) - 424 w_{20})) + 4 c^{12} (45 g^2 + 4 (-727 \\
&+ 19v - 82w_{10}) - 6 g (3+v- 3w_{20})) + (64 - 20 g^2 + g^4)^3 (2-2w_{10} +g (-1 +w_{20})) - c^2 (g - 2) (4 + \\
&g) (4096 (209 + 12v - 311w_{10}) + g (-1024 (320 +21v+ 311w_{10}) +g (-256 (1887 +67v - \\
&2358w_{10}) + g (256 (720+29v +434w_{10}) +g (32 (2685+47v -3024w_{10}) +g (-16 (2093 + 41v + \\
&644w_{10}) + g (-8 (559 + 3v - 734w_{10}) + g (2048 - 48g - 17g^2 + g^3 + 12 v - 90 (-2 + g) \\
&w_{10}))))))) + (-4 + g) g (2 + g) (-67712 + 22096 g^2 - 1788 g^4 + 15g^6) w_{20}) - c (-4+ g)(-2 + g)^2 \\
&(2 + g) (4 + g)^2 (128 (44 + v - 52 w_{10}) + g (-64 (26 + v + 26w_{10} - 48w_{20}) + g (-16 (142 + v - \\
&122w_{10} - 48 w_{20}) + g (8 (83 + v + 35 w_{10} - 108 w_{20}) + g (-4 (-53 + 37 w_{10} + 30w_{20}) + g (- \\
&62+g - 2 w_{10} + g w_{10} + 60 w_{20})))))) - 2 c^5 (-1536 (-256 + 645 v - 6644 w_{10}) + g (3072 (165 v \\
&- 1076 (-1 +w_{20})) +g (256 (-5162 + 721v - 13482w_{10}) + g (-448 (225v-2188 (-1+w_{20})) + g (8 \\
&(42096- 965 v +42868 w_{10}) + g (28 (175v - 2804(-1+w_{20})) + g (30 (-789 + 2v- 337w_{10}) + g \\
&(-1474 + 389g - 50v +43g w_{10} + 1474 w_{20})))))))) + c^4 (12288 (447 + 145v - 1870w_{10}) + g (- \\
&92160 (89 + 11 v - 89w_{20}) +g (-1024 (233 + 416 v - 9738 w_{10}) + g (3584 (75 v - 907 (-1 + \\
&w_{20})) + g (64 (-7015 + 389 v -21630 w_{10}) +g (-560 (35 v - 699 (-1 + w_{20})) + g (40 (1703 - 8 \\
&v + 1697 w_{10}) + g (4 (100 v -3681 (-1 + w_{20})) + g (-12 (217 + 72 w_{10}) + g (-87 + 15 g - v + \\
&87 w_{20})))))))))))/D
\end{aligned}
\tag{4.A1.7}
$$

$$
\begin{aligned}
&q_{20}^* = (2 c^5 (g (-3072 (-94 + 165v - 1076 w_{10}) + g (g (448 (225v - 4 (285 + 547w_{10})) + g (g \\
&(28 (3836 - 175v + 2804 w_{10}) + g (g (-5688 + 50 v - 1474 w_{10} + 43 g (1 + g - w_{20})) - 30 (2 v \\
&-337(-1 + w_{20})))) + 8 (965 v - 42868 (-1 + w_{20})))) - 1792 (103v - 1926 (-1 + w_{20})))) + 1536 \\
&(645 - 6644 (-1+ w_{20}))) - c^3 (g (4096 (778 +165v - 1770w_{10}) +g (g (-1024 (1023 +225v - \\
&3614w_{10}) + g (g (224 (74+105v-2790w_{10}) + g (g (32 (587- 25v + 1226w_{10}) + 3 g (g (2 (- \\
&230+v -116w_{10}) + 5 g (1 + g - w_{20})) + 1156 (w_{20}- 1))) +160 (3 v - 853 (w_{20}- 1)))) - 1088 (23 \\
&v - 1708 (w_{20}- 1)))) + 1024 (314 v - 9819 (w_{20}-1)))) - 4096 (263 v - 4532 (-1 + w_{20}))) - 2 c^7 \\
&(g (768 (-974 + 231 v -1098 w_{10}) +g (g (-32 (-7572 + 525 v - 3698 w_{10}) + g (-754 v +g (245 \\
&v - 6 (2567 + 471 w_{10}) + g (118 + 118 g + v - 118 w_{20})) + 23880 (w_{20}- 1))) + 64 (700 v - \\
&9333 (w_{20}- 1)))) -768 (581v - 4268 (w_{20}-1))) + c^8 (g (-768 (-1039 + 165 v - 695 w_{10}) + g (g \\
&(32 (-4947 + 225v - 1399w_{10}) + g (212 v + g (4807 - 35 v + 355 w_{10}) - 5872 (w_{20} - 1))) - \\
&256 (98 v - 1143 (-1 + w_{20})))) +1536 (243v - 1562 (w_{20}-1))) + 2c^6 (g (-1536 (-555 + 231 v - \\
&1267w_{10}) + g (g (224 (-2091 + 225v - 1835 w_{10}) + g (g (53914 - 1470 v + g2 (-1301 + 5 v - \\
&123 w_{10}) +19698 w_{10} - 2 g (835 + 6 v - 835 w_{20})) + 16 (191 v - 7063 (w_{20}-1)))) - 896 (122 v \\
&- 1899 (w_{20}-1)))) + 6144 (127 v - 1089(-1 + w_{20}))) +c^9 (g (-320 (-896 + 99 v - 376 w_{10}) +g \\
&(g (-32544 + 900 v - 5024 w_{10} + g (320 +320 g + 13 v - 320 w_{20})) - 16 (287 v - 2976 (-1 + \\
&w_{20})))) +640 (177 v - 1012 (w_{20}-1))) + 2c^{10} (g (35152 - 2640 v + g^2 (-1935 + 25 v - 127 w_{10}) \\
&+ 9168 w_{10} - 4 g (579 + 62 v - 579 w_{20})) + 64 (190 v - 979 (-1 + w_{20}))) - c^4 (g (30720 (53 + \\
&33 v - 267 w_{10}) + g (g (-512 (-429 + 525 v - 6349w_{10}) + g (g (560 (-375 + 35v - 699w_{10}) + \\
&g (40 (8v-1697 (w_{20}-1)) + g (23708 - 400v + g^2 (-603 + v - 87w_{10}) + 14724 w_{10} + 864 g \\
&(w_{20}-1)))) - 64 (389 v - 21630 (w_{20} -1)))) + 2048 (208 v - 4869 (-1 + w_{20})))) - 61440 (29 v - \\
&374 (-1 + w_{20}))) - 8 c12 (g (-137 + 3 v - 9 w_{10}) -2 (82 + 19 v - 82 w_{20})) - 4 c11 (-876 v + g \\
&(4 (-710 + 33 v - 106 w_{10}) + 3 g (17 + 17g + 2v - 17w_{20})) + 4112 (w_{20}-1)) + 12c^{13} (4 + 4 g \\
&+v - 4w_{20}) + (64 - 20g^2 + g^4)^3 (2 +g (w_{10}-1) -2w_{20}) +c^2 (-2 + g) (4 + g) (-4096 (311 + 12 v) - \\
&g (-1024 (32 + 21 v - 529w_{10}) +g (-256 (2701 +67v-529 w_{10}) + g (256 (73 + 29v - 955w_{10}) \\
&+ g (32 (3695 + 47v - 1381w_{10}) + g (-16 (133+ 41v-2275w_{10}) + g (-8 (787 + 3v-447w_{10}) +g \\
&(-328 + 45g^2 +12v + 3 (-636 + 5 (g-2) g) w_{10}))))))) + 2 (g-4) (2 + g) (-79616 + 27776 g^2 - \\
&2576 g^4 + 45 g^6) w_{20}) + c (-4 + g) (g-2)^2 (2 + g) (4 + g)^2 (-128 (52 + v - 52w_{20}) + g (64 (14 + \\
&v - 48w_{10} + 26w_{20}) + g (16 (162 + v - 48w_{10} - 122w_{20}) + g (-8 (45 + v - 108 w_{10} + 35 w_{20}) + \\
&g (4 (-57 + 30 w_{10} + 37 w_{20}) + g (30 + g2 - 60 w_{10} + 2w_{20} - g (1 + w_{20})))))))))/D
\end{aligned}
\tag{4.A1.8}
$$

$$D = (4 (2 + c)^{10} (4 + c)^2 (4 + 3 c)^2 - 4 (2 + c)^8 (1216 + c (2432 + c (1768 + 552 c + 63 c^2))) g^2 + (2 + c)^6 (9408 + c (18816 + c (14000 + c (4592 + 561 c)))) g^4 - (2 + c)^6 (2372 + c (2372 + 589 c)) g^6 + 332 (2 + c)^6 g^8 - 102 (2 +c)^4 g^{10} + 16 (2 + c)^2 g^{12} - g^{14})$$

Given the optimal products of both periods as well as the optimal wages of the first period each union's utilities are

$$U1(w_{10}, w_{20}) = (((2 + c) (2 + c - g)^3 (2 + c + g)^3 (-4 (2 + c)^2 + g^2)^2 ((-4 + g) (-2 + g)^3 (2 + g)^2 (4 + g)^2 v + 6 c^8 (3 + v - w_{10}) + c^5 (32 (102 v - 65 (-3 +w_{10})) + g (g (-1002 - 256 v + 94w_{10} + g (19 +19 g + 10v - 19w_{20})) - 16 (45 + 26v - 45 w_{20}))) - 4 c^2 (-32 (84 + 111v - 28w_{10}) + g (g (4 (294 +329 v - 50 w_{10}) + g (g (-2 (85 + 63 v - 3 w_{10}) + g (8 + 9 v + 2 g (4 + v) - 8 w_{20})) - 2 (76 + 91 v -76 w_{20}))) + 16 (36 + 43 v - 36 w_{20}))) - 2 c^3 (-32 (218 v - 83 (-3 + w_{10})) + g (g (4 (657 + 436 v - 99w_{10}) + g (g (6 (-41 - 14v + w_{10}) + g (4 + 4g + 3v - 4 w_{20})) - 6 (38 + 31v - 38w_{20}))) + 240 (6 + 5 v - 6 w_{20}))) + c^6 (8 (222 + 97 v - 74 w_{10}) - 3 g^2 (57 + 7 v - 3 w_{10}) - 4 g (36 + 19 v - 36 w_{20})) + c^4 (-4 g^2 (771 + 324 v - 97w_{10}) + 160 (81 + 53 v - 27w_{10}) + g^4 (158 + 21v - 2 w_{10}) + 8 g^3 (19 +12v - 19 w_{20}) - 640 g (3 + 2 v - 3 w_{20})) - 2 c^7 (-52v + 46 (-3 + w_{10}) + 3 g (2 + 2 g + v - 2w_{20})) + c (g-2) (2+g) (4+g) (-64 (3 + 8v - w_{10}) + g (16 (6 + 15 v - w10 - 3 w_{20}) +g (4 (9 + 19v +3w_{20}) + g (g (-3 + g + v - w_{20}) + 4 (-4 - 9 v + w_{20}))))))^2)/D + w_{10} (-36 c^{14} + 12 c^{13} (v - 4 (20 + w_{10})) +c^9 (128(-10768 + 885v - 5060w_{10}) + g (g (-4592v + 256 (859 +186w_{10}) + g (900v + g (-5340 +13 v - 320 w_{10}) - 5024 (-1 + w_{20}))) - 320 (99 v - 376 (-1+ w_{20})))) - 2 c^7 (-768 (-3428 + 581v - 4268w_{10}) +g (g (64 (-15545 +700v -9333w_{10}) + g (g (86072 -754v + 23880w_{10} + g (245v + g (-1540 + v - 118 w_{10}) - 2826 (-1 + w_{20}))) - 32 (525 v - 3698 (-1 + w_{20})))) +2304 (77 v - 366 (-1 + w_{20})))) + c^{10} (128 (-3196 + 190 v - 979 w_{10}) + g (g (8 (4799 - 62 v + 579 w_{10}) + g (254 - 307g + 50 v - 254 w_{20})) -96 (55 v - 191 (-1 + w_{20})))) -2c^6 (-1536 (-1643 + 508v - 4356 w_{10}) + g (g (896 (-1747 + 122 v - 1899 w_{10}) +g (g (8 (27777 - 382 v + 14126 w_{10}) +g (g (2 (-4190 + 6 v - 835 w_{10}) + g (43 g - 5 v + 123 (-1 + w_{20}))) + 294 (67 + 5 v - 67 w_{20}))) - 1120 (45 v - 367 (w_{20} -1)))) + 10752 (33 v - 181 (-1 + w_{20})))) + c^8 (1536 (-2125 + 243 v - 1562 w_{10}) +g (g (128 (6347 - 196v + 2286 w_{10}) + g (g (-40380 + 234g^2 + 212v - 5872w_{10} - 5g (71 + 7v - 71w_{20})) + 32 (225v - 1399 (w_{20}-1)))) - 3840 (33v -139 (w_{20}-1)))) + c^3 (4096 (2140 + 263v - 4532w_{10}) + g (64g^3 (4260 + 391 v - 29036w_{10}) - 1024 g (3377 + 314 v - 9819 w_{10}) + 15g^9 (5 + w_{10}) - 4g^7 (871 + 867w_{10}) + 32 g^5 (859 -15v +4265w_{10}) + 1024 g^2 (225v -3614 (w_{20}-1)) + 32 g^6 (25v -1226 (-1 + w_{20})) - 3360 g^4 (186 + 7 v - 186 w_{20}) - 61440 (118 + 11 v - 118 w_{20}) - 6 g^8 (116 + v - 116 w_{20}))) - 4 c^{11} (21136 - 876 v + 4112 w_{10} + g (424 + 132 v + g (-983 + 6 v - 51 w_{10}) - 424 w_{20})) + 4 c^{12} (45 g^2 + 4 (-727 + 19 v - 82 w_{10}) - 6 g (3 + v - 3 w_{20})) + (64 - 20 g^2 + g^4)^3 (2 - 2 w_{10} + g (-1 +w_{20})) - c^2 (-2 + g) (4 + g) (4096 (209 + 12v - 311w_{10}) + g (-1024 (320 + 21 v + 311 w_{10}) + g (-256 (1887 + 67 v - 2358 w_{10}) + g (256 (720 + 29 v + 434 w_{10}) + g (32 (2685 + 47v - 3024 w_{10}) + g (-16 (2093 + 41v + 644w_{10}) + g (-8 (559 + 3v -734w_{10}) + g (2048 - 48g -17g^2 + g^3 + 12v - 90 (g-2) w_{10}))))))) + (g-4) g (2+g) (-67712 +22096g^2 - 1788g^4 + 15g^6) w_{20}) - c (-4 + g) (-2 + g)^2 (2 + g) (4 + g)^2 (128 (44 + v - 52 w_{10}) + g (-64 (26 + v + 26 w10 - 48 w_{20}) + g (-16 (142 + v - 122 w10 - 48 w_{20}) + g (8 (83 + v + 35 w10 - 108 w_{20}) + g (-4 (-53 + 37 w_{10} +30w_{20}) + g (-62 + g - 2w_{10} + gw_{10} + 60w_{20})))))) - 2 c^5 (-1536 (-256 + 645v - 6644w_{10}) + g (3072 (165 v - 1076 (-1 + w_{20})) + g (256 (-5162 + 721 v - 13482 w_{10}) + g (-448 (225 v - 2188 (-1 + w_{20})) + g (8 (42096 - 965 v + 42868 w_{10}) + g (28 (175 v - 2804 (-1 + w_{20})) + g (30 (-789 + 2v - 337w_{10}) +g (-1474+ 389g -50v +43g w_{10} + 1474w_{20})))))))) + c^4 (12288 (447+145v -1870w_{10}) + g (-92160 (89 + 11 v - 89 w_{20}) + g (-1024 (233 + 416 v - 9738 w_{10}) + g (3584 (75 v - 907 (w_{20}-1)) +g (64 (-7015 +389v -21630w_{10}) +g (-560(35v - 699(w_{20}-1)) +g (40 (1703 -8v +1697w_{10}) + g (4 (100 v - 3681 (-1 + w_{20})) + g (-12 (217 + 72 w_{10}) + g (-87 + 15 g - v + 87 w_{20}))))))))))))/D \tag{4.A1.9}$$

$U2 (w_{10}, w_{20})= (((2 + c) (2 + c - g)^3 (2 + c + g)^3 (-4 (2 + c)^2 +g^2)^2 ((-4 + g) (-2 + g)^3 (2 + g)^2$
$(4 + g)^2 v + c^6 (g (160 - 76 v + 144 w_{10} - 3g (3 + 3 g + 7 v - 3 w_{20})) + 8 (74 + 97v - 74 w_{20}))$
$+ c^4 (g (-160 (9 + 8 v - 12 w_{10}) + g (g (68 + 96 - 152 w_{10} + g (2 + 2g + 21 v - 2 w_{20})) - 4 (97$
$+ 324 - 97 w_{20}))) + 160 (27 + 53 v - 27w_{20})) + c (-2+g) (2 + g) (4 + g) (g^4 (3 +v - w_{10}) + 4g^3$
$(-3 - 9v +w_{10}) + 4 g^2 (-5 + 19 v + 3 w_{10}) - 64 (1 + 8 v - w_{20}) + 16 g (6 + 15 v - 3 w_{10} - w_{20}))$
$+ 2 c^7 (46 +52 v + g (28 - 3 v + 6 w_{10}) - 46 w_{20}) + 6 c^8 (1 + g + v - w_{20}) + c^5 (32 (65 + 102 v$
$- 65 w_{20}) + g(-16 (5 + 26 v - 45 w_{10}) + g (-94 - 256 v + g (-37 + 10 v - 19 w_{10}) + 94 w_{20}))) -$
$2 c^3 (-32 (83 +218 v - 83 w_{20}) +g (16 (104 + 75 v - 90 w_{10}) + g (4 (99 + 436 v - 99 w_{20}) + g$
$(-6 (48 + 31 v - 38 w_{10}) + g (-6 - 84 v + g (6 + 3 v - 4 w_{10}) + 6 w_{20}))))) - 4 c^2 (-32 (28 +$
$111v - 28 w_{20}) + g (16 (52 + 43 v - 36w_{10}) + g (4 (50 + 329 v - 50 w_{20}) + g (-2 (128 + 91v -$
$76w_{10}) + g (g (18 + (9 + 2 g) v - 8 w_{10}) + 6 (-1 - 21 v + w_{20})))))))^2)/D+ w_{20} (2 c^5 (g (-3072 (-$
$94 + 165v - 1076 w_{10}) + g (g (448 (225v - 4 (285 +547w_{10})) + g (g (28 (3836 - 175v + 2804$
$w_{10}) + g (g (-5688 + 50v - 1474 w_{10} + 43 g (1 + g - w_{20})) - 30 (2 v - 337 (-1 + w_{20})))) + 8$
$(965 v - 42868 (-1 + w_{20})))) - 1792 (103 v - 1926 (w_{20}-1)))) + 1536 (645 v - 6644 (-1 +$
$w_{20}))) - c^3 (g (4096 (778 + 165v - 1770 w_{10}) + g (g (-1024 (1023 + 225 v - 3614 w_{10}) + g (g$
$(224 (74 + 105 v - 2790 w_{10}) + g (g (32 (587 - 25v + 1226 w_{10}) + 3 g (g (2 (-230 + v-$
$116w_{10}) + 5 g (1 + g- w_{20})) + 1156 (-1 + w_{20}))) + 160 (3 v - 853 (-1 + w_{20})))) - 1088 (23 v -$
$1708 (-1 + w_{20})))) + 1024 (314 v - 9819 (-1 + w_{20})))) - 4096 (263 v - 4532 (-1 + w_{20}))) - 2 c^7$
$(g (768 (-974 + 231 v - 1098 w_{10}) + g(g (-32 (-7572 + 525 v - 3698 w_{10}) + g (-754 v + g$
$(245 v - 6 (2567 + 471 w_{10}) + g (118 + 118g + v - 118 w_{20})) + 23880 (-1 + w_{20}))) + 64 (700$
$v - 9333 (-1 + w_{20})))) - 768 (581 v - 4268 (w_{20}-1))) + c^8 (g (-768 (-1039 +165v - 695w_{10}) +$
$g (g (32 (-4947+225v - 1399w_{10}) + g (212 v + g (4807 - 35 v + 355 w_{10}) - 5872 (-1 + w_{20})))$
$- 256 (98 v - 1143 (-1 + w_{20})))) + 1536 (243v -1562 (w_{20}-1))) + 2 c^6 (g (-1536 (-555 + 231v$
$-1267w_{10}) + g (g (224 (-2091 + 225 v - 1835 w_{10}) + g (g (53914 - 1470 v + g^2 (-1301 + 5 v -$
$123 w_{10}) + 19698 w_{10} - 2 g (835 + 6 v - 835 w_{20})) + 16 (191 v - 7063 (-1 + w_{20})))) - 896$
$(122 v - 1899 (-1 + w_{20})))) + 6144 (127 v - 1089 (w_{20}-1))) + c^9 (g (-320 (-896 + 99 v -$
$376w_{10}) + g (g (-32544 + 900v - 5024 w_{10} + g (320 + 320 g + 13 v - 320 w_{20})) - 16 (287 v -$
$2976 (w_{20}-1)))) + 640 (177 v-1012 (-1 + w_{20}))) + 2 c^{10} (g (35152 - 2640 v + g^2 (-1935 + 25$ (4.A1.10)
$v - 127 w_{10}) + 9168 w_{10} - 4g (579 + 62v - 579w_{20})) + 64 (190 v - 979 (-1 + w_{20}))) - c^4 (g$
$(30720 (53 + 33 v - 267 w_{10}) + g (g (-512 (-429 + 525 v - 6349 w_{10}) + g (g (560 (-375+35 v$
$- 699 w_{10}) + g (40 (8 v - 1697 (-1 + w_{20})) + g (23708 - 400 v + g^2 (-603 + v - 87 w_{10}) +$
$14724 w_{10} + 864 g (-1 + w_{20})))) - 64 (389 v - 21630 (-1 + w_{20})))) + 2048 (208 v - 4869 (w_{20}-$
$1)))) - 61440 (29 v - 374 (-1 + w_{20}))) - 8 c^{12} (g (-137 +3v - 9w_{10}) - 2 (82 + 19v -82w_{20})) -$
$4c^{11} (-876v +g (4 (-710 + 33v - 106 w_{10}) + 3 g (17 + 17 g + 2 v - 17 w_{20})) + 4112 (-1+ w_{20}))$
$+ 12 c^{13} (4 + 4 g + v - 4w_{20}) + (64 - 20g^2 + g^4)^3 (2 + g (w_{20} - 1) - 2w_{20}) + c^2 (-2 + g) (4 + g)$
$(-4096 (311 + 12 v) - g (-1024 (32 + 21 v - 529 w_{10}) + g (-256 (2701 + 67v - 529 w_{10}) + g$
$(256 (73 + 29 v - 955 w_{10}) + g (32 (3695 + 47v - 1381w_{10}) + g (-16 (133+ 41v - 2275 w_{10}) +$
$g (-8 (787 + 3 v - 447w_{10}) + g (-328 + 45g^2 + 12 v + 3 (-636 + 5 (-2 + g) g) w_{10}))))))) + 2$
$(g-4) (2 + g) (-79616 + 27776g^2 - 2576 g^4 + 45 g^6) w_{20}) + c (g-4) (-2 + g)^2 (2 + g) (4 + g)^2 (-$
$128 (52 + v - 52 w_{20}) + g (64(14 +v- 48w_{10} +26w_{20}) + g (16 (162 +v- 48w_{10} -122w_{20}) + g (-$
$8 (45 +v-108w_{10} + 35w_{20}) + g (4 (-57 + 30 w_{10}+ 37 w_{20}) + g (30 + g^2 - 60 w_{10} + 2 w_{20} - g (1$
$+ w_{20})))))))))/D$

The bargaining process of the first period results in wages that follows

$w_{10} = (-20736 c^{37} + (-4 + g)^8 (g-2)^{10} (2 +g)^9 (4 + g)^9 - 2304 c^{36} (738 +5v) + 96 c^{35} (-699360$
$-36 g + 4149g^2 - 9104 v + 33g v) + c^2 (g-4)^5 (g- 2)^7 (2 + g)6 (4 + g)^6 (36421632 - 620544g$
$-22127104g^2 + 503296g^3 + 4642624g^4 - 138528g^5 - 383616 g^6 + 14872g^7 + 8632g^8 - 504g^9$
$+129 g^{10} +4 (g-2) (4+ g) (17920 +g (17664 + g (-736 + 3g (-960 + g (-92 + g (12 + g))))))v)$
$+ c^4 (g - 4)^3(g - 2)^4 (2 + g)^3 (4 + g)^4 (-2(g - 4)(2 + g)(-729241616384 + g (216570658816 +$
$g(707254452224 + g(-195485319168 + g(-276497457152 + g$

(70110240768+g(55261260288+g (-12659847424 + g (-5816892416 + g (1194946688 + g (279297440 + g (-54538624 + g(-1209136 +g (929200+g (-279920+ g(-3857+2986g))))))))))))))+(g-2)(-218783285248+g (-263576354816+g (87323574272+g (192340754432+g (9843245056+g (-52473102336+g (-9469530112 + g (6556427264 +g (1662022656 + g (-358526720 + g (-116974848 +g (5745856 + g (3167360 + g (42304 + g (-28176 + g (-888 + g (48 + g)))))))))))))))))v)-16c^{33}(6g (192304 +g (-11063720 + 3 g (-541+10655g))) - g (118168 + 3g (228412 + 801 g)) v +128 (15196112 + 368943 v)) + 32 c^{34} (-32 (1661598 + 31277 v) + 3 g (-4056 + 1444 v + 3g(102844 + 545 v))) - c $(-4 + g)^{6}$ (-2 + $g)^{9}$ $(2 + g)^{7}$ $(4 +g)^{8}$ (-64 (-136 + v) + g (-64 (-33 +v) +g (g (-284 + g (156 + (g-5) g)) - 8 (296 + v)))) - 16 c^{32} (1024 (26732841+ 787609 v) + g(64(512811+ 19358 v) + 3g (-16 (31861281 + 482141v) + g (-245212 + 4353893g - 66104v + 1854 g v)))) + 16 c^{31} (-2048 (150664475 + 5189932 v) + g (-64 (10119340 + 1249917v) + g (32 (795119486 + 15732681 v) + g (g (-428843848 + 3 g (-27789 + 274614 g - 9787 v) - 3611590v) + 8(3324280+947521v))))) + c^{24} (-31457280 (35352327728+2472344121v) +g(-17039360 (1492095105+ 406912624 v) +g (327680 (1354388289993+71481180551v)+ g (-g^{9} (4611927601 + 10158169v) + 8g^{8} (165904501 + 48253808v) + 176g^{7} (8473139823 + 68454239v) - 128g^{6} (2971188693+887062508v) - 256g^{5} (492715879502 + 7875420701v) + 81920(89701820679 + 26116772482v) + 256 g^{4} (105652326959 + 31898233798v) + 3072g^{3} (1377908781776 +35813837449v) - 2048g^{2} (345088242301+ 103522482160v) - 4096g (15745990890557+601560261315v))))) + 32c^{30} (-1024(1397891046 + 54941155 v) +g (-128 (37449850+6702701v) +g (32(5089889720+123884033v) +g (96 (3198552 + 943561v) +g (-4520308160 -56619828v+ g (-2790861 - 968858v + 3g (8595536+30411 v))))))) - 8c^{29} (8192 (5443817751 + 240067601v) + g (512 (436482156 + 94191595v) +g (-256 (26089262792+752523381v) +g (-64 (318635074+95832515v)+g (288 (954932422 +15858695 v) + g (88 (4091695 + 1395883 v) +g (9 g (-78055 + 504108 g - 27957 v) - 8 (388603043 + 2754194 v)))))))) -16 c^{28} (4096 (36116787382 + 1766348579 v) + g (2048(517213409+124294557v) +g (-512 (54999590115+1834586072v) +g (-128 (1014420009 +307600280 v) + g (96 (16745706923 + 346717870 v) + g (8 (465840019 + 155725236 v) +g (-4 (7508759484 + 80133275 v) + g (-21453225 - 7447874 v +g (130565685 + 392741 v))))))))) + 2 c^{27} (-65536 (103194777646 + 5551997085 v) + g (-4096 (16324500164 + 4183624351 v) + g (4096 (388904462305 + 14752897172 v) + g (28672 (368527603 + 111686845 v) + g (-3072 (39128324293 + 972776236 v) + g (-128 (3477754499 + 1136060575v) + g (320 (10448858840 + 149617109v) + g (5049100080+1696747408v + g (g (-7298598 + 34575093 g - 2450869 v) - 16 (1805356909 + 10991836 v)))))))))) + 2c^{26} (-12976128 (2587359438 + 152311757 v) + g (-16384 (27266424390 + 7254011947v) + g (4096 (2338910559532 + 99740636343 v) + g (24576 (3591463927 + 1080158515 v) + g (-1024 (900766508756 + 26233786031 v) + g (-256 (19954882429 + 6352842430 v) + g (3840 (9293083349 + 167839065 v) + g (32 (2977428837 + 967083232 v) + g (-16(31933053839 + 295716573 v) + g (-409083052 + 1825955040 g - 132274876 v + 4712359 g v)))))))))) + c^{21} (-5788139520 (5065071927 + 462350458 v) + g (-241172480 (5856087900 + 1537897831 v) + g (120586240 (147645855648 + 10567650181 v) + g (30146560 (21425851194 + 5738270525 v) + g (-3014656 (1406409278474 + 76701144045 v) + g (-1081344 (101750617953 + 27357631940 v) + g (180224 (2807154806586 + 112118023445 v) + g (2048 (4287638860854 + 1143027213725 v) + g (-1024 (31326119486064 + 865831990193 v) + g (4 g (263948321134784 + g (1450352837456 + g (-4062897257592 + g (-8649632657 + g (22593656333 + (7731634 – 19449617 g) g))))) + g (18531686134784 + g (1472402477984 + g (-156487807584 + g(-8427835924 + g (344602216 + 7135229 g))))) v - 256 (1321521491906 + 345550565659 v))))))))))) + c^{25} (-20447232 (14256111132 + 915315755 v) + g (-425984 (12020049756 + 3259385035v) + g (16384 (6046046627836 + 287556384755v) + g (12288 (100523325282 + 29824164775 v) + g (-4096

(2891473730184 + 97005659645 v) + g (-1024 (91559583403 + 28332842405 v) + g (9216 (65914773343 + 1445880294 v) + g (128 (20244319849 + 6345140530 v) + g (-64 (202856933532 + 2545561519 v) + g (-8 (2756004256 + 856937695 v) + g (92414265488 + 486806412 v + g (25818832 - 95539682 g + 7831143 v)))))))))))) + 2 c^{23} (-31457280 (59478848916 + 4532813665 v) + g (-34078720 (1617396552 + 439050151 v) + g (7536640 (115003773100 + 6717241371 v) + g (245760 (76470592132 + 21746324487 v) + g (-4096 (36738709140008 + 1586165907875 v) + g (-4096 (544535829186 + 157937381815 v) + g (3072 (4003289543488 + 121252183455 v) + g (256 (440166981117 + 127767845161 v) + g (-256 (1902439799113 + 37299294000 v) + g (-32 (73692341135 + 21090155258 v) + g (8613435065632 + 95087516016 v + g (16357461036 + g (-53157411064 + g (-16406912 + 49350207 g - 4386559 v) - 240676273 v) + 4554802192 v)))))))))))) + c^3 $(g-4)^4$ $(-2 + g)^6$ $(2 + g)^4$ $(4 + g)^5$ (-262144 (-92904 + 2377 v) + g (-32768 (-349436 + 23211 v) + g (16384 (-1154126 + 8343 v) + g (8192 (-1048205 + 51681 v) + g (2048 (2777913 + 27860 v) + g (512 (4745650 – 149633 v) + g (-256 (3274021 + 74537 v) + g (256 (-1242096 + 18281 v) + g (64 (976432 +25621 v) + g (32 (564874 - 603 v) + g (-32 (67691 + 1073 v) + g (-8 (27118 + 171 v) + g (26408 – 9696 g - 41 g^2 + 43 g^3 + 6 (24 + g) v))))))))))))) + 2 c^{22} (-118656860160 (46911208 + 3904487 v) + g (-120586240 (1741300788 + 466580245 v) + g (30146560 (98329294316 + 6352686433 v) + g (45219840 (1843729057 + 509878458 v) + g (-753664 (804403401096 + 39069420775 v) + g (-327680 (36487156553 + 10203727600 v) + g (4096 (14742523491719 + 514562401485 v) + g (512 (1503190615947 + 418628445448 v) + g (-256 (11981581744427 + 281183337911 v) + g (-64 (349796604455 + 95786969266 v) + g (32 (2368696063123 + 33593549230 v) + g (8 (32164814001 + 8559249412 v) + g (-778438957380 - 5444561848 v + g (-770682478 + 2165517816 g – 196875802 v + 3991407 g v)))))))))))))) + c^{18} (-23152558080 (11041279522 + 1400273571 v) + g (-91645542400 (286389510 + 68312951 v) + g (4582277120 (48557788500 + 4893526127 v) + g (18329108480 (950176269 + 225565105 v) + g (-458227712 (173632636924 + 13618170075 v) + g (-54788096 (84064230581 + 19715441270 v) + g (4980736 (3044692495521 + 180642120515 v) + g (622592 (1002783506051 + 230791590208 v) + g (-311296 (5373632132485 + 232006010339 v) + g (-4096 (11341886069615 + 2544528330161 v) + g (1024 (105989701331716 + 3155642544309 v) + g (1024 (1841093711379 + 399663763490 v) + g (-128 (31437296741938 + 597208575817 v) + g (2 g (39305889215192 + g (172184068156 + g (-339662207084 + g (-423511983 + 838627726 g)))) + g (843996031264 + g (67976145024 + g (-3296807804 + g (-156178896 + 1907981 g)))) v - 96 (405174587707 + 84350667688 v))))))))))))))) + c^{14} (-32212254720 (19760825922 +5685409261 v) + g (-30870077440 (8248575654 + 1602271727 v) + g (7717519360 (116792072264 +24744146379 v) + g (23152558080 (10984255378 + 2086257207 v) + g (-192937984 (2773592468032 + 435091318615 v) + g (-39845888 (2688439402479 + 498022262350 v) + g(119537664 (1471406790742 + 169788847743 v) + g (169345024 (146659235289 + 26423200088 v) + g (-84672512 (418474063141 + 34901325261 v) + g (-1114112 (3114572107003 + 543833714428 v) + g (557056 (8187820455833 + 479253103689 v) + g (8192 (36717271751515 + 6183708871768 v) + g (-2048 (184842766235560 + 7268096543333 v) + g (3 g^9 (75057545 + 36106 v) - 2 g^8 (75308205 +8895778 v) - 8 g^7 (11487311299 + 30828825 v) + 144 g^6 (463955011 + 61145954 v) + 80 g^5 (140316229347 + 991900793 v) - 128 g^4 (67563883929 + 9686695336 v) - 640 g^3 (992784787445 + 14142497627 v) + 768 g^2 (659615259883 + 100995977832 v) + 256 g (77827949717683 + 1929763904304 v) - 512 (31362140008159 + 5060235810380 v)))))))))))))))) + 4 c^{20} (-1447034880 (11807086158 + 1189728331 v) + g (-482344960 (2181192453 + 557894195 v) + g (120586240 (97473416891 + 7759089858 v) + g (633077760 (867216353 + 224017588 v) + g (-1507328 (2141721142661 + 131246678960 v) + g (-16384 (6684944365207 + 1722916521020 v) + g (24576

(18517252046720 + 843278940933 v) + g (10240 (1035589007241 + 263615946754 v) +
g (-12288 (2869291095047 + 92434831812 v) + g (-256 (2039404965247 + 508046702300
v) + g (64 (23310613471087 + 494441955889 v) + g (32 (389722559909 + 94130315322
v) - g (8 (4019098796750 + 50115728659 v) + g (123447555022 + 28609788464 v + g (-
298082199424 - 1767665097 v + g (-329919691 – 72450435 v + g (770137915 + 1146501
v)))))))))))))))) + c^{19} (-11576279040 (12164082790 + 1365854957 v) + g (-241172480
(46191776124 + 11440785001 v) + g (241172480 (452664976235 + 40339684624 v) + g
(1447034880 (4546529109 + 1128203774 v) + g (-24117248 (1422856480377 +
98315006665 v) + g (-393216 (3862643139449 + 951264370485 v) + g (65536
(86217434851616 + 4475077480509 v) + g (16384 (10698357171583 + 2593378357801
v) + g (-16384 (32098660013885 + 1198217409439 v) +g (-1024 (10475205804078 +
2479890492173 v) + g (2560 (10877515303278 + 274589375891 v) +g (256
(1328288404282 + 304566985267 v) + g (-64 (12517036289266 + 194855634185 v) + g (-
16 (314337881523 + 69140331790 v) + g (11153228544656 + 91721922392 v + g
(26796901380 + 5586951756 v + g (g (-22024809 +47390079 g - 4286047 v) - 4
(14425642031 + 44707099 v)))))))))))))))))))) + c^{13} (-64424509440 (7190137751 +
3266330431 v) + g (-30870077440 (10859864772 + 1974054641 v) + g (77175193600
(10287593896 + 3113675675 v) + g (81033953280 (4523929150 + 801747269 v) + g (-
385875968 (1436900962578 + 302452790125 v) + g (-79691776 (2144033206403 +
369784694515 v) + g (79691776 (2662274880013 + 395544739860 v) + g (169345024
(262189524969 + 43910123557 v) + g (-5249695744 (9478117684 + 990350941 v) + g (-
4456448 (1585688963435 + 257093436451 v) + g (1114112 (6744267962954
+486257495339 v) + g (32768 (21746891481781 + 3398697105973 v) + g (-
4096(181335734868222 + 8714269221307 v) + g (-3072 (14823499000833 +
2219011275338 v) + g (8192 (5822995538353 + 176797991877 v) + g (768
(2333359432991 + 331513960518 v) + g (2 g (-20358796923072 + g (22975109796064 +
g (235566050000 + g (-284950053424 + g (-1060820560 + g (1408323322 + (793726 -
1187885 g) g)))))) + g (-5419420916224 + g (419874169952 + g (57686389808 + g (-
2182358576 + g (-233074564 + g (2895960 +151139 g)))))) v - 256 (7531531617407 +
133389353357 v)))))))))))))))))) + 2 c^{11} (-12884901888 (-5479905546 + 7390438705 v) + g
(-17448304640 (11513670300 + 1792406009 v) + g (1342177280 (11966535683 +
96071072124 v) + g (277830696960 (937437731 + 141640523 v) + g (-1543503872
(59398065589 + 48855256450 v) + g (-8388608 (17470798292503 + 2559941680835 v) +
g (12582912 (5288949947704 + 1986106571735 v) + g (19922944 (2358973254501 +
334770121175 v) + g (-537919488 (45129424297 + 9626393624 v) + g (-1114112
(8512255569646 + 1167413790869 v) + g (1114112 (4820371831357 + 626886925498 v)
+ g (196608 (6382758267543 + 842919414886 v) + g (-8192 (93963831775246 +
7560008815051 v) + g (-26624 (4135147819139 + 522887237236 v) + g (79872
(914551743727 + 44639931472 v) + g (13312 (475573922257 + 57089439627 v) + g (-
1280 (3566544122924 + 100482364125 v) + g (2 g (91144364644992 + g
(2482267826880 + g (-2182281869792 + g (-27884725016 + g (27719361800 + g
(124954360 + g (-143545316 + 3 g (-31870 + 43509 g)))))))) + g (2719089596608 + g
(514690030624 - g (29845369344 + g (5196655776 + g (-133560604 + g (-20225786 + g
(129106 + 12797 g))))))) v - 384 (600153131474 + 67546075817 v))))))))))))))))))) + c^{17} (-
416746045440 (975429352 + 142451577 v) + g (-18329108480 (3005728668 +
686783683 v) + g (9164554240 (43262984276 +5004539595 v) + g (6873415680
(5938022722 + 1342787143 v) + g (-916455424 (175258314451 +15772261105 v) + g (-
49807360 (246060114389 + 54754999355 v) + g (29884416 (1177644461539
+80405928840 v) + g (1245184 (1547924505411 + 337122110494 v) + g (-622592
(7311380004044 + 365897057049 v) + g (-4096 (41820019011800 + 8863323921749 v) +
g (10240 (34764269633052 + 1216074360317 v) + g (512 (16903311345616 +

3463282463289 v) + g (-256 (64702994940942 + 1479541596575 v) + g (-448 (529628592163 + 104039508908 v) + g (192 (2255350111972 + 30512489589 v) + g (400 (7855308431 + 1463488452 v) + g (-8 (703986730518 + 4774812473 v) + g (-4 (3855229353 + 671390032 v) + g (27897065388 + 66515262 v + g (11974169 - 22460603 g + 1911657 v)))))))))))))))))))))) + 2 c^{12} (-83751862272 (1030470678 + 1274979995 v) + g (-69793218560(2799693273 + 472861841 v) + g (1342177280 (176976293079 + 99108417076 v) + g (7717519360 (30198625347 + 4960594960 v) + g (-385875968 (560856523487 + 184114591770 v) + g (-62914560 (1901210999377 + 303354950540 v) + g (2097152 (48243392228748 + 10161899102815 v) + g (9961472 (3475863729663 + 537787655182 v) + g (-69730304 (405820630749 + 56683126265 v) + g (-4456448 (1397310713945 + 209099973622 v) + g (1114112 (4549153777403 + 422341025495 v) + g (32768 (22054317095083 + 3179502612040 v) + g (-4096 (145953859073676 + 8829331938193 v) + g (-13312 (4090691717985 + 564771881552 v) + g (6656 (6995706750773 + 264631105632 v) + g (16640 (157097016921 + 20587921580 v) + g (-128 (18343429328976 + 405867858461 v) + g (-128 (595639853133 + 73158106172 v) + g (2 g (616675556944 + g (-643918741544 + g (-4623161856 + g (5365671137 + (10367534 - 13710387 g) g)))) + g (139451794784 + g (-6670632288 + g (-938862016 + g (17814170 + (1826778 - 5645 g) g)))) v + 96 (762019513461 + 8885953919 v)))))))))))))))))))))) - c^5 $(-4 + g)^2$ $(-2 + g)^3$ $(2 + g)^2$ $(4 + g)^3$ (-134217728 (-15836496 + 831425 v) + g (-16777216 (8921508 + 4577389 v) + g (4194304 (-673743404 + 31341113 v) + g (48234496 (4605826 + 1819353 v) + g (-524288 (-3050618650 + 122327881 v) + g (-524288 (269156353 + 79828971 v) + g (65536 (-7641717482 + 254938975 v) + g (32768 (1528415682 + 329576945 v) + g (-8192 (-11484149468 + 304006343 v) + g (-12288 (884779436 + 132972923 v) + g (2048 (-5206789017 + 102321164 v) + g (1024 (1449223729 + 143442078 v) +g (676399284736 – 8893981184 v + g (-256 (491489765 + 29656256 v) + g (256 (-65605379 + 449027 v) + g (384 (16339032 + 541541 v) + g (64 (-6847821 + 40622 v) + g (-16 (10134418 + 163039 v) + g (32399432 - 65264 v + g (92 (17341 + 120 v) + g (-451990 + g (-2477 + 853 g - 6 v) + 252 v))))))))))))))))))))))) + c^{16} (-370440929280 (1513364661 + 261919802 v) + g (-7717519360 (13353983055 + 2907072218 v) + g (1929379840 (317838689503 + 43149752495 v) + g (27493662720 (3082913313 + 661080634 v) + g (-458227712 (613100406741 + 64375834345 v) + g (-338690048 (84672029087 + 17806203880 v) + g (508035072 (138663940216 + 11023751055 v) + g (42336256 (122304154421 + 25113112610 v) + g (-42336256 (250368106416 + 14622499027 v) + g (-1114112 (486021834184 + 96968038711 v) + g (139264 (7083844938413 + 291477888027 v) + g (4096 (8133241763139 + 1567393642342 v) + g (-512 (110247990060553 + 3014292879400 v) + g (-128 (9145353721643 + 1689319436288 v) + g (64 (29806390092177 + 497575857770 v) + g (96 (225472401901 + 39509306304 v) + g (-80 (436295268706 + 3898436259 v) + g (-8 (22082859251 + 3619391708 v) + g (4 (72356026378 + 272240591 v) + g (410784330 + 61782772 v - g (701860037 + 559504 v)))))))))))))))))))))))) + c^{15} (-740881858560 (889085795 + 190777257 v) + g (-7717519360 (22253180724 + 4589665859 v) + g (3858759680 (210633992586 + 34727833577 v) + g (18329108480 (8511478032 + 1722295291 v) + g (-916455424 (460397140964 + 57875760405 v) + g (-338690048 (174413357557 + 34509661615 v) + g (1016070144 (118992879610 + 11225101729 v) + g (423362560 (28661284211 + 5525599363 v) + g (-84672512 (248704452343 + 17170447000 v) + g (-557056 (2647097703459 + 495192630985 v) + g (278528 (8297613906096 + 404342896637 v) + g (4096 (26487975535201 + 4782594744774 v) + g (-2048 (78189063128645 + 2555217800691 v) + g (-256 (18560976989077 + 3211469207556 v) + g (768 (8877309482047 + 180850298337 v) + g (384 (304321884273 + 49958916200 v) + g (-32 (5233589913588 + 59581163119 v) + g (-16 (89232323480 + 13709608807 v) + g (8 (261973409852 + 1390106057 v) + g (6628268136 + 935346020 v

+ g (g (-4987533 + 8293839 g - 630121 v) - 8 (1277470213 + 2151999 v)))))))))))))))))))))
- 2 c^{6} (-4 + g) $(-2 + g)^{2}$ (2 + g) $(4 + g)^{2}$ (-2147483648 (-70916525 + 5128671 v) + g (-1879048192 (8393915 + 3933048 v) + g (67108864 (-3384428701 + 223056964 v) + g (16777216 (1542357139 + 584023086 v) + g (-4194304 (-35014874253 + 2077291298 v) +g (-69206016 (268008621 + 80609413 v) + g (524288 (-102609914547 + 5408665600 v) + g (262144 (29091398155 + 6807521702 v) + g (-327680 (-37261237781 + 1724320334 v) + g (-32768 (60582103135 + 10755615171 v) + g (24576 (-71354852021 + 2883563964 v) + g (16384 (20787756615 + 2715287744 v) + g (-2048 (-74844856165 + 2697989282 v) + g (-2048 (18970146433 + 1755442888 v) + g (1536 (-4403638057 + 164458484 v) + g (256 (11324922041 + 708715307 v) + g (-128 (174364634 + 45100329 v) + g (-128 (1057140831 + 42275980 v) + g (64 (275557085 + 447253 v) + g (3608318576 + 86230864 v + g (-775533848 + 944120 v + g (-8 (5709323 + 75455 v) + g (12592928 - 10244 v + g (175820 - 57023 g + 4 (289 + 3 g) v)))))))))))))))))))))))))) - c^{10} (51539607552 (-7190497310 + 2876339819 v) + g (27917287424 (12983759538 + 1845832921 v) + g (-536870912 (-702311163196 + 405094682905 v) + g (-16106127360 (31613358975 + 4354404107 v) + g (134217728 (-826052020044 + 1033556398675 v) + g (1107296256 (282569569833 + 37693073750 v) + g (-369098752 (56246180285 + 136965158109 v) + g (-20971520 (5274183459749 + 680697420896 v) g (39845888 (606242525905 + 291987251723 v) + g (524288 (47466750916959 + 5916163880695 v) + g (-11141120 (715794705384 + 158409799291 v) + g (-262144 (14206942953747 + 1704402014750 v) + g (32768 (45555225415130 + 5457545957087 v) + g (57344 (6562283206175 + 753714170488 v) + g (-12288 (14479999538195 + 977666436916 v) + g (-159744 (159835808663 + 17429956480 v) + g (22528 (617338962871 + 23097377693 v) + g (11264 (100715927845 + 10300807162 v) + g (-256 (2760783154629 + 53777384969 v) + g (-5632 (5572640227 + 525299410 v) + g (128 (175207286161 + 1577645209 v) + g (192 (2568340221 + 217764836 v) + g (-32 (12758803092 + 42560039 v) + g (-128 (28749429 + 2119166 v) + g (8 (448764095 + 333826 v) + g (8441408 + 515064 v + g (-9942199 + 111 v)))))))))))))))))))))))))))))) + c^{9} (-103079215104 (-4399570964 + 968837529 v) + g (-2147483648 (132736813044 + 17044876009 v) + g (1073741824 (-569686479084 + 147377889565 v) + g (1342177280 (322299712338 + 40040574991 v) + g (-268435456 (-1258546619956 + 408650148105 v) + g (-67108864 (4310237147599 + 517992618145 v) + g (67108864 (-1368480465069 + 652228576274 v) + g (8388608 (13377424580501 + 1554125209642 v) + g (-4194304 (-2064145939652 + 2653617814427 v) + g (-524288 (53396778585616 + 5987218955551 v) + g (262144 (8162103303172 + 7196352373731 v) + g (65536 (71783985715952 + 7744824159343 v) + g (-65536 (13606753030424 + 3305357334649 v) + g (-16384 (33103040479399 + 3418899701340 v) + g (65536 (2343102879070 + 256241117627 v) + g (159744 (268377610839 + 26319868528 v) + g (-4096 (3862120144767 + 209964851863 v) + g (-22528 (101342573410 + 9324312651 v) + g (512 (2031324753266 + 54373839575 v) + g (2816 (28007702966 + 2376258999 v) + g (-256 (169575843116 + 2054929167 v) + g (-256 (6448580486 + 492451547 v) + g (448 (2440312414 + 11132007 v) + g (160 (115410657 + 7668430 v) - g (32 (463276882 + 515487 v) + g (84664688 + 4661048 v + g (-83892940 + g (-68456 + 86276 g - 2895 v) + 1274 v)))))))))))))))))))))))))))))) + 2 c^{7} (-2 + g) (4 + g) (17179869184 (-1016678808 + 101919943 v) + g (6442450944 (414530100 +177558463 v) + g (-1073741824 (-26878806350 + 2519330361 v) + g (-134217728 (35890195304 + 12899199387 v) + g (67108864 (-312198196536 + 27296769521 v) + g (33554432 (114950938915 + 34236593117 v) + g (-8388608 (-1043897550307 + 85203699587 v) + g (-4194304 (430742733608 + 104804481715 v) + g (1048576 (-2207200826731 + 169426571626 v) + g (1048576 (523702195812 + 102398989405 v) + g (-917504 (-433296401281 + 32006741277 v) + g (-491520 (231065162572 + 35627829753 v) + g (16384 (-2633702796417 + 199629880435 v) + g (8192 (1989368976001 + 236689857741 v) + g (-

4096 (-617360183507 + 59382322145 v) + g (-2048 (792775513641 + 70987718302 v) + g (1024 (-4328709313 + 11408472093 v) + g (512 (215834721167 + 14133308804 v) + g (-768 (14888095485 + 438936449 v) + g (-128 (38718401501 + 1793213745 v) + g (256 (3469465011 + 19204553 v) + g (64 (2140815217 + 67394671 v) + g (-64 (502696223 + 236701 v) + g (-32 (64455356 + 1312335 v) + g (572739208 - 308552 v + g (2 (6495082 + 79851v) + g (g (-14769 + 5181g - 98v) + 22 (-185447+ 72v)))))))))))))))))))))))))))) + 4 c^{8} (-103079215104 (-974217588 + 139665073 v) + g (-4294967296 (11278155447 + 1290884996 v) + g (1073741824 (-146069569885 + 22882292937 v) + g (805306368 (98349411103 + 10877932858 v) + g (-67108864 (-1578963815333 + 274906279225 v) + g (-33554432 (1711779640633 + 182977886080 v) + g (50331648 (-795973804288 + 159510668925 v) + g (4194304 (5806971396003 + 599635709134 v) + g (-29360128 (-307018928762 + 76464600759 v) + g (-2097152 (3196479869815 + 318418736038 v) + g (262144 (-4238210236143 + 1612459471829 v) + g (131072 (9589729178693 + 918945501348 v) + g (-16384 (-1561998725259 + 3332440325143 v) + g (-8192 (20018646692281 + 1836202961408 v) + g (12288 (1260235990057 + 393703892874 v) + g (2048 (7294201869569 + 635376358224 v) + g (-1024 (2830811844912 + 282394324745 v) + g (-2048 (458566701957 + 37485838147 v) + g (2304 (117670738353 + 4879671401 v) + g (2816 (14068509125 + 1060959753 v) + g (-256 (59848967911 + 1039076370 v) + g (-256 (4161456372 + 282643643 v) + g (96 (5514603677 + 35144753 v) + g (112 (148859409 + 8803378 v) + g (-336 (31283496 + 50323 v) + g (-56 (2273793 + 111508 v) + g (102429807 - 1281 v + g (308712 + 11639 v +g (-323933 + 50 v))))))))))))))))))))))))))))))))))/G (4.A1.11)

w_{20} = ((-4 + g)^{8} (-2 + g)^{10} (2 + g)^{9} (4 + g)^{9} + 288 c^{36} (32 + 23 g - 40 v) + 96 c^{35} (8704 - 9104 v + g (5464 + 33 v)) - 16 c^{34} (64 (-35104 + 31277 v) + g (9 g (1676 + 815 g - 1090 v) - 8 (155957 + 1083 v))) + 16 c^{33} (61364224 - 47224704 v + g (8 (3803396 + 14771 v) + 3 g (-393680 - 175862 g + 228412 v + 801 g v))) - c (-4 + g)^{6} (-2 + g)^{9} (2 + g)^{7} (4 + g)^{8} (-64 (-144 + v) + g (-64 (-39 + v) + g (g (-372 + g (168 + g)) - 8 (308 + v)))) + 16 c^{32} (1024 (1171831 - 787609 v) + g (128 (4178306 - 9679 v) + 3 g (g (-6092812 + g (45389 + 18002 g - 18541 v) + 66104 v) + 16 (-925429 + 482141 v)))) + 16 c^{30} (-2048 (-104485724 + 54941155 v) + g (256 (304840711 - 6702701 v) + g (64 (-290399402 + 123884033 v) + g (-9 g (-36132272 + g (-12559334 + g (71264 + 25049 g))) + 2 g (-56619828 + g (-968858 + 91233 g)) v + 32 (-203611247 + 5661366 v))))) + 2 c^{23} (-31457280 (-17492801032 + 4532813665 v) + g (-2621440 (-37610989802 + 5707651963 v) +g (7536640 (-28884257584 + 6717241371 v) + g (4 g (7817103873294336 +g (1595533810743296 + g (-506157740601344 + g (-110653271032384 + g (14864222533952 + g (3474615433944 + g (-173086090672 + g (-43121763919 + g (526994571 + 139349941 g))))))))) - g (6496935558656000 + g (646911515914240 + g (-372486707573760 + g (-32708568361216 +g (9548619264000 + g (674884968256 + g (-95087516016 + g (-4554802192 + g (240676273 + 4386559 g))))))))) v + 737280 (-56319306028 + 7248774829 v))))) - 16 c^{31} (8192 (-2190601 + 1297483 v) + g (-7223632384 + 79994688 v + g (96 (11149128 - 5244227 v) + g (406364032 - 7580168 v + g (-9586256 + 3611590 v + g (-3560639 + 29361 v)))))) + c^{25} (-20447232 (-2930420848 + 915315755 v) + g (-425984 (-31387249484 + 3259385035 v) + g (16384 (-1049190221924 + 287556384755 v) + g (4096 (-970130988094 + 89472494325 v) + g (-8 g (-50121207920512 + g (7981139982080 + g (2036221863568 + g (-113627200064 +g (-30590014704 + g (403103642 + 114423339 g)))))) + g (-29012830622720 + g (13325232789504 + g (812177987840 + g (-162915937216 + g (-6855501560 + 486806412 g + 7831143 g^{2}))))) v - 4096 (-404208458852 + 97005659645 v)))))) + 8 c^{29} (-8192 (-511120784 + 240067601 v) + g (-2560 (-541373732 + 18838319 v) + g (256 (-1937621180 + 752523381 v) + g (64 (-2508643842 + 95832515 v) + g (14164309376 - 4567304160 v + g (88 (52292811 -

1395883 v) + g (-82395632 + 22033552 v + g (-27308207 + 251613 v)))))))) + 2 c^{27} (-
65536 (-14600691584 + 5551997085 v) + g (-4096 (-63725149608 + 4183624351 v) + g
(16384 (-11354706966 + 3688224293 v) + g (28672 (-1784019893 + 111686845 v) + g (-
2048 (-5255270695 + 1459164354 v) + g (3036969663872 - 145415753600 v + g (64 (-
3165678808 + 748085545 v) + g (-59281959280 + g (883662064 + g (269509578 -
2450869 v) - 175869376 v) + 1696747408 v)))))))) + 2 c^{28} (-32768 (-4188589956 +
1766348579 v) + g (-4096 (-10072022177 + 497178228 v) + g (4096 (-5172134921 +
1834586072 v) + g (5120 (-1235424243 + 61520056 v) + g (890460063488 -
266279324160 v + g (269923061056 - 9966415104 v + g (32 (-318769222 + 80133275 v)
+ g (g (15005960 + 4847729 g - 3141928 v) + 8 (-397590859 + 7447874 v))))))))) + c^{18} (-
439898603520 (-444939008 + 73698609 v) + g (-91645542400 (-177399487 + 68312951
v) + g (32075939840 (-4506039142 + 699075161 v) + g (11455692800 (-1236859317 +
360904168 v) + g (-458227712 (-94607050702 + 13618170075 v) + g (-383516672 (-
12835644337 + 2816491610 v) + g (4980736 (-1370413913011 + 180642120515 v) + g (-2
g (-304406624512303104 + g (-44267128873852928 + g (15479186017389568 + g
(2498007413649664 + g (-428467971596032 + g (-75883574439184 + g (5748610252328
+ g (1106179927644 + g (-28690348024 + g (-5944166659 +g (22741049 + 5031256
g))))))))))) + g (-72222542994489344 + g (-10422388040339456 + g (3231377965372416
+ g (409255693813760 + g (-76442697704576 + g (-8097664098048 + g (843996031264 +
g (67976145024 + g (-3296807804 + g (-156178896 + 1907981 g)))))))))) v + 622592 (-
1418821283783 + 230791590208 v))))))))) + c^{21} (-11576279040 (-1068786736 +
231175229 v) + g (-241172480 (-7130298164 + 1537897831 v) + g (120586240 (-
53406657596 + 10567650181 v) + g (30146560 (-32602845066 + 5738270525 v) + g (-
3014656 (-426543562532 + 76701144045 v) + g (-163840 (-1313257059185 +
180560370804 v) + g (180224 (-693551200028 + 112118023445 v) + g (2048 (-
11151822348886 + 1143027213725 v) + g (-4 g (-307450339832448 + g
(37381608402752 + g (8008598989328 + g (-375007940544 + g (-86188133901 + 2 g
(509643877 + 124868811 g)))))) + 1024 (6051713851168 - 865831990193 v) + g (-
88460944808704 + g (18531686134784 + g (1472402477984 + g (-156487807584 + g (-
8427835924 + g (344602216 + 7135229 g)))))) v))))))))) - c^2 $(-4 + g)^5$ $(-2 + g)^7$ $(2 + g)^6$ $(4 +
g)^6$ (114688 (-359 + 5 v) + g (2048 (-829 + 206 v) + g (-3584 (-7043 + 66 v) + g (256
(3396 - 613 v) + g (64 (-84901 + 268 v) + g (32 (-4513 + 465 v) + g (484864 + 912 v + g
(9224 - 168 v + g (g (-216 + g (41 + g)) - 12 (1272 + v)))))))))) + c^{26} (-2359296 (-
4868763408 + 1675429327 v) + g (-32768 (-86622216983 + 7254011947 v) + g (8192 (-
334582653826 + 99740636343 v) + g (12288 (-56333304799 + 4320634060 v) + g (2048
(101661728238 - 26233786031 v) + g (512 (106498599567 - 6352842430 v) + g (512 (-
11334907277 + 2517585975 v) + g (64 (-24820929801 + 967083232 v) + g (50075814880
- 9462930336 v + g (14374366384 - 264549752 v + g (-59725468 - 17987689 g + 9424718
v)))))))))))) +c^{24} (-408944640 (-669146282 + 190180317 v) + g (-17039360 (-3221290499 +
406912624 v) + g (327680 (-283280661937 + 71481180551 v) + g (81920 (-
238643785191 + 26116772482 v) + g (-4096 (-2694121927721 + 601560261315 v) + g
(2048 (1201305547183 - 103522482160 v) + g (1024 (-548144103850 + 107441512347 v)
+ g (256 (-519145944953 + 31898233798 v) + g (-256 (-46389662684 + 7875420701 v) +
g (64 (46659808491 - 1774125016 v) + g (g (-22278081680 + g (85381513 + 24040706 g -
10158169 v) + 386030464 v) + 16 (-5224446785 + 752996629 v)))))))))))) - 2 c^{11}
(167503724544 (-6728914624 + 568495285 v) + g (3489660928 (13016645032 +
8962030045 v) + g (-5368709120 (-292583136950 + 24017768031 v) + g (-4026531840
(10165293703 + 9773196087 v) + g (3087007744 (-310703329581 + 24427628225 v) + g
(41943040 (261789910205 + 511988336167 v) + g (-4194304 (-80478381621656 +
5958319715205 v) + g (-7340032 (-159195507573 + 908661757475 v) + g (19922944 (-
3803697314441 + 259912627848 v) + g (65536 (-21494003461074 + 19846034444773 v)

+ g (-4 g (-95957927122780160 + g (287632408857280512 + g (14112278963615232 + g (-19653578278816768 + g (-1260850043118848 + g (882690849554688 + g (69689789300160 + g (-24780288850816 + g (-2312194021728 + g (396617359728 + g (42471446732 + g (-3025880480 + g (-363607028 + g (7118477 + 942988 g)))))))))))))) + g (-165724700321906688 + g (61931592212897792 + g (13921349804171264 + g (-3565480606531584 + g (-759974620314624 + g (128617426080000 + g (25937693113728 + g (-2719089596608 + g (-514690030624 + g (29845369344 + g (5196655776 + g (-133560604 + g (-20225786 + g (129106 + 12797 g)))))))))))))) v - 2228224 (-5092015326120 + 313443462749 v)))))))))))) + c^{13} (-64424509440 (-31392118000 + 3266330431 v) + g (-30870077440 (338540036 + 1974054641 v) + g (15435038720 (-155263899028 + 15568378375 v) + g (3858759680 (-5949609398 + 16836692649 v) + g (-385875968 (-3171710498884 + 302452790125 v) + g (-4194304 (-7199050543453 + 7025909195785 v) + g (159383552 (-2216018074409 + 197772369930 v) + g (9961472 (-1418930224489 + 746472100469 v) + g (-169345024 (-374796863936 + 30700879171 v) + g (-4456448 (-790732517211 + 257093436451 v) + g (-4 g (131094425471426560 + g (-139711789742481408 + g (-12084075198658304 + g (6763382251323136 + g (687750662102848 + g (-199668454522368 + g (-23259433343968 + g (3282156743888 + g (429166634552 + g (-25098952016 + g (-3623320568 + 57615549 g + 9057282 g^{2}))))))))))) + g (111368506768523264 + g (-35693646730473472 + g (-6816802637838336 + g (1448329149456384 + g (254602721677824 + g (-34147674459392 + g (-5419420916224 + g (419874169952 +g (57686389808 + g (-2182358576 + g (-233074564 + g (2895960 + 151139 g)))))))))))) v + 1114112 (-6623941688706 + 486257495339 v)))))))))))) + c^{3} $(-4 + g)^{4}$ $(-2 + g)^{6}$ $(2 + g)^{4}$ $(4 + g)^{5}$ (-262144 (-113288 + 2377 v) + g (-294912 (-56516 + 2579 v) + g (16384 (-1342730 + 8343 v) + g (57344 (-221153 + 7383 v) + g (2048 (3017299 + 27860 v) + g (512 (7233834 - 149633 v) + g (-256 (3190811 + 74537 v) + g (256 (-2034819 + 18281 v) + g (64 (777380 + 25621 v) + g (35789632 - 19296 v + g (-1184 (831 + 29 v) + g (-8 (129722 + 171 v) + g (-11832 + 6832 g + 205 g2 + 6 (24 + g) v))))))))))))) + c^{22} (-5788139520 (-676564402 + 160083967 v) + g (-241172480 (-2576889281 + 466580245 v) + g (60293120 (-29619321238 + 6352686433 v) + g (60293120 (-5078196749 + 764817687 v) + g (-10551296 (-28869430294 + 5581345825 v) + g (-32768 (-1725225407369 + 204074552000 v) + g (8192 (-2982199424687 + 514562401485 v) + g (1024 (-4786032678311 + 418628445448 v) + g (512 (1853509938295 - 281183337911 v) + g (128 (1602145248039 - 95786969266 v) + g (320 (-51442411581 + 6718709846 v) + g (16 (-238398240489 + 8559249412 v) + g (99641655000 - 10889123696 v + g (24629329748 - 393751604 v + g (-90717600 - 23791307 g + 7982814 v))))))))))))))) + 2 c^{20} (-2894069760 (-6012917068 + 1189728331 v) + g (-120586240 (-17301451499 + 4463153560 v) + g (241172480 (-42504038453 + 7759089858 v) + g (180879360 (-7602786869 + 1568123116 v) + g (-3014656 (-785229741543 + 131246678960 v) + g (-32768 (-10783328248577 + 1722916521020 v) + g (180224 (-1520367837598 + 229985165709 v) + g (4096 (-11032012591359 + 1318079733770 v) + g (-8192 (-2058810088141 + 277304495436 v) + g (1024 (2977955173329 - 254023351150 v) + g (128 (-4213838193135 + 494441955889 v) + g (64 (-1657720278121 + 94130315322 v) + g (8073958737728 - 801851658544 v + g (1714685287820 - 57219576928 v + g (-43690557980 + 3535330194 v + g (-9945135430 + g (36606232 + 8869381 g - 2293002 v) + 144900870 v))))))))))))))) + c^{19} (-11576279040 (-7544146656 + 1365854957 v) + g (-241172480 (-36677316296 + 11440785001 v) + g (3858759680 (-14975951629 + 2521230289 v) + g (482344960 (-13895636905 + 3384611322 v) + g (-24117248 (-633422714386 + 98315006665 v) + g (-393216 (-5103288171217 + 951264370485 v) + g (65536 (-31662823696360 + 4475077480509 v) + g (16384 (-18637335630451 + 2593378357801 v) + g (-16384 (-9468465963843 + 1198217409439 v) + g (1024

(24714225153738 - 2479890492173 v) + g (512 (-12389278952656 + 1372946879455 v) + g (256 (-4437431387274 + 304566985267 v) + g (64 (2067004974744 - 194855634185 v) + g (25769690217456 - 1106245308640 v + g (-1187987991264 + 91721922392 v + g (-249595089908 + g (2974935728 + g (668820261 - 4286047 v) - 178828396 v) + 5586951756 v)))))))))))))))) + c^{4} (-4 + g)^{3} (-2 + g)^{4} (2 + g)^{3} (4 + g)^{4} (16777216 (-926789 + 26081 v) + g (2097152 (-599353 + 147042 v) + g (-524288 (-35140496 + 835845 v) + g (-131072 (-10482747 + 2268661 v) + g (65536 (-139559536 + 2634495 v) + g (262144 (-2365673 + 437887 v) + g (-8192 (-301156413 + 4093511 v) + g (-2048 (-73071429 + 11026555 v) + g (1024 (-382545776 + 3156623 v) + g (512 (-40858981 + 4646633 v) + g (256 (144468780 - 486629 v) + g (1719304576 - 128466560 v + g (-64 (31016485 + 9201 v) + g (128 (-631265 + 24084 v) + g (53426528 + 98656 v + g (48 (41917 - 550 v) + g (-522572 - 984 v + g (-21766 + 46 v + g (772 + 41 g + v)))))))))))))))))))) +c^{17} (-138915348480 (-2821832048 + 427354731 v) + g (-964689920 (-26466442004 + 13048889977 v) + g (9164554240 (-35045491652 + 5004539595 v) + g (6873415680 (-3749307994 + 1342787143 v) +g (-916455424 (-118347582708 + 15772261105 v) + g (-9961472 (-1043137485093 + 273774996775 v) +g (617611264 (-31715087383 + 3890609460 v) + g (1245184 (-1756570916163 + 337122110494 v) +g (-622592 (-3298787740280 + 365897057049 v) + g (-4096 (-64002613356856 + 8863323921749 v) + g (10240 (-12406174291876 + 1216074360317 v) + g (1536 (-11874491206920 + 1154427487763 v) + g (1792 (2512358971784 - 211363085225 v) + g (717098779236416 - 46609699990784 v + g (64 (-1316415639593 + 91537468767 v) + g (16 (-919395862807 + 36587211300 v) + g (698506610432 - 38198499784 v + g (132286822932 - 2685560128 v + 3 g (-552074096 + 22171754 v + g (-112342463 + 637219 v))))))))))))))))))))) + c^{16} (-370440929280 (-1893625237 + 261919802 v) + g (-15435038720 (-2154765905 + 1453536109 v) + g (36658216960 (-17306079377 + 2271039605 v) + g (9164554240 (-4329625543 + 1983241902 v) + g (-7789871104 (-30755056469 + 3786813785 v) + g (-1693450240 (-11046333371 + 3561240776 v) + g (169345024 (-290415146974 + 33071253165 v) + g (42336256 (-109052694059 + 25113112610 v) + g (-42336256 (-141409379566 + 14622499027 v) + g (-1114112 (-589930702053 + 96968038711 v) + g (139264 (-3176937106453 + 291477888027 v) + g (8192 (-6786069513644 + 783696821171 v) + g (-2560 (-7626092198463 + 602858575880 v) + g (2758868715699072 - 216232887844864 v + g (64 (-7584890117739 + 497575857770 v) + g (288 (-263993156797 + 13169768768 v) + g (6018525553856 - 311874900720 v + g (1031700560760 - 28955133664 v + g (g (-5282137786 + g (21901099 + 4357412 g - 559504 v) + 61782772 v) + 4 (-7116081328 + 272240591 v))))))))))))))))))))) +c^{15} (-740881858560 (-1512717692 + 190777257 v) + g (-7717519360 (-4332034984 + 4589665859 v) + g (3858759680 (-288599265256 + 34727833577 v) + g (964689920 (-51833873496 + 32723610529 v) + g (-916455424 (-510092422952 + 57875760405 v) + g (-19922944 (-1415807201789 + 586664247455 v) + g (338690048 (-319528178624 + 33675305187 v) + g (84672512 (-97165649347 + 27627996815 v) + g (-84672512 (-178718295991 + 17170447000 v) + g (-557056 (-2494379081691 + 495192630985 v) + g (278528 (-4726048125796 + 404342896637 v) + g (4096 (-34579150224293 + 4782594744774 v) + g (2048 (34539394962526 - 2555217800691 v) + g (8714964840059136 - 822136117134336 v + g (768 (-2935089719156 + 180850298337 v) + g (384 (-812916679829 + 49958916200 v) + g (39015549830528 - 1906597219808 v + g (5980562964224 - 219353740912 v + g (g (-51376236072 + g2 (127837321 - 630121 v) + 72 g (9813127 - 239111 v) + 935346020 v) + 8 (-38345947796 + 1390106057 v))))))))))))))))))))) + c^{14} (-740881858560 (-2154611780 + 247191707 v) + g (-30870077440 (-637234477 + 1602271727 v) + g (7717519360 (-224872482542 + 24744146379 v) + g (34728837120 (-1362254137 + 1390838138 v) + g (-3665821696 (-219722847894 + 22899543085 v) + g (-39845888 (-860057621297 + 498022262350 v) + g (677380096 (-

308290670890 + 29962737837 v) + g (169345024 (-71620166123 + 26423200088 v) + g (-
84672512 (-392521423073 + 34901325261 v) + g (-1114112 (-2199810000933 +
543833714428 v) + g (557056 (-6032577912257 + 479253103689 v) + g (8192 (-
36675505751717 + 6183708871768 v) + g (-2048 (-105460361547838 + 7268096543333
v) + g (-512 (-44264786344811 + 5060235810380 v) + g (768 (-11169144566435 +
643254634768 v) + g (1536 (-671115106295 + 50497988916 v) + g (197439228584064 -
9051198481280 v + g (256 (104101546043 - 4843347668 v) + g (-2323202797744 +
79352063440 v + g (g (10679676248 + g (1737950146 - 3 g (2728507 + 478771 g - 36106
v) - 17791556 v) - 246630600 v) + 8224 (-42149342 + 1070649 v)))))))))))))))))))))) +c^9 (-
103079215104 (-14496740048 + 968837529 v) + g (-2147483648 (52838649436 +
17044876009 v) + g (1073741824 (-2254666896092 + 147377889565 v) + g (1342177280
(112368164550 + 40040574991 v) + g (-268435456 (-6490799977172 + 408650148105 v)
+ g (-67108864 (1252633638493 + 517992618145 v) + g (1476395008 (-497589589945 +
29646753467 v) + g (8388608 (2962753206763 + 1554125209642 v) + g (-4194304 (-
48042629109672 + 2653617814427 v) + g (-524288 (7347800445184 + 5987218955551
v) + g (262144 (-144029927245796 + 7196352373731 v) + g (65536 (2417716297192 +
7744824159343 v) + g (-65536 (-75345818372520 + 3305357334649 v) + g (-49152 (-
1087282327449 + 1139633233780 v) + g (16384 (-27615138625743 + 1024964470508 v)
+ g (4096 (-2850924199969 + 1026474872592 v) + g (-4096 (-7011459508980 +
209964851863 v) + g (-6144 (-189851715910 + 34189146387 v) + g (512 (-
2398386407338 + 54373839575 v) + g (-4 g (-8400149633024 + g (-580266907520 + g
(134254282400 + g (11069849192 + g (-1050518912 + g (-99921788 + g (2616423 +
279959 g))))))) + g (-526061866752 + g (-126067596032 + g (4987139136 + g
(1226948800 + g (-16495584 + g (-4661048 + g (-1274 + 2895 g))))))) v + 8448 (-
8007498174 + 792086333 v)))))))))))))))))))))) + c^5 $(-4 + g)^2$ $(-2 + g)^3$ $(2 + g)^2$ $(4 + g)^3$
(134217728 (-23459456 + 831425 v) + g (16777216 (-18783796 + 4577389 v) + g (-
4194304 (-1016049588 + 31341113 v) + g (-2097152 (-190252526 + 41845119 v) + g
(524288 (-4729670710 + 122327881 v) + g (524288 (-410308267 + 79828971 v) + g (-
327680 (-2475582182 + 50987795 v) + g (-32768 (-1970214142 + 329576945 v) + g (8192
(-19987154044 + 304006343 v) + g (4096 (-2890654964 + 398918769 v) + g (2048
(10276768307 - 102321164 v) + g (-3072 (-446871877 + 47814026 v) + g (512 (-
3348402569 + 17371057 v) + g (256 (-393623991 + 29656256 v) + g (256 (332162352 -
449027 v) + g (4565579776 - 207951744 v + g (-64 (36667529 + 40622 v) + g (16 (-
7507710 + 163039 v) + g (8 (3722641 + 8158 v) + g (20 (79633 - 552 v) + g (-110882 -
6895 g + 6 (-42 + g) v)))))))))))))))))))))) - c^6 (-4 + g) $(-2 + g)^2$ (2 + g) $(4 + g)^2$ (-4294967296
(-119498287 + 5128671 v) + g (-536870912 (-113766041 + 27531336 v) + g (134217728
(-5839226647 + 223056964 v) + g (100663296 (-876031517 + 194674362 v) + g (-
8388608 (-62528582615 + 2077291298 v) + g (-4194304 (-13219458041 + 2660110629 v)
+ g (1048576 (-192879486469 + 5408665600 v) + g (524288 (-38059002301 +
6807521702 v) + g (-131072 (-378958599231 + 8621601670 v) + g (-196608 (-
23120710409 + 3585205057 v) + g (49152 (-164974023007 + 2883563964 v) + g (16384
(-41812258239 + 5430575488 v) + g (-4096 (-217549273973 + 2697989282 v) + g (4096
(16936465179 - 1755442888 v) + g (1024 (-63742476433 + 493375452 v) + g (512 (-
9183949747 + 708715307 v) + g (256 (12042876982 - 45100329 v) + g (-512 (-407294917
+ 21137990 v) + g (128 (-686515971 + 447253 v) + g (32 (-179910615 + 5389429 v) + g
(16 (83258563 + 118015 v) + g (89151696 + 765 g^4 - 1207280 v + 24 g^3 (369 + v) + 8 g^2 (-
75370 + 289 v) - 8 g (1027334 + 2561 v)))))))))))))))))))))))) + 2 c^{12} (-418759311360 (-
2712869828 + 254995999 v) + g (-17448304640 (1477508801 + 1891447364 v) + g
(30870077440 (-47366405515 + 4309061612 v) + g (7717519360 (1549925859 +
4960594960 v) + g (-385875968 (-2120191138961 + 184114591770 v) + g (-4194304 (-
1303548232681 + 4550324258100 v) + g (39845888 (-6559474233946 + 534836794885

v) + g (9961472 (-572542536989 + 537787655182 v) + g (-846725120 (-62193829051 + 4668022163 v) + g (-1114112 (-1755598922997 + 836399894488 v) + g (5570560 (-1252352760067 + 84468205099 v) + g (32768 (-11184716020859 + 3179502612040 v) + g (-53248 (-11542860217778 + 679179379861 v) + g (-13312 (-3133829445821 + 564771881552 v) + g (379392 (-93877492379 + 4642650976 v) + g (16640 (-177690569183 + 20587921580 v) + g (1311421870189824 - 51951085883008 v + g (256 (501090656919 - 36579053086 v) + g (32 (-896449115863 + 26657861757 v) + g (32 (-100515453414 + 4357868587 v) + g (328477325168 - 6670632288 v + g (41326470544 - 938862016 v + g (-1505740250 + 17814170 v + g (-208947866 + g (1182240 + 178423 g - 5645 v) + 1826778 v)))))))))))))))))))))))) - 2 c^7 (-2 + g) (4 + g) (-17179869184 (-2013694472 + 101919943 v) +g (-2147483648 (-2216972716 + 532675389 v) + g (1073741824 (-54947927114 + 2519330361 v) + g (134217728 (-57685753480 + 12899199387 v) + g (-67108864 (-667072054760 + 27296769521 v) + g (-33554432 (-166201628823 + 34236593117 v) + g (8388608 (-2376007801233 + 85203699587 v) + g (4194304 (-559468180028 + 104804481715 v) + g (-1048576 (-5515325351669 + 169426571626 v) + g (-1048576 (-611135881351 + 102398989405 v) + g (131072 (-8762800110889 + 224047188939 v) + g (32768 (-3643491371308 + 534417446295 v) + g (-16384 (-9739101499071 + 199629880435 v) + g (-8192 (-1896399023899 + 236689857741 v) + g (20480 (-760298831135 + 11876464429 v) + g (2048 (-694277563623 + 70987718302 v) + g (1024 (1031019941513 - 11408472093 v) + g (90978329420288 - 7236254107648 v + g (256 (-189050100341 + 1316809347 v) + g (384 (-10384203281 + 597737915 v) + g (1426160126336 - 4916365568 v + g (64 (1795675789 - 67394671 v) + g (64 (-385788979 + 236701 v) + g (32 (-62930482 + 1312335 v) + g (8 (26366325 + 38569 v) + g (18235884 - 159702 v - 22 g (26369 + 72 v) + g^2 (-56187 + 98 v)))))))))))))))))))))))))))) - c^{10} (51539607552 (-38121652464 + 2876339819 v) + g (2147483648 (53096936959 + 23995827973 v) + g (-2684354560 (-1101280146902 + 81018936581 v) + g (-1342177280 (97101170353 + 52252849284 v) + g (3087007744 (-635867490026 + 44937234725 v) + g (100663296 (580102248527 + 414623811250 v) + g (-4798283776 (-157899937951 + 10535781393 v) + g (-4194304 (2724318557027 + 3403487104480 v) + g (39845888 (-4730204518737 + 291987251723 v) + g (524288 (57191286894 + 5916163880695 v) + g (-2228224 (-14212353487142 + 792048996455 v) + g (-65536 (-7211389741621 + 6817608059000 v) + g (229376 (-15963875142744 + 779649422441 v) + g (8192 (-13432299112893 + 5275999193416 v) + g (-53248 (-5470801059263 + 225615331596 v) + g (-2555904 (-5138870853 + 1089372280 v) + g (22528 (-695647284555 + 23097377693 v) + g (11264 (-83320593673 + 10300807162 v) + g (256 (2144737339197 - 53777384969 v) + g (2816 (14486163513 - 1050598820 v) + g (6272 (-1865454045 + 32196841 v) + g (192 (-5373165031 + 217764836 v) + g (32 (4178830306 - 42560039 v) + g (32 (423713419 - 8476664 v) + g (8 (-78557309 + 333826 v) + g (g (522219 + 65561 g + 111 v) + 8 (-8945279 + 64383 v)))))))))))))))))))))))))))) + 2 c^8 (-206158430208 (-2384627198 + 139665073 v) + g (-8589934592 (5381930621 + 1290884996 v) + g (15032385536 (-56908973045 + 3268898991 v) + g (1610612736 (42876678329 + 10877932858 v) + g (-134217728 (-4957347077991 + 274906279225 v) + g (-67108864 (663950333493 + 18297788600 v) + g (33554432 (-9099958506946 + 478532006775 v) + g (8388608 (1922158505451 + 599635709134 v) + g (-8388608 (-10959368587192 + 535252205313 v) + g (-2097152 (1679243011859 + 636837472076 v) + g (524288 (-36435548165413 + 1612459471829 v) + g (524288 (857917181437 + 459472750674 v) + g (-32768 (-85620131077801 + 3332440325143 v) + g (-16384 (1478144880967 + 1836202961408 v) + g (8192 (-35806438463401 + 1181111678622 v) + g (4096 (-451792559611 + 635376358224 v) + g (2048 (10591185329002 - 282394324745 v) + g (-2048 (-223794397557 + 74971676294 v) + g (4608 (-241215393077 + 4879671401 v) + g (50688 (-782330408 + 117884417 v) + g (512

(4.A1.12)

$(74167968913 - 1039076370 v) + g (-512 (-3693000655 + 282643643 v) + g (1344 (-601481215 + 5020679 v) + g (224 (-228590187 + 8803378 v) + g (9479650880 - 33817056 v + g (448 (1612634 - 27877 v) + g (-47181878 - 2562 v + g (-4167570 + 23278 v + g (43238 + 4309 g + 100 v))))))))))))))))))))))))))))))))/G,$

where $G = (4096 (2 + c)^{28} (4 + c)^2 (4 + 3 c)^2 (8 + c (8 + c))^2 – 64 (2 + c)^{26} (4 + c) (4 + 3 c) (737280 + c (2211840 + c (2590720 + c (1495040 + c (443536 + 64656 c + 3651 c^2))))) g^2 + 16 (2 + c)^{24} (245366784 + c (981467136 + c (1659994112 + c (1544847360 + c (862717952 + c (295735296 + c (60869248 + c (6898304 + 330657 c)))))))) g^4 - 128 (2 + c)^{22} (97812480 + c (391249920 + c (666910208 + c (631355904 + c (362532304 + c (129263008 + c (27984591 + c (3370751 + 173412 c)))))))) g^6 + 8 (2 + c)^{20} (3425107968 + c (13700431872 + c (23501029376 + c (22551576576 + c (13231353344 + c (4860582912 + c (1092801464 + c (137700792 + 7460879 c)))))))) g^8 – 4 (2 + c)^{18} (10910269440 + c (43641077760 + c (75238145024 + c (72970662912 + c (43530827664 + c (16358474528 + c (3784215288 + c (493242216 + 27775975 c)))))))) g^{10} + (2 + c)^{16} (52351401984 + c (209405607936 + c (362457683968 + c (354453424128 + c (214166059456 + c (81882954624 + c (19354671696 + c (2587830416 + 150008481 c)))))))) g^{12} - 4 (2 + c)^{14} (12076830720 + c (48307322880 + c (83873186880 + c (82543930560 + c (50361263444 + c (19507852648 + c (4686574028 + c (638748984 + 37840027 c)))))))) g^{14} + 2 (2 + c)^{12} (17363907072 + c (69455628288 + c (120877007424 + c (119536323264 + c (73466841728 + c (28738044352 + c (6988477824 + 41 c (23566784 + 1419005 c)))))))) g^{16} - 8 (2 + c)^{10} (2447984800 + c (9791939200 + c (17071710640 + c (16943344720 + c (10470238702 + c (4125498604 + c (1012305703 + c (141457321 + 8620781 c)))))))) g^{18} + (2 + c)^8 (8681953536 + c (34727814144 + c (60625511296 + c (60329184384 + c (37429675264 + c (14826493056 + c (3662061568 + 5 c (103142016 + 6341429 c)))))))) g^{20} - (2 + c)^8 (754801920 + c (2264405760 + c (2822597600 + c (1871185600 + c (695832028 + c (137640188 + 11316817 c)))))) g^{22} + 4 (2 + c)^8 (12781104 + c (25562208 + c (19096030 + c (6314926 + 780391 c)))) g^{24} - (2 + c)^6 (10654560 + c (21309120 + c (15950536 + c (5295976 + 658213 c)))) g26 + 8 (2 + c)^4 (209052 + c (418104 + c (313333 + c (104281 + 13005 c)))) g^{28} - 2 (2 + c)^4 (23880 + c (23880 + 5963 c)) g^{30} +936 (2 + c)^4 g^{32} - 45 (2 + c)^2 g^{34} + g^{36})$

Note also that in (4.22) where A1, A2, B1, and B2 are

$$A1(c,g)= (-8(2 +c)^{15} (4 + c) (4 + 3 c) (128 + c (4 + c) (52 + 9 c)) – 4(2+c)^{13} (6144 + c (14720 + c (12480 +c (4384 + 537 c)))) g^3 - 2 (2 + c)^{11} (150528 +c (383296 + c (375936 + c (176536 + c (39688 + 3447 c))))) g^4 - (2 + c)^9 (89856 + c (198688 + c (162400 + c (57676 + 7467 c)))) g^7 - 2 (2 + c)^8 (123024 + c (246332 + c (181690 + c (58327 + 6914 c)))) g^8 - (2 + c)^6 (21288 + c (33564 + c (17590 + 3039 c))) g^{11} - (2 + c)^4 (71568 + c (142576 + c (106240 + 51 c (688 +85 c)))) g^{12} - 2 (2 +c)^4 (186 + 95 c) g^{15} - (2 + c)^3 (456 + 227 c) g^{16} -(2 +c) g^{19} - g^{20})<0; \quad (4.A1.12a)$$

$$A2(c,g) = (16 (2 + c)^{15} (4 +c) (4 + 3 c) (16 + c (24 + 7 c)) g + 4 (2+ c)^{13} (26624 + c (68736 + c (66688 + 9 c (3344 + 700 c + 55 c^2)))) g^2 + 8 (2 + c)^{11} (7872 + c (18096 + c (15124 + c (5387 +686 c)))) g^5 + (2 +c)^9 (485376 + c (1223008 + c (1206816 + c (581304 +c (136744 + 12627 c))))) g^6 + 4 (2 +c)^7 (19524 + c (41704 + c (33222 + c (11641 + 1508 c)))) g^9 + 4 (2 + c)^6 (40812 +c (81368 + c (60370 + 27 c (731 + 89 c)))) g^{10} + 4 (2+ c)^4 (1812 + c (2782 +c (1423 + 242c))) g^{13} + 4(2+c)^4 (1278+ c (1271 + 315c)) g^{14} + 21 (2+c)^3 g^{17} + 23 (2+c)^2 g^{18})>0; \quad (4.A1.12b)$$

$$B1(c,g)= -16 (2 + c)^{15} (4 + c) (4 + 3 c) (8 + 3 c) (8 + 5 c) g -16 (2 + c)^{14} (5888 + c (11776 + c (8056 + c (2168 + 201 c)))) g^{2} -8 (2 + c)^{11} (26368 + c (52736 + c (38766 + c (12398 + 1455 c)))) g^{5} - 8 (2 + c)^{10} (42176 + c (84352 + c (61023 + c (18847 + 2127 c)))) g^{6} - 2 (2 + c)^{7} (108832 + c (217664 + c (162266 + c (53434 + 6557 c)))) g^{9} -14 (2 + c)^{6} (13352 + c (26704 + c (19854 + c (6502 + 793 c)))) g10 - (2 + c)^{5} (8576 +c (8576 + 2127 c)) g^{13} - (2 + c)^{4} (5272 + c (5272 + 1313 c)) g^{14} - 23 (2 + c)^{2} g^{18} - 44 (2 + c)3\, g^{17}<0; \qquad (4.A1.12c)$$

$$B2(c,g)=(128 (2 + c)^{16} (4+c) (4 + 3c) (8 + c (8+c)) + 4(2+c)^{13} (22528 + c (45056 + c (32672 + c (10144 + 1137 c)))) g^{3} + 4 (2 + c)^{12} (58880 + c (117760 + c (83184 + c (24304 + 2543 c)))) g^{4} + (2 + c)^{9} (274432 + c (548864 + c (407024 + c (132592 + 16005 c)))) g^{7} + (2 + c)^{8} (307520 + c (615040 + c (452444 + 3 c (48308 + 5721 c)))) g^{8} + (2 + c)^{7} (27208 + c (27208 + 6679 c)) g^{11} +4 (2 + c)^{4} (19220 + c (38440 + c (28743 + c (9523 + 1180 c)))) g^{12} + (2 + c)^{3} (1648 + c (1648 + 411 c)) g15 + 230 (2 + c)^{4} g^{16} + 2 (2 + c) g^{19} + g^{20})>0. \qquad (4.A1.12d)$$

Given the optimal wages of the first period, the optimal products of the second period depend on c, g and v

$$q_{11}^{*}= ((2 + c) (3456 c^{31} - (-4 + g)^{7} (-2 + g)^{8} (2 + g)^{7} (4 + g)^{8} v + 1152 c^{30} (216 + 7 v) + c^{3} (-4 + g)^{3} (-2 + g)^{4} (2 + g)^{3} (4 + g)^{4} (2 (-4 + g) (2 + g) (136478720 + g (-12367872 + g (-98062848 + g (5810688 + g (27318208 + g (-912800 + g (-3665072 + g (52440 + g (235676 + g (-688 + g (-5937 + g + 18 g^{2})))))))))))) + (-8448376832 + g (-201654272 + g (6752059392 + g (106082304 + g (-2081100800 + g (-14566400 + g (309567616 + g (-527552 + g (-22496320 + g (211776 + g (690864 + 7 g (-1376 + g (-722 + 5 g))))))))))))) v) - c^{5} (-4 + g) (-2 + g)^{2} (2 + g) (4 + g)^{2} (2 (-4 + g) (2 + g) (-8325999624192 + g (680895971328 + g (8924397961216 + g (-569617711104 + g (-4022321741824 + g (190864572416 + g (989418983424 + g (-32708123136 + g (-144099771392 + g (3029590912 + g (12586458880 + g (-146822368 + g (-634370736 + g (3290424 + g (16540796 + g (-26224 + g (-170330 + g (32 + 295 g))))))))))))))))))) + (298139181383680 + g (11367360757760 + g (-343693099794432 + g (-10079446237184 + g (166864656793600 + g (3378391023616 + g (-44378637762560 + g (-492854706176 + g (7039955578880 + g (15725879296 + g (-679265375232 + g (3989154816 + g (38874062848 + g (-483874688 + g (-1221023104 + g (19844928 + g (17693112 + g (-268844 + g (-76886 + 553 g))))))))))))))))))) v) - 48 c^{29} (3 g (60 + 399 g + 73 v) - 32 (5571 + 353 v)) - 16 c^{28} (9 g^{2} (25492 + 763 v) + 6 g (5304 + 6583 v) - 32 (365932 + 34159 v)) + 8 c^{24} (-8 g (533795904 + g (5100465776 + g (-29403608 + g (-158216357 + g (252999 + 1019147 g))))) - g (5854414656 + g (6103568224 + 3 g (-111730144 + g (-37638996 + g (996046 + 80637 g))))) v + 1024 (316425852 + 66942925 v)) - c (-4 + g)^{6} (-2 + g)^{7} (2 + g)^{6} (4 + g)^{7} (80 + 960 v + g (12 + 8 v + g (g (-2 + g + v) - 6 (3 + 25 v)))) + 16 c^{27} (128 (1434133 + 176090 v) + g (-8 (112722 + 142985 v) + 3 g (3 g (559 + 2546 g + 738 v) - 56 (41765 + 2488 v)))) + 16 c^{26} (2432 (903144 + 137333 v) + g (-16 (1022584 + 1328313 v) + g (-16 (8552038 + 762895 v) + 3 g (90872 + 438507 g + 3 (40940 + 4283 g) v)))) - 8 c^{25} (-13312 (3136795 + 569443 v) + g (427073152 + 569286688 v + g (3824466144 + 455553472 v + g (-16 (885731 + 1227927 v) + g (-72292016 - 4262316 v + 3 g (13602 + 52423 g + 19551 v)))))) + c^{20} (786432 (2164574980 + 745387753 v) + g (-16384 (2698227752 + 4244302925 v) + g (-8192 (64804975028 + 18363795711 v) + g (32768 (263850643 + 429870379 v) + g (-8 g (64221697536 + g (305002640640 + g (-1265290304 + g (-4942217274 + g (5878256 + 20287565 g))))) - g (864510977536 + g$$

(388913300352 + g (-17539005632 + g (-3809408856 + g (83661386 + 5259115 g))))) v + 6144 (9188066342 + 2037056885 v)))))) + 4 c^{23} (16384 (255461817 + 61943888 v) + g (-1024 (66448011 + 93910126 v) + g (-512 (1350469660 + 244309443 v) + g (64 (87513171 + 128460872 v) + g (64 (495681103 + 59635773 v) + g (-16 (6001735 + 9121022 v) + g (3 g (63653 + 223850 g + 99772 v) - 4 (100954865 + 6074851 v)))))))) + 4 c^{22} (32768 (696580610 + 191317663 v) + g (-45056 (9842136 + 14375485 v) + g (-512 (9331424572 + 1991367065 v) + g (1024 (49726937 + 75374889 v) + g (32 (9468441144 + 1444325333 v) + g (-64 (22656656 + 35527359 v) + g (-6357818712 - 583404908 v + g (8621972 + 13936896 v + g (31423817 + 965695 v))))))))) + 2 c^{18} (9961472 (893849144 + 379327221 v) + g (-3735552 (77965352 + 133740107 v) + g (-2490368 (1549417845 + 562330472 v) + g (311296 (277959354 + 492753191 v) + g (4096 (150269786295 + 45064906802 v) + g (-4096 (2140176444 + 3911732347 v) + g (-512 (87647496730 + 20640263933 v) + g (2 g (752814267952 + g (-2780173216 + g (-10176641736 + g (10835121 + 36640457 g)))) + g (255736206192 + g (-10729392968 + g (-2096006426 + g (42834350 + 2545069 g)))) v + 1408 (256251348 + 481772173 v))))))))) + c^{21} (2883584 (147381101 + 45462617 v) + g (-73728 (130675292 + 197786465 v) + g (-8192 (13395615841 + 3314433468 v) + g (4096 (358919871 + 563241868 v) + g (2048 (4478511402 + 833981693 v) - g (256 (244499536 + 396603831 v) + g (64 (4465607345 + 556960908 v) + g (-112 (6625475 + 11072758 v) + g (7 g (164604 + 551225 g + 282503 v) - 8 (350107072 + 21967487 v)))))))))) + c^2 (-4 + $g)^4$ (-2 + $g)^5$ (2 + $g)^4$ (4 + $g)^5$ (-24576 (196 + 1157 v) + g (-4096 (182 + 113 v) + g (512 (6576 + 35045 v) + g (1024 (459 + 160 v) + g (-64 (13886 + 62379 v) + g (-32 (3251 + 140 v) + g (16 (6931 + 23022 v) + g (9424 - 2416 v + g (-32 (205 + 374 v) + g (-286 + 136 v + g (147 + 37 v))))))))))) + 2 c^{19} (524288 (5630265117 + 2155282430 v) + g (-32768 (2641049818 + 4330962419 v) + g (-32768 (33353313887 + 10729487086 v) + g (4096 (5150005423 + 8737679070 v) + g (16384 (8782054857 + 2278331459 v) + g (-2048 (813770637 + 1424750719 v) + g (-512 (16059609043 + 3150186255 v) + g (64 (767498230 + 1382946697 v) + g (32 (6199742243 + 817783085 v) + g (-455256864 - 842101364 v + g (g (592680 + 1952139 g + 1122833 v) - 2 (809274323 + 53894441 v)))))))))))) + c^{17} (79691776 (588091965 + 276258073 v) + g (-2490368 (680159324 + 1224987735 v) + g (-2490368 (9432039161 + 3845746550 v) + g (3735552 (161246173 + 299772225 v) + g (49152 (90757126772 + 31193685891 v) + g (-12288 (6192341630 + 11858995801 v) + g (-16384 (24639053883 + 6848852278 v) + g (512 (8136683746 + 16019973781 v) + g (128 (140210082740 + 29545394171 v) - g (96094506432 + 194145968624 v + g (48 (7540717841 + 1071170050 v) + g (-4 (186884057 + 386831618 v) + g (g (857306 + 2860230 g + 1815789 v) - 52 (49893073 + 3586669 v)))))))))))))) + 2 c^{16} (677380096 (79582276 + 41364247 v) + g (-42336256 (50445304 + 95779675 v) + g (-21168128 (1461239796 + 667625933 v) + g (21168128 (42300238 + 82805793 v) + g (1531904 (4499550274 + 1762088619 v) + g (-208896 (660303004 + 1330367073 v) + g (-2048 (369867355316 + 119942661765 v) + g (22016 (440067112 + 911035713 v) + g (64 (672675984298 + 171645910977 v) - g (312243048832 + 663196398896 v + g (8 (151191299830 + 27881976133 v) + g (-8 (505114480 + 1099309371 v) + g (-2 (7194080967 + 805954081 v) + g (13879924 + 30923345 v + g (47416921 + 1794908 v))))))))))))))) + c^{15} (2709520384 (80079285 + 46081592 v) + g (-2032140288 (4605723 + 9260735 v) + g (-338690048 (417155746 + 213329887 v) + g (42336256 (107901677 + 223420668 v) + g (2228224 (16378508249 + 7281189225 v) + g (-1671168 (504234467 + 1073642976 v) + g (-8192 (584054308551 + 219211694893 v) + g (6144 (12017833883 + 26279157603 v) + g (3072 (110070548063 + 33463236374 v) + g (-256 (12371009321 + 27748563670 v) + g (-64 (197011953556 + 45477552735 v) + g (160 (383591459 + 881741642 v) + g (8 (28012605871 + 4366107264 v) + g (-8 (52622711 + 123878295 v) + g (g (442377 + 1540460 g + 1066224 v) - 4 (368048934 + 29076649 v)))))))))))))))) + c^{14} (159383552 (2395920408 + 1528867885 v) + g (-1354760192 (13163832 + 28172261 v) +

g (-338690048 (826826932 + 473352141 v) + g (338690048 (29594196 + 65144737 v) + g
(52363264 (1584648556 + 798229809 v) + g (-8912896 (244686433 + 553404143 v) + g (-
8192 (1561052443810 + 674712909409 v) + g (28672 (8131058210 + 18876044453 v) + g
(1536 (715717918174 + 256453002369 v) + g (-2048 (6266804054 + 14920292873 v) + g
(-64 (820073385980 + 231499238957 v) + g (64 (5432411989 + 13255992368 v) + g (32
(40663859233 + 8306831153 v) + g (-80 (49613046 + 124032409 v) + g (-4 (3550951972
+ 441244721 v) + g (12496232 + 44475841 g + 32005536 v + 1869711 g v)))))))))))))))) -
c^{4} $(-4 + g)^{2}$ $(-2 + g)^{3}$ $(2 + g)^{2}$ $(4 + g)^{3}$ (4194304 (151448 + 431683 v) + g (262144 (397064
+ 215091 v) + g (-1835008 (359663 + 960233 v) + g (-98304 (1010070 + 403601 v) + g
(49152 (5810267 + 14186789 v) + g (4096 (9352281 + 2300321 v) + g (-2048 (32612902 +
70353075 v) + g (-512 (14943837 + 1254347 v) + g (256 (35747947 + 64703183 v) + g
(838039552 - 68252544 v + g (-64 (11586292 + 16226773 v) + g (64 (-759967 + 183578 v)
+ g (64 (527693 + 498117 v) + g (1284256 - 489392 v + g (-4 (185446 + 89427 v) + g (-
9126 + 4819 g + 7 (612 + 89 g) v))))))))))))))))) + 2 c^{13} (318767104 (920557031 +
653097469 v) + g (-169345024 (87190788 + 199748623 v) + g (-169345024 (1421507905
+ 912355112 v) + g (338690048 (27951104 + 65787099 v) + g (4456448 (18196281588 +
10384067977 v) + g (-2228224 (1076561161 + 2601215229 v) + g (-16384
(885629148235 + 439637669476 v) + g (12288 (25000229969 + 61971455017 v) + g
(3072 (485143521616 + 203485649589 v) + g (-128 (164899255716 + 419132197195 v) +
g (-128 (692186141907 + 235375884340 v) + g (32 (23790111527 + 61981608774 v) + g
(32 (91223197032 + 23576761223 v) - g (40 (325486453 + 869116642 v) + g
(47662473960 + 8326183488 v + g (-4 (20471319 + 56034328 v) + g (g (81646 + 306776
g + 229267 v) - 9 (33089179 + 2932876 v))))))))))))))))))) + 2 c^{12} (218103808 (1800831980
+ 1425797639 v) + g (-179306496 (118262808 + 291925235 v) + g (-9961472
(35968664876 + 25952835885 v) + g (677380096 (22709274 + 57526097 v) + g (4456448
(30550386066 + 19779533671 v) + g (-4456448 (1007607226 + 2618160447 v) + g (-
32768 (854112041700 + 486718765949 v) + g (106496 (6371800760 + 16977059353 v) +
g (13312 (254203224854 + 124352470011 v) + g (-3328 (17098194408 + 46703873939 v)
+ g (-1664 (147246384154 + 59751085731 v) + g (1664 (1583730424 + 4434633113 v) +
g (32 (323369521941 + 103453579453 v) - g (32 (1969480631 + 5653975446 v) + g (8
(29491577247 + 6833131918 v) + g (-16 (41245745 + 121446038 v) + g (-2451766578 -
345821468 v + g (1971996 + 5961130 v + g (7564391 + 361213 v))))))))))))))))))) + c^{11}
(402653184 (2264991179 + 2011385278 v) + g (-159383552 (330291742 + 885146755 v)
+ g (-159383552 (5774005859 + 4703390172 v) + g (338690048 (126657193 + 347948182
v) + g (106954752 (3662690369 + 2698763968 v) + g (-17825792 (801658407 +
2257247021 v) + g (-262144 (349716069909 + 229124119723 v) + g (425984
(5906218686 + 17044627823 v) + g (212992 (60500045957 + 34487216705 v) + g (-26624
(9497162730 + 28092649273 v) + g (-13312 (83841024608 + 40395379751 v) + g (13312
(1098319530 + 3330693557 v) + g (128 (459047575749 + 179396840032 v) + g (-448
(1039541979 + 3233015755 v) + g (-32 (55691411967 + 16551148262 v) + g (80
(91348137 + 291532780 v) + g (792 (35008425 + 7036432 v) + g (-8 (5454263 +
17882885 v) + g (g (42709 + 180185 g + 144188 v) - 4 (42708605 + 4357652
v))))))))))))))))))))) + c^{10} (1476395008 (618569640 + 620180393 v) + g (-16777216
(3348923512 + 9830527625 v) + g (-318767104 (3182640746 + 2944061287 v) + g
(79691776 (641245886 + 1927596105 v) + g (1048576 (458584725849 + 386459258605
v) + g (-35651584 (540455556 + 1663929601 v) + g (-786432 (161605973806 +
122181371319 v) + g (65536 (59653770684 + 188149533509 v) + g (8192
(2508170032690 + 1669016053029 v) + g (-53248 (8710316296 + 28153958645 v) + g (-
13312 (157543217728 + 90005080143 v) + g (732160 (44723313 + 148208572 v) + g (512
(262915671346 + 124629014379 v) + g (-384 (3488928128 + 11860731905 v) + g (-192
(27283131241 + 10199728427 v) + g (192 (153159854 + 534547363 v) + g (80

(1423822305 + 385642514 v) + g (-16 (18274816 + 65565821 v) + g (-8 (146018141 + 24083852 v) + g (857860 + 3171680 v + 9 g (410294 + 22915 v))))))))))))))))))))))) + c^{9} (536870912 (1457983939 + 1664478915 v) + g (-50331648 (1017035404 + 3305070567 v) + g (-16777216 (56814295683 + 60155906654 v) + g (159383552 (324182937 + 1077792533 v) + g (2097152 (238577622780 + 231597454619 v) + g (-151519232 (144484950 + 491641861 v) + g (-524288 (282878479879 + 248303640176 v) + g (131072 (38688388650 + 134806076551 v) + g (32768 (833891891764 + 650719068197 v) + g (-53248 (13167883364 + 47011699885 v) + g (-53248 (60953311377 + 41380414318 v) + g (439296 (135097299 + 494527126 v) + g (1024 (243593824927 + 139782642471 v) + g (-3840 (789777040 + 2966428267 v) + g (-384 (31538947209 + 14681530624 v) + g (1344 (65988302 + 254563911 v) + g (32 (10956325233 + 3879219314 v) + g (-16 (82612840 + 327795373 v) + g (-16 (336692591 + 80633234 v) + g (7750196 + 31711912 v + g (34029872 + 4137332 v - g (7717 + 38501 g + 32732 v))))))))))))))))))))))) + 4 c6 (-2 + g) (4 + g) (-134217728 (39959340 + 73072991 v) + g (-33554432 (27566412 + 13112231 v) + g (4194304 (1791549124 + 3128633863 v) + g (1048576 (1149658140 + 451111547 v) + g (-1048576 (4384776923 + 7229710607 v) + g (-131072 (5152850938 + 1562486495 v) + g (3670016 (440071861 + 675133242 v) + g (16384 (12992337652 + 2710274191 v) + g (-16384 (21979900294 + 30785578535 v) + g (-2048 (20247861528 + 2214387493 v) + g (1024 (51944255726 + 64795011495 v) + g (10752 (480291004 + 3315665 v) + g (-256 (20576500636 + 22119358113 v) + g (768 (-534576069 + 51686801 v) + g (256 (1350473620 + 1196378053 v) + g (224 (89971589 - 17865613 v) + g (-16 (904601291 + 619649201 v) + g (8 (-70631636 + 20853229 v) + g (128 (2788597 + 1336848 v) + g (7660334 - 2932868 v + g (-4396867 - 1221679 v + g (-32926 + 17837 g + 3 (5033 + 526 g) v))))))))))))))))))))))) + c^{8} (536870912 (1058703852 + 1391517113 v) + g (-33554432 (1174761896 + 4281691041 v) + g (-16777216 (45048178540 + 55173374743 v) + g (150994944 (290946358 + 1083903629 v) + g (6291456 (69492949598 + 78478404621 v) + g (-3145728 (6606693172 + 25174581887 v) + g (-2621440 (55104760704 + 56664546707 v) + g (262144 (20792853448 + 81099568825 v) + g (32768 (918210784510 + 846777246307 v) + g (-8192 (106062128808 + 423787890149 v) + g (-12288 (334831989470 + 271705850637 v) + g (53248 (1630928728 + 6681732193 v) + g (1024 (364231937249 + 253715500403 v) + g (-2560 (2117219080 + 8902180089 v) + g (-1536 (14385989292 + 8314055639 v) + g (384 (530348392 + 2291126279 v) + g (64 (12834032784 + 5849472139 v) + g (-3296 (1288424 + 5727771 v) - g (17651343536 + 5828423440 v + g (-16 (2591994 + 11890109 v) + g (-4 (46426863 + 9334477 v) + g (123788 + 589100 v + g (630055 + 42886 v)))))))))))))))))))))))))) + c^{7} (163208757248 (2113195 + 3244414 v) + g (-268435456 (94299263 + 391818800 v) + g (-134217728 (3717820464 + 5341652635 v) + g (16777216 (1848061279 + 7842456384 v) + g (16777216 (18879199495 + 25139589753 v) + g (-12582912 (1294121813 + 5613760710 v) + g (-1048576 (110630850697 + 134981903931 v) + g (20185088 (237836419 + 1055649306 v) + g (1310720 (20693702225 + 22819155276 v) + g (-65536 (13335004963 + 60623766803 v) + g (-16384 (258218812328 + 253029115913 v) + g (106496 (956265607 + 4457579802 v) + g (2048 (218101168713 + 185870417344 v) + g (-16384 (465235036 + 2226098237 v) + g (-1024 (30967684801 + 22301669063 v) + g (768 (465888465 + 2291103788 v) + g (1024 (1437292833 + 839496163 v) + g (-896 (11096654 + 56179531 v) + g (-224 (188109211 + 83550632 v) + g (16 (9097775 + 47553247 v) + g (8 (83074477 + 24951824 v) + g (-868412 - 4712236 v + g (-4508190 - 687188 v + g (915 + 5820 g + 5216 v))))))))))))))))))))))))))))/H (4.A1.13)

q_{21}* = -((2 + c) (($(-4 + g)^{7}$ $(-2 + g)^{8}$ (2 + g)7 $(4 + g)^{8}$ v + c $(-4 + g)^{6}$ $(-2 + g)^{7}$ (2 + g)6 $(4 + g)^{7}$ (16 + 2 (-5 + g) g (2 + g) + 960 v + g (8 + (-150 + g) g) v) - c^{3} $(-4 + g)^{3}$ $(-2 + g)^{4}$ $(2 + g)^{3}$ $(4 + g)^{4}$ (2 (-4 + g) (2 + g) (26542080 + g (-37115904 + g (-14158336 + g (21752832 + g

$$
(2874816 + g (-4567648 + g (-276976 + g (407032 + 3 g (3932 + g (-4568 + g (-33 + 34
g))))))))))) + (-8448376832 + g (-201654272 + g (6752059392 + g (106082304 + g (-
2081100800 + g (-14566400 + g (309567616 + g (-527552 + g (-22496320 + g (211776 + g
(690864 + 7 g (-1376 + g (-722 + 5 g))))))))))))) v) + 144 c^{30} (13 g - 8 (4 + 7 v)) + 48 c^{29} (g
(2738 + 219 v) - 32 (194 + 353 v)) - 16 c^{28} (g (-273928 + 9 g (-364 + 179 g - 763 v) - 39498
v) + 32 (18028 + 34159 v)) + 8 c^{25} (4 g (502780144 + g (48162880 + g (-28533316 + 3 g (-
138545 + 83746 g)))) +g (569286688 + g (455553472 + 3 g (-6548944 + g (-1420772 +
19551 g)))) v - 1024 (3428902 + 7402759 v)) + c^{21} (g9 (37787566 + 1977521 v) - 8 g^{8}
(6402651 + 21967487 v) - 2883584 (17283120 + 45462617 v) - 16 g^{7} (509921025 +
77509306 v) + 448 g^{6} (24475925 + 79565844 v) + 256 g^{5} (1614567494 + 396603831 v) -
4096 g^{3} (1687077759 + 563241868 v) + 24576 g (1422742658 + 593359395 v) - 2048 g^{4}
(273189713 + 833981693 v) + 16384 g^{2} (582722735 + 1657216734 v)) - 16 c^{27} (256
(44557 + 88045 v) + g (-40 (144836 + 28597 v) + 3 g (g (33625 + 2214 v) - 8 (7993 +
17416 v)))) + 8 c^{26} (-256 (1264308 + 2609327 v) + g (32 (5466757 + 1328313 v) + g (32
(336352 + 762895 v) + g (9 g (-3474 + 1983 g - 8566 v) - 8 (752546 + 92115 v))))) + c^{20} (-
2883584 (73189588 + 203287569 v) + g (16384 (9457878156 + 4244302925 v) + g (24576
(2044541780 + 6121265237 v) + g (-8192 (4643076219 + 1719481516 v) + g (2 g
(1508173998080 + g (57039822336 + g (-44449863104 + g (-529886744 + g (410357524
+ (700270 - 530153 g) g))))) + g (864510977536 + g (388913300352 + g (-17539005632 +
g (-3809408856 + g (83661386 + 5259115 g))))) v - 2048 (1905848966 + 6111170655
v)))))) + 4 c^{23} (-16384 (26089843 + 61943888 v) + g (1024 (265915367 + 93910126 v) + g
(512 (94575175 + 244309443 v) + g (-64 (497880403 + 128460872 v) + g (-96 (14268779
+ 39757182 v) + g (919752752 + 8162336 g - 5505883 g^{2} + 4 (36484088 + (6074851 -
74829 g) g) v)))))) + 4 c^{24} (-2048 (29593492 + 66942925 v) + g (128 (286421408 +
91475229 v) + g (64 (77242300 + 190736507 v) + g (-64 (48298603 + 10474701 v) + g (-8
(10571878 + 28229247 v) + g (53956576 + 5976276 v + 3 g (56428 - 36119 g + 161274
v))))))) + c^{22} (-131072 (76631676 + 191317663 v) + g (8192 (820527549 + 316260670 v) +
g (10240 (147094516 + 398273413 v) + g (-12288 (84682978 + 25124963 v) + g (-128
(495769464 + 1444325333 v) + g (256 (174760951 + 35527359 v) + g (750991136 +
825293 g^{3} + 2333619632 v - 736 g (723619 + 75744 v) - 20 g^{2} (58829 + 193139 v)))))))) +
2 c^{19} (-1048576 (366561281 + 1077641215 v) + g (32768 (8999371888 + 4330962419 v) +
g (32768 (3395568375 + 10729487086 v) + g (-61440 (1431622469 + 582511938 v) + g (-
4096 (2698738321 + 9113325836 v) + g (2048 (4356882443 + 1424750719 v) + g (-4 g
(91663302080 + g (1732519092 + g (-1406075850 + g (-6834723 + 5439197 g)))) + g (-
88508588608 + g (-26169058720 + g (842101364 + (107788882 - 1122833 g) g))) v +
4608 (97639892 + 350020695 v)))))))) + c^{2} (-4 + g)^{4} (-2 + g)^{5} (2 + g)^{4} (4 + g)^{5} (8192 (116 +
3471 v) + g (4096 (-278 + 113 v) + g (-512 (1568 + 35045 v) + g (1024 (643 - 160 v) + g
(64 (3586 + 62379 v) + g (224 (-589 + 20 v) + g (-144 (179 + 2558 v) + g (16 (682 + 151
v) + g (16 (59 + 748 v) + g (g (-3 + g - 37 v) - 2 (163 + 68 v))))))))))) + c^{18} (-19922944
(121582876 + 379327221 v) + g (2490368 (778724467 + 401220321 v) + g (9961472
(84074805 + 281165236 v) + g (-622592 (1109115666 + 492753191 v) + g (-8192
(12635425695 + 45064906802 v) + g (4096 (21280315401 + 7823464694 v) + g (1024
(5459413918 + 20640263933 v) + g (-256 (18585344898 + 5299493903 v) + g (-32
(4017158174 + 15983512887 v) + g (16 (6823259691 + 1341174121 v) + g (1009593344 +
g (-843130728 + g (-1185860 + 958483 g - 5090138 v) - 85668700 v) + 4192012852
v))))))))))) + c^{5} (-4 + g) (-2 + g)^{2} (2 + g) (4 + g)^{2} (335544320 (74881 + 888524 v) + g
(8388608 (-3155722 + 1355095 v) + g (-2097152 (16193867 + 163885641 v) + g (-
1048576 (-28019071 + 9612509 v) + g (131072 (145053219 + 1273076300 v) + g (65536
(-209397823 + 51550156 v) + g (-32768 (175266008 + 1354328545 v) + g (-8192 (-
428670327 + 60162928 v) + g (2048 (497971709 + 3437478310 v) + g (1024 (-524028727
+ 15357304 v) + g (7 g^{9} (726 + 79 v) - 2 g^{8} (7544 + 38443 v) - 4 g^{7} (299008 + 67211 v) +
$$

64 g^5 (1322164 + 310077 v) + 2304 g (21695875 + 1731404 v) + 8 g^6 (425885 + 2211639 v) - 128 g^3 (21648432 + 3780271 v) - 64 g^4 (3523939 + 19078486 v) + 128 g^2 (52657627 + 303703616 v) - 512 (211967203 + 1326690186 v)))))))))))) + c^{17} (-79691776 (83191506 + 276258073 v) + g (2490368 (2221527938 + 1224987735 v) + g (24903680 (108321008 + 384574655 v) + g (-1245184 (1860927389 + 899316675 v) + g (-49152 (8254625791 + 31193685891 v) + g (12288 (28974519976 + 11858995801 v) + g (4096 (6848388409 + 27395409112 v) + g (-14848 (1677240808 + 552412889 v) + g (-128 (7024237841 + 29545394171 v) + g (798001206560 + 194145968624 v + g (800 (14648986 + 64270203 v) + g (-4 (2563663621 + 386831618 v) + g (-41130820 + 34944148 g - 186506788 v + 1815789 g v))))))))))))) + c16 (-1354760192 (11664220 + 41364247 v) + g (84672512 (162165592 + 95779675 v) + g (42336256 (176518340 + 667625933 v) + g (-42336256 (157819229 + 82805793 v) + g (-835584 (1607903310 + 6460991603 v) + g (417792 (2937937040 + 1330367073 v) + g (4096 (28234834642 + 119942661765 v) + g (-1024 (104490828136 + 39174535659 v) + g (-384 (12820022026 + 57215303659 v) + g (32 (142535625344 + 41449774931 v) + g (95876350560 + 446111618128 v + g (-16 (5486616310 + 1099309371 v) + g (-4 (167626837 + 805954081 v) + g (597804520 + 61846690 v + g (729586 - 623847 g + 3589816 v))))))))))))))) + c^{15} (-10838081536 (3030641 + 11520398 v) + g (677380096 (43834027 + 27782205 v) + g (338690048 (52739705 + 213329887 v) + g (-42336256 (392306005 + 223420668 v) + g (-1114112 (3394542679 + 14562378450 v) + g (557056 (6452186761 + 3220928928 v) + g (8192 (48395434556 + 219211694893 v) + g (-6144 (62161103017 + 26279157603 v) + g (-512 (42225438689 + 200779418244 v) + g (256 (81598919873 + 27748563670 v) + g (320 (1835559151 + 9095510547 v) + g (-32 (17565118053 + 4408708210 v) + g (-8 (852880199 + 4366107264 v) + g (72 (88535149 + 13764255 v) + g (22202340 - 19951681 g + 116306596 v - 1066224 g v))))))))))))))) + c^{14} (-2709520384 (21976684 + 89933405 v) + g (338690048 (165213163 + 112689044 v) + g (338690048 (108936100 + 473352141 v) + g (-338690048 (105266186 + 65144737 v) + g (-10027008 (906036356 + 4168533447 v) + g (8912896 (1007013467 + 553404143 v) + g (57344 (19864862202 + 96387558487 v) + g (-4096 (278744757758 + 132132311171 v) + g (-512 (151161157418 + 769359007107 v) + g (256 (304367373911 + 119362342984 v) + g (64 (43665081156 + 231499238957 v) + g (-256 (10905977309 + 3313998092 v) + g (-32 (1516899735 + 8306831153 v) + g (80 (593243235 + 124032409 v) + g (4 (78746372 + 441244721 v) + g (-297000304 - 329251 g + 295242 g^2 - 3 (10668512 + 623237 g) v)))))))))))))))) - c^4 $(-4 + g)^2$ $(-2 + g)^3$ (2 + g)2 $(4 + g)^3$ (-4194304 (29016 + 431683 v) + g (-2359296 (-56752 + 23899 v) + g (262144 (546425 + 6721631 v) + g (32768 (-3787766 + 1210803 v) + g (-16384 (4090429 + 42560367 v) + g (4096 (11397545 - 2300321 v) + g (2048 (7828448 + 70353075 v) + g (512 (-17894365 + 1254347 v) + g (-256 (8164817 + 64703183 v) + g (128 (7826704 + 533223 v) + g (64 (2270230 + 16226773 v) + g (-128 (468687 + 91789 v) + g (-32 (150355 + 996234 v) + g (1771568 + 489392 v + g (56224 + 357708 v + g (g (-99 + 33 g - 623 v) - 18 (1073 + 238 v))))))))))))))))) + 2 c^{13} (-318767104 (147518274 + 653097469 v) + g (9961472 (4608167082 + 3395726591 v) + g (1354760192 (24308035 + 114044389 v) + g (-1016070144 (32528250 + 21929033 v) + g (-4456448 (2093548611 + 10384067977 v) + g (2228224 (4297667603 + 2601215229 v) + g (16384 (84138067377 + 439637669476 v) + g (-4096 (350737413325 + 185914365051 v) + g (-1024 (111458799491 + 610456948767 v) + g (128 (934646923522 + 419132197195 v) + g (256 (20636904067 + 117687942170 v) + g (-96 (57345683785 + 20660536258 v) + g (-32 (4003281474 + 23576761223 v) + g (40 (3273082305 + 869116642 v) + g (64 (21590192 + 130096617 v) + g (-28 (48753213 + 8004904 v) + g (-4322589 + 4070780 g - 26395884 v + 229267 g v))))))))))))))))) + c11 (-268435456 (571680065 + 3017077917 v) + g (8388608 (19238751324 + 16817788345 v) + g (478150656 (281314615 + 1567796724 v) + g (-19922944 (7298573157 + 5915119094 v) + g (-35651584 (1377895561 + 8096291904 v) +

g (18164482048 (2991763 + 2215159 v) + g (262144 (37089415678 + 229124119723 v) + g (-32768 (333772368568 + 221580161699 v) + g (-106496 (10666133827 + 68974433410 v) + g (26624 (48384447428 + 28092649273 v) + g (13312 (6004010265 + 40395379751 v) + g (-13312 (6790008775 + 3330693557 v) + g (-128 (25828911009 + 179396840032 v) + g (320 (11501523303 + 4526222057 v) + g (74540215776 + 529636744384 v + g (-16 (5033390793 + 1457663900 v) + g (-104 (7442345 + 53585136 v) + g (800290184 + 2410700 g - 2356529 g2 + 4 (35765770 + (4357652 - 36047 g) g) v)))))))))))))))) + c^{12} (-637534208 (202506996 + 975545753 v) + g (358612992 (364865036 + 291925235 v) + g (1693450240 (59918756 + 305327481 v) + g (-338690048 (312128085 + 230104388 v) + g (-8912896 (3676800694 + 19779533671 v) + g (8912896 (3918784454 + 2618160447 v) + g (851968 (6613325040 + 37439905073 v) + g (-212992 (28648619034 + 16977059353 v) + g (-26624 (20970026522 + 124352470011 v) + g (6656 (91480140580 + 46703873939 v) + g (3328 (9680906210 + 59751085731 v) + g (-3328 (10513226532 + 4434633113 v) + g (-64 (16233663895 + 103453579453 v) + g (192 (5774599021 + 1884658482 v) + g (16 (1048044209 + 6833131918 v) + g (-16 (1083414705 + 242892076 v) + g (-4 (26170723 + 172910734 v) + g (g (108874 - 101479 g + 722426 v) + 4 (25843242 + 2980565 v))))))))))))))))))) + c^{10} (-1476395008 (106397356 + 620180393 v) + g (16777216 (10213029001 + 9830527625 v) + g (318767104 (479030280 + 2944061287 v) + g (-717225984 (238718342 + 214177345 v) + g (-17825792 (3512546473 + 22732897565 v) + g (8912896 (8063489711 + 6655718404 v) + g (262144 (53917459726 + 366544113957 v) + g (-65536 (251889146486 + 188149533509 v) + g (-319488 (6017297382 + 42795283411 v) + g (53248 (42554132817 + 28153958645 v) + g (146432 (1106187800 + 8182280013 v) + g (-292864 (650953023 + 370521430 v) + g (-1536 (5441449398 + 41543004793 v) + g (384 (25255440398 + 11860731905 v) + g (192 (1306056723 + 10199728427 v) + g (-192 (1473189117 + 534547363 v) + g (-80 (48734155 + 385642514 v) + g (16 (263457349 + 65565821 v) + g (8 (3030887 + 24083852 v) + g (5 g (-5210 + 5003 g - 41247 v) - 8 (3102447 + 396460 v))))))))))))))))))))) + c^9 (-5905580032 (23273194 + 151316265 v) + g (16777216 (9264964502 + 9915211701 v) + g (33554432 (4393653900 + 30077953327 v) + g (-8388608 (20432706681 + 20478058127 v) + g (-2097152 (32161736715 + 231597454619 v) + g (524288 (153163482840 + 142084497829 v) + g (1048576 (16426704665 + 124151820088 v) + g (-131072 (159301819404 + 134806076551 v) + g (-32768 (82345209499 + 650719068197 v) + g (4096 (806634006414 + 611152098505 v) + g (106496 (2518313168 + 20690207159 v) + g (-39936 (8220474909 + 5439798386 v) + g (-1024 (16486980895 + 139782642471 v) + g (1280 (15939253138 + 8899284801 v) + g (384 (1692993365 + 14681530624 v) + g (-192 (3983473888 + 1781947377 v) + g (-32 (441498939 + 3879219314 v) + g (16 (997118322 + 327795373 v) + g (16 (9139607 + 80633234 v) + g (-4 (39130009 + 7927978 v) + g (-470736 - 4137332 v + g (473029 + 32732 v)))))))))))))))))))))))) + c^6 (-2 + g) (4 + g) (536870912 (7413620 + 73072991 v) + g (134217728 (-29905064 + 13112231 v) + g (-16777216 (363414148 + 3128633863 v) + g (-4194304 (-1242436892 + 451111547 v) + g (4194304 (949682641 + 7229710607 v) + g (524288 (-5571291702 + 1562486495 v) + g (-524288 (2776699033 + 18903730776 v) + g (-65536 (-14139758488 + 2710274191 v) + g (65536 (5002487296 + 30785578535 v) + g (8192 (-22373443804 + 2214387493 v) + g (-20480 (2304669232 + 12959002299 v) + g (-6144 (-3818848766 + 23209655 v) + g (3072 (1419226820 + 7373119371 v) + g (-3072 (635663601 + 51686801 v) + g (-1024 (246301746 + 1196378053 v) + g (896 (115215521 + 17865613 v) + g (64 (134686429 + 619649201 v) + g (-32 (102265200 + 20853229 v) + g (-64 (2419577 + 10694784 v) + g (8 (6920343 + 1466434 v) + g (1132452 + 4886716 v + g (-387232 - 1470 g + 489 g^2 - 12 (5033 + 526 g) v))))))))))))))))))))))) + c^8 (-536870912 (189538308 + 1391517113 v) + g (33554432 (3554918192 + 4281691041 v) + g (16777216 (7146386860 + 55173374743 v) + g (-16777216 (8612912327 + 9755132661 v) + g (-6291456 (9673109266 +

$$78478404621 v) + g (3145728 (23885248760 + 25174581887 v) + g (524288 (33297662986 + 283322733535 v) + g (-262144 (83764445747 + 81099568825 v) + g (-32768 (95230564042 + 846777246307 v) + g (8192 (484372223528 + 423787890149 v) + g (53248 (6784665106 + 62701350147 v) + g (-53248 (8633557118 + 6681732193 v) + g (-1024 (26608656371 + 253715500403 v) + g (24064 (1424121656 + 947040435 v) + g (1536 (852567178 + 8314055639 v) + g (-384 (4180779038 + 2291126279 v) + g (-448 (84579856 + 835638877 v) + g (32 (1395028228 + 589960413 v) + g (587575696 + 5828423440 v + g (-16 (41047466 + 11890109 v) + g (-4 (944693 + 9334477 v) + g (3967612 + 589100 v + g (4385 - 4301 g + 42886 v)))))))))))))))))))))))) + c^{7} (-4294967296 (14642039 + 123287732 v) + g (268435456 (284570591 + 391818800 v) + g (671088640 (120785735 + 1068330527 v) + g (-16777216 (6034957391 + 7842456384 v) + g (-25165824 (1804766419 + 16759726502 v) + g (12582912 (4620381281 + 5613760710 v) + g (7340032 (1981261952 + 19283129133 v) + g (-262144 (72385134743 + 81284996562 v) + g (-393216 (7482165881 + 76063850920 v) + g (65536 (59261898941 + 60623766803 v) + g (16384 (23955285405 + 253029115913 v) + g (-106496 (4872972841 + 4457579802 v) + g (-2048 (17058776905 + 185870417344 v) + g (8192 (5579093581 + 4452196474 v) + g (1024 (2001535892 + 22301669063 v) + g (-768 (3406498985 + 2291103788 v) + g (-1024 (74372801 + 839496163 v) + g (128 (730554632 + 393256717 v) + g (224 (7373097 + 83550632 v) - g (1925471568 + 760851952 v + g (8 (2210739 + 24951824 v) + g (-4 (4844129 + 1178059 v) + g (-61518 - 687188 v + g (62943 + 5216 v))))))))))))))))))))))))))))/H \tag{4.A1.14}$$

Given that the optimal wages of the first period as well as optimal quantities of the second period, depend on c, g, and v then the optimal quantities of the first period are

$$q_{10}^{*}= -((2 + c) (2304 c^{31} + (-4 + g)^{7} (-2 + g)^{8} (2 + g)^{7} (4 + g)^{8} - 768 c^{30} (-206 + 9 v) + c^{4} (-4 + g)^{2} (-2 + g)^{3} (2 + g)^{2} (4 + g)^{3} (-2 (-4 + g) (2 + g) (82085937152 + g (-24858591232 + g (-60293652480 + g (16874180608 + g (16903202816 + g (-4329181696 + g (-2197249664 + g (516976704 + g (118296640 + g (-28151120 + g (41896 + g (562440 + g (-180230 + g (-2759 + 2164 g)))))))))))))) + (182552887296 + g (-36436967424 + g (-158518083584 + g (28053438464 + g (54776610816 + g (-8334983168 + g (-9573185536 + g (1195178496 + g (887331584 + g (-84204928 + g (-41650752 + g (2634880 + g (853632 + g (-29800 + g (-5716 + 5 g (14 + g)))))))))))))))) v) - 32 c^{29} (9 g (-44 + 151 g - 45 v) + 16 (-10142 + 873 v)) + 4 c^{23} (609193 g^{8} + 32 g^{4} (686095076 - 57214719 v) - 6 g^{7} (49041 + 38686 v) + 16 g6 (-20236339 + 635867 v) + 8 g^{5} (18739424 + 15799511 v) - 24576 (-76009744 + 26089843 v) - 128 g^{3} (69349295 + 61742251 v) + 512 g (214142564 + 199344033 v) + 256 g^{2} (-1552626516 + 268810175 v)) + 96 c^{28} (32 (35326 - 4507 v) + g (7864 + 7956 v + g (-27736 + 777 v))) - c (-4 + g)^{5} (-2 + g)^{7} (2 + g)^{5} (4 + g)^{7} (64 (112 + 3 v) + g (1728 + g (g (-236 + g (132 + (-5 + g) g)) - 8 (248 + 5 v)))) + 16 c^{27} (-384 (-264118 + 44557 v) + g (8 (169244 + 169083 v) + 3 g (-1625024 + 6233 g^{2} + 90948 v - 6 g (407 + 393 v)))) - 16 c^{26} (768 (-1504994 + 316077 v) + g (-64 (389425 + 383469 v) + g (90971344 - 7651456 v + g (404268 - 1033139 g + 6 (63948 + 3479 g) v)))) - 32 c^{25} (256 (-20390242 + 5143353 v) + g (-176 (939452 + 909957 v) + g (607224496 - 68459936 v + g (5360408 + 4990932 v + g (-13638518 + 33999 g^{2} + 555128 v - 18 g (839 + 743 v)))))) + 32 c^{24} (-512 (-74915374 + 22195119 v) + g (1728 (974802 + 926729 v) + g (1072 (-5759116 + 819245 v) + g (-16 (5685227 + 5182042 v) + g (228983600 - 14124754 v + g (768444 + 664731 v + g (-1699153 + 26340 v))))))) + 2 c^{18} (-458227712 (-5547982 + 3964659 v) + g (14942208 (36426181 + 29237007 v) + g (1245184 (-1244535119 + 478131090 v) + g (-311296 (518225345 + 393516198 v) + g (-77824 (-4059916813 + 890950152 v) + g (32768 (498309941 + 354897786 v) + g (4 g (-168160426368 + g (273815462736 + g$$

(2599168100 + g (-4233632686 + g (-10183277 + 17162204 g))))) - g (444573098496 + g (74778504928 + g (-6294192704 + g (-539287288 + g (22224322 + 573823 g))))) v + 10240 (-2724106289 + 342231757 v)))))))) + c^{15} (-124637937664 (-258805 + 395301 v) + g (27433893888 (729100 + 511747 v) + g (6943145984 (-5304244 + 3659417 v) + g (-84672512 (115802057 + 76493833 v) + g (-21168128 (-639631472 + 239690563 v) + g (835584 (2177286304 + 1342471603 v) + g (2 g (-80310666905600 + g (99360249230848 + g (3477292914816 + g (-4381258008064 + g (-68168822960 + g (89925859664 + g (474046060 + g (-671093830 + g (-506237 + 788652 g))))))))) + g (-91520458825728 + g (-25273053450752 + g (3616936631040 + g (632206837184 + g (-63729063968 + g (-6676483608 + g (390594892 + 7 (2774580 - 51203 g) g))))))) v + 557056 (-4121860401 + 894783086 v)))))))) + 4 c^{22} (-1507328 (-6330391 + 2498859 v) + g (4096 (178615631 + 162395244 v) + g (1024 (-2536368909 + 522610513 v) + g (-256 (323785091 + 280878482 v) + g (-320 (-624556945 + 66292993 v) + g (64 (36442688 + 29875535 v) + g (-4889290768 + 234316432 v - g (13730204 - 27476607 g + 4 (2628329 + 84835 g) v)))))))) + c^2 $(-4 + g)^4$ $(-2 + g)^5$ $(2 + g)^4$ $(4 + g)^5$ (16384 (1502 + 87 v) + g (-165888 (3 + 2 v) + g (-1536 (9869 + 470 v) + g (256 (1608 + 505 v) + g (64 (50365 + 1868 v) + g (-32 (3605 + 417 v) + g (-16 (16717 + 425 v) + g (200 (63 + v) + g (5684 + 3 g (-144 + 37 g) + 60 v))))))))) + 2 c^{21} (-66322432 (-1244087 + 563580 v) + g (811008 (10061396 + 8909679 v) + g (4096 (-6847051230 + 1656373327 v) + g (-1024 (1203311648 + 1014382387 v) + g (1280 (2242611952 - 292376365 v) + g (64 (812385032 + 645888335 v) + g (64 (-1641080072 + 107044959 v) + g (-8 (76506236 + 56717537 v) + g (1176886976 - 29615568 v + g (945856 - 1821487 g + 644763 v)))))))))) + 2 c^{20} (-18087936 (-16782310 + 8750929 v) + g (360448 (106972310 + 91985037 v) + g (8192 (-15455479988 + 4359075519 v) + g (-32768 (228385223 + 186547817 v) + g (-2048 (-8140226747 + 1275421868 v) + g (1024 (431633053 + 331871568 v) + g (128 (-6665766487 + 557485701 v) + g (-64 (135481885 + 96972566 v) + g (15910057480 - 614090176 v + g (40200008 - 73741070 g + 26423008 v + 743457 g v)))))))))) + 2 c^{19} (-12058624 (-78965086 + 47812341 v) + g (360448 (432725956 + 360143157 v) + g (16384 (-29395128584 + 9653891735 v) + g (-8192 (4618154343 + 3643414103 v) + g (-4096 (-19435642065 + 3613824092 v) + g (57344 (51943846 + 38503737 v) + g (256 (-21253887656 + 2201137353 v) + g (-320 (273895308 + 188701403 v) + g (152227091616 - 8047566656 v + g (812745464 + 513542464 v + g (-1410511272 + g (-1061204 + 1904987 g - 604869 v) + 29119898 v))))))))))) - 4 c^6 (-2 + g) (4 + g) (-268435456 (21121648 + 5560215 v) + g (33554432 (18273667 + 7468962 v) + g (41943040 (170930666 + 44502491 v) + g (-2097152 (428554663 + 140389039 v) + g (-1048576 (3661551089 + 951929387 v) + g (3145728 (178877422 + 46609757 v) + g (3407872 (330756873 + 87559114 v) + g (-32768 (6011019221 + 1231665313 v) + g (-16384 (11889123308 + 3341084831 v) + g (-2 g (-9659601046528 + g (2892159663104 + g (435798686464 + g (-250440580160 + g (8565844192 + g (13290769568 + g (-2002350816 + g (-399547608 + g (90760750 + g (5754127 + 6 g (-269678 + g (-4336 + 1413 g)))))))))))) + g (6372458039296 + g (-694258625536 + g (-470784214016 + g (44197266816 + g (21466373312 + g (-1659026880 + g (-567543968 + g (33521600 + g (7693352 + g (-302506 + g (-40887 + 766 g + 36 g^2))))))))))) v + 16384 (2585002373 + 409853994 v))))))))))) + c^3 $(-4 + g)^3$ $(-2 + g)^4$ $(2 + g)^3$ $(4 + g)^4$ (2621440 (2566 + 243 v) + g (-16384 (14180 + 8397 v) + g (-16384 (317733 + 26626 v) + g (4096 (55327 + 19374 v) + g (1024 (1487945 + 108426 v) + g (-256 (323488 + 61991 v) + g (-128 (1610729 + 99830 v) + g (64 (220259 + 19667 v) + g (128 (94213 + 4953 v) + g (-224 (4977 + 142 v) + g (-64 (2227 + 156 v) + g (33472 - 7174 g - 109 g^2 + 37 g^3 + 30 (6 + g) v)))))))))))) + 2 c^{17} (-2749366272 (-2086174 + 1808511 v) + g (2490368 (661619932 + 510119493 v) + g (2490368 (-1693851157 + 770141344 v) + g (-61636608 (9506242 + 6921329 v) + g (-1556480 (-673770155 + 174698919 v) + g (61440 (1206510868 + 822507005 v) + g (4096

(-28504120718 + 4297287319 v) + g (-512 (7952355544 + 5023661051 v) + g (64 (96000327362 - 8186277385 v) + g (94411191200 + 54563802848 v + g (16 (-8934571387 + 392312855 v) + g (-6 (123325966 + 64175411 v) + g (1161380430 - 19983508 v + g (857056 - 1433569 g + 393553 v)))))))))))))) + 2 c^{16} (-31159484416 (-345142 + 380355 v) + g (169345024 (25652866 + 18916989 v) + g (42336256 (-228333700 + 125521241 v) + g (-465698816 (3919921 + 2724894 v) + g (-26460160 (-110205368 + 34045169 v) + g (417792 (675601924 + 439032875 v) + g (139264 (-2890916774 + 521577049 v) + g (-303104 (65767492 + 39547849 v) + g (27483078451456 - 2873215486848 v + g (64 (10112908235 + 5556902978 v) + g (-900445141632 + 51480531120 v + g (-8 (1057073203 + 522515576 v) + g (12241027718 - 326965506 v + g (29393932 + 12813558 v + g (-45458787 + 317600 v))))))))))))))) + 2 c^{11} (-4630511616 (14140974 + 24855655 v) + g (79691776 (914920772 + 495437613 v) + g (39845888 (356264320 + 2402348541 v) + g (-338690048 (177416081 + 89921759 v) + g (-4233625600 (-3850033 + 7795654 v) + g (4456448 (4549578272 + 2141524005 v) + g (2228224 (-4080912196 + 2750915083 v) + g (-14057472 (257626058 + 111555371 v) + g (-53248 (-38052874614 + 12527046857 v) + g (26624 (13893905639 + 5469857503 v) + g (6656 (-36182349092 + 6507902775 v) + g (-768 (28357959617 + 10002316764 v) + g (-192 (-84136682456 + 8511360957 v) + g (96 (7371739563 + 2286402517 v) + g (2 g (-5675710920 + g (5495270696 + g (34727220 + g (-39269767 + g (-34959 + 47360 g))))) + g (-3021378720 + g (-301889536 + g (15350356 + (801839 - 12242 g) g))) v + 96 (-6218484163 + 344596025 v)))))))))))))))) + c^{14} (-21994930176 (-1579054 + 4060909 v) + g (327851966464 (122797 + 81594 v) + g (677380096 (-83644731 + 77488985 v) + g (-169345024 (134462629 + 83955134 v) + g (-42336256 (-613134655 + 288014277 v) + g (2228224 (2240893744 + 1304165047 v) + g (557056 (-9699531383 + 2573064991 v) + g (-40960 (13155657553 + 7066458899 v) + g (1024 (570684633368 - 88472031443 v) + g (512 (58600013999 + 28699524000 v) + g (256 (-131295433317 + 11773444046 v) + g (-576 (1429999361 + 628916637 v) + g (975568277664 - 47640543040 v + g (9549610800 + 3699439232 v - g (12223232068 - 276721540 v + g (30604868 - 43333205 g + 3 (3394828 + 84387 g) v)))))))))))))))) + c^{13} (-14663286784 (-986366 + 9620757 v) + g (677380096 (104594708 + 65393091 v) + g (677380096 (-96571521 + 138349030 v) + g (-338690048 (135215860 + 79324321 v) + g (-84672512 (-473048956 + 295899709 v) + g (1114112 (10516510792 + 5743069969 v) + g (1114112 (-9321996568 + 3116138649 v) + g (-24576 (61746788068 + 31085066441 v) + g (-2048 (-682090880650 + 130629466877 v) + g (1024 (103161483265 + 47304411726 v) + g (512 (-200578677891 + 22298556358 v) + g (-448 (8633532038 + 3552200017 v) + g (64 (63046203197 - 3939622371 v) + g (67286572560 + 24371086496 v + g (40 (-1916628230 + 60928871 v) + g (-4 (107857940 + 33547161 v) + g (548056032 - 6683090 v + g (439710 - 638290 g + 113581 v)))))))))))))))))) + c^{12} (-1543503872 (26726146 + 125466291 v) + g (1434451968 (75914582 + 44348553 v) + g (39845888 (-1063359672 + 3623957269 v) + g (-2709520384 (29441569 + 16117658 v) + g (-1693450240 (-27554218 + 25993537 v) + g (62390272 (377467414 + 192125605 v) + g (2228224 (-7135493237 + 3186306539 v) + g (-851968 (4231601529 + 1983346766 v) + g (-1224704 (-2194509276 + 534871465 v) + g (26624 (11523531175 + 4914883006 v) + g (6656 (-37599250176 + 5238902995 v) + g (-256 (56432759643 + 21579802328 v) + g (64 (202170814425 - 16014031441 v) + g (32 (10998963279 + 3700489882 v) + g (16 (-21863693427 + 927431680 v) + g (-160 (23515791 + 6791740 v) + g (4218154168 - 81313408 v + g (11507452 - 14873595 g + 2759976 v + 72001 g v)))))))))))))))))) - c^5 (-4 + g) $(-2 + g)^2$ (2 + g) $(4 + g)^2$ (-100663296 (1933312 + 374405 v) + g (33554432 (444196 + 206211 v) + g (1048576 (204003152 + 37989709 v) + g (-1048576 (18410866 + 6395187 v) + g (-131072 (741652640 + 133403359 v) + g (65536 (158336056 + 40543795 v) + g (442368 (52982150 + 9329633 v) + g (-16384 (183462704 + 33817457 v) + g (-2048 (1547199706 + 276808005 v) + g (1024

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\begin{aligned}
&(497323606 + 63667623\ v) + g\ (512\ (439847971 + 89526402\ v) + g\ (-256\ (199705787 + 16846545\ v) + g\ (-4992\ (1065776 + 421737\ v) + g\ (64\ (45707134 + 2357569\ v) + g\ (32\ (-8447469 + 1569286\ v) + g\ (-16\ (5336081 + 152712\ v) + g\ (18007384 - 517912\ v + g\ (28 \\
&(34391 + 486\ v) + g\ (-276154 + 617\ g^2 + 1468\ v - 5\ g\ (357 + 2\ v)))))))))))))))))))) + c10\ (-67914170368\ (3400570 + 3469479\ v) + g\ (738197504\ (229592095 + 114167847\ v) + g \\
&(16777216\ (8354875367 + 12955836964\ v) + g\ (-79691776\ (1958972037 + 910774958\ v) + g\ (-19922944\ (516056523 + 4224298618\ v) + g\ (142606336\ (418656640 + 180584781\ v) \\
&+ g\ (17825792\ (-692928409 + 1000327201\ v) + g\ (-262144\ (46946261005 + 18611139669\ v) + g\ (-212992\ (-20964853598 + 10610135377\ v) + g\ (106496\ (13952287753 + 5024786948\ v) + g\ (53248\ (-13349159817 + 3301665191\ v) + g\ (-11264\ (9494810975 + 3061553193\ v) + g\ (-11264\ (-5522981003 + 734933568\ v) + g\ (2688\ (1662755435 + 471217536\ v) + g\ (960\ (-3205184728 + 232473861\ v) + g\ (-768\ (130727272 + 31782943\ v) + g\ (81491168160 - 3052713856\ v + g\ (64\ (16001489 + 3230012\ v) - g\ (989726316 - 16214004\ v + g\ (3093264 - 3633823\ g + 494696\ v + 14243\ g\ v)))))))))))))))))))) + c^9\ (-12348030976\ (24503978 + 16695987\ v) + g\ (1107296256\ (153559684 + 69343323\ v) + g \\
&(33554432\ (7633825801 + 6254979870\ v) + g\ (-159383552\ (1091893154 + 460557459\ v) + g\ (-5658116096\ (13091465 + 16027206\ v) + g\ (35651584\ (2100326006 + 821177601\ v) + g\ (35651584\ (103266325 + 609833143\ v) + g\ (-1310720\ (13460146304 + 4832693991\ v) + g\ (-32768\ (-83936426758 + 96873998963\ v) + g\ (212992\ (11669806723 + 3803526677\ v) + g\ (106496\ (-6954999985 + 2736418487\ v) + g\ (-11264\ (19064302842 + 5559839213\ v) + g\ (-11264\ (-7924185507 + 1486963295\ v) + g\ (1536\ (7305196745 + 1871579893\ v) + g\ (384\ (-15448131708 + 1510253455\ v) + g\ (-192\ (1750871321 + 384711628\ v) + g\ (128\ (1711186433 - 86706450\ v) + g\ (5144780368 + 938453120\ v + g\ (-4112776080 + 98230712\ v + g\ (-8\ (3885503 + 561553\ v) + g\ (30865340 - 40677\ g2 - 258978\ v + 67\ g\ (479 + 51\ v)))))))))))))))))))))) + c^8\ (-3221225472\ (97729294 + 47384577\ v) + g \\
&(10334765056\ (14141734 + 5721243\ v) + g\ (33554432\ (9716792476 + 5087472035\ v) + g\ (-100663296\ (1642462745 + 620141694\ v) + g\ (-4194304\ (31813971976 + 19513083201\ v) + g\ (6291456\ (12638522980 + 4419689129\ v) + g\ (2097152\ (12096278177 + 10511957174\ v) + g\ (-4194304\ (5062847333 + 1624655527\ v) + g\ (-65536\ (19724834482 + 56054573317\ v) + g\ (163840\ (21073041101 + 6134726294\ v) + g\ (8192\ (-46442512256 + 47967580141\ v) + g\ (-12288\ (28694614403 + 7470502912\ v) + g\ (1024\ (83979685917 - 26435796211\ v) + g\ (1024\ (21996440455 + 5028676621\ v) + g\ (33280\ (-246630715 + 35172408\ v) + g\ (-384\ (2260015375 + 442992744\ v) + g\ (64\ (6616399781 - 466409504\ v) + g\ (32\ (581206605 + 94564576\ v) + g\ (16\ (-732002235 + 24765046\ v) + g\ (-16\ (11706835 + 1509196\ v) + g\ (151591780 - 2090052\ v + g\ (580150 - 614211\ g + 20\ (2755 + 93\ g)\ v))))))))))))))))))))))) + 2\ c^7\ (-1073741824\ (123556532 + 43926117\ v) + g \\
&(67108864\ (793805716 + 282897789\ v) + g\ (33554432\ (4759270828 + 1719889339\ v) + g\ (-16777216\ (3951357179 + 1313264943\ v) + g\ (-20971520\ (3821574596 + 1456578523\ v) + g\ (3145728\ (11262015008 + 3464360485\ v) + g\ (2097152\ (10168395229 + 4379983741\ v) + g\ (-262144\ (40607706931 + 11455420474\ v) + g\ (-327680\ (9321492032 + 5288043135\ v) + g\ (32768\ (60374460875 + 15442303289\ v) + g\ (319488\ (523946264 + 668667335\ v) + g\ (-4096\ (57558874009 + 13159664369\ v) + g\ (1024\ (15348952420 - 16968352593\ v) + g\ (512\ (35306456152 + 7085591841\ v) + g\ (512\ (-6913866393 + 1792110065\ v) + g\ (-1920\ (453549766 + 78022827\ v) + g\ (-512\ (-545819394 + 58728319\ v) + g\ (448\ (55553749 + 7931812\ v) + g\ (32\ (-355772330 + 17460093\ v) + g\ (-40\ (9400615 + 1063474\ v) + g\ (234551656 - 4913244\ v + g\ (2329258 + 194134\ v + g\ (g\ (-2581 + 3070\ g - 139\ v) + 2\ (-988477 + 6563\ v))))))))))))))))))))))))))/H
\end{aligned}
\tag{4.A1.15}
$$

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q_{20}^* = -((2 + c)\ ((-4 + g)^7\ (-2 + g)^8\ (2 + g)^7\ (4 + g)^8 + 96\ c^{29}\ (-6400 + 224\ g - 4656\ v + 135\ g\ v) + 288\ c^{30}\ (g - 8\ (4 + 3\ v)) - 32\ c^{27}\ (-3\ g\ (162128 + g\ (74440 + 2209\ g)) + 9\ g\ (-75148 + g\ (-15158 + 393\ g))\ v + 64\ (196256 + 133671\ v)) + 32\ c^{28}\ (-32\ (19192 + 13521\ v) + g\ (9\ g
$$

(410 + 14 g + 259 v) + 4 (5737 + 5967 v))) - c $(-4 + g)^5$ $(-2 + g)^7$ $(2 + g)^5$ $(4 + g)^7$ (192 (40 + v) + g (2112 + g (g (-324 + g (144 + g)) - 40 (52 + v)))) + 2 c^{21} (-2387607552 (29287 + 15655 v) + g (90112 (30603472 + 80187111 v) + g (-8 g (-17342124032 + g (108510368512 + g (6270389824 + g (-2224269280 + g (-231568316 + g (10808231 + 1603893 g)))))) + g (-1038727564288 + g (-374241747200 + g (41336853440 + g (6850877376 + g (-453740296 + 3 g (-9871856 + 214921 g)))))) v + 4096 (3443931920 + 1656373327 v)))) - 16 c^{26} (256 (1442536 + 948231 v) + g (-32 (454405 + 766938 v) + g (-16 (810823 + 478216 v) + g (-334684 + 39573 g + 3456 g^2 + 6 (63948 + 3479 g) v)))) + 8 c^{24} (-6144 (12150432 + 7398373 v) + g (256 (11521123 + 25021683 v) + g (64 (100367958 + 54889415 v) + g (3 g (-38473008 + g (-2895292 + g (80452 + 10355 g))) + 64 (1921817 - 5182042 v) + 4 g (-14124754 + 3 g (221577 + 8780 g)) v)))) + 32 c^{25} (-256 (8122112 + 5143353 v) + g (82273408 + 160152432 v + g (32 (3763506 + 2139373 v) + g (2690552 - 4990932 v + g (-1091484 - 88611 g - 555128 v + 13374 g v))))) + 8 c^{23} (-4096 (133952864 + 78269529 v) + g (256 (84379468 + 199344033 v) + g (640 (102384908 + 53762035 v) + g (64 (16463923 - 61742251 v) + g (-16 (121674308 + 57214719 v) + g (-135191200 + 3 g2 (487107 - 38686 v) + 63198044 v + 8 g (1515613 + 635867 v))))))) + 2 c^{19} (-12058624 (98753504 + 47812341 v) + g (32768 (1481722172 + 3961574727 v) + g (16384 (22133915276 + 9653891735 v) + g (-385024 (-2570069 + 77519449 v) + g (-16384 (2308599737 + 903456023 v) + g (8192 (-210041698 + 269526159 v) + g (2 g (70211054208 + g (-12917533568 + g (-1644867116 + g (52712956 + 8723791 g)))) + g (-60384448960 + g (-8047566656 + g (513542464 + (29119898 - 604869 g) g))) v + 2304 (699171320 + 244570817 v)))))))) + 2 c^{22} (-3014656 (4466153 + 2498859 v) + g (32768 (16148912 + 40598811 v) + g (2048 (1038748277 + 522610513 v) + g (-512 (-54073431 + 280878482 v) + g (-128 (735349657 + 331464965 v) + g (64 (-93611321 + 59751070 v) + g (32 (36396729 + 14644777 v) - g (-130849184 + 1897066 g + 301093 g^2 + 8 (2628329 + 84835 g) v)))))))) + 2 c^{17} (-916455424 (12520256 + 5425533 v) + g (2490368 (203580296 + 510119493 v) + g (9961472 (492620329 + 192535336 v) + g (-6848512 (4294342 + 62291961 v) + g (-2801664 (276494302 + 97054955 v) + g (12288 (-2010793326 + 4112535025 v) + g (4096 (13684784280 + 4297287319 v) + g (-2 g (936012133632 + g (98586572624 + g (-12633380448 + g (-1768252610 + 45555567 g + 7861659 g2)))) + g (-523921752640 + g (54563802848 + g (6277005680 + g (-385052466 + g (-19983508 + 393553 g))))) v - 512 (-7569673112 + 5023661051 v))))))))) - c^2 $(-4 + g)^4$ $(-2 + g)^5$ $(2 + g)^4$ $(4 + g)^5$ (-16384 (1736 + 87 v) + g (2048 (-665 + 162 v) + g (512 (34603 + 1410 v) + g (256 (2778 - 505 v) + g (-64 (60669 + 1868 v) + g (32 (-3781 + 417 v) + g (80 (4409 + 85 v) + g (7912 - 200 v + g (g (-192 + g (35 + g)) - 4 (2831 + 15 v)))))))))) + c^{15} (-373913812992 (344736 + 131767 v) + g (338690048 (19172596 + 41451507 v) + g (169345024 (434738052 + 150036097 v) + g (-84672512 (14787067 + 76493833 v) + g (-148176896 (110370060 + 34241509 v) + g (835584 (-320656888 + 1342471603 v) + g (557056 (3221455071 + 894783086 v) + g (4096 (21677793715 - 22343862018 v) + g (2 g (-4177232524928 + g (1436248994432 + g (162543001552 + g (-17158726592 + g (-2451039724 + g (56920986 + 9703955 g)))))) + g (3616936631040 + g (632206837184 + g (-63729063968 + g (-6676483608 + g (390594892 + 7 (2774580 - 51203 g) g))))) v - 512 (199253382352 + 49361432521 v)))))))))) + 2 c^{20} (-66322432 (4685036 + 2386617 v) + g (360448 (34389511 + 91985037 v) + g (139264 (559092182 + 256416207 v) + g (-32768 (-15299837 + 186547817 v) + g (-14336 (442988995 + 182203124 v) + g (1024 (-321013051 + 331871568 v) + g (128 (1517042567 + 557485701 v) + g (18591208960 - 6206244224 v + g (-8 (234680375 + 76761272 v) + g (-259002992 + 26423008 v + g (2564944 + 456335 g + 743457 v))))))))))) + c^3 $(-4 + g)^3$ $(-2 + g)^4$ $(2 + g)^3$ $(4 + g)^4$ (4718592 (1792 + 135 v) + g (-16384 (-35492 + 8397 v) + g (-16384 (406335 + 26626 v) + g (12288 (-32893 + 6458 v) + g (1024 (1970387 + 108426 v) + g (-256 (-408196 + 61991 v) + g (-640 (460527 + 19966 v) + g (64 (-196697 + 19667 v) + g (192 (109539 + 3302 v)

+ g (729056 - 31808 v + g (-96 (6625 + 104 v) + g (-19168 + 4630 g + 175 g^2 + 30 (6 + g) v)))))))))))) + c^{13} (-14663286784 (29071312 + 9620757 v) + g (39845888 (641732992 + 1111682547 v) + g (6773800960 (46253576 + 13834903 v) + g (-338690048 (28624572 + 79324321 v) + g (-592707584 (157075664 + 42271387 v) + g (1114112 (137369488 + 5743069969 v) + g (3342336 (4308031524 + 1038712883 v) + g (-8192 (-48578131148 + 93255199323 v) + g (-2048 (607052638260 + 130629466877 v) + g (1024 (-68663369129 + 47304411726 v) + g (1024 (58278815884 + 11149278179 v) + g (-2 g (744756912480 + g (83132288072 + g (-8196569040 + g (-1122208200 + g (25923380 + 4153883 g))))) + 64 (78977104994 - 24865400119 v) + g (-252135831744 + g (24371086496 + g (2437154840 + g (-134188644 + g (-6683090 + 113581 g))))) v))))))))))))) + 2 c^{18} (-1374683136 (2880488 + 1321553 v) + g (2490368 (67056647 + 175422042 v) + g (6225920 (231228965 + 95626218 v) + g (-311296 (6412571 + 393516198 v) + g (-77824 (2399899679 + 890950152 v) + g (4096 (-1773578417 + 2839182288 v) + g (2048 (5154462477 + 1711158785 v) - g (512 (-1614941162 + 868306833 v) + g (32 (7901270740 + 2336828279 v) + g (29450715952 - 6294192704 v + g (-8 (256903059 + 67410911 v) + g (-314170222 + 22224322 v + g (2478810 + 463503 g + 573823 v))))))))))))) + c^{16} (-62318968832 (932672 + 380355 v) + g (4402970624 (619021 + 1455153 v) + g (84672512 (341070210 + 125521241 v) + g (-84672512 (3810907 + 29973834 v) + g (-10584064 (514772874 + 170225845 v) + g (835584 (-159212777 + 439032875 v) + g (278528 (1762427196 + 521577049 v) + g (-4096 (-7136741991 + 5853081652 v) + g (-256 (85072349260 + 22446995991 v) + g (128 (-15962476129 + 5556902978 v) + g (32 (13731277138 + 3217533195 v) + g (55695410704 - 8360249216 v + g (-12 (263242845 + 54494251 v) + g (-498709364 + 25627116 v + g (3502338 + 654053 g + 635200 v))))))))))))))) + c^{14} (-124637937664 (2009644 + 716631 v) + g (677380096 (20232131 + 39491496 v) + g (677380096 (240541999 + 77488985 v) + g (-169345024 (22679551 + 83955134 v) + g (-42336256 (994125223 + 288014277 v) + g (1114112 (-291653335 + 2608330094 v) + g (557056 (9916380463 + 2573064991 v) + g (-8192 (-26206877116 + 35332294495 v) + g (-1024 (382161709398 + 88472031443 v) + g (1536 (-17704709693 + 9566508000 v) + g (1792 (8174673929 + 1681920578 v) - g (64 (-22699329898 + 5660249733 v) + g (32 (8181553611 + 1488766970 v) + g (64 (521341801 - 57803738 v) + g (-4 (433187793 + 69180385 v) + g (-266638248 + 1830465 g + 326129 g^2 + 3 (3394828 + 84387 g) v)))))))))))))))) + c^4 $(-4 + g)^2$ $(-2 + g)^3$ $(2 + g)^2$ (4 + g)3 (4194304 (432437 + 43524 v) + g (-1048576 (-151828 + 34749 v) + g (-131072 (13253697 + 1209397 v) + g (32768 (-4240561 + 856123 v) + g (16384 (41227360 + 3343299 v) + g (-8192 (-5902925 + 1017454 v) + g (-2048 (66859802 + 4674407 v) + g (512 (-16845071 + 2334333 v) + g (256 (60265644 + 3466139 v) + g (845741184 - 84204928 v + g (-64 (14813663 + 650793 v) + g (64 (-713349 + 41170 v) + g (28592816 + 853632 v + g (1285592 - 29800 v + g (-4 (79388 + 1429 v) + g (-15762 + 70 v + g (552 + 35 g + 5 v))))))))))))))))) + c^{12} (-263939162112 (2403592 + 733721 v) + g (159383552 (263748079 + 399136977 v) + g (677380096 (772281238 + 213173957 v) + g (-1354760192 (15135977 + 32235316 v) + g (-338690048 (523112336 + 129967685 v) + g (8912896 (234217631 + 1344879235 v) + g (28966912 (1100719393 + 245100503 v) + g (-1703936 (-294365571 + 991673383 v) + g (-266240 (12376897346 + 2460408739 v) + g (26624 (-5324565097 + 4914883006 v) + g (6656 (29634003738 + 5238902995 v) + g (256 (53701097475 - 21579802328 v) + g (-64 (102333888153 + 16014031441 v) + g (32 (-19513058997 + 3700489882 v) + g (80 (1348714677 + 185486336 v) + g (-640 (-20152437 + 1697935 v) + g (-8 (85152025 + 10164176 v) + g (g (710291 + 115352 g + 72001 v) + 36 (-2686131 + 76666 v))))))))))))))))))) + 2 c^{11} (-1543503872 (266711008 + 74566965 v) + g (4194304 (7226160844 + 9413314647 v) + g (39845888 (9498418900 + 2402348541 v) + g (-19922944 (919679219 + 1528669903 v) + g (-338690048 (427887388 + 97445675 v) + g (4456448 (740428528 + 2141524005 v) + g (2228224 (13473337756 + 2750915083 v) + g

(-32768 (-2818906462 + 47857254159 v) + g (-53248 (68706146896 + 12527046857 v) + g (26624 (-3961095971 + 5469857503 v) + g (19968 (13374843244 + 2169300925 v) + g (256 (56457792353 - 30006950292 v) + g (-576 (19754048240 + 2837120319 v) + g (96 (-9309791023 + 2286402517 v) + g (96 (2728534926 + 344596025 v) + g (-816 (-32650981 + 3702670 v) + g (-64 (42993355 + 4717024 v) + g (28 (-12115458 + 548227 v) + g (8593782 + 2 g (614683 - 6121 v) + 801839 v)))))))))))))))))) + c^{5} (-4 + g) $(-2 + g)^{2}$ (2 + g) $(4 + g)^{2}$ (167772160 (1782248 + 224643 v) + g (-33554432 (-935708 + 206211 v) + g (-1048576 (323416592 + 37989709 v) + g (3145728 (-10533186 + 2131729 v) + g (131072 (1233939312 + 133403359 v) + g (-65536 (-222051152 + 40543795 v) + g (-49152 (858360586 + 83966697 v) + g (16384 (-211358034 + 33817457 v) + g (2048 (3201201662 + 276808005 v) + g (3072 (158791150 - 21222541 v) + g (-512 (1209265621 + 89526402 v) + g (256 (-163374073 + 16846545 v) + g (128 (270975182 + 16447743 v) + g (64 (33821238 - 2357569 v) + g (-32 (33386529 + 1569286 v) + g (-64472048 + 2443392 v + g (8 (1906673 + 64739 v) + g (969756 - 13608 v + g (-65782 - 4887 g + 2 (-734 + 5 g) v)))))))))))))))))))) + c^{10} (-67914170368 (13664392 + 3469479 v) + g (369098752 (206535107 + 228335694 v) + g (318767104 (2967424801 + 681886156 v) + g (-79691776 (696300647 + 910774958 v) + g (-338690048 (1200513755 + 248488154 v) + g (35651584 (395478155 + 722339124 v) + g (17825792 (5388889689 + 1000327201 v) + g (-786432 (1269666480 + 6203713223 v) + g (-212992 (63989646132 + 10610135377 v) + g (106496 (-1780272339 + 5024786948 v) + g (585728 (2034398627 + 300151381 v) + g (-11264 (-4009652654 + 3061553193 v) + g (-33792 (1874509983 + 244977856 v) + g (384 (-10095179363 + 3298522752 v) + g (192 (10109388394 + 1162369305 v) - g (192 (-843250605 + 127131772 v) + g (32 (954279565 + 95397308 v) + g (3182231952 - 206720768 v + g (-4 (47620159 + 4053501 v) + g (g (203683 + 28676 g + 14243 v) + 8 (-2937292 + 61837 v)))))))))))))))))))))) + c^{9} (-407485022208 (2216576 + 505939 v) + g (369098752 (224905304 + 208029969 v) + g (67108864 (15134038898 + 3127489935 v) + g (-8388608 (8478117314 + 8750591721 v) + g (-159383552 (3055550477 + 568965813 v) + g (2097152 (11049040549 + 13960019217 v) + g (35651584 (3650742192 + 609833143 v) + g (-786432 (4096554208 + 8054489985 v) + g (-32768 (649078247208 + 96873998963 v) + g (212992 (178795333 + 3803526677 v) + g (106496 (20600190844 + 2736418487 v) + g (-11264 (-4089273330 + 5559839213 v) + g (-799744 (177936257 + 20943145 v) + g (1536 (-4006645811 + 1871579893 v) + g (384 (14577909472 + 1510253455 v) + g (361569396672 - 73864632576 v + g (-320 (384770437 + 34682580 v) + g (80 (-130347057 + 11730664 v) + g (1278437664 + 98230712 v + g (132078200 - 4492424 v + g (-4096484 - 492957 g + 51 (-5078 + 67 g) v)))))))))))))))))))))) - c^{6} (-2 + g) (4 + g) (-3221225472 (12232566 + 1853405 v) + g (134217728 (-35393077 + 7468962 v) + g (33554432 (1545871556 + 222512455 v) + g (-8388608 (-703671133 + 140389039 v) + g (-4194304 (7019000492 + 951929387 v) + g (4194304 (-749234116 + 139829271 v) + g (524288 (18004311965 + 2276536964 v) + g (-131072 (-7170965889 + 1231665313 v) + g (-65536 (28724817585 + 3341084831 v) + g (131072 (-1329247421 + 204926997 v) + g (8192 (29580598440 + 3111551777 v) + g (-4096 (-5090840553 + 677986939 v) + g (-2048 (9874585181 + 919500418 v) + g (1536 (-1060997003 + 115097049 v) + g (256 (4178357941 + 335412083 v) + g (82196268032 - 6636107520 v + g (-128 (264860910 + 17735749 v) + g (64 (-40124177 + 2095100 v) + g (574558064 + 30773408 v + g (45144808 - 1210024 v + g (-132 (30670 + 1239 v) + g (-352712 + 5202 g + 541 g^{2} + 8 (383 + 18 g) v))))))))))))))))))))))) + c^{8} (-24696061952 (30502112 + 6180597 v) + g (134217728 (578005591 + 440535711 v) + g (167772160 (5544344806 + 1017494407 v) + g (-33554432 (2289295999 + 1860425082 v) + g (-239075328 (2069537106 + 342334793 v) + g (6291456 (4840868457 + 4419689129 v) + g (35651584 (4165826289 + 618350422 v) + g (-524288 (11350492207 + 12997244216 v) + g (-65536 (422561910300 + 56054573317 v) + g (32768 (15500890333 + 30673631470 v) + g (106496 (31245004686

+ 3689813857 v) + g (-4096 (-2714576071 + 22411508736 v) + g (-11264 (22960913955 + 2403254201 v) + g (1024 (-6224423856 + 5028676621 v) + g (7680 (1653715827 + 152413768 v) + g (-3456 (-166139335 + 49221416 v) + g (-64 (5812634555 + 466409504 v) + g (32 (-752207631 + 94564576 v) + g (5787454480 + 396240736 v + g (479661920 - 24147136 v + g (-4 (9262969 + 522513 v) + g (-3690098 + 55100 v + 3 g (14179 + 1628 g + 620 v)))))))))))))))))))))))) + 2 c^{7} (-1073741824 (248106464 + 43926117 v) + g (67108864 (458385148 + 282897789 v) + g (33554432 (10722355116 + 1719889339 v) + g (-16777216 (2067525361 + 1313264943 v) + g (-4194304 (50357714148 + 7282892615 v) + g (15728640 (1029736520 + 692872097 v) + g (2097152 (33741368109 + 4379983741 v) + g (-262144 (15228291687 + 11455420474 v) + g (-65536 (227858753584 + 26440215675 v) + g (32768 (16318489073 + 15442303289 v) + g (57344 (36055973236 + 3725432295 v) + g (-4096 (7631332625 + 13159664369 v) + g (-3072 (61747740768 + 5656117531 v) + g (512 (-1979317936 + 7085591841 v) + g (2560 (4442095401 + 358422013 v) + g (-640 (-448312790 + 234068481 v) + g (-512 (835542965 + 58728319 v) + g (64 (-295580319 + 55522684 v) + g (9308468096 + 558722976 v + g (577555192 - 42538960 v + g (-99235072 - 4913244 v + g (-7794402 + 194134 v + g (341500 + g (31983 - 139 v) + 13126 v)))))))))))))))))))))))))))/H (4.A1.16)

The optimal wages of the second period also depend on c, g, and v

w_{11}* = ((2 + c - g) (2 + c + g) (3456 c^{31} - $(-4 + g)^{7}$ $(-2 + g)^{8}$ $(2 + g)^{7}$ $(4 + g)^{8}$ v + 1152 c^{30} (216 + 7 v) + c^{3} $(-4 + g)^{3}$ $(-2 + g)^{4}$ $(2 + g)^{3}$ $(4 + g)^{4}$ (2 (-4 + g) (2 + g) (136478720 + g (-12367872 + g (-98062848 + g (5810688 + g (27318208 + g (-912800 + g (-3665072 + g (52440 + g (235676 + g (-688 + g (-5937 + g + 18 g^{2}))))))))))) + (-8448376832 + g (-201654272 + g (6752059392 + g (106082304 + g (-2081100800 + g (-14566400 + g (309567616 + g (-527552 + g (-22496320 + g (211776 + g (690864 + 7 g (-1376 + g (-722 + 5 g))))))))))))) v) - c^{5} (-4 + g) $(-2 + g)^{2}$ (2 + g) $(4 + g)^{2}$ (2 (-4 + g) (2 + g) (-8325999624192 + g (680895971328 + g (8924397961216 + g (-569617711104 + g (-4022321741824 + g (190864572416 + g (989418983424 + g (-32708123136 + g (-144099771392 + g (3029590912 + g (12586458880 + g (-146822368 + g (-634370736 + g (3290424 + g (16540796 + g (-26224 + g (-170330 + g (32 + 295 g))))))))))))))))) + (298139181383680 + g (11367360757760 + g (-343693099794432 + g (-10079446237184 + g (166864656793600 + g (3378391023616 + g (-44378637762560 + g (-492854706176 + g (7039955578880 + g (15725879296 + g (-679265375232 + g (3989154816 + g (38874062848 + g (-483874688 + g (-1221023104 + g (19844928 + g (17693112 + g (-268844 + g (-76886 + 553 g))))))))))))))))))) v) - 48 c^{29} (3 g (60 + 399 g + 73 v) - 32 (5571 + 353 v)) - 16 c28 (9 g2 (25492 + 763 v) + 6 g (5304 + 6583 v) - 32 (365932 + 34159 v)) + 8 c^{24} (-8 g (533795904 + g (5100465776 + g (-29403608 + g (-158216357 + g (252999 + 1019147 g))))) - g (5854414656 + g (6103568224 + 3 g (-111730144 + g (-37638996 + g (996046 + 80637 g))))) v + 1024 (316425852 + 66942925 v)) - c $(-4 + g)^{6}$ $(-2 + g)^{7}$ $(2 + g)^{6}$ $(4 + g)^{7}$ (80 + 960 v + g (12 + 8 v + g (g (-2 + g + v) - 6 (3 + 25 v)))) + 16 c^{27} (128 (1434133 + 176090 v) + g (-8 (112722 + 142985 v) + 3 g (3 g (559 + 2546 g + 738 v) - 56 (41765 + 2488 v)))) + 16 c^{26} (2432 (903144 + 137333 v) + g (-16 (1022584 + 1328313 v) + g (-16 (8552038 + 762895 v) + 3 g (90872 + 438507 g + 3 (40940 + 4283 g) v)))) - 8 c^{25} (-13312 (3136795 + 569443 v) + g (427073152 + 569286688 v + g (3824466144 + 455553472 v + g (-16 (885731 + 1227927 v) + g (-72292016 - 4262316 v + 3 g (13602 + 52423 g + 19551 v)))))) + c^{20} (786432 (2164574980 + 745387753 v) + g (-16384 (2698227752 + 4244302925 v) + g (-8192 (64804975028 + 18363795711 v) + g (32768 (263850643 + 429870379 v) + g (-8 g (64221697536 + g (305002640640 + g (-1265290304 + g (-4942217274 + g (5878256 + 20287565 g))))) - g (864510977536 + g (388913300352 + g (-17539005632 + g (-3809408856 + g (83661386 + 5259115 g))))) v + 6144 (9188066342 + 2037056885 v)))))) + 4 c^{23} (16384 (255461817 + 61943888 v) + g (-

1024 (66448011 + 93910126 v) + g (-512 (1350469660 + 244309443 v) + g (64 (87513171 + 128460872 v) + g (64 (495681103 + 59635773 v) + g (-16 (6001735 + 9121022 v) + g (3 g (63653 + 223850 g + 99772 v) - 4 (100954865 + 6074851 v)))))))) + 4 c22 (32768 (696580610 + 191317663 v) + g (-45056 (9842136 + 14375485 v) + g (-512 (9331424572 + 1991367065 v) + g (1024 (49726937 + 75374889 v) + g (32 (9468441144 + 1444325333 v) + g (-64 (22656656 + 35527359 v) + g (-6357818712 - 583404908 v + g (8621972 + 13936896 v + g (31423817 + 965695 v))))))))) + 2 c^{18} (9961472 (893849144 + 379327221 v) + g (-3735552 (77965352 + 133740107 v) + g (-2490368 (1549417845 + 562330472 v) + g (311296 (277959354 + 492753191 v) + g (4096 (150269786295 + 45064906802 v) + g (-4096 (2140176444 + 3911732347 v) + g (-512 (87647496730 + 20640263933 v) + g (2 g (752814267952 + g (-2780173216 + g (-10176641736 + g (10835121 + 36640457 g)))) + g (255736206192 + g (-10729392968 + g (-2096006426 + g (42834350 + 2545069 g)))) v + 1408 (256251348 + 481772173 v))))))))) + c^{21} (2883584 (147381101 + 45462617 v) + g (-73728 (130675292 + 197786465 v) + g (-8192 (13395615841 + 3314433468 v) + g (4096 (358919871 + 563241868 v) + g (2048 (4478511402 + 833981693 v) - g (256 (244499536 + 396603831 v) + g (64 (4465607345 + 556960908 v) + g (-112 (6625475 + 11072758 v) + g (7 g (164604 + 551225 g + 282503 v) - 8 (350107072 + 21967487 v)))))))))) + c^{2} $(-4 + g)^{4}$ $(-2 + g)^{5}$ $(2 + g)^{4}$ $(4 + g)^{5}$ (-24576 (196 + 1157 v) + g (-4096 (182 + 113 v) + g (512 (6576 + 35045 v) + g (1024 (459 + 160 v) + g (-64 (13886 + 62379 v) + g (-32 (3251 + 140 v) + g (16 (6931 + 23022 v) + g (9424 - 2416 v + g (-32 (205 + 374 v) + g (-286 + 136 v + g (147 + 37 v))))))))))) + 2 c^{19} (524288 (5630265117 + 2155282430 v) + g (-32768 (2641049818 + 4330962419 v) + g (-32768 (33353313887 + 10729487086 v) + g (4096 (5150005423 + 8737679070 v) + g (16384 (8782054857 + 2278331459 v) + g (-2048 (813770637 + 1424750719 v) + g (-512 (16059609043 + 3150186255 v) + g (64 (767498230 + 1382946697 v) + g (32 (6199742243 + 817783085 v) + g (-455256864 - 842101364 v + g (g (592680 + 1952139 g + 1122833 v) - 2 (809274323 + 53894441 v)))))))))))) + c^{17} (79691776 (588091965 + 276258073 v) + g (-2490368 (680159324 + 1224987735 v) + g (-2490368 (9432039161 + 3845746550 v) + g (3735552 (161246173 + 299772225 v) + g (49152 (90757126772 + 31193685891 v) + g (-12288 (6192341630 + 11858995801 v) + g (-16384 (24639053883 + 6848852278 v) + g (512 (8136683746 + 16019973781 v) + g (128 (140210082740 + 29545394171 v) - g (96094506432 + 194145968624 v + g (48 (7540717841 + 1071170050 v) + g (-4 (186884057 + 386831618 v) + g (g (857306 + 2860230 g + 1815789 v) - 52 (49893073 + 3586669 v)))))))))))))) + 2 c^{16} (677380096 (79582276 + 41364247 v) + g (-42336256 (50445304 + 95779675 v) + g (-21168128 (1461239796 + 667625933 v) + g (21168128 (42300238 + 82805793 v) + g (1531904 (4499550274 + 1762088619 v) + g (-208896 (660303004 + 1330367073 v) + g (-2048 (369867355316 + 119942661765 v) + g (22016 (440067112 + 911035713 v) + g (64 (672675984298 + 171645910977 v) - g (312243048832 + 663196398896 v + g (8 (151191299830 + 27881976133 v) + g (-8 (505114480 + 1099309371 v) + g (-2 (7194080967 + 805954081 v) + g (13879924 + 30923345 v + g (47416921 + 1794908 v))))))))))))))) + c^{15} (2709520384 (80079285 + 46081592 v) + g (-2032140288 (4605723 + 9260735 v) + g (-338690048 (417155746 + 213329887 v) + g (42336256 (107901677 + 223420668 v) + g (2228224 (16378508249 + 7281189225 v) + g (-1671168 (504234467 + 1073642976 v) + g (-8192 (584054308551 + 219211694893 v) + g (6144 (12017833883 + 26279157603 v) + g (3072 (110070548063 + 33463236374 v) + g (-256 (12371009321 + 27748563670 v) + g (-64 (197011953556 + 45477552735 v) + g (160 (383591459 + 881741642 v) + g (8 (28012605871 + 4366107264 v) + g (-8 (52622711 + 123878295 v) + g (g (442377 + 1540460 g + 1066224 v) - 4 (368048934 + 29076649 v)))))))))))))))) + c^{14} (159383552 (2395920408 + 1528867885 v) + g (-1354760192 (13163832 + 28172261 v) + g (-338690048 (826826932 + 473352141 v) + g (338690048 (29594196 + 65144737 v) + g (52363264 (1584648556 + 798229809 v) + g (-8912896 (244686433 + 553404143 v) + g (-

8192 (1561052443810 + 674712909409 v) + g (28672 (8131058210 + 18876044453 v) + g (1536 (715717918174 + 256453002369 v) + g (-2048 (6266804054 + 14920292873 v) + g (-64 (820073385980 + 231499238957 v) + g (64 (5432411989 + 13255992368 v) + g (32 (40663859233 + 8306831153 v) + g (-80 (49613046 + 124032409 v) + g (-4 (3550951972 + 441244721 v) + g (12496232 + 44475841 g + 32005536 v + 1869711 g v))))))))))))))))) - c^4 $(-4 + g)^2$ $(-2 + g)^3$ $(2 + g)^2$ $(4 + g)^3$ (4194304 (151448 + 431683 v) + g (262144 (397064 + 215091 v) + g (-1835008 (359663 + 960233 v) + g (-98304 (1010070 + 403601 v) + g (49152 (5810267 + 14186789 v) + g (4096 (9352281 + 2300321 v) + g (-2048 (32612902 + 70353075 v) + g (-512 (14943837 + 1254347 v) + g (256 (35747947 + 64703183 v) + g (838039552 - 68252544 v + g (-64 (11586292 + 16226773 v) + g (64 (-759967 + 183578 v) + g (64 (527693 + 498117 v) + g (1284256 - 489392 v + g (-4 (185446 + 89427 v) + g (-9126 + 4819 g + 7 (612 + 89 g) v)))))))))))))))))) + 2 c^{13} (318767104 (920557031 + 653097469 v) + g (-169345024 (87190788 + 199748623 v) + g (-169345024 (1421507905 + 912355112 v) + g (338690048 (27951104 + 65787099 v) + g (4456448 (18196281588 + 10384067977 v) + g (-2228224 (1076561161 + 2601215229 v) + g (-16384 (885629148235 + 439637669476 v) + g (12288 (25000229969 + 61971455017 v) + g (3072 (485143521616 + 203485649589 v) + g (-128 (164899255716 + 419132197195 v) + g (-128 (692186141907 + 235375884340 v) + g (32 (23790111527 + 61981608774 v) + g (32 (91223197032 + 23576761223 v) - g (40 (325486453 + 869116642 v) + g (47662473960 + 8326183488 v + g (-4 (20471319 + 56034328 v) + g (g (81646 + 306776 g + 229267 v) - 9 (33089179 + 2932876 v)))))))))))))))))))) + 2 c^{12} (218103808 (1800831980 + 1425797639 v) + g (-179306496 (118262808 + 291925235 v) + g (-9961472 (35968664876 + 25952835885 v) + g (677380096 (22709274 + 57526097 v) + g (4456448 (30550386066 + 19779533671 v) + g (-4456448 (1007607226 + 2618160447 v) + g (-32768 (854112041700 + 486718765949 v) + g (106496 (6371800760 + 16977059353 v) + g (13312 (254203224854 + 124352470011 v) + g (-3328 (17098194408 + 46703873939 v) + g (-1664 (147246384154 + 59751085731 v) + g (1664 (1583730424 + 4434633113 v) + g (32 (323369521941 + 103453579453 v) - g (32 (1969480631 + 5653975446 v) + g (8 (29491577247 + 6833131918 v) + g (-16 (41245745 + 121446038 v) + g (-2451766578 - 345821468 v + g (1971996 + 5961130 v + g (7564391 + 361213 v))))))))))))))))))))) + c^{11} (402653184 (2264991179 + 2011385278 v) + g (-159383552 (330291742 + 885146755 v) + g (-159383552 (5774005859 + 4703390172 v) + g (338690048 (126657193 + 347948182 v) + g (106954752 (3662690369 + 2698763968 v) + g (-17825792 (801658407 + 2257247021 v) + g (-262144 (349716069909 + 229124119723 v) + g (425984 (5906218686 + 17044627823 v) + g (212992 (60500045957 + 34487216705 v) + g (-26624 (9497162730 + 28092649273 v) + g (-13312 (83841024608 + 40395379751 v) + g (13312 (1098319530 + 3330693557 v) + g (128 (459047575749 + 179396840032 v) + g (-448 (1039541979 + 3233015755 v) + g (-32 (55691411967 + 16551148262 v) + g (80 (91348137 + 291532780 v) + g (792 (35008425 + 7036432 v) + g (-8 (5454263 + 17882885 v) + g (g (42709 + 180185 g + 144188 v) - 4 (42708605 + 4357652 v)))))))))))))))))))))) + c^{10} (1476395008 (618569640 + 620180393 v) + g (-16777216 (3348923512 + 9830527625 v) + g (-318767104 (3182640746 + 2944061287 v) + g (79691776 (641245886 + 1927596105 v) + g (1048576 (458584725849 + 386459258605 v) + g (-35651584 (540455556 + 1663929601 v) + g (-786432 (161605973806 + 122181371319 v) + g (65536 (59653770684 + 188149533509 v) + g (8192 (2508170032690 + 1669016053029 v) + g (-53248 (8710316296 + 28153958645 v) + g (-13312 (157543217728 + 90005080143 v) + g (732160 (44723313 + 148208572 v) + g (512 (262915671346 + 124629014379 v) + g (-384 (3488928128 + 11860731905 v) + g (-192 (27283131241 + 10199728427 v) + g (192 (153159854 + 534547363 v) + g (80 (1423822305 + 385642514 v) + g (-16 (18274816 + 65565821 v) + g (-8 (146018141 + 24083852 v) + g (857860 + 3171680 v + 9 g (410294 + 22915 v)))))))))))))))))))))) + c^9

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(536870912 (1457983939 + 1664478915 v) + g (-50331648 (1017035404 + 3305070567 v) + g (-16777216 (56814295683 + 60155906654 v) + g (159383552 (324182937 + 1077792533 v) + g (2097152 (238577622780 + 231597454619 v) + g (-151519232 (144484950 + 491641861 v) + g (-524288 (282878479879 + 248303640176 v) + g (131072 (38688388650 + 134806076551 v) + g (32768 (833891891764 + 650719068197 v) + g (-53248 (13167883364 + 47011699885 v) + g (-53248 (60953311377 + 41380414318 v) + g (439296 (135097299 + 494527126 v) + g (1024 (243593824927 + 139782642471 v) + g (-3840 (789777040 + 2966428267 v) + g (-384 (31538947209 + 14681530624 v) + g (1344 (65988302 + 254563911 v) + g (32 (10956325233 + 3879219314 v) + g (-16 (82612840 + 327795373 v) + g (-16 (336692591 + 80633234 v) + g (7750196 + 31711912 v + g (34029872 + 4137332 v - g (7717 + 38501 g + 32732 v)))))))))))))))))))))) + 4 c^{6} (-2 + g) (4 + g) (-134217728 (39959340 + 73072991 v) + g (-33554432 (27566412 + 13112231 v) + g (4194304 (1791549124 + 3128633863 v) + g (1048576 (1149658140 + 451111547 v) + g (-1048576 (4384776923 + 7229710607 v) + g (-131072 (5152850938 + 1562486495 v) + g (3670016 (440071861 + 675133242 v) + g (16384 (12992337652 + 2710274191 v) + g (-16384 (21979900294 + 30785578535 v) + g (-2048 (20247861528 + 2214387493 v) + g (1024 (51944255726 + 64795011495 v) + g (10752 (480291004 + 3315665 v) + g (-256 (20576500636 + 22119358113 v) + g (768 (-534576069 + 51686801 v) + g (256 (1350473620 + 1196378053 v) + g (224 (89971589 - 17865613 v) + g (-16 (904601291 + 619649201 v) + g (8 (-70631636 + 20853229 v) + g (128 (2788597 + 1336848 v) + g (7660334 - 2932868 v + g (-4396867 - 1221679 v + g (-32926 + 17837 g + 3 (5033 + 526 g) v)))))))))))))))))))))) + c^{8} (536870912 (1058703852 + 1391517113 v) + g (-33554432 (1174761896 + 4281691041 v) + g (-16777216 (45048178540 + 55173374743 v) + g (150994944 (290946358 + 1083903629 v) + g (6291456 (69492949598 + 78478404621 v) + g (-3145728 (6606693172 + 25174581887 v) + g (-2621440 (55104760704 + 56664546707 v) + g (262144 (20792853448 + 81099568825 v) + g (32768 (918210784510 + 846777246307 v) + g (-8192 (106062128808 + 423787890149 v) + g (-12288 (334831989470 + 271705850637 v) + g (53248 (1630928728 + 6681732193 v) + g (1024 (364231937249 + 253715500403 v) + g (-2560 (2117219080 + 8902180089 v) + g (-1536 (14385989292 + 8314055639 v) + g (384 (530348392 + 2291126279 v) + g (64 (12834032784 + 5849472139 v) + g (-3296 (1288424 + 5727771 v) - g (17651343536 + 5828423440 v + g (-16 (2591994 + 11890109 v) + g (-4 (46426863 + 9334477 v) + g (123788 + 589100 v + g (630055 + 42886 v))))))))))))))))))))))) + c^{7} (163208757248 (2113195 + 3244414 v) + g (-268435456 (94299263 + 391818800 v) + g (-134217728 (3717820464 + 5341652635 v) + g (16777216 (1848061279 + 7842456384 v) + g (16777216 (18879199495 + 25139589753 v) + g (-12582912 (1294121813 + 5613760710 v) + g (-1048576 (110630850697 + 134981903931 v) + g (20185088 (237836419 + 1055649306 v) + g (1310720 (20693702225 + 22819155276 v) + g (-65536 (13335004963 + 60623766803 v) + g (-16384 (258218812328 + 253029115913 v) + g (106496 (956265607 + 4457579802 v) + g (2048 (218101168713 + 185870417344 v) + g (-16384 (465235036 + 2226098237 v) + g (-1024 (30967684801 + 22301669063 v) + g (768 (465888465 + 2291103788 v) + g (1024 (1437292833 + 839496163 v) + g (-896 (11096654 + 56179531 v) + g (-224 (188109211 + 83550632 v) + g (16 (9097775 + 47553247 v) + g (8 (83074477 + 24951824 v) + g (-868412 - 4712236 v + g (-4508190 - 687188 v + g (915 + 5820 g + 5216 v)))))))))))))))))))))))))/H \qquad (4.A1.17)
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w_{21}^* = -((2 + c - g) (2 + c + g) ((-4 + g)^{7} (-2 + g)^{8} (2 + g)^{7} (4 + g)^{8} v + c (-4 + g)^{6} (-2 + g)^{7} (2 + g)^{6} (4 + g)^{7} (16 + 2 (-5 + g) g (2 + g) + 960 v + g (8 + (-150 + g) g) v) - c^{3} (-4 + g)^{3} (-2 + g)4 (2 + g)^{3} (4 + g)^{4} (2 (-4 + g) (2 + g) (26542080 + g (-37115904 + g (-14158336 + g (21752832 + g (2874816 + g (-4567648 + g (-276976 + g (407032 + 3 g (3932 + g (-4568 +
$$

$$
g (-33 + 34 g)))))))))) + (-8448376832 + g (-201654272 + g (6752059392 + g (106082304 + g (-2081100800 + g (-14566400 + g (309567616 + g (-527552 + g (-22496320 + g (211776 + g (690864 + 7 g (-1376 + g (-722 + 5 g))))))))))))) v) + 144 c^{30} (13 g - 8 (4 + 7 v)) + 48 c^{29} (g (2738 + 219 v) - 32 (194 + 353 v)) - 16 c^{28} (g (-273928 + 9 g (-364 + 179 g - 763 v) - 39498 v) + 32 (18028 + 34159 v)) + 8 c^{25} (4 g (502780144 + g (48162880 + g (-28533316 + 3 g (-138545 + 83746 g)))) +g (569286688 + g (455553472 + 3 g (-6548944 + g (-1420772 + 19551 g)))) v - 1024 (3428902 + 7402759 v)) + c^{21} (g^{9} (37787566 + 1977521 v) - 8 g^{8} (6402651 + 21967487 v) - 2883584 (17283120 + 45462617 v) - 16 g^{7} (509921025 + 77509306 v) + 448 g^{6} (24475925 + 79565844 v) + 256 g^{5} (1614567494 + 396603831 v) - 4096 g^{3} (1687077759 + 563241868 v) + 24576 g (1422742658 + 593359395 v) - 2048 g^{4} (273189713 + 833981693 v) + 16384 g^{2} (582722735 + 1657216734 v)) - 16 c^{27} (256 (44557 + 88045 v) + g (-40 (144836 + 28597 v) + 3 g (g (33625 + 2214 v) - 8 (7993 + 17416 v)))) + 8 c26 (-256 (1264308 + 2609327 v) + g (32 (5466757 + 1328313 v) + g (32 (336352 + 762895 v) + g (9 g (-3474 + 1983 g - 8566 v) - 8 (752546 + 92115 v))))) + c^{20} (-2883584 (73189588 + 203287569 v) + g (16384 (9457878156 + 4244302925 v) + g (24576 (2044541780 + 6121265237 v) + g (-8192 (4643076219 + 1719481516 v) + g (2 g (1508173998080 + g (57039822336 + g (-44449863104 + g (-529886744 + g (410357524 + (700270 - 530153 g) g))))) + g (864510977536 + g (388913300352 + g (-17539005632 + g (-3809408856 + g (83661386 + 5259115 g))))) v - 2048 (1905848966 + 6111170655 v)))))) + 4 c^{23} (-16384 (26089843 + 61943888 v) + g (1024 (265915367 + 93910126 v) + g (512 (94575175 + 244309443 v) + g (-64 (497880403 + 128460872 v) + g (-96 (14268779 + 39757182 v) + g (919752752 + 8162336 g - 5505883 g^{2} + 4 (36484088 + (6074851 - 74829 g) g) v)))))) + 4 c^{24} (-2048 (29593492 + 66942925 v) + g (128 (286421408 + 91475229 v) + g (64 (77242300 + 190736507 v) + g (-64 (48298603 + 10474701 v) + g (-8 (10571878 + 28229247 v) + g (53956576 + 5976276 v + 3 g (56428 - 36119 g + 161274 v))))))) + c^{22} (-131072 (76631676 + 191317663 v) + g (8192 (820527549 + 316260670 v) + g (10240 (147094516 + 398273413 v) + g (-12288 (84682978 + 25124963 v) + g (-128 (495769464 + 1444325333 v) + g (256 (174760951 + 35527359 v) + g (750991136 + 825293 g^{3} + 2333619632 v - 736 g (723619 + 75744 v) - 20 g^{2} (58829 + 193139 v)))))))) + 2 c^{19} (-1048576 (366561281 + 1077641215 v) + g (32768 (8999371888 + 4330962419 v) + g (32768 (3395568375 + 10729487086 v) + g (-61440 (1431622469 + 582511938 v) + g (-4096 (2698738321 + 9113325836 v) + g (2048 (4356882443 + 1424750719 v) + g (-4 g (91663302080 + g (1732519092 + g (-1406075850 + g (-6834723 + 5439197 g)))) + g (-88508588608 + g (-26169058720 + g (842101364 + (107788882 - 1122833 g) g))) v + 4608 (97639892 + 350020695 v)))))))) + c^{2} (-4 + g)^{4} (-2 + g)^{5} (2 + g)^{4} (4 + g)^{5} (8192 (116 + 3471 v) + g (4096 (-278 + 113 v) + g (-512 (1568 + 35045 v) + g (1024 (643 - 160 v) + g (64 (3586 + 62379 v) + g (224 (-589 + 20 v) + g (-144 (179 + 2558 v) + g (16 (682 + 151 v) + g (16 (59 + 748 v) + g (g (-3 + g - 37 v) - 2 (163 + 68 v)))))))))))) + c^{18} (-19922944 (121582876 + 379327221 v) + g (2490368 (778724467 + 401220321 v) + g (9961472 (84074805 + 281165236 v) + g (-622592 (1109115666 + 492753191 v) + g (-8192 (12635425695 + 45064906802 v) + g (4096 (21280315401 + 7823464694 v) + g (1024 (5459413918 + 20640263933 v) + g (-256 (18585344898 + 5299493903 v) + g (-32 (4017158174 + 15983512887 v) + g (16 (6823259691 + 1341174121 v) + g (1009593344 + g (-843130728 + g (-1185860 + 958483 g - 5090138 v) - 85668700 v) + 4192012852 v))))))))))) + c^{5} (-4 + g) (-2 + g)^{2} (2 + g) (4 + g)^{2} (335544320 (74881 + 888524 v) + g (8388608 (-3155722 + 1355095 v) + g (-2097152 (16193867 + 163885641 v) + g (-1048576 (-28019071 + 9612509 v) + g (131072 (145053219 + 1273076300 v) + g (65536 (-209397823 + 51550156 v) + g (-32768 (175266008 + 1354328545 v) + g (-8192 (-428670327 + 60162928 v) + g (2048 (497971709 + 3437478310 v) + g (1024 (-524028727 + 15357304 v) + g (7 g^{9} (726 + 79 v) - 2 g^{8} (7544 + 38443 v) - 4 g^{7} (299008 + 67211 v) +
$$

64 g^5 (1322164 + 310077 v) + 2304 g (21695875 + 1731404 v) + 8 g^6 (425885 + 2211639 v) - 128 g^3 (21648432 + 3780271 v) - 64 g^4 (3523939 + 19078486 v) + 128 g^2 (52657627 + 303703616 v) - 512 (211967203 + 1326690186 v)))))))))))) + c^{17} (-79691776 (83191506 + 276258073 v) + g (2490368 (2221527938 + 1224987735 v) + g (24903680 (108321008 + 384574655 v) + g (-1245184 (1860927389 + 899316675 v) + g (-49152 (8254625791 + 31193685891 v) + g (12288 (28974519976 + 11858995801 v) + g (4096 (6848388409 + 27395409112 v) + g (-14848 (1677240808 + 552412889 v) + g (-128 (7024237841 + 29545394171 v) + g (798001206560 + 194145968624 v + g (800 (14648986 + 64270203 v) + g (-4 (2563663621 + 386831618 v) + g (-41130820 + 34944148 g - 186506788 v + 1815789 g v))))))))))))) + c^{16} (-1354760192 (11664220 + 41364247 v) + g (84672512 (162165592 + 95779675 v) + g (42336256 (176518340 + 667625933 v) + g (-42336256 (157819229 + 82805793 v) + g (-835584 (1607903310 + 6460991603 v) + g (417792 (2937937040 + 1330367073 v) + g (4096 (28234834642 + 119942661765 v) + g (-1024 (104490828136 + 39174535659 v) + g (-384 (12820022026 + 57215303659 v) + g (32 (142535625344 + 41449774931 v) + g (95876350560 + 446111618128 v + g (-16 (5486616310 + 1099309371 v) + g (-4 (167626837 + 805954081 v) + g (597804520 + 61846690 v + g (729586 - 623847 g + 3589816 v))))))))))))))) + c^{15} (-10838081536 (3030641 + 11520398 v) + g (677380096 (43834027 + 27782205 v) + g (338690048 (52739705 + 213329887 v) + g (-42336256 (392306005 + 223420668 v) + g (-1114112 (3394542679 + 14562378450 v) + g (557056 (6452186761 + 3220928928 v) + g (8192 (48395434556 + 219211694893 v) + g (-6144 (62161103017 + 26279157603 v) + g (-512 (42225438689 + 200779418244 v) + g (256 (81598919873 + 27748563670 v) + g (320 (1835559151 + 9095510547 v) + g (-32 (17565118053 + 4408708210 v) + g (-8 (852880199 + 4366107264 v) + g (72 (88535149 + 13764255 v) + g (22202340 - 19951681 g + 116306596 v - 1066224 g v))))))))))))))) + c^{14} (-2709520384 (21976684 + 89933405 v) + g (338690048 (165213163 + 112689044 v) + g (338690048 (108936100 + 473352141 v) + g (-338690048 (105266186 + 65144737 v) + g (-10027008 (906036356 + 4168533447 v) + g (8912896 (1007013467 + 553404143 v) + g (57344 (19864862202 + 96387558487 v) + g (-4096 (278744757758 + 132132311171 v) + g (-512 (151161157418 + 769359007107 v) + g (256 (304367373911 + 119362342984 v) + g (64 (43665081156 + 231499238957 v) + g (-256 (10905977309 + 3313998092 v) + g (-32 (1516899735 + 8306831153 v) + g (80 (593243235 + 124032409 v) + g (4 (78746372 + 441244721 v) + g (-297000304 - 329251 g + 295242 g^2 - 3 (10668512 + 623237 g) v)))))))))))))))) - c^4 (-4 + g)2 (-2 + g)3 (2 + g)2 (4 + g)^3 (-4194304 (29016 + 431683 v) + g (-2359296 (-56752 + 23899 v) + g (262144 (546425 + 6721631 v) + g (32768 (-3787766 + 1210803 v) + g (-16384 (4090429 + 42560367 v) + g (4096 (11397545 - 2300321 v) + g (2048 (7828448 + 70353075 v) + g (512 (-17894365 + 1254347 v) + g (-256 (8164817 + 64703183 v) + g (128 (7826704 + 533223 v) + g (64 (2270230 + 16226773 v) + g (-128 (468687 + 91789 v) + g (-32 (150355 + 996234 v) + g (1771568 + 489392 v + g (56224 + 357708 v + g (g (-99 + 33 g - 623 v) - 18 (1073 + 238 v))))))))))))))))) + 2 c^{13} (-318767104 (147518274 + 653097469 v) + g (9961472 (4608167082 + 3395726591 v) + g (1354760192 (24308035 + 114044389 v) + g (-1016070144 (32528250 + 21929033 v) + g (-4456448 (2093548611 + 10384067977 v) + g (2228224 (4297667603 + 2601215229 v) + g (16384 (84138067377 + 439637669476 v) + g (-4096 (350737413325 + 185914365051 v) + g (-1024 (111458799491 + 610456948767 v) + g (128 (934646923522 + 419132197195 v) + g (256 (20636904067 + 117687942170 v) + g (-96 (57345683785 + 20660536258 v) + g (-32 (4003281474 + 23576761223 v) + g (40 (3273082305 + 869116642 v) + g (64 (21590192 + 130096617 v) + g (-28 (48753213 + 8004904 v) + g (-4322589 + 4070780 g - 26395884 v + 229267 g v))))))))))))))))) + c^{11} (-268435456 (571680065 + 3017077917 v) + g (8388608 (19238751324 + 16817788345 v) + g (478150656 (281314615 + 1567796724 v) + g (-19922944 (7298573157 + 5915119094 v) + g (-35651584 (1377895561 + 8096291904 v) +

g (18164482048 (2991763 + 2215159 v) + g (262144 (37089415678 + 229124119723 v) + g (-32768 (333772368568 + 221580161699 v) + g (-106496 (10666133827 + 68974433410 v) + g (26624 (48384447428 + 28092649273 v) + g (13312 (6004010265 + 40395379751 v) + g (-13312 (6790008775 + 3330693557 v) + g (-128 (25828911009 + 179396840032 v) + g (320 (11501523303 + 4526222057 v) + g (74540215776 + 529636744384 v + g (-16 (5033390793 + 1457663900 v) + g (-104 (7442345 + 53585136 v) + g (800290184 + 2410700 g - 2356529 g2 + 4 (35765770 + (4357652 - 36047 g) g) v))))))))))))))))) + c^{12} (-637534208 (202506996 + 975545753 v) + g (358612992 (364865036 + 291925235 v) + g (1693450240 (59918756 + 305327481 v) + g (-338690048 (312128085 + 230104388 v) + g (-8912896 (3676800694 + 19779533671 v) + g (8912896 (3918784454 + 2618160447 v) + g (851968 (6613325040 + 37439905073 v) + g (-212992 (28648619034 + 16977059353 v) + g (-26624 (20970026522 + 124352470011 v) + g (6656 (91480140580 + 46703873939 v) + g (3328 (9680906210 + 59751085731 v) + g (-3328 (10513226532 + 4434633113 v) + g (-64 (16233663895 + 103453579453 v) + g (192 (5774599021 + 1884658482 v) + g (16 (1048044209 + 6833131918 v) + g (-16 (1083414705 + 242892076 v) + g (-4 (26170723 + 172910734 v) + g (g (108874 - 101479 g + 722426 v) + 4 (25843242 + 2980565 v))))))))))))))))))))) + c^{10} (-1476395008 (106397356 + 620180393 v) + g (16777216 (10213029001 + 9830527625 v) + g (318767104 (479030280 + 2944061287 v) + g (-717225984 (238718342 + 214177345 v) + g (-17825792 (3512546473 + 22732897565 v) + g (8912896 (8063489711 + 6655718404 v) + g (262144 (53917459726 + 366544113957 v) + g (-65536 (251889146486 + 188149533509 v) + g (-319488 (6017297382 + 42795283411 v) + g (53248 (42554132817 + 28153958645 v) + g (146432 (1106187800 + 8182280013 v) + g (-292864 (650953023 + 370521430 v) + g (-1536 (5441449398 + 41543004793 v) + g (384 (25255440398 + 11860731905 v) + g (192 (1306056723 + 10199728427 v) + g (-192 (1473189117 + 534547363 v) + g (-80 (48734155 + 385642514 v) + g (16 (263457349 + 65565821 v) + g (8 (3030887 + 24083852 v) + g (5 g (-5210 + 5003 g - 41247 v) - 8 (3102447 + 396460 v))))))))))))))))))))))) + c^{9} (-5905580032 (23273194 + 151316265 v) + g (16777216 (9264964502 + 9915211701 v) + g (33554432 (4393653900 + 30077953327 v) + g (-8388608 (20432706681 + 20478058127 v) + g (-2097152 (32161736715 + 231597454619 v) + g (524288 (153163482840 + 142084497829 v) + g (1048576 (16426704665 + 124151820088 v) + g (-131072 (159301819404 + 134806076551 v) + g (-32768 (82345209499 + 650719068197 v) + g (4096 (806634006414 + 611152098505 v) + g (106496 (2518313168 + 20690207159 v) + g (-39936 (8220474909 + 5439798386 v) + g (-1024 (16486980895 + 139782642471 v) + g (1280 (15939253138 + 8899284801 v) + g (384 (1692993365 + 14681530624 v) + g (-192 (3983473888 + 1781947377 v) + g (-32 (441498939 + 3879219314 v) + g (16 (997118322 + 327795373 v) + g (16 (9139607 + 80633234 v) + g (-4 (39130009 + 7927978 v) + g (-470736 - 4137332 v + g (473029 + 32732 v)))))))))))))))))))))))) + c^{6} (-2 + g) (4 + g) (536870912 (7413620 + 73072991 v) + g (134217728 (-29905064 + 13112231 v) + g (-16777216 (363414148 + 3128633863 v) + g (-4194304 (-1242436892 + 451111547 v) + g (4194304 (949682641 + 7229710607 v) + g (524288 (-5571291702 + 1562486495 v) + g (-524288 (2776699033 + 18903730776 v) + g (-65536 (-14139758488 + 2710274191 v) + g (65536 (5002487296 + 30785578535 v) + g (8192 (-22373443804 + 2214387493 v) + g (-20480 (2304669232 + 12959002299 v) + g (-6144 (-3818848766 + 23209655 v) + g (3072 (1419226820 + 7373119371 v) + g (-3072 (635663601 + 51686801 v) + g (-1024 (246301746 + 1196378053 v) + g (896 (115215521 + 17865613 v) + g (64 (134686429 + 619649201 v) + g (-32 (102265200 + 20853229 v) + g (-64 (2419577 + 10694784 v) + g (8 (6920343 + 1466434 v) + g (1132452 + 4886716 v + g (-387232 - 1470 g + 489 g^{2} - 12 (5033 + 526 g) v)))))))))))))))))))))))) + c^{8} (-536870912 (189538308 + 1391517113 v) + g (33554432 (3554918192 + 4281691041 v) + g (16777216 (7146386860 + 55173374743 v) + g (-16777216 (8612912327 + 9755132661 v) + g (-6291456 (9673109266 +

$$78478404621\ v) + g\ (3145728\ (23885248760 + 25174581887\ v) + g\ (524288\ (33297662986 + 283322733535\ v) + g\ (-262144\ (83764445747 + 81099568825\ v) + g\ (-32768\ (95230564042 + 846777246307\ v) + g\ (8192\ (484372223528 + 423787890149\ v) + g\ (53248\ (6784665106 + 62701350147\ v) + g\ (-53248\ (8633557118 + 6681732193\ v) + g\ (-1024\ (26608656371 + 253715500403\ v) + g\ (24064\ (1424121656 + 947040435\ v) + g\ (1536\ (852567178 + 8314055639\ v) + g\ (-384\ (4180779038 + 2291126279\ v) + g\ (-448\ (84579856 + 835638877\ v) + g\ (32\ (1395028228 + 589960413\ v) + g\ (587575696 + 5828423440\ v + g\ (-16\ (41047466 + 11890109\ v) + g\ (-4\ (944693 + 9334477\ v) + g\ (3967612 + 589100\ v + g\ (4385 - 4301\ g + 42886\ v)))))))))))))))))))))))) + c^7\ (-4294967296\ (14642039 + 123287732\ v) + g\ (268435456\ (284570591 + 391818800\ v) + g\ (671088640\ (120785735 + 1068330527\ v) + g\ (-16777216\ (6034957391 + 7842456384\ v) + g\ (-25165824\ (1804766419 + 16759726502\ v) + g\ (12582912\ (4620381281 + 5613760710\ v) + g\ (7340032\ (1981261952 + 19283129133\ v) + g\ (-262144\ (72385134743 + 81284996562\ v) + g\ (-393216\ (7482165881 + 76063850920\ v) + g\ (65536\ (59261898941 + 60623766803\ v) + g\ (16384\ (23955285405 + 253029115913\ v) + g\ (-106496\ (4872972841 + 4457579802\ v) + g\ (-2048\ (17058776905 + 185870417344\ v) + g\ (8192\ (5579093581 + 4452196474\ v) + g\ (1024\ (2001535892 + 22301669063\ v) + g\ (-768\ (3406498985 + 2291103788\ v) + g\ (-1024\ (74372801 + 839496163\ v) + g\ (128\ (730554632 + 393256717\ v) + g\ (224\ (7373097 + 83550632\ v) - g\ (1925471568 + 760851952\ v + g\ (8\ (2210739 + 24951824\ v) + g\ (-4\ (4844129 + 1178059\ v) + g\ (-61518 - 687188\ v + g\ (62943 + 5216\ v))))))))))))))))))))))))))))/H, \quad (4.A1.18)$$

where

$$H = (1024\ (2 + c)2^4\ (4 + c)^2\ (4 + 3\ c)^2\ (8 + c\ (8 + c))^2 - 16\ (2 + c)^{22}\ (4 + c)\ (4 + 3\ c)\ (64 + c\ (64 + 9\ c))\ (10240 + c\ (20480 + c\ (14160 + c\ (3920 + 379\ c))))\ g^2 + 16\ (2 + c)^{20}\ (47972352 + c\ (191889408 + c\ (325300224 + c\ (304287744 + c\ (171348416 + c\ (59421568 + c\ (12406984 + c\ (1429256 + 69729\ c))))))))\ g^4 - 8\ (2 + c)^{18}\ (266076160 + c\ (1064304640 + c\ (1819205632 + c\ (1732550656 + c\ (1004342368 + c\ (362789056 + 3\ c\ (26604488 + c\ (3264424 + 171403\ c))))))))\ g^6 + 16\ (2 + c)^{16}\ (249847808 + c\ (999391232 + c\ (1719300096 + c\ (1660030976 + c\ (983368084 + c\ (365974312 + c\ (83615022 + c\ (10734490 + 593797\ c))))))))\ g^8 - (2 + c)^{14}\ (5381160960 + c\ (21524643840 + c\ (37213731840 + c\ (36304942080 + c\ (21854781248 + c\ (8313410176 + c\ (1952287920 + c\ (258965872 + 14871617\ c))))))))\ g^{10} + (2 + c)^{12}\ (5362008064 + c\ (21448032256 + c\ (37220055808 + c\ (36592054528 + c\ (22289565968 + c\ (8615078688 + c\ (2063847936 + c\ (280312304 + 16537411\ c))))))))\ g^{12} - (2 + c)^{10}\ (4029030400 + c\ (16116121600 + c\ (28044684160 + c\ (27727626880 + c\ (17035610672 + c\ (6660651744 + c\ (1618692128 + c\ (223617136 + 13450359\ c))))))))\ g^{14} + (2 + c)^8\ (2305163520 + c\ (9220654080 + c\ (16077634560 + c\ (15960614400 + c\ (9866516032 + c\ (3889437824 + c\ (954911768 + c\ (133519576 + 8142301\ c))))))))\ g^{16} - 8\ (2 + c)^8\ (31476800 + c\ (94430400 + c\ (117550620 + c\ (77717240 + c\ (28783533 + c\ (5663313 + 462637\ c))))))\ g^{18} + (2 + c)^8\ (20945344 + c\ (41890688 + c\ (31235632 + c\ (10290288 + 1264841\ c))))\ g^{20} - (2 + c)^6\ (5255040 + c\ (10510080 + c\ (7859880 + c\ (2604840 + 322879\ c))))\ g^{22} + 2\ (2 + c)^4\ (487984 + c\ (975968 + c\ (731136 + c\ (243152 + 30291\ c))))\ g^{24} - 14\ (2 + c)^4\ (2320 + c\ (2320 + 579\ c))\ g^{26} + 732\ (2 + c)^4\ g^{28} - 40\ (2 + c)^2\ g^{30} + g^{32}).$$

Public-private product differentials in the first period do not depend on v

$$q_{10}^* - q_{20}^* = (c\ (2 + c)\ (-8\ (2 + c)^{11}\ (4 + c)\ (8 + c)\ (4 + 3\ c) + 16\ (2 + c)^{11}\ (4 + c)\ (4 + 3\ c)\ g + 16\ (2 + c)^9\ (4 + c)2\ (17 + 12\ c)\ g^2 - 12\ (2 + c)^9\ (80 + c\ (80 + 17\ c))\ g^3 - 2\ (2 + c)^7\ (3744 + c\ (4820 + c\ (1920 + 247\ c)))\ g^4 + 16\ (2 + c)^7\ (87 + c\ (87 + 20\ c))\ g^5 + 4\ (2 + c)^5\ (1676 + c\ (2289 + c\ (1009 + 146\ c)))\ g^6 - 4\ (2 + c)^5\ (245 + c\ (245 + 59\ c))\ g^7 - 2\ (2 + c)^3\ (1676 + c\ (2402 + 3\ c\ (378 + 59\ c)))\ g^8 + 2\ (2 + c)^3\ (174 + c\ (174 + 43\ c))\ g^9 + (2 + c)^2\ (468 + c\ (458 + 111\ c))\ g^{10} - 15\ (2 + c)^3\ g^{11} - 17\ (2 + c)^2\ g^{12} + (2 + c)\ g^{13} + g^{14}))/I \quad (4.A1.19)$$

$I = (32 (2 + c)^{12} (4 + c) (4 + 3 c) (8 + c (8 + c)) - 4 (2 + c)^{11} (4 + c) (4 + 3 c) (8 + 3 c) (8 + 5 c) g - 4 (2 + c)^{10} (4608 + c (9216 + c (6376 + c (1768 + 171 c)))) g^2 + 16 (2 + c)^9 (1088 + c (2176 + c (1587 + c (499 + 57 c)))) g^3 + 8 (2 + c)^8 (4352 + c (8704 + c (6245 + c (1893 + 208 c)))) g^4 - (2 + c)^7 (29952 + c (59904 + c (44336 + c (14384 + 1725 c)))) g^5 - (2 + c)^6 (36224 + c (72448 + c (53220 + c (16996 + 2003 c)))) g^6 + (2 + c)^7 (6704 + c (6704 + 1617 c)) g^7 + (2 + c)^4 (22896 + c (45792 + c (34096 + c (11200 + 1371 c)))) g^8 - 2 (2 + c)^5 (1676 + c (1676 + 413 c)) g^9 - (2 + c)^4 (2264 + c (2264 + 561 c)) g^{10} + (2 + c)^3 (936 + c (936 + 233 c)) g^{11} + 136 (2 + c)^4 g^{12} - 34 (2 + c)^3 g^{13} - 18 (2 + c)^2 g^{14} + 2 (2 + c) g^{15} + g^{16})$

APPENDIX 4.A2

Second period/ fifth stage, each firm maximizes its profits as for q_{11} and q_{21} respectively;

$$\underset{q_{11}}{\text{Max}}\,\Pi_1(q_{10},q_{20},q_{11},q_{21},w_{10},w_{11})=-c\left(q_{10}-q_{11}+\frac{1}{2}(q_{10}-q_{11})^2\right)+(1-q_{10}-gq_{20})q_{10}+(v-q_{11}-gq_{21})q_{11}-q_{10}w_{10}-q_{11}w_{11} \quad (4.A2.1)$$

$$\underset{q_{21}}{\text{Max}}\,\Pi_2(q_{10},q_{20},q_{11},q_{21},w_{10},w_{20})=q_{20}(1-gq_{10}-q_{20})-q_{20}w_{20}-q_{21}w_{21}+q_{21}(v-gq_{11}-q_{21}). \quad (4.A2.2)$$

Reaction functions of the first period accrues solving the *foc* of the (4.A2.1) and (4.A2.2) as for q_{11}, and q_{21} respectively;

$$RF_{11}(q_{21}) = \frac{(v+c)+cq_{10}-gq_{21}-w_{11}}{2+c} \quad (4.A2.3)$$

$$RF_{21}(q_{11})=\frac{v-gq_{11}-w_{21}}{2}. \quad (4.A2.4)$$

We solve the system of the second period RFs to get the optimal q_{11}^* and q_{21}^* -rules in the second period;

$$q_{11}^* = \frac{-(g-2)v+2c(1+q_{10})-2w_{11}+gw_{21}}{4+2c-g^2} \quad (4.A2.5)$$

$$q_{21}^* = \frac{-(g-2)v+gw_{11}-2w_{21}-c(g+gq_{10}-v+w_{21})}{4+2c-g^2}. \quad (4.A2.6)$$

Substituting the later into (4.A2.1) and (4.A2.2) accrues profits that depend on products of the first period and wages;

$U1(q_{10}, q_{20}, w_{10}, w_{11}, w_{21}) = q_{10}\, w_{10} + (w_{11}\, (2\, c\, (1 + q_{10}) - (-2 + g)\, v - 2\, w_{11} + g\, w_{21}))/(4 + 2\, c - g^2)$ (4.A2.7)

$U2(q_{10}, q_{20}, w_{10}, w_{11}, w_{21}) = q_{20}w_{20} - (w_{21}((g-2)v - gw_{11} + 2w_{21} + c\,(g + gq_{10} – v + w_{21})))/(4 + 2c – g^2).$ (4.A2.8)

Second period/ fourth stage under universal RTM the f1/U1 and f2/U2 bargaining units, independently and simultaneously bargain about w_{11} and w_{21}, respectively. I make the assumption that unions possess all the bargaining power (b=1). The bargaining problem is set as follow;

$$w_{j1}^* = \max_{w_{j1}} U_j\left(=\left(w_{j0}q_{j0}+w_{j1}q_{j1}^*\right)\right),\ j=1,2.$$

Solving the system that accrues from the first order conditions of the above problem as for w_{11} and w_{21} accrues a unique stable solution for the equilibrium firm-specific wage contracts w_{11}^* and w_{21}^*, respectively;

$w_{11}^* = (c\,(8 + 4\,c – g^2)\,(1 + q_{10}) - (c\,(-4 + g) + (-2 + g)\,(4 + g))\,v)/(16 + 8\,c – g^2)$ (4.A2.9)

$$w_{21}^{*} = -((2\, c\, (g + g\, q_{10} - 2\, v) + (-2 + g)\, (4 + g)\, v)/(16 + 8\, c - g^2)). \qquad (4.A2.10)$$

Substituting the optimal products and wages of the second period into (4.A2.1) and (4.A2.2) accrues profits that depend on products and wages of the first period;

$$\Pi 1(q_{10}, q_{20}, w_{10}, w_{20}) = -(1/(2\, (-4 - 2\, c + g^2)^2\, (-8\, (2 + c) + g^2)^2))\, (64\, c^5\, (-1 + 3\, q_{10}\, (2 + q_{10})) - 32\, c^4\, (g^2\, (-1 + 9\, q_{10}\, (2 + q_{10})) - g\, (16\, q_{10}\, q_{20} + v + q_{10}\, v) + 4\, (3 + v + q_{10}\, (-22 - 17\, q_{10} + v - 4\, w_{10}))) + 2\, (-2 + g)^2\, (4 + g)2\, (-4\, v^2 + (-4 + g)^2\, (2 + g)^2\, q_{10}\, (-1 + q_{10} + g\, q_{20} + w_{10})) - 4\, c^2\, (5\, g^6\, q_{10}\, (2 + q_{10}) - 66\, g^5\, q_{10}\, q_{20} - 48\, g\, (v\, (2 + v) + 2\, q_{10}\, (32\, q_{20} + v)) + 2\, g^3\, (v\, (4 + v) + 4\, q_{10}\, (120\, q_{20} + v)) + 2\, g^2\, (-16 + 944\, q_{10}^2 - v\, (32 + v) - 32\, q_{10}\, (-14 + v - 15\, w_{10})) - 32\, (-4 + 156\, q_{10}^2 - 3\, v\, (4 + v) - 12\, q_{10}\, (-2 + v - 8\, w_{10})) + 2\, g^4\, ((1 + q_{10})\, (1 - 98\, q_{10} + v) - 33\, q_{10}\, w_{10})) + 4\, c^3\, (g^4\, (-1 + 32\, q_{10}\, (2 + q_{10})) + 8\, g\, (128\, q_{10}\, q_{20} + 6\, (1 + q_{10})\, v + v^2) - 2\, g^3\, (v + q_{10}\, (80\, q_{20} + v)) - g^2\, (-32 + 608\, q_{10}^2 + (-16 + v)\, v - 16\, q_{10}\, (-46 + v - 10\, w_{10})) + 16\, (-12 + 148\, q_{10}^2 - v\, (12 + v) + 4\, q_{10}\, (26 - 3\, v + 16\, w_{10}))) + c\, (-2 + g)\, (4 + g)\, ((-4 + g)\, (2 + g)\, (320 - 60\, g^2 + g^4)\, q_{10}^2 - 4\, v\, (4\, (-8 + g^2) + (-24 + g\, (6 + g))\, v) + 2\, q_{10}\, (64\, (8 + v - 16\, w_{10}) + g\, (-128\, (-1 + 8\, q_{20} + 2\, w_{10}) + g\, (-8\, (8 + 32\, q_{20} + v - 36\, w_{10}) + g\, (g\, (-8 + g\, (-2 + g - 20\, q_{20}) + 40\, q_{20} - 20\, w_{10}) + 8\, (36\, q_{20} + 5\, w_{10})))))))) \qquad (4.A2.11)$$

$$\Pi 2\, (q_{10}, q_{20}, w_{10}, w_{20}) = 1/((-4 - 2\, c + g^2)^2\, (-8\, (2 + c) + g^2)^2)\, (4\, c\, (-2 + g)\, (4 + g)\, ((-4 + g)\, (2 + g)\, (-32 + 5\, g^2)\, q_{20}^2 + v\, (4\, g\, (1 + q_{10}) + (-16 + g\, (2 + g))\, v) + (-4 + g)\, (2 + g)\, (-32 + 5\, g^2)\, q_{20}\, (-1 + g\, q_{10} + w_{20})) + 4\, c^4\, (g^2\, (1 + q_{10})^2 + 4\, v^2 - 4\, g\, (16\, q_{10}\, q_{20} + v + q_{10}\, v) - 64\, q_{20}\, (-1 + q_{20} + w_{20})) - (-2 + g)^2\, (4 + g)^2\, (-4\, v^2 + (-4 + g)^2\, (2 + g)^2\, q_{20}\, (-1 + g\, q_{10} + q_{20} + w_{20})) + 4\, c^3\, (g^3\, (80\, q_{10}\, q_{20} + v + q_{10}\, v) - 4\, g\, (128\, q_{10}\, q_{20} + 6\, (1 + q_{10})\, v + v^2) + 32\, (v^2 - 16\, q_{20}\, (-1 + q_{20} + w_{20})) + 2\, g^2\, (2 + 2\, q_{10}\, (2 + q_{10}) + v + q_{10}\, v - v^2 + 40\, q_{20}\, (-1 + q_{20} + w_{20}))) + c^2\, (-132\, g^5\, q_{10}\, q_{20} - 96\, g\, (64\, q_{10}\, q_{20} + 2\, (1 + q_{10})\, v + v^2) + 4\, g^3\, (480\, q_{10}\, q_{20} + 4\, (1 + q_{10})\, v + v^2) + g^4\, (v^2 - 132\, q_{20}\, (-1 + q_{20} + w_{20})) - 384\, (-v^2 + 16\, q_{20}\, (-1 + q_{20} + w_{20})) + 4\, g^2\, (4 + 4\, q_{10}\, (2 + q_{10}) + 8\, v + 8\, q_{10}\, v - 11\, v^2 + 480\, q_{20}\, (-1 + q_{20} + w_{20})))) \qquad (4.A2.12)$$

First period/ third stage each firm maximizes its profits as for q_{10} and q_{20} respectively. Reaction functions of the first period are given below;

$$RF_{10}\left(q_{20}\right) = K + Lq_{20} + Mw_{10} \qquad (4.A2.13)$$

$$RF_{20}\left(q_{10}\right) = \frac{1 - gq_{10} - w_{20}}{2}, \qquad (4.A2.14)$$

where, $L = \frac{dRF_{10}}{q_{20}} = -\frac{g\left(16(2+c)^2 - 10(2+c)g^2 + g^4\right)^2}{(2+c)\left(8(2+c)(4+c) - 4(5+2c)g^2 + g^4\right)\left(8(2+c)(4+3c) - 4(5+3c)g^2 + g^4\right)} < 0$

$$M = \frac{dRF_{10}}{w_{10}} = gL < 0$$

$$K = \left(-192c^5 + \left(64 - 20g^2 + g^4\right)^2 + 16c^4\left(-88 + 18g^2 + 4v - gv\right) + 4c^3\left(-16\left(52 - 23g^2 + 2g^4\right) + (4+g)\left(24 + (-12+g)g\right)v\right) + 4c^2\left(-384 + 448g^2 - 97g^4 + 5g^6 + (-4+g)\left(-48 + g^2(8+g)\right)v\right) - c(-2+g)(4+g)\left(g^2 - 8\right)\left(-16g - 2g^3 + g^4 - 8(8+v)\right)\right) / \left((2+c)\left(8(2+c)(4+c) - 4(5+2c)g^2 + g^4\right)\left(8(2+c)(4+3c) - 4(5+3c)g^2 + g^4\right)\right) > 0$$

We solve the system of the first period RFs to get the optimal q_{10}^{*} and q_{20}^{*} -rules in the first period;

$$q_{10}^* = (-384 c^5 + (64 - 20 g^2 + g^4)^2 (2 - 2 w_{10} + g (-1 + w_{20})) + 4 c^2 (10 g^6 + 2 g^4 (-97 + v - 33 w_{10}) - 64 g^2 (-14 + v - 15 w_{10}) + 384 (-2 + v - 8 w_{10}) + 8 g^3 (60 + v - 60 w_{20}) - 96 g (16 + v - 16 w_{20}) + 33 g^5 (-1 + w_{20})) + 32 c^4 (4 (-22 + v - 4 w_{10}) + g (-8 + 18 g - v + 8 w_{20})) - 8 c^3 (832 - 96 v + 512 w_{10} + g (8 (32 + 3 v - 32 w_{20}) + g (8 (-46 + v - 10 w_{10}) + g (-40 + 32 g - v + 40 w_{20})))) - 2 c (-2 + g) (4 + g) (64 (8 + v - 16 w_{10}) + g (-128 (3 + 2 w_{10} - 4 w_{20}) + g (-8 (v - 4 (-6 + 9 w_{10} + 4 w_{20})) + g (8 (18 + 5 w_{10} - 18 w_{20}) + g (12 - 12 g + g^2 - 20 w_{10} + 10 (-2 + g) w_{20}))))))/I \tag{4.A2.15}$$

$$q_{20}^* = (4 c^3 (g (832 - 96 v + 512 w_{10} + g (g (8 (-46 + v - 10 w_{10}) + g (32 + 32 g - v- 32 w_{20})) + 8 (3 v + 76 (-1 + w_{20})))) - 2368 (-1 + w_{20})) - 16 c^4 (g (4 (-22 + v - 4 w_{10}) + g (18 + 18 g - v - 18 w_{20})) + 136 (-1 + w_{20})) - 4 c^2 (g (192 (-2 + v - 8 w_{10}) + g (g (-32 (-14 + v - 15 w_{10}) + g (g (-97 + v - 33 w_{10} + 5 g (1 + g - w_{20})) + 4 (v + 49 (-1 + w_{20})))) - 16 (3 v + 118 (-1 + w_{20})))) + 4992 (-1 + w_{20})) + (64 - 20 g^2 + g^4)^2 (2 + g (-1 + w_{10}) - 2 w_{20}) + 192 c^5 (1 + g - w_{20}) + c (-2 + g) (4 + g) (2560 (-1 + w_{20}) + g (64 (-2 + v - 16 w10 + 10 w_{20}) + g (-32 (-29 + 8 w_{10} + 25 w_{20}) + g (-8 (-7 + v - 36 w10 + 15 w_{20}) + g (-68 + g^3 + 40 w_{10} + 68 w_{20} - g2 (1 + w_{20}) + 2 g (-5 - 10 w_{10} + w_{20})))))))/I \tag{4.A2.16}$$

Substituting the later into (4.A2.7) and (4.A2.8) accrues union's utilities that depend on wages of the first period only;

$$U1 (w_{10}, w_{20})= ((2 (4 + 2 c - g^2)^3 (-8 (2 + c) + g^2)^2 ((-2 + g)^2 (2 + g) (4 + g) v + c (16 (3 + 2 v - w_{10}) + g (g (2 (-7 - 3 v + w_{10}) + g (1 + g + v - w_{20})) - 8 (1 + v - w_{20}))) - 2 c^2 (-4 (3 + v - w_{10}) + g (2 + 2 g + v - 2 w_{20})))^2)/I + w_{10} (-384 c^5 + (64 - 20 g^2 + g^4)^2 (2 - 2 w_{10} + g (-1 + w_{20})) + 4 c^2 (10 g^6 + 2 g^4 (-97 + v - 33 w_{10}) - 64 g^2 (-14 + v - 15 w_{10}) + 384 (-2 + v - 8 w_{10}) + 8 g^3 (60 + v - 60 w_{20}) - 96 g (16 + v - 16 w_{20}) + 33 g^5 (-1 + w_{20})) + 32 c^4 (4 (-22 + v - 4 w_{10}) + g (-8 + 18 g - v + 8 w_{20})) - 8 c^3 (832 - 96 v + 512 w_{10} + g (8 (32 + 3 v - 32 w_{20}) + g (8 (-46 + v - 10 w_{10}) + g (-40 + 32 g - v + 40 w_{20})))) - 2 c (-2 + g) (4 + g) (64 (8 + v - 16 w_{10}) + g (-128 (3 + 2 w_{10} - 4 w_{20}) + g (-8 (v - 4 (-6 + 9 w_{10} + 4 w_{20})) + g (8 (18 + 5 w_{10} - 18 w_{20}) + g (12 - 12 g + g^2 - 20 w_{10} + 10 (-2 + g) w2_0)))))))/I \tag{4.A2.17}$$

$$U2(w_{10}, w_{20}) = (((2 + c) (4 + 2 c - g^2) (96 c^4 v + (-4 + g) (-2 + g)^3 (2 + g)^2 (4 + g)^2 v + 8 c^3 (112 v + g (-8 (3 + v - w_{10}) + g (4 + 4 g - 21 v - 4 w_{20}))) + 2 c (-2 + g) (2 + g) (4 + g) (g^4 - 128 v + 8 g (3 + 7 v - w_{10}) - g^3 (3 + 8 v + w_{20}) + 2 g^2 (-5 + 10 v + w10 + 2 w_{20})) - 4 c^2 (-736 v + g (32 (6 + 3 v - 2 w_{10}) + g (g (-2 (31 + 9 v - 5 w_{10}) + g (5 + 5 g - 21 v - 5 w_{20})) + 4 (-8 + 65 v + 8 w_{20}))))))^2)/I+w_{20} (4 c3 (g (832 - 96 v + 512 w_{10} + g (g (8 (-46 + v - 10 w_{10}) + g (32 + 32 g - v - 32 w_{20})) + 8 (3 v + 76 (-1 + w_{20})))) - 2368 (-1 + w_{20})) - 16 c^4 (g (4 (-22 + v - 4 w10) + g (18 + 18 g - v - 18 w_{20})) + 136 (-1 + w_{20})) - 4 c^2 (g (192 (-2 + v - 8 w_{10}) + g (g (-32 (-14 + v - 15 w_{10}) + g (g (-97 + v - 33 w10 + 5 g (1 + g - w_{20})) + 4 (v + 49 (-1 + w_{20})))) - 16 (3 v + 118 (-1 + w_{20})))) + 4992 (-1 + w_{20})) + (64 - 20 g^2 + g^4)^2 (2 + g (-1 + w_{10}) - 2 w_{20}) + 192 c^5 (1 + g - w_{20}) + c (-2 + g) (4 + g) (2560 (-1 + w_{20}) + g (64 (-2 + v - 16 w_{10} + 10 w_{20}) + g (-32 (-29 + 8 w_{10}+ 25 w_{20}) + g (-8 (-7 + v - 36 w_{10} + 15 w_{20}) + g (-68 + g^3 + 40 w_{10} + 68 w_{20} - g^2 (1 + w_{20}) + 2 g (-5 - 10 w_{10} + w_{20}))))))))/I \tag{4.A2.18}$$

$$I = (128 (2 + c)^3 (4 + c) (4 +3 c) - 64 (2 + c)^2 (56 + c (56 + 13 c)) g^2 + 32 (2 + c) (73 + c (73 + 18 c)) g^4 - 172 (2 + c)^2 g^6 + 22 (2 + c) g^8 - g^{10})$$

In the first period/ second stage, each firm bargains with the union that represents employees of the same sector the wage. I make the assumption that unions possess all the bargaining power (b=1). The bargaining problem is set as follow;

$$w_{j0} = \max_{w_{j0}} U_j \left(= w_{j0} q_{j0}^* + w_{j1}^* q_{j1}^* \right), \;\; j = 1, 2 \,. \qquad (4.A2.19)$$

Solving the system that accrues from the first order conditions of the above problem as for w_{10} and w_{20} accrues a unique stable solution for the equilibrium firm-specific wage contracts w_{10}^* and w_{20}^*, respectively.

w_{10} = (-56623104 c^{15} + $(-4 + g)^6$ $(-2 + g)^8$ $(2 + g)^7$ $(4 + g)^7$ - 4 c^3 $(-4 + g)^2$ $(-2 + g)^3$ $(2 + g)^2$
$(4 + g)^3$ (2 (-4 + g) $(-2 + g)^2$ (2 + g) (-5423104 + g (-2539520 + g (2373632 + g (885248 + g
(-412352 + g (-53472 + g (49664 + g (-8448 + g (-5072 + g (767 + 245 g)))))))))) + (-
40894464 +g^2 (42811392 + g (-2334720 + g (-17393664 + g (1894400 + g (3458816 + g (-
567040 + g (-344704 + g (75328 + g (15144 + g (-4076 + 3 g (-56 + 17 g))))))))))) v) +
1572864 c^{14} (-4 (294 + 5 v) + g (-4 + 206 g + 5 v)) - 131072 c^{13} (32 (6456 + 199 v) + g
(1984 - 1592 v + g (-8 (8974 + 127 v) + g (-348 + 6060 g + 247 v)))) - c $(-4 + g)^4$ $(-2 + g)^6$
$(2 + g)^5$ $(4 + g)^5$ (512 (-48 + v) + g (1024 + g (-256 (-43 + v) + g (32 (-16 + v) + g (g (76 +
g (32 + (-3 + g) g) - 8 v) + 8 (-170 + 3 v)))))) + 32768 c^{12} (-1024 (7039 + 303 v) + g (256 (-
535 + 303 v) + g (1152 (3243 + 86 v) + g (46048 - 24448 v + g (-627212 - 7620 v + g (-
3707 + 34001 g + 1878 v)))))) + 2048 c10 (-32768 (83478 + 5657 v) + g (8192 (-17452 +
5657 v) + g (16384 (143835 + 7258 v) + g (-4096 (-22378 + 7345 v) + g (-256 (3081778 +
107565 v) + g (2 g (63974464 + g (1049136 + g (-5013104 + g (-37161 + 151493 g)))) + g
(2710016 + g (-720992 + g (-94892 + 26339 g))) v + 384 (-55294 + 18513 v))))))) - 8192
c^{11} (8192 (20445 + 1123 v) + g (2048 (2664 - 1123 v) + g (-1024 (112696 + 4311 v) + g
(1024 (-2613 + 1078 v) + g (2080 (13930 + 327 v) + g (420544 - 171856 v + g (3 g (-7036
+ 40903 g + 2879 v) - 4 (783164 + 8357 v)))))))) + 64 c^7 (-125829120 (6429 + 914 v) + g
(62914560 (-2840 + 457 v) + g (880803840 (1323 + 148 v) + g (-524288 (-369564 + 64603
v) + g (-589824 (1201054 + 103005 v) + g (98304 (-884924 + 167293 v) + g (-4 g (-
5219833856 + g (11950739456 + g (720183808 + g (-1482513120 + g (-56847696 + g
(110454488 + g (2365140 + g (-4516807 + g (-39803 + 77635 g))))))))) + g (-4251033600
+ g (-2105869312 + g (629321472 + g (166626048 + g (-53127264 + g (-6817568 + g
(2356856 + (110080 - 42147 g) g))))))) v + 16384 (14478214 + 915765 v)))))))) + 2 c^2 (-4
+ g)^3 $(-2 + g)^5$ $(2 + g)^4$ $(4 + g)^4$ (-8192 (-246 + 13 v) + g (-233472 + g (512 (-2254 + 135 v)
+ g (256 (589 - 25 v) + g (-384 (-554 + 35 v) + g (32 (-1066 + 81 v) + g (-11552 + 856 v +
g (3144 - 252 v + g (-6 (64 + v) + g (-97 + 33 g + 2 v)))))))))) + 1024 c^9 (-131072 (122716
+ 10233 v) + g (32768 (-40232 + 10233 v) + g (98304 (169966 + 10975 v) + g (-4096 (-
254212 + 67325 v) + g (-5120 (1371194 + 65199 v) + g (512 (-619860 + 171461 v) + g
(512 (2975905 + 96372 v) + g (64 (726250 - 210267 v) + g (-80 (2246526 + 43225 v) + g
(-3256784 + 988356 v + g (10901046 + g (86935 - 265199 g - 27687 v) + 91126 v)))))))))))
+ 32 c^6 (-150994944 (10018 + 2287 v) + g (12582912 (-53684 + 6861 v) + g (25165824
(104531 + 17828 v) + g (-4194304 (-198294 + 27967 v) + g (-3932160 (490402 + 62153 v)
+ g (917504 (-471894 + 72883 v) + g (393216 (1998443 + 184384 v) + g (-98304 (-
1261183 + 211913 v) + g (-90112 (2182407 + 141241 v) + g (2048 (-10383595 + 1887569
v) + g (2 g (1113358720 + g (-1597195328 + g (-69147504 + g (100075104 + g (2316628
+ g (-3505169 + g (-31876 + 52359 g))))))) + g (-435823616 + g (-82463680 + g
(29015152 + g (2664016 + g (-1036968 + g (-34246 + 15115 g)))))) v + 512 (61289513 +
2623480 v)))))))))))) + 256 c8 (-6291456 (21672 + 2281 v) + g (1572864 (-11298 + 2281
v) + g (1572864 (106341 + 8836 v) + g (-131072 (-127089 + 27343 v) + g (-122880
(693496 + 43837 v) + g (4096 (-1533705 + 351329 v) + g (8192 (2847950 + 129757 v) + g
(1024 (1186723 - 289072 v) + g (-128 (28903808 + 873603 v) + g (32 (-3956786 +
1023469 v) + g (339216272 + 5896128 v + g (6705896 - 1839512 v + g (-12 (1385609 +
10048 v) + g (-140338 + 335863 g + 40778 v)))))))))))))) + 8 c^4 (-4 + g) $(-2 + g)^2$ (2 + g) (4
+ g)^2 (2097152 (-3075 + 1144 v) + g (6232735744 + g (-131072 (-39805 + 21852 v) + g
(65536 (-113433 + 1964 v) + g (32768 (-28194 + 42617 v) + g (-16384 (-225075 + 7609 v)

$$+ g (-2048 (191060 + 174723 v) + g (1024 (-959579 + 46439 v) + g (3584 (59289 + 14356 v) + g (151291648 - 9007488 v + g (-32 (1317079 + 127160 v) + g (32 (-418886 + 27269 v) + g (16 (265699 + 9830 v) + g (630232 - 39200 v + g (-215624 - 2050 v + g (-12089 + 4333 g + 569 v))))))))))))))) - 16 c5 (-2 + g) (4 + g) (-75497472 (1632 + 1271 v) + g (-270230618112 + g (3145728 (84784 + 40885 v) + g (-1048576 (-346077 + 4570 v) + g (-1310720 (175458 + 55535 v) + g (262144 (-791655 + 20581 v) + g (32768 (3270406 + 690291 v) + g (-16384 (-4018156 + 152151 v) + g (-12288 (2456055 + 341498 v) + g (1024 (-12338499 + 593846 v) + g (1024 (5238025 + 461033 v) + g (256 (5851133 - 327170 v) + g (-32 (18855798 + 971297 v) + g (16 (-6677779 + 401754 v) + g (8 (5183722 + 134243 v) + g (4183544 - 249656 v + g (g (-68761 + 25445 g + 3662 v) - 2 (790148 + 7191 v))))))))))))))))))))/ ((16 (2 + c)^2 - 10 (2 + c) g^2 + g^4)^2 N \qquad (4.A2.20)$$

$$N= (131072 (2 + c)^6 (4 + c) (4 + 3 c) (8 + c (8 + c)) - 131072 (2 + c)^5 (400 + c (800 + c (547 + c (147 + 13 c)))) g^2 + 20480 (2 + c)^4 (3456 + c (6912 + c (4868 + c (1412 + 141 c)))) g^4 - 512 (2 + c)^3 (105600 + c (211200 + c (151908 + c (46308 + 5017 c)))) g^6 + 128 (2 + c)^2 (201120 + c (402240 + c (293644 + c (92524 + 10581 c)))) g^8 - 128 (2 + c) (62100 + c (124200 + c (91625 + c (29525 + 3501 c)))) g^{10} + 48 (2 + c)^2 (8380 + c (8380 + 1981 c)) g^{12} - 8 (2 + c) (6600 + c (6600 + 1609 c)) g^{14} + 16 (270 + c (270 + 67 c)) g^{16} - 50 (2 + c) g^{18} + g^{20}))$$

$$w_{20} = (49152 c^{10} (4 + g) + (-4 + g)^4 (-2 + g)^6 (2 + g)^5 (4 + g)^5 - 32 c^5 (-110493696 + 4 g (-1392640 + g (22874112 + g (1745408 + g (-7295488 + g (-749760 + g (1115552 + g (143896 + g (-81476 + g (-12545 + g (2268 + 401 g)))))))))) + g (6512640 + g (419840 + g (-4556800 + g (-332800 + g (1151616 + g (89952 + g (-123632 + g (-9688 + 5 g (940 + 69 g))))))))) v) - 4096 c^9 (-1216 + g (53 g (4 + g) - g v + 4 (-64 + 9 v))) + 2048 c^8 (26496 + g (416 (11 - 3 v) + g (-8 (1138 + v) + g (-1876 + 749 g + 179 g^2 + 5 (46 + g) v)))) - 2 c (-4 + g)^2 (-2 + g)^4 (2 + g)^3 (4 + g)^3 (-10240 + g (192 (-4 + v) + g (64 (71 + v) + g (304 - 56 v + g (-580 - 32 g + 21 g^2 + g^3 + 4 (-4 + g) v))))) - 256 c^7 (-1318912 + g (256 (-692 + 291 v) + g (64 (10472 + 37 v) + g (32 (3436 - 839 v) + g (-16 (6806 + 73 v) + g (g (5630 + 1246 g + 125 v) + 4 (-5275 + 577 v))))))) + 4 c^2 (-4 + g) (-2 + g)^3 (2 + g)^2 (4 + g)^2 (724992 + g (-4096 (-25 + 6 v) + g (-256 (1641 + 32 v) + g (128 (-431 + 83 v) + g (32 (2617 + 99 v) + g (40 (250 - 37 v) + g (-4 (1697 + 73 v) + g (-736 + 19 g^2 + 70 v + 2 g (94 + v))))))))) + 128 c^6 (10444800 + g (2048 (479 - 306 v) + g (-1024 (6826 + 31 v) + g (256 (-3314 + 1301 v) + g (64 (26343 + 314 v) + g (251376 - 56608 v + g (-8 (21619 + 479 v) + g (-30406 + 3058 v + g (6345 + 1270 g + 216 v))))))))) - 8 c^3 (-2 + g) (4 + g) (119537664 + g (-16384 (-1460 + 333 v) + g (-16384 (7067 + 110 v) + g (4096 (-5461 + 1101 v) + g (1024 (43579 + 1366 v) + g (256 (32312 - 5619 v) + g (-128 (68000 + 3109 v) + g (32 (-48134 + 6931 v) + g (900752 + 49696 v + g (151984 - 16672 v + g (-4 (11668 + 619 v) + g (-7556 + 496 v + g (943 + 149 g + 27 v))))))))))))) + 16 c^4 (396623872 + g (-65536 (-17 + 327 v) + g (-16384 (23866 + 97 v) + g (4096 (-2626 + 4535 v) + g (10240 (15127 + 149 v) + g (1024 (8033 - 6074 v) + g (-512 (61385 + 1041 v) + g (64 (-38816 + 15577 v) + g (3431072 + 83792 v + g (358656 - 75536 v + g (-8 (23809 + 728 v) + g (-24506 + 2130 v + g (4193 + 632 g + 139 v))))))))))))))/N. \qquad (4.A2.21)$$

Wage differential is formed in favor of the private sector in the first period;

$$w_{10}-w_{20} = -(c (56623104 c^{14} - 1572864 c^{13} (2 (-604 + g (-6 + 103 g)) + 5 (-4 + g) v) + 4 c^2 (-4 + g)^2 (-2 + g)^3 (2 + g)^3 (4 + g)^3 (2 (-4 + g) (-2 + g) (2 + g) (-6209536 + g (-1689600 + g (4586496 + g (1040512 + g (-1281664 + g (-222944 + g (168928 + g (19552 + 5 g (-2096 + g (-117 + 49 g))))))))) + (-20447232 + g (23150592 + g (14123008 + g (-19021824 + g (-2567168 + g (5734400 + g (-159232 + g (-758208 + g (82592 + g (39416 + g (-6004 + g (-$$

$$
364 + 51 g)))))))))))) v) + (-16 + g^2)^5 (-4 + g2)^6 (-64 (8 + v) + g (64 (-3 + v) + g (g (60 + (-7
+ g) g (4 + g) - 16 v) + 8 (28 + 3 v)))) + 131072 c^{12} (32 (6856 + 199 v) + g (4800 - 1880 v
+ g (-48 (1541 + 21 v) + g (-892 + 6060 g + 247 v)))) - 32768 c^{11} (-1024 (7793 + 303 v) + g
(768 (-371 + 139 v) + g (128 (31219 + 773 v) + g (32 (3211 - 924 v) + g (-648468 - 7660 v
+ g (-8889 + 34001 g + 1878 v)))))) + 2048 c^9 (32768 (104470 + 5657 v) + g (-8192 (-
28860 + 10217 v) + g (-16384 (171669 + 7328 v) + g (4096 (-39960 + 12049 v) + g (2 g
(20525184 + g (-69714720 + g (-2203112 + g (5230186 + (85127 - 151493 g) g)))) + g (-
10323840 + g (-2859648 + g (894040 + (103968 - 26339 g) g))) v + 256 (3512258 +
110509 v)))))) - 2 c (-16 + g^2)^4 (-4 + g2)5 (1024 (115 + 13 v) + g (-12544 (-3 + v) + g (-
6912 (10 + v) + g (32 (-545 + 156 v) + g (8 (1752 + 97 v) + g (2352 - 468 v + g (-1160 - 89
g + 33 g^2 + 2 (-3 + g) v))))))) + 8192 c^{10} (8192 (23869 + 1123 v) + g (-8192 (-1227 + 445
v) + g (-1536 (84280 + 2887 v) + g (1024 (-5189 + 1534 v) + g (32 (976682 + 21617 v) + g
(902016 - 211096 v + g (-14 (232292 + 2483 v) + 3 g (-16338 + 40903 g + 2879 v)))))))) +
32 c^5 (50331648 (86318 + 6861 v) + g (-37748736 (-18836 + 6351 v) + g (-25165824
(250587 + 18256 v) + g (4194304 (-225186 + 70975 v) + g (262144 (15114886 + 984543
v) + g (-26607616 (-20034 + 5843 v) + g (-196608 (7177861 + 404678 v) + g (24576 (-
6773466 + 1803019 v) + g (2 g (15683274240 + g (-22430407168 + g (-1820713280 + g
(2067945728 + g (126873744 + g (-118214696 + g (-4844466 + g (3800204 + (77576 -
52359 g) g)))))))) + g (-7476725760 + g (-1634363392 + g (756630528 + g (107383648 + g
(-44185040 + g (-3764384 + g (1330148 + (53408 - 15115 g) g))))))) v + 4096 (76486804
+ 3573285 v)))))))))) + 64 c^6 (25165824 (62577 + 4570 v) + g (-12582912 (-16992 + 5771
v) + g (-6291456 (324028 + 21161 v) + g (3670016 (-68196 + 21325 v) + g (458752
(2450354 + 139143 v) + g (-937230336 (-130 + 37 v) + g (-106496 (3230698 + 153495 v)
+ g (2048 (-15566530 + 3968409 v) + g (8192 (7786247 + 293010 v) + g (4 g (-
1829650368 + g (-105473680 + g (126648648 + g (4914262 + g (-4829363 + g (-94047 +
77635 g)))))) + 256 (18857975 - 4208913 v) + g (-200735232 + g (79721600 + g (8785712
+ g (-2986608 + g (-153974 + 42147 g))))) v)))))))))) + 1024 c^8 (393216 (56260 + 3411 v)
+ g (-2064384 (-952 + 331 v) + g (-196608 (110263 + 5565 v) + g (4096 (-409380 +
125309 v) + g (1024 (8410706 + 337851 v) + g (512 (1081908 - 286847 v) + g (-768
(2306221 + 68649 v) + g (96 (-919642 + 206101 v) + g (32 (6198799 + 120252 v) + g
(6767532 - 1233784 v + g (-11444020 - 106847 v + g (-199057 + 265199 g + 27687
v)))))))))))) + 256 c^7 (6291456 (34132 + 2281 v) + g (-1572864 (-15150 + 5197 v) + g (-
1572864 (155293 + 8995 v) + g (131072 (-184053 + 57007 v) + g (8192 (14146660 +
686559 v) + g (-4096 (-2408283 + 664541 v) + g (-2048 (14482827 + 560494 v) + g (256
(-8120578 + 1958179 v) + g (1536 (2872469 + 82081 v) + g (237424608 - 48619040 v + g
(-96 (3964495 + 73088 v) + g (4 (-3473497 + 577855 v) + g (g (324782 - 335863 g - 40778
v) + 2 (8797319 + 76698 v)))))))))))))) - 8 c^3 (-4 + g) (-2 + g)^2 (2 + g)^2 (4 + g)^2 (1048576
(12151 + 1144 v) + g (-524288 (461 + 2449 v) + g (-65536 (233237 + 15496 v) + g (32768
(19434 + 37997 v) + g (8192 (932114 + 34915 v) + g (-4096 (120691 + 115679 v) + g (-
1024 (2003393 + 18519 v) + g (512 (351227 + 173659 v) + g (318826624 - 4874624 v + g
(-64 (546761 + 131354 v) + g (32 (-891864 + 28741 v) + g (3719504 + 354624 v + g
(1356736 - 49488 v + g (-201796 - 4138 v + g (-26333 + 4333 g + 569 v))))))))))))))) + 16
c^4 (-2 + g) (4 + g) (-75497472 (14704 + 1271 v) + g (226492416 (-2182 + 215 v) + g
(3145728 (484376 + 45925 v) + g (-1048576 (-611463 + 57193 v) + g (-524288 (1733846
+ 171455 v) + g (65536 (-5398402 + 480649 v) + g (49152 (6303488 + 613501 v) + g (-
24576 (-4403033 + 373294 v) + g (-6144 (10844625 + 966112 v) + g (1024 (-19553193 +
1570112 v) + g (256 (36713946 + 2717525 v) + g (1600 (1431682 - 107095 v) + g (-96
(9076131 + 486113 v) + g (48 (-3291365 + 220841 v) + g (8 (6401198 + 198903 v) + g
(5999128 - 333180 v + g (g (-95995 + 25445 g + 3662 v) - 2 (864277 + 9897
v)))))))))))))))))))))/P \qquad (4.A2.22)
$$

$$P = ((16 (2 + c)^2 - 10 (2 + c) g^2 + g^4)^2 (131072 (2 + c)^6 (4 + c) (4 + 3 c) (8 + c (8 + c)) - 131072 (2 + c)^5 (400 + c (800 + c (547 + c (147 + 13 c)))) g^2 + 20480 (2 + c)^4 (3456 + c (6912 + c (4868 + c (1412 + 141 c)))) g^4 - 512 (2 + c)^3 (105600 + c (211200 + c (151908 + c (46308 + 5017 c)))) g^6 + 128 (2 + c)^2 (201120 + c (402240 + c (293644 + c (92524 + 10581 c)))) g^8 - 128 (2 + c) (62100 + c (124200 + c (91625 + c (29525 + 3501 c)))) g^{10} + 48 (2 + c)^2 (8380 + c (8380 + 1981 c)) g^{12} - 8 (2 + c) (6600 + c (6600 + 1609 c)) g14 + 16 (270 + c (270 + 67 c)) g^{16} - 50 (2 + c) g^{18} + g^{20}))$$

For g=1 then

$$w_{10}* - w_{20}* = (2 c (-16 c (64572541875 + c (245974350375 + c (570538948725 + 2 c (450204150330 + c (511392495491 + 4 c (107777329215 + c (68476605089 + 8 c (4116502523 + 4 c (372518354 + c (99796263 + 16 c (1197383 + 8 c (19424 + 9 c (169 + 6 c))))))))))))) - 2 c (-5215995000 + c (-5164570125 + 2 c (10274795325 + 4 c (10037030540 + c (17560980073 + 2 c (9664256365 + 8 c (914672611 + 4 c (122891480 + c (47040149 + 4 c (3145083 + 8 c (69963 + 7454 c + 360 c^2))))))))))) v + 553584375 (-225 + 4 v)))/S \quad (4.A2.22a)$$

where, $S = ((3 + 2 c)^2 (15 + 8 c)^2 (184528125 + 2 c (553584375 + 4 c (365690700 + c (560524455 +2 c (275949631 + 8 c (22785395 + c (10215661 + 4 c (766023 + 8 c (18337 + 96 c (21 + c)))))))))))$

For g=0.5, then

$$w_{10}* - w_{20}* = (-0.5625 c (1.00589 + c) (1.26938 + c) (1.82946 + c) (2.06774 + c) (3.75538 + c) (6.74884 + c) (3.4546 + c (3.71386 + c)) (3.72951 + c (3.85207 + c)) (3.93805 + c (3.95941 + c)) (4.18675 + c (4.08965 + c)) - 0.273438 c (1.30938 + c) (1.83095 + c) (3.78235 + c) (1.32812 + c (2.10796 + c)) (3.46365 + c (3.71905 + c)) (3.7469 + c (3.86429 + c)) (4.07799 + c (4.03345 + c)) (4.30155 + c (4.14721 + c)) v)/((1.10438 + c) (1.25224 + c) (1.875 + c)^2 (1.94625 + c) (1.96875 + c)^2 (2.00026 + c) (3.74727 + c) (6.45841 + c) (3.71921 + c (3.85703 + c)) (3.77177 + c (3.88416 + c))) \quad (4.A2.22b)$$

Substituting the wages with the above optimal wages in the equations (4.A2.15) and (4.A2.16) accrues the optimal quantities of the first period, depending only on *c*, *g*, and *v*;

$$q_{10} = -(2 (49152 c^{10} + (-4 + g)^4 (-2 + g)^5 (2 + g)^4 (4 + g)^5 + 4 c^2 (-4 + g) (-2 + g)^2 (2 + g) (4 + g)^2 (2 (-4 + g) (2 + g) (-19712 + g (9152 + g (6144 + g (-3184 + g (-204 + g (297 + g (-72 + 5 (-1 + g) g))))))) + (98304 + g^2 (-57344 + g (-1408 + g (12096 + g (576 + g (-1096 + g (-54 + 37 g))))))) v) - 4096 c^9 (g (-12 + 52 g - 9 v) + 4 (-64 + 9 v)) + 1024 c^8 (832 (11 - 3 v) + g (928 + 624 v + g (g (-132 + 334 g - 79 v) + 48 (-77 + 8 v)))) + 16 c^5 (-81920 (-136 + 159 v) + g (4096 (1876 + 795 v) + g (16384 (-853 + 470 v) + g (-15360 (280 + 103 v) + g (4 g (210784 + g (-247072 + g (-16912 + g (19468 + (461 - 561 g) g)))) + 256 (21998 - 6501 v) + g (259776 + g (156064 + g (-16600 - 5340 g + 333 g^2))) v))))) - 256 c^7 (256 (-692 + 291 v) + g (-192 (164 + 97 v) + g (-2048 (-53 + 11 v) + g (8896 + 4632 v + g (g (-587 + 1077 g - 258 v) + 4 (-4927 + 413 v)))))) - c (-4 + g)^2 (-2 + g)^3 (2 + g)^2 (4 + g)^3 (-512 (28 + 3 v) + g (1024 + g (640 (10 + v) + g (32 (-16 + v) + g (-8 (95 + 11 v) + g (76 - 8 v + g (10 + (-3 + g) g + 4 v))))))) + 64 c^6 (-4096 (-479 + 306 v) + g (2048 (301 + 153 v) + g (1024 (-1641 + 548 v) + g (-128 (2030 + 901 v) + g (469856 - 81472 v + g (24 (1421 + 530 v) + g (-52352 + g (-1374 + 1989 g - 409 v) + 3844 v))))))) - 4 c3 (-2 + g) (4 + g) (-32768 (364 + 333 v) + g (7798784 + g (4096 (1568 + 2003 v) + g (28672 (-224 + 3 v) + g (-1536 (311 + 1542 v) + g$$

$$(256 (7825 - 189 v) + g (64 (-4790 + 5119 v) + g (64 (-4579 + 129 v) + g (74456 - 21824 v + g (19940 - 388 v + g (-6118 + g (-503 + 169 g - 5 v) + 570 v))))))))))) + 16 c^4 (-65536 (-17 + 327 v) + g (49152 (322 + 109 v) + g (16384 (-701 + 960 v) + g (-5120 (2156 + 631 v) + g (1024 (7749 - 4402 v) + g (768 (3745 + 916 v) + g (128 (-16760 + 4931 v) + g (-16 (21564 + 4195 v) + g (276592 - 43024 v + g (4 (4687 + 670 v) + g (-16933 + 395 g^2 + 1129 v - 33 g (11 + v)))))))))))))))/L \tag{4.A2.23}$$

$$q_{20} = ((2 + c) (24576 c^9 (4 + g) - (-4 + g)^4 (-2 + g)^5 (2 + g)^4 (4 + g)^5 + 2 c (-4 + g)^2 (-2 + g)^3 (2 + g)^2 (4 + g)^3 ((-4 + g) (-2 + g) (2 + g) (4 + g) (-144 + g (-12 + g (20 + g))) - 2 g (-8 + g^2) (12 + g (-4 + (-2 + g) g)) v) - 128 c^6 (-966656 + 2 g (-59392 + g (227072 + g (40640 + g (-34024 + g (-7918 + g (1615 + 454 g)))))) + g (56832 + g (-30592 + g (-15744 + g (9088 + (996 - 641 g) g)))) v) + 8 c^3 (-2 + g) (4 + g) (2 (-4 + g) (2 + g) (1290240 + g (-38912 + g (-784640 + g (-15744 + g (170880 + g (11928 + g (-15708 + g (-1820 + g (512 + 79 g))))))))) + g (1019904 + g (-335872 + g (-655360 + g (231424 + g (152576 + g (-56512 + g (-15160 + g (5756 + (538 - 205 g) g)))))))) v) + 32 c^4 (28934144 - 2 g (-380928 + g (11151360 + g (734720 + g (-3290880 + g (-347840 + g (462176 + g (66548 + g (-30736 + g (-5485 + 4 g (193 + 40 g)))))))))) + g (-1658880 + g (947200 + g (911360 + g (-561152 + g (-172800 + g (118912 + g (13072 + g (-10644 + g (-320 + 339 g))))))))) v) - 2048 c^8 (-1120 + g (g (180 + 51 g - 17 v) + 4 (-58 + 9 v))) + 1024 c^7 (22016 + g (48 (76 - 23 v) + g (-6976 + 564 v + g (g (529 + 155 g - 84 v) + 2 (-808 + 77 v))))) - 4 c^2 (-4 + g) (-2 + g)^2 (2 + g) (4 + g)^2 (577536 + g (1024 (88 - 21 v) + g (256 (-1341 + 28 v) + g (64 (-794 + 157 v) + g (70304 - 3808 v + g (9712 - 1520 v + g (-5812 + 628 v + g (-748 + g (163 + 20 g - 32 v) + 74 v)))))))) + 64 c^5 (6578176 + g (-26624 (-19 + 15 v) + g (512 (-7976 + 435 v) + g (3840 (-132 + 43 v) + g (909696 - 99072 v + g (48 (3232 - 435 v) + g (-85664 + 13988 v + g (-18314 + g (2862 + 719 g - 625 v) + 786 v)))))))))))/L \tag{4.A2.24}$$

$$L=(131072 (2 + c)^6 (4 + c) (4 + 3 c) (8 + c (8 + c)) - 131072 (2 + c)^5 (400 + c (800 + c (547 + c (147 + 13 c)))) g^2 + 20480 (2 + c)^4 (3456 + c (6912 + c (4868 + c (1412 + 141 c)))) g^4 - 512 (2 + c)^3 (105600 + c (211200 + c (151908 + c (46308 + 5017 c)))) g^6 + 128 (2 + c)^2 (201120 + c (402240 + c (293644 + c (92524 + 10581 c)))) g^8 - 128 (2 + c) (62100 + c (124200 + c (91625 + c (29525 + 3501 c)))) g^{10} + 48 (2 + c)^2 (8380 + c (8380 + 1981 c)) g^{12} - 8 (2 + c) (6600 + c (6600 + 1609 c)) g^{14} + 16 (270 + c (270 + 67 c)) g^{16} - 50 (2 + c) g^{18} + g^{20})$$

Also, substituting the wages with the optimal wages in the equations (4.A2.9) and (4.A2.10) accrues the optimal wages of the second period, depending only on c, g, and v;

$$w_{11} = 1/(16 + 8 c - g^2) (-(c (-4 + g) + (-2 + g) (4 + g)) v + c (8 + 4 c - g^2) (1 - (2 (49152 c^{10} + (-4 + g)^4 (-2 + g)^5 (2 + g)^4 (4 +g)^5 + 4 c^2 (-4 + g) (-2 + g)2 (2 + g) (4 + g)^2 (2 (-4 + g) (2 + g) (-19712 + g (9152 + g (6144 + g (-3184 + g (-204 + g (297 + g (-72 + 5 (-1 + g) g))))))) + (98304 + g^2 (-57344 + g (-1408 + g (12096 + g (576 + g (-1096 + g (-54 + 37 g))))))) v) - 4096 c^9 (g (-12 + 52 g - 9 v) + 4 (-64 + 9 v)) + 1024 c^8 (832 (11 - 3 v) + g (928 + 624 v + g (g (-132 + 334 g - 79 v) + 48 (-77 + 8 v)))) + 16 c^5 (-81920 (-136 + 159 v) + g (4096 (1876 + 795 v) + g (16384 (-853 + 470 v) + g (-15360 (280 + 103 v) + g (4 g (210784 + g (-247072 + g (-16912 + g (19468 + (461 - 561 g) g)))) + 256 (21998 - 6501 v) + g (259776 + g (156064 + g (-16600 - 5340 g + 333 g^2))) v))))) - 256 c^7 (256 (-692 + 291 v) + g (-192 (164 + 97 v) + g (-2048 (-53 + 11 v) + g (8896 + 4632 v + g (g (-587 + 1077 g - 258 v) + 4 (-4927 + 413 v)))))) - c (-4 + g)^2 (-2 + g)^3 (2 + g)^2 (4 + g)^3 (-512 (28 + 3 v) + g (1024 + g (640 (10 + v) + g (32 (-16 + v) + g (-8 (95 + 11 v) + g (76 - 8 v + g (10 + (-3 + g) g + 4 v))))))) + 64 c^6 (-4096 (-479 + 306 v) + g (2048 (301 + 153 v) + g (1024 (-1641 + 548 v) + g (-128 (2030 + 901 v) + g (469856 - 81472 v + g (24 (1421 + 530 v) + g (-52352 + g (-1374 + 1989 g - 409 v) + 3844 v))))))) - 4 c^3 (-2 + g) (4 + g) (-32768 (364 + 333 v) + g \tag{4.A2.25}$$

$$(7798784 + g (4096 (1568 + 2003 v) + g (28672 (-224 + 3 v) + g (-1536 (311 + 1542 v) + g (256 (7825 - 189 v) + g (64 (-4790 + 5119 v) + g (64 (-4579 + 129 v) + g (74456 - 21824 v + g (19940 - 388 v + g (-6118 + g (-503 + 169 g - 5 v) + 570 v))))))))))) + 16 c^4 (-65536 (-17 + 327 v) + g (49152 (322 + 109 v) + g (16384 (-701 + 960 v) + g (-5120 (2156 + 631 v) + g (1024 (7749 - 4402 v) + g (768 (3745 + 916 v) + g (128 (-16760 + 4931 v) + g (-16 (21564 + 4195 v) + g (276592 - 43024 v + g (4 (4687 + 670 v) + g (-16933 + 395 g^2 + 1129 v - 33 g (11 + v)))))))))))))))/M$$

$$w_{21} = -(1/(16 + 8 c - g^2)) ((-2 + g) (4 + g) v + 2 c (g - 2 v - (2 g (49152 c10 + (-4 + g)^4 (-2 + g)^5 (2 + g)^4 (4 + g)^5 + 4 c^2 (-4 + g) (-2 + g)^2 (2 + g) (4 + g)^2 (2 (-4 + g) (2 + g) (-19712 + g (9152 + g (6144 + g (-3184 + g (-204 + g (297 + g (-72 + 5 (-1 + g) g))))))) + (98304 + g^2 (-57344 + g (-1408 + g (12096 + g (576 + g (-1096 + g (-54 + 37 g))))))) v) - 4096 c^9 (g (-12 + 52 g - 9 v) + 4 (-64 + 9 v)) + 1024 c^8 (832 (11 - 3 v) + g (928 + 624 v + g (g (-132 + 334 g - 79 v) + 48 (-77 + 8 v)))) + 16 c^5 (-81920 (-136 + 159 v) + g (4096 (1876 + 795 v) + g (16384 (-853 + 470 v) + g (-15360 (280 + 103 v) + g (4 g (210784 + g (-247072 + g (-16912 + g (19468 + (461 - 561 g) g)))) + 256 (21998 - 6501 v) + g (259776 + g (156064 + g (-16600 - 5340 g + 333 g2))) v))))) - 256 c^7 (256 (-692 + 291 v) + g (-192 (164 + 97 v) + g (-2048 (-53 + 11 v) + g (8896 + 4632 v + g (g (-587 + 1077 g - 258 v) + 4 (-4927 + 413 v)))))) - c (-4 + g)^2 (-2 + g)^3 (2 + g)^2 (4 + g)^3 (-512 (28 + 3 v) + g (1024 + g (640 (10 + v) + g (32 (-16 + v) + g (-8 (95 + 11 v) + g (76 - 8 v + g (10 + (-3 + g) g + 4 v))))))) + 64 c^6 (-4096 (-479 + 306 v) + g (2048 (301 + 153 v) + g (1024 (-1641 + 548 v) + g (-128 (2030 + 901 v) + g (469856 - 81472 v + g (24 (1421 + 530 v) + g (-52352 + g (-1374 + 1989 g - 409 v) + 3844 v))))))) - 4 c^3 (-2 + g) (4 + g) (-32768 (364 + 333 v) + g (7798784 + g (4096 (1568 + 2003 v) + g (28672 (-224 + 3 v) + g (-1536 (311 + 1542 v) + g (256 (7825 - 189 v) + g (64 (-4790 + 5119 v) + g (64 (-4579 + 129 v) + g (74456 - 21824 v + g (19940 - 388 v + g (-6118 + g (-503 + 169 g - 5 v) + 570 v))))))))))) + 16 c^4 (-65536 (-17 + 327 v) + g (49152 (322 + 109 v) + g (16384 (-701 + 960 v) + g (-5120 (2156 + 631 v) + g (1024 (7749 - 4402 v) + g (768 (3745 + 916 v) + g (128 (-16760 + 4931 v) + g (-16 (21564 + 4195 v) + g (276592 - 43024 v + g (4 (4687 + 670 v) + g (-16933 + 395 g^2 + 1129 v - 33 g (11 + v)))))))))))))))/M \qquad (4.A2.26)$$

$$M=(131072 (2 + c)^6 (4 + c) (4 + 3 c) (8 + c (8 + c)) - 131072 (2 + c)^5 (400 + c (800 + c (547 + c (147 + 13 c)))) g^2 + 20480 (2 + c)^4 (3456 + c (6912 + c (4868 + c (1412 + 141 c)))) g^4 - 512 (2 + c)^3 (105600 + c (211200 + c (151908 + c (46308 + 5017 c)))) g^6 + 128 (2 + c)^2 (201120 + c (402240 + c (293644 + c (92524 + 10581 c)))) g^8 - 128 (2 + c) (62100 + c (124200 + c (91625 + c (29525 + 3501 c)))) g^{10} + 48 (2 + c)^2 (8380 + c (8380 + 1981 c)) g^{12} - 8 (2 + c) (6600 + c (6600 + 1609 c)) g^{14} + 16 (270 + c (270 + 67 c)) g^{16} - 50 (2 + c) g^{18} + g^{20}))).$$

In the second period wage differential is formed in favor of the public sector;

$$w_{11}^{*} - w_{21}^{*} = \frac{c\left(4c-(g-4)(g+2)\right)\left(1+q_{10}^{*}\right)-cgv}{16+8c-g^2} > 0,$$

note that we suppose that $g \in [0,1]$ and $0.03 \le v \le 1.97$; also, in **figure (4.B2.1c)** and **(4.B2.2c)** it is clear that $0 < q_{10} < 1$.

substituting now the optimal quantities of the first period as well as the optimal wages of the second period in equations (4.A2.5) and (4.A2.6) accrues the optimal quantities of the second period depending on *c*, *g*, and *v*;

$$q_{11} = (2 (1179648 c^{12} - (-4 + g)^5 (-2 + g)^6 (2 + g)^5 (4 + g)^6 v - 98304 c^{11} (-344 + 4 g + 55 g^2 + 7 (-4 + g) v) + 32768 c^{10} (4 (3220 + g (-64 + g (-1015 + g (9 + 77 g)))) + (2008 + g (-502 + g (-325 + 74 g))) v) - c (-4 + g)^4 (-2 + g)^5 (2 + g)^4 (4 + g)^5 (80 + 352 v + g (16 + g (g (-2 + g + v) - 2 (9 + 28 v)))) - 2048 c^9 (-256 (5830 + 1349 v) + g (1216 (32 + 71 v) + g (64 (10891 + 1727 v) + g (-320 (34 + 79 v) + g (-12 (8711 + 709 v) + g (719 + 5016 g + 1747 v)))))) + 512 c^8 (2048 (13702 + 4307 v) + g (-512 (1696 + 4307 v) + g (-256 (67560 + 16379 v) + g (768 (472 + 1253 v) + g (3853888 + 639648 v + g (-8 (5962 + 16537 v) + g (g (1961 + 12532 g + 5684 v) - 4 (91675 + 7801 v)))))))) + 2 c^2 (-4 + g)^2 (-2 + g)^3 (2 + g)^2 (4 + g)^3 (-4096 (208 + 437 v) + g (-172032 + g (1024 (582 + 1105 v) + g (256 (400 - 39 v) + g (-384 (410 + 653 v) + g (32 (-661 + 147 v) + g (8 (2457 + 2839 v) + g (1780 - 652 v + g (g (-51 + 26 g + 25 v) - 2 (581 + 345 v)))))))))) + 128 c^7 (16384 (22050 + 9083 v) + g (-4096 (3080 + 9083 v) + g (-4096 (67387 + 22834 v) + g (8192 (854 + 2627 v) + g (256 (317895 + 82957 v) + g (-64 (21497 + 69021 v) + g (-32 (360475 + 64306 v) + g (8 (14093 + 47296 v) + g (8 (97943 + 8947 v) - g (3218 + 20352 g + 11323 v)))))))))) - 8 c^3 (-4 + g) (-2 + g)^2 (2 + g) (4 + g)^2 (32768 (973 + 1293 v) + g (6504448 + g (-2048 (13435 + 16421 v) + g (1024 (-4795 + 341 v) + g (256 (37051 + 39786 v) + g (1406592 - 220288 v + g (-64 (26095 + 22818 v) + g (32 (-5939 + 1521 v) + g (158364 + 98240 v + g (12042 - 4396 v + g (g (-285 + 148 g + 134 v) - 6 (1278 + 409 v)))))))))))) + 32 c^6 (917504 (3554 + 1899 v) + g (-229376 (544 + 1899 v) + g (-16384 (181276 + 82955 v) + g (16384 (5264 + 19125 v) + g (10240 (106252 + 39927 v) + g (-512 (44044 + 166835 v) + g (-256 (798838 + 230805 v) + g (384 (7199 + 28504 v) + g (192 (107989 + 21300 v) + g (-24 (6558 + 27251 v) + g (g (3296 + 22066 g + 14469 v) - 4 (268075 + 27049 v)))))))))))) + 16 c^5 (524288 (19730 + 13783 v) + g (-917504 (464 + 1969 v) + g (-131072 (83445 + 51311 v) + g (393216 (896 + 3949 v) + g (8192 (584219 + 307105 v) + g (-10240 (11221 + 51505 v) + g (-10240 (109347 + 47122 v) + g (512 (36605 + 175608 v) + g (384 (392677 + 129925 v) - g (96 (16639 + 83831 v) + g (16 (728573 + 164384 v) + g (-8 (8345 + 44463 v) + g (-478270 - 55158 v + g (1056 + 8033 g + 6019 v)))))))))))))) + 8 c^4 (-2 + g) (4 + g) (-262144 (10800 + 10129 v) + g (-584056832 + g (393216 (7398 + 6455 v) + g (-65536 (-8163 + 440 v) + g (-4096 (302030 + 237567 v) + g (4096 (-47803 + 5570 v) + g (512 (550531 + 372248 v) + g (1792 (20371 - 3834 v) + g (-192 (193382 + 104245 v) + g (64 (-56916 + 15265 v) + g (8 (353321 + 132939 v) + g (183924 - 65080 v + g (g (-3667 + 1936 g + 1614 v) - 2 (57571 + 11129 v))))))))))))))))/K \qquad (4.A2.27)$$

$$q_{21} = -((2 + c) (196608 c^{11} (3 g - 8 v) + (-4 + g)^5 (-2 + g)^6 (2 + g)^5 (4 + g)^6 v - 16384 c^{10} (12 g (-80 + g + 13 g2) + (2624 - g (84 + 431 g)) v) - 4 c^2 (-4 + g)^2 (-2 + g)^3 (2 + g)^2 (4 + g)^3 ((-4 + g) (-2 + g) g (2 + g) (4 + g) (1504 + g (304 + g (-390 + g (-47 + 24 g)))) - 2 (444416 + g (19712 + g (-276096 + g (-9280 + g (59968 + g (1276 + 3 g (-1774 + g (-16 + 53 g)))))))) v) + 8 c^3 (-4 + g) (-2 + g)^2 (2 + g) (4 + g)^2 (4 (-4 + g) g (2 + g) (200960 + g (-9152 + g (-119872 + g (3184 + g (25140 + g (-297 + g (-2157 + g (5 + 62 g)))))))) + (41484288 + g (2703360 + g (-32276480 + g (-1704960 + g (9559040 + g (374784 + g (-1338240 + g (-33472 + g (87872 + (998 - 2145 g) g))))))))) v) - 16 c^4 (-2 + g) (4 + g) ((-4 + g) g (2 + g) (-31375360 + g (1347584 + g (23827456 + g (-704256 + g (-6899200 + g (124544 + g (944400 + g (-8428 + g (-60548 + 15 g (11 + 96 g)))))))))) - 2 (635699200 + g (54312960 + g (-594837504 + g (-42993664 + g (222730240 + g (12916736 + g (-42529920 + g (-1826080 + g (4348480 + g (120376 + g (-224818 + 7 g (-419 + 654 g)))))))))))) v) + 2 c (-4 + g)^4 (-2 + g)^5 (2 + g)^4 (4 + g)^5 (176 v + g (-10 + g (-2 + g - 27 v) + 4 v)) + 4096 c9 (-125440 v + g (320 (137 + 23 v) + g (-928 + 40592 v + g (g (132 + 1076 g - 3157 v) - 16 (877 + 76 v))))) + 1024 c^8 (-3485696 v + g (256 (4460 + 1119 v) + g (64 (-492 + 26093 v) + g (-256 (2113 + 365 v) + g (8896 - 256584 v + g (81892 + 7292 v + g (-587 - 3940 g + 12596 v))))))) + 64 c^6 (-776208384 v + g (16384 (12808 + 5895 v) + g (4096 (-1876 + 148325 v) + g (-8192 (19834 + 7535 v) + g (-537600 (-8 + 341 v) + g (-4 g (210784 + g$$

$$(1712768 + g (-16912 + g (-115380 + g (461 + 2940 g))))) + g (26564544 + g (-1378624 + g (-1842760 + 47664 g + 48837 g^2))) v + 256 (188642 + 55371 v))))))) + 256 c^7 (-62881792 v + g (4096 (4621 + 1594 v) + g (2048 (-301 + 19399 v) + g (-1024 (11549 + 3085 v) + g (259840 - 9064576 v + g (32 (83057 + 15246 v) + g (-34104 + 882032 v + g (g (1374 + 8592 g - 30787 v) - 56 (4523 + 427 v))))))))) + 32 c^5 (-3354394624 v + g (131072 (6035 + 3699 v) + g (32768 (-966 + 95465 v) + g (-32768 (22283 + 11730 v) + g (-71680 (-308 + 16355 v) + g (2048 (131641 + 57062 v) + g (1536 (-3745 + 146624 v) + g (-512 (99025 + 33088 v) + g (690048 - 23317280 v + g (32 (159229 + 36401 v) + g (g (-258350 + g (726 + 5153 g - 25824 v) - 30266 v) + 8 (-4687 + 153812 v))))))))))))))/K \tag{4.A2.28}$$

$$K = (2097152 (2 + c)^8 (4 + c) (4 + 3 c) (8 + c (8 + c)) - 262144 (2 + c)^7 (48 + c (48 + 7 c)) (80 + c (80 + 17 c)) g^2 + 65536 (2 + c)^6 (25536 + c (51072 + c (35616 + c (10080 + 971 c)))) g^4 - 81920 (2 + c)^5 (19840 + c (39680 + c (28236 + c (8396 + 875 c)))) g^6 + 1024 (2 + c)4 (999360 + c (1998720 + c (1444188 + c (444828 + 49067 c)))) g^8 - 256 (2 + c)^3 (1713600 + c (3427200 + c (2505036 + c (791436 + 90947 c)))) g^{10} + 384 (2 + c)^2 (341080 + c (682160 + c (502946 + c (161866 + 19159 c)))) g^{12} - 32 (2 + c) (856800 + c (1713600 + c (1271604 + c (414804 + 50155 c)))) g^{14} + 960 (2 + c)^2 (1041 + c (1041 + 251 c)) g^{16} - 8 (2 + c) (12400 + c (12400 + 3049 c)) g^{18} + 4 (1596 + c (1596 + 397 c)) g^{20} - 60 (2 + c) g^{22} + g^{24})$$

The optimal yields are also depending on *c*, *g*, and *v*;

$$\Pi_1^*(v, c, g) = -c\left(q_{10}^* - q_{11}^* + \frac{\left(q_{10}^* - q_{11}^*\right)^2}{2}\right) + q_{10}^*\left(1 - q_{10}^* - gq_{20}^*\right) + q_{11}^*\left(v - q_{11}^* - gq_{21}^*\right) - q_{10}^* w_{10}^* - q_{11}^* w_{11}^* \tag{4.A2.29}$$

$$\Pi_2^*(v, c, g) = \left(1 - gq_{10}^* - q_{20}^*\right) q_{20}^* + q_{21}^*\left(v - gq_{11}^* - q_{21}^*\right) - q_{20}^* w_{20}^* - q_{21}^* w_{21}^* \tag{4.A2.30}$$

$$U_j^* = w_{j0}^* q_{j0}^* + w_{j1}^* q_{j1}^*,\ j = 1, 2 \tag{4.A2.31}$$

APPENDIX 4.B1

frr$_1$ under re-bargaining: all the figures concern the optimal point

Figure 4.B1.1a | g=1
Contour Plot q_{11} (*frr*$_1$)

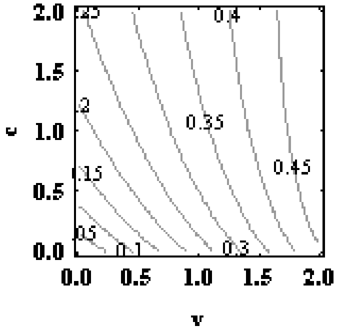


Figure 4.B1.1b | g=1
Contour Plot q_{21} (*frr*$_1$)

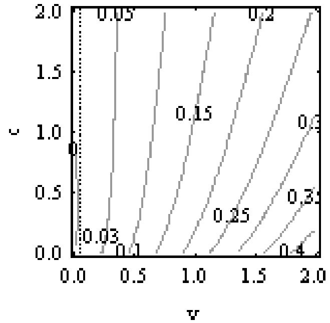


Figure 4.B1.1c | g=1
Contour Plot q_{10} (*frr*$_1$)

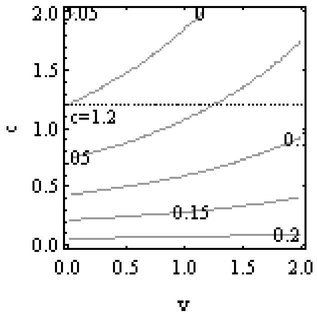


Figure 4.B1.1d | g=1
Contour Plot q_{20} (*frr*$_1$)

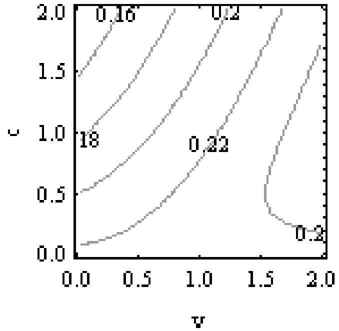


Figure 4.B1.1: Re-bargaining: positive isoquants, in each period simultaneously, provided that $v > 0.03$ and $c \leq 1.2$

Figure 4.B1.2a | g=0.5
Contour Plot q_{11} (*frr*$_1$)

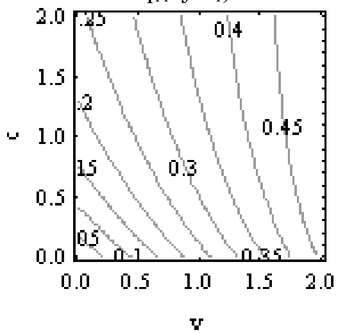


Figure 4.B1.2b | g=0.5
Contour Plot q_{21} (*frr*$_1$)

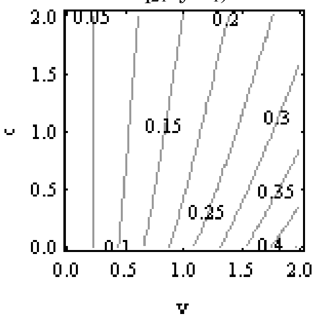


Figure 4.B1.2c | g=0.5
Contour Plot q_{10} (*frr*$_1$)

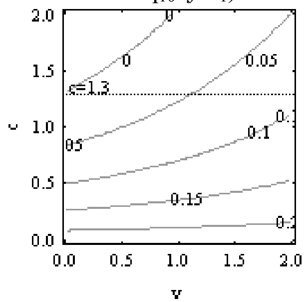


Figure 4.B1.2d | g=0.5
Contour Plot q_{20} (*frr*$_1$)

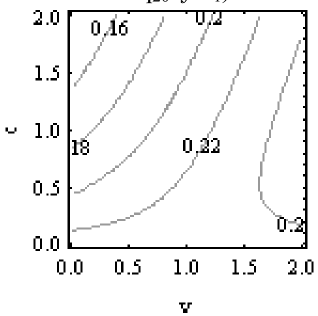


Figure 4.B1.2: Re-bargaining: positive isoquants, in each period simultaneously, provided that $c \leq 1.3$

Figure 4.B1.3a | g=1
Contour Plot w_{11} (*frr*$_1$)

Figure 4.B1.3b | g=1
Contour Plot w_{21} (*frr*$_1$)

Figure 4.B1.3c | g=1
Contour Plot w_{10} (*frr*$_1$)

Figure 4.B1.3d | g=1
Contour Plot w_{20} (*frr*$_1$)

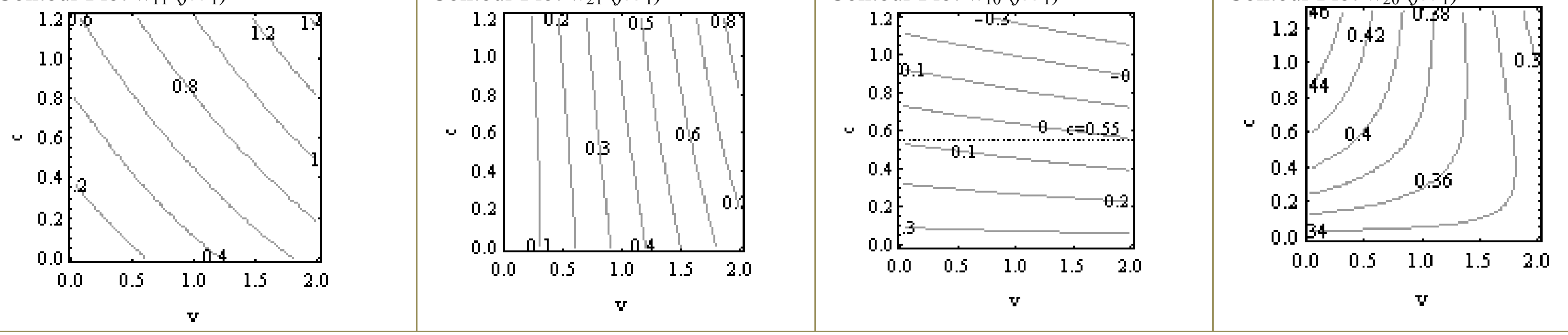


Figure 4.B1.3: Re-bargaining: positive wages, in each period, provided that $c \leq 0.5$

Figure 4.B1.4a | g=0.5
Contour Plot w_{11} (*frr*$_1$)

Figure 4.B1.4b | g=0.5
Contour Plot w_{21} (*frr*$_1$)

Figure 4.B1.4c | g=0.5
Contour Plot w_{10} (*frr*$_1$)

Figure 4.B1.4d | g=0.5
Contour Plot w_{20} (*frr*$_1$)

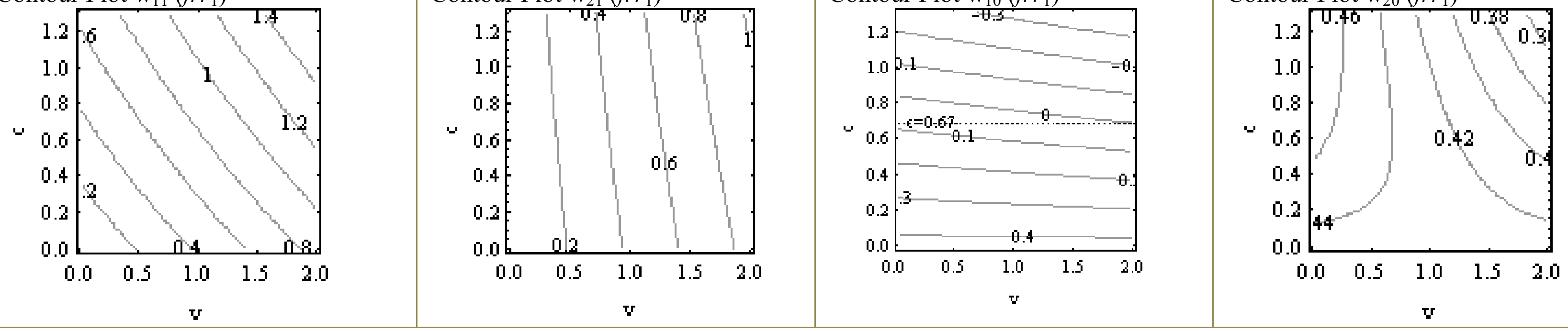


Figure 4.B1.4: Re-bargaining: positive wages, in each period, provided that $c \leq 0.6$

Figure 4.B1.5a | g=1
Contour Plot P_{10} (frr_1)

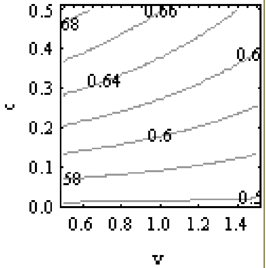


Figure 4.B1.5b | g=1
Contour Plot P_{20} (frr_1)

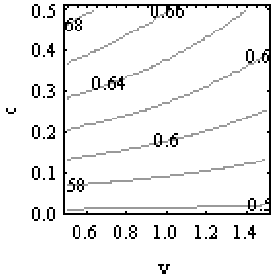


Figure 4.B1.5c | g=1
Contour Plot P_{11} (frr_1)

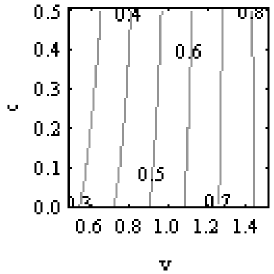


Figure 4.B1.5d | g=1
Contour Plot P_{21} (frr_1)

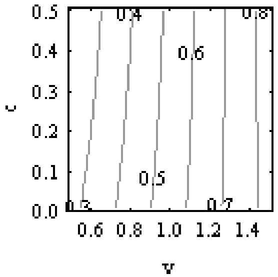


Figure 4.B1.5e | g=1
Contour Plot TC_1 (frr_1)

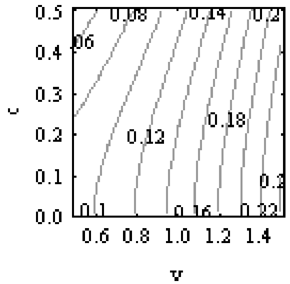


Figure 4.B1.5f | g=1
Contour Plot TC_2 (frr_1)

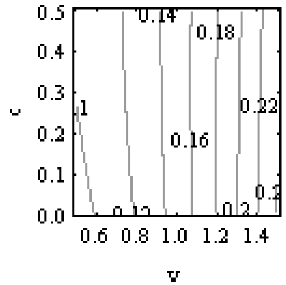


Figure 4.B1.5: Re-bargaining: positive prices and total costs

Figure 4.B1.6a | g=0.5
Contour Plot P_{10} (frr_1)

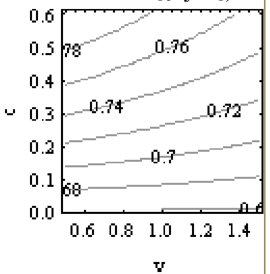


Figure 4.B1.6b | g=0.5
Contour Plot P_{20} (frr_1)

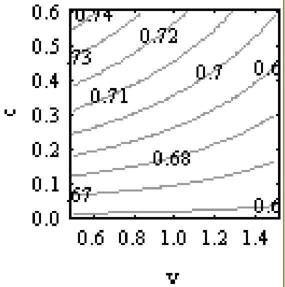


Figure 4.B1.dc | g=0.5
Contour Plot P_{11} (frr_1)

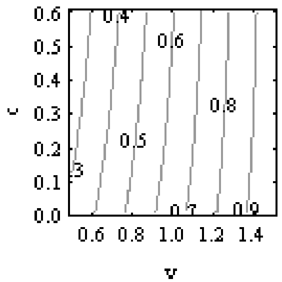


Figure 4.B1.6d | g=0.5
Contour Plot P_{21} (frr_1)

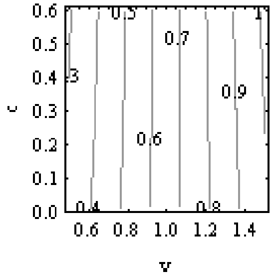


Figure 4.B1.de | g=0.5
Contour Plot TC_1 (frr_1)

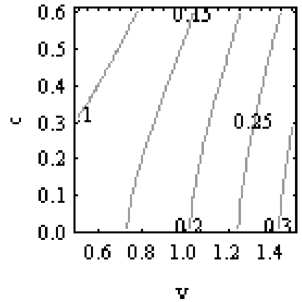


Figure 4.B1.6f | g=0.5
Contour Plot TC_2 (frr_1)

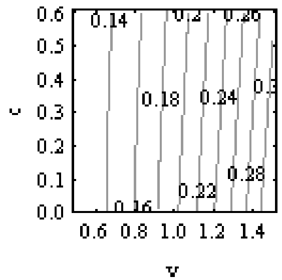


Figure 4.B1.6: Re-bargaining: positive prices and total costs

Figure 4.B1.7a | g=1
Contour Plot Π1 (*frr*$_1$)

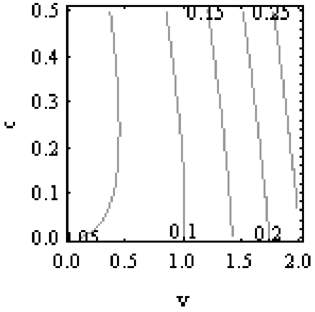


Figure 4.B1.7b | g=1
Contour Plot Π2 (*frr*$_1$)

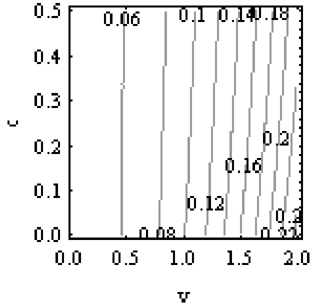


Figure 4.B1.7c | g=1
Contour Plot U1 (*frr*$_1$)

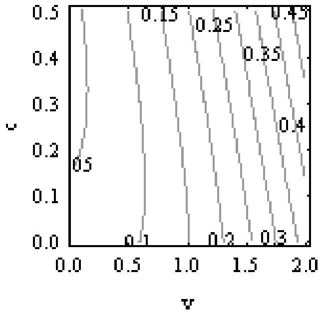


Figure 4.B1.7d | g=1
Contour Plot U2 (*frr*$_1$)

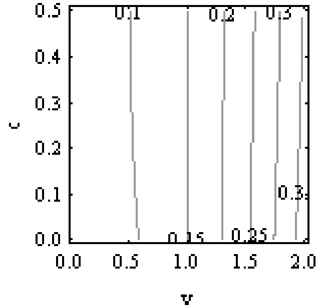


Figure 4.B1.7e | g=1
Contour Plot CS (*frr*$_1$)

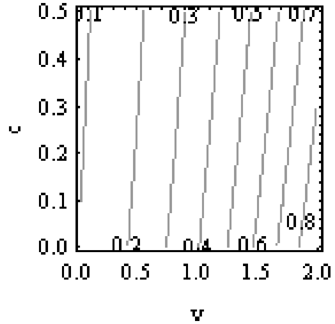


Figure 4.B1.7: Re-bargaining: Π_1, Π_2 U_1, U_2 and CS are positive

Figure 4.B1.8a | g=0.5
Contour Plot Π1 (*frr*$_1$)

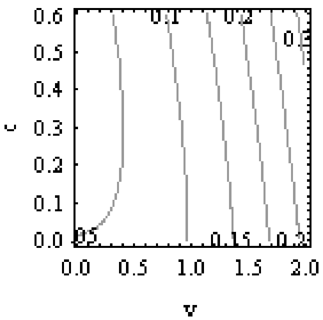


Figure 4.B1.8b | g=0.5
Contour Plot Π2 (*frr*$_1$)

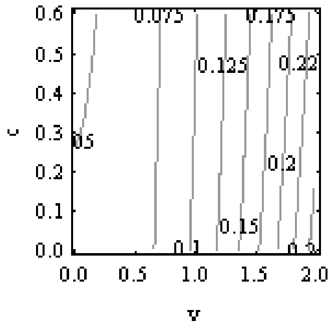


Figure 4.B1.8c | g=0.5
Contour Plot U1 (*frr*$_1$)

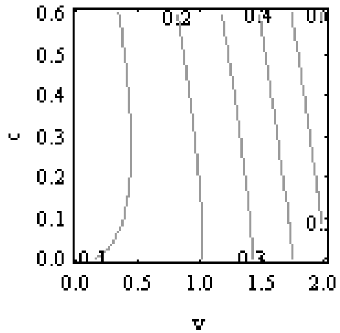


Figure 4.B1.8d | g=0.5
Contour Plot U2 (*frr*$_1$)

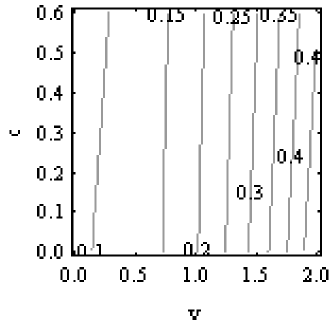


Figure 4.B1.8e | g=0.5
Contour Plot CS (*frr*$_1$)

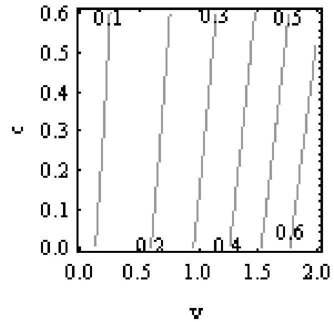


Figure 4.B1.8: Re-bargaining: Π_1, Π_2 U_1, U_2 and CS are positive

Figure 4.B1.9a
Contour Plot (q_{10}-q_{20}) (*frr*$_1$)

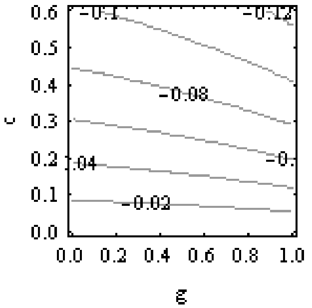


Figure 4.B1.9b
Contour Plot (q_{11}-q_{21}) (*frr*$_1$)

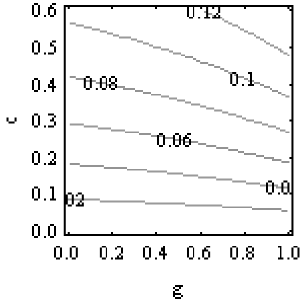


Figure 4.B1.9c
Contour Plot (tq_1-tq_2) (*frr*$_1$)

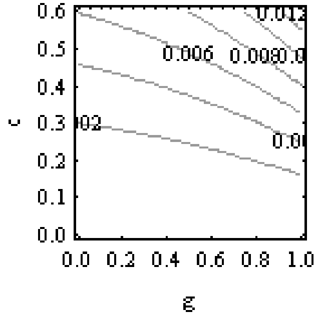


Figure 4.B1.9d | g=1
Contour Plot (tq_A-tq_B) (*frr*$_1$)

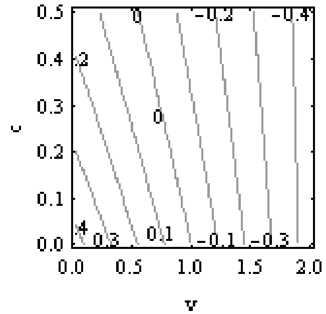


Figure 4.B1.9e | g=0.5
Contour Plot (tq_A-tq_B) (*frr*$_1$)

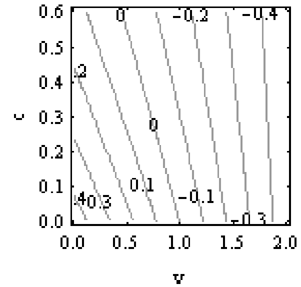


Figure 4.B1.9: Re-bargaining: $q_{10}<q_{20}$, $q_{11}>q_{21}$, $tq_1>tq_2$, $tq_B=q_{11}+q_{21}>tq_A=q_{10}+q_{20}$, if v>1 and $tq_A>tq_B$ if v<1

Figure 4.B1.10d | g=1
Contour Plot Π1-Π2 (*frr*$_1$)

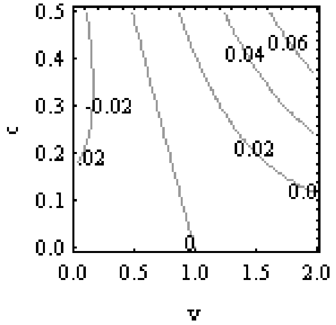


Figure 4.B1.10d | g=1
Contour Plot U1-U2 (*frr*$_1$)

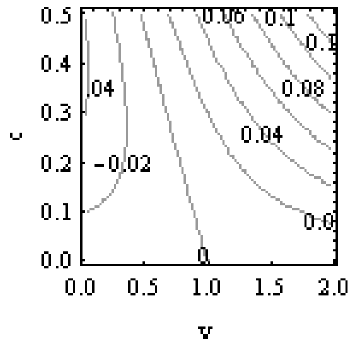


Figure 4.B1.10d | g=0.5
Contour Plot Π1-Π2 (*frr*$_1$)

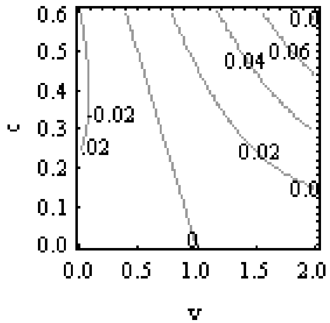


Figure 4.B1.10d | g=0.5
Contour Plot U1-U2 (*frr*$_1$)

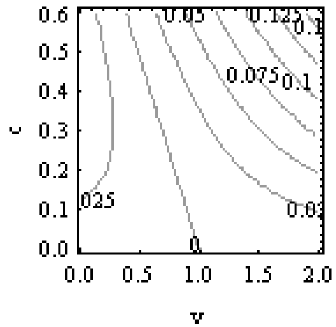


Figure 4.B1.10: Re-bargaining: Π1>Π2 and U1>U2 if v>1 and Π1<Π2 and U1<U2 if v>1

Figure 4.B1.11a | g=1
Contour Plot $q_{11}(frr_1)$-q_{10} (frr_1)

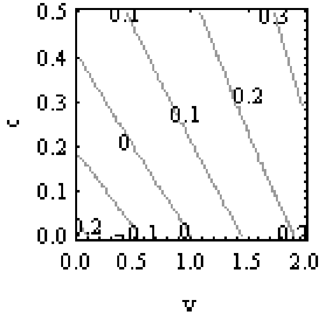


Figure 4.B1.11b | g=1
Contour Plot $q_{21}(frr_1)$-q_{20} (frr_1)

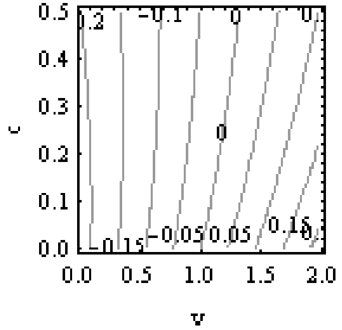


Figure 4.B1.11c | g=0.5
Contour Plot $q_{11}(frr_1)$-q_{10} (frr_1)

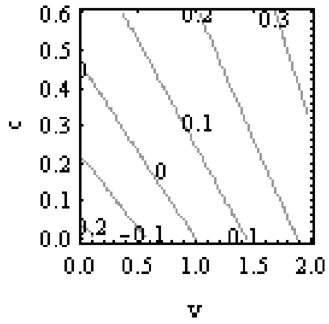


Figure 4.B1.11d | g=0.5
Contour Plot $q_{21}(frr_1)$-q_{20} (frr_1)

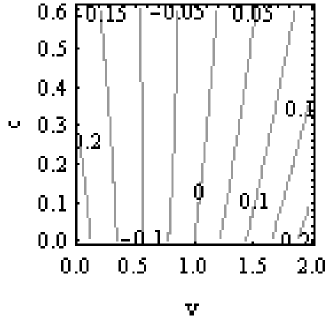


Figure 4.B1.11: Re-bargaining: $q_{11}>q_{20}$ and q_{21}-q_{20} if v>; $q_{11}>q_{20}$ and q_{21}-q_{20} if v>1

Figure 4.B1.12a
Contour Plot $q_{10}(frr_0)$

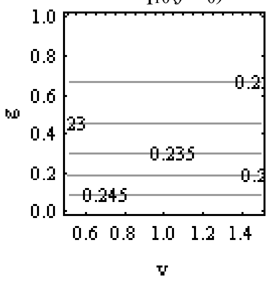


Figure 4.B1.12b
Contour Plot $q_{11}(frr_0)$

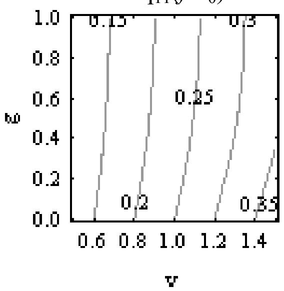


Figure 4.B1.12c
Contour Plot $P_{10}(frr_0)$

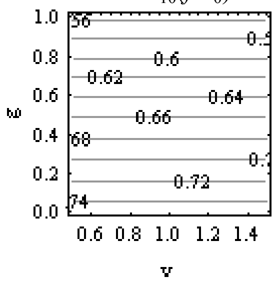


Figure 4.B1.12d
Contour Plot $P_{11}(frr_0)$

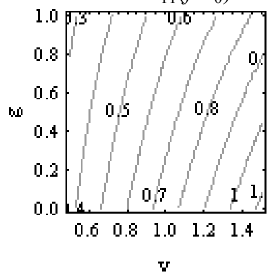


Figure 4.B1.12e
Contour Plot $w_{10}(frr_0)$

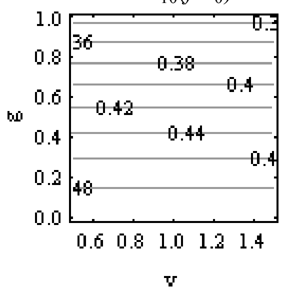


Figure 4.B1.12f
Contour Plot $w_{11}(frr_0)$

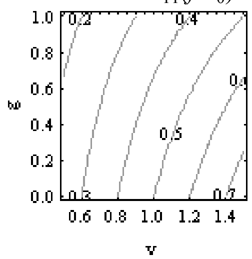


Figure 4.B1.12: Re-bargaining: interior solution is ensured under frr_0

Figure 4.B1.13a |g=1
Contour Plot $q_{10}(frr_1)$-$q_{10}(frr_0)$

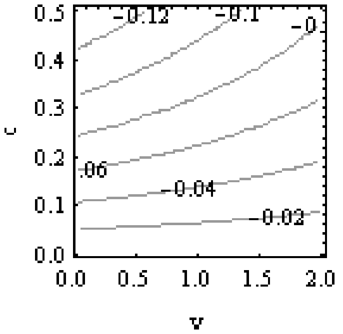


Figure 4.B1.13b |g=1
Contour Plot $q_{20}(frr_1)$-$q_{20}(frr_0)$

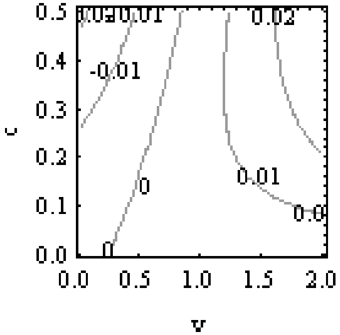


Figure 4.B1.13c |g=1
Contour Plot $q_{11}(frr_1)$-$q_{11}(frr_0)$

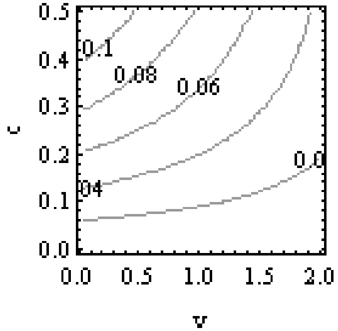


Figure 4.B1.13d |g=1
Contour Plot $q_{21}(frr_1)$-$q_{21}(frr_0)$

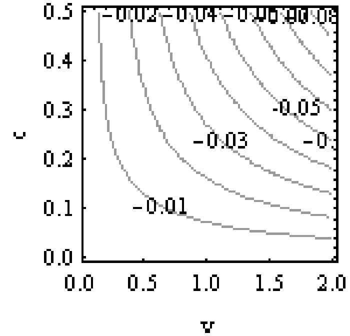


Figure 4.B1.13: Re-bargaining: $q_{10}(frr_1)<q_{10}(frr_0)$, $q_{20}(frr_1)>q_{20}(frr_0)$, $q_{11}(frr_1)>q_{11}(frr_0)$ and $q_{21}(frr_1)<q_{21}(frr_0)$

Figure 4.B1.14a |g=0.5
Contour Plot $q_{10}(frr_1)$-$q_{10}(frr_0)$

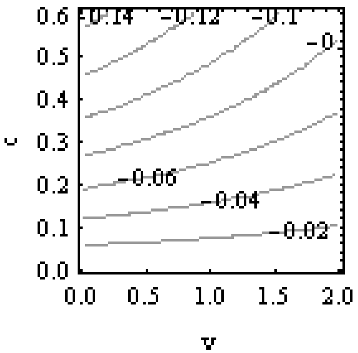


Figure 4.B1.14b |g=0.5
Contour Plot $q_{20}(frr_1)$-$q_{20}(frr_0)$

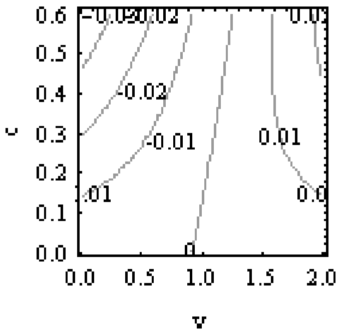


Figure 4.B1.14c |g=0.5
Contour Plot $q_{11}(frr_1)$-$q_{11}(frr_0)$

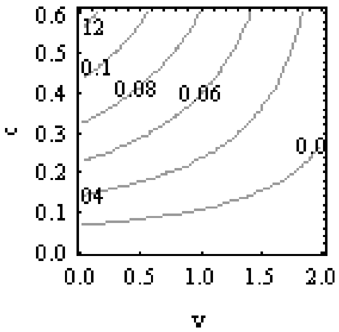


Figure 4.B1.14d |g=0.5
Contour Plot $q_{21}(frr_1)$-$q_{21}(frr_0)$

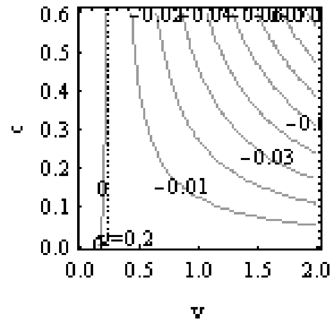


Figure 4.B1.14: Re-bargaining: $q_{10}(frr_1)<q_{10}(frr_0)$, $q_{20}(frr_1)>q_{20}(frr_0)$ if $v>1$, $q_{11}(frr_1)>q_{11}(frr_0)$ and $q_{21}(frr_1)<q_{21}(frr_0)$

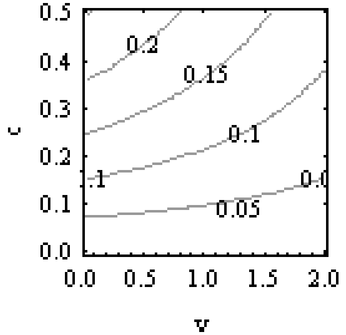
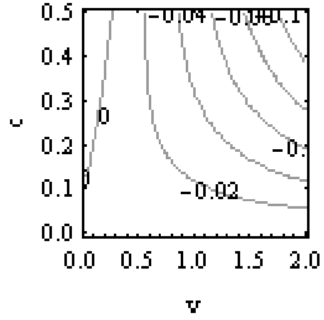
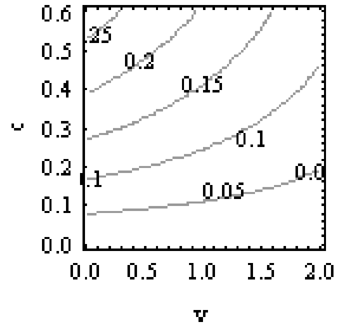
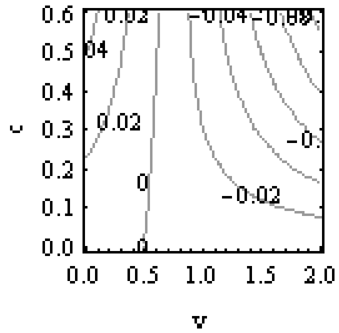
Figure 4.B1.15a |g=1 Contour Plot $(q_{11}-q_{10})$ (*frr*$_1$)-$(q_{11}-q_{10})$ (*frr*$_0$) 	**Figure 4.B1.15b** |g=1 Contour Plot $(q_{21}-q_{20})$ (*frr*$_1$)-$(q_{21}-q_{20})$ (*frr*$_0$) 	**Figure 4.B1.15a** |g=0.5 Contour Plot $(q_{11}-q_{10})$ (*frr*$_1$)-$(q_{11}-q_{10})$ (*frr*$_0$) 	**Figure 4.B1.15b** |g=0.5 Contour Plot $(q_{21}-q_{20})$ (*frr*$_1$)-$(q_{21}-q_{20})$ (*frr*$_0$) 

Figure 4.B1.15: Re-bargaining: $(q_{11}-q_{10})$ (*frr*$_1$) > $(q_{11}-q_{10})$ (*frr*$_0$), $(q_{21}-q_{20})$ (*frr*$_1$) < $(q_{21}-q_{20})$ (*frr*$_0$)

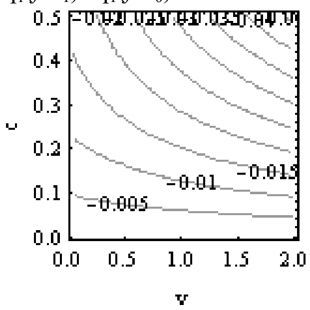
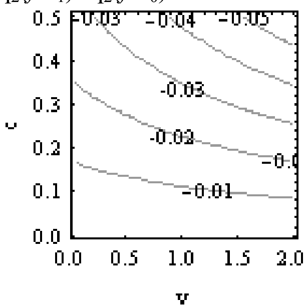
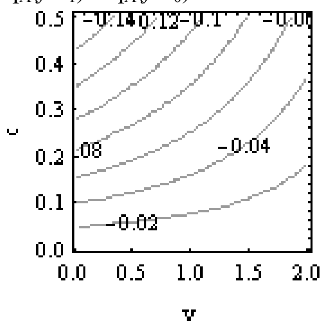
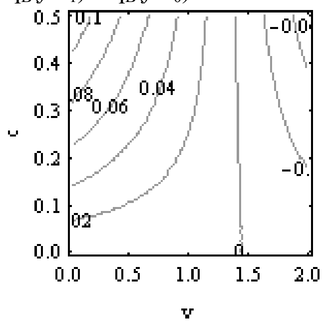
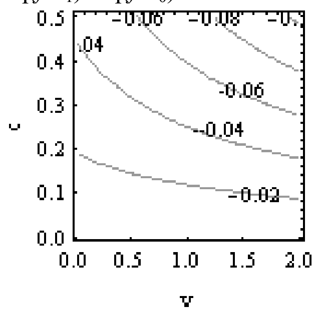
Figure 4.B1.16a |g=1 Contour Plot tq_1(*frr*$_1$)-tq_1(*frr*$_0$) 	**Figure 4.B1.16b** |g=1 Contour Plot tq_2(*frr*$_1$)-tq_2(*frr*$_0$) 	**Figure 4.B1.16c** |g=1 Contour Plot Tq_A(*frr*$_1$)-Tq_A(*frr*$_0$) 	**Figure 4.B1.16d** |g=1 Contour Plot Tq_B(*frr*$_1$)-Tq_B(*frr*$_0$) 	**Figure 4.B1.16e** |g=1 Contour Plot Tq(*frr*$_1$)-Tq(*frr*$_0$) 

Figure 4.B1.16: Re-bargaining: $tq_1=q_{10}+q_{11}$ and $tq_2= q_{20}+q_{21}$; tq_1(*frr*$_1$)<tq_1(*frr*$_0$) and tq_2(*frr*$_1$)<tq_2(*frr*$_0$);$Tq_A=q_{10}+q_{20}$ and $Tq_B=q_{11}+q_{21}$; Tq_A(*frr*$_1$)<Tq_A(*frr*$_0$) and Tq_B(*frr*$_1$)>Tq_B(*frr*$_0$), if v<1.4; Tq(*frr*$_1$)<Tq(*frr*$_0$)

Figure 4.B1.17a |g=0.5
Contour Plot
$tq_1(frr_1)-tq_1(frr_0)$

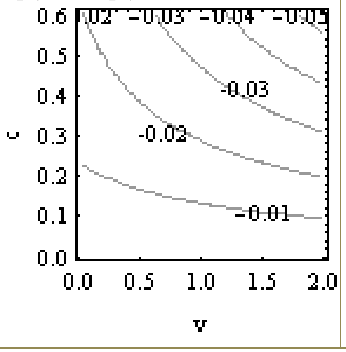


Figure 4.B1.17b |g=0.5
Contour Plot
$tq_2(frr_1)-tq_2(frr_0)$

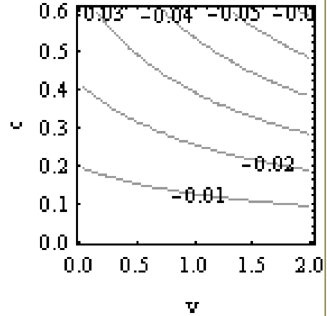


Figure 4.B1.17c |g=0.5
Contour Plot
$Tq_A(frr_1)-Tq_A(frr_0)$

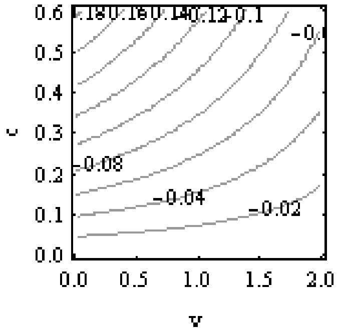


Figure 4.B1.17d |g=0.5
Contour Plot
$Tq_B(frr_1)-Tq_B(frr_0)$

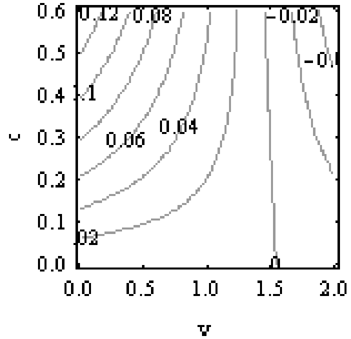


Figure 4.B1.17e |g=0.5
Contour Plot
$Tq(frr_1)-Tq(frr_0)$

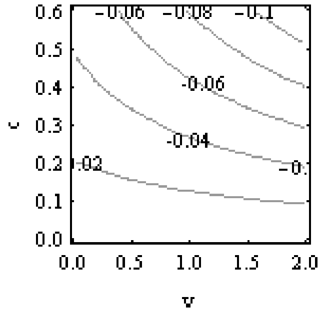


Figure 4.B1.17: Re-bargaining: $tq_1=q_{10}+q_{11}$ and $tq_2= q_{20}+q_{21}$; $tq_1(frr_1)<tq_1(frr_0)$ and $tq_2(frr_1)<tq_2(frr_0)$;$Tq_A=q_{10}+q_{20}$ and $Tq_B=q_{11}+q_{21}$; $Tq_A(frr_1)<Tq_A(frr_0)$ and $Tq_B(frr_1)>Tq_B(frr_0)$ if v<1.4; $Tq(frr_1)<Tq(frr_0)$

Figure 4.B1.18a |g=1
Contour Plot $\Pi1(frr_1)-\Pi1(frr_0)$

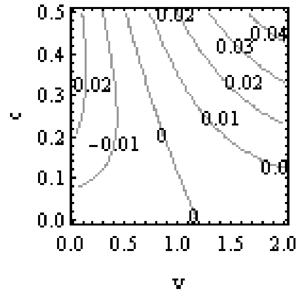


Figure 4.B1.18b |g=1
Contour Plot $\Pi2(frr_1)-\Pi2(frr_0)$

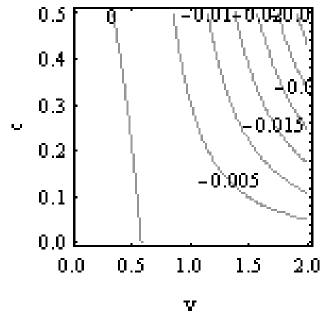


Figure 4.B1.18c |g=1
Contour Plot $U1(frr_1)-U1(frr_0)$

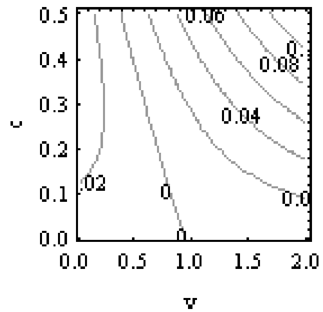


Figure 4.B1.18d |g=1
Contour Plot $U2(frr_1)-U2(frr_0)$

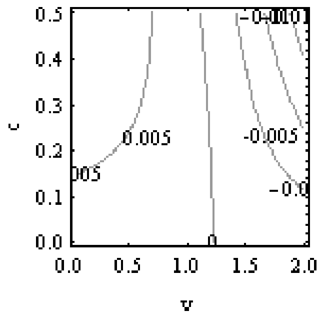


Figure 4.B1.18e |g=1
Contour Plot $CS(frr_1)-CS(frr_0)$

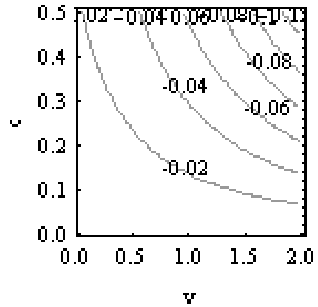


Figure 4.B1.18: Re-bargaining: $\Pi1(frr_1)>\Pi1(frr_0)$ but $\Pi2(frr_1)<\Pi2(frr_0)$ and $U1(frr_1)>U1(frr_0)$ and $U2(frr_1)<U2(frr_0)$ under positive demand shock; $CS(frr_1)<CS(frr_0)$

Figure 4.B1.19a |g=0.5	Figure 4.B1.19b |g=0.5	Figure 4.B1.19c |g=0.5	Figure 4.B1.19d |g=0.5	Figure 4.B1.19e |g=0.5
Contour Plot Π1(frr_1)-Π1(frr_0)	Contour Plot Π2(frr_1)-Π2(frr_0)	Contour Plot U1(frr_1)-U1(frr_0)	Contour Plot U2(frr_1)-U2(frr_0)	Contour Plot CS(frr_1)-CS(frr_0)

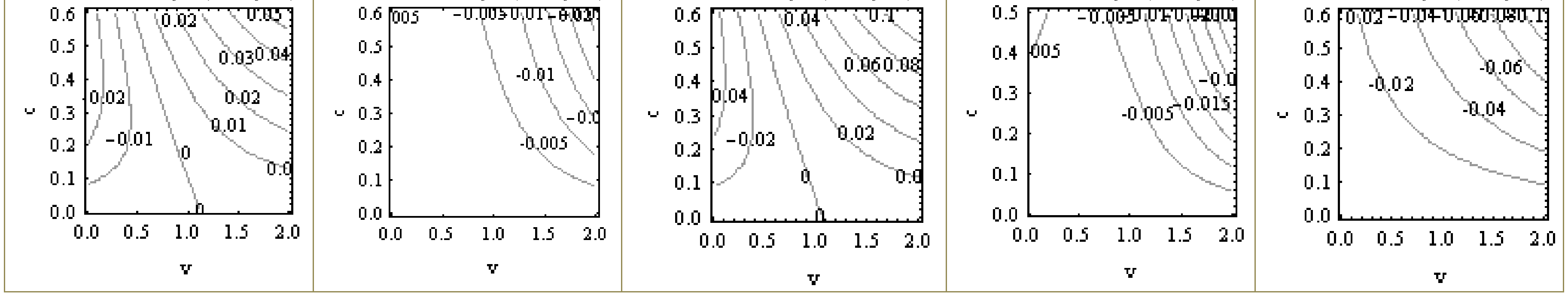


Figure 4.B1.19: Re-bargaining: Π1(frr_1)>Π1(frr_0) and U1(frr_1)>U1(frr_0); under positive demand shock; Π2(frr_1)<Π2(frr_0) U2(frr_1)<U2(frr_0) CS(frr_1)<CS(frr_0)

Figure 4.B1.20a |g=1	Figure 4.B1.20b |g=1	Figure 4.B1.20c |g=1	Figure 4.B1.20d |g=1	Figure 4.B1.20e |g=1
Contour Plot $\frac{d\Pi 1}{dc}(frr_1)$	Contour Plot $\frac{d\Pi 2}{dc}(frr_1)$	Contour Plot $\frac{dU1}{dc}(frr_1)$	Contour Plot $\frac{dU2}{dc}(frr_1)$	Contour Plot $\frac{dCS}{dc}(frr_1)$

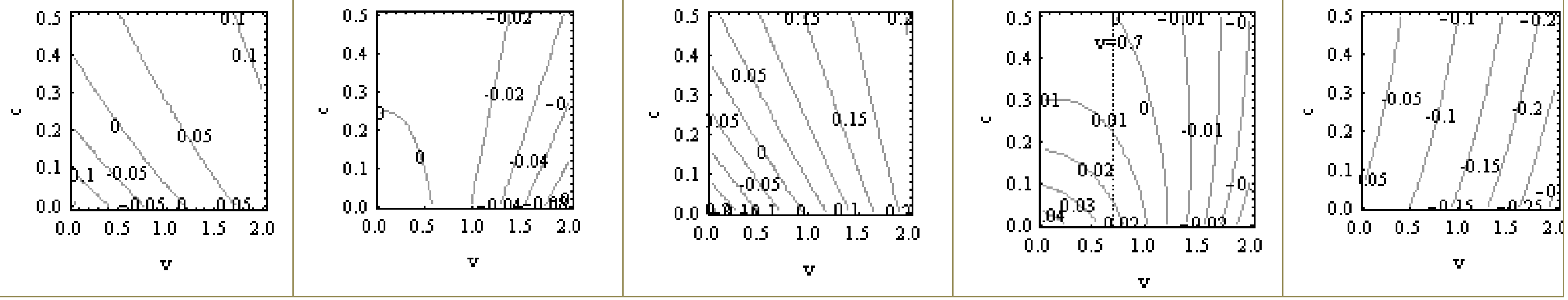


Figure 4.B1.20: Re-bargaining: Π1, U1 and U2 increasing simultaneously with c id c>0.4 and v<1.1. Π2 and CS decreasing with c

Figure 4.B1.21a |g=0.5

Contour Plot $\frac{d\Pi 1}{dc}$ (frr_1)

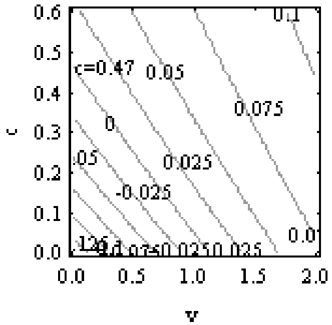


Figure 4.B1.21b |g=0.5

Contour Plot $\frac{d\Pi 2}{dc}$ (frr_1)

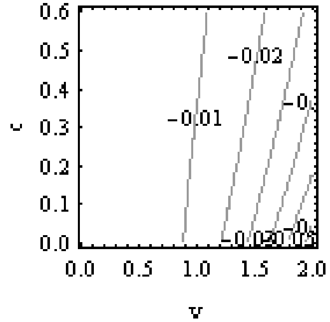


Figure 4.B1.21c |g=0.5

Contour Plot $\frac{dU1}{dc}$ (frr_1)

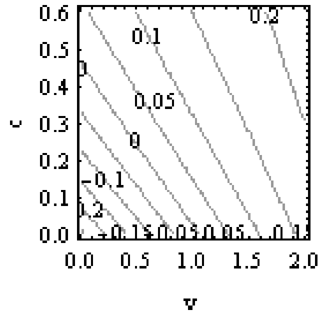


Figure 4.B1.21d |g=0.5

Contour Plot $\frac{dU2}{dc}$ (frr_1)

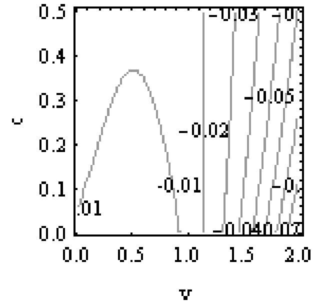


Figure 4.B1.21e |g=0.5

Contour Plot $\frac{dCS}{dc}$ (frr_1)

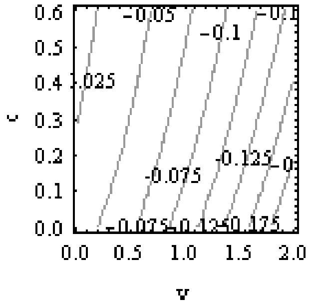


Figure 4.B1.21: Re-bargaining: Π1 and U1 increasing simultaneously with c if c>0.47. Π2, U2, and CS decreasing with c

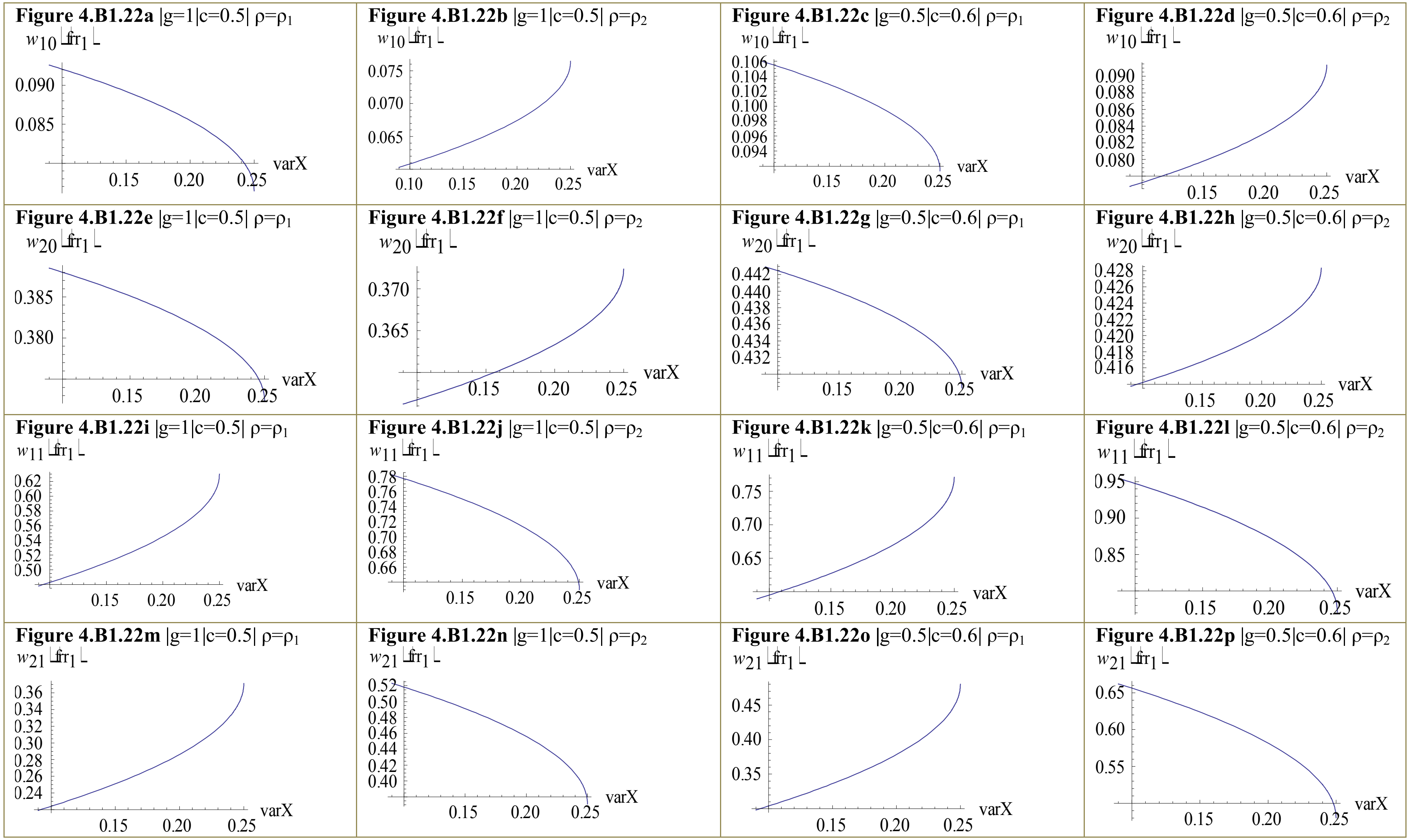


Figure 4.B1.22: w_{10}, w_{20} decreasing (increasing) with varX under ρ_1 (ρ_2); the stimulus is reversed in the second period under frr_1 and re-bargaining

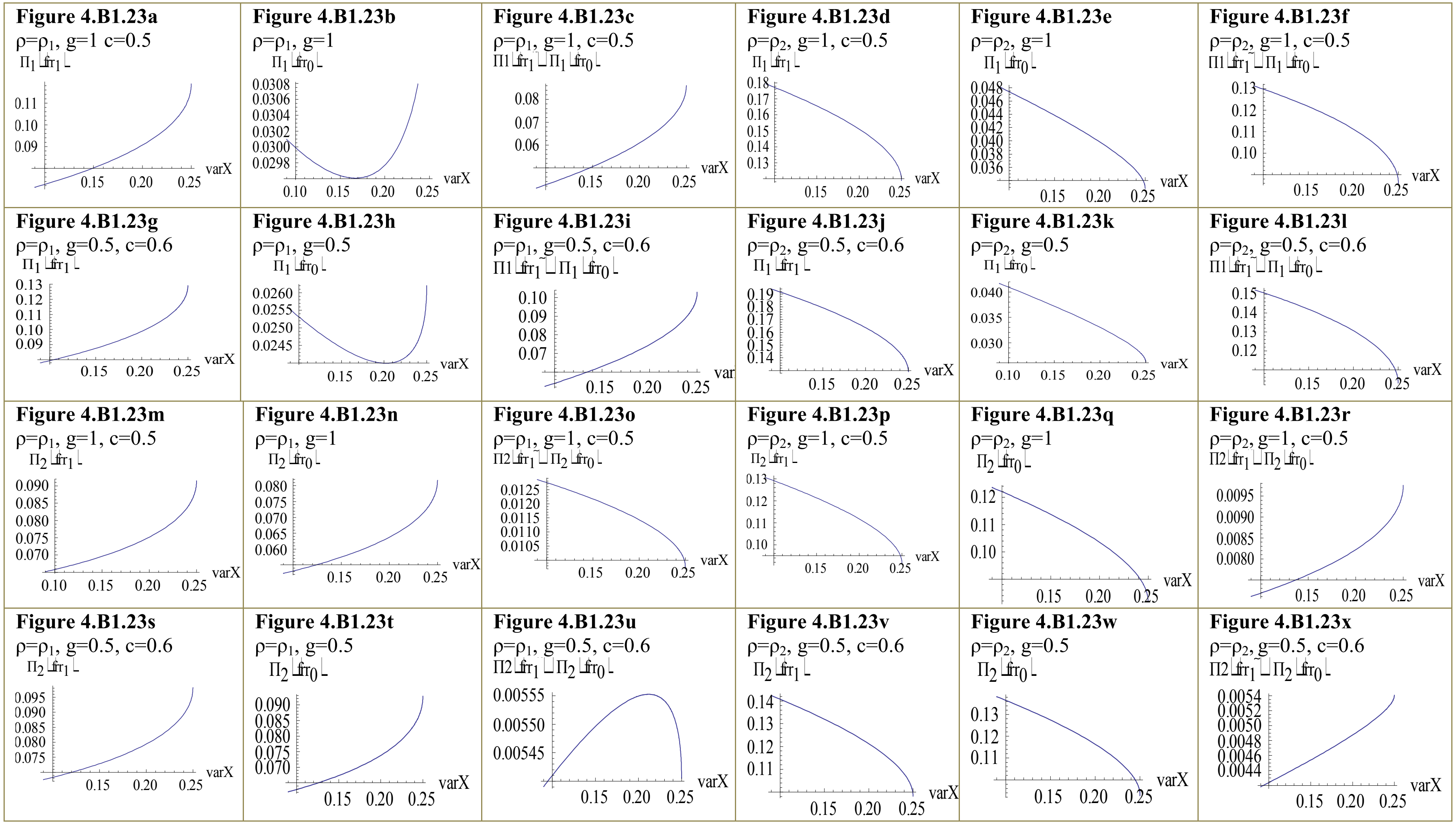


Figure 4.B1.23: How varX affects Π1(frr_1), Π1(frr_0), Π1(frr_1)-Π1(frr_0), Π2(frr_1), Π2(frr_0), Π2(frr_1)-Π2(frr_0), under ρ_1 and ρ_2 under re-bargaining

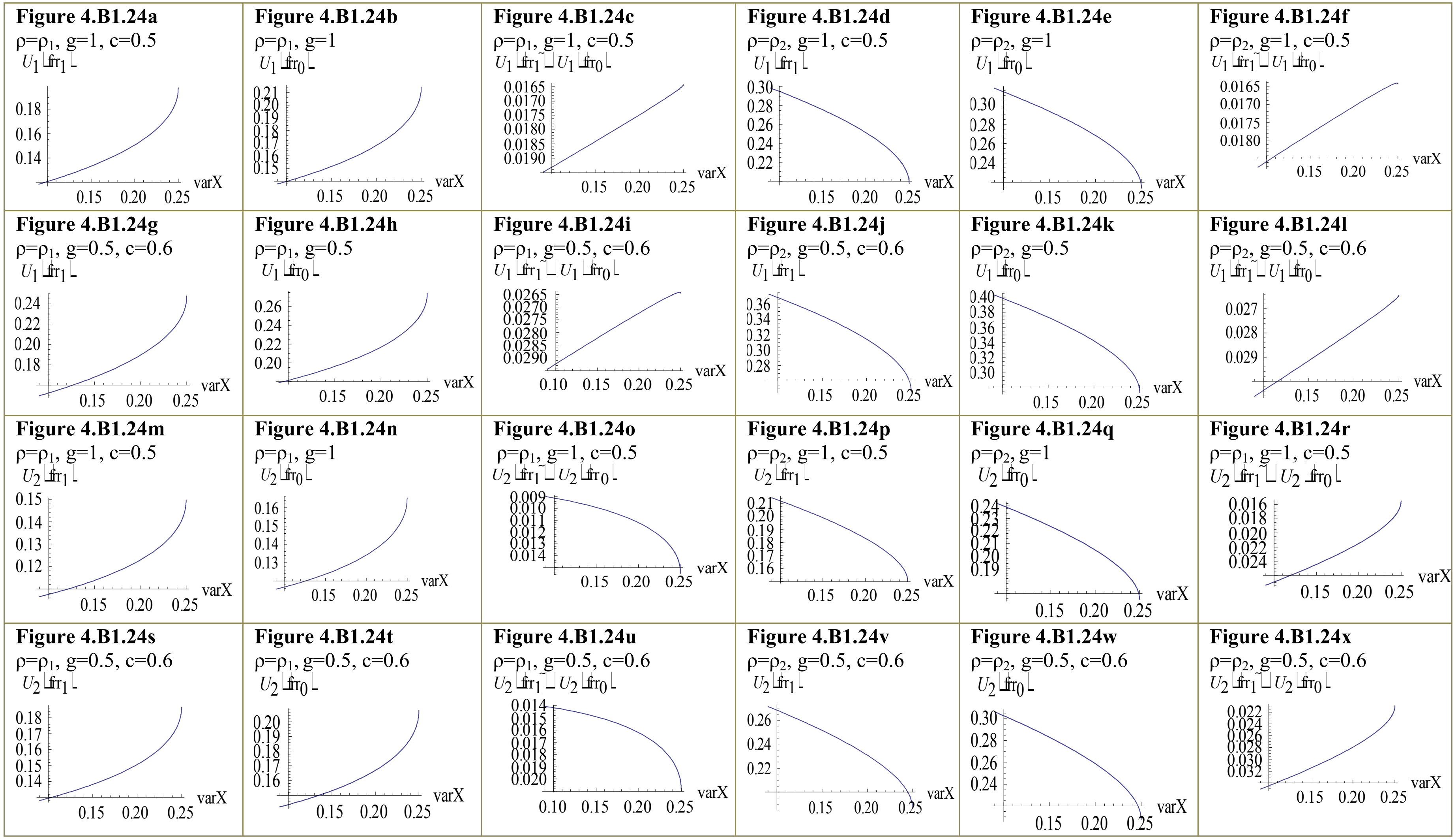


Figure 4.B1.24: How varX affects U1(frr_1), U1(frr_0), U1(frr_1)-U1(frr_0), U2(frr_1), U2(frr_0), U2(frr_1)-U2(frr_0), under ρ_1 and ρ_2 under re-bargaining

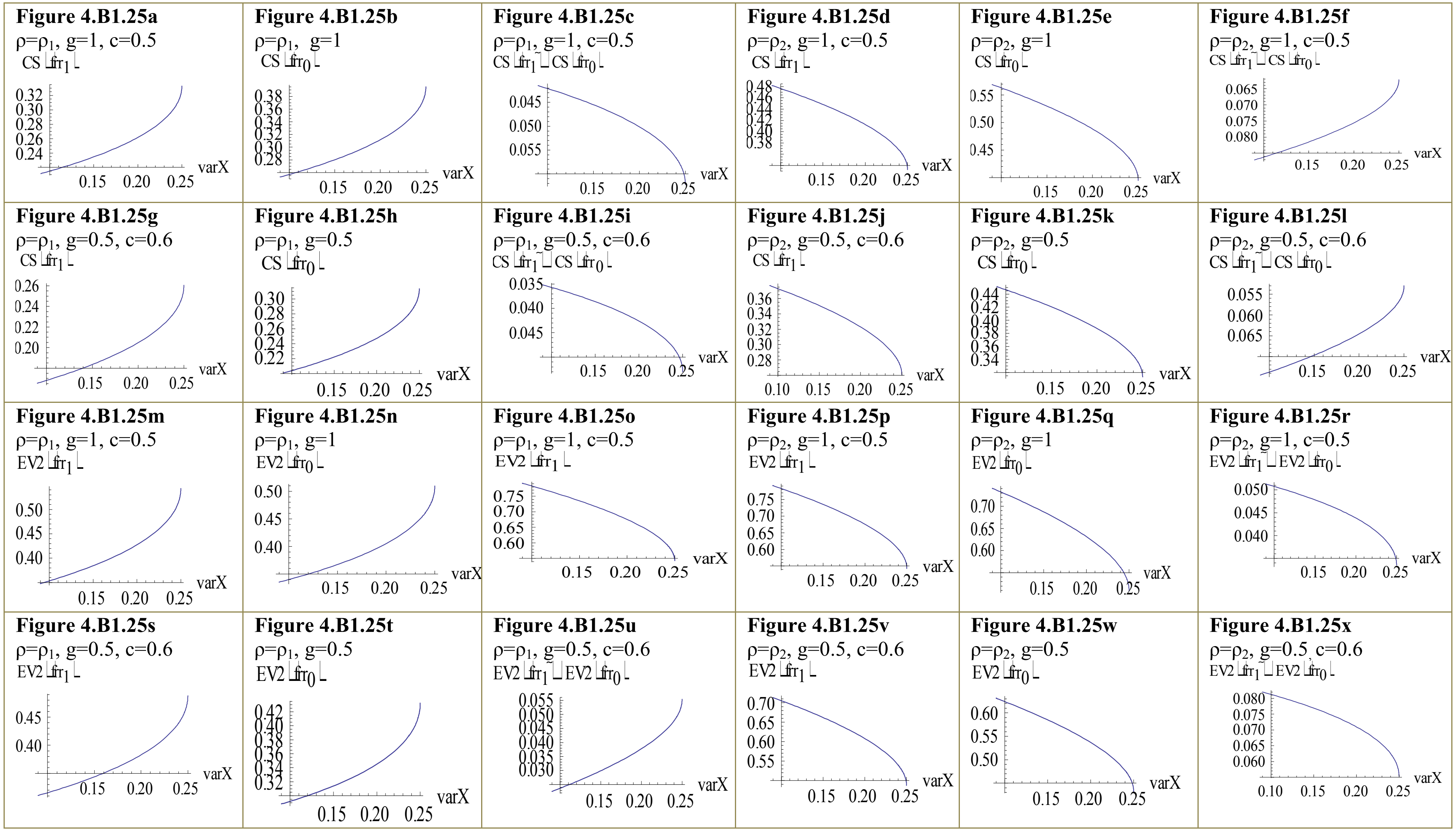


Figure 4.B1.25: How varX affects CS(*frr*$_1$), CS(*frr*$_0$), CS(*frr*$_1$)-CS(*frr*$_0$), EV2(*frr*$_1$), EV2(*frr*$_0$), EV2(*frr*$_1$)-EV2(*frr*$_0$), under ρ_1 and ρ_2 under re-bargaining

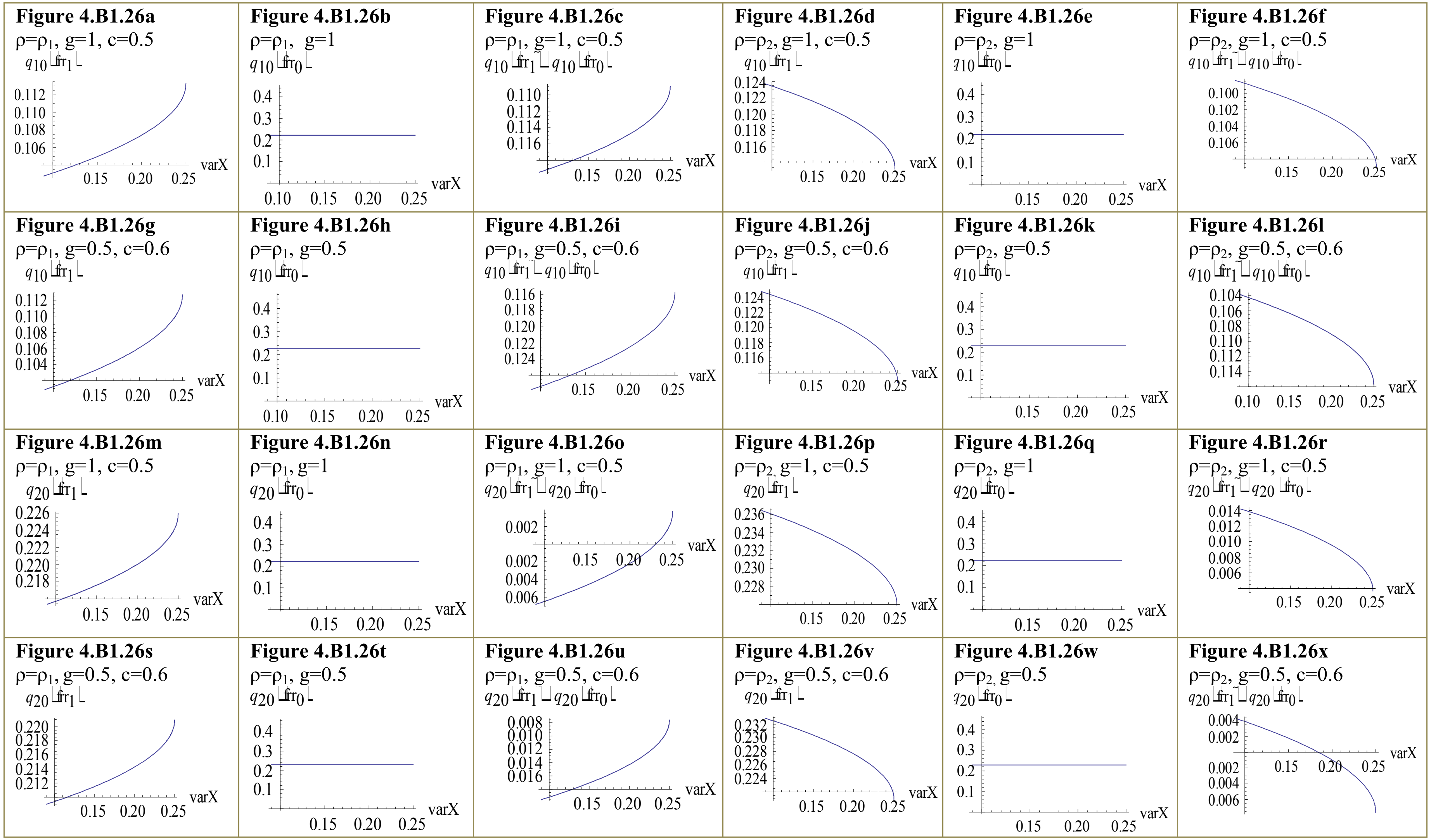


Figure 4.B1.26: How varX affects $q_{10}(\mathit{frr}_1)$, $q_{10}(\mathit{frr}_0)$, $q_{10}(\mathit{frr}_1)$-$q_{10}(\mathit{frr}_0)$, $q_{20}(\mathit{frr}_1)$, $q_{20}(\mathit{frr}_0)$, $q_{20}(\mathit{frr}_1)$-$q_{20}(\mathit{frr}_0)$, under ρ_1 and ρ_2 under re-bargaining

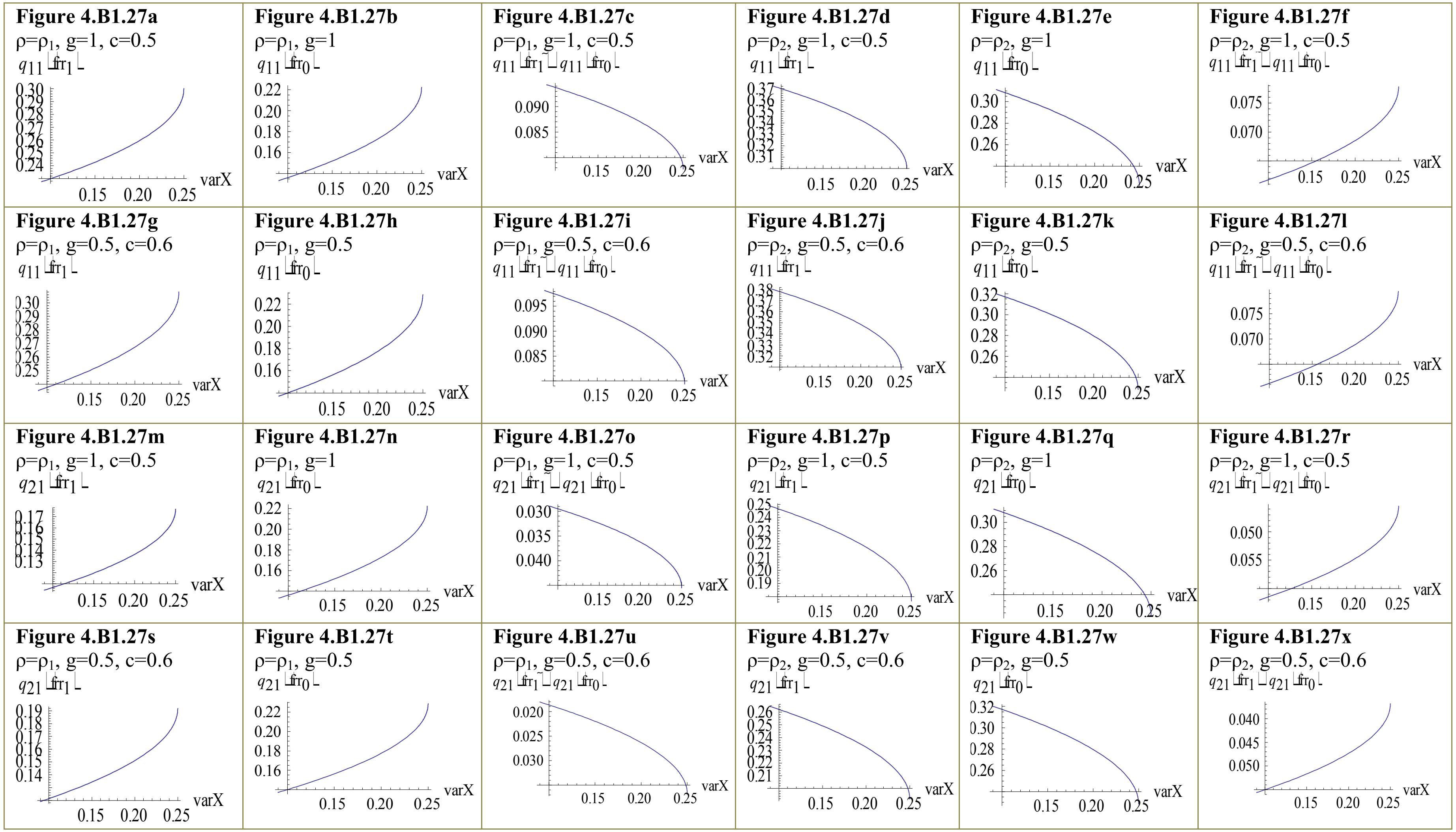


Figure 4.B1.27: How varX affects $q_{11}(frr_1)$, $q_{11}(frr_0)$, $q_{11}(frr_1)$-$q_{11}(frr_0)$, $q_{21}(frr_{21})$, $q_{21}(frr_0)$, $q_{21}(frr_1)$-$q_{21}(frr_0)$, under ρ_1 and ρ_2 under re-bargaining

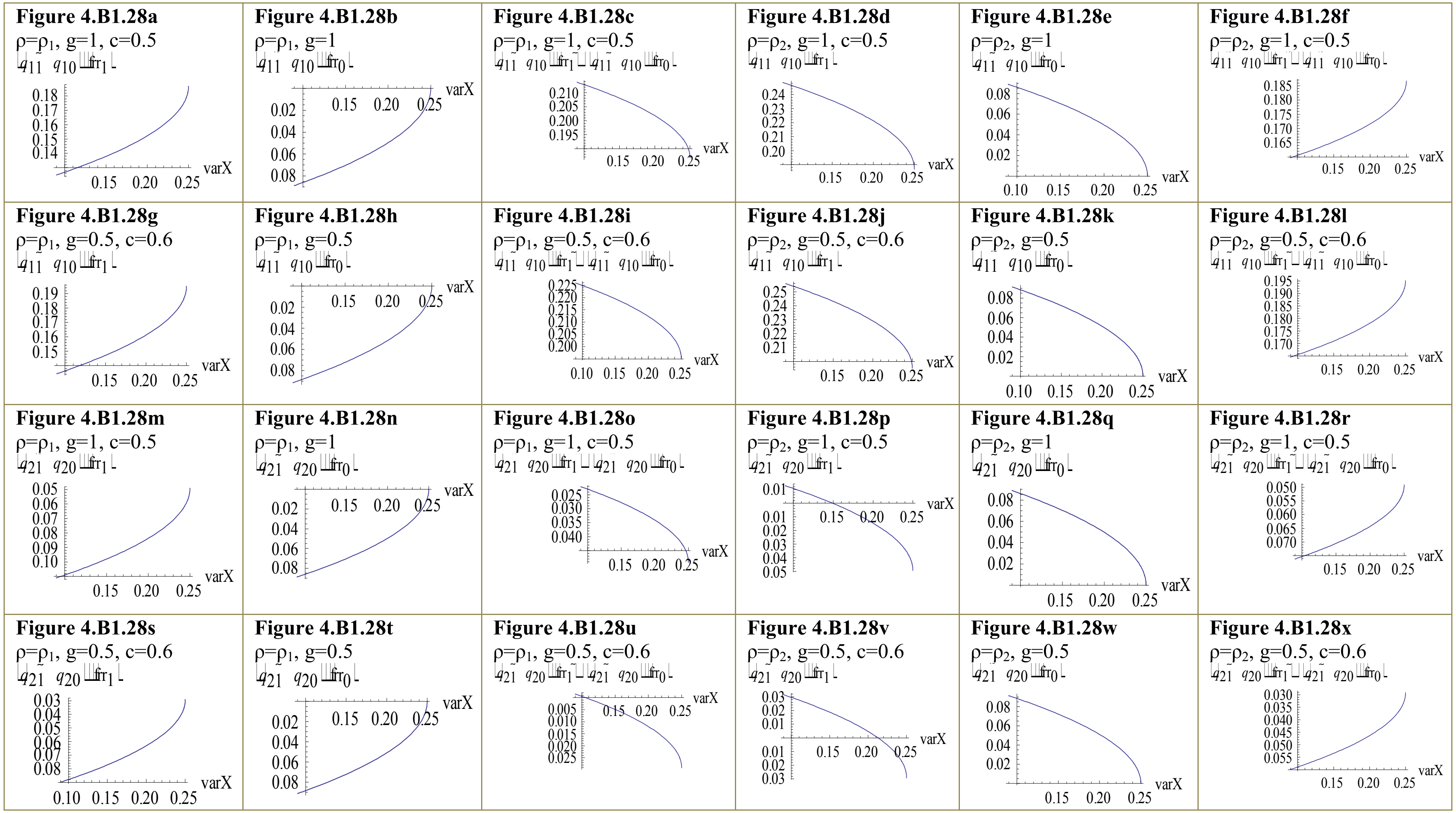


Figure 4.B1.28: How varX affects $(q_{11}-q_{10})(frr_1)$, $(q_{11}-q_{10})(frr_0)$, $(q_{11}-q_{10})(frr_1)-(q_{11}-q_{10})(frr_0)$, $(q_{21}-q_{20})(frr_1)$, $(q_{21}-q_{20})(frr_0)$, $(q_{21}-q_{20})(frr_1)-(q_{21}-q_{20})(frr_0)$, under ρ_1 and ρ_2 under re-bargaining

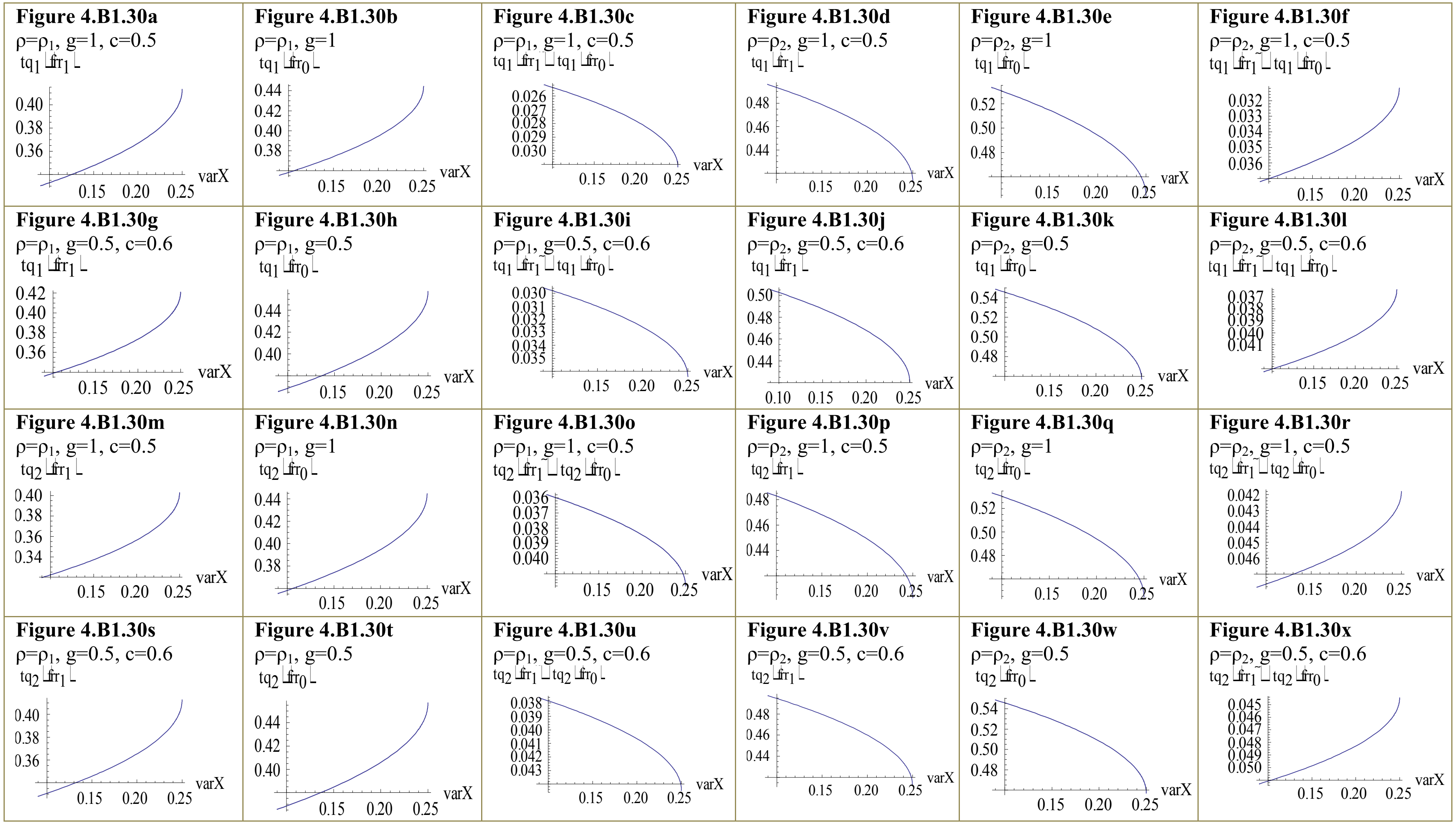


Figure 4.B1.30: How varX affects $tq_1(frr_1)$, $tq_1(frr_0)$, $tq_1(frr_1)-tq_1(frr_0)$, $tq_2(frr_1)$, $tq_2(frr_0)$, $tq_2(frr_1)-tq_2(frr_0)$, under ρ_1 and ρ_2 under re-bargaining

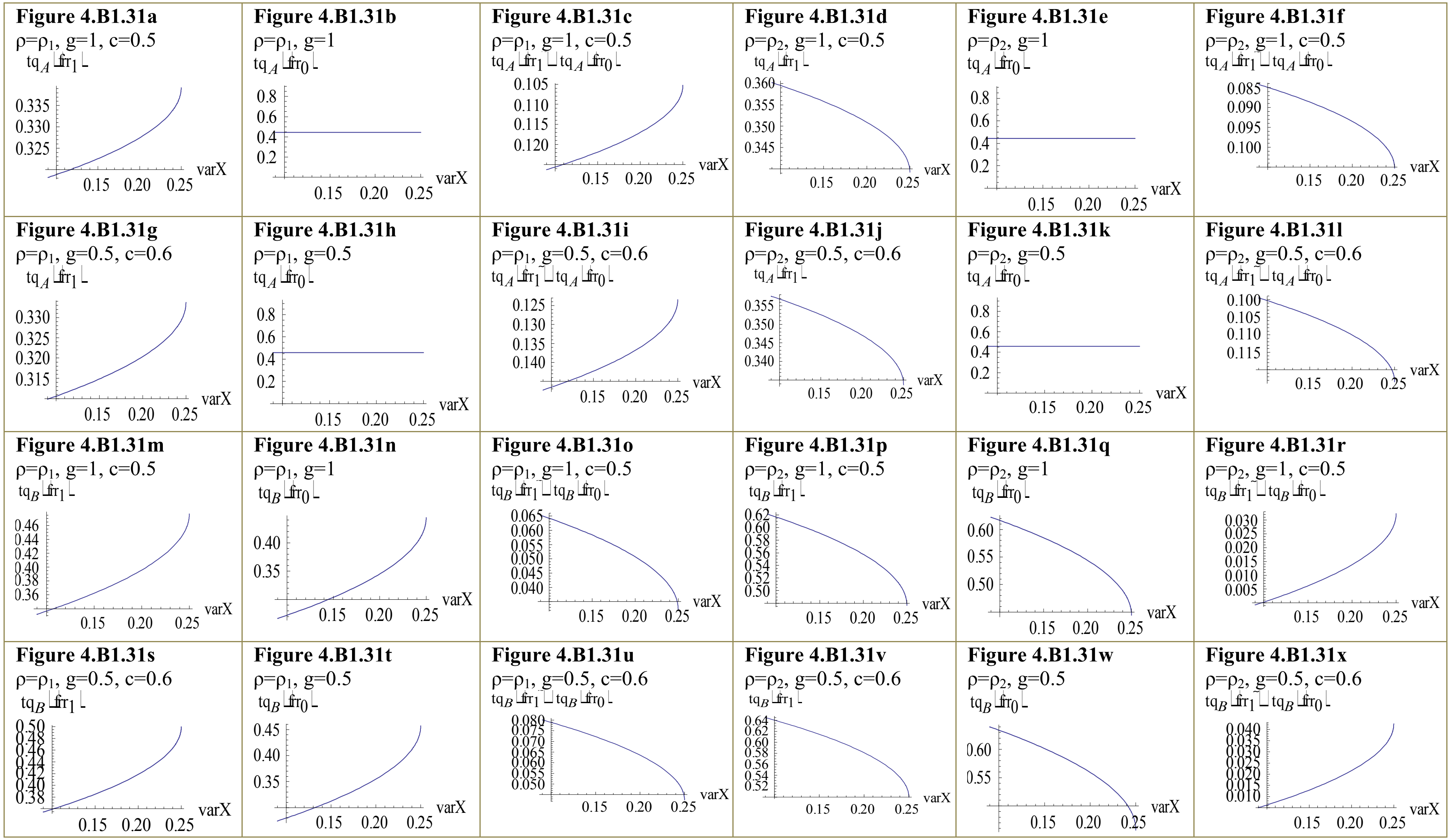


Figure 4.B1.31: How varX affects $tq_A(frr_1)$, $tq_A(frr_0)$, $tq_A(frr_1)$-$tq_A(frr_0)$, $tq_B(frr_1)$, $tq_B(frr_0)$, $tq_B(frr_1)$-$tq_B(frr_0)$, under ρ_1 and ρ_2 under re-bargaining

Figure 4.B1.32a $|\rho=\rho_1$, g=1, c=0.5

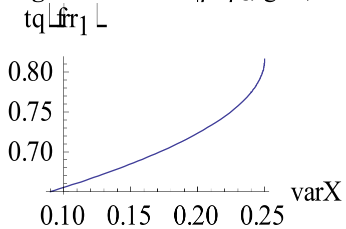


Figure 4.B1.32b $|\rho=\rho_1$, g=1

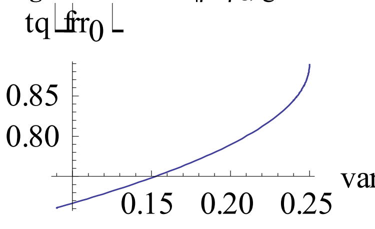


Figure 4.B1.32c $|\rho=\rho_1$, g=1, c=0.5

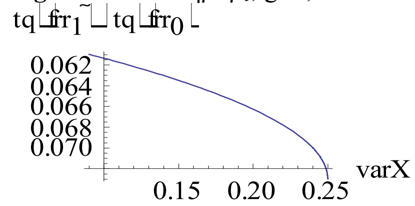


Figure 4.B1.32d $|\rho=\rho_2$, g=1, c=0.5

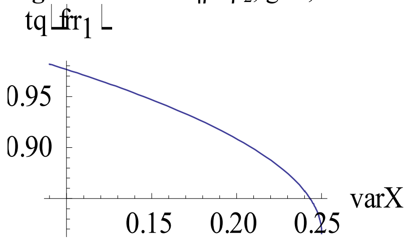


Figure 4.B1.32e $|\rho=\rho_2$, g=1

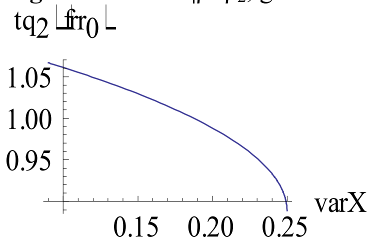


Figure 4.B1.32f $|\rho=\rho_2$, g=1, c=0.5

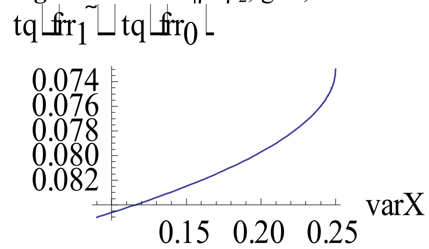


Figure 4.B1.32g $|\rho=\rho_1$, g=0.5, c=0.6

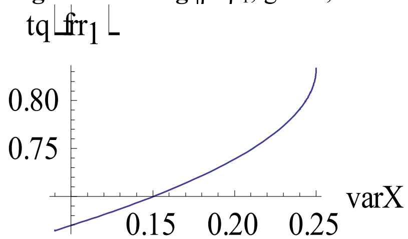


Figure 4.B1.32h $|\rho=\rho_1$, g=0.5

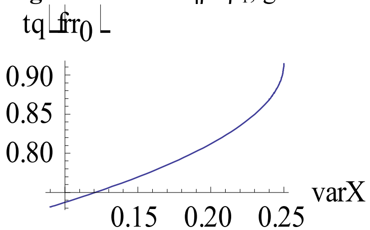


Figure 4.B1.32i $|\rho=\rho_1$, g=0.5, c=0.6

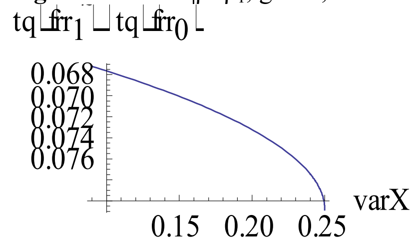


Figure 4.B1.32j $|\rho=\rho_2$, g=0.5, c=0.6

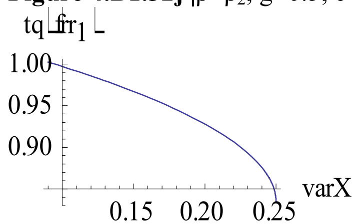


Figure 4.B1.32k $|\rho=\rho_2$, g=0.5

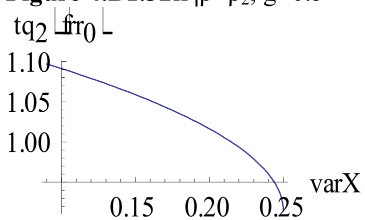


Figure 4.B1.32l $|\rho=\rho_2$, g=0.5, c=0.6

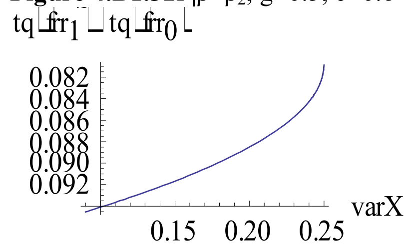


Figure 4.B1.32: How varX affects tq (*frr*$_1$), tq (*frr*$_0$), tq (*frr*$_1$)-tq(*frr*$_0$), under ρ_1 and ρ_2 and re-bargaining

APPENDIX 4.B2

***frr*$_2$**: Re-bargaining: all the figures the optimal point

Figure 4.B2.1a |g=1
Contour Plot q_{11} (frr_2)

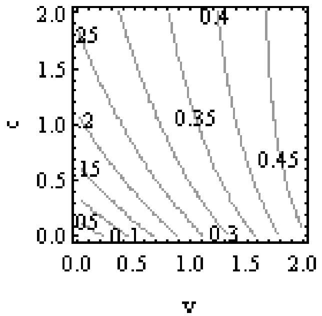


Figure 4.B2.1b |g=1
Contour Plot q_{21} (frr_2)

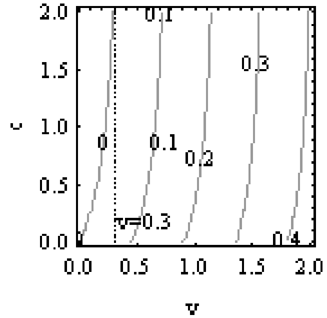


Figure 4.B2.1c g=1
Contour Plot q_{10} (frr_2)

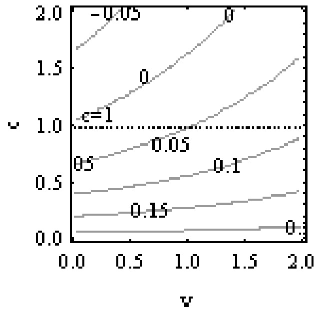


Figure 4.B2.1d g=1
Contour Plot q_{20} (frr_2)

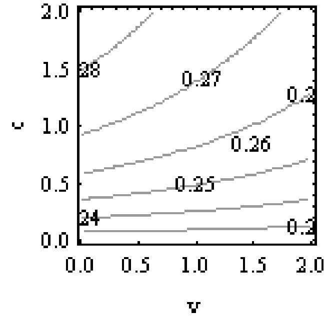


Figure 4.B2.1: Re-bargaining: positive isoquants, in each period simultaneously, provided that $v \geq 0.3$ and $c \leq 1$

Figure 4.B2.2a |g=0.5
Contour Plot q_{11} (frr_2)

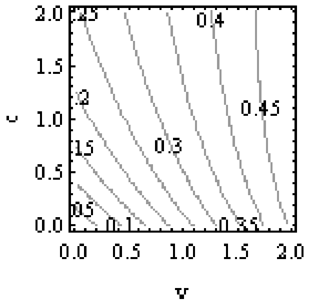


Figure 4.B2.2b |g=0.5
Contour Plot q_{21} (frr_2)

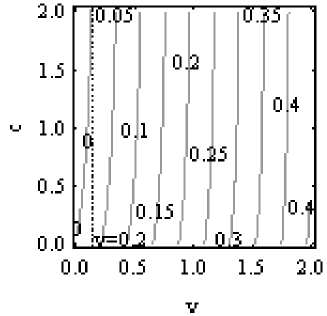


Figure 4.B2.2c |g=0.5
Contour Plot q_{10} (frr_2)

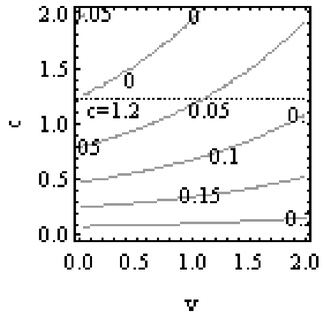


Figure 4.B2.2d |g=0.5
Contour Plot q_{20} (frr_2)

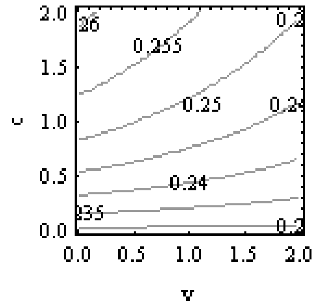


Figure 4.B2.2: Re-bargaining: positive isoquants, in each period simultaneously, provided that $v \geq 0.2$ and $c \leq 1.2$

Figure 4.B2.3a |g=1
Contour Plot w_{11} (*frr*$_2$)

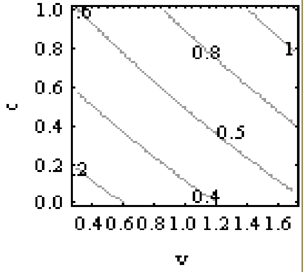


Figure 4.B2.3b |g=1
Contour Plot w_{21} (*frr*$_2$)

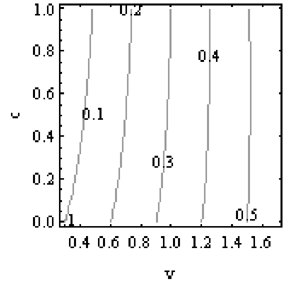


Figure 4.B2.3c |g=1
Contour Plot w_{10} (*frr*$_2$)

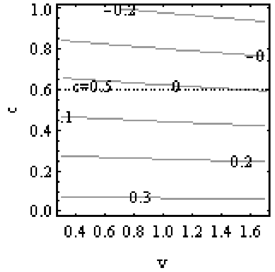


Figure 4.B2.3d |g=1
Contour Plot w_{20} (*frr*$_2$)

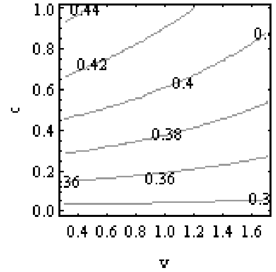


Figure 4.B2.3e | g=1
Contour Plot TC_1 (*frr*$_2$)

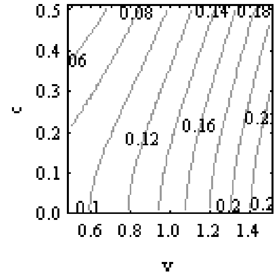


Figure 4.B2.3f |g=1
Contour Plot TC_2 (*frr*$_2$)

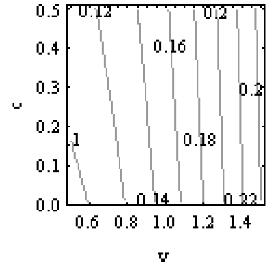


Figure 4.B2.3: Re-bargaining: positive wages and total costs, simultaneously, provided that $c \leq 0.5$

Figure 4.B2.4a |g=0.5
Contour Plot w_{11} (*frr*$_2$)

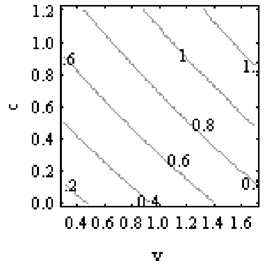


Figure 4.B2.4b |g=0.5
Contour Plot w_{21} (*frr*$_2$)

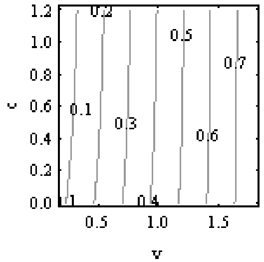


Figure 4.B2.4c |g=0.5
Contour Plot w_{10} (*frr*$_2$)

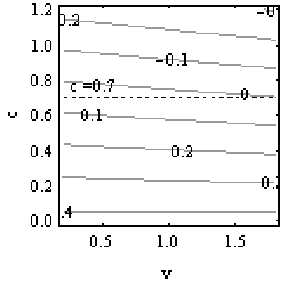


Figure 4.B2.4d |g=0.5
Contour Plot w_{20} (*frr*$_2$)

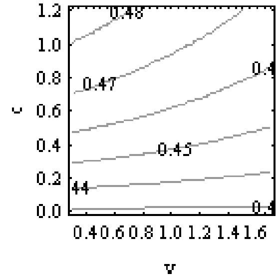


Figure 4.B2.4e |g=0.5
Contour Plot TC_1(*frr*$_2$)

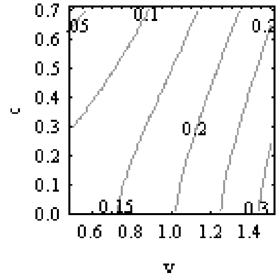


Figure 4.B2.4e |g=0.5
Contour Plot TC_2 (*frr*$_2$)

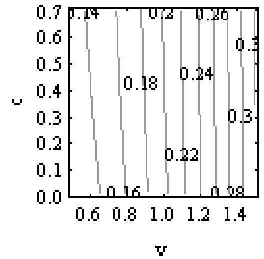


Figure 4.B2.4: Re-bargaining: positive wages and total costs, simultaneously, provided that $c \leq 0.7$

Figure 4.B2.5a |g=1
Contour Plot P_{11} (frr_2)

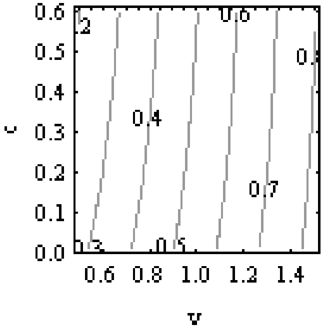


Figure 4.B2.5b |g=1
Contour Plot P_{21} (frr_2)

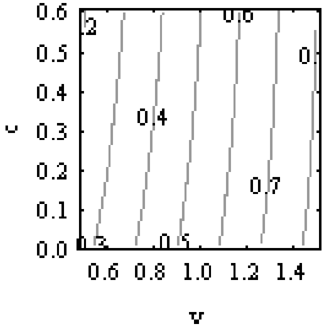


Figure 4.B2.5c |g=1
Contour Plot P_{10} (frr_2)

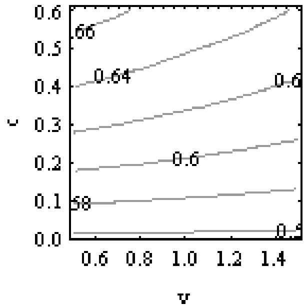


Figure 4.B2.5d |g=1
Contour Plot P_{20} (frr_2)

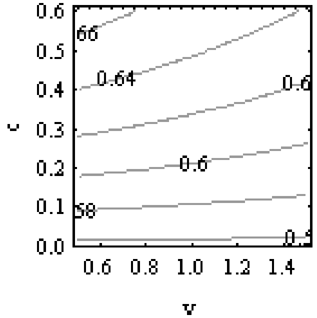


Figure 4.B2.5: Re-bargaining: positive prices in each period

Figure 4.B2.6a |g=0.5
Contour Plot P_{11} (frr_2)

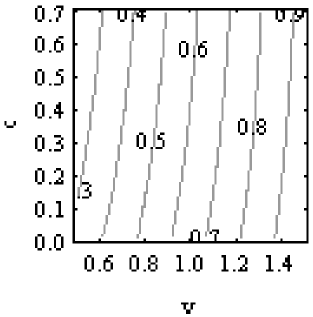


Figure 4.B2.6b |g=0.5
Contour Plot P_{21} (frr_2)

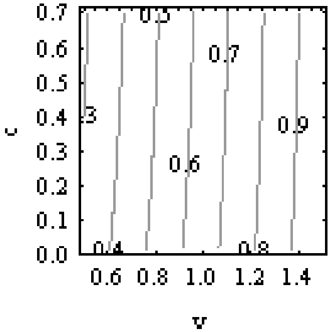


Figure 4.B2.6c |g=0.5
Contour Plot P_{10} (frr_2)

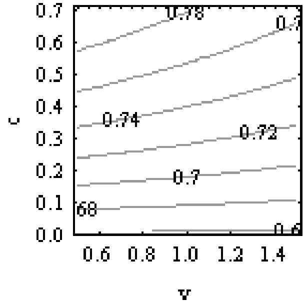


Figure 4.B2.6d |g=0.5
Contour Plot P_{20} (frr_2)

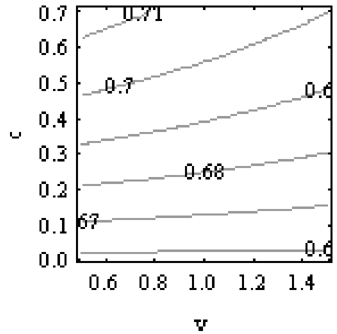


Figure 4.B2.6: Re-bargaining: positive prices in each period

Figure 4.B2.7a |g=1
Contour Plot Π1 (*frr*$_2$)

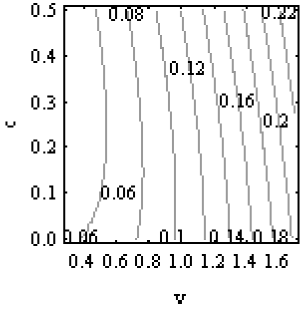


Figure 4.B2.7b |g=1
Contour Plot Π2 (*frr*$_2$)

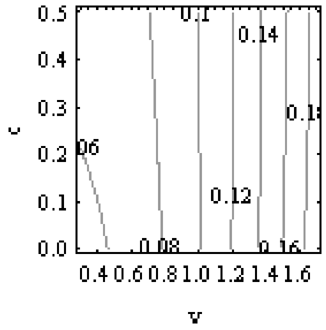


Figure 4.B2.7c |g=1
Contour Plot U1 (*frr*$_2$)

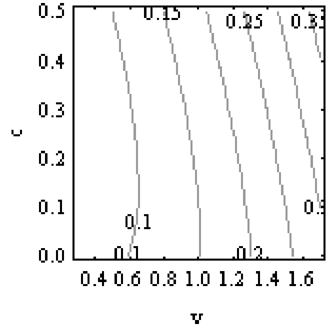


Figure 4.B2.7d |g=1
Contour Plot U2 (*frr*$_2$)

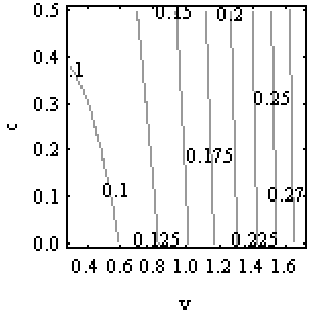


Figure 4.B2.7e |g=1
Contour Plot CS (*frr*$_2$)

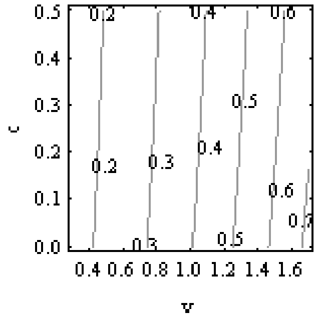


Figure 4.B2.7: Re-bargaining: Π_1, Π_2 U_1, U_2 and CS are positive

Figure 4.B2.8a |g=0.5
Contour Plot Π1 (*frr*$_2$)

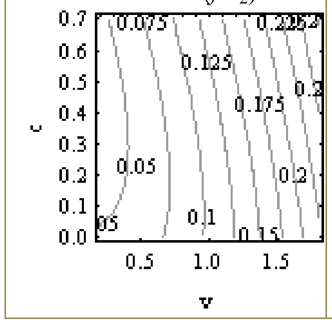


Figure 4.B2.8b |g=0.5
Contour Plot Π2 (*frr*$_2$)

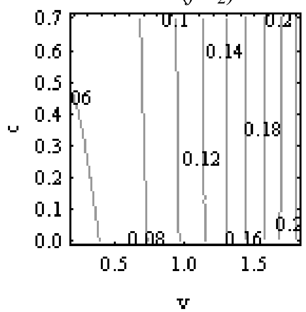


Figure 4.B2.8c |g=0.5
Contour Plot U1 (*frr*$_2$)

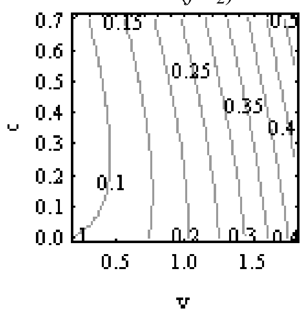


Figure 4.B2.8d |g=0.5
Contour Plot U2 (*frr*$_2$)

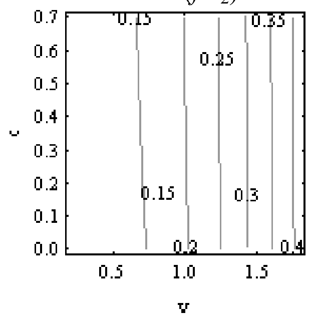


Figure 4.B2.8e |g=0.5
Contour Plot CS (*frr*$_2$)

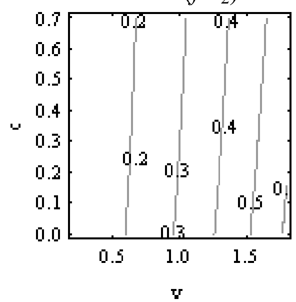


Figure 4.B2.8: Re-bargaining:Π_1, Π_2 U_1, U_2 and CS are positive

Figure 4.B2.9a | g=1
Contour Plot (q_{10}-q_{20}) (frr_2)

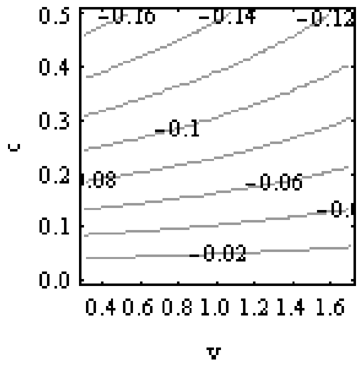


Figure 4.B2.9b | g=1
Contour Plot (q_{11}-q_{21}) (frr_2)

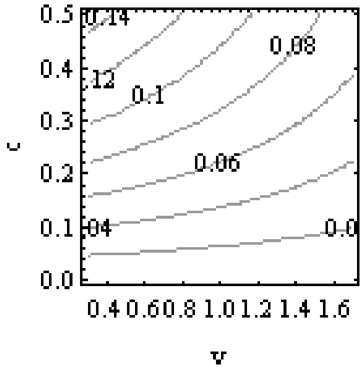


Figure 4.B2.9c | g=1
Contour Plot (q_{11}-q_{10}) (frr_2)

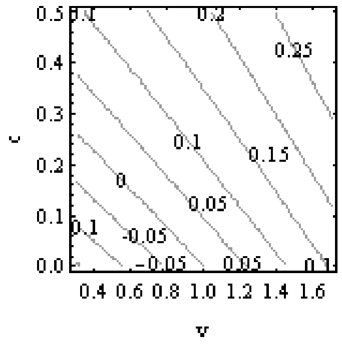


Figure 4.B2.9d |g=1
Contour Plot (q_{21}-q_{20}) (frr_2)

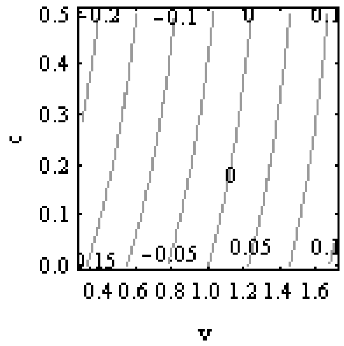


Figure 4.B2.9: Re-bargaining: $q_{10}<q_{20}$ and $q_{11}>q_{21}$ and $q_{11}>q_{10}$ if v>1 and $q_{20}>q_{21}$ if v<1

Figure 4.B2.10a |g=1
Contour Plot (tq_1-tq_2) (frr_2)

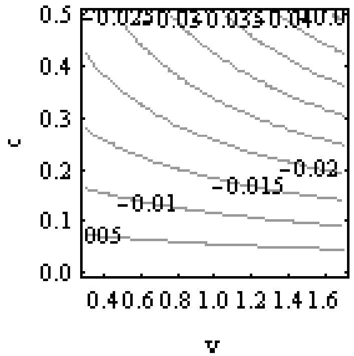


Figure 4.B2.10b | g=1
Contour Plot (Π1-Π2) (frr_2)

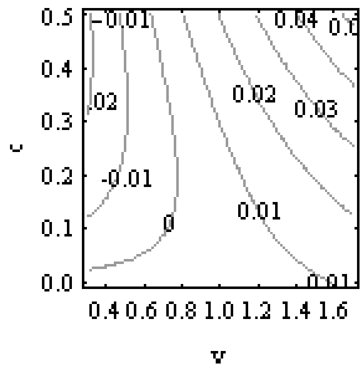


Figure 4.B2.10c | g=1
Contour Plot (U1-U2) (frr_2)

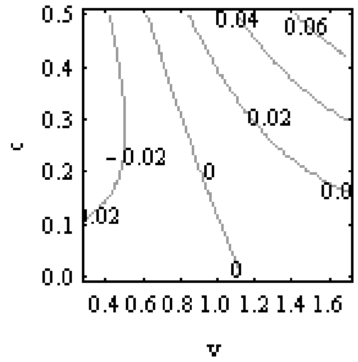


Figure 4.B2.10: Re-bargaining: $tq_1<tq_2$, $\Pi_1>\Pi_2$ and U1>U2 if v>1

Figure 4.B2.11a |g=0.5
Contour Plot (q_{10}-q_{20}) (*frr*$_2$)

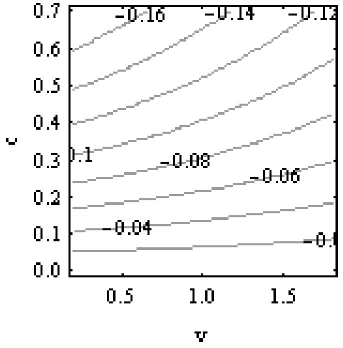


Figure 4.B2.11b |g=0.5
Contour Plot (q_{11}-q_{21}) (*frr*$_2$)

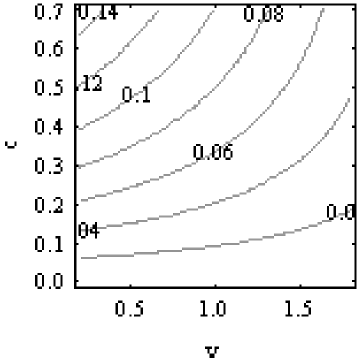


Figure 4.B2.11c |g=0.5
Contour Plot (q_{11}-q_{10}) (*frr*$_2$)

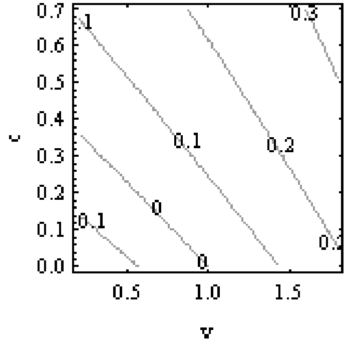


Figure 4.B2.11d |g=0.5
Contour Plot (q_{21}-q_{20}) (*frr*$_2$)

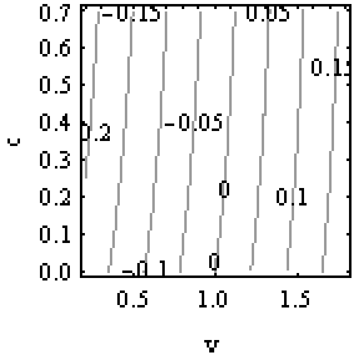


Figure 4.B2.11: Re-bargaining: $q_{10}<q_{20}$ and $q_{11}>q_{21}$ and $q_{11}>q_{10}$ if v>1 and $q_{20}>q_{21}$ if v<1

Figure 4.B2.12a |g=0.5
Contour Plot (tq_1-tq_2) (*frr*$_2$)

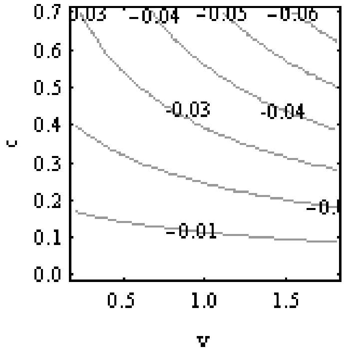


Figure 4.B2.12b| g=0.5
Contour Plot (Π1-Π2) (*frr*$_2$)

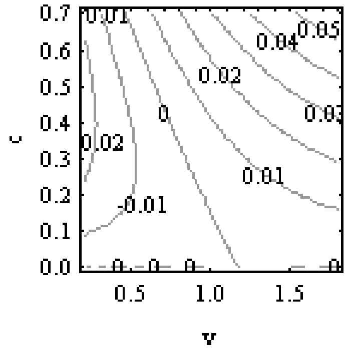


Figure 4.B2.12c |g=0.5
Contour Plot (U1-U2) (*frr*$_2$)

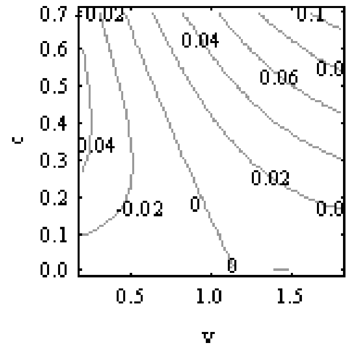


Figure 4.B2.12: Re-bargaining: $tq_1<tq_2$, $\Pi_1>\Pi_2$ and U1>U2 if v>1

Figure 4.B2.13a |g=1
Contour Plot $q_{10}(frr_2)-q_{10}(frr_0)$

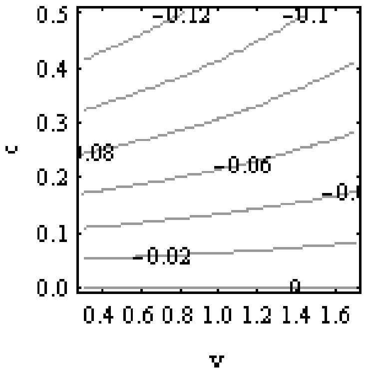


Figure 3.B2.13b |g=1
Contour Plot $q_{20}(frr_2)-q_{20}(frr_0)$

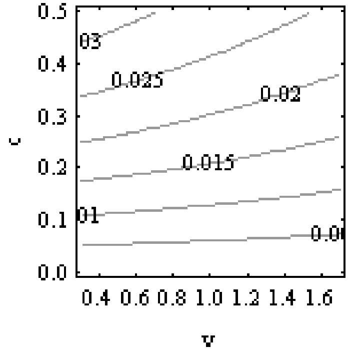


Figure 4.B2.13c |g=1
Contour Plot $q_{11}(frr_2)-q_{11}(frr_0)$

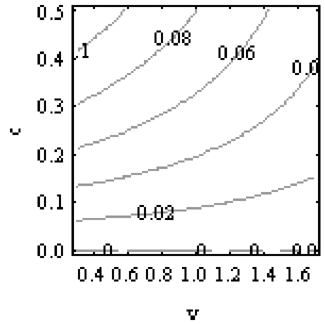


Figure 4.B2.13d |g=1
Contour Plot $q_{21}(frr_2)-q_{21}(frr_0)$

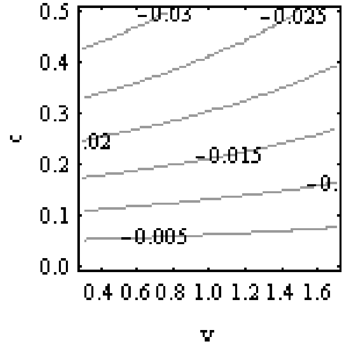


Figure 4.B2.13: Re-bargaining: $q_{10}(frr_2)<q_{10}(frr_0)$ and $q_{20}(frr_2)>q_{20}(frr_0)$; $q_{11}(frr_2)>q_{11}(frr_0)$, and $q_{21}(frr_2)<q_{21}(frr_0)$

Figure 4.B2.14a |g=0.5
Contour Plot $[q_{10}(frr_2)-q_{10}(frr_0)$,

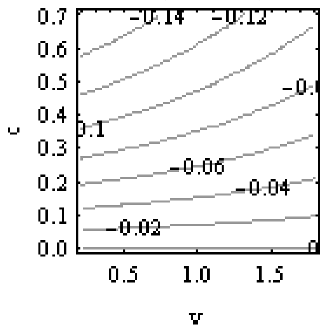


Figure 4.B2.14b |g=0.5
Contour Plot $[q_{20}(frr_2)-q_{20}(frr_0)$,

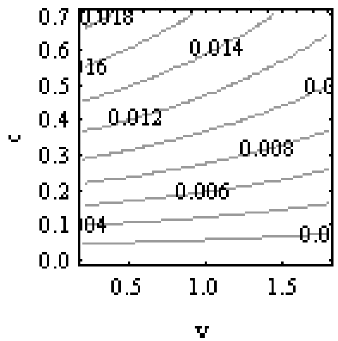


Figure 4.B2.14c |g=0.5
Contour Plot $[q_{11}(frr_2)-q_{11}(frr_0)$,

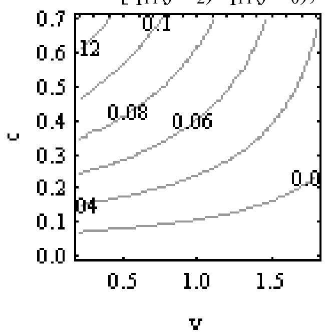


Figure 4.B2.14d |g=0.5
Contour Plot $[q_{21}(frr_2)-q_{21}(frr_0)$,

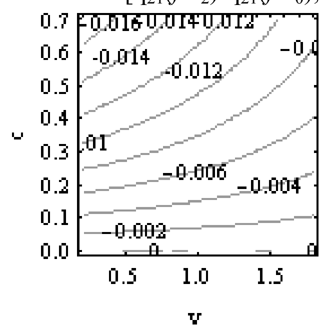


Figure 4.B2.14: Re-bargaining:$q_{10}(frr_2)<q_{10}(frr_0)$ and $q_{20}(frr_2)>q_{20}(frr_0)$; $q_{11}(frr_2)>q_{11}(frr_0)$, and $q_{21}(frr_2)<q_{21}(frr_0)$

Figure 4.B2.15a |g=1
Contour Plot
$(q_{11}-q_{10})\ (frr_2)-(q_{11}-q_{10})(frr_0)$

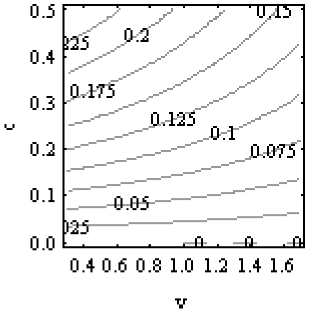


Figure 4.B2.15b |g=1
Contour Plot
$(q_{21}-q_{20})\ (frr_2)-(q_{21}-q_{20})\ (frr_0)$

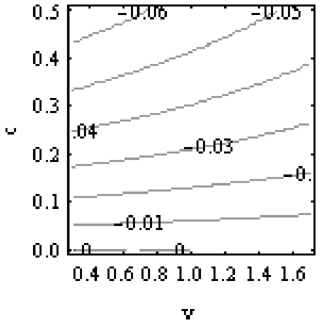


Figure 4.B2.15c |g=0.5
Contour Plot
$(q_{11}-q_{10})\ (frr_2)-(q_{11}-q_{10})(frr_0)$

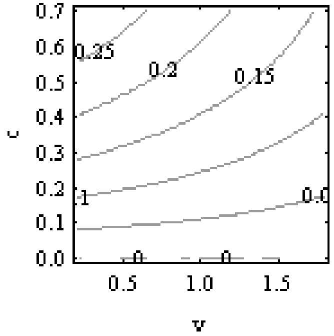


Figure 4.B2.15d |g=0.5
Contour Plot
$(q_{21}-q_{20})\ (frr_2)-(q_{21}-q_{20})\ (frr_0)$

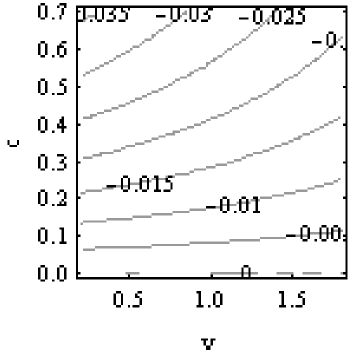


Figure 4.B2.15: Re-bargaining: $(q_{11}-q_{10})\ (frr_2) > (q_{11}-q_{10})\ (frr_0)$ and $(q_{21}-q_{20})\ (frr_2) < (q_{21}-q_{20})\ (frr_0)$

Figure 4.B2.16a |g=1
Contour Plot
$tq_1(frr_2)-tq_1(frr_0)$

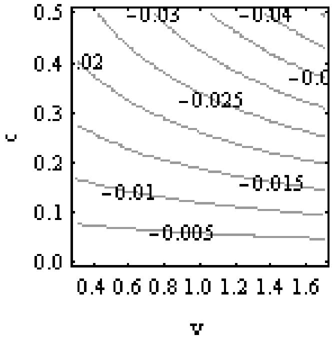


Figure 4.B2.16b |g=1
Contour Plot
$tq_2(frr_2)-tq_2(frr_0)$

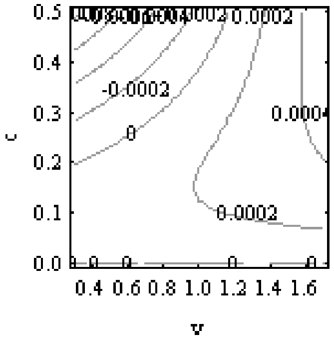


Figure 4.B2.16c |g=1
Contour Plot
$Tq_A(frr_2)-Tq_A(frr_0)$

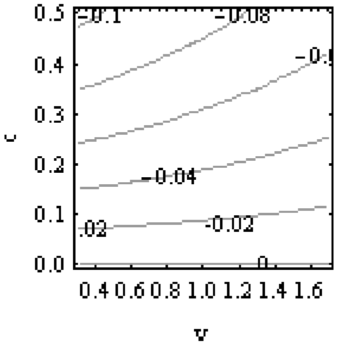


Figure 4.B2.16d |g=1
Contour Plot
$Tq_B(frr_2)-Tq_B(frr_0)$

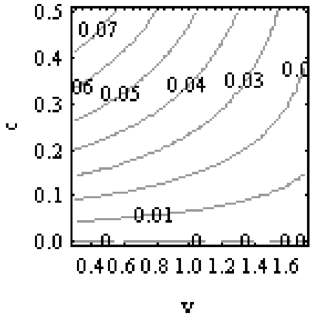


Figure 4.B2.16e |g=1
Contour Plot
$Tq(frr_2)-Tq(frr_0)$

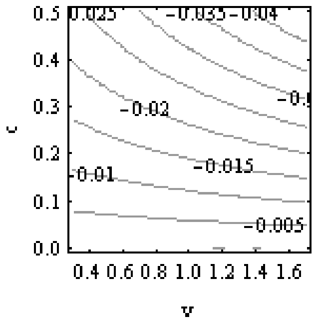


Figure 4.B2.16: Re-bargaining: $tq_1(frr_2)<tq_1(frr_0)$; $tq_2(frr_2)>tq_2(frr_0)$ if $v>1$; $Tq_A(frr_2)<Tq_A(frr_0)$ and $Tq_B(frr_2)>Tq_B(frr_0)$; $Tq(frr_2)<Tq(frr_0)$

Figure 4.B2.17a |g=0.5
Contour Plot
$tq_1(frr_2)$-$tq_1(frr_0)$

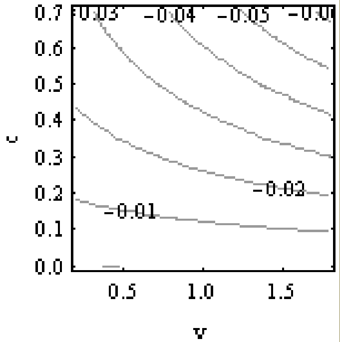


Figure 4.B2.17b |g=0.5
Contour Plot
$tq_2(frr_2)$-$tq_2(frr_0)$

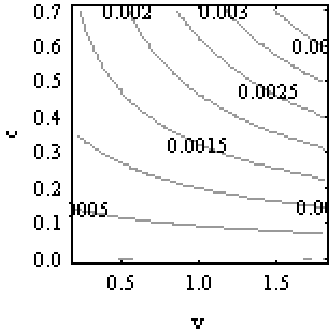


Figure 4.B2.17c |g=0.5
Contour Plot
$Tq_A(frr_2)$-$Tq_A(frr_0)$

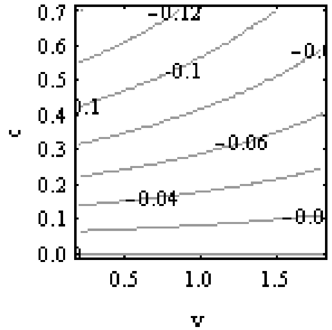


Figure 4.B2.17d |g=0.5
Contour Plot
$Tq_B(frr_2)$-$Tq_B(frr_0)$

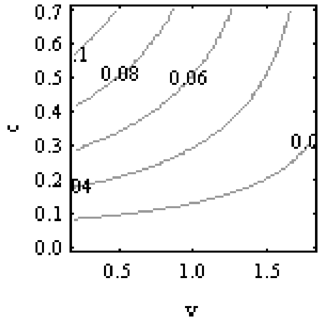


Figure 4.B2.17e |g=0.5
Contour Plot
$Tq(frr_2)$-$Tq(frr_0)$

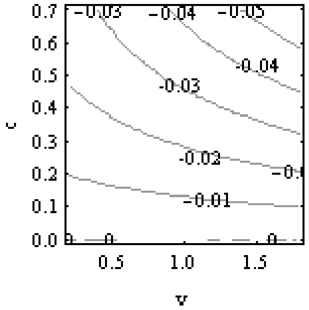


Figure 4.B2.17: Re-bargaining: $tq_1(frr_2)<tq_1(frr_0)$; $tq_2(frr_2)>tq_2(frr_0)$; $Tq_A(frr_2)<Tq_A(frr_0)$ and $Tq_B(frr_2)>Tq_B(frr_0)$; $Tq(frr_2)<Tq(frr_0)$

Figure 4.B2.18a |g=1
Contour Plot
Π1(frr_2)-Π1(frr_0)

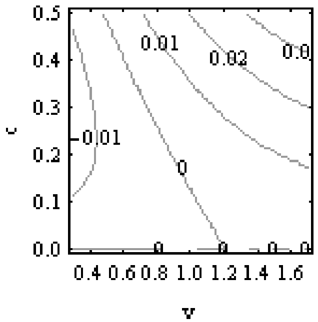


Figure 4.B2.18b |g=1
Contour Plot
Π2(frr_2)-Π2(frr_0)

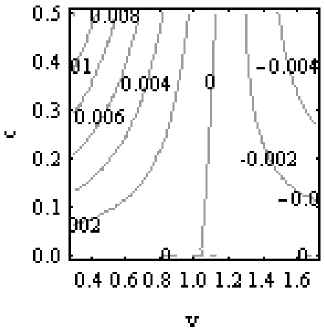


Figure 4.B2.18c |g=1
Contour Plot
U1(frr_2)-U1(frr_0)

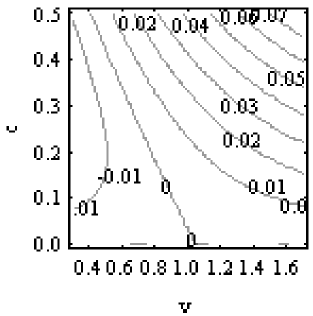


Figure 4.B2.18d |g=1
Contour Plot
U2(frr_2)-U2(frr_0)

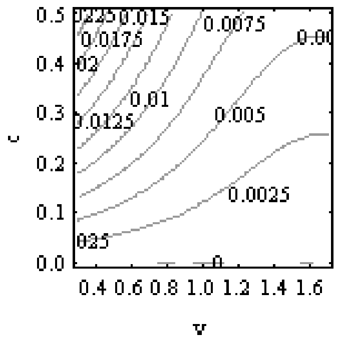


Figure 4.B2.18e |g=1
Contour Plot
CS(frr_2)-CS(frr_0)

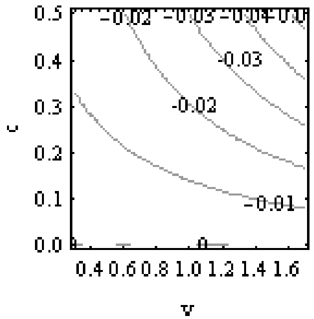


Figure 4.B2.18: Re-bargaining: Π1(frr_2)>Π1(frr_0), Π2(frr_2)<Π2(frr_0) and U1(frr_2)>U1(frr_0) if v>1; and U2(frr_2)>U2(frr_0), CS(frr_2)<CS(frr_0)

Figure 4.B2.19a |g=0.5
Contour Plot
Π1(frr_2)-Π1(frr_0)

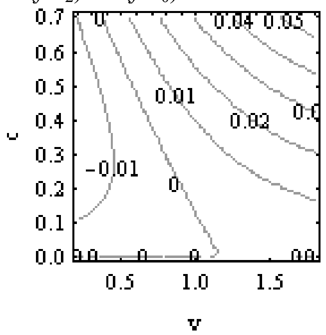


Figure 4.B2.19b |g=0.5
Contour Plot
Π2(frr_2)-Π2(frr_0)

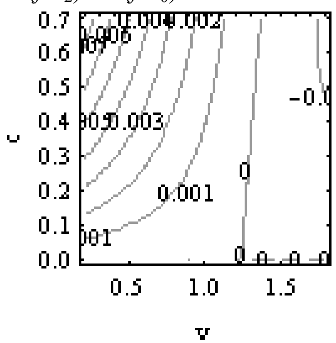


Figure 4.B2.19c |g=0.5
Contour Plot
U1(frr_2)-U1(frr_0)

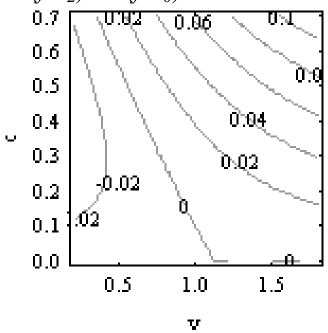


Figure 4.B2.19d |g=0.5
Contour Plot
U2(frr_2)-U2(frr_0)

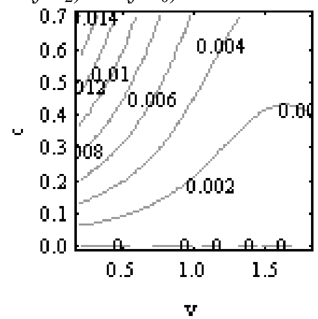


Figure 4.B2.19e |g=0.5
Contour Plot
CS(frr_2)-CS(frr_0)

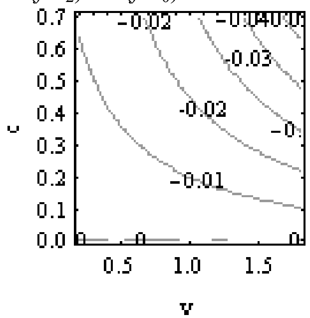


Figure 4.B2.19: Re-bargaining: Π1(frr_2)>Π1(frr_0), Π2(frr_2)<Π2(frr_0) and U1(frr_2)>U1(frr_0) if v>1; and U2(frr_2)>U2(frr_0), CS(frr_2)<CS(frr_0)

Figure 4.B2.20a |g=1
Contour Plot $\frac{d\Pi 1}{dc}$(frr_2)

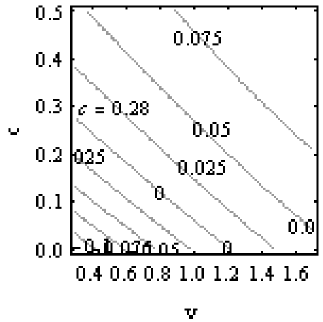


Figure 4.B2.20b |g=1
Contour Plot $\frac{d\Pi 2}{dc}$(frr_2)

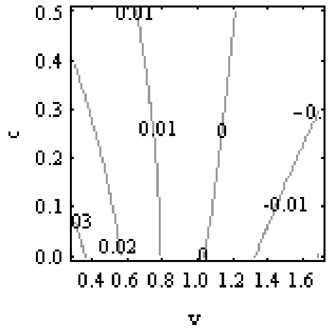


Figure 4.B2.20c |g=1
Contour Plot $\frac{dU1}{dc}$(frr_2)

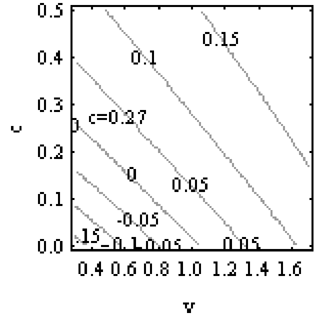


Figure 4.B2.20d |g=1
Contour Plot $\frac{dU2}{dc}$(frr_2)

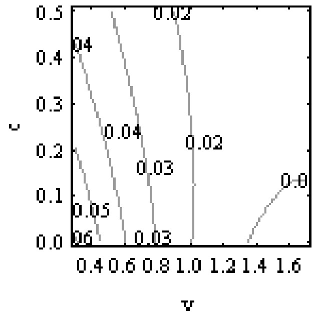


Figure 4.B2.20e |g=1
Contour Plot $\frac{dCS}{dc}$(frr_2)

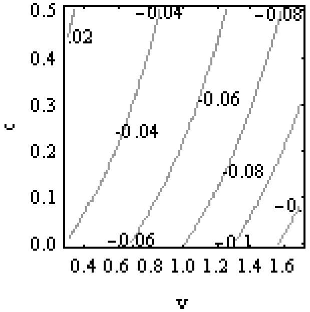


Figure 4.B2.20: Re-bargaining: Π1, U1 and U2 are increasing with c provided that $0.28 \le c \le 0.5$

Figure 4.B2.21a |g=0.5

Contour Plot $\frac{d\Pi 1}{dc}(frr_2)$

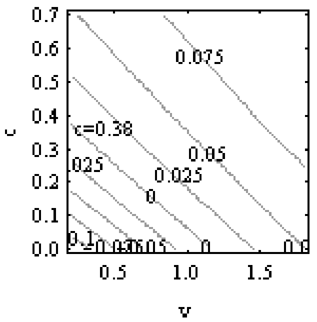


Figure 4.B2.21b |g=0.5

Contour Plot $\frac{d\Pi 2}{dc}(frr_2)$

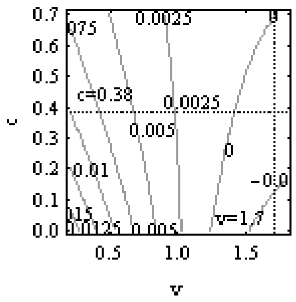


Figure 4.B2.21c |g=0.5

Contour Plot $\frac{dU1}{dc}(frr_2)$

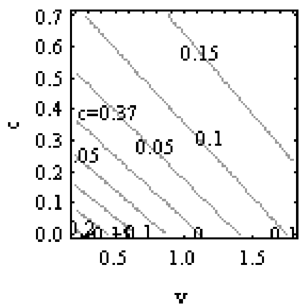


Figure 4.B2.21d |g=0.5

Contour Plot $\frac{dU2}{dc}(frr_2)$

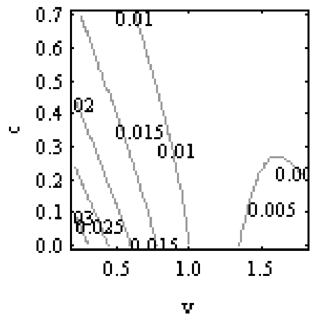


Figure 4.B2.21e |g=0.5

Contour Plot $\frac{dCS}{dc}$ (frr_2)

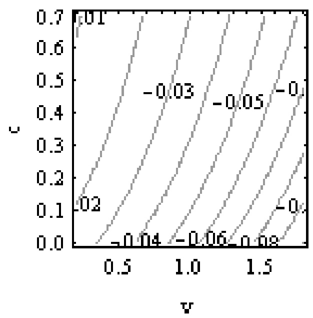


Figure 4.B2.21: Re-bargaining: Π1, U1, U2 are increasing with c provided that $0.38 \le c \le 0.7$

Figure 4.B2.22.a |g=1| c=0.5| ρ=ρ_1

$\tilde{w}_{10} - w_{20}$ (frr$_2$)

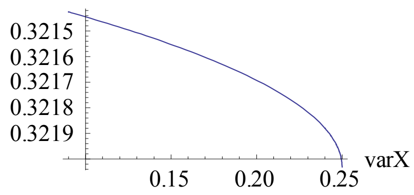


Figure 4.B2.22.b |g=1| c=0.5| ρ=ρ_2

$\tilde{w}_{10} - w_{20}$ (frr$_2$)

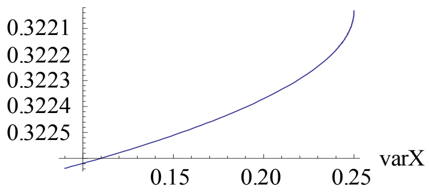


Figure 4.B2.22.c |g=0.5| c=0.7| ρ=ρ_1

$\tilde{w}_{10} - w_{20}$ (frr$_2$)

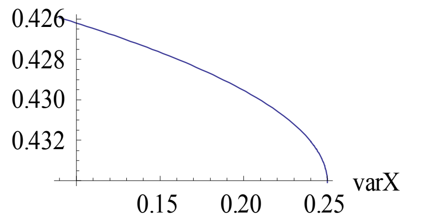


Figure 4.B2.22.d |g=0.5| c=0.7| ρ=ρ_2

$\tilde{w}_{10} - w_{20}$ (frr$_2$)

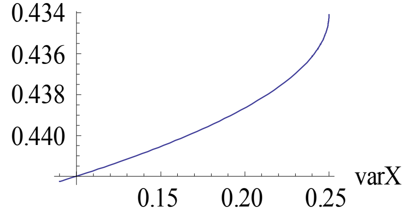


Figure 4.B2.22.e |g=1| c=0.5| ρ=ρ_1

$\tilde{w}_{11} - w_{21}$ (frr$_2$)

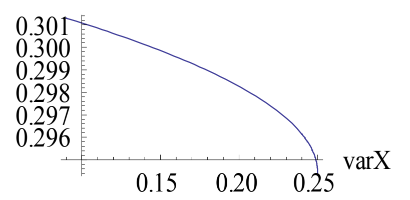


Figure 4.B2.22.f |g=1| c=0.5| ρ=ρ_2

$\tilde{w}_{11} - w_{21}$ (frr$_2$)

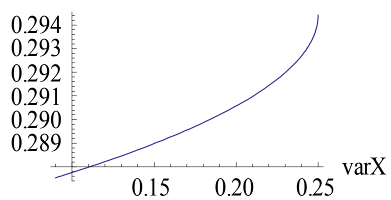


Figure 4.B2.22.g |g=0.5| c=0.7| ρ=ρ_1

$\tilde{w}_{11} - w_{21}$ (frr$_2$)

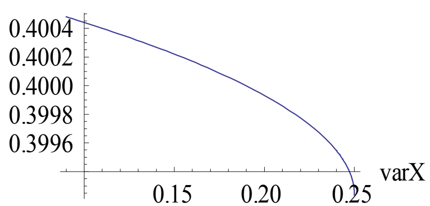


Figure 4.B2.22.h |g=0.5| c=0.7| ρ=ρ_2

$\tilde{w}_{11} - w_{21}$ (frr$_2$)

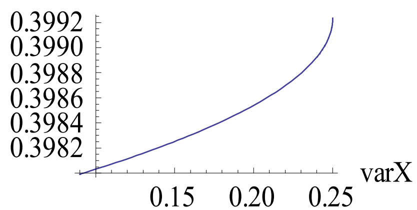


Figure 4.B2.22: **(**w_{10}-w_{20}) increasing (decreasing) with varX under ρ_1 (ρ_2); the stimulus is reversed in the second period, under re-bargaining

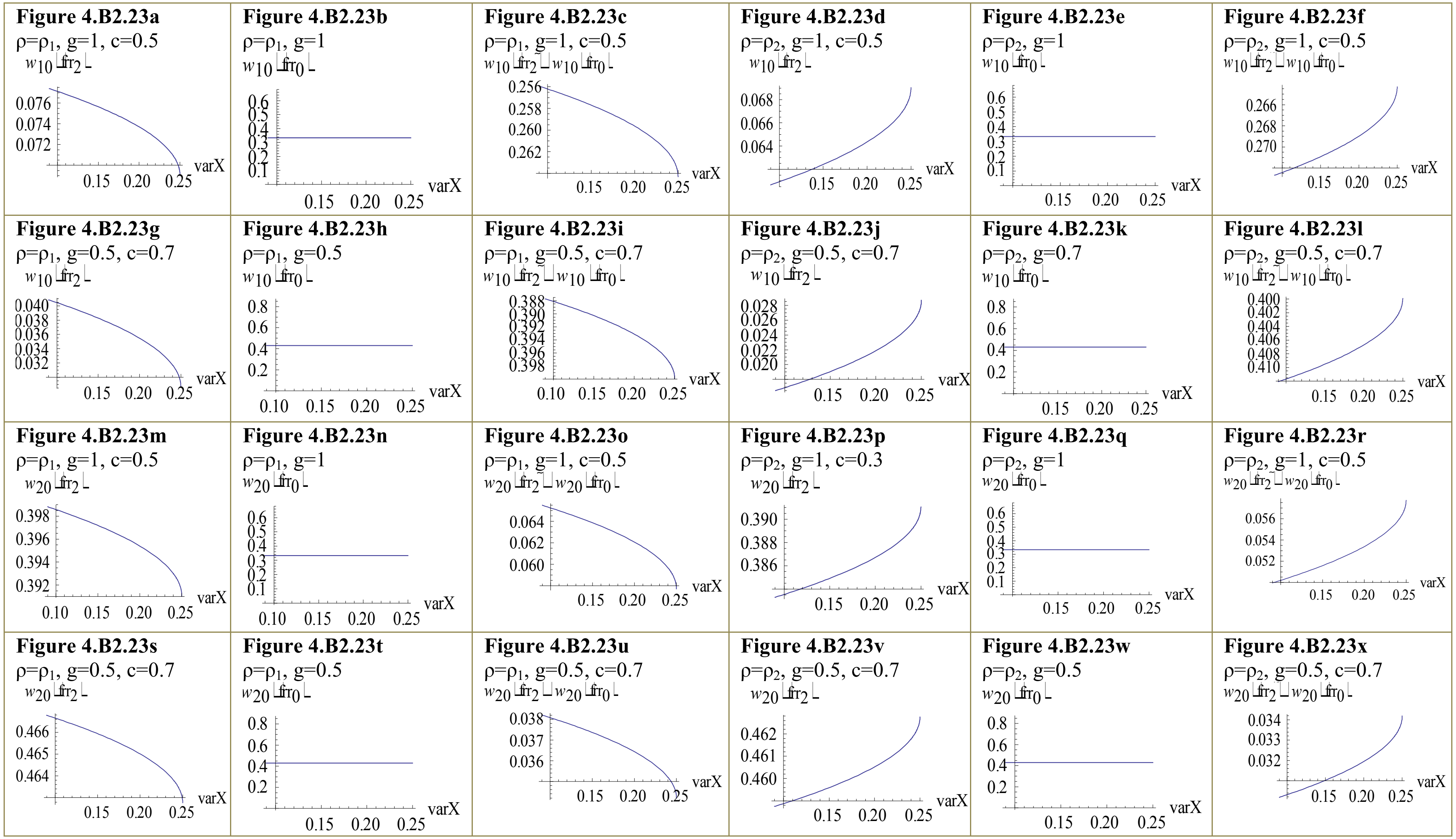


Figure 3.B2.23: w_{10}, w_{20} decreasing (increasing) with varX under ρ_1 (ρ_2) under re-bargaining

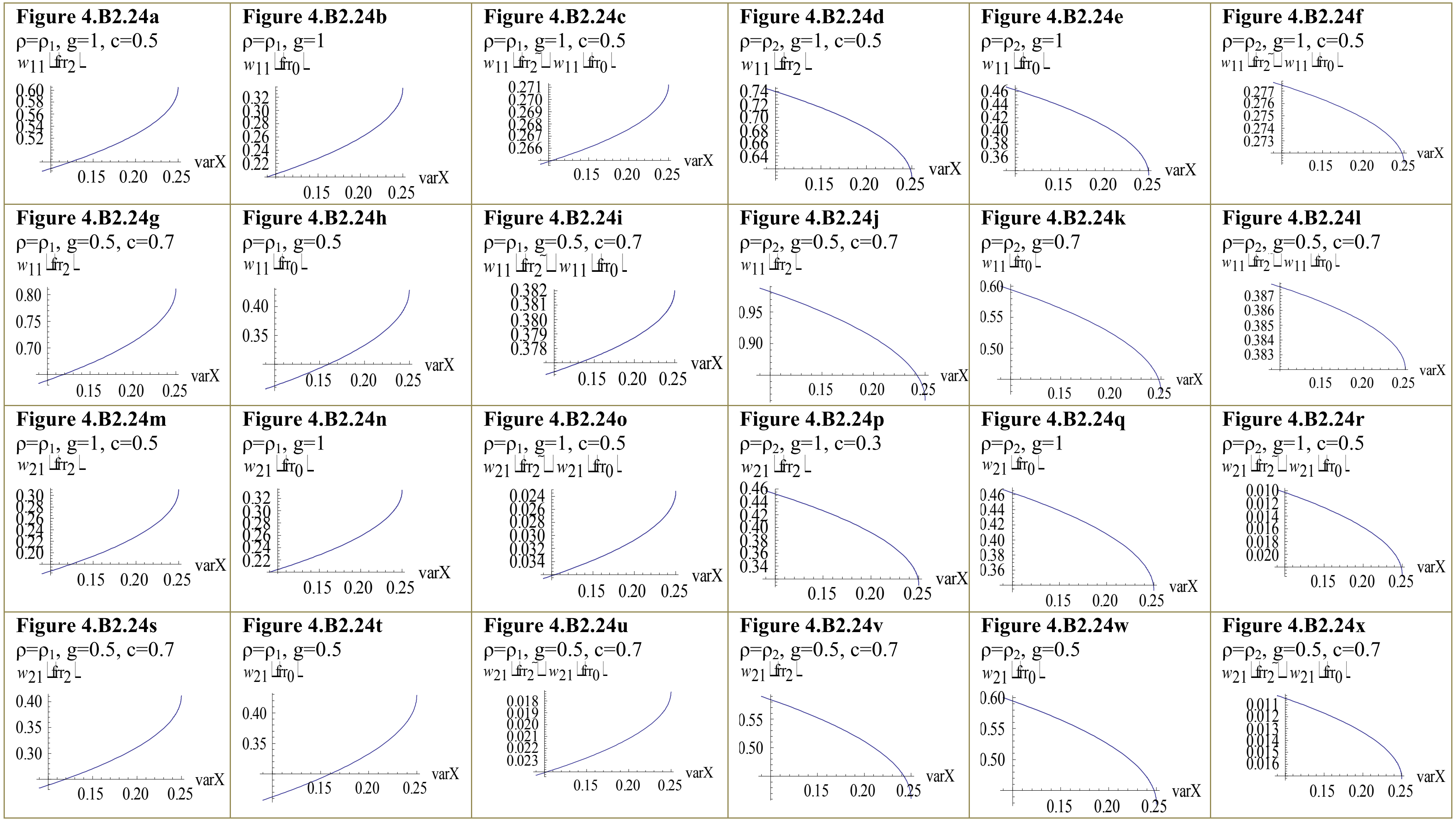


Figure 3.B2.24: w_{11}, w_{21} increasing (decreasing) with varX under ρ_1 (ρ_2) under re-bargaining

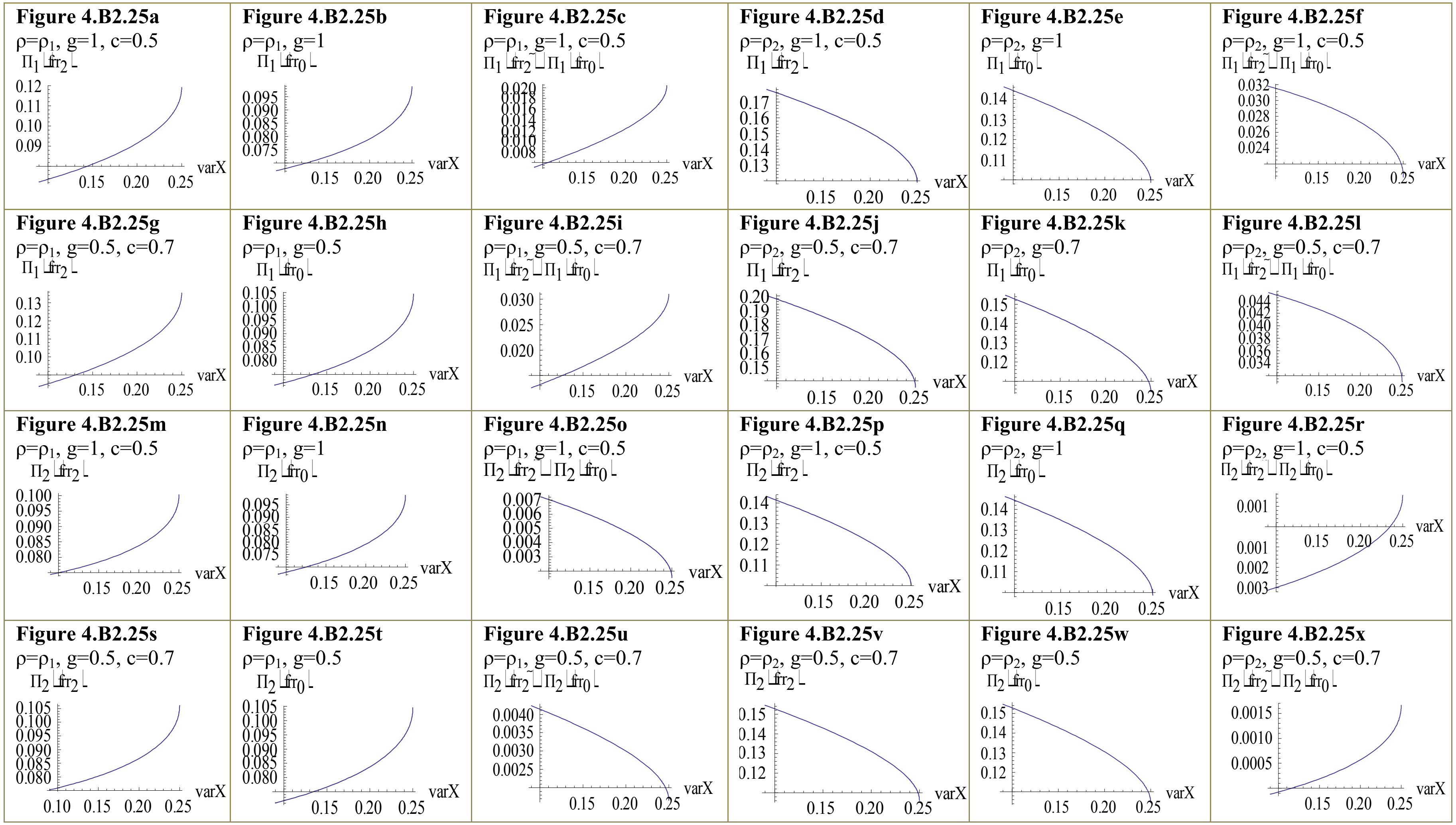


Figure 4.B2.25: How varX affects Π1(frr_2), Π1(frr_0), Π1(frr_2)-Π1(frr_0), Π2(frr_2), Π2(frr_0), Π2(frr_2)-Π2(frr_0), under ρ_1 or ρ_2 under re-bargaining

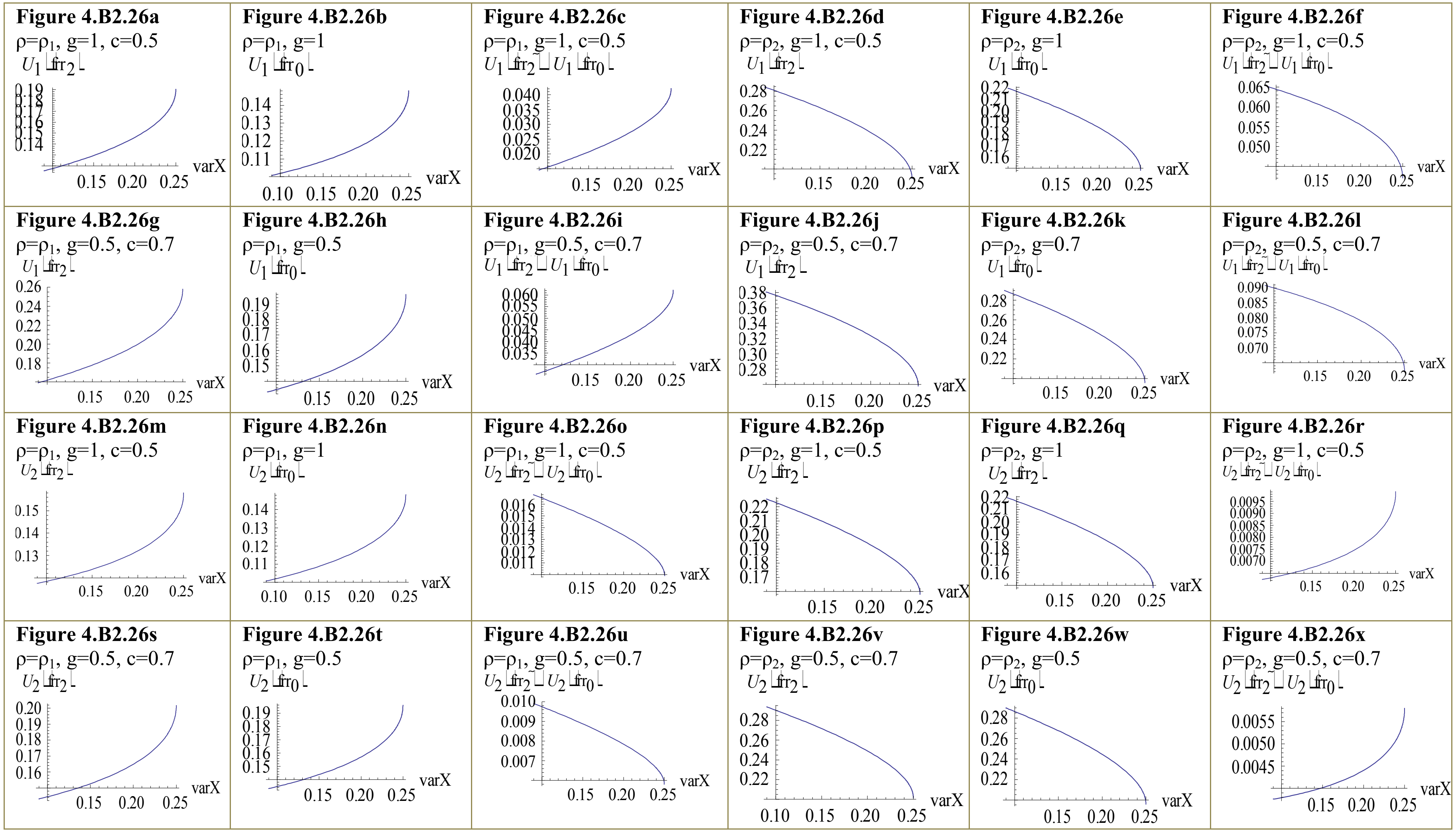


Figure 4.B2.26: How varX affects U1(*frr*$_2$), U1(*frr*$_0$), U1(*frr*$_2$)-U1(*frr*$_0$), U2(*frr*$_2$), U2(*frr*$_0$), U2(*frr*$_2$)-U2(*frr*$_0$), under ρ_1 or ρ_2 under re-bargaining

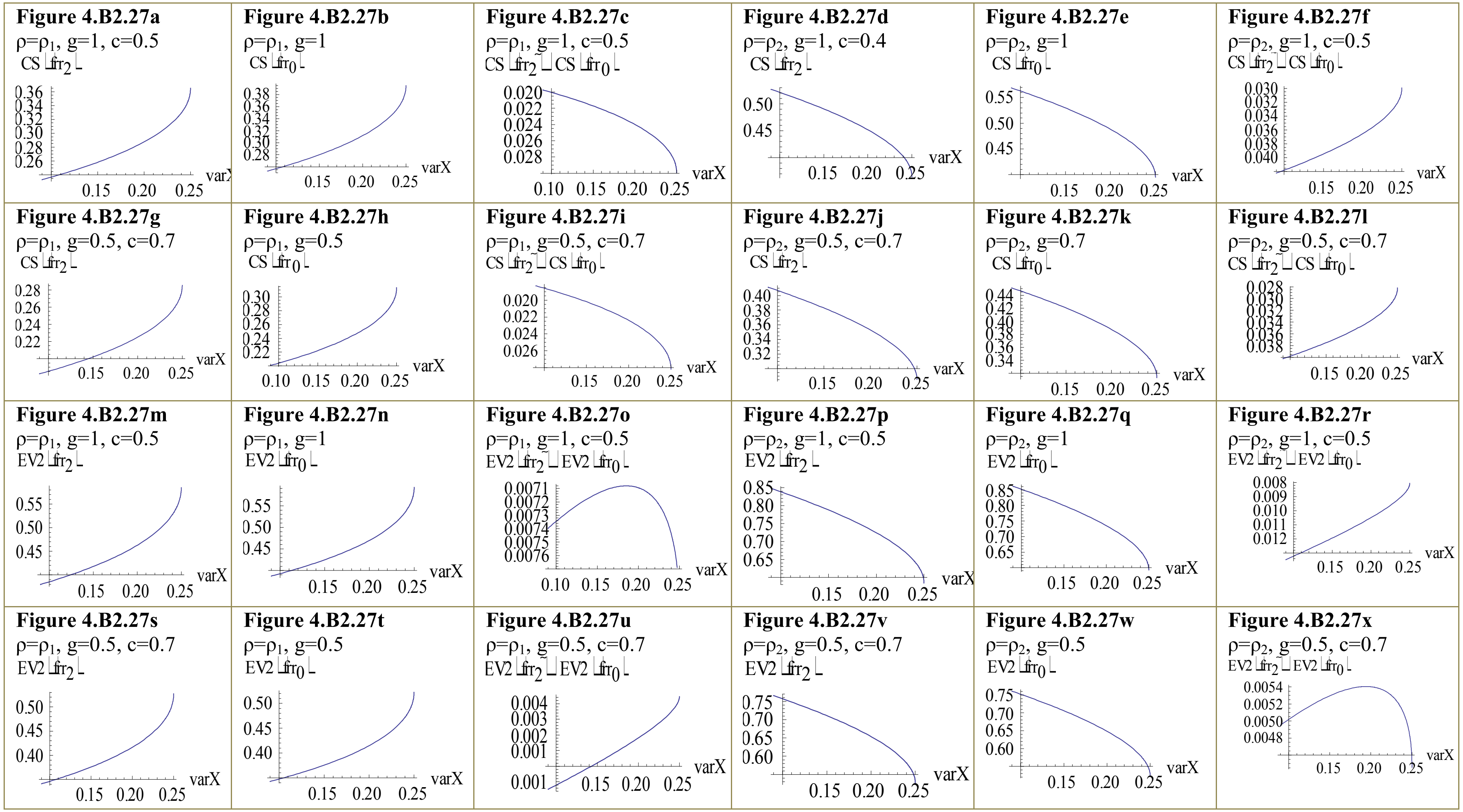


Figure 4.B2.27: How varX affects CS(*frr*$_2$), CS(*frr*$_0$), CS(*frr*$_2$)-CS(*frr*$_0$), EV2(*frr*$_2$), EV2(*frr*$_0$), EV2(*frr*$_2$)-EV2(*frr*$_0$), under ρ_1 or ρ_2 under re-bargaining

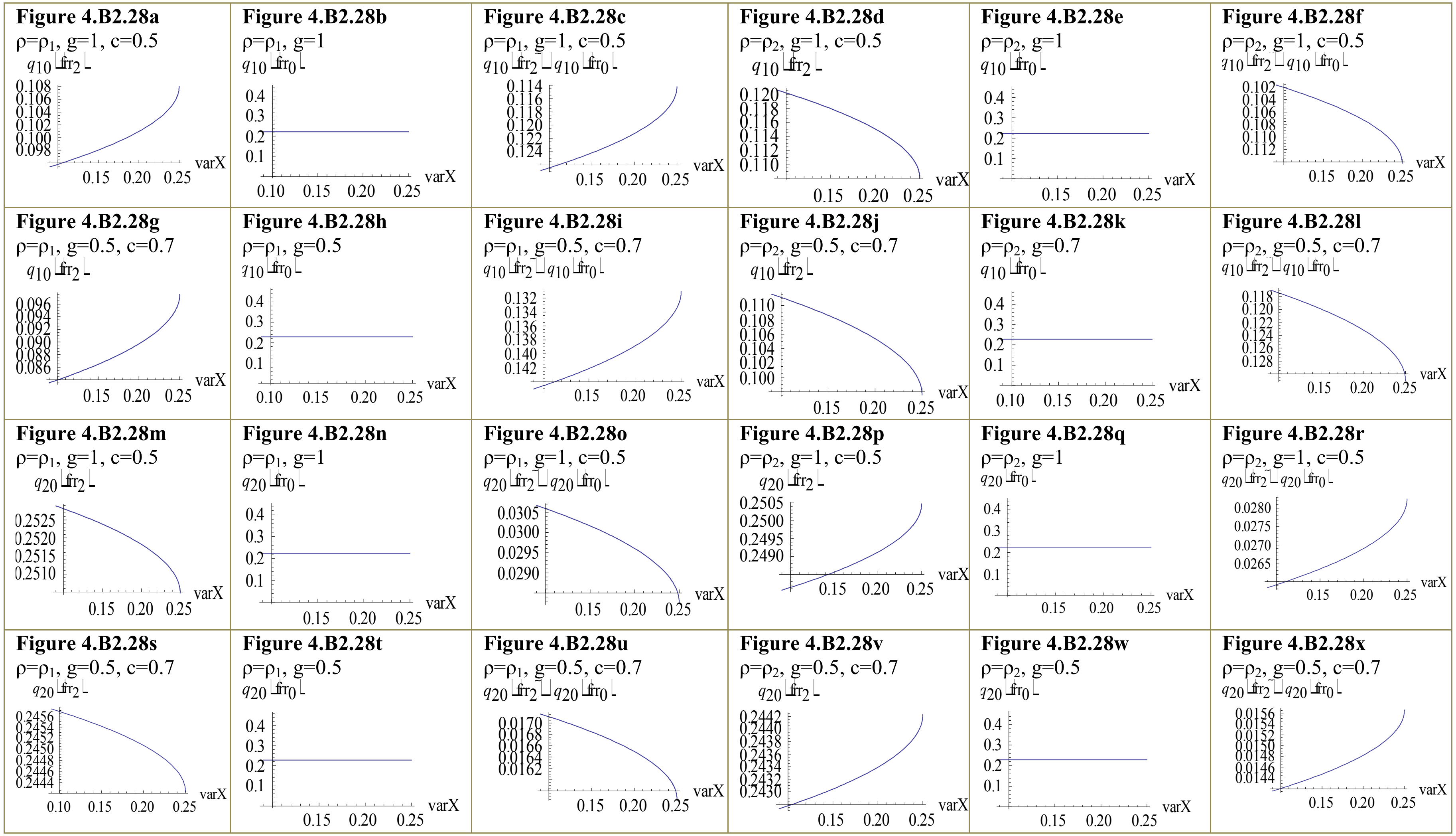


Figure 4.B2.28: How varX affects $q_{10}(frr_2)$, $q_{10}(frr_0)$, $q_{10}(frr_2)$-$q_{10}(frr_0)$, $q_{20}(frr_2)$, $q_{20}(frr_0)$, $q_{20}(frr_2)$-$q_{20}(frr_0)$, under ρ_1 or ρ_2 under re-bargaining

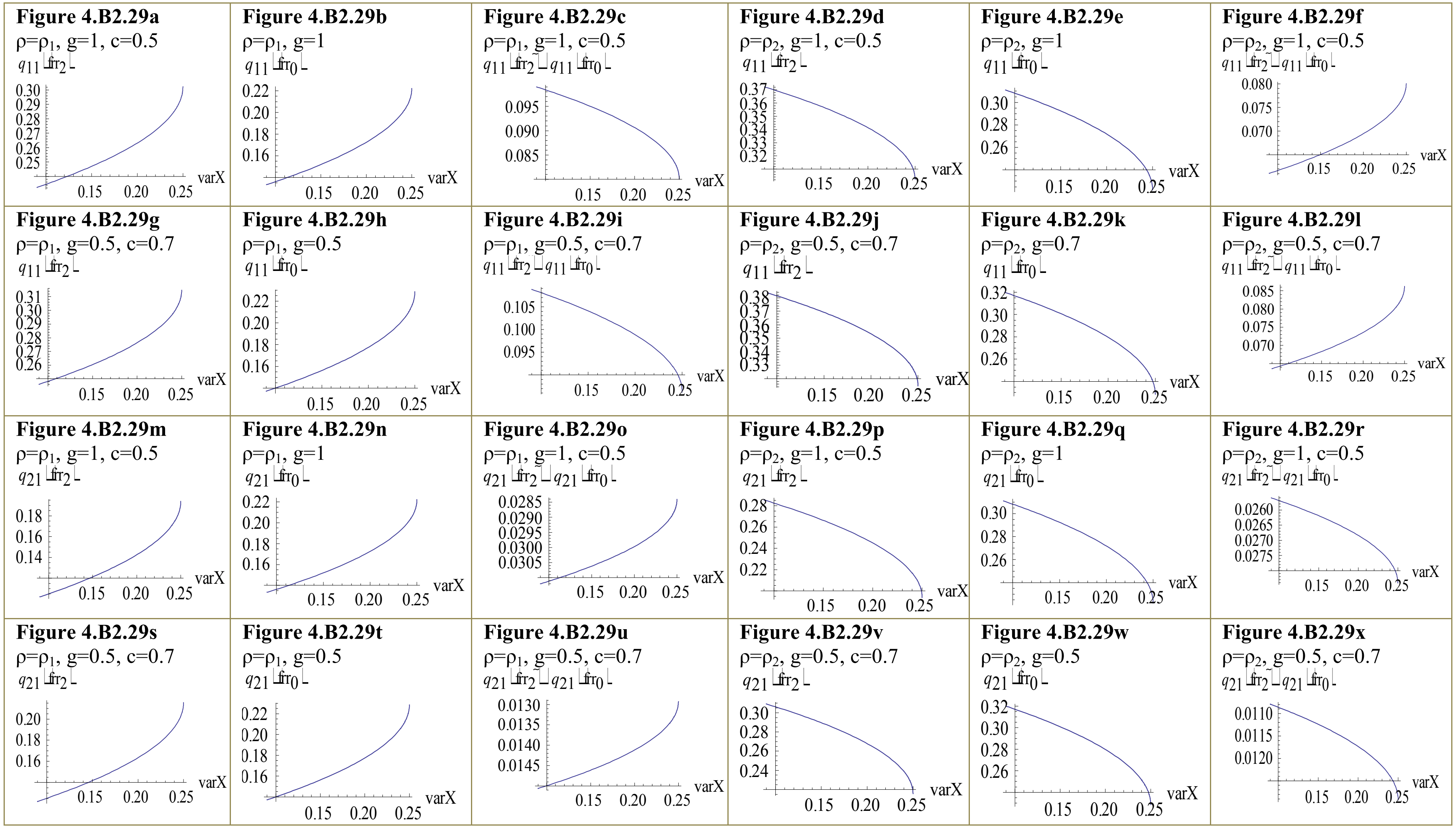


Figure 4.B2.29: How varX affects $q_{11}(frr_2)$, $q_{11}(frr_0)$, $q_{11}(frr_2)$-$q_{11}(frr_0)$, $q_{21}(frr_2)$, $q_{21}(frr_0)$, $q_{21}(frr_2)$-$q_{21}(frr_0)$, under ρ_1 or ρ_2 under re-bargaining

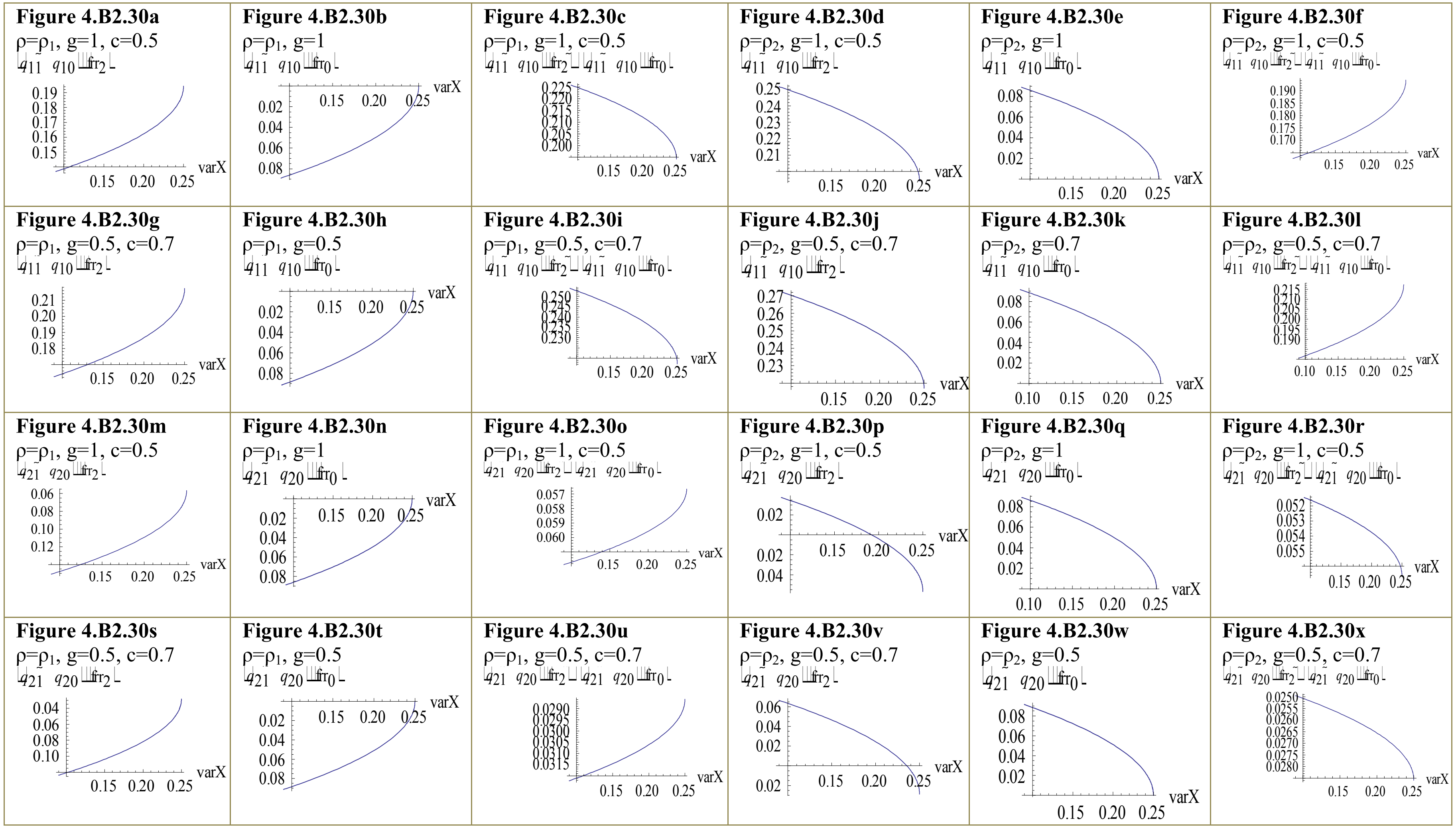


Figure 4.B2.30: How varX affects $(q_{11}-q_{10})(frr_2)$, $(q_{11}-q_{10})(frr_0)$, $(q_{11}-q_{10})(frr_2)-(q_{11}-q_{10})(frr_0)$, $(q_{21}-q_{20})(frr_2)$, $(q_{21}-q_{20})(frr_0)$, $(q_{21}-q_{20})(frr_2)-(q_{21}-q_{20})(frr_0)$, under ρ_1 or ρ_2 under re-bargaining

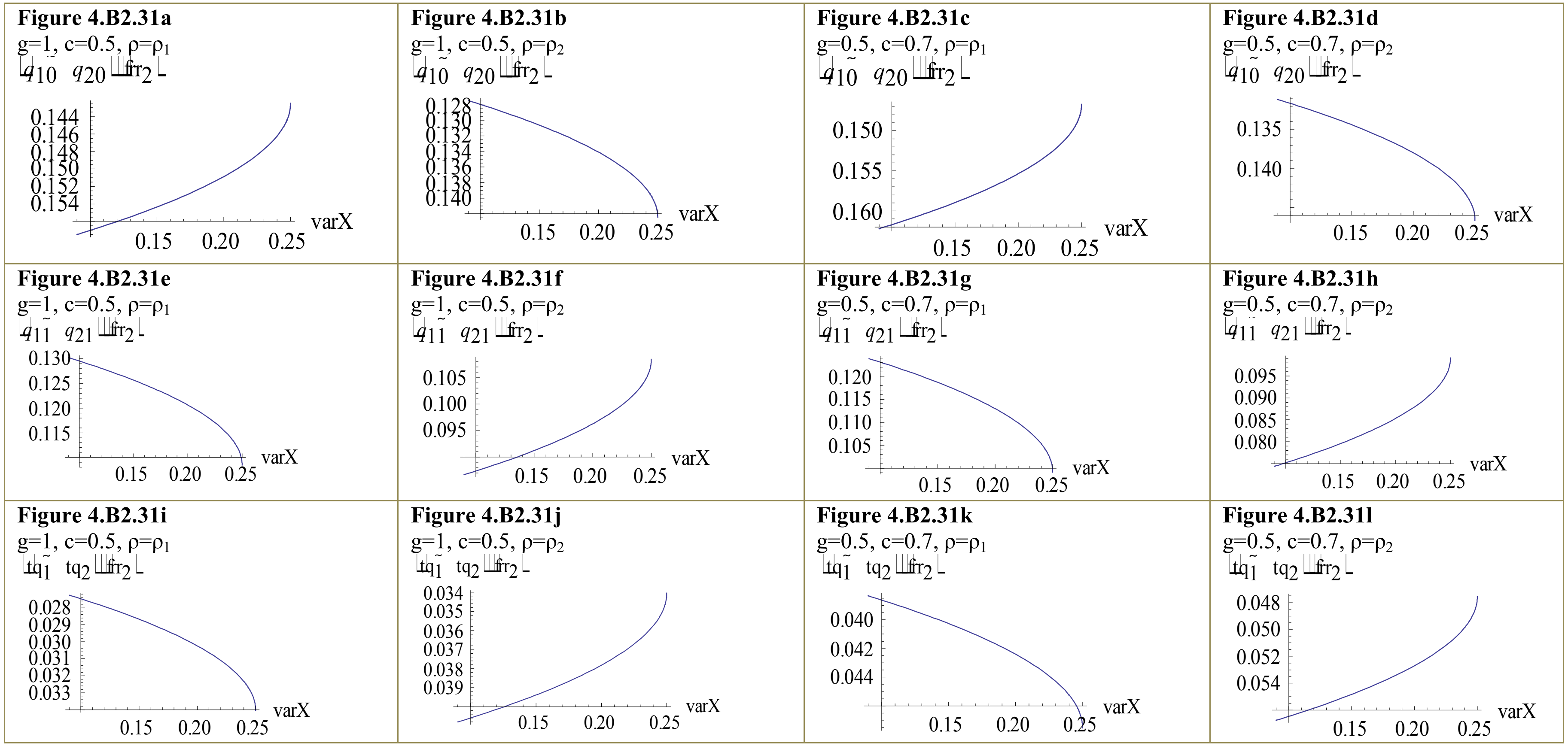


Figure 4.B2.31: How varX affects $(q_{10}-q_{20})(frr_2)$, $(q_{10}-q_{20})(frr_0)$; $(q_{11}-q_{21})(frr_2)$, $(q_{11}-q_{21})(frr_0)$; $(tq_1-tq_2)(frr_2)$, $(tq_1-tq_2)(frr_0)$, under ρ_1 or ρ_2 under re-bargaining

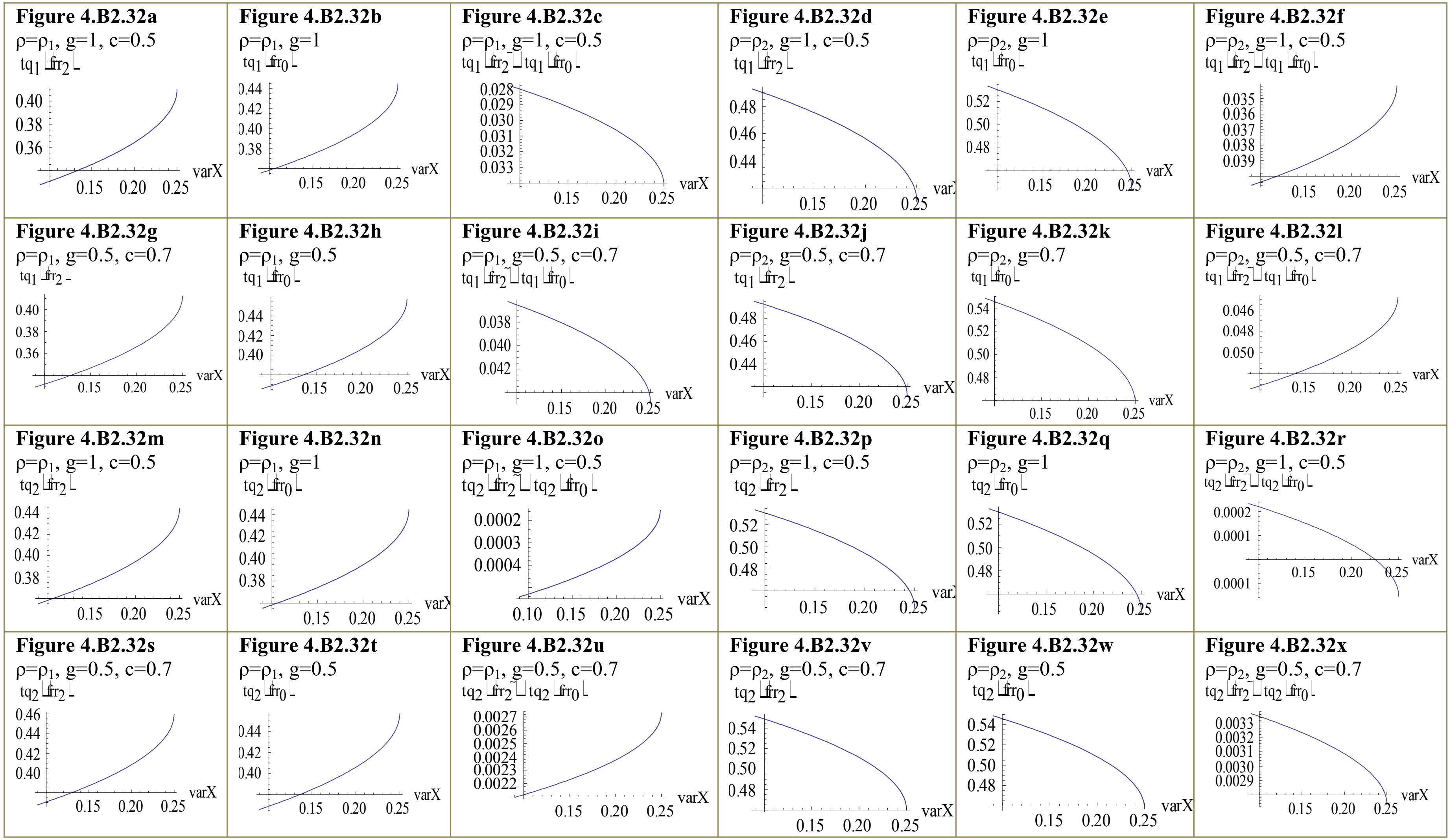


Figure 4.B2.32: Re-bargaining:How varX affects $tq_1(frr_2)$,$tq_1(frr_0)$, $tq_1(frr_2)$-$tq_1(frr_0)$, $tq_2(frr_2)$, $tq_2(frr_0)$, $tq_2(frr_2)$-$tq_2(frr_0)$, under ρ_1 and ρ_2

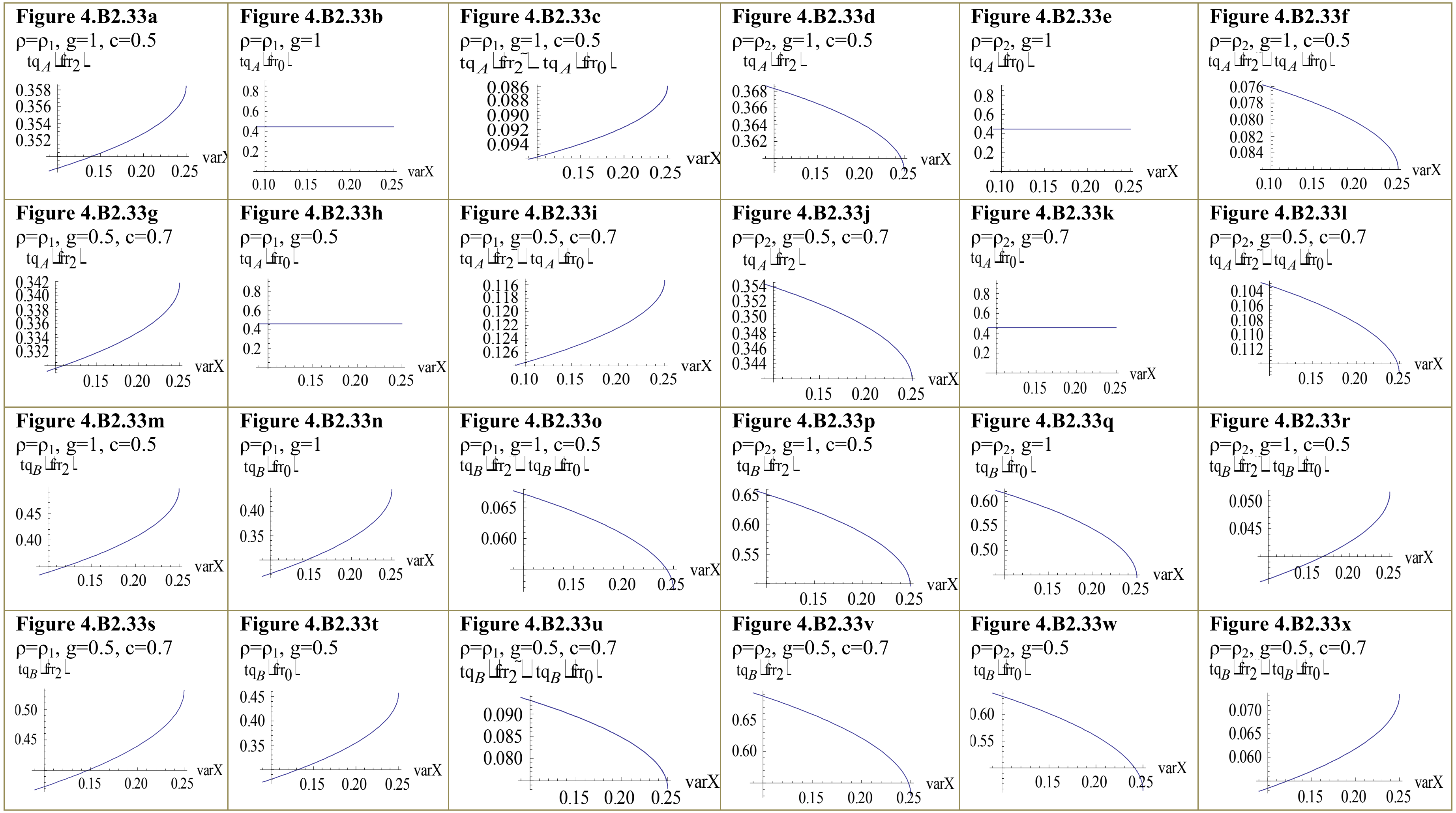


Figure 4.B2.33: RE-bargaining: How varX affects $tq_A(frr_2)$,$tq_A(frr_0)$, $tq_A(frr_2)$-$tq_A(frr_0)$, $tq_B(frr_2)$, $tq_B(frr_0)$, $tq_B(frr_2)$-$tq_B(frr_0)$, under ρ_1 and ρ_2

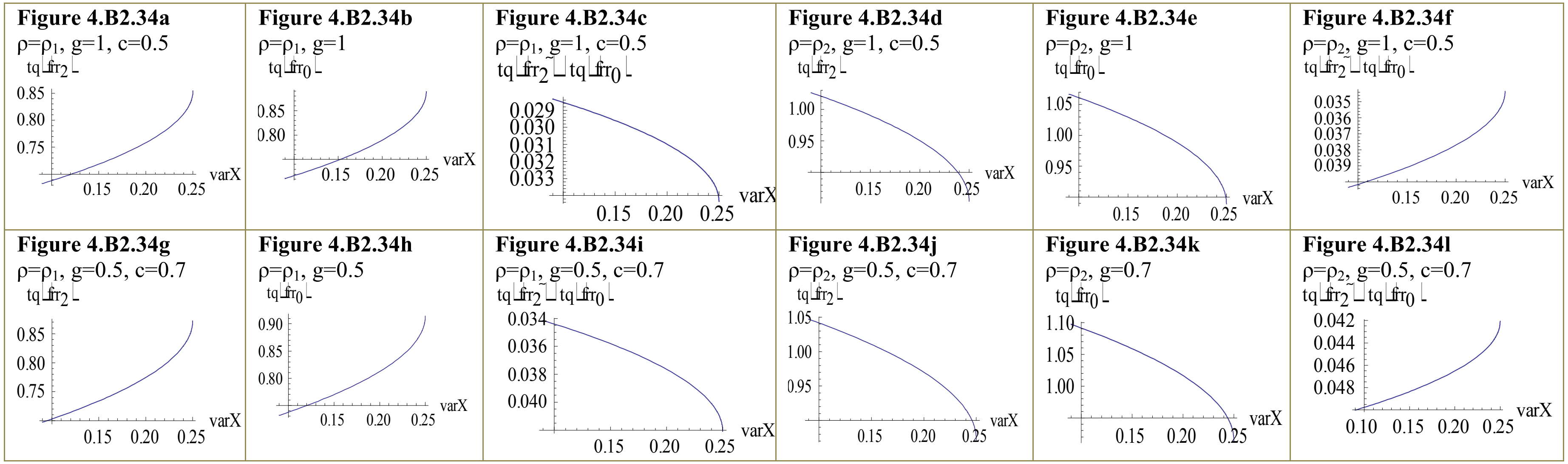


Figure 4.B2.34: Re-bargaining: How varX affects tq(*frr*$_2$),tq(*frr*$_0$), tq(*frr*$_2$)-tq(*frr*$_0$), under ρ_1 and ρ_2

APPENDIX 5.A1

Case *frr*$_1$

Second period/ fourth stage public firm maximizes $EV_1=\Pi_1+CS$ as for q_{11} and private firms maximazizes its profits as for q_{21};

$$\underset{q_{11}}{Max}\, EV_1\left(q_{10},q_{20},q_{11},q_{21},w_{10},w_{20}\right) = -c\left(\frac{(q_{10}-q_{11})^2}{2} - (q_{10}-q_{11})\right) + q_{10}(1-q_{10}-gq_{20}) + \tag{5.A1.1}$$
$$q_{11}(-q_{11}-gq_{21}+v) - (q_{10}+q_{11})w_{10} + (1+g)\frac{(q_{10}+q_{11}+q_{20}+q_{21})^2}{4},$$

$$\underset{q_{21}}{Max}\, \Pi_2\left(q_{10},q_{20},q_{11},q_{21},w_{10},w_{20}\right) = (1-gq_{10}-q_{20})q_{20} - c\frac{(q_{20}-q_{21})^2}{2} + \tag{5.A1.2}$$
$$q_{21}(v-gq_{11}-q_{21}) - (q_{20}+q_{21})w_{20}$$

From the *foc* accrues reaction functions of the second period;

$$RF_{11}\left(q_{21}\right) = \frac{2(v+c)+(1+2c+g)q_{10}+(1+g)q_{20}+(1-g)q_{21} - 2w_{10}}{3+2c-g}\ , \tag{5.A1.3}$$

$$RF_{21}\left(q_{11}\right) = \frac{v-gq_{11}+cq_{20}-w_{20}}{2+c}\ . \tag{5.A1.4}$$

I solve the system of the second period RFs to get the optimal q_{11}^{*} and q_{21}^{*} -rules in the second period;

$$q_{11}^{*} = (2(1+g)q_{10}+2c^2(1+q_{10})+2q_{20}+2gq_{20}+5v-gv+c((5+g)q_{10}+2(2+q_{20}+v-w_{10}))- \tag{5.A1.5}$$
$$4w_{10}+(g-1)w_{20})/((2+c-g)(3+2c+g)),$$

$$q_{21}^{*} = -((-2c^2q_{20}-3v+g(q_{10}+gq_{10}+q_{20}+gq_{20}+3v-2w_{10}-w_{20})+3w_{20}+c(-3q_{20}+ \tag{5.A1.6}$$
$$g(2+2q_{10}+q_{20})-2v+2w_{20}))/((2+c-g)(3+2c+g))).$$

Substituting the later into (5.A1.1) and (5.A1.2) accrues EV1 and Π2 that depend on products of the first period and wages;

$$EV_1(q_{10},q_{20},w_{10},w_{20})=-q_{10}\ (-1 + q_{10} + g\ q_{20}) + ((1 + g)\ ((2 + g)\ q_{10} + (2 + g)\ q_{20} + c\ (1 + 2\ q_{10} + 2\ q_{20}) + 2\ v - w_{10} - w_{20})^2)/(3 + 2\ c + g)^2 - (\ w_{10}\ (8\ q_{10} + c^2\ (2 + 4\ q_{10}) + 2\ q_{20} + 5\ v + 2\ c\ (2 + 6\ q_{10} + q_{20} + v - w_{10}) - 4\ w_{10} - w_{20} + g\ (q_{10} - g\ q_{10} + 2\ q_{20} - v + w_{20})))/((2 + c - g)\ (3 + 2\ c + g)) - (1/((2 + c - g)^2\ (3 + 2\ c + g)^2))(2\ (1 + g)\ q_{10} + 2\ c2\ (1 + q_{10}) + 2\ q_{20} + 2\ g\ q_{20} + 5\ v - g\ v + c\ ((5 + g)\ q_{10} + 2\ (2 + q_{20} + v - w_{10})) - 4\ w_{10} + (-1 + g)\ w_{20})\ (-(1 + g)\ (-2 + g^2)\ q_{10} + 2\ q_{20} + 2\ c^2\ (1 + q_{10} + g\ q_{20} - v) - v - 4\ w_{10} + c\ (4 + 5\ q_{10} + 2\ q_{20} - 5\ v - 2\ w_{10} + g\ (q_{10} - g\ (2 + 2\ q_{10} + q_{20}) + 3\ (q_{20} + v) - 2\ w_{20})) - w_{20} + g\ (-(-2 + g + g^2)\ q_{20} + 3\ v - 2\ g\ v + 2\ g\ w_{10} + (-2 + g)\ w_{20})) - c\ (q_{10} - (2\ (1 + g)\ q_{10} + 2\ c^2\ (1 + q_{10}) + 2\ q_{20} + 2\ g\ q_{20} + 5\ v - g\ v + c\ ((5 + g)\ q_{10} + 2\ (2 + q_{20} + v - w_{10})) - 4\ w_{10} + (-1 + g)\ w_{20})/((2 + c - g)\ (3 + 2\ c + g)) + (2\ c^2 + (-1 + g)\ (4 + g)\ q_{10} + 2\ q_{20} + 2\ g\ q_{20} + 5\ v - g\ v + 2\ c\ (2 + (-1 + g)\ q_{10} + q_{20} + v - w_{10}) - 4\ w_{10} + (-1 + g)\ w_{20})^2/(2\ (2 + c - g)^2\ (3 + 2\ c + g)^2)), \tag{5.A1.7}$$

$$\Pi_2(q_{10},q_{20},w_{10},w_{20})=-q_{20}\ (-1 + g\ q_{10} + q_{20}) - (\ c\ (g\ (q_{10} + g\ q_{10} + 3\ v - 2\ w_{10} - w_{20}) + 3\ (2\ q_{20} - v + w_{20}) + 2\ c\ (g + g\ q_{10} + 2\ q_{20} - v + w_{20}))^2)/(\ 2\ (2 + c - g)^2\ (3 + 2\ c + g)^2) + (\ w_{20}\ (-4\ c^2\ q_{20} - 3\ (2\ q_{20} + v - w_{20}) + g\ ((1 + g)\ q_{10} + 2\ (1 + g)\ q_{20} + 3\ v - 2\ w_{10} - w_{20}) + 2\ c\ (-5\ q_{20} + g\ (1 + q_{10} + q_{20}) - v + w_{20})))/((2 + c - g)\ (3 + 2\ c + g)) - (1/((2 + c - g)^2\ (3 + 2\ c + g)^2))(2\ c^2\ (g + g\ q_{10} + q_{20} - v) + g\ (q_{10} + q_{20} + 3\ v - 2\ w_{10}) +c\ (3\ q_{20} - 5\ v + g\ (2 + (3 + g)\ q_{10} + q_{20} + 3\ v - 2 \tag{5.A1.8}$$

$w_{10}) - 2 w_{20}) + g^2 (q_{10} + q_{20} + w_{20}) - 3 (v + w_{20})) (2 c^2 q_{20} + 3 v - g (q_{10} + g q_{10} + q_{20} + g q_{20} +$
$3 v - 2 w_{10} - w_{20}) - 3 w_{20} - c (-3 q_{20} + g (2 + 2 q_{10} + q_{20}) - 2 v + 2 w_{20}))$.

First period/ third stage public firm maximizes $EV_1=\Pi_1+CS$ as for q_{10} and private firms maximazizes its profits as for q_{20}, respectively. Reaction functions of the first period are

$$RF_{10}\left(q_{20}\right) = A1 + B1q_{20} + C1w_{10} + D1w_{20}, \tag{5.A1.9}$$

note that $A1>0$, $B1>0$, $C1<0$, and $D1<0$, where

$$A1 = -(((-6+g+g^2)^2 + (1+g)(24+g(-20+g+3g^2))v + 4c^4(g+v) + 2c^3(3+7g+14v) +$$
$$c^2(33+69v+g(9+g+g^2-2v-7gv)) + c(60+70v+g(-12-v+g(-9-20v+g(4+g+3v))))) / E1 > 0,$$

where

$$E1 = (-4(1+c)(2+c)^2(3+2c) + 4(1+c)(2+c)(5+2c(3+c))g - (-2+c(7+2c(5+2c)))g^2 -$$
$$2(2+c)(3+c)g^3 + (4+5c)g^4 + 2g^5),$$

$$B1 = (-24-8c^4 - c^2(88+(-30-14g)g) + 2c^3(-22+2g(2+g)) + g(20-g(-8+g(3+g(4+g)))) -$$
$$c(76+g(-40+g(-19+g(4+3g))))) / E1 > 0,$$

$$C1 = (48+8c^4+2c^3(26-2g) - c^2(-124+g(18+6g)) - c(-128+g(25+(20-5g)g)) + g(-10-$$
$$g(19+(-6-3g)g))) / E1 < 0,$$

$$D1 = (12+4c^3 - c^2(-18+g(2+4g)) - c(-26+g(2+12g)) + g(2-g(11-g^2))) / E1 < 0 .$$

$$RF_{20}\left(q_{10}\right) = A2 + B2q_{10} + C2w_{20} + D2w_{10}, \tag{5.A1.10}$$

note that $A2>0$, $B2<0$, $C2<0$, and $D2>0$, note also that $A1 > A2$ and $C1<C2$ where

$$A2 = ((-6+g+g^2)^2 + 6g(-1+g^2)v + 4c^4(1-g+v) + 2c^3(g(-9+g-4v) + 2(7+5v)) + c(84+18v+g(-26+$$
$$2g(-4+v) - 31v + 3g^2(2+v))) + c^2(73+33v+g(-30(1+v)+g(3+2g+v)))) / E2 > 0,$$

$$E2 = (4(2+c)(3+2c)(3+5c+2c^2-(1+c)g-g^2)),$$

$$B2 = (-8c^4g + 2c^3g(-22+2g) + c^2g(-92+g(18+6g)) + cg(-90+g(27+(20-g)g)) - g(36+g(-14+$$
$$g(-15+g^2)))) / E2 < 0,$$

$$C2 = (-36-8c^4+2c^3(-24+4g) + c^2(-106+g(34+4g)) + c(-102+g(45+(16-3g)g)) - g(-18+$$
$$g(-15+g(4+g)))) / E2 < 0,$$

$$D2 = (4c^3g + c^2(14-2g)g - g^2(4+4g) + cg(12+(-6-2g)g)) / E2 > 0,$$

We solve the system of the first period RFs to get the optimal q_{10}^{*} and q_{20}^{*} -rules in the first period;

$$q_{10}^{*} = (-16\, c^{6} (1 + g + 3 v - 4 w_{10} - 2 w_{20}) - 288 (2 + v - 2 w_{10} - w_{20}) - 8 c^{5} (4 (6 + 13 v - 18 w_{10} - 9 w_{20}) + g (10 + g + g^{2} - 5 v - g v + 6 w_{10} + 2 (3 + g) w_{20})) - 4 c^{4} (229 + 367 v - 532 w_{10} - 266 w_{20} + g (14 - 66 v + 86 w_{10} + 82 w_{20} + g (3 - 18 v + 8 w_{10} + 30 w_{20} + g (3 v - 2 (-5 + w_{10} + w_{20}))))) + 2 c^{3} (2 (-561 - 673 v + 1032 w_{10} + 516 w_{20}) + g (197 + 325 v - 480 w_{10} - 436 w_{20} + g (65 + 149 v - 118 w_{10} - 196 w_{20} + g (-54 - 41 v + 34 w_{10} + 32 w_{20} + g (-1 + 3 g - 3 v + 8 w_{20}))))) + c^{2} (-8 (374 + 337 v - 554 w_{10} - 277 w_{20}) + g (4 (262 + 179 v - 325 w_{10} - 281 w_{20}) + g (467 + 645 v - 632 w_{10} - 680 w_{20} + g (-209 - 211 v + 202 w_{10} + 178 w_{20} + g (-25 - 29 v + 8 w_{10} + g (21 + 2 g + 7 v - 6 w_{10} - 2 w_{20}) + 68 w_{20}))))) + c (-48 (43 + 29 v - 52 w_{10} - 26 w_{20}) + g (4 (255 + 79 v - 213 w_{10} - 175 w_{20}) + g (570 + 684 v - 728 w_{10} - 606 w_{20} + g (-224 - 235 v + 248 w_{10} + 203 w_{20} + g (-74 - 55 v + 34 w_{10} + g (20 + 23 v - 20 w_{10} + g (8 + 3 v - 2 w_{10} - 5 w_{20}) - 7 w_{20}) + 107 w_{20}))))) + g (24 (15 + v - 9 w_{10} - 7 w_{20}) + g (4 (61 + 69 v - 76 w_{10} - 54 w_{20}) + g (-94 - 90 v + 104 w_{10} + 80 w_{20} + g (-53 - 42 v + 36 w_{10} + 59 w_{20} + g (1 + 18 v - 16 w_{10} - 3 w_{20} + g (5 + g + 6 v - 4 w_{10} - (7 + g) w_{20})))))))/Z, \quad (5.A2.11)$$

$$q_{20}^{*} = (288 (-1 + w_{20}) + 16 c^{6} (-1 - v + 2 g (1 + v - w_{10} - w_{20}) + 2 w_{20}) + 8 c^{5} (-4 (5 + 4 v - 9 w_{20}) + g (36 (1 + v - w_{10}) - 38 w_{20} + g (-3 + g - 4 v + 4 w_{10} + 4 w_{20}))) + 4 c^{4} (-165 - 101 v + 266 w_{20} + g (272 + 264 v - 266 w_{10} - 298 w_{20} + g (-39 - 53 v + 56 w_{10} + 50 w_{20} + g (5 + g - 6 v + 6 w_{10} + 6 w_{20})))) - 2 c^{3} (718 + 314 v - 1032 w_{20} + g (-2 (558 + 500 v - 516 w_{10} - 615 w_{20}) + g (201 + 268 v - 312 w_{10} - 232 w_{20} + g (41 + 84 v - 82 w_{10} - 88 w_{20} + g (-23 + 3 g - 14 v + 14 w_{10} + 14 w_{20}))))) + c^{2} (-8 (217 + 60 v - 277 w_{20}) + g (2654 + 2034 v - 2216 w_{10} - 2808 w_{20} - g (522 + 628 v - 864 w_{10} - 468 w_{20} + g (367 + 438 v - 416 w_{10} - 472 w_{20} + g (-147 - 120 v + 120 w_{10} + 116 w_{20} + g (11 + 5 g - 8 v + 8 w_{10} + 8 w_{20})))))) + c (-48 (23 + 3 v - 26 w_{20}) + g (4 (438 + 257 v - 312 w_{10} - 419 w_{20}) + g (-350 - 316 v + 590 w_{10} + 160 w_{20} - g (491 + 499 v - 465 w_{10} - 555 w_{20} + g (-172 - 165 v + 167 w_{10} + 153 w_{20} + g (-32 + g^{2} - 31 v + 33 w_{10} + 9 g (2 + v - w_{10} - w_{20}) + 39 w_{20})))))) + g (24 (21 + 8 v - 12 w_{10} - 17 w_{20}) + g (-4 (25 + 10 v - 39 w_{10} + 4 w_{20}) + g (-226 - 208 v + 194 w_{10} + 240 w_{20} + g (63 + 69 v - 75 w_{10} - 57 w_{20} + g (41 + 35 v - 33 w_{10} - 43 w_{20} + g (-7 - 13 v + 11 w_{10} + 9 w_{20} + 3 g (-1 - v + w_{10} + w_{20}))))))))/Z, \quad (5.A2.12)$$

where $Z=(-16 (1 + c)^{2} (2 + c)^{2} (3 + 2 c)^{2} + 8 (1 + c)^{2} (2 + c) (3 + 2 c) (9 + 2 c (4 + c)) g + 4 (2 + c) (26 + c (46 + c (25 + 4 c))) g^{2} + 2 (-72 + c (-121 + c (-27 + 8 c (7 + c (5 + c))))) g^{3} + (12 + c (39 + 4 c (6 + c))) g^{4} - (7 + c (53 + 2 c (27 + 8 c))) g^{5} - (9 + c (11 + 4 c)) g^{6} + 3 (1 + c) g^{7} + g^{8})$.

Substituting the later into (5.A1.7) and (5.A1.8) accrues EV_1 and Π_2 that depend on wages only;

$$EV_{1} (w_{10},w_{20}) = (1024\, c^{13} (-2 + g)^{2} + 1024\, c^{12} (69 + g^{4} + 5 v^{2} - 12 w_{10} + 8 w_{10}^{2} + 2 g^{3} (v - w_{10} - w_{20}) (2 + v - w_{10} - w_{20}) + 8 (-1 + w_{10}) w_{20} + 4 w_{20}^{2} - 2 v (-7 + 6 w_{10} + 4 w_{20}) - 2 g (36 + 3 v^{2} - 7 w_{10} + v (8 - 7 w_{10} - 5 w_{20}) - 5 w_{20} + 2 (w_{10} + w_{20}) (2 w_{10} + w_{20})) + g^{2} (17 + 2 (-1 + w_{10}) w_{10} - 2 (-1 + w_{20}) w_{20} + v (1 - 2 w_{10} + 2 w_{20}))) - 32 c^{8} (10 g9 + g^{8} (38 - 30 v + 44 w_{10} + 44 w_{20}) + 2 g (416043 + 277415 v^{2} + 72 w_{10} (-8411 + 4842 w_{10}) + v (650110 - 608592 w_{10} - 448668 w_{20}) - 445668 w_{20} + 516936 w_{10} w_{20} + 188700 w_{20}^{2}) + g^{7} (421 + 69 v^{2} + 28 w_{10}^{2} - 668 w_{20} + 60 w_{20}^{2} + 8 w_{10} (-74 + 15 w_{20}) - 8 v (-79 + 13 w_{10} + 17 w_{20})) - g^{6} (99 + 2105 v^{2} + 2316 w_{10}^{2} + 4 w_{20} (-800 + 599 w_{20}) - 4 v (-660 + 1097 w_{10} + 1121 w_{20}) + 4 w_{10} (-649 + 1168 w_{20})) + 2 g^{5} (-4607 + 944 v^{2} + 288 w_{10}^{2} + 4 w_{10} (461 + 776 w_{20}) + 2 w_{20} (93 + 1363 w_{20}) - v (1515 + 1270 w_{10} + 3584 w_{20})) + 4 g^{4} (11223 + 14718 v^{2} + 15958 w_{10}^{2} + v (25929 - 30765 w_{10} - 28781 w_{20}) + 4 w_{10} (-7133 + 7422 w_{20}) + w_{20} (-27966 + 13657 w_{20})) + g^{2} (-$$

$175881 - 124549 v^2 - 255816 w_{10}^2 + 4 w_{10} (93887 - 60784 w_{20}) + 64532 w_{20} + 54252 w_{20}^2 +$
$v (-312386 + 381788 w_{10} + 70100 w_{20})) - g^3 (118939 + 139243 v^2 + 73576 w_{10}^2 + v$
$(249374 - 221096 w_{10} - 348884 w_{20}) + 4 w_{20} (-83926 + 49961 w_{20}) + 4 w_{10} (-52191 +$
$74962 w_{20})) + 6 (-109935 - 67319 v^2 + 102616 w_{20} + 2 v (-80237 + 76962 w_{10} + 51308 w_{20})$
$- 51308 (w_{10} (-3 + 2 w_{10}) + 2 w_{10} w_{20} + w_{20}^2))) + 2 c^2 (2 g^{14} (7 + 5 v - 6 w_{10} - 6 w_{20}) + g^{13}$
$(312 + 44 v^2 + 4 w_{10} (-72 + 7 w_{10}) + v (275 - 74 w_{10} - 75 w_{20}) - 277 w_{20} + 58 w_{10} w_{20} + 29$
$w_{20}^2) + g^{10} (33147 + 22085 v^2 + 68 w_{10} (-832 + 363 w_{10}) + v (52576 - 46388 w_{10} - 54540$
$w_{20}) - 66024 w_{20} + 58692 w_{10} w_{20} + 31444 w_{20}^2) - g^{12} (1262 + 569 v^2 + 768 w_{10}^2 - 2 v (-758$
$+ 677 w_{10} + 813 w_{20}) + 2 w_{10} (-957 + 994 w_{20}) + w_{20} (-2404 + 1023 w_{20})) - g^{11} (6119 +$
$3530 v^2 + 2540 w_{10}^2 + v (8617 - 6212 w_{10} - 5315 w_{20}) + w_{20} (-6835 + 1248 w_{20}) + w_{10} (-$
$8154 + 4414 w_{20})) + g^9 (32878 + 42384 v^2 + 42664 w_{10}^2 + v (75381 - 85058 w_{10} - 41097$
$w_{20}) + 6 w_{10} (-12814 + 6789 w_{20}) - w_{20} (24219 + 7183 w_{20})) - 2304 g (5067 + 6298 v^2 +$
$6906 w_{10}^2 + v (9250 - 11935 w_{10} - 8873 w_{20}) + 4 w_{20} (-2143 + 961 w_{20}) + w_{10} (-11634 +$
$9757 w_{20})) + 16 g^2 (96783 + 209361 v^2 + 174238 w_{10}^2 + v (42428 - 346138 w_{10} - 36004$
$w_{20}) - 2 w_{20} (5224 + 59369 w_{20}) + w_{10} (-291882 + 283928 w_{20})) + g^7 (115721 - 9790 v^2 -$
$222608 w_{10}^2 + v (54925 + 223808 w_{10} - 390475 w_{20}) + 2 w_{10} (58577 + 109731 w_{20}) + w_{20} (-$
$510079 + 397994 w_{20})) - 8 g^4 (501995 + 627488 v^2 + 749910 w_{10}^2 + 121 w_{20} (-7232 + 2689$
$w_{20}) - 2 v (-575299 + 700385 w_{10} + 482390 w_{20}) + 2 w_{10} (-645276 + 590155 w_{20})) - g^8$
$(357196 + 318675 v^2 + 328356 w_{10}^2 + v (671820 - 641146 w_{10} - 684090 w_{20}) + w_{20} (-$
$706804 + 349683 w_{20}) + w_{10} (-679402 + 684428 w_{20})) - 4 g^5 (482850 + 444169 v^2 +$
$136546 w_{10}^2 + v (954347 - 609766 w_{10} - 1301931 w_{20}) + 2 w_{10} (-320141 + 505452 w_{20}) +$
$w_{20} (-1277011 + 796562 w_{20})) + g^6 (1800025 + 1957767 v^2 + 2109864 w_{10}^2 + v (3894302 -$
$4105600 w_{10} - 3716840 w_{20}) + 2 w_{20} (-1735970 + 824619 w_{20}) + 4 w_{10} (-985278 + 956831$
$w_{20})) + 16 g^3 (482068 + 525868 v^2 + 482290 w_{10}^2 + v (1103038 - 1020121 w_{10} - 1131749$
$w_{20}) + 3 w_{20} (-358721 + 193476 w_{20}) + w_{10} (-989499 + 1047296 w_{20})) + 27648 (256 + 296$
$v^2 + v (479 - 615 w_{10} - 410 w_{20}) - 410 w_{20} + 205 (w_{10} (-3 + 2 w_{10}) + 2 w_{10} w_{20} + w_{20}^2))) +$
$128 c^{10} (g^7 + g^6 (5 - 6 v + 8 w_{10} + 8 w_{20}) + g^4 (147 - 786 v - 481 v^2 + 856 w_{10} + 968 v w_{10} -$
$488 w_{10}^2 + 16 (59 + 60 v - 60 w_{10}) w_{20} - 476 w_{20}^2) - g^5 (-113 + v^2 + 8 w_{10}^2 + 4 w_{20} (24 +$
$w_{20}) - 2 v (45 + 4 w_{10} + 2 w_{20}) + w10 (84 + 8 w_{20})) + g^3 (905 + 2087 v^2 + 1444 w_{10}^2 + v$
$(3718 - 3596 w_{10} - 4788 w_{20}) + 56 w20 (-82 + 47 w_{20}) + 40 w_{10} (-85 + 108 w_{20})) + g^2 (5377$
$+ 1081 v^2 + 3324 w_{10}^2 + 64 (5 - 22 w_{20}) w_{20} + 4 v (853 - 1108 w_{10} + 76 w_{20}) + 16 w_{10} (-276$
$+ 137 w_{20})) - 2 g (12017 + 3855 v^2 + 5000 w_{10}^2 + v (9754 - 8746 w_{10} - 6370 w_{20}) - 6362 w_{20}$
$+ 2624 w_{20}^2 + w_{10} (-8738 + 7484 w_{20})) + 4 (5241 + 1509 v^2 + v (3926 - 3540 w_{10} - 2360$
$w_{20}) - 2360 w_{20} + 1180 (w_{10} (-3 + 2 w_{10}) + 2 w_{10} w_{20} + w_{20}^2))) + 256 c^{11} (g^6 + 4 g^5 (1 + v -$
$w_{10} - w_{20}) - 2 g (1191 + 225 v^2 + 74 w_{10} (-7 + 4 w_{10}) + v (586 - 518 w_{10} - 374 w_{20}) - 374 w_{20}$
$+ 444 w_{10} w_{20} + 152 w_{20}^2) + 2 g^4 (19 - 8 v^2 + 14 w_{10} + 16 w2_0 - 8 (w_{10} + w_{20})^2 + v (-13 + 16$
$w_{10} + 16 w_{20})) + g^2 (35 v2 + v (146 - 212 w_{10} + 76 w_{20}) + 4 (136 + w_{10} (-53 + 44 w_{10}) + 19$
$w_{20} + 18 w_{10} w_{20} - 28 w_{20}^2)) + 4 (545 + 91 v^2 - 144 w_{20} - 6 v (-41 + 36 w_{10} + 24 w_{20}) + 72$
$(w_{10} (-3 + 2 w_{10}) + 2 w_{10} w_{20} + w_{20}^2)) + 2 g^3 (67 v^2 + v (125 - 124 w_{10} - 144 w_{20}) + 2 (7 + 4$
$w_{10} (-15 + 7 w_{10}) - 70 w_{20} + 68 w_{10} w_{20} + 38 w_{20}^2))) + 2 (g^{15} (8 + 2 v^2 + 4 (-3 + w_{10}) w_{10} + v$
$(9 - 6 w1_0 - 7 w_{20}) - 13 w_{20} + 10 w_{10} w_{20} + 5 w_{20}^2) - g^8 (37979 + 45833 v^2 + 6 w_{10} (-14423 +$
$7556 w_{10}) + v (87862 - 90738 w_{10} - 88790 w_{20}) - 77282 w_{20} + 86604 w_{10} w_{20} + 39734 w_{20}^2)$
$+ 82944 (3 + 3 v^2 + 4 w_{10}^2 + v (4 - 6 w_{10} - 4 w_{20}) + 2 (-2 + w_{20}) w_{20} + w_{10} (-6 + 4 w_{20})) -$
$13824 g (33 + 35 v^2 + 36 w_{10}^2 + v (38 - 62 w_{10} - 46 w_{20}) + 4 w_{20} (-11 + 5 w_{20}) + 10 w_{10} (-6$
$+ 5 w_{20})) - g^{11} (-1160 + 1096 v^2 + 756 w_{10}^2 + v (173 - 1854 w_{10} - 511 w_{20}) + (1895 - 737$
$w_{20}) w_{20} + 18 w_{10} (14 + 5 w_{20})) + g^{13} (81 v2 + 8 w_{10} (14 + 3 w_{10}) - 54 w_{10} w_{20} - 72 w_{20}^2 + 71$
$(-2 + 3 w_{20}) - v (41 + 106 w_{10} + 15 w_{20})) + g^{14} (41 + 36 v^2 + 40 w_{10}^2 + v (84 - 78 w_{10} - 78$
$w_{20}) + w20 (-80 + 37 w_{20}) + w_{10} (-86 + 84 w_{20})) - g^{12} (869 + 665 v^2 + 720 w_{10}^2 + 2 w_{20} (-$
$845 + 408 w_{20}) - 2 v (-827 + 695 w_{10} + 797 w_{20}) + 2 w_{10} (-851 + 826 w_{20})) + 192 g^3 (1863 +$
$2136 v^2 + 2166 w_{10}^2 + v (4270 - 4291 w_{10} - 4251 w_{20}) + w_{20} (-3931 + 2079 w_{20}) + w_{10} (-$
$4065 + 4024 w_{20})) + g^9 (-6838 + 3749 v^2 + 8 w_{10} (175 + 666 w_{10}) + 14843 w_{20} - 2962 w_{10}$

$$
w_{20} - 8022\, w_{20}^2 + v\, (-2567 - 9094\, w_{10} + 4163\, w_{20})) - 16\, g^4\, (10953 + 14546\, v^2 + 8\, w_{10}\, (-3333 + 2029\, w_{10}) - 16506\, w_{20} + 25248\, w_{10}\, w_{20} + 5283\, w_{20}^2 - 4\, v\, (-5316 + 7762\, w_{10} + 4827\, w_{20})) + 4\, g^7\, (8717 + 4412\, v2 - 1282\, w_{10}^2 + v\, (14245 - 3170\, w_{10} - 19899\, w_{20}) + 9\, w_{20}\, (-2579 + 1606\, w_{20}) + 2\, w_{10}\, (-4234 + 7101\, w_{20})) + g^{10}\, (7771 + 7330\, v^2 + 7324\, w_{10}^2 + 3\, w_{20}\, (-5140 + 2607\, w_{20}) + 2\, w_{10}\, (-7901 + 7818\, w_{20}) - 2\, v\, (-7840 + 7241\, w_{10} + 7929\, w_{20})) - 16\, g^5\, (8630 + 9640\, v^2 + 6071\, w_{10}^2 - 4\, v\, (-5381 + 4044\, w_{10} + 6157\, w_{20}) + w_{20}\, (-22781 + 13686\, w_{20}) + w_{10}\, (-16003 + 20037\, w_{20})) + 4\, g^6\, (27339 + 37671\, v^2 + 40393\, w_{10}^2 + v\, (68870 - 79328\, w_{10} - 64884\, w_{20}) + w_{20}\, (-53940 + 25337\, w_{20}) + w_{10}\, (-69608 + 68150\, w_{20})) + 576\, g^2\, (105 + 147\, v^2 - 82\, v\, (4 + w_{10}) + 136\, w_{20} + 116\, v\, w_{20} - 2\, (w_{10}\, (9 + w_{10}) - 52\, w_{10}\, w_{20} + 89\, w_{20}^2))) + 2\, c^4\, (4\, g^{13} + g^{12}\, (274 + 66\, v - 76\, w_{10} - 76\, w_{20}) - 2\, g^8\, (95677 + 95238\, v^2 + 2\, w_{10}\, (-96493 + 51922\, w_{10}) + v\, (189275 - 196896\, w_{10} - 217756\, w_{20}) - 226246\, w_{20} + 226408\, w_{10}\, w_{20} + 121210\, w_{20}^2) + g^6\, (2647675 + 2565387\, v^2 + 32\, w_{10}\, (-162761 + 85444\, w_{10}) + v\, (5066430 - 5303080\, w_{10} - 5220264\, w_{20}) - 5170116\, w_{20} + 5295184\, w_{10}\, w_{20} + 2551700\, w_{20}^2) + g^{10}\, (-1357 + 3743\, v^2 + 4776\, w_{10}^2 + 384\, w_{10}\, (-14 + 31\, w_{20}) + 12\, w_{20}\, (-699 + 520\, w_{20}) - 2\, v\, (-2665 + 4256\, w_{10} + 5004\, w_{20})) + 2\, g^9\, (25527 + 11941\, v^2 - 35708\, w_{10} + 8952\, w_{10}^2 + 38\, (-927 + 490\, w_{10})\, w_{20} + 6362\, w_{20}^2 - 2\, v\, (-18043 + 10662\, w_{10} + 9897\, w_{20})) + g^7\, (-267001 - 151675\, v^2 - 267048\, w_{10}^2 + w_{10}\, (511176 - 93984\, w_{20}) + v\, (-427172 + 411304\, w_{10} + 4520\, w_{20}) + 12\, w_{20}\, (7761 + 14225\, w_{20})) - 32\, g\, (1176919 + 1508171\, v^2 + 1745488\, w_{10}^2 + v\, (2730762 - 3027440\, w_{10} - 2251580\, w_{20}) + 4\, w_{20}\, (-549313 + 242407\, w_{20}) + 8\, w_{10}\, (-371639 + 313697\, w_{20})) + 32\, g^2\, (174800 + 395773\, v^2 + 509165\, w_{10}^2 + v\, (541636 - 877357\, w_{10} - 228398\, w_{20}) - 9\, w_{20}\, (20974 + 13461\, w_{20}) + w_{10}\, (-813685 + 659462\, w_{20})) - 4\, g^5\, (533056 + 523863\, v^2 - 54066\, w_{10}^2 + v\, (958883 - 517968\, w_{10} - 1768011\, w_{20}) + 2\, w_{10}\, (-254104 + 656635\, w_{20}) + w_{20}\, (-1731875 + 1184508\, w_{20})) - 2\, g^4\, (5258987 + 5683455\, v^2 + 6896528\, w_{10}^2 + 4\, w_{20}\, (-2307649 + 947582\, w_{20}) + 4\, w_{10}\, (-3071131 + 2819108\, w_{20}) - 2\, v\, (-5640919 + 6397914\, w_{10} + 4866570\, w_{20})) + 8\, g^3\, (2422123 + 2385531\, v^2 + 1821332\, w_{10}^2 + v\, (4994646 - 4323696\, w_{10} - 5591110\, w_{20}) + 14\, w_{20}\, (-387821 + 216136\, w_{20}) + w_{10}\, (-4258736 + 5008540\, w_{20})) + 64\, (392451 + 465075\, v^2 - 667204\, w_{20} - 2\, v\, (-439586 + 500403\, w_{10} + 333602\, w_{20}) + 333602\, (w_{10}\, (-3 + 2\, w_{10}) + 2\, w_{10}\, w_{20} + w_{20}^2)) - g^{11}\, (299\, v^2 + 208\, w_{10}^2 + 144\, w_{10}\, (-15 + 4\, w_{20}) - 4\, v\, (-459 + 132\, w_{10} + 152\, w_{20}) + 3\, (491 - 772\, w_{20} + 96\, w_{20}^2))) + 4\, c^5\, (16\, g^{12} + 16\, g^{11}\, (5 - 6\, v + 8\, w_{10} + 8\, w_{20}) + 2\, g^9\, (4543 + 1775\, v^2 + 68\, w_{10}\, (-105 + 17\, w_{10}) + v\, (7138 - 3040\, w_{10} - 3334\, w_{20}) - 7906\, w_{20} + 3132\, w_{10}\, w_{20} + 1368\, w_{20}^2) - 2\, g^4\, (2404697 + 2456927\, v^2 + 6\, w_{10}\, (-883347 + 490888\, w_{10}) + v\, (4870296 - 5482246\, w_{10} - 4396724\, w_{20}) - 4226460\, w_{20} + 4949712\, w_{10}\, w_{20} + 1819166\, w_{20}^2) + 4\, g^3\, (2234737 + 2121259\, v^2 + 22\, w_{10}\, (-164381 + 64838\, w_{10}) + v\, (4332840 - 3673734\, w_{10} - 5167670\, w_{20}) - 5040610\, w_{20} + 4546252\, w_{10}\, w_{20} + 2867976\, w_{20}^2) + g^{10}\, (-1238 + 231\, v^2 + 316\, w_{10}^2 + 72\, w_{10}\, (7 + 11\, w_{20}) + 4\, w_{20}\, (74 + 99\, w_{20}) - 2\, v\, (143 + 274\, w_{10} + 314\, w_{20})) - 2\, g^8\, (11595 + 17744\, v^2 + 20214\, w_{10}^2 + v\, (30626 - 37564\, w_{10} - 41312\, w_{20}) + 152\, w_{10}\, (-200 + 291\, w_{20}) + 4\, w_{20}\, (-9537 + 5902\, w_{20})) - 4\, g^7\, (33496 + 13763\, v2 + 16410\, w_{10}^2 + v\, (45138 - 30065\, w_{10} - 12424\, w_{20}) - 2\, w_{20}\, (15396 + 1681\, w_{20}) + w_{10}\, (-46679 + 15230\, w_{20})) + 2\, g^5\, (-218225 - 320031\, v^2 + 84900\, w_{10}^2 + w_{10}\, (204962 - 844016\, w_{20}) + 2\, (533899 - 398444\, w_{20})\, w_{20} + 2\, v\, (-247519 + 134245\, w_{10} + 572301\, w_{20})) - 16\, g\, (1327071 + 1592953\, v^2 + 1887488\, w_{10}^2 + 4\, w_{20}\, (-597533 + 261296\, w_{20}) - 4\, v\, (-781959 + 819887\, w_{10} + 609311\, w_{20}) + 4\, w_{10}\, (-808109 + 684252\, w_{20})) + 16\, g^2\, (218277 + 425073\, v^2 + 621499\, w_{10}^2 + v\, (739389 - 1030273\, w_{10} - 284546\, w_{20}) - w_{20}\, (247414 + 109115\, w_{20}) + w_{10}\, (-976829 + 750190\, w_{20})) + 16\, (924019 + 1025975\, v^2 - 1496160\, w_{20} - 30\, v\, (-69101 + 74808\, w_{10} + 49872\, w_{20}) + 748080\, (w_{10}\, (-3 + 2\, w_{10}) + 2\, w10\, w_{20} + w_{20}^2)) + g^6\, (808727\, v^2 - 2\, v\, (-769617 + 830856\, w_{10} + 841778\, w_{20}) + 2\, (402887 + 26\, w_{10}\, (-30663 + 16510\, w_{10}) - 834756\, w_{20} + 851204\, w_{10}\, w_{20} + 426428\, w_{20}^2))) + c^3\, (24\, g^{13}\, (9 + 5\, v - 6\, w_{10} - 6\, w_{20}) - 256\, g\, (252391\, v^2 + 6\, (32579 + w_{10}\, (-80369 + 47445\, w_{10})) + v\, (415288 - 492853\, w_{10} - 366575\, w_{20}) + 7\, (-50848 + 57961\, w_{10})\, w_{20} + 158392\, w_{20}^2) - 2\, g^{11}\, (5533 + 1673\, v^2 + 1184\, w_{10}^2 + v\, (6022 - 2954\, w_{10} - 2996\, w_{20}) + 4\, w_{20}\, (-1634 + 281\, w_{20}) + w_{10}\, (-6374 + 2688\, w_{20})) + g^{10}\, (27833 + 25281\, v^2 + 30084\, w_{10}^2 + v\, (54874 - 55112\, w_{10} - 65388
$$

$w_{20}) + 56\ w_{10}\ (-1064 + 1317\ w_{20}) + 4\ w_{20}\ (-18913 + 9879\ w_{20})) + 8\ g^9\ (16360 + 10698\ v^2 +$
$9304\ w_{10}^2 + v\ (25502 - 20157\ w_{10} - 14683\ w_{20}) + 7\ w_{20}\ (-2731 + 360\ w_{20}) + w_{10}\ (-25435 +$
$13998\ w_{20})) + 16\ g^2\ (430755 + 1010115\ v^2 + 1095956\ w_{10}^2 + 32\ w_{10}\ (-55765 + 48473\ w_{20})$
$- 4\ w_{20}\ (77389 + 102385\ w_{20}) - 2\ v\ (-443293 + 995340\ w_{10} + 211250\ w_{20})) - 2\ g^7\ (75001 +$
$107461\ v^2 + 324888\ w_{10}^2 + 52\ (5486 - 6899\ w_{20})\ w_{20} - 34\ w_{10}\ (11827 + 2048\ w_{20}) + v$
$(236206 - 420482\ w_{10} + 235532\ w_{20})) + 16\ g^3\ (1858201 + 1928087\ v^2 + 44\ w_{10}\ (-80832 +$
$37123\ w_{10}) - 4167708\ w_{20} + 3946932\ w_{10}\ w_{20} + 2280972\ w_{20}^2 - 8\ v\ (-508803 + 453945\ w_{10}$
$+ 541285\ w_{20})) - g^8\ (715259 + 628659\ v^2 + 663520\ w_{10}^2 + 4\ w_{20}\ (-371731 + 187664\ w_{20}) +$
$4\ w_{10}\ (-335853 + 355856\ w_{20}) - 2\ v\ (-658519 + 639282\ w_{10} + 699694\ w_{20})) - 16\ g^4\ (994669$
$+ 1157763\ v^2 + 1403326\ w_{10}^2 + 16\ w_{10}\ (-153867 + 140290\ w_{20}) + 2\ w_{20}\ (-876593 + 343169$
$w_{20}) - 2\ v\ (-1120784 + 1303521\ w_{10} + 942682\ w_{20})) + g^6\ (5529775 + 5545239\ v^2 +$
$5947076\ w_{10}^2 + 4\ w_{20}\ (-2652331 + 1278887\ w_{20}) + 8\ w_{10}\ (-1415751 + 1396331\ w_{20}) - 2\ v\ (-$
$5555643 + 5775224\ w_{10} + 5470154\ w_{20})) + 1536\ (20751 + 24819\ v^2 + v\ (43586 - 52500$
$w_{10} - 35000\ w_{20}) - 35000\ w_{20} + 17500\ (w_{10}\ (-3 + 2\ w_{10}) + 2\ w_{10}\ w_{20} + w_{20}^2)) + g^{12}\ (467 -$
$229\ v^2 + 644\ w_{20} + 2\ v\ (-97 + 282\ w_{10} + 338\ w_{20}) - 4\ (w_{10}\ (-97 + 82\ w_{10}) + 220\ w_{10}\ w_{20} +$
$110\ w_{20}^2)) - 4\ g^5\ (1327383 + 1188713\ v^2 - 3711926\ w_{20} - 2\ v\ (-1203152 + 701994\ w_{10} +$
$1872721\ w_{20}) + 4\ (w_{10}\ (-368109 + 30677\ w_{10}) + 709343\ w_{10}\ w_{20} + 600501\ w_{20}^2))) + c\ (4\ g^6$
$(334765 + 406935\ v^2 + w_{10}\ (-792142 + 438831\ w_{10}) + v\ (785648 - 856854\ w_{10} - 738344$
$w_{20}) - 654900\ w_{20} + 767506\ w_{10}\ w_{20} + 308555\ w_{20}^2) + g^{14}\ (159 + 43\ v^2 + 56\ w_{10}^2 + 68\ w_{10}$
$(-3 + 2\ w_{20}) + 4\ w_{20}\ (-54 + 17\ w_{20}) - 2\ v\ (-85 + 50\ w_{10} + 56\ w_{20})) + 2\ g^{13}\ (87 + 127\ v^2 + 64$
$w_{10}^2 - 2\ w_{20}\ (40 + w_{20}) + 2\ w_{10}\ (-93 + 32\ w_{20}) - 2\ v\ (-134 + 99\ w10 + 72\ w_{20})) - 27648\ g$
$(245 + 284\ v2 + 302\ w_{10}2 + v\ (364 - 521\ w_{1}0 - 387\ w_{20}) + 12\ w_{20}\ (-31 + 14\ w_{20}) + w_{10}\ (-$
$506 + 423\ w_{20})) - g^{12}\ (4499 + 2055\ v^2 + 2524\ w_{10}^2 + v\ (6006 - 4628\ w_{10} - 5524\ w_{20}) + 12$
$w_{20}\ (-639 + 269\ w_{20}) + w_{10}\ (-6748 + 6256\ w_{20})) + 2\ g^9\ (-4517 + 21195\ v^2 + 24736\ w_{10}^2 + v$
$(20252 - 45834\ w_{10} - 6692\ w_{20}) + 2\ (10918 - 8509\ w_{20})\ w_{20} + w_{10}\ (-22494 + 8536\ w_{20})) +$
$192\ g^2\ (4647 + 8403\ v^2 + 4112\ w_{10}^2 - 2\ v\ (3217 + 5098\ w_{10} - 854\ w_{20}) + 4\ (701 - 1718\ w_{20})$
$w_{20} + w_{10}\ (-7404 + 9232\ w_{20})) + 4\ g^7\ (59929 + 18465\ v^2 - 35710\ w_{10}^2 + v\ (73404 + 15921$
$w_{10} - 140431\ w_{20}) + 29\ w_{10}\ (-873 + 3196\ w_{20}) + 5\ w_{20}\ (-35205 + 23416\ w_{20})) + 64\ g^3$
$(76314 + 86271\ v^2 + 83880\ w_{10}^2 + 22\ w_{10}\ (-7449 + 7594\ w_{20}) - 2\ v\ (-88790 + 85457\ w_{10} +$
$89145\ w_{20}) + w_{20}\ (-167212 + 89181\ w_{20})) - 16\ g^4\ (153877 + 202060\ v^2 + 235004\ w_{10}^2 + 3$
$w_{20}\ (-85382 + 29827\ w_{20}) - 4\ v\ (-84694 + 110799\ w_{10} + 72655\ w_{20}) + 4\ w_{10}\ (-98888 +$
$91579\ w_{20})) + g^{10}\ (55661 + 39705\ v^2 + 41964\ w_{10}^2 + v\ (93150 - 80820\ w_{10} - 92552\ w_{20}) + 8$
$w_{20}\ (-13124 + 6245\ w_{20}) + w_{10}\ (-97388 + 95312\ w_{20})) - 16\ g^5\ (97909 + 97996\ v^2 + 47581$
$w_{10}^2 + v\ (217055 - 151026\ w_{10} - 267613\ w_{20}) + 2\ w_{20}\ (-128059 + 78004\ w_{20}) + w_{10}\ (-$
$154781 + 212989\ w_{20})) - g^8\ (367613 + 366457\ v^2 + 370116\ w_{10}^2 + v\ (749906 - 731420\ w_{10}$
$- 751108\ w_{20}) + 4\ w_{20}\ (-181437 + 90379\ w_{20}) + w_{10}\ (-748820 + 741488\ w_{20})) + 27648\ (141$
$+ 153\ v^2 + v\ (226 - 312\ w_{10} - 208\ w_{20}) - 208\ w_{20} + 104\ (w_{10}\ (-3 + 2\ w_{10}) + 2\ w_{10}\ w_{20} +$
$w_{20}^2)) + 4\ g^{11}\ (-454 - 1646\ v^2 - 1182\ w_{10}^2 + 3\ w_{10}\ (607 - 428\ w_{20}) + w_{20}\ (307 + 102\ w_{20}) + v$
$(2863\ w_{10} + 31\ (-76 + 59\ w_{20})))) + 8\ c^6\ (21\ g^{11} + g^{10}\ (-99 - 66\ v + 96\ w10 + 96\ w_{20}) + g^9$
$(381 + 221\ v^2 + 128\ w_{10}^2 + 4\ w20\ (-431 + 54\ w_{20}) + 4\ w_{10}\ (-385 + 108\ w_{20}) - 2\ v\ (-739 +$
$184\ w_{10} + 228\ w_{20})) + g^8\ (49 - 3763\ v2 - 4528\ w_{10}^2 + 40\ w_{10}\ (85 - 246\ w_{20}) + 16\ (326 - 321$
$w_{20})\ w_{20} + v\ (-4162 + 8216\ w_{10} + 8848\ w_{20})) - 2\ g^4\ (765189 + 770840\ v^2 + 901346\ w_{10}^2 + v$
$(1490773 - 1690938\ w_{10} - 1431862\ w_{20}) + 8\ w_{20}\ (-173323 + 78157\ w_{20}) + 8\ w_{10}\ (-203573 +$
$195497\ w_{20})) + 2\ g^2\ (846033 + 1281481\ v2 + 2091456\ w_{10}^2 + v\ (2604234 - 3352432\ w_{10} -$
$901112\ w_{20}) - 128\ w_{20}\ (6302 + 2547\ w_{20}) + 8\ w_{10}\ (-403606 + 294975\ w_{20})) - 32\ g\ (289497$
$+ 305018\ v^2 + 369244\ w_{10}^2 + v\ (641610 - 642664\ w_{10} - 476846\ w_{20}) - 469820\ w_{20} + 203426$
$w_{20}^2 + w_{10}\ (-635638 + 539814\ w_{20})) + 2\ g^3\ (1484691 + 1409519\ v^2 + 829352\ w_{10}^2 + v$
$(2765670 - 2331592\ w_{10} - 3533030\ w_{20}) - 3443506\ w_{20} + 2000672\ w_{20}^2 + 4\ w_{10}\ (-568753 +$
$764785\ w_{20})) + 2\ g^5\ (-68749\ v^2 + v\ (-76707 + 54578\ w_{10} + 255966\ w_{20}) + 2\ (2083 + 10801$
$w_{10}^2 + w_{10}\ (8000 - 95752\ w_{20}) + 8\ (13171 - 11503\ w_{20})\ w_{20})) + 8\ (840801 + 819721\ v^2 -$
$1214128\ w_{20} - 2\ v\ (-879689 + 910596\ w_{10} + 607064\ w_{20}) + 607064\ (w_{10}\ (-3 + 2\ w_{10}) + 2$

$$
\begin{aligned}
& w_{10}\, w_{20} + w_{20}^2)) + g^6 (174325\, v^2 + v (311314 - 357532\, w_{10} - 368220\, w_{20}) + 2 (74739 + \\
& 1686\, w_{10} (-96 + 55\, w_{10}) - 177844\, w_{20} + 186608\, w_{10}\, w_{20} + 95784\, w_{20}^2)) - g^7 (11605\, v^2 - 4\, v \\
& (-11425 + 5395\, w_{10} + 4058\, w_{20}) + 4 (9028 + 2405\, w_{10}^2 + 2\, w_{20} (-5241 + 385\, w_{20}) + w_{10} (- \\
& 10889 + 4010\, w_{20})))) - 64\, c^9 (6\, g^8 + 4\, g^7 (1 + 6\, v - 6\, w_{10} - 6\, w_{20}) + 2\, g (82905 + 39885\, v^2 + \\
& 6\, w_{10} (-14803 + 8492\, w_{10}) + v (97454 - 89050\, w_{10} - 65306\, w_{20}) - 65074\, w_{20} + 75964\, w_{10} \\
& w_{20} + 27208\, w_{20}^2) - g^6 (-65 + 81\, v^2 + 8\, w_{10} - 16\, w_{20} + 92 (w_{10} + w_{20})^2 - 2\, v (-9 + 86\, w_{10} + \\
& 86\, w_{20})) + g^2 (-36311 - 15051\, v^2 - 36232\, w_{10}^2 + w_{10} (51104 - 30000\, w_{20}) + 8\, w_{20} (523 + \\
& 1324\, w_{20}) + v (-41626 + 51576\, w_{10} + 4632\, w_{20})) + 2\, g^4 (1391 + 3378\, v^2 + 3526\, w_{10}^2 + v \\
& (5683 - 6904\, w_{10} - 6696\, w_{20}) - 6526\, w_{20} + 3264\, w_{20}^2 + w_{10} (-6238 + 6768\, w_{20})) + 12 (- \\
& 11483 - 5024\, v^2 + 7760\, w_{20} + 8\, v (-1567 + 1455\, w_{10} + 970\, w_{20}) - 3880 (w_{10} (-3 + 2\, w_{10}) + 2 \\
& w_{10}\, w_{20} + w_{20}^2)) + 2\, g^5 (45\, v^2 - 2\, v (202 + 54\, w_{10} + 85\, w_{20}) + 2 (-346 + 34\, w_{10}^2 + 2\, w_{10} (94 \\
& + 45\, w_{20}) + w_{20} (197 + 68\, w_{20}))) - 2\, g^3 (10148\, v^2 + v (17825 - 16548\, w_{10} - 24558\, w_{20}) + 2 \\
& (3341 + 2962\, w_{10}^2 + 2\, w_{10} (-3878 + 5365\, w_{20}) + w_{20} (-11753 + 6938\, w_{20})))) + 16\, c^7 (5\, g^{10} + \\
& g^9 (-96 + 60\, v - 68\, w_{10} - 68\, w_{20}) - 2\, g^7 (2772 + 3565\, v + 680\, v^2 - 3286\, w_{10} - 1118\, v\, w_{10} + \\
& 392\, w_{10}^2 - 8 (460 + 147\, v - 134\, w_{10})\, w_{20} + 416\, w_{20}^2) + 2\, g^3 (354341 + 357801\, v^2 + 6\, w_{10} (- \\
& 91571 + 31794\, w_{10}) + v (668718 - 572214\, w_{10} - 906904\, w_{20}) - 879188\, w_{20} + 777936\, w_{10} \\
& w_{20} + 519544\, w_{20}^2) - g^8 (-137 + 175\, v^2 + 224\, w_{10}^2 + 48\, w_{10} (7 + 10\, w_{20}) + 12\, w_{20} (21 + 20 \\
& w_{20}) - 4\, v (40 + 99\, w_{10} + 103\, w_{20})) - 4\, g (789041 + 682975\, v^2 + 843096\, w_{10}^2 + 120\, w_{20} (- \\
& 8969 + 3842\, w_{20}) + 12\, w_{10} (-121528 + 103489\, w_{20}) - 4\, v (-380792 + 367431\, w_{10} + 271917 \\
& w_{20})) + 2\, g^2 (314293 + 339809\, v^2 + 617880\, w_{10}^2 + v (774188 - 957690\, w_{10} - 229250\, w_{20}) - \\
& 6\, w_{20} (34843 + 17348\, w_{20}) + w_{10} (-933374 + 646484\, w_{20})) - g^4 (330966 + 354311\, v^2 + \\
& 399964\, w_{10}^2 + v (656710 - 759524\, w_{10} - 678048\, w_{20}) + 4\, w_{20} (-164579 + 77517\, w_{20}) + w_{10} \\
& (-720884 + 718608\, w_{20})) + g^5 (-20274\, v^2 + 2\, v (-3367 + 9400\, w_{10} + 38460\, w_{20}) + 4 (7889 + \\
& 840\, w_{10}^2 + (11111 - 14212\, w_{20})\, w_{20} - 15\, w_{10} (135 + 1006\, w_{20}))) + 2\, g^6 (12394\, v^2 + v \\
& (19843 - 25534\, w_{10} - 26368\, w_{20}) + 2 (3527 + 6669\, w_{10}^2 + 13\, w_{20} (-907 + 537\, w_{20}) + 2\, w_{10} \\
& (-5123 + 6746\, w_{20}))) + 4 (478029\, v^2 - 18\, v (-60217 + 59880\, w_{10} + 39920\, w_{20}) + 5 (119593 \\
& - 143712\, w_{20} + 71856 (w_{10} (-3 + 2\, w_{10}) + 2\, w_{10}\, w_{20} + w_{20}^2)))))/(2\, Z^2),
\end{aligned}
\qquad (5.A1.13)
$$

$$
\begin{aligned}
& \Pi_2 (w_{10},w_{20}) = -(c (-16\, c^5 (-2 + g) (1 + g - v) - 8\, c^4 (32 (-1 + v) + g (-2 (4 + 9\, v + w_{20}) + g (g \\
& + g^2 - v - g\, v + 2 (6 + w_{10} + w_{20})))) + 2\, c^2 (-628 (-1 + v) + g (-230 + 478\, v + 88\, w_{20} + g (-31 \\
& + 29\, v - 120\, w_{10} - 86\, w_{20} + g (-27 + 41\, v - 10\, w_{10} - 24\, w_{20} + g (-24 + g (5 + 3\, g - 3\, v) - 25\, v \\
& + 22\, w_{10} + 22\, w_{20}))))) - 4\, c^3 (202 (-1 + v) + g (7 - 131\, v - 22\, w_{20} + g (46 - 10\, v + 26\, w_{10} + \\
& 22\, w_{20} + g (11 - 10\, v + 2\, w_{20} + g (3\, v - 2 (-3 + w_{10} + w_{20})))))) + (4 + g) (-72 (-1 + v) + g (12 \\
& (-9 + 8\, v + w_{20}) + g (-2 (-29 + 15\, v + 9\, w_{10} + 5\, w_{20}) + g (7 + 15\, v - 8\, w_{10} - 14\, w_{20} + g (g (1 \\
& - 3\, v + g (1 + 3\, v - 2\, w_{10} - 2\, w_{20}) + 2\, w_{20}) + 3 (-5 - 3\, v + 4\, w_{10} + 4\, w_{20})))))) + c (-960 (-1 + \\
& v) + g (-732 + 868\, v + 152\, w_{20} + g (4 (40 + 2\, v - 57\, w_{10} - 33\, w_{20}) + g (51 + 81\, v - 64\, w_{10} - \\
& 102\, w_{20} + g (-81 - 67\, v + 78\, w_{10} + 78\, w_{20} + g (3 - 17\, v + 4\, w_{10} + g (13 + 2\, g + 7\, v - 6\, w_{10} - \\
& 6\, w_{20}) + 10\, w_{20})))))))^2 - 2 (-16 (1 + c)^2 (2 + c)^2 (3 + 2\, c)^2 + 8 (1 + c)^2 (2 + c) (3 + 2\, c) (9 + 2 \\
& c (4 + c))\, g + 4 (2 + c) (26 + c (46 + c (25 + 4\, c)))\, g^2 + 2 (-72 + c (-121 + c (-27 + 8\, c (7 + c \\
& (5 + c)))))\, g^3 + (12 + c (39 + 4\, c (6 + c)))\, g^4 - (7 + c (53 + 2\, c (27 + 8\, c)))\, g^5 - (9 + c (11 + 4 \\
& c))\, g^6 + 3 (1 + c)\, g^7 + g^8)\, w_{20} (288 (1 + v - 2\, w_{20}) - 32\, c^6 (-1 - v + 2\, g (1 + v - w_{10} - w_{20}) + 2 \\
& w_{20}) - 16\, c^5 (-18 (1 + v - 2\, w_{20}) + g (37 + g^2 + 37\, v - 36\, w_{10} - 38\, w_{20} + 4\, g (-1 - v + w_{10} + \\
& w_{20}))) + 8\, c^4 (133 (1 + v - 2\, w_{20}) + g (-280 - 282\, v + 266\, w_{10} + 296\, w_{20} + g (51 - 4\, g + 52\, v \\
& + 5\, g\, v - 54\, w_{10} - 6\, g\, w_{10} - 6 (8 + g)\, w_{20}))) + 4\, c^3 (516 (1 + v - 2\, w_{20}) + g (-1109 - 1131\, v + \\
& 1032\, w_{10} + 1208\, w_{20} + g (247 + 258\, v - 286\, w_{10} - 210\, w_{20} + g (52 + 74\, v - 82\, w_{10} - 86\, w_{20} \\
& + g (-17 + 3\, g - 11\, v + 12\, w_{10} + 12\, w_{20}))))) + 2\, c^2 (1108 (1 + v - 2\, w_{20}) + g (-8 (303 + 314\, v \\
& - 277\, w_{10} - 340\, w_{20}) + g (553 + 599\, v - 744\, w_{10} - 382\, w_{20} + g (394 + 397\, v - 406\, w_{10} - 448 \\
& w_{20} + g (-123 - 95\, v + 98\, w_{10} + 94\, w_{20} + g (6 + 2\, g - 5\, v + 8\, w_{10} + 8\, w_{20})))))) + c (1248 (1 + \\
& v - 2\, w_{20}) + g (-4 (693 + 731\, v - 624\, w_{10} - 800\, w_{20}) + g (540 + 624\, v - 952\, w_{10} - 188\, w_{20} + g
\end{aligned}
$$

$$
(931 + 917 v - 866 w_{10} - 1008 w_{2}0 + g (-263 - 263 v + 256 w_{10} + 228 w_{20} + g (-67 - 45 v +
62 w_{10} + g (23 + 11 v - 12 w_{10} - 12 w_{20}) + 68 w_{20})))))) + g (24 (-27 - 29 v + 24 w_{10} + 32
w_{20}) + g (4 (19 + 26 v - 60 w_{10} + 15 w_{20}) + g (366 + 386 v - 338 w_{10} - 414 w_{20} + g (-73 -
117 v + 110 w_{10} + 80 w_{20} + g (-71 - 49 v + 54 w_{10} + 66 w_{20} + g (9 + 5 g + 17 v + 3 g v - 14
w_{10} - 4 g w_{10} - 4 (3 + g) w_{20}))))))) + 2 (-288 (-1 + w_{20}) - 16 c^{6} (-1 - v + 2 g (1 + v - w_{10} - w_{20})
+ 2 w_{20}) - 8 c^{5} (-4 (5 + 4 v - 9 w_{20}) + g (36 (1 + v - w_{10}) - 38 w_{20} + g (-3 + g - 4 v + 4 w_{10} +
4 w_{20}))) - 4 c^{4} (-165 - 101 v + 266 w_{20} + g (272 + 264 v - 266 w_{10} - 298 w_{20} + g (-39 - 53 v
+ 56 w_{10} + 50 w_{20} + g (5 + g - 6 v + 6 w_{10} + 6 w_{20})))) + 2 c^{3} (718 + 314 v - 1032 w_{20} + g (-2
(558 + 500 v - 516 w_{10} - 615 w_{20}) + g (201 + 268 v - 312 w_{10} - 232 w_{20} + g (41 + 84 v - 82
w_{10} - 88 w_{20} + g (-23 + 3 g - 14 v + 14 w_{10} + 14 w_{20}))))) + g (-24 (21 + 8 v - 12 w_{10} - 17
w_{20}) + g (4 (25 + 10 v - 39 w_{10} + 4 w_{20}) + g (226 + 208 v - 194 w_{10} - 240 w_{20} + g (-63 - 69 v
+ 75 w_{10} + 57 w_{20} + g (-41 - 35 v + 33 w_{10} + g (7 + 13 v - 11 w_{10} + 3 g (1 + v - w_{10} - w_{20}) - 9
w_{20}) + 43 w_{20}))))) + c^{2} (8 (217 + 60 v - 277 w_{20}) + g (-2654 - 2034 v + 2216 w_{10} + 2808 w_{20}
+ g (522 + 628 v - 864 w_{10} - 468 w_{20} + g (367 + 438 v - 416 w_{10} - 472 w_{20} + g (-147 - 120 v
+ 120 w_{10} + 116 w_{20} + g (11 + 5 g - 8 v + 8 w_{10} + 8 w_{20})))))) + c (48 (23 + 3 v - 26 w_{20}) + g
(-4 (438 + 257 v - 312 w_{10} - 419 w_{20}) + g (350 + 316 v - 590 w_{10} - 160 w_{20} + g (491 + 499 v
- 465 w_{10} - 555 w_{20} + g (-172 - 165 v + 167 w_{10} + 153 w_{20} + g (-32 + g^{2} - 31 v + 33 w_{10} + 9
g (2 + v - w_{10} - w_{20}) + 39 w_{20}))))))) (16 c6 (-3 + v + g (1 + g + v - 2 w_{10}) - 2 w_{20}) - 288 (1 +
w_{20}) + 8 c^{5} (4 (-13 + 4 v - 9 w_{20}) + g (2 (13 + 8 v - 18 w_{10} + w_{20}) + g (13 - v + 2 w_{10} + 2 w_{20}
+ g (2 + g - v + 2 w_{20})))) + 4 c^{4} (-367 + 101 v - 266 w_{20} + g (257 + 103 v - 266 w_{10} + 32 w_{20}
+ g (57 - 13 v + 30 w_{10} + g^{2} (9 + 3 v - 2 w_{10} - 2 w_{20}) + 32 w_{20} + 2 g (9 - 6 v + w_{10} + 12
w_{20})))) + 2 c^{3} (2 (-673 + 157 v - 516 w_{20}) + g (2 (629 + 173 v - 516 w_{10} + 99 w_{20}) + g (70 -
57 v + 168 w_{10} + 204 w_{20} + g (-65 v + 4 (8 + 9 w_{10} + 27 w_{20}) - g (-33 - 27 v + 20 w_{10} + 18
w_{20} + g (4 + 3 g - 3 v + 8 w_{20})))))) + c^{2} (8 (-337 + 60 v - 277 w_{20}) + g (3234 + 662 v - 2216
w_{10} + 592 w_{20} + g (-142 - 88 v + 436 w_{10} + 656 w_{20} + g (-154 - 207 v + 216 w_{10} + 208 w_{20} -
g (-86 - 91 v + 82 w_{10} + 62 w_{20} + g (18 - 21 v + g (20 + 2 g + 7 v - 6 w_{10} - 2 w_{20}) + 60
w_{20})))))) + c (48 (-29 + 3 v - 26 w_{20}) + g (4 (516 + 91 v - 312 w_{10} + 107 w_{20}) + g (-198 +
262 w_{10} + 540 w_{20} - g (321 + 185 v - 263 w_{10} - 51 w_{20} + g (-91 - 70 v + 81 w_{10} + 50 w_{20} + g
(11 - 24 v + w_{10} + 68 w_{20} + g (13 + 14 v - 11 w_{10} + g (4 + 3 v - 2 w_{10} - 5 w_{20}) + 2 w_{20})))))))
+ g (24 (21 + 4 v - 12 w_{10} + 5 w_{20}) + g (4 (-13 + 4 v + 15 w_{10} + 46 w_{20}) + g (-2 (81 + 34 v -
55 w_{10} + 12 w_{20}) + g (43 + 21 v - 29 w_{10} - 23 w_{20} + g (5 + 7 v - 3 w_{10} - 16 w_{20} + g (-3 - 5 v +
5 w_{10} - 6 w_{20} + g (1 - 3 v + w_{10} + (4 + g) w_{20})))))))) + 2 (2 + c - g) (3 + 2 c + g) (8 c^{4} (-1 - v
+ 2 g (1 + v - w_{10} - w_{20}) + 2 w_{20}) - 2 c2 (26 + 62 v - 88 w_{20} + g (-92 - 100 v + 88 w_{10} + g (17
+ 12 v - 12 w_{10} + g (3 + 2 v - 4 w_{10} - 4 w_{20}) - 10 w_{20}) + 90 w_{20})) + 4 c^{3} (-9 - 13 v + 22 w_{20} +
g (23 + 23 v - 22 w_{10} - 22 w_{20} + g (-3 + g - 2 v + 2 w_{10} + 2 w_{20}))) + (-2 + g) (24 (v - w_{20}) - g
(2 (6 + 13 v - 12 w_{10} - 7 w_{20}) + g (10 + 10 v - 9 w_{10} - 11 w_{20} + g^{2} (-2 + w_{10} + w_{20}) + 4 g (-1 -
v + w_{10} + w_{20})))) + c (-8 (3 + 16 v - 19 w_{20}) + g (2 (69 + 99 v - 76 w_{10} - 80 w_{20}) - g (18 + 20
v - 20 w_{10} - 4 w_{20} + g (31 + g^{2} + 20 v - 26 w_{10} - 26 w_{20} + 2 g (-2 - v + w_{10} + w_{20})))))) (16 c^{6}
(-1 + 3 v + g (-3 + g - 3 v + 2 w_{10}) + 2 w_{20}) + 4 c^{4} (-101 + 367 v + g (-281 - 431 v + 266 w_{10}
+ g (109 + 41 v - 26 w_{10} + g (8 - 6 v - 6 w_{10} + g (7 + 3 v - 2 w_{10} - 2 w_{20}) - 24 w_{20}) - 28 w_{20}) -
28 w_{20}) + 266 w_{20}) + 8 c^{5} (4 (-4 + 13 v + 9 w_{20}) + g (-2 (23 + 28 v - 18 w_{10} + w_{20}) + g (17 +
g^{2} + 3 v - g v - 2 w_{10} - 2 (1 + g) w_{20}))) - (-2 + g)^{2} (3 + g) (-24 (v + w_{20}) + g (2 (6 + 13 v - 12
w_{10} - 5 w_{20}) + g (10 + 10 v - 9 w_{10} + 5 w_{20} + g (-4 - 4 v + 4 w_{10} + 7 w_{20} + g (-2 + w_{10} + (5 +
g) w_{20}))))) + 2 c^{3} (2 (-157 + 673 v + 516 w_{20}) + g (-2 (436 + 870 v - 516 w_{10} + 77 w_{20}) + g
(219 v - 4 (-81 + 35 w_{10} + 42 w_{20}) + g (90 + 23 v - 72 w_{10} - 116 w_{20} + g (13 + 17 v - 12 w_{10}
- 14 w_{20} + g (-2 - 3 g + 3 v + 8 w_{20})))))) - c^{2} (-8 (-60 + 337 v + 277 w_{20}) + g (1462 + 3858 v
- 2216 w_{10} + 416 w_{20} + g (-424 - 562 v + 396 w_{10} + 548 w_{20} + g (-370 - 275 v + 332 w10 +
268 w_{20} + g (3 v + g (2 - 15 v - 4 w_{10} + g (14 + 2 g + 7 v - 6 w_{10} - 10 w_{20}) - 52 w_{20}) + 2 (9 +
w_{10} + 15 w_{20})))))) + c (48 (-3 + 29 v + 26 w_{20}) + g (-12 (55 + 184 v - 104 w_{10} + 23 w_{20}) + g
(66 + 332 v - 290 w_{10} - 472 w_{20} + g (354 + 388 v - 351 w_{10} - 145 w_{20} + g (-38 - 77 v + 49
\tag{5.A1.14}
$$

$w_{10} + 2\, w_{20} + g\, (-24 + 7\, v + 17\, w_{10} + 62\, w_{20} - g\, (7\, v - 5\, w_{10} - 14\, w_{20} + g\, (2 + 3\, v - 2\, w_{10} + w_{20}))))))))))/(2Z^2)$.

Union's utility is also depending on wages only. Union's utility accrues substituting optimal quantities in the equation below;

$$U_j\left(q_{j0}, q_{j1}, w_{j0}\right) = \left(q_{j0} + q_{j1}\right) w_{j0}\,, \qquad (5.A1.15)$$

Analytically,

$$U_1(w_{10},w_{20}) = (w_{10}\, (-32\, c^6\, (3 + 3\, v - 4\, w_{10} - 2\, w_{20}) +16\, c^5\, (18\, (-3 - 3\, v + 4\, w_{10} + 2\, w_{20}) +g\, (6 - g + 6\, v + g\, v - 6\, w_{10} - 2\, (3 + g)\, w_{20})) - 8\, c^4\, (133\, (3 + 3\, v - 4\, w_{10} - 2\, w_{20}) + g\, (-79 - 81\, v + 82\, w_{10} + 78\, w_{20} + g\, (-1 + g - 18\, v + 10\, w_{10} + 30\, w_{20}))) + 4\, c^3\, (-516\, (3 + 3\, v - 4\, w10 - 2\, w_{20}) + g\, (11\, (37 + 39\, v - 40\, w_{10} - 36\, w_{20}) + g\, (109 + 146\, v - 136\, w_{10} - 198\, w_{20} + g\, (-26 - 15\, v - 4\, g\, v + 16\, w_{10} + 14\, w_{20} + 8\, g\, w_{20})))) + (4 + g)\, (-72\, (3 + 3\, v - 4\, w_{10} - 2\, w_{20}) + g\, (1 + g)\, (12\, (11 + 12\, v - 14\, w_{10} - 9\, w_{20}) + g\, (-34 - 58\, v + 62\, w_{10} + 30\, w_{20} + g\, (-19 - 7\, v + 8\, w_{10} + 18\, w_{20} + g\, (4 + g + 4\, v + g\, v - 6\, w_{10} - 2\, (1 + g)\, w_{20}))))) + 2\, c^2\, (-1108\, (3 + 3\, v - 4\, w_{10} - 2\, w_{20}) + g\, (4\, (257 + 279\, v - 290\, w_{10} - 246\, w_{20}) + g\, (629 + 611\, v - 680\, w_{10} - 686\, w_{20} + g\, (-163 - 130\, v + 138\, w_{10} + 110\, w_{20} + g\, (-13 - 35\, v + 10\, w_{10} + 64\, w_{20} + g\, (3 - 2\, v + 4\, w_{20})))))) + c\, (-1248\, (3 + 3\, v - 4\, w_{10} - 2\, w_{20}) + g\, (4\, (319 + 357\, v - 376\, w_{10} - 300\, w_{20}) + g\, (4\, (341 + 312\, v - 370\, w_{10} - 301\, w_{20}) + g\, (-353 - 371\, v + 386\, w_{10} + 280\, w_{20} + g\, (-133 - 115\, v + 80\, w_{10} + 198\, w_{20} + g\, (25 + 7\, v - 18\, w_{10} + g\, (5 + 3\, v - 6\, w_{20}) + 4\, w_{20}))))))))/Z, \qquad (5.A1.16)$$

$$U_2\, (w_{10},w_{20}) = (w_{20}\, (-288\, (1 + v - 2\, w_{20}) + 32\, c^6\, (-1 - v + 2\, g\, (1 + v - w_{10} - w_{20}) + 2\, w_{20}) + 16\, c^5\, (-18\, (1 + v - 2\, w_{20}) + g\, (37 + g^2 + 37\, v - 36\, w_{10} - 38\, w_{20} + 4\, g\, (-1 - v + w10 + w_{20}))) - 8\, c^4\, (133\, (1 + v - 2\, w_{20}) + g\, (-280 - 282\, v + 266\, w_{10} + 296\, w_{20} + g\, (51 - 4\, g + 52\, v + 5\, g\, v - 54\, w_{10} - 6\, g\, w_{10} - 6\, (8 + g)\, w_{20}))) - 4\, c^3\, (516\, (1 + v - 2\, w_{20}) + g\, (-1109 - 1131\, v + 1032\, w_{10} + 1208\, w_{20} + g\, (247 + 258\, v - 286\, w_{10} - 210\, w_{20} + g\, (52 + 74\, v - 82\, w_{10} - 86\, w_{20} + g\, (-17 + 3\, g - 11\, v + 12\, w_{10} + 12\, w_{20}))))) - 2\, c^2\, (1108\, (1 + v - 2\, w_{20}) + g\, (-8\, (303 + 314\, v - 277\, w_{10} - 340\, w_{20}) + g\, (553 + 599\, v - 744\, w_{10} - 382\, w_{20} + g\, (394 + 397\, v - 406\, w_{10} - 448\, w_{20} + g\, (-123 - 95\, v + 98\, w_{10} + 94\, w_{20} + g\, (6 + 2\, g - 5\, v + 8\, w_{10} + 8\, w_{20})))))) + g\, (24\, (27 + 29\, v - 24\, w_{10} - 32\, w_{20}) + g\, (-4\, (19 + 26\, v - 60\, w_{10} + 15\, w_{20}) + g\, (-366 - 386\, v + 338\, w_{10} + 414\, w_{20} + g\, (73 + 117\, v - 110\, w_{10} - 80\, w_{20} + g\, (71 + 49\, v - 54\, w_{10} - 66\, w_{20} + g\, (-9 - 17\, v + 14\, w_{10} + 12\, w_{20} + g\, (-5 - 3\, v + 4\, w_{10} + 4\, w_{20}))))))) + c\, (-1248\, (1 + v - 2\, w_{20}) + g\, (4\, (693 + 731\, v - 624\, w_{10} - 800\, w_{20}) + g\, (-540 - 624\, v + 952\, w_{10} + 188\, w_{20} + g\, (-931 - 917\, v + 866\, w_{10} + 1008\, w_{20} + g\, (263 + 263\, v - 256\, w_{10} - 228\, w_{20} + g\, (67 + 45\, v - 62\, w_{10} - 68\, w_{20} + g\, (-23 - 11\, v + 12\, w_{10} + 12\, w_{20})))))))))/Z. \qquad (5.A1.17)$$

In the first period/ second stage, each firm bargains the wage with the union that represents employees of the same sector. I make the assumption that unions possess all the bargaining power (b=1). The bargaining problem is

$$w_{j0} = \max_{w_{j0}} B_j = \max_{w_{j0}}\left(\mathrm{b}\ \mathrm{Log}\left[\mathrm{U}_j\right] + (1-\mathrm{b})\ \mathrm{Log}\left[EV_1\right]\right). \qquad (5.A1.18)$$

Solving the system that accrues from the first order conditions of the above problem as for w_{10} and w_{20} accrues a unique stable solution for the equilibrium firm-specific wage contracts w_{10}^* and w_{20}^*, depending only on *c*, *g*, and *v*;

$$w_{10}^{*}= (1024 c^{12} (-5 + 4 g) (1 + v) + (-2 + g) (4 + g) (-(-2 + g)^2 (2 + g) (3 + g) (-2160 + g (2304 + g (759 + g (-1284 + g (51 + g (208 + g (-27 + (-12 + g) g))))))) + (51840 + g (-65664 + g (-25704 + g (59808 + g (-6198 + g (-18855 + g (5241 + g (2593 + g (-871 + g (-197 + g (43 + g (11 + g)))))))))))) v) + 256 c^{10} (-2950 (1 + v) + g (2807 + g (-501 + g (73 + (-3 + g) g - 8 v) - 398 v) + 2809 v)) + 512 c^{11} (-180 (1 + v) + g (g (-17 + 3 g - 13 v) + 158 (1 + v))) + 128 c^9 (-29100 (1 + v) + g (29719 + 29777 v + g (-6548 - 5415 v + g (552 - 378 v + g (31 + 66 v + g (4 + v)))))) - 64 c^8 (192405 (1 + v) + g (-3 (69719 + 69969 v) + g (49801 + 43002 v + g (1119 + 6933 v + g (-1874 - 1797 v + g (212 - v + g (-15 + 7 g + v))))))) - 32 c^7 (898200 (1 + v) + g (-3 (344005 + 345903 v) + g (7 (34653 + 31400 v) + g (49048 + 69553 v + g (-24707 - 21384 v + g (2626 - 107 v + g (g (69 + 4 g + 7 v) + 4 (-5 + 43 v)))))))) + 16 c^6 (-3035320 (1 + v) + g (4 (915989 + 923015 v) + g (-781340 - 749934 v + g (-401293 - 434696 v + g (168320 + 145711 v + g (-9788 + 4487 v + g (-4 (803 + 894 v) + g (106 - 35 v + g (-88 + 15 g + 3 v))))))))) + 8 c^5 (-7480800 (1 + v) + g (9437416 + 9531640 v + g (-2 (821045 + 851777 v) + g (-8 (227409 + 224243 v) + g (694078 + 625081 v + g (18885 + 51501 v + g (-37549 - 32951 v + g (3479 + 90 v + g (-339 + 258 v + g (88 + 16 g + 19 v)))))))))) + 4 c^4 (-13344080 (1 + v) + g (17513296 + 17730608 v + g (-4 (516521 + 612697 v) + g (-8 (653659 + 623953 v) + g (1818665 + 1745117 v + g (3 (98227 + 97494 v) + g (-2 (97943 + 83904 v) + g (12136 - 997 v + g (2948 + 3790 v + g (-320 + 131 v + g (153 - 5 g + v))))))))))) + 2 c^3 (-16800000 (1 + v) + g (22846448 + 23186896 v + g (-8 (129653 + 234631 v) + g (-9783036 - 9294116 v + g (4 (755215 + 789067 v) + g (1129055 + 946777 v - g (559297 + 505201 v + g (8414 + 19181 v + g (-4 (7027 + 5927 v) + g (2879 - 389 v + 3 g (-47 + 4 g^2 + 7 g (-2 + v) + 61 v))))))))))) + c^2 (-14169600 (1 + v) + g (576 (34541 + 35143 v) + g (872528 - 122832 v + g (-16 (724699 + 694992 v) + g (4 (756589 + 885383 v) + g (4 (546639 + 446258 v) + g (-895652 - 894630 v - g (129431 + 87453 v + g (-88367 - 74349 v + g (2742 - 1862 v + g (2638 + 1868 v + g (-373 + 113 v + g (17 + 4 g + 7 v))))))))))))) +c (-3594240 (1 + v) + g (10368 (501 + 511 v) + g (768 (1016 + 603 v) + g (-8 (495731 + 483617 v) + g (803204 + 1105980 v + g (1096438 + 915918 v + g (-7 (53113 + 61363 v) + g (-132035 - 83083 v + g (59870 + 57524 v + g (5914 + 3656 v + g (-4195 - 3057 v + g (49 - 199 v + 2 g (56 + 19 v + g (-3 + 2 v)))))))))))))))/G, \qquad (5.A1.19)$$

where

$$G=(2 (-512 (1 + c)^4 (2 +c)^4 (3 + 2 c)^4 + 64 (1 + c)^4 (2 + c)^3 (3 + 2 c)^3 (74 + c (69 + 14 c)) g - 32 (1 + c)^2 (2 + c)^2 (3 + 2 c)^2 (-166 + c (139 + c (1060 + c (1235 + 564 c + 92 c^2)))) g^2 + 8 (1 + c)^2 (2 + c) (3 + 2 c) (-20062 + c (-67785 + c (-92125 + 2 c (-31769 + 2 c (-5665 - 878 c - 4 c2 + 8 c^3))))) g^3 - 8 (2 + c) (3 + 2 c) (-4636 + c (-31129 + c (-81262 + c (-110461 + 4 c (-21409 + c (-9496 + c (-2199 + 2 c (-91 + 4 c)))))))) g^4 - 2 (3 + 2 c) (-54797 + c (-219774 + c (-356018 + c (-290581 + 2 c (-57603 + 2 c (-2479 + 4 c (509 + c (165 + 14 c)))))))) g^5 + (-133517 + c (-689369 + 2 c (-734431 + c (-839349 + 4 c (-138419 + c (-51797 + 16 c (-592 + c (-19 + 6 c)))))))) g^6 + (-50077 + c (-137813 + c (-119753 + 4 c (-407 + c (14695 + 4 c (2301 + c (559 + 46 c))))))) g^7 + (28303 + 2 c (48332 + c (62795 + 2 c (19051 + 2 c (2469 + (59 - 56 c) c))))) g^8 - (-3485 + 2 c (-697 + c (3661 + c (4133 + 2 c (769 + 90 c))))) g9 + (-2897 + c (-5737 + 2 c (-1675 + c (-193 + 56 c)))) g^{10} + (-115 + c (359 + c (447 + 104 c))) g^{11} + (145 - 2 (-61 + c) c) g^{12} + (5 - 12 c) g^{13} - 2 g^{14})),$$

$$w_{20}^{*}= -(-2 g (-8 (1 + c)^2 (2 + c)^2 (3 + 2 c)^2 + 4 (1 + c)^2 (2 + c) (3 + 2 c) (5 + 4 c) g + (169 + c (433 + 2 c (203 + 2 c (41 + 6 c)))) g^2 - (55 + 2 c (64 + c (49 + 12 c))) g^3 - (27 + c (31 + 8 c)) g^4 + (7 + 6 c) g^5 + 2 g^6) (-96 c^6 (1 + v) + 16 c5 (-54 (1 + v) + g (6 - g + (6 + g) v)) - 8 c^4 (399 (1 + v) + g (-79 - g + g2 - 9 (9 + 2 g) v)) - 4 c^3 (1548 (1 + v) + g (-11 (37 + 39 v) + g (-109 + 26 g + (-146 + g (15 + 4 g)) v))) + (4 + g) (-216 (1 + v) + g (1 + g) (132 + 144 v + g (-34 - 58 v + g (-19 - 7 v + g (4 + g) (1 + v))))) - 2 c^2 (3324 (1 + v) + g (-4 (257 + 279 v) + g (-629 - 611 v + g (163 + 130 v + g (13 + 35 v + g (-3 + 2 v)))))) + c (-3744 (1 + v) + g (4$$

$$(319 + 357 v) + g (4 (341 + 312 v) + g (-353 - 371 v + g (-133 - 115 v + g (25 + 7 v + g (5 + 3 v)))))))) + 4 (16 (1 + c)^{2} (2 + c)^{2} (3 + 2 c)^{2} - 8 (1 + c)^{2} (2 + c) (3 + 2 c) (4 + 3 c) g - 4 (2 + c) (37 + 2 c (37 + c (24 + 5 c))) g^{2} + (1 + c) (3 + 2 c) (29 + 16 c) g3 + (39 + 10 c (4 + c)) g^{4} - (11 + 9 c) g5 - 3 g^{6}) (-(-2 + g)^{2} (2 + g) (3 + g) (12 + g (-25 + g (4 + 5 g))) - (288 + g (-696 + g (104 + g (386 + g (-117 + g (-49 + g (17 + 3 g))))))) v + 32 c^{6} (-1 + 2 g) (1 + v) + 16 c^{5} (g^{3} - 18 (1 + v) + 37 g (1 + v) - 4 g^{2} (1 + v)) - 8 c^{4} (133 (1 + v) + g (-280 - 282 v + g (51 + 52 v + g (-4 + 5 v)))) - 4 c^{3} (516 (1 + v) + g (-1109 - 1131 v + g (247 + 258 v + g (52 + g (-17 + 3 g - 11 v) + 74 v)))) - 2 c^{2} (1108 (1 + v) + g (-8 (303 + 314 v) + g (553 + 599 v + g (394 + 397 v + g (-123 + 6 g + 2 g2 - 5 (19 + g) v))))) + c (-1248 (1 + v) + g (4 (693 + 731 v) - g (540 + 624 v + g (931 + 917 v + g (-263 (1 + v) + g (-67 - 45 v + g (23 + 11 v)))))))))/(-16 (16 (1 + c)^{2} (2 + c)^{2} (3 + 2 c)^{2} - 8 (1 + c)2 (2 + c) (3 + 2 c) (4 + 3 c) g - 4 (2 + c) (37 + 2 c (37 + c (24 + 5 c))) g^{2} + (1 + c) (3 + 2 c) (29 + 16 c) g^{3} + (39 + 10 c (4 + c)) g^{4} - (11 + 9 c) g5 - 3 g^{6}) (-8 (1 + c)^{2} (2 + c)^{2} (3 + 2 c)^{2} + 16 (1 + c)^{2} (2 + c)^{3} (3 + 2 c) g - 2 (3 + 2 c) (-5 + c (19 + c (51 + 4 c (9 + 2 c)))) g^{2} - (3 + 2 c) (69 + 2 c (61 + 34 c + 6 c^{2})) g^{3} + 2 (5 + 3 c) (4 + c (9 + 4 c)) g4 + (3 + 2 c) (11 + 4 c) g^{5} - 6 (1 + c) g^{6} - 2 g^{7}) + 4 g (8 (1 + c)^{2} (2 + c)2 (3 + 2 c)^{2} - 4 (1 + c)^{2} (2 + c) (3 + 2 c) (5 + 4 c) g - (169 + c (433 + 2 c (203 + 2 c (41 + 6 c)))) g^{2} + (55 + 2 c (64 + c (49 + 12 c))) g^{3} + (27 + c (31 + 8 c)) g^{4} - (7 + 6 c) g5 - 2 g^{6}) (8 (1 + c)^{2} (2 + c)^{2} (3 + 2 c)^{2} - 24 (1 + c)^{3} (2 + c) (3 + 2 c) g - 2 (3 + 2 c) (35 + c (77 + c (63 + 4 c (6 + c)))) g^{2} + (3 + 2 c) (19 + 2 c (17 + 7 c)) g^{3} + (56 + c (99 + 16 c (4 + c))) g^{4} + 2 c (1 + 2 c) g^{5} - 3 (2 + c) g^{6} - g^{7})). \tag{5.A1.20}$$

The wage differential is formed in favor of the public sector

$$w_{10}^{*}-w_{20}^{*} = (-1024 c^{12} (1 + g) (1 + v) - 512 c^{11} (36 + 32 g + g^{2} + (1 + g) (36 + (-4 + g) g) v) + (1 + g) (4 + g) ((-2 + g)^{2} (2 + g) (3 + g) (-864 + g (1008 + g (54 + g (-452 + g (139 + g (46 + g (-26 + (-2 + g) g))))))) + (-3 +g) (6912 + g (-3456 + g (-6960 + g (4048 + g (1732 + g (-1742 + g (178 + g (351 + g (-116 + g (-36 + g (14 + 3 g))))))))))) v) + 256 c^{10} (-590 (1 + v) + g (-461 - 467 v + g (-9 + 96 v + g ((1 + g)^{2} + (-25 + 2 g) v)))) + 128 c^{9} (-5820 (1 + v) + g (-3943 - 4117 v + g (206 + 1409 v + g (5 - 268 v + g (52 + 34 v + g (21 + 8 v)))))) - 64 c^{8} (38481 (1 + v) + g (6 (3703 + 4078 v) + g (-4675 - 12528 v + g (220 + 1557 v + g (-2 (261 + 65 v) + g (g (11 + 7 g + 13 v) - 3 (74 + 63 v))))))) - 32 c^{7} (179640 (1 + v) + g (86421 + 103503 v + g (-43103 - 74956 v + g (3625 + 4797 v + g (-2158 + 1446 v + g (-1692 - 1975 v + g (2 g (55 + 2 g + 15 v) + 5 (41 + 45 v)))))))) + 16 c^{6} (-607064 (1 + v) + g (-4 (58985 + 80063 v) + g (235176 + 316486 v + g (-21 (1262 + 153 v) + g (-3 (682 + 6695 v) + g (10259 + 11930 v + g (-1499 - 1474 v + g (-824 - 571 v + g (-27 + 15 g + 31 v))))))))) + 8 c^{5} (-1496160 (1 + v) + g (-8 (56135 + 91469 v) + g (838446 + 960454 v + g (-114337 + 35695 v + g (-3 (21214 + 38709 v) + g (47111 + 45630 v + g (-4623 - 3637 v + g (-4269 - 4691 v + g (69 + 416 v + g (153 + 16 g + 64 v)))))))))) + 4 c^{4} (-2668816 (1 + v) + g (-32 (17981 + 38354 v) + g (4 (504099 + 522523 v) + g (-315374 + 164114 v + g (-313154 - 399132 v + g (2 (76025 + 56778 v) + g (1297 + 5591 v + g (-17283 - 21470 v + g (942 + 2032 v + g (780 + g (79 - 5 g - 27 v) + 868 v)))))))))) + 2 c^{3} (-3360000 (1 + v) + g (-16 (28349 + 92183 v) + g (8 (407259 + 398557 v) + g (40 (-14147 + 9162 v) + g (-808962 - 872790 v + g (325514 + 181064 v + g (53137 + 59410 v + g (-50880 - 58827 v + g (4 (389 + 912 v) + g (3010 + 4788 v - g (-9 + 220 v + g (44 + 12 g + 73 v))))))))))))) + c^{2} (-2833920 (1 + v) + g (-4032 (41 + 299 v) + g (3400784 + 3239728 v + g (8 (-80393 + 59797 v) + g (-8 (150859 + 149889 v) + g (434954 + 173150 v + g (3 (53295 + 52957 v) - g (94782 + 95912 v + g (5653 + 2307 v + g (-2 (4239 + 6667 v) + g (233 + 455 v + g (286 + 580 v + g (7 + 4 g + 5 v))))))))))))) + 2 c (1 + g) (-359424 (1 + v) + g (1728 (211 + 121 v) + g (960 (161 + 295 v) + g (-4 (65083 + 48905 v) + g (14284 - 41548 v + g (67337 + 62525 v + g (- \tag{5.A1.21}$$

15989 - 13123 v + g (-7913 - 8243 v + g (2813 + 4197 v + g (421 + 462 v + g (-201 - 382
v + g (-9 - 20 v + g (5 + 8 v)))))))))))))/G.

Substituting the wages with the optimal in the equations (5.A1.11) and (5.A1.12) accrues the optimal quantities of the first period depending on *c*, *g*, and *v*;

q_{10}*= ((-2 + g) (4 + g) (-(-2 + g)^2 (3 + g) (4478976 + g (-6117120 + g (-2709504 + g
(6155280 + g (50064 + g (-2341164 + g (218362 + g (431313 + g (-26732 + g (-43230 + g
(-5938 + g (2564 + g (1554 + g (-86 + g (-128 + g (-5 + 2 g))))))))))))))) + (-5971968 + g (-
11197440 + g (47319552 + g (-26502336 + g (-29658528 + g (33714480 + g (167328 + g
(-13450364 + g (4368704 + g (2321431 + g (-1462567 + g (-143022 + g (224256 + g (-
8442 + g (-18486 + g (1776 + g (808 + g (-85 + g (-11 + 2 g))))))))))))))))))) v) - 32768 c^{18}
(6 - 10 v + g (-14 + 7 g + 8 v)) - 16384 c^{17} (276 - 524 v + g (-695 + g (384 + g (-27 + 8 g) -
31 v) + 453 v)) - 8192 c^{16} (5730 - 12878 v + g (-15877 + 11921 v + g (9623 - 1431 v + g (-
1156 - 41 v + g (341 + (-23 + g) g + 13 v))))) + 4096 c^{15} (-69648 + 197168 v + g (220030 -
193798 v + g (-146184 + 30322 v + g (22675 + 2457 v + g (-6400 - 874 v + g (741 + 2 g (9
+ g - v) + 47 v)))))) + 2048 c^{14} (-526290 + 2106590 v + g (2045558 - 2181108 v + g (-
1502435 + 388924 v + g (270003 + 65749 v + g (g (9921 + g (1718 + 11 g^2 + 5 g (-23 + v)
- 13 v) + 1951 v) - 3 (22973 + 8334 v)))))) + 1024 c^{13} (4 (-552201 + 4167935 v) + g
(13211615 - 18040421 v + g (-11005378 + 3342853 v + g (2187311 + 1056178 v + g (-
461941 - 417125 v + g (64436 + 35036 v + g (38475 + 2537 v + g (-4827 - 193 v + g (187 -
16 g + 19 v))))))))) - 512 c^{12} (-6 (26971 + 16872211 v) + g (-58016177 + 113565969 v + g
(58642197 - 20003099 v + g (-12912022 - 11482877 v + g (1948568 + 4610228 v + g (-
92364 - 343761 v + g (-470274 - 85805 v + g (74149 + 11168 v + g (1651 - 564 v + g (-192
+ 58 g + 43 v)))))))))) + 256 c^{11} (8 (9400665 + 60243689 v) + g (146408864 - 555667020 v
+ g (-225227012 + 82430968 v + g (59271015 + 89995513 v + g (-5345931 - 36040853 v
+ g (-1930397 + 1768782 v + g (19 (198003 + 73232 v) + g (-635983 - 210669 v + g (-
76886 + 3151 v + g (12645 + 44 (-29 + g) g + 14 v - 90 g v)))))))))) + 128 c^{10} (248
(2465841 + 7348169 v) + g (-49966522 - 2138343490 v + g (-583785502 + 208967414 v +
g (230963469 + 527429731 v + g (-17880564 - 206623997 v + g (-18937665 + 846238 v +
g (21579353 + 14116658 v + g (-3418837 - 2200989 v + g (-1021169 - 89067 v + g
(188898 + 25330 v + g (-7432 - 379 g + 174 g^2 + 4 (-687 + 38 g) v))))))))))) + 64 c9 (320
(9407775 + 17215343 v) + g (-4 (601029601 + 1625647589 v) + g (-736700418 +
112795010 v + g (844553848 + 2361883432 v + g (-2 (71883991 + 442452273 v) + g (-
97510092 - 59693525 v + g (95382560 + 98486907 v + g (-11867833 - 14486363 v + g (-3
(2627397 + 656146 v) + g (1485101 + 485581 v + g (52957 - 29761 v + g (-23032 + 991 v
+ 2 g (1695 - 12 g + 134 v))))))))))))) + 32 c^8 (32 (332163867 + 416595635 v) + g (-16
(787190843 + 974916741 v) + g (1307416628 - 1629271780 v + g (3002080246 +
8175339226 v + g (-4 (255752669 + 711255114 v) + g (-5 (69434104 + 102022033 v) + g
(349441930 + 494040643 v + g (-25779405 - 61825424 v + g (-41179771 - 19670131 v + g
(7211322 + 4609833 v + g (1100397 - 80725 v + g (-284213 - 34230 v + g (21513 + g
(1634 - 285 g - 264 v) + 6865 v))))))))))))) + 16 c^7 (128 (221121273 + 200997353 v) + g (-
32 (1272523933 + 914364791 v) + g (9824327920 - 8438791216 v + g (9709769012 +
21953380156 v + g (-18 (268319369 + 378078387 v) + g (-969636371 - 2450135345 v + g
(1125995917 + 1819713836 v + g (-2 (15177887 + 80322968 v) + g (-3 (52339930 +
40773297 v) + g (22879234 + 26719783 v - g (-8141745 - 1197812 v + g (1776634 +
635160 v + g (5344 - 66345 v + g (-35003 + 4054 g + 94 g^2 + 18 (65 + 28 g) v)))))))))))))
+ 8 c^6 (128 (451233405 + 306514261 v) + g (-32 (2934969953 + 1317173549 v) + g (-320
(-89043533 + 75715141 v) + g (96 (271736991 + 474728428 v) + g (-4 (3916408311 +
2940287189 v) + g (-4 (589749949 + 1997481018 v) + g (3215253723 + 4949778373 v + g

$$
\begin{aligned}
&(-8100695 - 160842491 v + g (-11 (41937535 + 46920141 v) + g (5 (9690853 + 20090251 \\
&v) + g (36037108 + 14591978 v + g (-6532139 - 5014310 v + g (9 (-90858 + 28573 v) + g \\
&(279147 + 41159 v + g (-15883 - 9601 v + g (-3166 + 215 g + 187 v)))))))))))))))) + 4 c^5 \\
&(9216 (9789957 + 5048173 v) + g (-64 (2502915481 + 704982981 v) + g (53164402016 - \\
&47514441824 v + g (320 (170930063 + 225988550 v) + g (-416 (86619031 + 32768558 v) \\
&+ g (-8 (652649604 + 2321038315 v) + g (7736238260 + 9869520696 v + g (6 (2854479 + \\
&81738971 v) + g (-1087557455 - 1522004932 v + g (70759228 + 244261282 v + g \\
&(107409010 + 79950265 v + g (-4 (3621761 + 5687895 v) + g (-3 (1713336 + 87583 v) + g \\
&(1134340 + 549146 v + g (40914 - 66737 v + g (g (1567 + 152 g + 545 v) - 2 (15543 + 613 \\
&v))))))))))))))))) + 2 c^4 (829440 (128167 + 50415 v) + g (-768 (263128733 + 43878057 v) + \\
&g (68888337984 - 66334370624 v + g (85933661856 + 85866948320 v + g (-32 \\
&(1849083291 + 257069347 v) + g (-8 (1238162855 + 3868685873 v) + g (8 (1825807821 + \\
&1763534788 v) + g (8 (2628282 + 304070281 v) + g (-2082628463 - 3140876503 v + g \\
&(82484059 + 354938220 v + g (227465620 + 263505437 v + g (-181 (101024 + 350463 v) \\
&+ g (-16598678 - 6646729 v + g (2411378 + 3043973 v + g (544956 - 171273 v + g (- \\
&132112 - 41905 v + g (-2603 + 6604 v + g (2155 - 40 g + 43 v)))))))))))))))))) + c^3 (7077888 \\
&(12942 + 3865 v) + g (-18432 (9991553 + 820485 v) + g (6144 (10083957 - 10642327 v) + \\
&g (97220193728 + 73588462144 v + g (32 (-2130124691 + 65386251 v) + g (-32 \\
&(451366186 + 1133380043 v) + g (8 (2525556767 + 1729931553 v) + g (8 (14499304 + \\
&631414711 v) + g (-50 (62584871 + 88703861 v) + g (101188007 + 210635299 v + g \\
&(351996630 + 550620926 v + g (-2 (5214829 + 53618927 v) + g (-32205438 - 28297378 v \\
&+ g (2065972 + 9035976 v + g (34 (55837 + 5711 v) + g (-236246 - 282586 v + g (-51852 \\
&+ 24228 v + g (9301 + 2613 v - 4 g (-88 + 16 g + 67 v))))))))))))))))))) + c^2 (3981312 (6849 \\
&+ 1529 v) + g (-884736 (64993 + 1083 v) + g (-27648 (-668437 + 786249 v) + g (1152 \\
&(32418353 + 18567103 v) + g (96 (-273413429 + 44048053 v) + g (-96 (75606734 + \\
&147956819 v) + g (9493757744 + 4287357536 v + g (8 (23601854 + 374507057 v) + g (-4 \\
&(423824898 + 507695257 v) + g (59153159 - 89296037 v + g (197275533 + 357285455 v \\
&+ g (-21266 - 49817452 v + g (-2 (9662865 + 14901746 v) + g (-573048 + 7318162 v + g \\
&(2 (785628 + 513725 v) - g (11242 + 413412 v + g (81182 - 3024 v + g (-4545 - 9643 v + g \\
&(-1979 + 657 v + 4 g (33 + 2 g + 12 v))))))))))))))))))))) + c (47775744 (105 + 17 v) + g \\
&(331776 (-33245 + 1191 v) + g (-55296 (-59319 + 79351 v) + g (6912 (1261583 + 542145 \\
&v) + g (2304 (-2638123 + 707663 v) + g (-32 (68717045 + 104309408 v) + g (32 \\
&(84103891 + 21993836 v) + g (122796744 + 988783960 v + g (-12 (47862060 + 44855201 \\
&v) + g (20938496 - 94637558 v + g (72997294 + 131154930 v + g (79719 - 8993661 v + g \\
&(-8 (868835 + 2012216 v) + g (-818024 + 2890672 v + g (609856 + 1045108 v + g (105078 \\
&- 270478 v + g (-20 (2195 + 1551 v) + g (-5408 + 11710 v + g (2 (953 + 69 v) + g (131 - \\
&197 v + 4 g (-7 + 2 v))))))))))))))))))))))))/W,
\end{aligned}
\qquad (5.A1.22)
$$

where

$$
\begin{aligned}
&W=(2 (-16 (1 + c)^2 (2 + c)^2 (3 + 2 c)^2 + 8 (1 + c)^2 (2 + c) (3 + 2 c) (9 + 2 c (4 + c)) g + 4 (2 + c) (26 + c (46 + \\
&c (25 + 4 c))) g^2 + 2 (-72 + c (-121 + c (-27 + 8 c (7 + c (5 + c))))) g^3 + (12 + c (39 + 4 c (6 + c))) g^4 - (7 + c \\
&(53 + 2 c (27 + 8 c))) g^5 - (9 + c (11 + 4 c)) g^6 + 3 (1 + c) g^7 + g^8) (-512 (1 + c)^4 (2 + c)^4 (3 + 2 c)^4 + 64 (1 + \\
&c)^4 (2 + c)^3 (3 + 2 c)^3 (74 + c (69 + 14 c)) g - 32 (1 + c)^2 (2 + c)^2 (3 + 2 c)^2 (-166 + c (139 + c (1060 + c (1235 \\
&+ 564 c + 92 c^2)))) g^2 + 8 (1 + c)^2 (2 + c) (3 + 2 c) (-20062 + c (-67785 + c (-92125 + 2 c (-31769 + 2 c (- \\
&5665 - 878 c - 4 c^2 + 8 c^3))))) g^3 - 8 (2 + c) (3 + 2 c) (-4636 + c (-31129 + c (-81262 + c (-110461 + 4 c (- \\
&21409 + c (-9496 + c (-2199 + 2 c (-91 + 4 c)))))))) g^4 - 2 (3 + 2 c) (-54797 + c (-219774 + c (-356018 + c (- \\
&290581 + 2 c (-57603 + 2 c (-2479 + 4 c (509 + c (165 + 14 c)))))))) g^5 + (-133517 + c (-689369 + 2 c (- \\
&734431 + c (-839349 + 4 c (-138419 + c (-51797 + 16 c (-592 + c (-19 + 6 c)))))))) g^6 + (-50077 + c (-137813 \\
&+ c (-119753 + 4 c (-407 + c (14695 + 4 c (2301 + c (559 + 46 c))))))) g^7 + (28303 + 2 c (48332 + c (62795 + \\
&2 c (19051 + 2 c (2469 + (59 - 56 c) c))))) g^8 - (-3485 + 2 c (-697 + c (3661 + c (4133 + 2 c (769 + 90 c)))))
\end{aligned}
$$

$g^{9} + (-2897 + c (-5737 + 2 c (-1675 + c (-193 + 56 c)))) g^{10} + (-115 + c (359 + c (447 + 104 c))) g^{11} + (145 - 2 (-61 + c) c) g^{12} + (5 - 12 c) g^{13} - 2 g^{14}))$,

q_{20}*=$(32768 c^{18} (-1 + g) (-4 + 5 g) (1 + v) + (-2 + g) (4 + g) ((-2 + g)^{2} (3 + g) (-2985984 + g (5889024 + g (-736128 + g (-5555232 + g (3026592 + g (1814396 + g (-1637694 + g (-200190 + g (402901 + g (-14446 + g (-54294 + g (5486 + g (4264 + g (-486 + g (-204 + g (8 + 3 g)))))))))))))))) + (11943936 + g (-20901888 + g (4769280 + g (715392 + g (8280000 + g (4337856 + g (-15825728 + g (3035776 + g (7316498 + g (-3030556 + g (-1427621 + g (904157 + g (114916 + g (-133384 + g (46 + g (10866 + g (-526 + g (-20 + 3 g) (25 + 3 g))))))))))))))))))) v) - 8192 c^{16} (-6316 - 4652 v + g (13453 + 11845 v + g (-8102 + 3 g (344 + (-42 + g) g) - 8342 v + g (1286 + g (-47 + 4 g)) v))) + 16384 c^{17} (8 (29 + 25 v) + g (-506 - 478 v + g (294 + 302 v + g (-22 - 27 v + g (3 + v))))) + 4096 c^{15} (16 (6703 + 4169 v) + g (-38 (5913 + 4783 v) + g (139233 + 140693 v + g (-5 (4490 + 5657 v) + g (2034 + 537 v + g (75 + g (-40 + g - 7 v) + 58 v)))))) - 1024 c^{13} (-8 (1401917 + 590017 v) + g (6 (3865669 + 2513495 v) + g (-2 (7481665 + 6872833 v) + g (7 (390245 + 499286 v) + g (155572 + 11 g^{5} + 306134 v - 6 g^{4} (25 + 3 v) - 7 g^{3} (171 + 103 v) + 17 g^{2} (2069 + 817 v) - 2 g (77315 + 85812 v)))))) - 2048 c^{14} (-20 (63647 + 32911 v) + g (221 (11951 + 8743 v) + g (-1673905 - 1628049 v + g (299645 + 380543 v + g (-10479 + 7615 v + 2 g (-3238 - 3453 v + g (889 + g (-37 + g - 21 v) + 275 v))))))) + 512 c^{12} (76019348 + 25267860 v + g (-157638103 - 89303199 v + g (103139260 + 87790044 v + g (-22 (812207 + 1051698 v) + g (-3728271 - 4530960 v + g (2091764 + 2338742 v + g (-405568 - 176159 v + g (-2603 - 8077 v + g (7623 + g (-528 + 16 g - 203 v) + 2170 v))))))))) + 256 c^{11} (32 (12666622 + 3177689 v) + g (-847455074 - 408527246 v + g (560752709 + 433445385 v + g (-5 (17047316 + 22697725 v) + g (-40233086 - 41564995 v + g (18922580 + 20938699 v + g (-5 (592402 + 246801 v) + g (-313204 - 374966 v + g (g (-8842 + g (-220 + 58 g - 21 v) - 4650 v) + 17 (8545 + 3428 v)))))))))) - 128 c^{10} (-496 (3474767 + 610291 v) + g (8 (456433354 + 181805503 v) + g (-48 (50801260 + 34887281 v) + g (291170317 + 416451907 v + g (7 (41109455 + 38110439 v) + g (-3 (41047539 + 44260783 v) + g (13460856 + 3860570 v + g (4819706 + 5400407 v + g (-1578593 - 763507 v + g (58125 + 25245 v + g (17084 + g (-1995 + 44 g - 594 v) + 5128 v))))))))))) + 64 c^{9} (1280 (4599844 + 487973 v) + g (-16 (793634247 + 251193871 v) + g (8545486674 + 5086472818 v + g (-3 (215632323 + 378271655 v) + g (-1503107345 - 1260168427 v + g (599056554 + 619300816 v + g (19 (-1509227 + 578765 v) + g (-42002841 - 45717656 v + g (11080814 + 5918459 v + g (126079 + 325423 v + g (-2 (151363 + 65377 v) + g (27291 + 12994 v + g (380 - 174 g + 45 v))))))))))))) + 32 c^{8} (64 (253104491 + 10553971 v) + g (-176 (202883005 + 48217697 v) + g (8 (3023809957 + 1515107901 v) + g (-4 (125793961 + 555278495 v) + g (-5964989853 - 4487457575 v + g (2223383126 + 2163652803 v + g (72291346 + 187941107 v + g (-246608593 - 260327365 v + g (8 (6579454 + 3607185 v) + g (5282524 + 6399682 v + g (-4 (714110 + 381047 v) + g (176165 + 94550 v + g (28011 + 2 g (-2149 + 12 g - 571 v) + 9316 v))))))))))))) + 16 c^{7} (512 (69933856 - 1257745 v) + g (-96 (843367799 + 137103977 v) + g (64 (862088953 + 350614678 v) + g (2521583288 - 2827845176 v + g (-2 (9146956037 + 6079306847 v) + g (6349795777 + 5692566971 v + g (876810017 + 1063555527 v + g (-3 (346238839 + 350329099 v) + g (25 (6766777 + 3446721 v) + g (46134071 + 51577196 v + g (-2 (8441515 + 5133367 v) + g (309400 - 7071 v + g (401365 + g (-43120 + g (-1591 + 285 g - 229 v) - 20519 v) + 193049 v))))))))))))) + 8 c^{6} (63117463296 - 4631012608 v + g (-128 (1147335589 + 105329934 v) + g (256 (393007857 + 123540598 v) + g (16 (783233309 - 90641137 v) + g (-32 (1360481339 + 782039322 v) + g (4 (3484555849 + 2802867743 v) + g (3875482997 + 3728087033 v + g (-3222998553 - 3071911832 v + g (343638852 + 121488950 v + g (8 (29248801 + 31576736 v) + g (-66172656 - 43228660 v + g (-3412616 - 4566334 v + g (2950498 + 1779466 v + g (-4 (50663 + 31024 v) + g (-$

$$38377 - 14053 v + g (4737 + 94 g + 1418 v)))))))))))))))) + 4 c^5 (-18432 (-4748469 + 592723 v) + g (-512 (410405428 + 10570335 v) + g (32 (4519953029 + 1020845765 v) + g (31366866608 + 2252416240 v + g (-112 (713270897 + 345129253 v) + g (24 (969141795 + 675911147 v) + g (10861764156 + 8955860744 v + g (-7418275813 - 6513214999 v + g (306975862 - 136707775 v + g (789806072 + 822631379 v + g (-173809679 - 114783569 v + g (-29494269 - 34105969 v + g (13058495 + 9149627 v + g (-233793 - 84419 v + g (-350173 - 194095 v + g (29962 + 17728 v + g (3019 - 215 g + 588 v))))))))))))))))) + 2 c^4 (-552960 (-168863 + 29157 v) + g (12288 (-18868054 + 667263 v) + g (128 (1252006943 + 178703303 v) + g (64 (811528141 + 89701175 v) + g (-80 (1388557013 + 546165791 v) + g (28834339424 + 16611350192 v + g (8 (2630068943 + 1881960665 v) + g (-2 (6277507771 + 4936294749 v) + g (-4 (112181917 + 263615101 v) + g (1840414967 + 1822731538 v + g (-296472716 - 179333751 v + g (-11 (10559910 + 11909873 v) + g (36398708 + 28027233 v + g (2130728 + 2540989 v + g (-1684212 - 1208693 v + g (67288 + 61297 v + g (31536 + 14759 v - g (1919 + 152 g + 987 v)))))))))))))))))) + c^2 (-7962624 (-2571 + 665 v) + g (442368 (-123607 + 17634 v) + g (-497664 (-76409 + 239 v) + g (9216 (2439245 + 152094 v) + g (-64 (625355869 + 137775883 v) + g (192 (38908694 + 10996389 v) + g (8 (1596703661 + 827165349 v) + g (-4 (1563076021 + 873741217 v) + g (-28 (53527603 + 52046999 v) + g (2 (779529329 + 647903724 v) + g (-49803067 + 43814925 v + g (-195812577 - 209131567 v + g (4 (7967443 + 6015890 v) + g (4 (3338394 + 4058159 v) + g (-14 (252023 + 243829 v) + g (-474234 - 539918 v + g (g (8142 + g (-4579 + 32 g^2 + 105 g (-1 + v) - 3703 v) + 2820 v) + 4 (46120 + 45737 v)))))))))))))))))) + c^3 (-1769472 (-41793 + 9077 v) + g (73728 (-2580949 + 236127 v) + g (3072 (42964231 + 2841511 v) + g (2560 (23102737 + 2253642 v) + g (-32 (3541304939 + 1079907221 v) + g (16 (1593010027 + 700021941 v) + g (8 (3543375917 + 2180050073 v) + g (-8 (1901720176 + 1291852451 v) + g (-2 (973703407 + 1191201899 v) + g (2948296335 + 2714947019 v + g (-293450661 - 116110495 v + g (-20 (13625275 + 15076863 v) + g (63226772 + 50525192 v + g (2 (5814973 + 6838745 v) + g (-4608690 - 3954266 v + g (-4 (30845 + 23557 v) + g (g (-3793 + g (-1993 + 40 g - 695 v) - 5685 v) + 2 (76849 + 56299 v)))))))))))))))))) + c (-95551488 (-37 + 11 v) + g (2654208 (-3698 + 699 v) + g (-55296 (-123635 + 7981 v) + g (27648 (186953 + 3239 v) + g (-384 (22779731 + 3207053 v) + g (64 (19700143 + 1757685 v) + g (16 (217401700 + 91952517 v) + g (-72 (21678529 + 9279941 v) + g (-12 (49047388 + 40292157 v) + g (6 (82007595 + 59200949 v) + g (6 (3642929 + 8408898 v) + g (-80220215 - 80458157 v + g (7582373 + 3897677 v + g (7669796 + 9479254 v + g (-2 (672398 + 682397 v) + g (-448050 - 603776 v + g (2 (50453 + 62890 v) + g (2 (8383 + 9974 v) + g (-3698 - 5170 v + g (-439 - 299 v + g (41 + 4 g + 75 v))))))))))))))))))))))))/W. \qquad (5.A1.23)$$

Substituting the product of the first period and wages with the optimal in the equations (5.A1.5) and (5.A1.6) accrues the optimal quantities of the second period depending on *c*, *g*, and *v*;

$$q_{11}^*=(c (-2 + g)^2 (3 + g) (99532800 + g (63949824 + g (-352673280 + g (108254784 + g (222605568 + g (-125111736 + g (-45301972 + g (43852642 + g (-150750 + g (-6803817 + g (1157935 + g (456307 + g (-147585 + g (-3823 + g (7073 + g (-1057 + g (-157 + 28 g))))))))))))))))) + c (5398659072 - g (11642351616 + g (-3463464960 + g (-8903008512 + g (6392506368 + g (1917576576 + g (-2699691152 + g (60287872 + g (498561208 + g (-58555200 + g (-41673076 + g (1034449 + g (1177984 + g (1518380 + g (-25712 + g (-222138 + g (7040 + g (12780 + g (-332 + g (-239 + 8 g)))))))))))))))))))) v + 32768 c^{18} (26 + g (-30 + 7 g - 8 v) + 10 v) - (-2 + g)^2 (3 + g) (4 + g) (-(-2 + g)2 (3 + g) (82944 + g (286848 + g (-490176 + g (-101016 + g (366564 + g (-67850 + g (-95481 + g (40875 + g (8217 + g (-7157 + g (251 + g (505 + g (-65 + g (-13 + 2 g)))))))))))))) + (-8957952 + g (16298496 + g$$

(-815616 + g (-14112864 + g (6166944 + g (3970272 + g (-2752088 + g (-335708 + g
(428507 + g (-3308 + g (-10838 + g (-5406 + g (-3544 + g (1778 + g (414 + g (-142 + g (-
19 + 2 g))))))))))))))))) v) + 16384 c^{17} (1356 + 556 v + g (g (476 + g (-39 + 8 g) + 45 v) - 7
(235 + 71 v))) + 8192 c^{16} (33150 + 14542 v + g (-42099 - 14309 v + g (14077 + 2323 v + g
(-1726 - 47 v + g (371 + (-27 + g) g + 13 v))))) - 4096 c^{15} (-16 (31533 + 14857 v) + g
(667982 + 254530 v + g (-10 (24657 + 5600 v) + g (33931 + 1619 v + g (-6942 - 368 v + g
(817 + 2 g (6 + g - v) + 51 v)))))) - 2048 c^{14} (-10 (535419 + 272131 v) + g (7360618 +
3142192 v + g (-3 (966661 + 279104 v) + g (382301 + 21523 v + g (-61053 - 1360 v + g
(8583 + 1543 v + g (1750 - 23 v + g (-153 + 11 g + 5 v)))))))) + 1024 c^{13} (42047484 +
23166940 v + g (-3 (19926103 + 9546011 v) + g (24448554 + 8696771 v + g (-2577813 -
86300 v + g (80799 - 92231 v + g (-2 (2267 + 9285 v) + g (-41577 - 699 v + g (5891 + g (-
229 + 16 g - 19 v) + 225 v)))))))) + 512 c^{12} (253056194 + 151984754 v + g (-370494167 -
199961981 v + g (153303347 + 66584375 v + g (-8460272 + 1300289 v + g (-4240194 -
2104030 v + g (801662 - 91721 v + g (-486884 - 15343 v + g (81561 + 9682 v + g (1335 -
644 v + g (-356 + 58 g + 43 v)))))))))) + 256 c^{11} (8 (149042445 + 98199421 v) + g (-4
(447410033 + 273702220 v) + g (729388742 + 389059050 v + g (19242461 + 24984269 v
+ g (-55297569 - 24525845 v + g (9399987 + 211722 v + g (-13 (245291 + 4508 v) + g
(550865 + 143159 v + g (84470 - 8153 v + g (-16953 - 136 v + 2 g (696 - 22 g + 45
v))))))))))) + 128 c^{10} (248 (17959449 + 13077121 v) + g (-6818903406 - 4768501702 v + g
(18 (147328103 + 98428745 v) + g (408754261 + 223752695 v + g (-5 (78448228 +
37877709 v) + g (56353743 + 5967154 v + g (-9791369 + 1368360 v + g (1366875 +
1129307 v + g (1116507 - 48925 v + g (-225472 - 22838 v + g (8002 + g (791 - 174 g - 152
v) + 3028 v))))))))))) + 64 c^{9} (320 (41470395 + 33662827 v) + g (-4 (5150119699 +
4164065591 v) + g (7322673786 + 6363814678 v + g (6 (438807797 + 221293345 v) + g
(-2 (941211329 + 526784357 v) + g (191931386 + 40298949 v + g (18075510 + 23302487
v + g (-6730891 + 5062815 v + g (7598035 - 160724 v + g (-1462707 - 359335 v + g (-
73371 + g (31902 + 2 g (-1769 + 12 g - 134 v) - 695 v) + 39933 v))))))))))) + 32 c^{8} (32
(986128443 + 901696675 v) + g (-16 (3081496283 + 2924438945 v) + g (14957372028 +
18124565556 v + g (10781360182 + 5711775266 v + g (-6497268758 - 4385993766 v + g
(246950984 + 144726011 v + g (344096020 + 188268461 v + g (-72204525 + 10207268 v
+ g (28284963 - 957601 v + g (-4475748 - 2801417 v + g (-1331345 + 276607 v + g (7
(49271 + 4616 v) + g (-20681 - 7425 v + g (-2302 + 285 g + 264 v)))))))))))))) + 16 c^{7} (128
(465639837 + 485763757 v) + g (-96 (966839219 + 1099001233 v) + g (20983615344 +
40906474384 v + g (4 (7870267235 + 4612067649 v) + g (-2 (8201388633 + 6954013571
v) + g (-985958949 + 240763129 v + g (1818188723 + 978163678 v + g (-2 (143835871 +
9344926 v) + g (37427830 - 11675761 v + g (689812 - 12898629 v + g (-8389239 +
1115564 v + g (1742726 + 499850 v + g (45988 - 80779 v + g (-45337 + 650 v + 2 g (2026
+ 47 g + 252 v))))))))))))))) + 8 c6 (128 (691080085 + 835799229 v) + g (-32 (4231531643
+ 5924345311 v) + g (64 (236337657 + 1135812517 v) + g (67770113824 + 45304958912
v + g (-4 (7502534229 + 8442021335 v) + g (-12 (544711288 + 23356167 v) + g
(5778050281 + 3548364459 v + g (-560697839 - 198720885 v + g (-160206171 -
88996895 v + g (56973411 - 35643883 v + g (-26123066 + 2886568 v + g (3729119 +
3377746 v + g (1089658 - 457075 v + g (-29 (11035 + 1439 v) + g (12093 + g (3836 - 215
g - 187 v) + 10267 v))))))))))))))) + 2 c^{4} (276480 (314029 + 547285 v) + g (-2304
(51431069 + 126335121 v) + g (64 (-656678245 + 1644726089 v) + g (32 (3915790849 +
3684134471 v) + g (-192 (157754048 + 450920775 v) + g (-8 (4407245029 + 973171719
v) + g (8 (2176853708 + 2164716511 v) + g (2213632156 - 1301379900 v + g (-49
(54117799 + 24454935 v) + g (324141575 + 7736342 v + g (85361644 + 23932805 v + g
(-35871214 + 29739653 v + g (9787446 - 2643105 v + g (-510040 - 2276879 v + g (-
652300 + 306263 v + g (130306 + g (6809 + g (-2391 + 40 g - 43 v) - 6988 v) + 44723
v)))))))))))))))) + 4 c^{5} (9216 (10988773 + 15730557 v) + g (-192 (776793529 +

$$1395066717 v) + g (32 (-295900631 + 3136324647 v) + g (32 (3373858653 + 2639729971 v) + g (-32 (1194362032 + 1951703865 v) + g (-8 (2397451616 + 342683225 v) + g (12238277676 + 9243331224 v + g (-4 (29125854 + 167606371 v) + g (-1002557437 - 407851970 v + g (207181474 - 50357464 v + g (-27134222 + 6913861 v - g (3169760 - 12972662 v + g (-5182394 + 1459869 v + g (971878 + 468180 v + g (83322 - 79971 v + g (g (1281 + 152 g + 545 v) - 2 (18410 + 949 v)))))))))))))))) + c^{3} (3538944 (15011 + 33165 v) + g (-55296 (1124793 + 4245989 v) + g (6144 (-9375785 + 13159591 v) + g (192 (531988831 + 622451057 v) + g (-32 (279365575 + 2724459769 v) + g (-32 (1334638758 + 400938913 v) + g (8 (1986285267 + 2857740017 v) + g (5852945976 - 1528071696 v + g (-4071026054 - 2300851562 v + g (85793165 + 167315285 v + g (8 (45468715 + 9628083 v) + g (72 (-1070407 + 536558 v) + g (-4 (341213 + 695774 v) + g (4314434 - 5971870 v + g (-1457912 + 594900 v + g (86872 + 264712 v + g (65178 - 30974 v + g (-9127 - 3135 v + 4 g (-142 + 16 g + 67 v))))))))))))))))))) + c^{2} (27869184 (383 + 1143 v) + g (-2654208 (3244 + 24939 v) + g (27648 (-805473 + 773237 v) + g (3456 (7892337 + 12037823 v) + g (3914901088 - 30127537504 v + g (-32 (518538303 + 204021662 v) + g (16 (253590842 + 632389295 v) + g (72 (51612332 - 6890699 v) + g (-4 (458423823 + 351461810 v) + g (-236354273 + 148266027 v + g (292154515 + 77231513 v + g (-29818296 + 10875854 v + g (-2 (8975896 + 723775 v) + g (5184906 - 4289460 v + g (-144106 + 256188 v + g (-229436 + 353914 v + g (64396 - 27706 v + g (223 - 10647 v + g (-2257 + 667 v + 4 g (27 + 2 g + 12 v)))))))))))))))))))))/W, \qquad (5.A1.24)$$

$$q_{21}^{*}=((2 + c - g) (3 + 2 c + g) (16384 c^{16} (-1 + g) (-4 + 5 g) (1 + v) + 2 c^{3} (2 g (203649024 + g (575065984 + g (133119296 + g (-874275800 + g (209813488 + g (331388940 + g (-144075423 + g (-37493768 + g (26633931 + g (-467147 + g (-1654107 + g (252566 + g (5162 + g (-6371 + g (1584 + 5 (-36 + g) g))))))))))))))) + g (-11337658368 + g (7857550080 + g (2640126464 + g (-5308008016 + g (1240559632 + g (911030104 + g (-463464234 + g (-13807516 + g (47147267 + g (-8100374 + g (-1135177 + g (579194 + g (-39367 + g (-9938 + g (935 + 2 g))))))))))))))) v +368640 (-2461 + 12087 v)) + 8192 c^{15} (4 (43 + 51 v) + g (-7 (61 + 65 v) + g (8 (35 + 34 v) + g (-31 - 22 v + g (3 + v))))) - 4096 c^{14} (-40 (85 + 121 v) + g (9352 + 10748 v + g (-7005 - 6813 v + g (1299 + 938 v + g (-134 + 16 g - 44 v + 3 g v))))) - 256 c^{10} (-32 (247639 + 939088 v) + g (37551470 + 67948018 v + g (-41153367 - 49051459 v + g (12145881 + 9582246 v + g (3 (478259 + 685495 v) + g (-851965 + 2 g (58627 + g (-12040 + g (-58 + g (63 + 2 g)))) - 847373 v + g (81323 + g (-803 + 3 g (-336 + 23 g))) v)))))) - 2048 c^{13} (-40 (1021 + 1773 v) + g (2 (62671 + 78711 v) + g (-2 (52483 + 52269 v) + g (24832 + 18318 v + g (-2249 - 561 v + g (359 - 7 v + g (2 + g + 6 v))))))) + 1024 c^{12} (331540 + 718420 v + g (-3 (382827 + 533219 v) + g (9 (117781 + 122393 v) + g (-286363 - 216371 v + g (15756 - 2026 v + g (-1174 + 2912 v + g (-415 - 357 v + g (85 + g + 22 v)))))))) + 512 c^{11} (92 (20753 + 57993 v) + g (-7608327 - 11954659 v + g (7669752 + 8461500 v + g (-9 (246455 + 190756 v) + g (-23207 - 144075 v + g (49356 + 69112 v + g (-2 (5253 + 3770 v) + g (2505 + 464 v + 2 g (-17 + 5 g + 8 v)))))))) + 128 c^{9} (80 (291295 + 1636899 v) + g (-8 (17504560 + 37442903 v) + g (166849098 + 219006826 v + g (-48103986 - 38059936 v + g (-2 (6989595 + 8572831 v) + g (7170840 + 6644173 v - g (676903 + 482139 v + g (-55480 + 72602 v + g (-17379 - 20801 v + g (5738 + 1681 v + 14 g (-11 + 3 g + 2 v)))))))))) + 64 c^{8} (640 (69509 + 697205 v) + g (-32 (12345457 + 32338160 v) + g (514948602 + 761336634 v + g (-138119609 - 104720653 v + g (-79255339 - 97481243 v + g (38276053 + 36100838 v - g (1585319 + 1235972 v + g (810375 + 1097794 v + g (-230718 - 219782 v + g (56512 + 13995 v + g (-1244 + g (89 + 6 g - 116 v) + 1625 v)))))))))) + 32 c^{7} (320 (104881 + 3727809 v) + g (-16 (52006375 + 175636911 v) + g (16 (75414157 + 129234645 v) + g (-8 (35211139 + 22063146 v) + g (-303185819 - 399914901 v + g (139406468 + 140990839 v + g (4833113 + 3407365 v + g$$

(-8767457 - 8392174 v + g (3 (466783 + 458141 v) + g (-191215 - 16312 v + g (-2 (8573 + 14600 v) + g (6396 + 2651 v + g (-517 + 84 g + 37 v)))))))))))) + 16 c^6 (512 (-182243 + 4870822 v) + g (-96 (13143605 + 62267379 v) + g (96 (22045577 + 45549859 v) + g (-8 (47920159 + 8333289 v) + g (-2 (408350757 + 605771167 v) + g (353261705 + 401361639 v + g (52852299 + 44301695 v + g (-43472451 - 40638748 v + g (4238062 + 5304669 v + g (463208 + 542941 v + g (-253367 - 260982 v + g (56719 + 22093 v + g (g (227 + 48 g - 105 v) + 4 (-788 + 419 v))))))))))))))) + 8 c^5 (256 (-1537303 + 15759069 v) + g (-128 (9789309 + 76978108 v) + g (96 (28049333 + 74201061 v) + g (96 (-2952855 + 5608784 v) + g (-616 (2526131 + 4421137 v) + g (614038808 + 830790248 v + g (205649675 + 197223923 v + g (-130415217 - 133258892 v + g (3384386 + 12067356 v + g (6513529 + 4628164 v + g (-1268655 - 1350289 v + g (126162 + 72246 v + g (13322 + 23572 v - g (5293 + 2118 v + g (-848 + 69 g + 26 v)))))))))))))))) + $(-2 + g)^2$ (4 + g) (995328 (-1 + 3 v) + g (-82944 (-17 + 71 v) + g (3456 (-77 + 171 v) + g (1440 (-175 + 3769 v) + g (48 (7676 - 56847 v) + g (-4 (150316 + 405461 v) + g (8 (18333 + 172000 v) + g (4 (79009 + 25328 v) + g (-151457 - 281075 v + g (-60264 + 36908 v + g (8 (5019 + 2953 v) + g (4880 - 7900 v + g (-5010 - 326 v + g (g (312 + g (12 + g (-5 + v) - 20 v) - 64 v) + 8 (-21 + 71 v)))))))))))))))) + 4 c^4 (15360 (-49249 + 322323 v) + g (-512 (1077833 + 24093199 v) + g (224 (10387151 + 39106239 v) + g (23129904 + 1672731376 v + g (-16 (127994833 + 281059675 v) + g (24 (28771461 + 51122935 v) + g (468088872 + 526452772 v + g (-247822939 - 301201893 v + g (-19660037 + 11664872 v + g (25731867 + 19155608 v - g (2778108 + 4257019 v + g (484761 + 51935 v + g (-2 (84092 + 79139 v) + g (15 (1873 + 962 v) + g (-1972 + g (-208 + 67 g - 50 v) + 967 v))))))))))))))) + c^2 (4423680 (-157 + 627 v) + g (-221184 (-3737 + 32613 v) + g (18432 (5107 + 262881 v) + g (1024 (221177 + 2449549 v) + g (-32 (25934595 + 132615037 v) + g (22654832 + 792314704 v + g (8 (70249815 + 125934211 v) + g (-24 (7402718 + 19325055 v) + g (-2 (59404907 + 28124703 v) + g (59759993 + 70302649 v + g (7331370 - 8550734 v + g (-7305594 - 3588524 v + g (390294 + 1179854 v + g (324316 - 17282 v + g (-51998 - 43042 v + g (786 + 4316 v + g (832 + g (-269 + 28 g - 15 v) + 296 v))))))))))))))))) + 2 c (1327104 (-59 + 201 v) + g (-221184 (-613 + 3208 v) + g (82944 (-645 + 5527 v) + g (1536 (9533 + 222490 v) + g (-128 (222743 + 4023819 v) + g (80 (-411731 + 841481 v) + g (62006492 + 162714972 v + g (-7331034 - 67937578 v + g (-4 (5722996 + 4162553 v) + g (7826826 + 14727965 v + g (3239754 - 942065 v + g (-2 (873587 + 629385 v) + g (-145343 + 310088 v + g (168316 + 26930 v + g (-6867 - 21296 v + g (-6842 + 1624 v + g (787 + 480 v + g (34 - 59 v + g (-23 + 2 g + v)))))))))))))))))))))/W. (5.A1.25)

Substituting now the above optimal quantities of the first period as well as the optimal wages in equations (5.A1.1), (5.A1.2), (5.A1.16), and (5.A1.17) accrues the optimal yields for the participants of the game.

$$EV_1^*\left(v,c,g\right) = -c\Big(\frac{(q_{10}^* - q_{11}^*)^2}{2} - (q_{10}^* - q_{11}^*)\Big) + q_{10}^*(1 - q_{10}^* - gq_{20}^*) + q_{11}^*(-q_{11}^* - gq_{21}^* + v) \tag{5.A1.26}$$

$$-(q_{10}^* + q_{11}^*)w_{10}^* + (1+g)\frac{(q_{10}^* + q_{11}^* + q_{20}^* + q_{21}^*)^2}{4},$$

$$\Pi_2^*\left(v,c,g\right) = (1 - gq_{10}^* - q_{20}^*)q_{20}^* - c\frac{(q_{20}^* - q_{21}^*)^2}{2} + q_{21}^*(v - gq_{11}^* - q_{21}^*) - (q_{20}^* + q_{21}^*)w_{20}^*, \tag{5.A1.27}$$

$$U_1^*\left(v,c,g\right) = \left(q_{10}^* + q_{11}^*\right)w_{10}^*, \tag{5.A1.28}$$

$$U_2^*\left(v,c,g\right) = \left(q_{20}^* + q_{21}^*\right)w_{20}^*. \tag{5.A1.29}$$

APPENDIX 5.A2

Case *frr_2*

Second period/ fourth stage public firm maximizes $EV_1=\Pi_1+CS$ as for q_{11} and private firms maximazizes its profits as for q_{21};

$$\underset{q_{11}}{Max}\, EV_1\left(q_{10},q_{20},q_{11},q_{21},w_{10},w_{20}\right)=-c(\frac{(q_{10}-q_{11})^2}{2}-(q_{10}-q_{11}))+q_{10}(1-q_{10}-gq_{20})+ \tag{5.A2.1}$$

$$q_{11}(-q_{11}-gq_{21}+v)-(q_{10}+q_{11})w_{10}+(1+g)\frac{(q_{10}+q_{11}+q_{20}+q_{21})^2}{4},$$

$$\underset{q_{21}}{Max}\,\Pi_2\left(q_{10},q_{20},q_{11},q_{21},w_{10},w_{20}\right)=(1-gq_{10}-q_{20})q_{20}+q_{21}(v-gq_{11}-q_{21})-(q_{20}+q_{21})w_{20}\,. \tag{5.A2.2}$$

From the *foc* accrues reaction functions of the second period;

$$RF_{11}\left(q_{21}\right)=\frac{2\left(v+c\right)+\left(1+2c+g\right)q_{10}+\left(1+g\right)q_{20}+\left(1-g\right)q_{21}-2w_{10}}{3+2c-g}\ , \tag{5.A2.3}$$

$$RF_{21}\left(q_{11}\right)=\ \frac{v-gq_{11}-w_{20}}{2}\,. \tag{5.A2.4}$$

I solve the system of the second period RFs to get the optimal q_{11}^* and q_{21}^* -rules in the second period;

$$q_{11}^*=\frac{\left(5-g\right)v+4c-2\left(1+g-2c\right)q_{10}+2\left(1+g\right)q_{20}-4w_{10}+\left(g-1\right)w_{20}}{-6-4c+g+g^2}, \tag{5.A2.5}$$

$$q_{21}^*=\ \frac{3v\left(1-g\right)-2c(g-v)-g\left(1+g+2c\right)q_{10}-g\left(1+g\right)q_{20}+2gw_{10}-\left(3-g+2c\right)w_{20}}{4c-\left(-2+g\right)\left(3+g\right)}\,. \tag{5.A2.6}$$

Substituting the later into (5.A2.1) and (5.A2.2) accrues EV_1 and Π_2 that depend on products of the first period and wages;

$$\begin{aligned}&EV_1(q_{10},q_{20},w_{10},w_{20})=-q_{10}\ (-1+q_{10}+g\ q_{20})+((1+g)\ ((-2+g)\ ((2+g)\ q_{10}+(2+g)\ q_{20}+2\ v\\&-w_{10}-w_{20})+c\ (-2+g-4\ q_{10}+g\ q_{10}-2\ q_{20}-v+w_{20}))^2)/K^2-w_{10}\ (q_{10}-(2\ (1+g)\ q_{10}+4\ c\ (1\\&+q_{10})+2\ q_{20}+2\ g\ q_{20}+5\ v-g\ v-4\ w_{10}+(-1+g)\ w_{20})/K)-c\ (q_{10}+(2\ (1+g)\ q_{10}+4\ c\ (1+\\&q_{10})+2\ q_{20}+2\ g\ q_{20}+5\ v-g\ v-4\ w_{10}+(-1+g)\ w_{20})/K+(4\ c+(-1+g)\ (4+g)\ q_{10}+2\ (1\\&+g)\ q_{20}+5\ v-g\ v-4\ w_{10}+(-1+g)\ w_{20})^2/(2\ (-6-4\ c+g+g^2)^2))+(1/(K^2))(2\ (1+g)\ q_{10}+\\&4\ c\ (1+q_{10})+2\ q_{20}+2\ g\ q_{20}+5\ v-g\ v-4\ w_{10}+(-1+g)\ w_{20})\ ((1+g)\ (-2+g^2)\ q_{10}-2\ q_{20}+\\&v+4\ w_{10}+w_{20}+2\ c\ (-2\ (1+q_{10}-v)+g\ (g+g\ q_{10}-v+w_{20}))+g\ ((-2+g+g^2)\ q_{20}+(-3+2\\&g)\ v+2\ w_{20}-g\ (2\ w_{10}+w_{20}))),\end{aligned} \tag{5.A2.7}$$

where $K=(-6-4c+g+g^2)$,

$$\begin{aligned}&\Pi_2(q_{10},q_{20},w_{10},w_{20})=-q_{20}\ (-1+g\ q_{10}+q_{20})+((-3\ v+g\ (q_{10}+g\ q_{10}+q_{20}+g\ q_{20}+3\ v-2\ w_{10}\\&-w_{20})+3\ w_{20}+2\ c\ (g+g\ q_{10}-v+w_{20}))\ (g\ (q_{10}+q_{20}+3\ v-2\ w_{10})+2\ c\ (g+g\ q_{10}-v-\end{aligned} \tag{5.A2.8}$$

$w_{20}) + g^2 (q_{10} + q_{20} + w_{20}) - 3 (v + w_{20})))/(-6 - 4 c + g + g^2)^2 - w_{20} (q_{20} - (-3 v + g (q_{10} + g q_{10} + q_{20} + g q_{20} + 3 v - 2 w_{10} - w_{20}) + 3 w_{20} + 2 c (g + g q_{10} - v + w_{20}))/K)$.

First period/ third stage e public firm maximizes $EV_1=\Pi_1+CS$ as for q_{10} and private firms maximazizes its profits as for q_{20}, respectively. Reaction functions of the first period are

$$RF_{10}\left(q_{20}\right) = L1 + M1q_{20} + N1w_{10} + O1w_{20}, \quad (5.A2.9)$$

where $L1>0$, $M1>0$, $N1<0$, and $O1<0$, where

$$L1 = -(((-6+g+g^2)^2 + (1+g)(24+g(-20+g+3g^2))v + 2c^2(g(1+g)(2+g-v)+12v) + c(24+52v+g(6-7v+g(g(2+g+v)-5(1+2v)))))) / P1 > 0,$$

$$P1 = 2(-3+g+g^2)(8+g(-4+g+g^2)) + 2c^2(-16+g(8+g+g^2)) + c(-80+g(56+(-1+g)g(1+3g))),$$

$$M_1 = -24 - 2c^2(8-2g(1+g)) + g(20-g(-8+g(3+g(4+g)))) - c(40+g(-22+g(-12+g(4+2g)))) / P1 > 0,$$

$$N1 = (48+32c^2+g(-10-g(19+(-6-3g)g)) - c(-80+g(12+(10-2g)g))) / P1,$$

$$O1 = (12-2c^2(-4+g(1+g)) + g(2-g(11-g^2)) - c(-20+g(3+g(8+g)))) / O1.$$

$$RF_{20}\left(q_{10}\right) = L2 + M2q_{10} + N2w_{20} + O2w_{10}, \quad (5.A2.10)$$

where, $L2>0$, $M2<0$, $O2<0$, and $N2<0$, where $A2>L2$

$$L2 = -\left(16c^2 + (-6+g+g^2)^2 + 4c(12+g(1+g)(-2+g-v)) + 6g(-1+g^2)v\right) / P2 > 0,$$

$$M2 = -(-16c^2g + 4cg(-12+g(3+3g)) - g(36+g(-14+g(-15+g^2)))) / P2 < 0,$$

$$P2 = 8(3+2c)(-3-2c+g+g^2),$$

$$N2 = -((-36-16c^2+4c(-12+g(3+3g)) - g(-18+g(-15+g(4+g)))) / P2) < 0,$$

$$O2 = (g^2(4+4g)) / P2 < 0.$$

We solve the system of the first period RFs to get the optimal q_{10}^* and q_{20}^* -rules in the first period;

$$q_{10}^* = (-288 (2 + v - 2 w_{10} - w_{20}) - 16 c^3 (4 (1 + 3 v - 4 w_{10} - 2 w_{20}) + g (1 + g) (1 + g - v + 2 w_{20})) + 4 c^2 (-16 (7 + 11 v - 16 w_{10} - 8 w_{20}) + g (8 + 36 v - 40 w_{10} - 52 w_{20} + g (11 + 40 v - 36 w_{10} - 49 w_{20} + g (-14 - 6 v + 4 w_{10} + 8 w_{20} + g (1 + 2 g - 2 v + 5 w_{20}))))) + 2 c (24 (-19 - 17 v + 28 w_{10} + 14 w_{20}) + g (2 (81 + 49 v - 96 w_{10} - 88 w_{20}) + g (157 + 186 v - 204 w_{10} - 179 w_{20} + g (-55 - 50 v + 48 w_{10} + 53 w_{20} + g (-20 - 14 v + 8 w_{10} + 32 w_{20} - g (-9 - 4 v + 4 w_{10} + g (-3 + w_{20}) + w_{20})))))) + g (24 (15 + v - 9 w_{10} - 7 w_{20}) + g (4 (61 + 69 v - 76 w_{10} - 54 w_{20}) + g (-94 - 90 v + 104 w_{10} + 80 w_{20} + g (-53 - 42 v + 36 w_{10} + 59 w_{20} + g (1 + 18 v - 16 w_{10} - 3 w_{20} + g (5 + g + 6 v - 4 w_{10} - (7 + g) w_{20})))))))/Q, \quad (5.A2.11)$$

where $Q= (32 c^3 (-8 + g (2 + g + g^2)) + (-2 + g)^2 (3 + g) (4 + g) (-12 + g (4 + 3 g + g^3)) - 4 c^2 (256 + g (-112 + g (-48 + g (-17 + g (4 + 5 g))))) + 2 c (-672 + g (408 + g (172 + g (-33 + g (-5 + g (-20 + (-3 + g) g)))))))$,

$$q_{20}^* = (8 c^3 (g (g (1 + g) (3 + g - v) + 4 (2 + 3 v - 4 w_{10} - 3 w_{20})) + 16 (-1 + w_{20})) + 288 (-1 + w_{20}) - 2 c^2 (-256 (-1 + w_{20}) + g (-32 (7 + 6 v - 8 w_{10} - 8 w_{20}) + g (4 (-5 + 9 v - 14 w_{10}) + g (11 + 57 v - 52 w_{10} - 44 w_{20} + g (-13 + g (5 + g - v) - 4 v + 4 w_{10} + 4 w_{20}))))) + g (24 (21 + 8 v - 12 w_{10} - 17 w_{20}) + g (-4 (25 + 10 v - 39 w_{10} + 4 w_{20}) + g (-226 - 208 v + 194 w_{10} + 240 w_{20} + g (63 + 69 v - 75 w10 - 57 w_{20} + g (41 + 35 v - 33 w_{10} - 43 w_{20} + g (-7 - 13 v + 11 w_{10} + 9 w_{20} + 3 g (-1 - v + w_{10} + w_{20}))))))) + c (672 (-1 + w_{20}) + g (8 (108 + 61 v - 84 w_{10} - 103 w_{20}) + g (-66 - 122 v + 272 w_{10} + g (-253 - 289 v + 280 w_{10} + 292 w_{20} - g (-78 - 59 v + 66 w_{10} + 54 w_{20} + g (-14 - 25 v + 24 w_{10} + 24 w_{20} + g (g + v - 2 (-6 + w10 + w20)))))))))/Q. \qquad (5.A2.12)$$

Substituting the later into (5.A2.7) and (5.A2.8) accrues EV_1 and Π_2 that depend on wages only;

$$EV_1(w_{10},w_{20})=(1024 c^7 (-8 + g (2 + g + g^2))^2 -c (-192 g^2 (-243 - 5174 v + 2265 v^2 - 408 w_{10} - 1496 v w_{10} +20 w_{10}^2 + 8 (541 + 503 v + 215 w_{10}) w_{20} - 5036 w_{20}^2) -64 g^3 (39429 + 46203 v^2 + 6 w_{10} (-14327 + 7338 w_{10}) +v (93076 - 91168 w_{10} - 93396 w_{20}) - 86854 w_{20} +89110 w_{10} w_{20} + 45534 w_{20}^2) +g^{14} (-93 + 7 v^2 + 80 w_{10} - 2 v (25 + 4 w_{10}) + 88 w_{20} -8 w_{20} (2 w10 + w_{20})) +13824 g (221 + 285 v^2 + 304 w_{10}^2 +v (398 - 528 w_{10} - 392 w_{20}) + 8 w_{20} (-47 + 21 w_{20}) +16 w_{10} (-32 + 27 w_{20})) +2 g^{13} (-91 + 41 v_2 + 56 w_{10}^2 + 22 w_{20} + 60 w_{20}^2 -2 v (39 + 47 w_{10} + 45 w_{20}) + 2 w_{10} (31 + 56 w_{20})) -27648 (69 + 81 v^2 + 112 w_{10}^2 + 56 (-2 + w_{20}) w_{20} +56 w_{10} (-3 + 2 w_{20}) - 2 v (-65 + 84 w_{10} + 56 w_{20})) +4 g^{11} (445 + 11 v^2 - 126 w_{10}^2 + (163 - 554 w_{20}) w_{20} +v (420 + 89 w_{10} + 425 w_{20}) - w_{10} (337 + 536 w_{20})) +g^{12} (2169 + 789 v^2 + 1004 w_{10}^2 + 56 w_{10} (-48 + 43 w_{20}) +4 w_{20} (-794 + 277 w_{20}) - 2 v (-1057 + 900 w_{10} + 964 w_{20})) - g^{10} (25303 + 17779 v2 + 19916 w_{10}^2 +v (38462 - 37112 w_{10} - 37720 w_{20}) +20 w_{20} (-2220 + 973 w_{20}) + 8 w_{10} (-5322 + 5111 w_{20})) +g^9 (4410 + 3042 v^2 - 3712 w_{10}^2 +12 v (471 + 61 w_{10} - 2725 w_{20}) + 4 w_{10} (109 + 6724 w_{20}) +4 w20 (-9007 + 7818 w_{20})) +g^8 (166535 + 164251 v^2 + 179028 w_{10}^2 +v (326238 - 340408 w_{10} - 314040 w_{20}) +4 w_{20} (-77226 + 35867 w_{20}) + 8 w_{10} (-42480 + 40459 w_{20})) +16 g^5 (56503 + 62919 v^2 + 33522 w_{10}^2 +v (121198 - 99401 w_{10} - 153227 w_{20}) +5 w_{20} (-27954 + 16711 w^{20}) + 3 w_{10} (-30820 + 42387 w_{20})) +16 g^4 (59007 + 82374 v^2 + 103048 w_{10}^2 +w_{20} (-97412 + 25827 w_{20}) + 4 w_{10} (-42547 + 38253 w_{20}) -2 v (-72209 + 95672 w_{10} + 54371 w_{20})) -4 g^7 (36866 + 34076 v^2 + 2150 w_{10}^2 +v (63034 - 37155 w_{10} - 108207 w_{20}) +w_{20} (-106649 + 71162 w_{20}) + w_{10} (-38197 + 82920 w_{20})) -4 g^6 (147879 + 178401 v^2 + 205725 w_{10}^2 +v (343284 - 390868 w_{10} - 304898 w_{20}) +w_{20} (-273362 + 118249 w_{20}) + w_{10} (-357544 + 333134 w_{20}))) -256 c^6 (-11 g^6 + 3 g^8 + g^7 (1 + 2 v) +4 g^2 (83 + 34 v + 19 v^2 - 20 w_{10} - 20 v w_{10} -8 (7 + 7 v - 5 w10) w_{20} + 36 w_{20}^2) -8 g^3 (5 v^2 + v (3 - 7 w10 - 13 w_{20}) - 3 (7 + w_{10} + 3 w_{20}) +2 w_{20} (7 w_{10} + 3 w_{20})) -64 (29 + 5 v^2 + 8 w1_0^2 + 4 (-2 + w_{20}) w_{20} +4 w_{10} (-3 + 2 w_{20}) - 2 v (-7 + 6 w_{10} + 4 w_{20})) +32 g (39 + 7 v^2 + 8 w_{10}^2 + 16 w_{10} (-1 + w_{20}) +4 (-3 + w_{20}) w_{20} - 2 v (-11 + 8 w_{10} + 6 w_{20})) +g^5 (-89 - 6 v^2 + 8 w_{10} (3 - 2 w_{20}) - 8 (-4 + w_{20}) w_{20} +2 v (-13 + 4 w_{10} + 8 w_{20})) -2 g^4 (5 v^2 - 4 v (-9 + 2 w_{10} + 3 w_{20}) +2 (37 + 8 w_{10} (-1 + w_{20}) + 2 (-3 + w_{20}) w_{20}))) +2 (g^{15} (8 + 2 v^2 + 4 (-3 + w_{10}) w_{10} + v (9 - 6 w_{10} - 7 w_{20}) -13 w_{20} + 10 w_{10} w_{20} + 5 w_{20}^2) -g^8 (37979 + 45833 v^2 + 6 w_{10} (-14423 + 7556 w_{10}) +v (87862 - 90738 w_{10} - 88790 w_{20}) - 77282 w_{20} +86604 w_{10} w_{20} + 39734 w_{20}^2) +82944 (3 + 3 v^2 + 4 w10^2 + v (4 - 6 w_{10} - 4 w_{20}) +2 (-2 + w_{20}) w_{20} + w_{10} (-6 + 4 w_{20})) -13824 g (33 + 35 v2 + 36 w_{10}^2 + v (38 - 62 w_{10} - 46 w_{20}) +4 w_{20} (-11 + 5 w_{20}) + 10 w_{10} (-6 + 5 w_{20})) -g^{11} (-1160 + 1096 v^2 + 756 w_{10}^2 +v (173 - 1854 w_{10} - 511 w_{20}) + (1895 - 737 w_{20}) w_{20} +18 w_{10} (14 + 5 w_{20})) +g^{13} (81 v^2 + 8 w_{10} (14 + 3 w_{10}) - 54 w_{10} w_{20} - 72 w_{20}2 +71 (-2 + 3 w_{20}) - v (41 + 106 w_{10} + 15 w_{20})) +g^{14} (41 + 36 v^2 + 40 w_{10}^2 + v (84 - 78 w_{10} - 78 w_{20}) +w_{20} (-80 + 37 w_{20}) + w_{10} (-86 + 84 w_{20})) -g^{12} (869 + 665 v2 + 720 w_{10}^2 + 2 w_{20} (-845 + 408 w_{20}) -2 v (-827 + 695 w_{10} + 797 w_{20}) + 2 w_{10} (-851 + 826 w_{20})) +192 g^3 (1863 + 2136 v^2 + 2166 w_{10}^2 +v (4270 - 4291 w_{10} - 4251 w_{20}) + w_{20} (-3931 + 2079 w_{20}) +w_{10} (-4065 + 4024 w_{20})) +g^9 (-6838 + 3749 v^2 + 8 w_{10} (175 + 666 w_{10}) + 14843 w_{20} -2962 w_{10} w_{20} - 8022 w_{20}^2 +v (-2567 - 9094 w_{10} + 4163 w_{20})) -16 g^4 (10953 + 14546 v^2 + 8 w10 (-3333 + 2029 w_{10}) -16506 w_{20} +$$

$25248 w_{10} w_{20} + 5283 w_{20}^2 -4 v (-5316 + 7762 w_{10} + 4827 w_{20})) +4 g^7 (8717 + 4412 v^2 -$
$1282 w_{10}^2 +v (14245 - 3170 w_{10} - 19899 w_{20}) +9 w20 (-2579 + 1606 w_{20}) + 2 w_{10} (-4234 +$
$7101 w_{20})) +g^{10} (7771 + 7330 v^2 + 7324 w_{10}^2 + 3 w_{20} (-5140 + 2607 w_{20}) +2 w_{10} (-7901 +$
$7818 w_{20}) -2 v (-7840 + 7241 w_{10} + 7929 w_{20})) -16 g^5 (8630 + 9640 v^2 + 6071 w_{10}^2 -4 v (-$
$5381 + 4044 w_{10} + 6157 w_{20}) +w_{20} (-22781 + 13686 w_{20}) + w_{10} (-16003 + 20037 w_{20})) +4$
$g^6 (27339 + 37671 v^2 + 40393 w_{10}^2 +v (68870 - 79328 w_{10} - 64884 w_{20}) +w_{20} (-53940 +$
$25337 w_{20}) + w_{10} (-69608 + 68150 w_{20})) +576 g^2 (105 + 147 v^2 - 82 v (4 + w_{10}) + 136 w20$
$+ 116 v w_{20} -2 (w_{10} (9 + w_{10}) - 52 w_{10} w_{20} + 89 w_{20}^2))) +4 c^3 (g^{12} (90 - 6 v + v^2 - 16 w_{10} - 16$
$w_{20}) +32 g^3 (22607 + 24361 v^2 + 8 w_{10} (-5340 + 2401 w_{10}) +v (48152 - 45682 w_{10} - 51044$
$w_{20}) - 47930 w_{20} +50306 w_{10} w_{20} + 24530 w_{20}^2) -2 g^{11} (65 + 16 v^2 + 2 w_{20} (-77 + 10 w_{20}) -2$
$v (-48 + 7 w_{10} + 17 w_{20}) + 2 w_{10} (-63 + 20 w_{20})) +g^{10} (-995 - 278 v^2 - 156 w_{10}^2 + w_{10} (528 -$
$72 w_{20}) +4 w_{20} (50 + w_{20}) + 8 v (-22 + 51 w_{10} + 32 w_{20})) -4 g^9 (-595 + 138 v^2 + 396 w_{10}^2 +$
$10 w_{20} (82 + 7 w_{20}) -4 v (124 + 131 w_{10} + 37 w_{20}) + 4 w_{10} (116 + 39 w_{20})) -64 g^2 (4261 +$
$978 v^2 + 4 w_{10} (-299 + 90 w_{10}) - 8544 w_{20} +1816 w_{10} w_{20} + 7684 w_{20}^2 -4 v (-826 + 245 w_{10}$
$+ 2160 w_{20})) +2 g^7 (-1337 + 9408 v^2 + 6648 w_{10}^2 +6 w_{20} (-1605 + 1732 w_{20}) + 2 w_{10} (-2803$
$+ 8872 w_{20}) -2 v (-3502 + 8235 w_{10} + 9623 w_{20})) +g^6 (60257 + 43948 v2 + 79428 w_{10}^2 +88$
$w_{10} (-1345 + 1027 w_{20}) + 4 w_{20} (-19834 + 6475 w_{20}) -4 v (-21587 + 30476 w_{10} + 16948$
$w_{20})) -8 g^4 (21121 + 17695 v^2 + 36552 w_{10}^2 -4 w_{20} (4747 + 483 w_{20}) -2 v (-19618 + 28889$
$w_{10} + 8674 w_{20}) +w_{10} (-55906 + 39640 w_{20})) -8 g^5 (16195 + 23967 v^2 + 10708 w_{10}^2 +v$
$(36575 - 36508 w_{10} - 52937 w_{20}) +w_{20} (-45891 + 27166 w_{20}) + w_{10} (-29258 + 47644 w_{20}))$
$+3072 (291 + 336 v^2 - 496 w_{20} - 8 v (-89 + 93 w10 + 62 w_{20}) +248 (w_{10} (-3 + 2 w_{10}) + 2 w_{10}$
$w_{20} + w_{20}^2)) -g^8 (1535 v^2 + v (5430 - 6728 w_{10} - 4016 w_{20}) +8 (238 + w_{10} (-817 + 713 w_{10}) -$
$874 w_{20} + 824 w_{10} w_{20} +370 w_{20}^2)) -512 g (2741 v^2 + v (5552 - 5522 w_{10} - 4108 w_{20}) +2$
$(993 + 6 w_{10} (-449 + 258 w_{10}) - 1987 w_{20} + 2359 w_{10} w_{20} +841 w_{20}^2))) -16 c^4 (g^{11} (-20 + 3$
$v) -16 g^3 (4463 + 4591 v^2 + 8 w_{10} (-956 + 385 w_{10}) +v (8754 - 8314 w_{10} - 9914 w_{20}) - 9244$
$w_{20} + 10034 w_{10} w_{20} +4704 w_{20}^2) +g^{10} (2 (-1 + v) v - 15 (-3 + 2 w10 + 2 w_{20})) -2 g^8 (165 +$
$131 v^2 + 48 w_{10}^2 + v (217 - 186 w_{10} - 174 w_{20}) +4 w_{10} (-77 + 36 w_{20}) + w_{20} (-220 + 39 w_{20}))$
$+2 g^9 (-31 v^2 - 33 w_{20} (2 w_{10} + w_{20}) +9 (3 + 12 w_{10} + 17 w_{20}) + v (-153 + 34 w_{10} + 67 w_{20}))$
$+g^6 (-1043 + 16 v^2 - 2848 w_{10}^2 + w_{10} (1502 - 304 w_{20}) +4 v (355 + 573 w_{10} - 169 w_{20}) + 2$
$w_{20} (139 + 69 w_{20})) +512 g (328 + 320 v^2 + 368 w_{10}^2 + v (753 - 671 w10 - 500 w_{20}) +w_{20} (-$
$487 + 197 w_{20}) + w_{10} (-658 + 593 w_{20})) +4 g^4 (2260 + 789 v^2 + 5064 w_{10}^2 + (492 - 1919$
$w_{20}) w_{20} +30 w_{10} (-199 + 82 w_{20}) + v (910 - 6134 w_{10} + 1062 w_{20})) +32 g^2 (1397 + 687 v^2 +$
$8 w_{10} (-83 + 19 w_{10}) - 2720 w_{20} +1144 w_{10} w_{20} + 2178 w_{20}^2 -2 v (-533 + 348 w_{10} + 1390$
$w_{20})) +2 g^5 (711 + 6344 v2 + 3008 w_{10}^2 +v (8165 - 9968 w_{10} - 13179 w_{20}) + 2 w_{10} (-3383 +$
$6371 w_{20}) +w_{20} (-10081 + 6462 w20)) -512 (339 + 279 v^2 + v (658 - 636 w_{10} - 424 w_{20}) -$
$424 w_{20} +212 (w_{10} (-3 + 2 w_{10}) + 2 w_{10} w_{20} + w_{20}^2)) -g^7 (354 v^2 - 3 v (455 + 388 w_{10} + 96$
$w_{20}) +2 (-736 + 432 w_{10}^2 + w_{20} (776 + 61 w_{20}) +w_{10} (334 + 228 w_{20})))) +4 c^2 (g^7 (20741 +$
$41872 v^2 + 4 w_{10} (-8698 + 3689 w_{10}) +v (53641 - 58488 w_{10} - 105102 w_{20}) - 84550 w_{20}$
$+87444 w_{10} w_{20} + 62534 w_{20}^2) +g^{11} (-999 + 156 v^2 + 284 w_{10}^2 + 4 w_{10} (132 + 65 w_{20}) +2$
$w_{20} (331 + 72 w_{20}) - v (309 + 440 w_{10} + 262 w_{20})) -g^9 (-4286 + 2749 v^2 + 2944 w_{10}^2 +v (129$
$- 5712 w_{10} - 6736 w_{20}) + 108 w_{10} (11 + 56 w_{20}) +12 w_{20} (-52 + 375 w_{20})) +16 g^2 (-11979 +$
$3777 v^2 - 448 w_{10}^2 -2 v (8581 + 1306 w_{10} - 14288 w_{20}) +8 (3637 - 3712 w_{20}) w_{20} + 4 w_{10}$
$(123 + 430 w_{20})) -1152 g (969 + 1417 v^2 + 1560 w_{10}^2 +v (2422 - 2740 w_{10} - 2036 w_{20}) + 8$
$w_{20} (-245 + 107 w_{20}) +4 w_{10} (-666 + 571 w_{20})) +2 g^{10} (724 + 473 v^2 + 754 w_{10}^2 -6 v (-227 +$
$201 w_{10} + 200 w_{20}) + w_{20} (-1941 + 754 w_{20}) +w_{10} (-1569 + 1624 w_{20})) +16 g^3 (58503 +$
$66779 v^2 + 58664 w_{10}^2 +2 w_{20} (-64159 + 33244 w_{20}) -4 v (-33471 + 32182 w_{10} + 34284$
$w_{20}) +2 w_{10} (-60485 + 66281 w_{20})) -4 g^4 (69406 + 84473 v^2 + 125464 w_{10}^2 +w_{20} (-100948 +$
$16853 w_{20}) -2 v (-85267 + 110409 w_{10} + 52673 w_{20}) +2 w_{10} (-102375 + 84814 w20)) +2 g^6$
$(74413 + 73465 v^2 + 96910 w_{10}^2 +w_{20} (-120829 + 46895 w_{20}) -2 v (-73250 + 86261 w_{10} +$
$61172 w_{20}) +w_{10} (-164091 + 142360 w_{20})) -2 g^5 (134181 + 159856 v^2 + 75392 w_{10}^2 +v$
$(271585 - 244432 w_{10} - 370819 w_{20}) +w_{20} (-332193 + 196850 w_{20}) + w_{10} (-212094 +$

$$318062\ w_{20})) +g^{13} (v^2 + v (15 - 2 w_{20}) +2 (31 + 2 w_{10} (-8 + w_{20}) + (-17 + w_{20}) w_{20})) +g^{12} (29 v^2 + 16 w_{10}^2 + w_{10} (38 - 32 w_{20}) -4 v (2 + 11 w_{10} + 3 w_{20}) + 3 (5 + 34 w_{20} - 6 w_{20}^2)) +6912 (119 + 151 v^2 - 216 w_{20} - 6 v (-47 + 54 w_{10} + 36 w_{20}) +108 (w_{10} (-3 + 2 w_{10}) + 2 w_{10} w_{20} + w_{20}^2)) -g^8 (21869 v^2 + 29168 w_{10}^2 +v (46668 - 50084 w_{10} - 41844 w_{20}) +w_{10} (-51930 + 47248 w_{20}) +3 (8675 + 2 w_{20} (-7997 + 3370 w_{20})))) +16 c^5 (9 g^{10} + 4 g^9 (-16 + 5 v) +32 g^3 (281 + 397 v^2 + 4 w_{10} (-145 + 48 w_{10}) +v (668 - 678 w10 - 904 w_{20}) - 806 w_{20} + 946 w_{10} w_{20} +422 w_{20}^2) + 2 g^8 (2 v (1 + v) - 3 (23 + 8 w_{10} + 8 w_{20})) -8 g^5 (-538 + 101 v^2 + 80 w_{10}^2 + 64 w_{20} + 68 w_{20}^2 +24 w_{10} (1 + 8 w_{20}) - 6 v (6 + 31 w_{10} + 27 w_{20})) +1024 (91 + 41 v^2 + 64 w_{10}2 + 32 (-2 + w_{20}) w_{20} +32 w_{10} (-3 + 2 w_{20}) - 2 v (-53 + 48 w_{10} + 32 w_{20})) -1024 g (75 + 38 v^2 + 44 w_{10}^2 + v (103 - 83 w_{10} - 62 w_{20}) +w_{20} (-61 + 23 w_{20}) + w_{10} (-82 + 77 w_{20})) +8 g^4 (475 + 159 v^2 - 264 w_{10}^2 + v (820 + 54 w_{10} - 500 w_{20}) +36 w_{20} (-13 + 8 w_{20}) + w_{10} (-58 + 344 w_{20})) -g^6 (375 + 340 v^2 - 656 w_{20} - 4 v (-305 + 132 w_{10} + 160 w_{20}) +64 ((-14 + w_{10}) w_{10} + 11 w_{10} w_{20} + 3 w_{20}^2)) -64 g^2 (287 + 141 v^2 - 440 w_{20} -2 v (-107 + 74 w_{10} + 224 w_{20}) +4 (5 (-7 + w_{10}) w_{10} + 64 w_{10} w_{20} + 79 w_{20}^2)) -4 g^7 (36 v^2 + v (181 - 44 w_{10} - 84 w_{20}) +4 (4 w_{10} (-7 + 5 w_{20}) +5 (5 + 2 (-4 + w_{20}) w_{20})))))/2Q^2, \tag{5.A2.13}$$

$$\Pi_2(w_{10},w_{20})=((4 c - (-2 + g) (3 + g)) (g^3 (18 + 18 v - 17 w_{10} +c (15 - 2 (-4 + c) v - 14 w_{10} - 10 w_{20}) - 19 w_{20}) +16 (1 + c) (3 + 2 c) (v - w_{20}) + g^5 (-2 + c + w_{10} + w_{20}) +g^4 (c (-1 + 2 c + 3 v - 2 w_{10} - 2 w_{20}) +2 (-2 v + w_{10} + w_{20})) +g^2 (-8 + 6 v - 6 w_{10} + 8 w_{20} +c (13 - 2 c (-3 + v) + v - 4 w_{10} + 4 w_{20})) +4 g (-6 - 19 v + 12 w_{10} + 13 w_{20} +c (-19 - 26 v + 20 w_{10} + 19 w_{20} +c (-10 - 8 v + 8 w_{10} + 6 w_{20})))) (-g^8 w_{20} +32 (1 + c) (3 + 2 c)^2 (v + w_{20}) -g^7 (-2 + c + w_{10} + 2 (2 + c) w_{20}) -8 (3 + 2 c) g (3 (2 + 7 v - 4 w_{10} + w_{20}) +c (19 + 28 v - 20 w_{10} + 2 c (5 + 4 v - 4 w_{10} - w_{20}) + w_{20})) +g^6 (2 + 4 v - 3 w_{10} + 6 w_{20} +c (-2 c - 3 v + 2 w_{10} + 8 w_{20})) +g^4 (-10 - 48 v + 35 w_{10} + 11 w_{20} +c (-34 - 7 v + 14 w_{10} + 12 w_{20} +2 c (1 + 4 c + 8 v - 4 w_{10} + 4 w_{20}))) +g^5 (-30 - 14 v + 21 w_{10} + 30 w_{20} +c (-16 - 11 v + 20 w_{10} + 56 w_{20} + 2 c (1 + v + 10 w_{20}))) -g^3 (2 (-70 - 89 v + 72 w_{10} + 15 w_{20}) +c (-223 v + 3 (-75 + 76 w_{10} + 50 w_{20}) +2 c (-47 + (-27 + 4 c) v + 44 w_{10} + 66 w_{20} +16 c w_{20}))) -2 g^2 (12 - 32 v + 42 w_{10} + 82 w_{20} +c (-61 - 27 v + 64 w_{10} + 142 w_{20} +4 c (-16 + v + 6 w_{10} + 21 w_{20} +c (-3 + v + 4 w_{20}))))) + (32 c^3 (-8 +g (2 + g + g^2)) + (-2 + g)^2 (3 + g) (4 + g) (-12 +g (4 + 3 g + g^3)) -4 c^2 (256 + g (-112 + g (-48 + g (-17 + g (4 + 5 g))))) +2 c (-672 +g (408 +g (172 +g (-33 +g (-5 +g (-20 + (-3 + g) g))))))) w_{20} (-32 c^3 (g^3 +g (7 + 7 v - 8 w_{10} - 6 w_{20}) - 4 (1 + v - 2 w_{20})) +288 (1 + v - 2 w_{20}) +4 c^2 (128 (1 + v - 2 w_{20}) +g (-8 (31 + 32 v - 32 w_{10} - 31 w_{20}) +g (22 + 16 v - 40 w_{10} +g (29 + 42 v + g (-6 + 3 g + 2 v) - 48 w_{10} - 38 w_{20}) +6 w_{20}))) +2 c (336 (1 + v - 2 w_{20}) +g (-708 - 748 v + 672 w_{10} + 784 w_{20} +g (94 + 88 v - 200 w_{10} + 30 w_{20} +g (239 + 256 v - 254 w_{10} - 254 w_{20} +g (-56 - 33 v + 40 w_{10} + 28 w_{20} +g (-15 - g (-6 + v) - 18 v + 22 w_{10} +20 w_{20})))))) +g (24 (-27 - 29 v + 24 w_{10} + 32 w_{20}) +g (4 (19 + 26 v - 60 w_{10} + 15 w_{20}) +g (366 + 386 v - 338 w_{10} - 414 w_{20} +g (-73 - 117 v + 110 w_{10} + 80 w_{20} +g (-71 - 49 v + 54 w_{10} + 66 w_{20} +g (9 + 5 g + 17 v + 3 g v - 14 w_{10} - 4 g w_{10} -4 (3 + g) w_{20}))))))) + (8 c^3 (g (g (1 + g) (3 +g - v) + 4 (2 + 3 v - 4 w_{10} - 3 w_{20})) +16 (-1 + w_{20})) + 288 (-1 + w_{20}) -2 c^2 (-256 (-1 + w_{20}) +g (-32 (7 + 6 v - 8 w_{10} - 8 w_{20}) +g (4 (-5 + 9 v - 14 w_{10}) +g (11 + 57 v - 52 w_{10} - 44 w_{20} +g (-13 + g (5 + g - v) - 4 v + 4 w_{10} + 4 w_{20}))))) +g (24 (21 + 8 v - 12 w_{10} - 17 w_{20}) +g (-4 (25 + 10 v - 39 w_{10} + 4 w_{20}) +g (-226 - 208 v + 194 w_{10} + 240 w_{20} +g (63 + 69 v - 75 w_{10} - 57 w_{20} +g (41 + 35 v - 33 w_{10} - 43 w_{20} +g (-7 - 13 v + 11 w_{10} + 9 w_{20} +3 g (-1 - v + w_{10} + w_{20}))))))) +c (672 (-1 + w_{20}) +g (8 (108 + 61 v - 84 w_{10} - 103 w_{20}) +g (-66 - 122 v + 272 w_{10} +g (-253 - 289 v + 280 w_{10} + 292 w_{20} -g (-78 - 59 v + 66 w_{10} + 54 w_{20} +g (-14 - 25 v + 24 w_{10} + 24 w_{20} +g (g + v - 2 (-6 + w_{10} + w_{20}))))))))) (-288 (1 +w_{20}) + 8 c^3 (-16 (1 + w_{20}) +g (4 (2 + 3 v - 4 w_{10} - w_{20}) +g (1 + g) (3 + g - v + 4 w_{20}))) -2 c^2 (256 (1 + w_{20}) +g (-32 (7 + 5 v - 8 w_{10}) +g (4 (-15 + 9 v - 6 w_{10} - 26 w_{20}) +g (-23 + 23 v - 20 w_{10} - 54 w_{20} +g (-7 - 8 v + 4 w_{10} + 12 w_{20} +g (7 + 3 g - 3 v + 10 w_{20})))))) +g (24 (21 + 4 v - 12 w_{10} + 5 w_{20}) +g (4 (-13 + 4 v + 15 w_{10} + 46 w_{20}) +g (-2 (81 + 34 v - 55 w_{10} + 12 w_{20}) +g (43 + 21 v - 29 w_{10} - 23 w_{20} +g (5 + 7 v - 3 w_{10} - 16 w_{20} +g (-3 - \tag{5.A2.14}$$

$5 v + 5 w_{10} - 6 w_{20} + g (1 - 3 v + w_{10} + (4 + g) w_{20})))))) + c (-672 (1 + w_{20}) + g (8 (108 + 41 v - 84 w_{10} + 19 w_{20}) + g (86 - 74 v + 112 w_{10} + 352 w_{20} + g (-127 - 83 v + 128 w_{10} + 66 w_{20} + g (22 + 41 v - 30 w_{10} - 52 w_{20} + g (-14 + 3 v + 8 w_{10} - 40 w_{20} + g (-12 - 7 v + 6 w_{10} + g (-3 + 2 w_{20})))))))))/Q^2.$

Union's utility is also depending on wages only. Union's utility accrues substituting optimal quantities in the equation below;

$$U_j \left(q_{j0}, q_{j1}, w_{j0} \right) = \left(q_{j0} + q_{j1} \right) w_{j0} \,, \tag{5.A2.15}$$

Analytically,

$$U_1(w_{10},w_{20})=(w_{10} (32 c^3 (g (1 + v + g (-1 + v - 2 w_{20}) - 2 w_{20}) +4 (-3 - 3 v + 4 w_{10} + 2 w_{20})) + 4 c^2 (-128 (3 + 3 v - 4 w_{10} - 2 w_{20}) +g (8 (9 + 10 v - 8 w_{10} - 11 w_{20}) + g (58 + 84 v - 80 w_{10} - 98 w_{20} +g (-13 - 2 v + 8 w_{20} + g (2 + g - 6 v + 10 w_{20}))))) + (4 + g) (-72 (3 + 3 v - 4 w_{10} - 2 w_{20}) +g (1 + g) (12 (11 + 12 v - 14 w_{10} - 9 w_{20}) + g (-34 - 58 v + 62 w_{10} + 30 w_{20} +g (-19 - 7 v + 8 w_{10} + 18 w_{20} + g (4 + g + 4 v + g v - 6 w_{10} - 2 (1 + g) w_{20}))))) + 2 c (-336 (3 + 3 v - 4 w_{10} - 2 w_{20}) +g (4 (71 + 81 v - 80 w_{10} - 72 w_{20}) + g (382 + 376 v - 432 w_{10} - 362 w_{20} +g (-89 - 62 v + 62 w_{10} + 60 w_{20} + g (-36 - 45 v + 28 w_{10} + 68 w_{20} +g (9 - 2 v - 2 w_{10} + g (2 + v - 2 w_{20}) + 4 w_{20}))))))))/Q, \tag{5.A2.16}$$

$$U_1(w_{10},w_{20})=(w_{20} (32 c^3 (g^3 + g (7 + 7 v - 8 w_{10} - 6 w_{20}) - 4 (1 + v - 2 w_{20})) - 288 (1 + v - 2 w_{20}) - 4 c^2 (128 (1 + v - 2 w_{20}) + g (-8 (31 + 32 v - 32 w_{10} - 31 w_{20}) + g (22 + 16 v - 40 w_{10} + g (29 + 42 v + g (-6 + 3 g + 2 v) - 48 w_{10} - 38 w_{20}) + 6 w_{20}))) + 2 c (-336 (1 + v - 2 w_{20}) + g (708 + 748 v - 672 w_{10} - 784 w_{20} + g (-94 - 88 v + 200 w_{10} - 30 w_{20} + g (-239 - 256 v + 254 w_{10} + g (56 + 33 v - 40 w_{10} + g (15 + g (-6 + v) + 18 v - 22 w_{10} - 20 w_{20}) - 28 w_{20}) + 254 w_{20})))) + g (24 (27 + 29 v - 24 w_{10} - 32 w_{20}) + g (-4 (19 + 26 v - 60 w_{10} + 15 w_{20}) + g (-366 - 386 v + 338 w_{10} + 414 w_{20} + g (73 + 117 v - 110 w_{10} - 80 w_{20} + g (71 + 49 v - 54 w_{10} - 66 w_{20} + g (-9 - 17 v + 14 w_{10} + 12 w_{20} + g (-5 - 3 v + 4 w_{10} + 4 w_{20})))))))))/Q. \tag{5.A2.17}$$

In the first period/ second stage, each firm bargains the wage with the union that represents employees of the same sector. I make the assumption that unions possess all the bargaining power (b=1). The bargaining problem is

$$w_{j0} = \max_{w_{j0}} B_j = \max_{w_{j0}} \left(\text{b Log}\left[\text{U}_j \right] + (1 - \text{b}) \text{ Log}\left[EV_1 \right] \right). \tag{5.A2.18}$$

Solving the system that accrues from the first order conditions of the above problem as for w_{10} and w_{20} accrues a unique stable solution for the equilibrium firm-specific wage contracts w_{10}^* and w_{20}^*, depending only on *c*, *g*, and *v*; Provided that g=0.2, namely interior solution is ensured then

$$w_{10}^* =(0.316463 (0.985184 + c) (0.986533 + c) (0.991123 + c) (1.38671 + c) (1.39953+ c)(1.41117+c) (1.4715+c) (2.0722+c (2.879 +c)) +0.313895 (0.983155 +c) (0.985183+ c)(0.986546+c) (1.38713+c)(1.39643+c)(1.4102+c)(1.42282 +c)(1.47153+c)(1.47803+ c) v)/((0.984886 + c) (0.985273 + c)(0.986538 +c) (1.38699 +c)(1.39816+c)(1.41091 + c) (1.43522 + c) (1.45784 + c) (1.47146 + c)), \tag{5.A2.19}$$

$$w^*_{20}=(0.227819118488838(0.9637401239548367+c)(0.9852560673235647+c)(1.3976696032169678+c)(1.4356767212900055+c)(1.4552746821118971+c)(1.4741566714494632+c)+0.22810532341156273(0.9852552543343167+c)(0.9930945070975669+c)(1.3667082339401477+c)(1.3942098740296278+c)(1.4379979745445282+c)(1.4568056742470366+c)v)/((0.9848903271784706+c)(0.9852668670308729+c)(1.3981239126544593+c)(1.410950104082433+c)(1.435194995505853+c)(1.457849122683574+c)). \quad (5.A2.20)$$

The wage differential is formed in favor of the public sector

$$w_{10}^*-w_{20}^*=-((1.(-164.229-399.956c-318.013c^2-82.944c^3-155.846v-383.819cv-311.117c^2v-83.2c^3v))/(-103.088-248.878c-196.864c^2-51.2c^3))+(1.((-103.088-248.878c-196.864c^2-51.2c^3)(-781.731-1873.37c-1469.52c^2-378.88c^3-773.989v-1857.46cv-1458.66c^2v-376.32c^3v)-1.(2106.06+5053.04c+3968.c^2+1024.c^3)(-164.229-399.956c-318.013c^2-82.944c^3-155.846v-383.819cv-311.117c^2v-83.2c^3v)))/(-1.83424*10^6-8.87219*10^6c-1.77605*10^7c^2-1.88395*10^7c^3-1.11726*10^7c^4-3.51364*106c^5-457966.c^6)+(1.(846.327+2063.92c+1651.71c2+435.2c^3)((-103.088-248.878c-196.864c^2-51.2c^3)(-781.731-1873.37c-1469.52c^2-378.88c^3-773.989v-1857.46cv-1458.66c^2v-376.32c^3v)-1.(2106.06+5053.04c+3968.c^2+1024.c^3)(-164.229-399.956c-318.013c^2-82.944c^3-155.846v-383.819cv-311.117c^2v-83.2c^3v)))/((-103.088-248.878c-196.864c^2-51.2c^3)(-1.83424*10^6-8.87219*10^6c-1.77605*10^7c^2-1.88395*10^7c^3-1.11726*10^7c^4-3.51364*10^6c^5-457966.c^6)). \quad (5.A2.21)$$

Substituting the wages with the optimal in the equations (5.A2.11) and (5.A2.12) accrues the optimal quantities of the first period depending on *c*, *g*, and *v*; Substituting the product of the first period and wages with the optimal in the equations (5.A2.5) and (5.A2.6) accrues the optimal quantities of the second period depending on *c*, *g*, and *v*; Substituting now the optimal quantities of the first period as well as the optimal wages in equations (5.A1.1), (5.A1.2), (5.A1.16), and (5.A1.17)[37] accrues the optimal yields for the participants of the game.

[37] There is a very large amount for quantities and yields generated by mathematica, thus it is omitted. However, in the appendix B2 I present the figures that accompany this analysis.

APPENDIX 5.A3

Case *frr*$_0$

Second period/ fourth stage each firm maximazizes its maximization function as for q_{11} and q_{21} respectively;

$$\underset{q_{11}}{Max}\, EV_1\left(q_{10},q_{20},q_{11},q_{21},w_{10},w_{20}\right) = q_{10}(1-q_{10}-gq_{20})+q_{11}(-q_{11}-gq_{21}+v)-(q_{10}+q_{11})w_{10}+ \quad (5.A3.1)$$
$$(1+g)\frac{(q_{10}+q_{11}+q_{20}+q_{21})^2}{4},$$

$$\underset{q_{21}}{Max}\, \Pi_2\left(q_{10},q_{20},q_{11},q_{21},w_{10},w_{20}\right) = (1-gq_{10}-q_{20})q_{20}+q_{21}(v-gq_{11}-q_{21})-(q_{20}+q_{21})w_{20}\,. \quad (5.A3.2)$$

From the *foc* accrues reaction functions of the second period;

$$RF_{11}(q_{21}) \;=\; \frac{2v+(1+g)q_{10}+(1+g)q_{20}+(1-g)q_{21}-2w_{10}}{3-g}, \quad (5.A3.3)$$

$$RF_{21}\left(q_{11}\right) = \frac{v-gq_{11}-w_{20}}{2}. \quad (5.A3.4)$$

We solve the system of the second period RFs to get the optimal q_{11}^{*} and q_{21}^{*} -rules in the second period;

$$q_{11}{*} = -\frac{(5-g)v+2(1+g)q_{10}+2(1+g)q_{20}-4w_{10}+(1-g)w_{20}}{R}, \quad (5.A3.5)$$

where $R=-6+g+g^2$

$$q_{21}^{*} = -\frac{3v(1-g)-g(g+1)q_{10}-g(g+1)q_{20}+2gw_{10}+(g-3)w_{20}}{R}. \quad (5.A3.6)$$

Substituting the later into (5.A3.1) and (5.A3.2) accrues profits that depend on products of the first period and wages;

$EV_1(q_{10},q_{20},w_{10},w_{20}) = -q_{10}$ $(-1 + q_{10} + g\ q_{20}) + ((1 +g)\ (-(2 + g)\ q_{10} - (2 + g)\ q_{20} - 2\ v + w_{10} + w_{20})^2)/(3 +g)^2 + (w_{10}\ ((8 + g - g^2)\ q_{10} + 2\ (1 + g)\ q_{20} + 5\ v - g\ v -4\ w_{10} + (-1 + g)\ w_{20}))/R + ((2\ (1 + g)\ q_{10} + 2\ (1 + g)\ q_{20} + 5\ v - g\ v -4\ w_{10} + (-1 + g)\ w_{20})\ ((1 + g)\ (-2 + g^2)\ q_{10} + (1 + g)\ (-2 +g^2)\ q_{20} + v + 4\ w_{10} + w_{20} +g\ ((-3 + 2\ g)\ v + 2\ w_{20} - g\ (2\ w10 + w_{20}))))/R^2$, (5.A3.7)

$\Pi_2(q_{10},q_{20},w_{10},w_{20}) = 1/R^2\ (-g^5\ q_{10}\ q_{20} +g^3\ (2\ q_{10}^2 + q_{10}\ (15\ q_{20} + 6\ v - 4\ w_{10} - 2\ w_{20}) + 2\ q_{20}\ (1 + 3\ v - 2\ w_{10} - 2\ w_{20})) -6\ g\ (-2\ q_{20}^2 +q^{20}\ (2 + 6\ q^{10} + v - 3\ w^{20}) + (v - w^{20})\ (q_{10} + 3\ v - 2\ w^{10} -w_{20})) - 9\ (4\ q_{20}^2 - (v - w_{20})^2 + 4\ q^{20}\ (-1 + w^{20})) +g^4\ (q_{10}^2 + q_{20} - q_{20}\ w_{20}) +g^2\ (q_{10}^2 + 12\ q_{20}^2 + (-3\ v + 2\ w_{10} + w_{20})^2 +2\ q10\ (7\ q_{20} - 2\ w_{10} + 2\ w_{20}) + q_{20}\ (-11 - 4\ w_{10} + 15\ w_{20})))$. (5.A3.8)

First period/ third stage e public firm maximizes $EV_1=\Pi_1+CS$ as for q_{10} and private firms maximazizes its profits as for q_{20}, respectively. Reaction functions of the first period are

$$RF_{10}\left(q_{20}\right) = H1+F1q_{20}+I1w_{10}+S1w_{20}, \quad (5.A3.9)$$

where $H1>0$, $F1>0$, $I1<0$, and $S1<0$, analytically

$$H1 = -\frac{(-6+g+g^2)^2+(1+g)(24+g(-20+g+3g^2))v}{2(-3+g+g^2)(8+g(-4+g+g^2))} > 0,$$

$$F1 = \frac{-24+g(20-g(-8+g(3+g(4+g))))}{2(-3+g+g^2)(8+g(-4+g+g^2))} > 0,$$

$$I1 = \frac{48+g(-10-g(19+(-6-3g)g))}{2(-3+g+g^2)(8+g(-4+g+g^2))} < 0,$$

$$S1 = \frac{12+g(2-g(11-g^2))}{2(-3+g+g^2)(8+g(-4+g+g^2))} < 0.$$

$$RF_{20}(q_{10}) = H2 + F2q_{10} + I2w_{20} + S2w_{10}, \tag{5.A3.10}$$

where $H2>0$, $F2<0$, $I2<0$, and $S2<0$, note that $H1>H2$ and $I1<I2$ analytically

$$H2 = -\frac{(-6+g+g^2)^2+6g(-1+g^2)v}{24(-3+g+g^2)} > 0,$$

$$F2 = \frac{g(36+g(-14+g(-15+g^2)))}{24(-3+g+g^2)} < 0,$$

$$I2 = \frac{36+g(-18+g(-15+g(4+g)))}{24(-3+g+g^2)} < 0,$$

$$S2 = \frac{g^2(4+4g)}{24(-3+g+g^2)} < 0.$$

We solve the system of the first period RFs to get the optimal q_{10}^* and q_{20}^* -rules in the first period;

$$q_{10}^* = (-288(2+v-2w_{10}-w_{20})+g(24(15+v-9w_{10}-7w_{20})+g(4(61+69v-76w_{10}-54w_{20})+ g(-94-90v+104w_{10}+80w_{20}+g(-53-42v+36w_{10}+59w_{20}+g(1+18v-16w_{10}-3w_{20}+g(5+ g+6v-4w_{10}-(7+g)w_{20}))))))/T, \tag{5.A3.11}$$

where $T = (-2+g)^2(3+g)(4+g)(-12+g(4+3g+g^3))$,

$$q_{20}^* = (288(-1+w_{20})+g(24(21+8v-12w_{10}-17w_{20})+g(-4(25+10v-39w_{10}+4w_{20})+g(-226- 208v+194w_{10}+240w_{20}+g(63+69v-75w_{10}-57w_{20}+g(41+35v-33w_{10}-43w_{20}+g(-7-13v- 11w_{10}+9w_{20}+3g(-1-v+w10+w_{20}))))))))/T. \tag{5.A3.12}$$

Substituting the later into (5.A3.7) and (5.A3.8) accrues EV_1 and Π_1 that depend on wages only

$EV_1(w_{10},w_{20}) = (g^{15} (8 + 2 v^2 + 4 (-3 + w_{10}) w_{10} + v (9 - 6 w_{10} - 7 w_{20}) -13 w_{20} + 10 w_{10} w_{20} + 5 w_{20}^2) -g^8 (37979 + 45833 v^2 + 6 w_{10} (-14423 + 7556 w_{10}) +v (87862 - 90738 w_{10} - 88790 w_{20}) - 77282 w_{20} +86604 w_{10} w_{20} + 39734 w_{20}^2) +82944 (3 + 3 v^2 + 4 w_{10}^2 + v (4 - 6 w_{10} - 4 w_{20}) +2 (-2 + w_{20}) w_{20} + w_{10} (-6 + 4 w_{20})) -13824 g (33 + 35 v^2 + 36 w_{10}^2 + v (38 - 62 w_{10} - 46 w_{20}) +4 w_{20} (-11 + 5 w_{20}) + 10 w_{10} (-6 + 5 w_{20})) -g^{11} (-1160 + 1096 v^2 + 756$

$$w_{10}^2 + v(173 - 1854 w_{10} - 511 w_{20}) + (1895 - 737 w_{20}) w_{20} + 18 w_{10} (14 + 5 w_{20})) + g^{13} (81 v^2 + 8 w_{10} (14 + 3 w_{10}) - 54 w_{10} w_{20} - 72 w_{20}^2 + 71 (-2 + 3 w_{20}) - v (41 + 106 w_{10} + 15 w_{20})) + g^{14} (41 + 36 v^2 + 40 w_{10}^2 + v (84 - 78 w_{10} - 78 w_{20}) + w_{20} (-80 + 37 w_{20}) + w_{10} (-86 + 84 w_{20})) - g^{12} (869 + 665 v2 + 720 w_{10}^2 + 2 w_{20} (-845 + 408 w20) - 2 v (-827 + 695 w_{10} + 797 w_{20}) + 2 w_{10} (-851 + 826 w_{20})) + 192 g^3 (1863 + 2136 v^2 + 2166 w_{10}^2 + v (4270 - 4291 w_{10} - 4251 w_{20}) + w_{20} (-3931 + 2079 w_{20}) + w_{10} (-4065 + 4024 w_{20})) + g^9 (-6838 + 3749 v^2 + 8 w_{10} (175 + 666 w_{10}) + 14843 w_{20} - 2962 w_{10} w_{20} - 8022 w_{20}^2 + v (-2567 - 9094 w_{10} + 4163 w_{20})) - 16 g^4 (10953 + 14546 v^2 + 8 w_{10} (-3333 + 2029 w_{10}) - 16506 w_{20} + 25248 w_{10} w_{20} + 5283 w_{20}^2 - 4 v (-5316 + 7762 w_{10} + 4827 w_{20})) + 4 g^7 (8717 + 4412 v^2 - 1282 w_{10}^2 + v (14245 - 3170 w_{10} - 19899 w_{20}) + 9 w_{20} (-2579 + 1606 w_{20}) + 2 w_{10} (-4234 + 7101 w_{20})) + g^{10} (7771 + 7330 v^2 + 7324 w_{10}^2 + 3 w_{20} (-5140 + 2607 w_{20}) + 2 w_{10} (-7901 + 7818 w_{20}) - 2 v (-7840 + 7241 w_{10} + 7929 w_{20})) - 16 g^5 (8630 + 9640 v^2 + 6071 w_{10}^2 - 4 v (-5381 + 4044 w_{10} + 6157 w_{20}) + w_{20} (-22781 + 13686 w_{20}) + w_{10} (-16003 + 20037 w_{20})) + 4 g^6 (27339 + 37671 v^2 + 40393 w_{10}^2 + v (68870 - 79328 w_{10} - 64884 w_{20}) + w_{20} (-53940 + 25337 w_{20}) + w_{10} (-69608 + 68150 w_{20})) + 576 g^2 (105 + 147 v^2 - 82 v (4 + w_{10}) + 136 w_{20} + 116 v w_{20} - 2 (w_{10} (9 + w_{10}) - 52 w_{10} w_{20} + 89 w_{20}^2)))/T^4, \tag{5.A3.13}$$

$$\Pi_2 (w_{10},w_{20}) = (2 g^{13} (5 + 9 v - 8 w_{10} - 6 w_{20}) (1 + 3 v - 2 w10 - 2 w_{20}) + g^{14} (1 + 3 v - 2 w_{10} - 2 w_{20})^2 + 82944 (1 + v^2 - 2 v w_{20} + 2 (-1 + w_{20}) w_{20}) - 41472 g (7 + 7 v^2 + 2 w_{20} (-7 + 5 w_{20}) - 2 v (-2 + 2 w_{10} + 7 w_{20}) + w_{10} (-4 + 8 w_{20})) - g^{12} (69 + 45 v^2 + 20 w_{10}^2 + 8 w10 (-12 + 13 w_{20}) + 4 w_{20} (-33 + 20 w_{20}) - 6 v (-15 + 8 w_{10} + 22 w_{20})) + 576 g^2 (553 + 537 v^2 + 36 w_{10} (-21 + 8 w_{10}) - 842 w_{20} + 912 w_{10} w_{20} + 382 w_{20}^2 - 6 v (-82 + 122 w_{10} + 139 w_{20})) - 4 g^{11} (96 + 162 v2 + 139 w_{10}^2 - 6 v (-51 + 52 w_{10} + 53 w_{20}) + w_{20} (-246 + 139 w_{20}) + w_{10} (-252 + 286 w_{20})) + g^{10} (1395 + 267 v^2 + 448 w_{10}^2 - 6 v (-279 + 118 w_{10} + 250 w_{20}) + 4 w_{20} (-657 + 310 w_{20}) + 4 w_{10} (-459 + 412 w_{20})) + 192 g^3 (219 + 194 v^2 - 576 w_{10}^2 + v (699 + 375 w_{10} - 1462 w_{20}) + w_{10} (393 + 384 w_{20}) + 2 w_{20} (-765 + 652 w_{20})) - g^8 (14379 + 5427 v^2 + 6644 w_{10}^2 + 8 w_{10} (-2769 + 2761 w_{20}) - 6 v (-4053 + 2204 w_{10} + 3658 w_{20}) + 4 w_{20} (-7731 + 3848 w_{20})) + 2 g^9 (2505 + 2991 v^2 + 3100 w_{10}^2 + v (6552 - 6306 w_{10} - 6228 w_{20}) + 8 w_{20} (-699 + 359 w_{20}) + w_{10} (-5970 + 6076 w_{20})) + 16 g^5 (4018 + 4572 v^2 + 6444 w_{10}^2 - 2 w_{20} (1490 + 81 w_{20}) - 3 v (-1751 + 3380 w_{10} + 1419 w_{20}) + w_{10} (-10309 + 7561 w_{20})) - 4 g^7 (7239 + 8475 v^2 + 9067 w_{10}^2 - 6 v (-2583 + 2869 w_{10} + 2539 w_{20}) + w_{20} (-13038 + 6127 w_{20}) + 2 w_{10} (-8469 + 8009 w_{20})) - 16 g^4 (16289 + 14790 v^2 + 9630 w_{10}^2 + 54 w_{10} (-471 + 550 w_{20}) - 6 v (-4360 + 3921 w_{10} + 5369 w_{20}) + w_{20} (-33304 + 17909 w_{20})) + 4 g^6 (20955 + 15267 v^2 + 12409 w10^2 - 6 v (-6675 + 4759 w_{10} + 7005 w_{20}) + w_{20} (-47298 + 25465 w_{20}) + w_{10} (-34662 + 38398 w_{20})))/T^4. \tag{5.A3.14}$$

Union's utility is also depending on wages only. Union's utility accrues substituting optimal quantities in the equation below;

$$U_j\left(q_{j0}, q_{j1}, w_{j0}\right) = \left(q_{j0} + q_{j1}\right) w_{j0}, \tag{5.A3.15}$$

Analytically,

$$U_1\left(w_{10}, w_{20}\right) = (w_{10}(-72(3+3v-4w_{10}-2w_{20}) + g(1+g)(12(11+12v-14w_{10}-9w_{20}) + g(-34-58v+62w_{10}+30w_{20}+g(-19-7v+8w_{10}+18w_{20}+g(4+g+4v+gv-6w_{10}-2(1+g)w_{20}))))))/T, \tag{5.A3.16}$$

$$U_2\left(w_{10}, w_{20}\right) = w_{20}(-288(1+v-2w_{20}) + g(24(27+29v-24w_{10}-32w_{20}) + g(-4(19+26v-60w_{10}+15w_{20}) + g(-366-386v+338w_{10}+414w_{20}+g(73+117v-110w_{10}-80w_{20}+g(71+49v-54w_{10}-66w_{20}+g(-9-17v+14w_{10}+12w_{20}+g(-5-3v+4w_{10}+4w_{20}))))))))/T. \tag{5.A3.17}$$

In the first period/ second stage, each firm bargains the wage with the union that represents employees of the same sector. I make the assumption that unions possess all the bargaining power (b=1). The bargaining problem is set as follows;

$$w_{j0} = \max_{w_{j0}} B_j = \max_{w_{j0}} \left(\text{b Log}\left[\text{U}_j\right] + (1-\text{b})\ \text{Log}\left[EV_1\right] \right)$$

Solving the system that accrues from the first order conditions of the above problem as for w_{10} and w_{20} accrues a unique stable solution for the equilibrium firm-specific wage contracts w_{10}^* and w_{20}^*, respectively;

$$w_{10}^* = -(51840 - 63936 g - 30600 g^2 + 59052 g^3 + 1086 g^4 - 20691 g^5 + 2515 g^6 + 3493 g^7 - 601 g^8 - 297 g^9 + 49 g^{10} +11 g^{11} - g^{12} + 51840 v - 65664 g v - 25704 g^2 v +59808 g^3 v - 6198 g^4 v - 18855 g^5 v + 5241 g^6 v +2593 g^7 v - 871 g^8 v - 197 g^9 v + 43 g^{10} v + 11 g^{11} v +g^{12} v)/(2 (-82944 + 107136 g + 40320 g^2 - 96900 g^3 + 8631 g^4 + 31143 g^5 - 7825 g^6 - 4323 g^7 + 1479 g^8 + 265 g^9 - 111 g^{10} - 9 g^{11} + 2 g^{12})), \qquad (5.A3.18)$$

$$w_{20}^* = -(-82944 + 172800 g - 36288 g^2 - 132600 g^3 + 74140 g^4 +31590 g^5 - 29255 g^6 - 1158 g^7 + 4935 g^8 - 520 g^9 - 393 g^{10} + 66 g^{11} + 13 g^{12} - 2 g^{13} - 82944 v + 186624 g v -48960 g^2 v - 143640 g^3 v + 92456 g^4 v + 25702 g^5 v -34019 g^6 v + 3206 g^7 v + 4511 g^8 v - 1168 g^9 v - 197 g^{10} v + 94 g^{11} v + g^{12} v -2 g^{13} v)/(2 (165888 - 297216 g + 26496 g^2 + 234120 g^3 - 114162 g^4 - 53655 g^5 + 46793 g^6 + 821 g^7 - 7281 g^8 + 949 g^9 + 487 g^{10} - 93 g^{11} - 13 g^{12} + 2 g^{13})). \qquad (5.A3.19)$$

Substituting the wages with the optimal in the equations (5.A3.11) and (5.A3.12) accrues the optimal quantities of the first and the second period, depending only on *g* and *v*;

$$q_{11}^* = (-(-2 + g)^2 (3 + g) (82944 +g (286848 + g (-490176 +g (-101016 +g (366564 +g (-67850 +g (-95481 +g (40875 +g (8217 +g (-7157 +g (251 +g (505 +g (-65 + g (-13 + 2 g)))))))))))))) + (-8957952 +g (16298496 +g (-815616 +g (-14112864 +g (6166944 +g (3970272 +g (-2752088 +g (-335708 +g (428507 +g (-3308 +g (-10838 +g (-5406 +g (-3544 +g (1778 +g (414 +g (-142 +g (-19 + 2 g))))))))))))))))) v)/(2 (-2 + g) (4 + g) (-12 + g (4 + 3 g + g^3)) (-82944 + g (107136 + g (40320 + g (-96900 + g (8631 + g (31143 +g (-7825 +g (-4323 +g (1479 +g (265 + g (-111 + g (-9 + 2 g))))))))))))) \qquad (5.A3.20)$$

$$q_{21}^* = (995328 (-1 + 3 v) +g (-82944 (-17 + 71 v) +g (3456 (-77 + 171 v) +g (1440 (-175 + 3769 v) +g (48 (7676 - 56847 v) +g (-4 (150316 + 405461 v) +g (8 (18333 + 172000 v) +g (4 (79009 + 25328 v) +g (-151457 - 281075 v +g (-60264 + 36908 v +g (8 (5019 + 2953 v) +g (4880 - 7900 v +g (-5010 - 326 v +g (g (312 + g (12 + g (-5 + v) - 20 v) - 64 v) +8 (-21 + 71 v)))))))))))))))/(2 (4 + g) (-12 + g (4 + 3 g + g^3)) (-82944 + g (107136 + g (40320 + g (-96900 + g (8631 + g (31143 +g (-7825 +g (-4323 +g (1479 +g (265 + g (-111 + g (-9 + 2 g))))))))))))). \qquad (5.A3.21)$$

Substituting now the above optimal quantities of the first period as well as the optimal wages in equations (5.A3.5) and (5.A3.6) accrue the optimal quantities of the second period depending on *g* and *v*;

$$q_{10}^* = ((-2 + g)^2 (3 + g) (4478976 +g (-6117120 +g (-2709504 +g (6155280 +g (50064 +g (-2341164 +g (218362 +g (431313 +g (-26732 +g (-43230 +g (-5938 +g (2564 +g (1554 +g \qquad (5.A3.22)$$

(-86 +g (-128 + g (-5 + 2 g)))))))))))))))) + (5971968 +g (11197440 +g (-47319552 +g (26502336 +g (29658528 +g (-33714480 +g (-167328 +g (13450364 +g (-4368704 +g (-2321431 +g (1462567 +g (143022 +g (-224256 +g (8442 +g (18486 +g (-1776 +g (-808 +g (85 + (11 - 2 g) g))))))))))))))))))) v)/V

V=(2 (-2 + g)^2 (3 + g) (4 + g) (-12 + g (4 + 3 g + g^3)) (-82944 + g (107136 + g (40320 + g (-96900 + g (8631 + g (31143 +g (-7825 +g (-4323 +g (1479 +g (265 + g (-111 + g (-9 + 2 g)))))))))))))

q_{20}* = -((-2 + g)^2 (3 + g) (-2985984 +g (5889024 +g (-736128 +g (-5555232 +g (3026592 +g (1814396 +g (-1637694 +g (-200190 +g (402901 +g (-14446 +g (-54294 +g (5486 +g (4264 +g (-486 +g (-204 + g (8 + 3 g)))))))))))))))) + (11943936 +g (-20901888 +g (4769280 +g (715392 +g (8280000 +g (4337856 +g (-15825728 +g (3035776 +g (7316498 +g (-3030556 +g (-1427621 +g (904157 +g (114916 +g (-133384 +g (46 +g (10866 +g (-526 +g (-20 + 3 g) (25 +3 g))))))))))))))))))) v)/V (5.A3.23)

The optimal yields are also depending on *g* and *v*;

$$EV_1^*\left(v,c,g\right) = q_{10}^*(1-q_{10}^*-gq_{20}^*)+q_{11}^*(-q_{11}^*-gq_{21}^*+v)-(q_{10}^*+q_{11}^*)w_{10}^* + (1+g)\frac{(q_{10}^*+q_{11}^*+q_{20}^*+q_{21}^*)^2}{4}, \quad (5.A3.24)$$

$$\Pi_2^*\left(v,c,g\right) = (1-gq_{10}^*-q_{20}^*)q_{20}^*+q_{21}^*(v-gq_{11}^*-q_{21}^*)-(q_{20}^*+q_{21}^*)w_{20}^*, \quad (5.A3.25)$$

$$U_1^*\left(v,c,g\right) = \left(q_{10}^*+q_{11}^*\right)w_{10}^*, \quad (5.A3.26)$$

$$U_2^*\left(v,c,g\right) = \left(q_{20}^*+q_{21}^*\right)w_{20}^*. \quad (5.A3.27)$$

APPENDIX 5.B1

Case *frr*$_1$

Interior solution is ensured provided that both firms produce in each period. I test this assumption for different values of *g*, as I did in the previous chapters, and I find out that interior solution is ensured provided that products are imperfect substitutes. In particular, if $g \in [0.3,1]$, then there are combinations c and v that yields negative contour lines for q_{20} and P_{11}.Thus, in this appendix, I present the case where *g*=0.2. P_{11} imposes restrictions related to v, namely $v \geq 0.7$.

Figure 5.B1.1a | g=0.2
Contour Plot q_{11}* (*frr*$_1$)

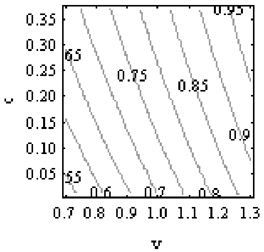


Figure 5.B1.1b | g=0.2
Contour Plot q_{21}* (*frr*$_1$)

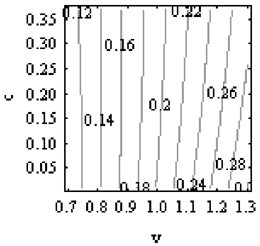


Figure 5.B1.1c | g=0.2
Contour Plot q_{10}* (*frr*$_1$)

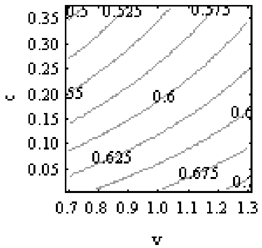


Figure 5.B1.1d | g=0.2
Contour Plot q_{20}* (*frr*$_1$)

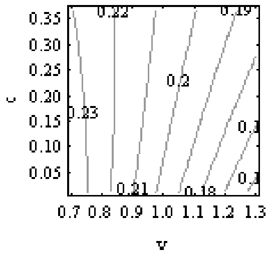


Figure 5.B1.1e| g=0.2
Contour Plot TC1 (*frr*$_1$)

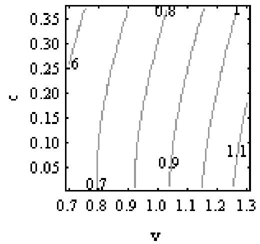


Figure 5.B1.1f| g=0.2
Contour Plot TC_2(*frr*$_1$)

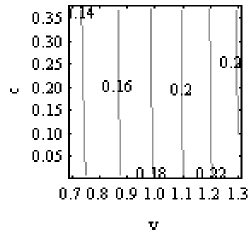


Figure 5.B1.1: Positive isoquants, in each period and total costs simultaneously, provided that $v > 0.7$ and $c \leq 0.37$

Figure 5.B1.2a | g=0.2
Contour Plot P_{11}* (*frr*$_1$)

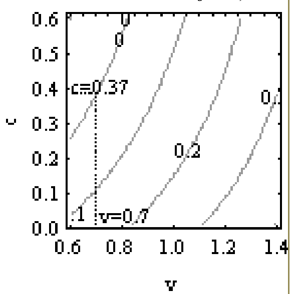


Figure 5.B1.2b | g=0.2
Contour Plot P_{21}* (*frr*$_1$)

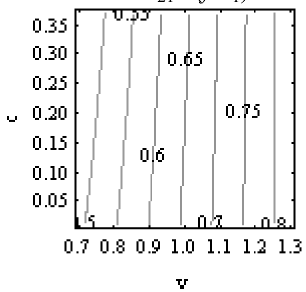


Figure 5.B1.2c | g=0.2
Contour Plot P_{10}* (*frr*$_1$)

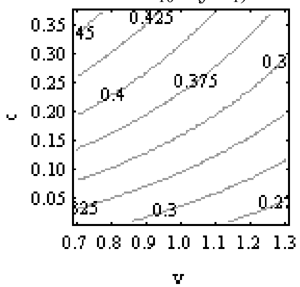


Figure 5.B1.2d | g=0.2
Contour Plot P_{20}* (*frr*$_1$)

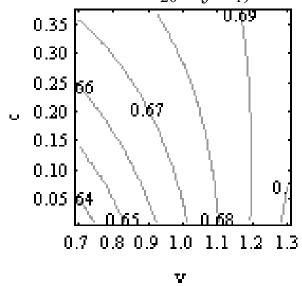


Figure 5.B1.2: Positive prices in each period simultaneously, provided that $v > 0.7$ [38] and $c \leq 0.37$

[38] Note that v is restricted even for positive demand shock, because we assume that θ is positive or negative demand shock of equal magnitude.

Figure 5.B1.3a | g=0.2

Contour Plot $w_{10}^{*}(frr_1)$,

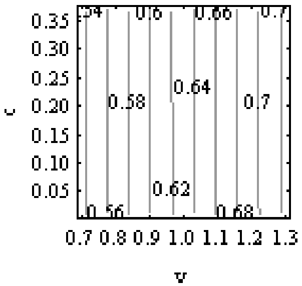


Figure 5.B1.3b | g=0.2

Contour Plot $w_{20}^{*}(frr_1)$,

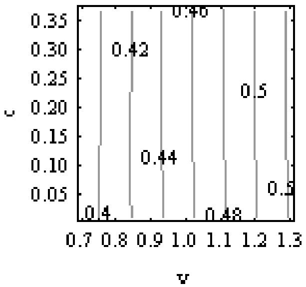


Figure 5.B1.3c | g=0.2

Contour Plot $\left(w_{10}^{*}-w_{20}^{*}\right)(frr_1)$,

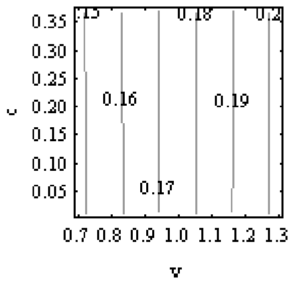


Figure 5.B1.3d | g=0.2

Contour Plot $EV1(frr_1)$

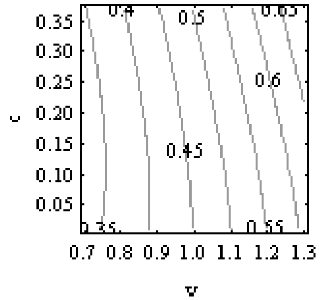


Figure 5.B1.3: Positive wages (w_{10}*>w_{20}*) and positive $EV1(frr_1)$

Figure 5.B1.4a | g=0.2

Contour Plot $\Pi_1(frr_1)$

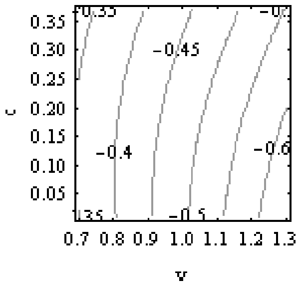


Figure 5.B1.4b | g=0.2

Contour Plot $\Pi_2(frr_1)$

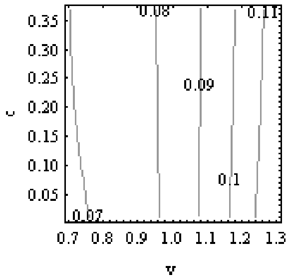


Figure 5.B1.4c | g=0.2

Contour Plot $U_1(frr_1)$

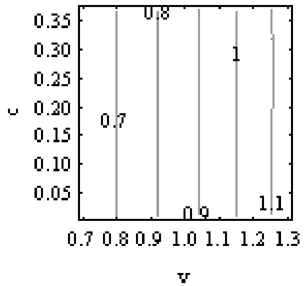


Figure 5.B1.4d | g=0.2

Contour Plot $U_2(frr_1)$

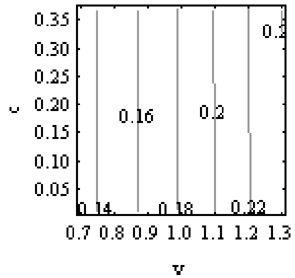


Figure 5.B1.4e | g=0.2

Contour Plot $CS(frr_1)$

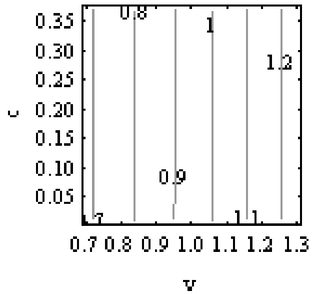


Figure 5.B1.4: Π_2 U_1, U_2 and CS are positive, provided we ensure interior solution, Π_1<0

Figure 5.B1.5a | g=0.2

Contour Plot $\left(q_{10}^{*}-q_{20}^{*}\right)\left(frr_{1}\right)$

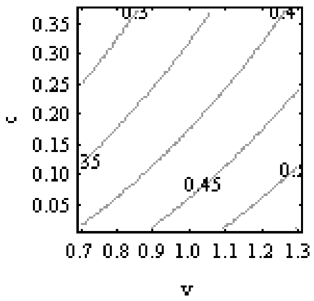


Figure 5.B1.5b | g=0.2

Contour Plot $\left(q_{11}^{*}-q_{21}^{*}\right)\left(frr_{1}\right)$

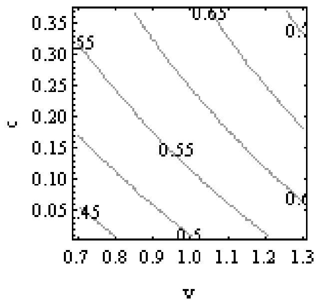


Figure 5.B1.5c | g=0.2

Contour Plot $\left(q_{11}^{*}-q_{10}^{*}\right)\left(frr_{1}\right)$

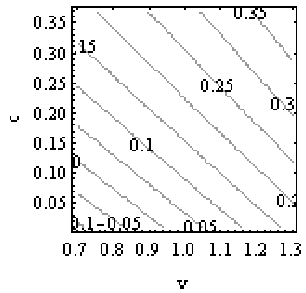


Figure 5.B1.5d | g=0.2

Contour Plot $\left(q_{21}^{*}-q_{20}^{*}\right)\left(frr_{1}\right)$

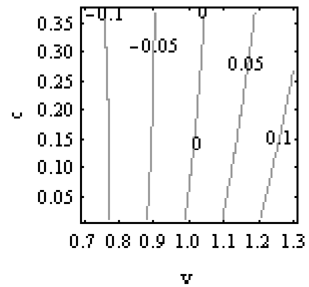


Figure 5.B1.5: q_{10}*>q_{20}* and q_{11}*>q_{21}*, irrespective of v and q_{11}*>q_{10}* and q_{20}*>q_{21}*, if v>1

Figure 5.B1.6a | g=0.2

Contour Plot (tq_1*-tq_2*) (frr_1)

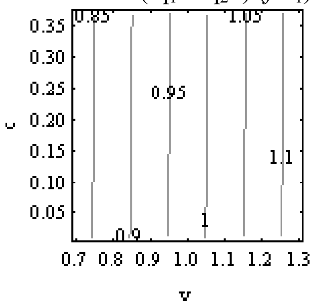


Figure 5.B1.6b | g=0.2

Contour Plot (tq_A*-tq_B*) (frr_1)

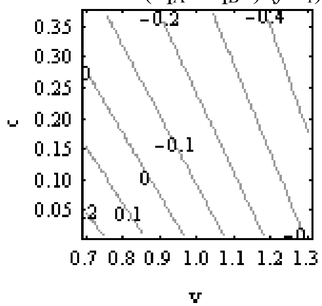


Figure 5.B1.6c |g=0.2

Contour Plot (Π_1-Π_2) (frr_1)

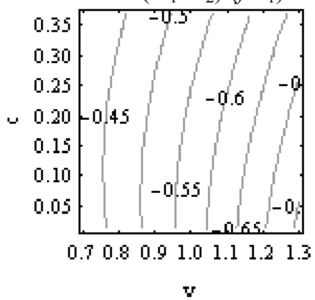


Figure 5.B1.6d |g=0.2

Contour Plot (U_1-U_2) (frr_1)

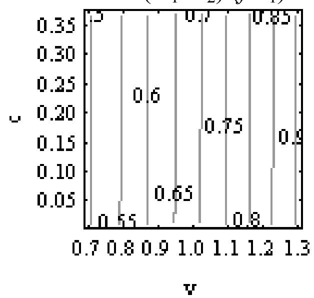


Figure 5.B1.6e | g=0.2

Contour Plot (EV_1-Π_2) (frr_1)

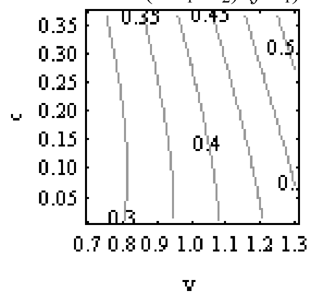


Figure 5.B1.6: tq_1*>tq_2*, Π_1<Π_2 , U_1>U_2, and EV_1>Π_2, tq_A*<tq_B* if v>1

Figure 5.B1.7a |g=0.2

Contour Plot $q_{10}^*(frr_1)-q_{10}^*(frr_0)$

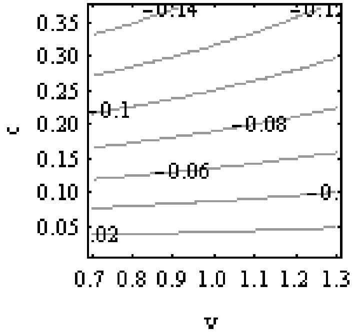


Figure 5.B1.7b |g=0.2

Contour Plot $q_{20}^*(frr_1)-q_{20}^*(frr_0)$

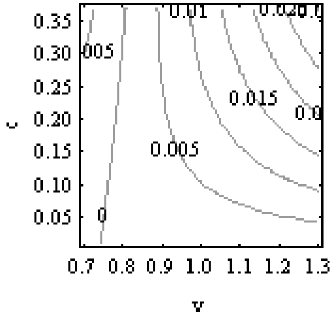


Figure 5.B1.7c |g=0.2

Contour Plot $q_{11}^*(frr_1)-q_{11}^*(frr_0)$

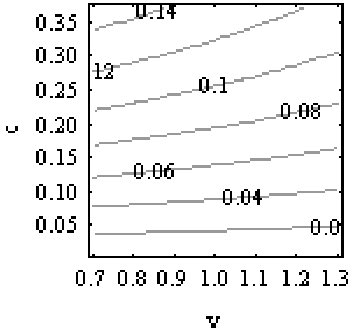


Figure 5.B1.7d |g=0.2

Contour Plot $q_{21}^*(frr_1)-q_{21}^*(frr_0)$

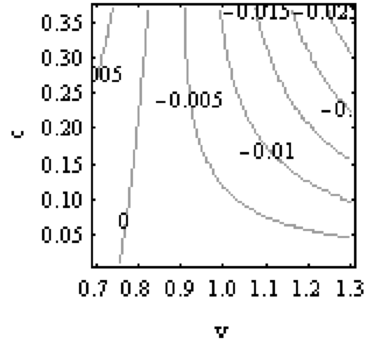


Figure 5.B1.7: $q_{10}^*(frr_1)<q_{10}^*(frr_0)$; $q_{20}^*(frr_1)>q_{20}^*(frr_0)$ v>0.8; $q_{11}^*(frr_1)>q_{11}^*(frr_0)$ and $q_{21}^*(frr_1)<q_{21}^*(frr_0)$ v<0.8

Figure 5.B1.8a |g=0.2

Contour Plot

$(w_{10}^*)(frr_1)-(w_{10}^*)(frr_0)$

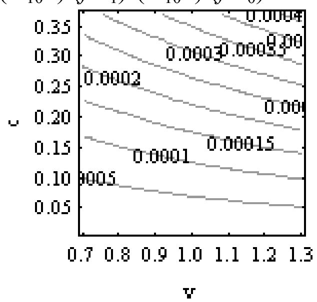


Figure 5.B1.8b |g=0.2

Contour Plot

$(w_{20}^*)(frr_1)-(w_{20}^*)(frr_0)$

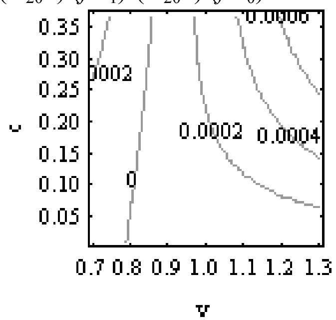


Figure 5.B1.8c |g=0.2

Contour Plot

$(w_{10}^*-w_{20}^*)(frr_1)-(w_{10}^*-w_{20}^*)(frr_0)$

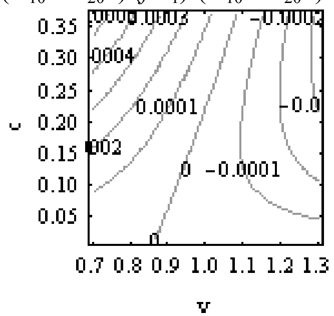


Figure 5.B1.8d |g=0.2

Contour Plot

$(w_{10}^*-w_{20}^*)(frr_0)$

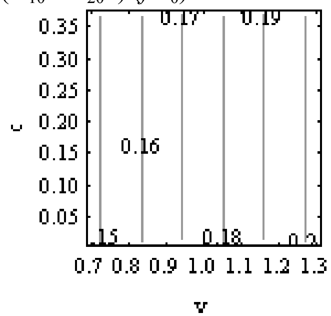


Figure 5.B1.8: $(w_{10}^*)(frr_1)>(w_{10}^*)(frr_0)$; $(w_{20}^*)(frr_1)>(w_{20}^*)(frr_0)$ if v>0.8; $(w_{10}^*-w_{20}^*)(frr_1)>(w_{10}^*-w_{20}^*)(frr_0)$ if v<1; $(w_{10}^*-w_{20}^*)(frr_0)>0$

Figure 5.B1.9a |g=0.2
Contour Plot
(q_{11}*-q_{10}*) (*frr*$_1$)-(q_{11}*-q_{10}*) (*frr*$_0$)

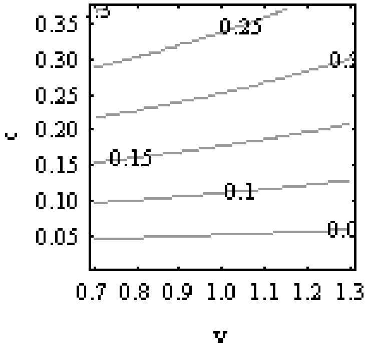


Figure 5.B1.9b |g=0.2
Contour Plot
(q_{21}*-q_{20}*) (*frr*$_1$)-(q_{21}*-q_{20}*) (*frr*$_0$)

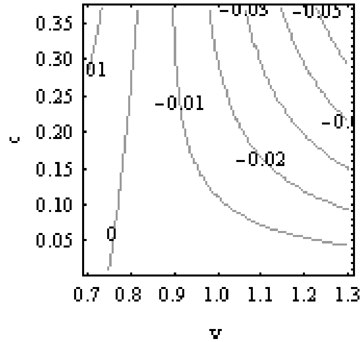


Figure 5.B1.9c |g=0.2
Contour Plot
(q_{10}-*q_{20}*) (*frr*$_1$)-(q_{10}*-q_{20}*) (*frr*$_0$)

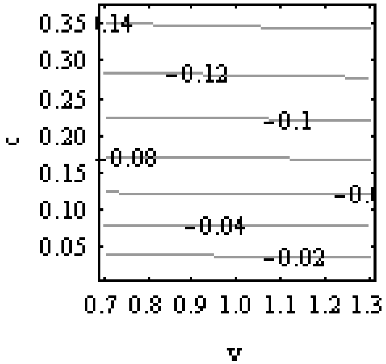


Figure 5.B1.9d |g=0.2
Contour Plot
(q_{11}*-q_{21}*) (*frr*$_1$)-(q_{11}*-q_{21}*) (*frr*$_0$)

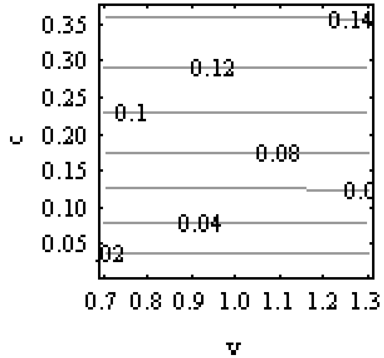


Figure 5.B1.9: (q_{11}*-q_{10}*) (*frr*$_1$)>(q_{11}*-q_{10}*) (*frr*$_0$); (q_{21}*-q_{20}*) (*frr*$_1$)<(q_{21}*-q_{20}*) (*frr*$_0$); (q_{10}*-q_{20}*) (*frr*$_1$)<(q_{10}*-q_{20}*) (*frr*$_0$); (q_{11}*-q_{21}*) (*frr*$_1$)>(q_{11}*-q_{21}*) (*frr*$_0$)

Figure 5.B1.10a |g=0.2
Contour Plot
tq_1* (*frr*$_1$)-tq_1* (*frr*$_0$)

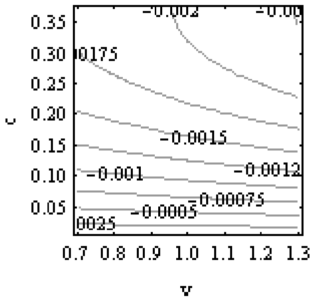


Figure 5.B1.10b |g=0.2
Contour Plot
tq_2* (*frr*$_1$)-tq_2* (*frr*$_0$)

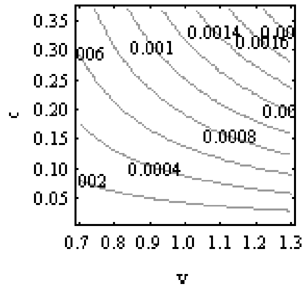


Figure 5.B1.10c |g=0.2
Contour Plot
Tq_A* (*frr*$_1$)-Tq_A* (*frr*$_0$)

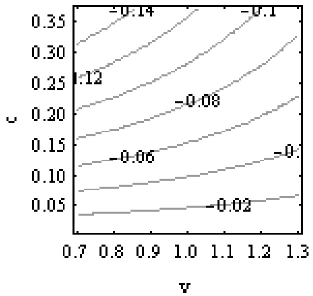


Figure 5.B1.10d |g=0.2
Contour Plot
Tq_B* (*frr*$_1$)-Tq_B* (*frr*$_0$)

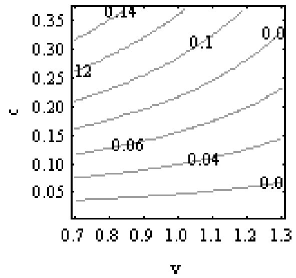


Figure 5.B1.10e |g=0.2
Contour Plot
Tq* (*frr*$_1$)-Tq* (*frr*$_0$)

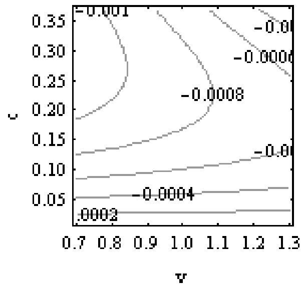


Figure 5.B1.10: tq_1*=q_{10}*+q_{11}* and tq_2*= q_{20}*+q_{21}*; tq_1*(*frr*$_1$)<tq_1*(*frr*$_0$) and tq_2*(*frr*$_1$)>tq_2*(*frr*$_0$) v>0.8;Tq_A*=q_{10}*+q_{20}* and Tq_B*=q_{11}*+q_{21}*; Tq_A* (*frr*$_1$)<Tq_A* (*frr*$_0$) and Tq_B* (*frr*$_1$)>Tq_B* (*frr*$_0$); Tq* (*frr*$_1$)<Tq* (*frr*$_0$)

Figure 5.B1.11a |g=0.2

Contour Plot $\Pi_2(frr_1)$-$\Pi_2(frr_0)$

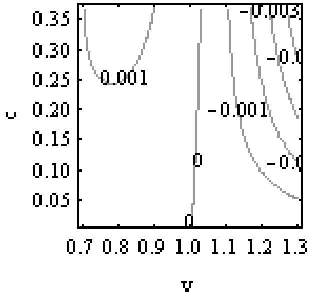


Figure 5.B1.11b |g=0.2

Contour Plot $U_1(frr_1)$-$U_1(frr_0)$

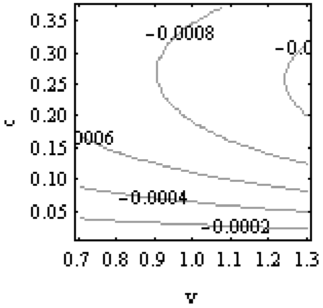


Figure 5.B1.11c |g=0.2

Contour Plot $U_2(frr_1)$-$U_2(frr_0)$

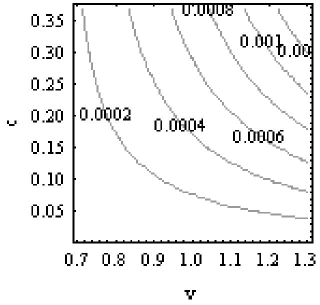


Figure 5.B1.11: $\Pi_2(frr_1)<\Pi_2(frr_0)$ v>1; $U_1(frr_1)<U_1(frr_0)$; $U_2(frr_1)>U_2(frr_0)$

Figure 5.B1.12a |g=0.2

Contour Plot

$\Pi_1(frr_1)$-$\Pi_1(frr_0)$

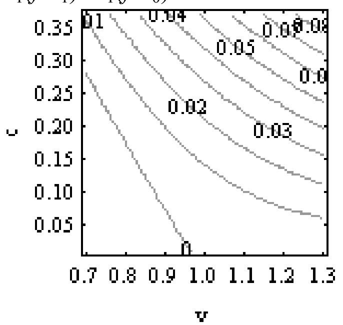


Figure 5.B1.12b |g=0.2

Contour Plot

$CS(frr_1)$-$CS(frr_0)$

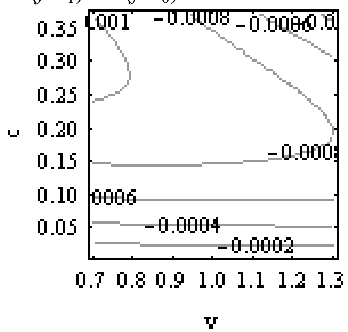


Figure 5.B1.12c |g=0.2

Contour Plot

$EV_1(frr_1)$-$EV_1(frr_0)$

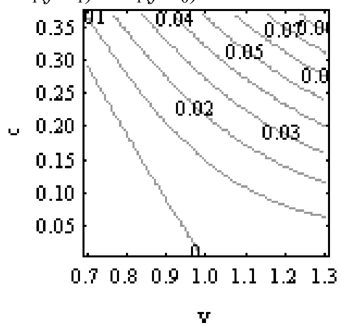


Figure 5.B1.12d |g=0.2

Contour Plot $\frac{dEV_1}{dc}(frr_1)$

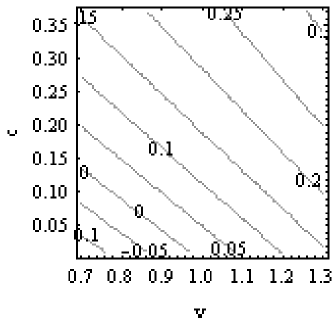


Figure 5.B1.12: $\Pi_1(frr_1)>\Pi_1(frr_0)$ c>0.5; $CS(frr_1)<CS(frr_0)$; $EV_1(frr_1)>EV_1(frr_0)$, c>0.3; $EV1$ is increasing with c, if c>1.5

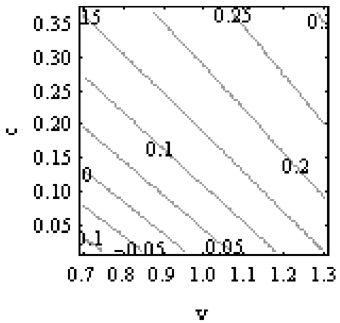
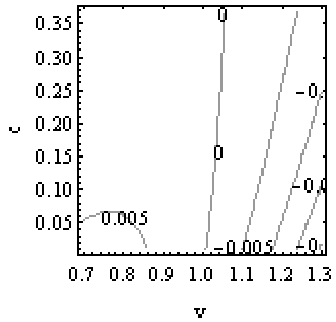
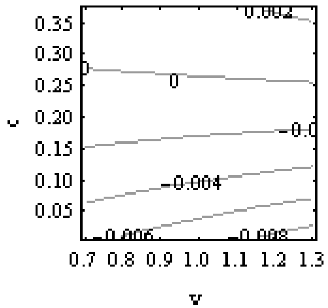
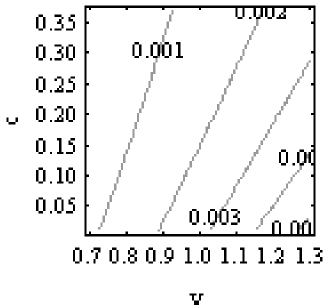
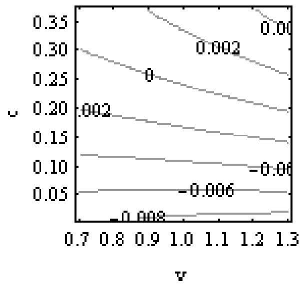
Figure 5.B1.13a |g=0.2	Figure 5.B1.13b |g=0.2	Figure 5.B1.13c |g=0.2	Figure 5.B1.13d |g=0.2	Figure 5.B1.13e |g=0.2
Contour Plot $\frac{d\Pi_1}{dc}$ (frr_1)	Contour Plot $\frac{d\Pi_2}{dc}$ (frr_1)	Contour Plot $\frac{dU_1}{dc}$ (frr_1)	Contour Plot $\frac{dU_2}{dc}$ (frr_1)	Contour Plot $\frac{dCS}{dc}$ (frr_1)
				

Figure 5.B1.13: Π_1, U_1, $U2$, CS are increasing simultaneously with c provided that $0.27 < c < 0.37$, irrespective of v

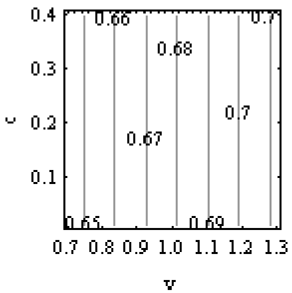
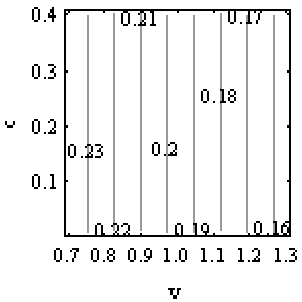
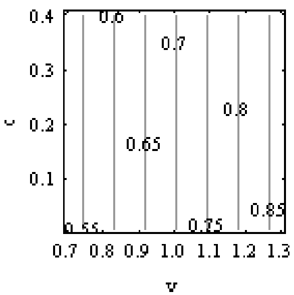
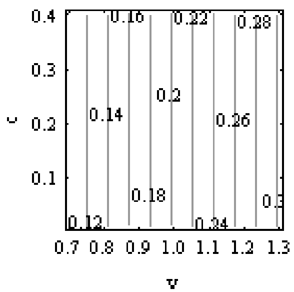
Figure 5.B1.14a |g=0.2	Figure 5.B1.14b |g=0.2	Figure 5.B1.14c |g=0.2	Figure 5.B1.14d |g=0.2
Contour Plot q^*_{10} (frr_0)	Contour Plot q^*_{20} (frr_0)	Contour Plot q^*_{11} (frr_0)	Contour Plot q^*_{21} (frr_0)
			

Figure 5.B1.14: Interior solution is ensured under frr_0 for g=0.2 and $0.7 \le v \le 1.3$ that interior solution is ensured under frr_1, frr_2, and frr_3

Figure 5.B1.15a |g=0.2

Contour Plot $P^*_{10}\,(frr_0)$

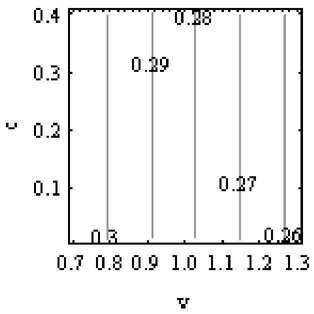


Figure 5.B1.15b |g=0.2

Contour Plot $P^*_{20}\,(frr_0)$

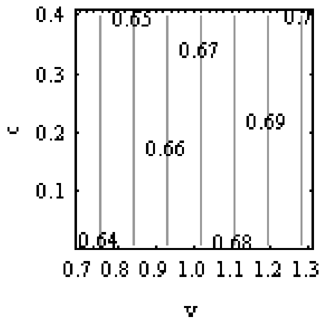


Figure 5.B1.15c |g=0.2

Contour Plot $P^*_{11}\,(frr_0)$

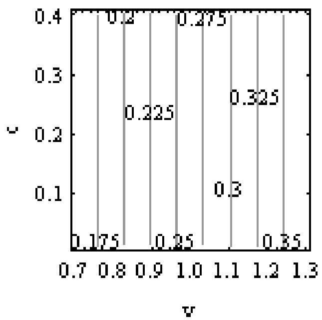


Figure 5.B1.15d |g=0.2

Contour Plot $P^*_{21}\,(frr_0)$

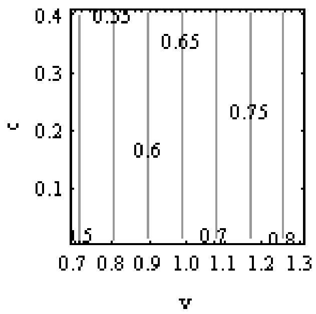


Figure 5.B1.15: Interior solution is ensured under frr_0 for g=0.2 and $0.7 \le v \le 1.3$ that interior solution is ensured under frr_1, frr_2, and frr_3

Figure 5.B1.16a |g=0.2

Contour Plot $w^*_{10}\,(frr_0)$

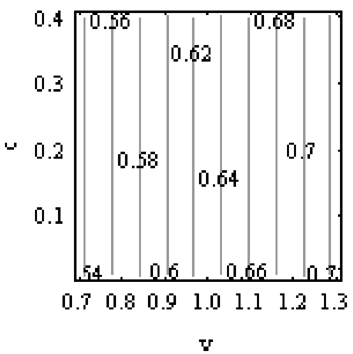


Figure 5.B1.16b |g=0.2

Contour Plot $w^*_{20}\,(frr_0)$

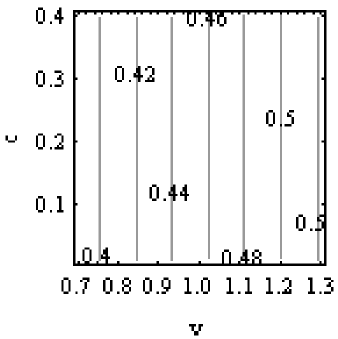


Figure 5.B1.16: Interior solution is ensured under frr_0 for g=0.2 and $0.7 \le v \le 1.3$ that interior solution is ensured under frr_1, frr_2, and frr_3

APPENDIX 5.B2

Case *frr*$_2$

Interior solution is ensured provided that both firms produce in each period. I test this assumption for different values of *g*, as I did in the previous chapters, and I find out that interior solution is ensured provided that products are imperfect substitutes. In particular, if $g \in [0.3,1]$, then there are combinations c and v that yields negative contour lines for q_{20} and P_{11}.Thus, in this appendix, I present the case where *g*=0.2. P_{11} imposes restrictions related to *v* and *c*, namely $v \geq 0.7$ and $c \in (0, 0.4]$

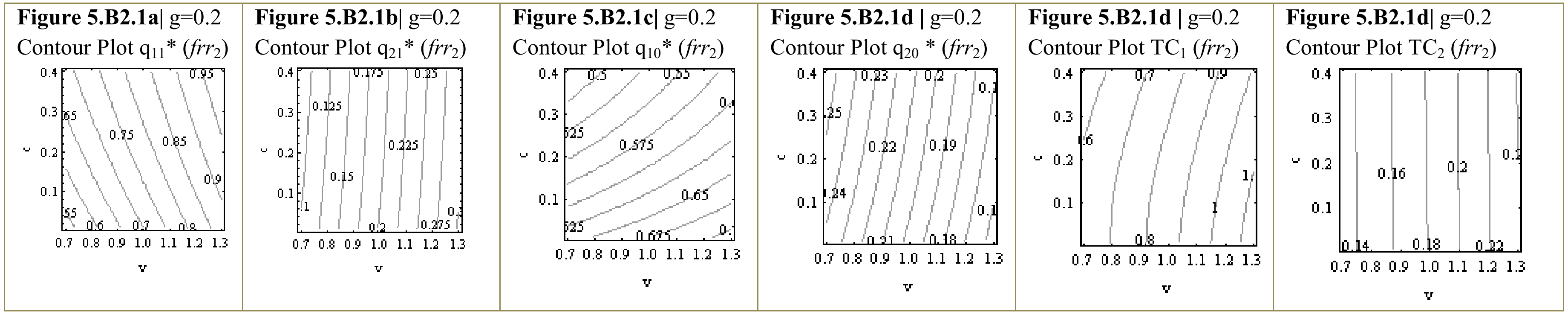


Figure 5.B2.1: Positive isoquants, in each period and total costs, simultaneously if $0 < c \leq 0.4$ and $0.7 \leq v \leq 1.3$

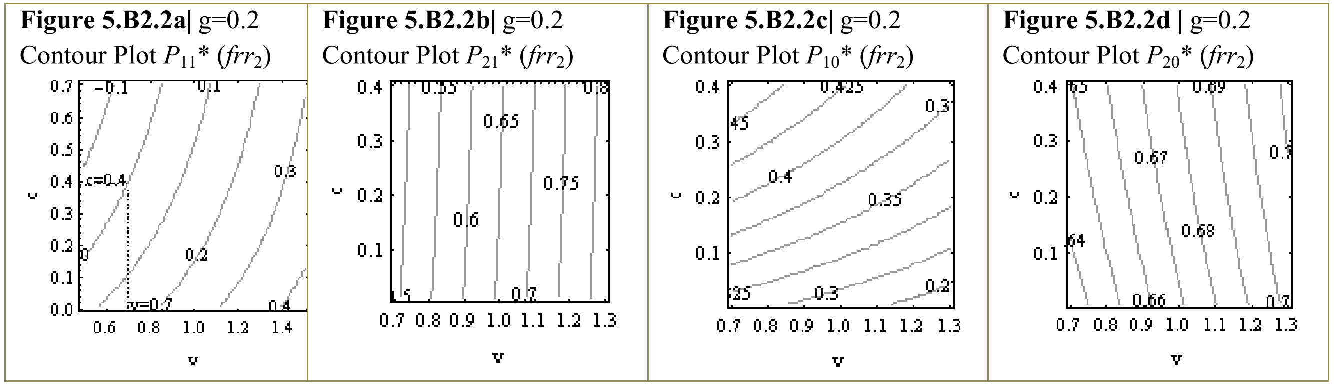


Figure 5.B2.2: Positive prices, in each period simultaneously, provided that $v > 0.7$, $c \leq 0.4$

Figure 5.B2.3a | g=0.2

Contour Plot $w_{10}^{*}(frr_2)$

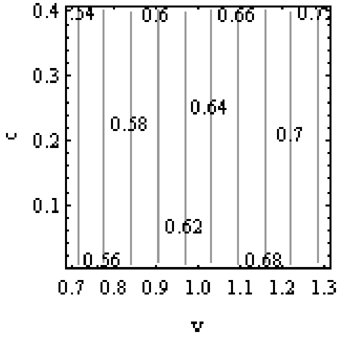


Figure 5.B2.3b | g=0.2

Contour Plot $w_{20}^{*}(frr_2)$

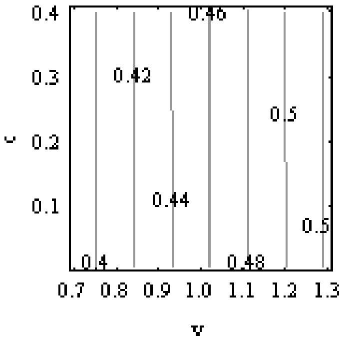


Figure 5.B2.3c | g=0.2

Contour Plot $\left(w_{10}^{*} - w_{20}^{*}\right)(frr_2)$

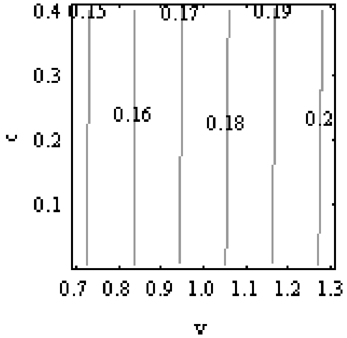


Figure 5.B2.3d | g=0.2

Contour Plot $EV_1(frr_2)$

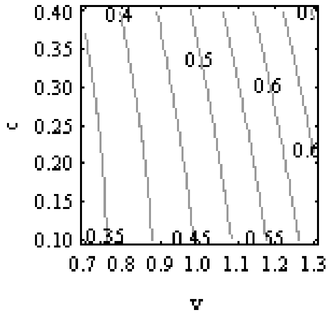


Figure 5.B2.3: Positive wages and positive $EV_1(frr_2)$ provided that interior solution is ensured

Figure 5.B2.4a | g=0.2

Contour Plot $\Pi_1(frr_2)$

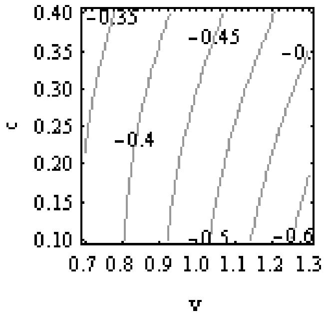


Figure 5.B2.4b | g=0.2

Contour Plot $\Pi_2(frr_2)$

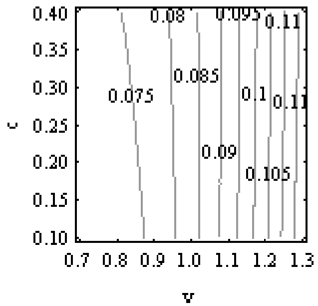


Figure 5.B2.4c | g=0.2

Contour Plot $U_1(frr_2)$

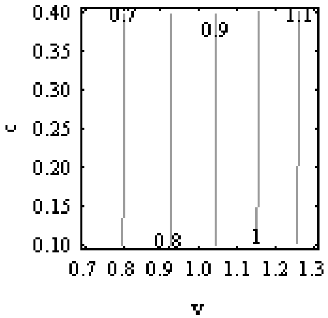


Figure 5.B2.4d | g=0.2

Contour Plot $U_2(frr_2)$

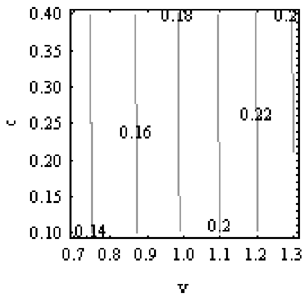


Figure 5.B2.4e | g=0.2

Contour Plot $CS(frr_2)$

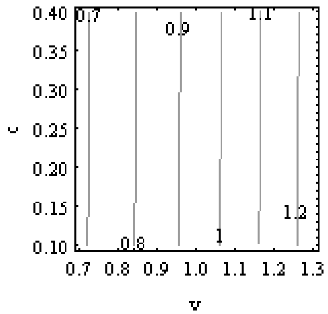


Figure 5.B2.4: Π_2 U_1, U_2 and CS are positive; Π_1<0, provided interior solution is ensured

Figure 5.B2.5a | g=0.2

Contour Plot $\left(q_{10}^{*}-q_{20}^{*}\right)\left(frr_{2}\right)$

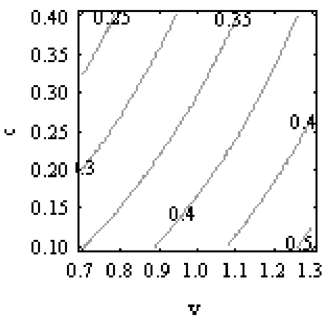


Figure 5.B2.5b | g=0.2

Contour Plot $\left(q_{11}^{*}-q_{21}^{*}\right)\left(frr_{2}\right)$

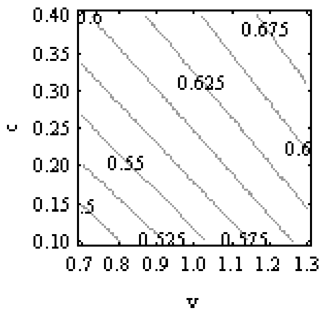


Figure 5.B2.5c | g=0.2

Contour Plot $\left(q_{11}^{*}-q_{10}^{*}\right)\left(frr_{2}\right)$

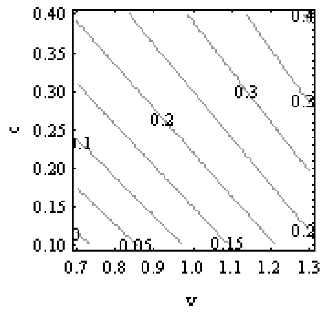


Figure 5.B2.5d | g=0.2

Contour Plot $\left(q_{21}^{*}-q_{20}^{*}\right)\left(frr_{2}\right)$

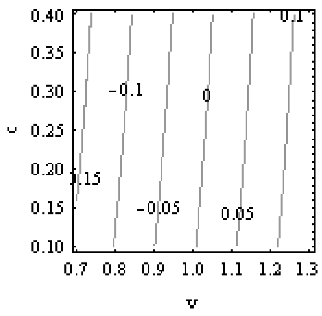


Figure 5.B2.5: $q_{10}^{*}>q_{20}^{*}$, $q_{11}^{*}>q_{21}^{*}$; $q_{11}^{*}>q_{10}^{*}$; $q_{20}^{*}>q_{21}^{*}$, if v>1

Figure 5.B2.6a | g=0.2

Contour Plot (tq_1^*-tq_2^*) (*frr*$_2$)

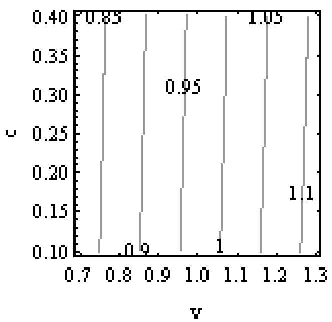


Figure 5.B2.6b | g=0.2

Contour Plot (tq_A^*-tq_B^*) (*frr*$_2$)

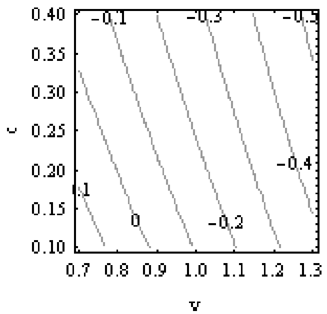


Figure 5.B2.6c | g=0.2

Contour Plot (Π_1-Π_2) (*frr*$_2$)

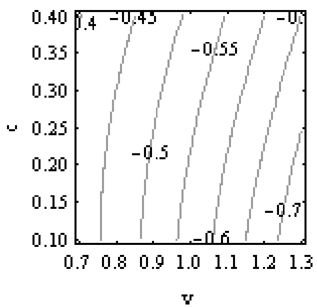


Figure 5.B2.6d | g=0.2

Contour Plot (U_1-U_2) (*frr*$_2$)

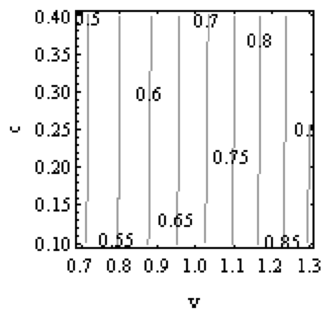


Figure 5.B2.6e | g=0.2

Contour Plot (EV_1-Π_2) (*frr*$_2$)

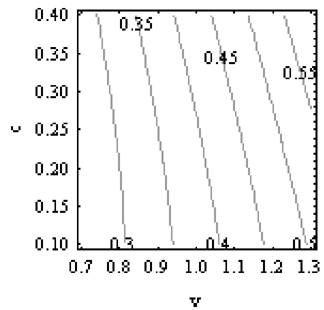


Figure 5.B2.6: $tq_1^*>tq_2^*$, $tq_A^*<tq_B^*$, if c>0.9, $\Pi_1<\Pi_2$, $U1>U2$, and $EV_1>\Pi_2$

Figure 5.B2.7a |g=0.2

Contour Plot $q_{10}^*(frr_2)-q_{10}^*(frr_0)$

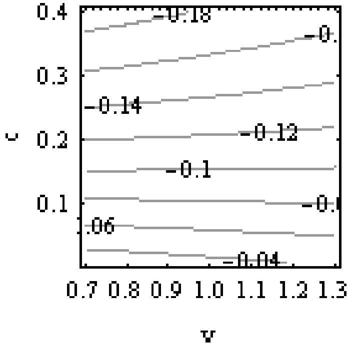


Figure 5.B2.7b |g=0.2

Contour Plot $q_{20}^*(frr_2)-q_{20}^*(frr_0)$

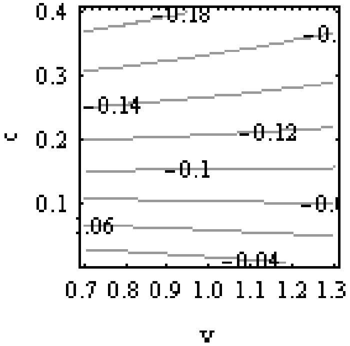


Figure 5.B2.7c |g=0.2

Contour Plot $q_{11}^*(frr_2)-q_{11}^*(frr_0)$

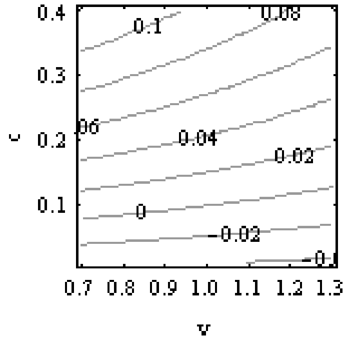


Figure 5.B2.7d |g=0.2

Contour Plot $q_{21}^*(frr_2)-q_{21}^*(frr_0)$

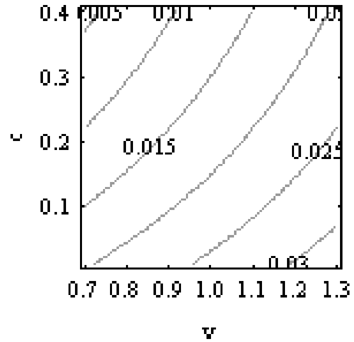


Figure 5.B2.7: $q_{10}^*(frr_2)<q_{10}^*(frr_0)$; $q_{20}^*(frr_2)<q_{20}^*(frr_0)$; $q_{11}^*(frr_2)>q_{11}^*(frr_0)$ if $c>0.1$ and $q_{21}^*(frr_2)>q_{21}^*(frr_0)$

Figure 5.B2.8a |g=0.2

Contour Plot

$(w_{10}^*)(frr_2)-(w_{10}^*)(frr_0)$

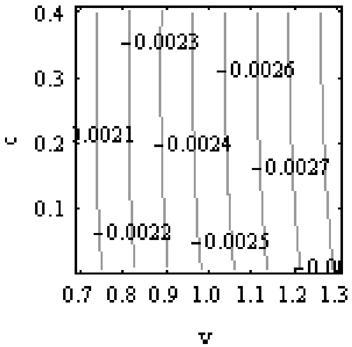


Figure 5.B2.8b |g=0.2

Contour Plot

$(w_{20}^*)(frr_2)-(w_{20}^*)(frr_0)$

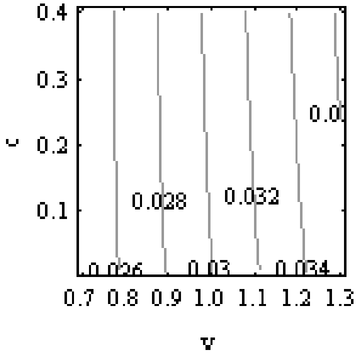


Figure 5.B2.8c |g=0.2

Contour Plot

$(w_{10}^*-w_{20}^*)(frr_2)-(w_{10}^*-w_{20}^*)(frr_0)$

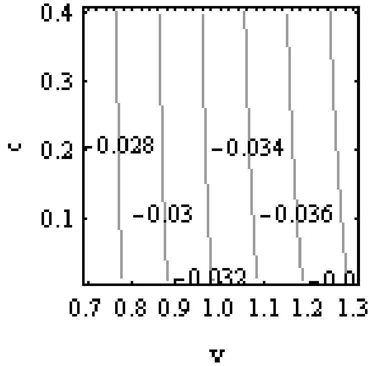


Figure 5.B2.8d |g=0.2

Contour Plot

$(w_{10}^*-w_{20}^*)(frr_0)$

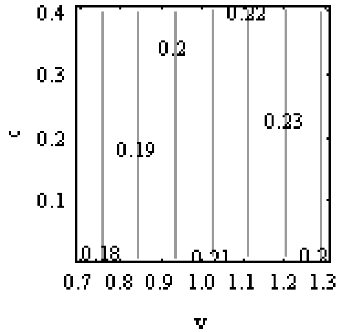


Figure 5.B2.8: $(w_{10}^*)(frr_2)<(w_{10}^*)(frr_0)$; $(w_{20}^*)(frr_2)>(w_{20}^*)(frr_0)$; $(w_{10}^*-w_{20}^*)(frr_2)<(w_{10}^*-w_{20}^*)(frr_0)$; $(w_{10}^*-w_{20}^*)(frr_0)>0$

Figure 5.B2.9a |g=0.2
Contour Plot
$(q_{11}{*}-q_{10}{*})\ (frr_2)-(q_{11}{*}-q_{10}{*})\ (frr_0)$

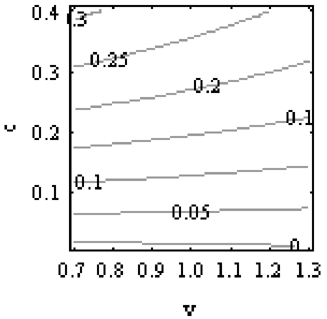


Figure 5.B2.9b |g=0.2
Contour Plot
$(q_{21}{*}-q_{20}{*})\ (frr_2)-(q_{21}{*}-q_{20}{*})\ (frr_0)$

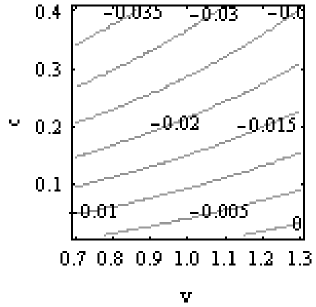


Figure 5.B2.9c |g=0.2
Contour Plot
$(q_{10}-{*}q_{20}{*})\ (frr_2)-(q_{10}{*}-q_{20}{*})\ (frr_0)$

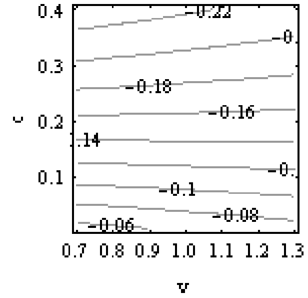


Figure 5.B2.9d |g=0.2
Contour Plot
$(q_{11}{*}-q_{21}{*})\ (frr_2)-(q_{11}{*}-q_{21}{*})\ (frr_0)$

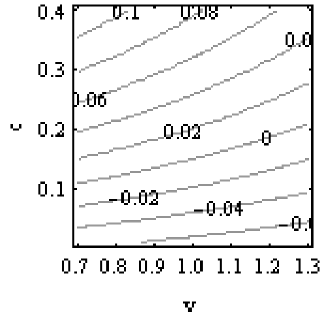


Figure 5.B2.9: $(q_{11}{*}-q_{10}{*})\ (frr_2)>(q_{11}{*}-q_{10}{*})\ (frr_0)$; $(q_{21}{*}-q_{20}{*})\ (frr_2)<(q_{21}{*}-q_{20}{*})\ (frr_0)$; $(q_{10}-{*}q_{20}{*})\ (frr_2)<(q_{10}{*}-q_{20}{*})\ (frr_0)$; $(q_{11}{*}-q_{21}{*})\ (frr_2)>(q_{11}{*}-q_{21}{*})\ (frr_0)$ if $c>0.1$

Figure 5.B2.10a |g=0.2
Contour Plot
$tq_1{*}\ (frr_2)-tq_1{*}\ (frr_0)$

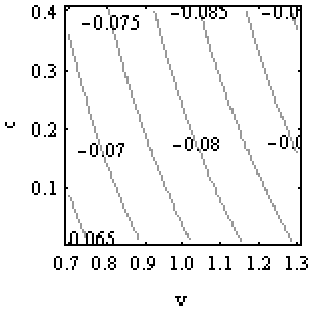


Figure 5.B2.10b |g=0.2
Contour Plot
$tq_2{*}\ (frr_2)-tq_2{*}\ (frr_0)$

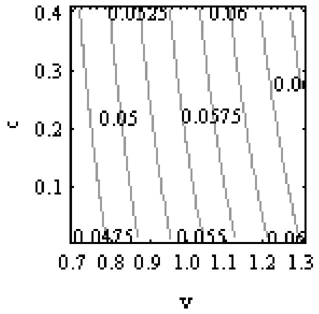


Figure 5.B2.10c |g=0.2
Contour Plot
$Tq_A{*}\ (frr_2)-Tq_A{*}\ (frr_0)$

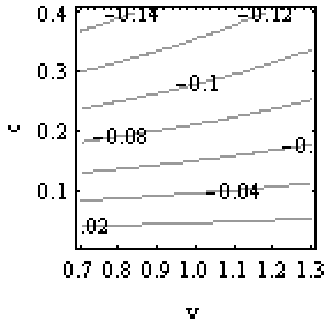


Figure 5.B2.10d |g=0.2
Contour Plot
$Tq_B{*}\ (frr_2)-Tq_B{*}\ (frr_0)$

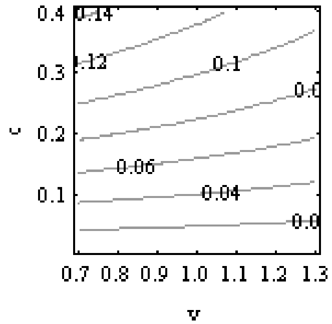


Figure 5.B2.10e |g=0.2
Contour Plot
$Tq{*}\ (frr_2)-Tq{*}\ (frr_0)$

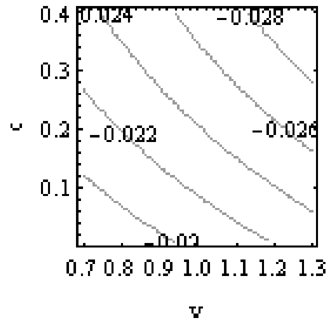


Figure 5.B2.10: $tq_1{*}=q_{10}{*}+q_{11}{*}$ and $tq_2{*}= q_{20}{*}+q_{21}{*}$; $tq_1{*}(frr_2)<tq_1{*}(frr_0)$ and $tq_2{*}(frr_2)>tq_2{*}(frr_0)$;$Tq_A{*}=q_{10}{*}+q_{20}{*}$ and $Tq_B{*}=q_{11}{*}+q_{21}{*}$; $Tq_A{*}\ (frr_2)<Tq_A{*}\ (frr_0)$ and $Tq_B{*}\ (frr_2)>Tq_B{*}\ (frr_0)$; $Tq{*}\ (frr_2)< Tq{*}\ (frr_0)$

Figure 5.B2.11a |g=0.2

Contour Plot $\Pi_2(frr_2)$-$\Pi_2(frr_0)$

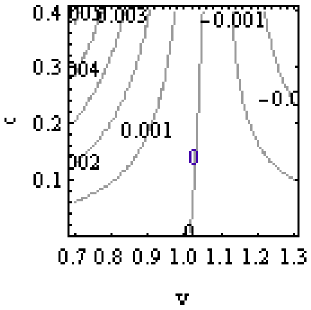


Figure 5.B2.11b |g=0.2

Contour Plot $U_1(frr_2)$-$U_1(frr_0)$

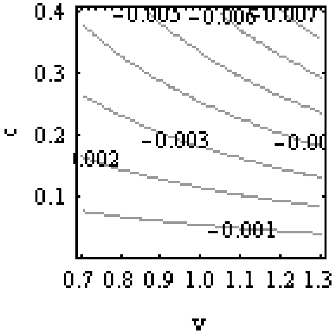


Figure 5.B2.11c |g=0.2

Contour Plot $U_2(frr_2)$-$U_2(frr_0)$

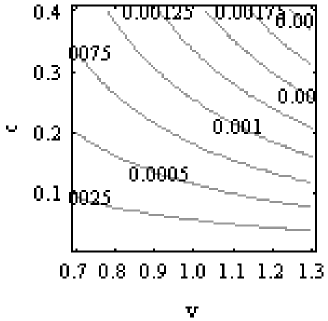


Figure 5.B2.11: $\Pi_2(frr_2)<\Pi_2(frr_0)$ v>1; $U_1(frr_2)<U_1(frr_0)$; $U_2(frr_2)>U_2(frr_0)$

Figure 5.B2.12a |g=0.2

Contour Plot

$\Pi_1(frr_2)$-$\Pi_1(frr_0)$

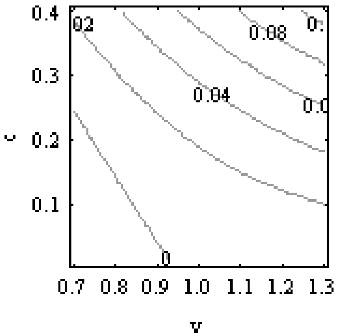


Figure 5.B2.12b |g=0.2

Contour Plot

$CS(frr_2)$-$CS(frr_0)$

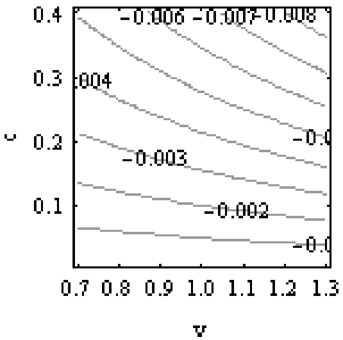


Figure 5.B2.12c |g=0.2

Contour Plot

$EV_1(frr_2)$-$EV_1(frr_0)$

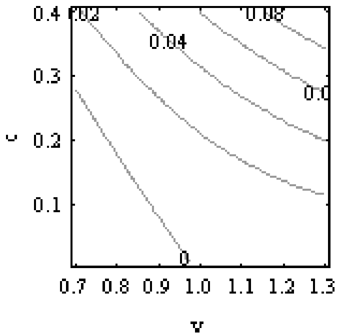


Figure 5.B2.12d |g=0.2

Contour Plot $\frac{dEV_1}{dc}(frr_2)$

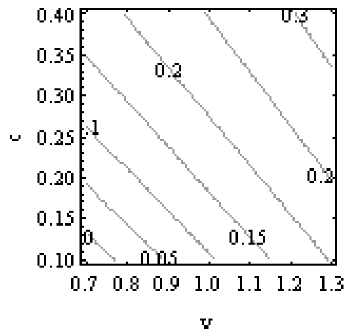


Figure 5.B2.12: $\Pi_1(frr_2)>\Pi_1(frr_0)$ if c >0.25, $CS(frr_2)<CS(frr_0)$; $EV_1(frr_2)>EV_1(frr_0)$ if c >0.25; EV_1 is increasing with c

Figure 5.B2.13a |g=0.2

Contour Plot $\frac{d\Pi_1}{dc}$ (frr_2)

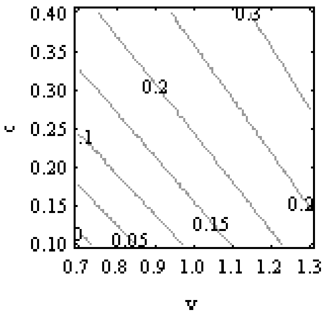


Figure 5.B2.13b |g=0.2

Contour Plot $\frac{d\Pi_2}{dc}$ (frr_2)

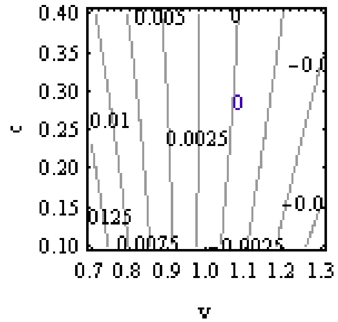


Figure 5.B2.13c |g=0.2

Contour Plot $\frac{dU_1}{dc}$ (frr_2)

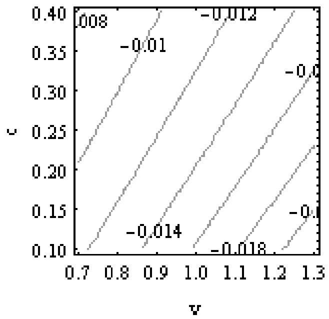


Figure 5.B2.13d |g=0.2

Contour Plot $\frac{dU_2}{dc}$ (frr_2)

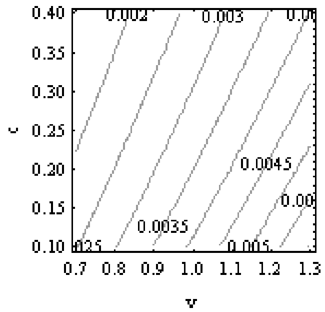


Figure 5.B2.13e |g=0.2

Contour Plot $\frac{dCS}{dc}$ (frr_2)

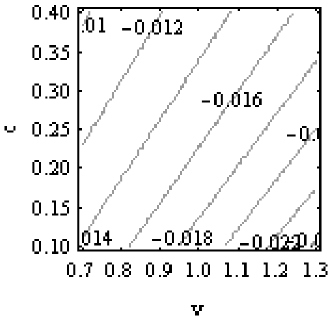


Figure 5.B2.13: Π_1 and U_2 are increasing with c, Π_2 is increasing with *c* if v<1.1

APPENDIX 5.B3

Case *frr*$_3$

Interior solution is ensured provided that both firms produce in each period. I test this assumption for different values of g, as I did in the previous chapters, and I find out that interior solution is ensured provided that products are imperfect substitutes. In particular, if $g \in [0.3,1]$, then there are combinations c and v that yields negative contour lines for q_{20} and P_{10}.Thus, in this appendix, I present the case where $g=0.2$. P_{10} imposes restrictions related to v, namely $v \geq 0.7$ [39] and restrictions related to $c \leq 0.33$.

Figure 5.B3.1a g=0.2
Contour Plot q_{11}*(*frr*$_3$)

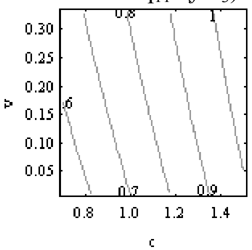


Figure 5.B3.1b g=0.2
Contour Plot q_{21}* (*frr*$_3$)

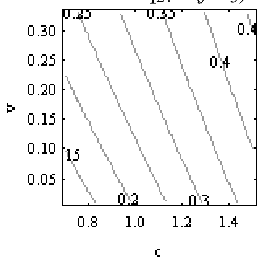


Figure 5.B3.1c g=0.2
Contour Plot q_{10}*(*frr*$_3$)

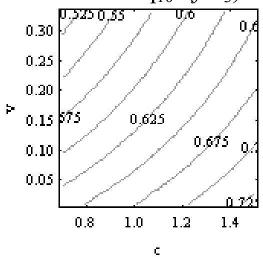


Figure 5.B3.1d g=0.2
Contour Plot q_{20}* (*frr*$_3$)

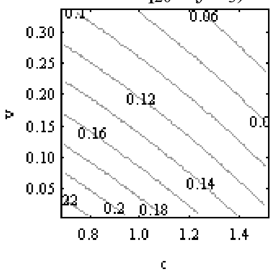


Figure 5.B3.1d g=0.2
Contour Plot TC_1 (*frr*$_3$)

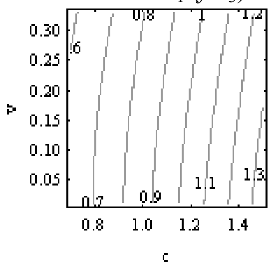


Figure 5.B3.1d g=0.2
Contour Plot TC_2 (*frr*$_3$)

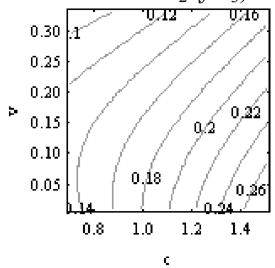


Figure 5.B3.1: Positive isoquants, in each period and total costs, simultaneously, provided that $v \geq 0.7$ and $c \leq 0.33$

Figure 5.B3.2a g=0.2
Contour Plot P_{11}*(*frr*$_3$)

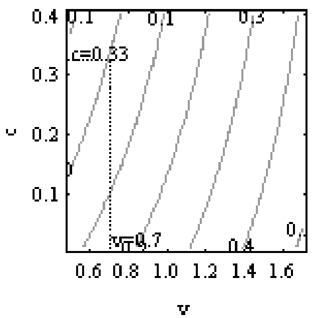


Figure 5.B3.2b g=0.2
Contour Plot P_{21}* (*frr*$_3$)

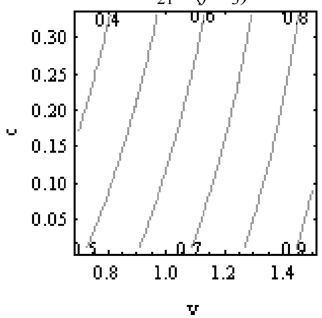


Figure 5.B3.2c g=0.2
Contour Plot P_{10}*(*frr*$_3$)

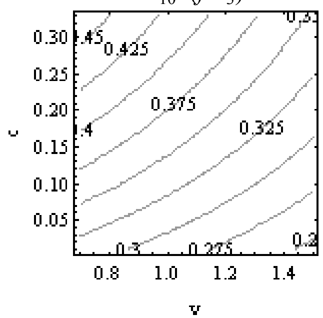
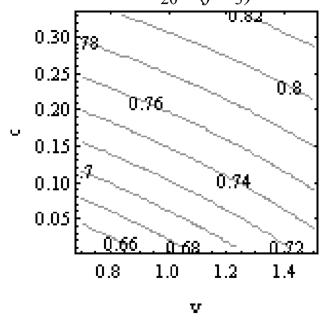


Figure 5.B3.2d g=0.2
Contour Plot P_{20}* (*frr*$_3$)



[39] Note that v is restricted even for positive demand shock, because we assume that θ is positive or negative demand shock of equal magnitude.

Figure 5.B3.2: Positive prices, in each period, simultaneously, provided that $v \geq 0.7$ and $c \leq 0.33$

Figure 5.B3.3a g=0.2

Contour Plot $w_{10}^{*}(frr_3)$

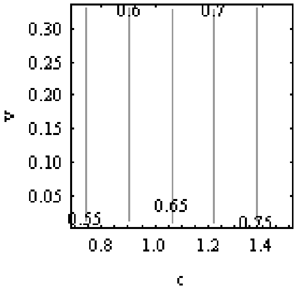


Figure 5.B3.3b g=0.2

Contour Plot $w_{20}^{*}(frr_3)$

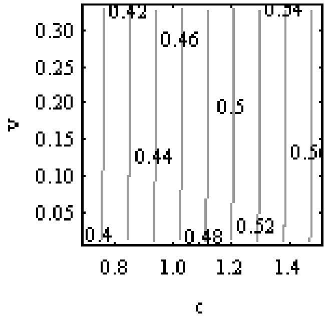


Figure 5.B3.3c g=0.2

Contour Plot $\left(w_{10}^{*} - w_{20}^{*}\right)(frr_3)$

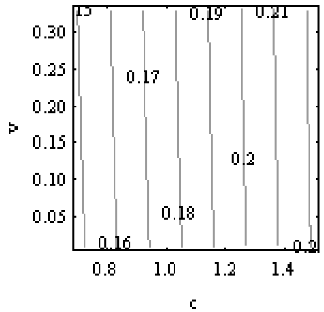


Figure 5.B3.3d g=0.2

Contour Plot $EV_1(frr_3)$

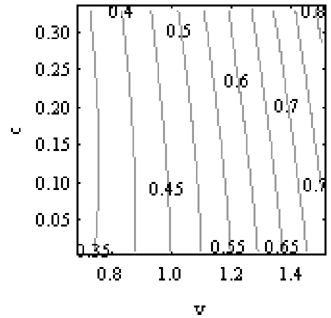


Figure 5.B3.3: Positive wages ($w_{10}^{*} > w_{20}^{*}$) and positive $EV_1(frr_3)$ under frr_3

Figure 5.B3.4a g=0.2

Contour Plot $\Pi_1(frr_3)$

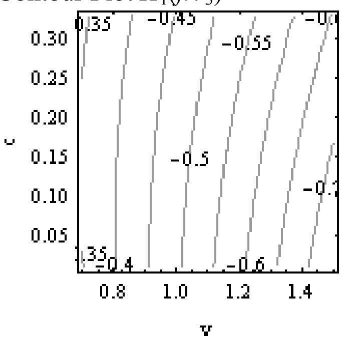


Figure 5.B3.4b g=0.2

Contour Plot $\Pi_2(frr_3)$

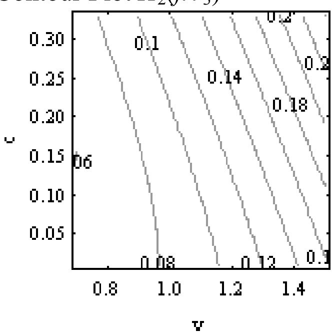


Figure 5.B3.4c g=0.2

Contour Plot $U_1(frr_3)$

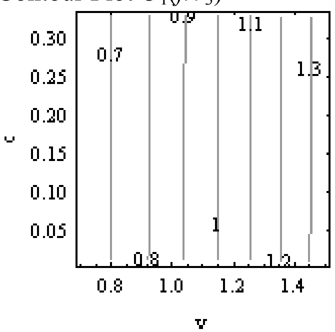


Figure 5.B3.4d g=0.2

Contour Plot $U_2(frr_3)$

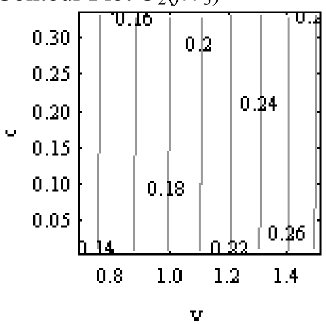


Figure 5.B3.4e g=0.2

Contour Plot $CS(frr_3)$

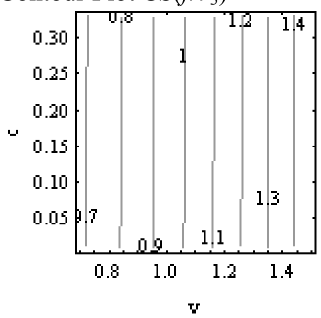


Figure 5.B3.4: Π_2 U_1, U_2 and CS are positive, Π_1<0, provided that interior solution is ensured

Figure 5.B3.5a g=0.2

Contour Plot $\left(q_{10}^{*}-q_{20}^{*}\right)\left(frr_{3}\right)$,

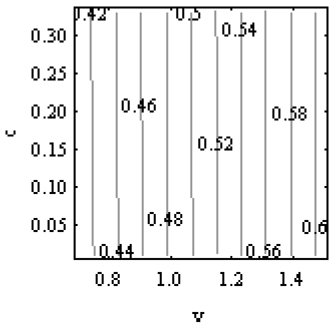


Figure 5.B3.5b g=0.2

Contour Plot $\left(q_{11}^{*}-q_{21}^{*}\right)\left(frr_{3}\right)$,

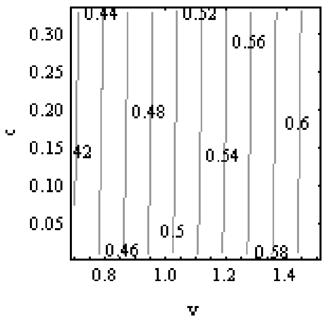


Figure 5.B3.5c g=0.2

Contour Plot $\left(q_{11}^{*}-q_{10}^{*}\right)\left(frr_{3}\right)$,

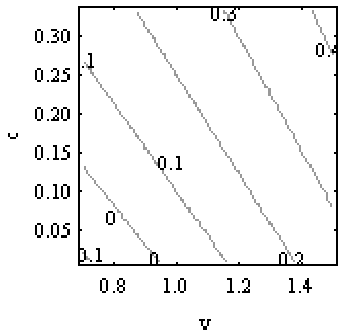


Figure 5.B3.5d g=0.2

Contour Plot $\left(q_{21}^{*}-q_{20}^{*}\right)\left(frr_{3}\right)$,

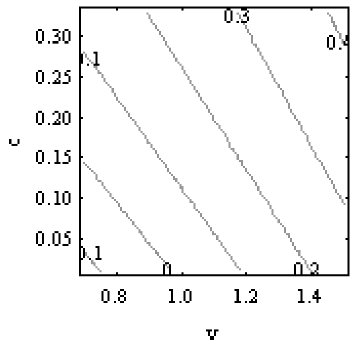


Figure 5.B3.5: $q_{10}^{*}>q_{20}^{*}$ and $q_{11}^{*}>q_{21}^{*}$, $q_{11}^{*}>q_{10}^{*}$, c>0.15 and $q_{21}^{*}>q_{20}^{*}$, c>0.15

Figure 5.B3.6a g=0.2

Contour Plot (tq_1^*-tq_2^*) (frr_3)

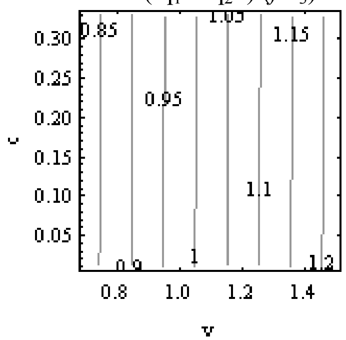


Figure 5.B3.6b g=0.2

Contour Plot (tq_A^*-tq_B^*) (frr_3)

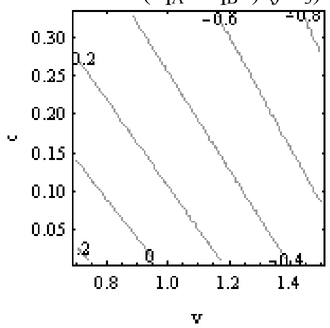


Figure 5.B3.6c g=0.2

Contour Plot (Π_1-Π_2) (frr_3)

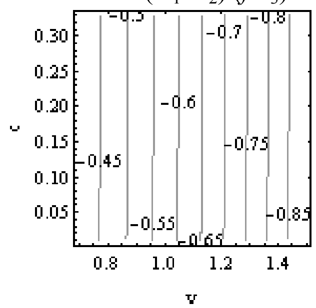


Figure 5.B3.6d g=0.2

Contour Plot (U_1-U_2) (frr_3)

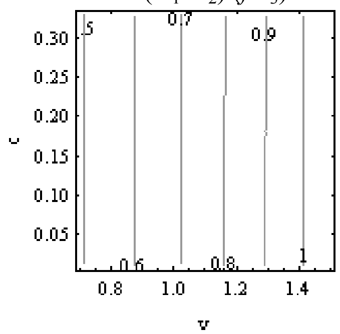


Figure 5.B3.6e g=0.2

Contour Plot (EV_1-Π_2) (frr_3)

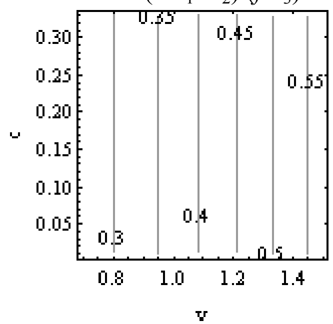


Figure 5.B3.6: $tq_1^*>tq_2^*$, $tq_A^*<tq_B^*$ if c>0.15, $\Pi_1<\Pi_2$, $U_1>U_2$, and $EV_1>\Pi_2$

Figure 5.B3.7a |g=0.2

Contour Plot $q_{10}*(frr_3)-q_{10}*(frr_0)$

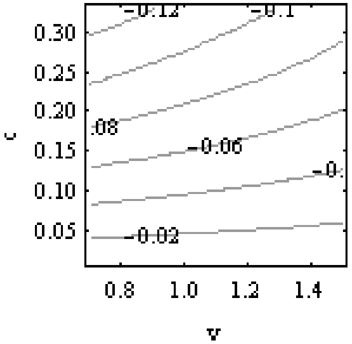


Figure 5.B3.7b |g=0.2

Contour Plot $q_{20}*(frr_3)-q_{20}*(frr_0)$

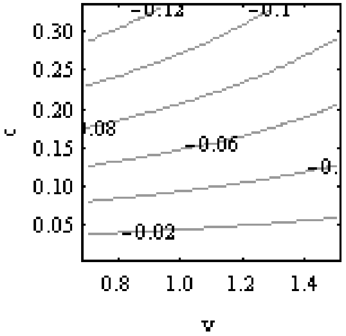


Figure 5.B3.7c |g=0.2

Contour Plot $q_{11}*(frr_3)-q_{11}*(frr_0)$

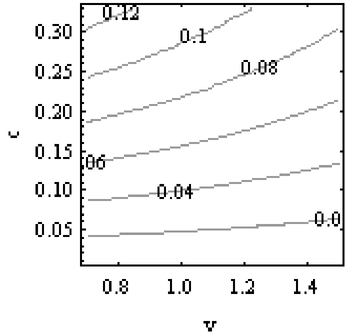


Figure 5.B3.7d |g=0.2

Contour Plot $q_{21}*(frr_3)-q_{21}*(frr_0)$

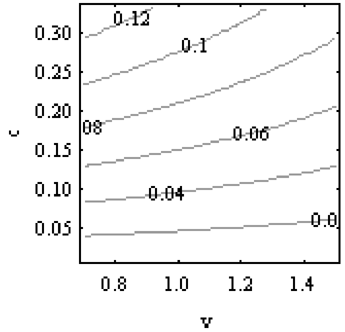


Figure 5.B3.7: $q_{10}*(frr_3)<q_{10}*(frr_0)$; $q_{20}*(frr_3)<q_{20}*(frr_0)$; $q_{11}*(frr_3)>q_{11}*(frr_0)$ and $q_{21}*(frr_3)>q_{21}*(frr_0)$

Figure 5.B3.8a |g=0.2

Contour Plot

$(w_{10}*)(frr_3)-(w_{10}*)(frr_0)$

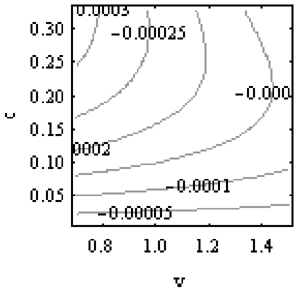


Figure 5.B3.8b |g=0.2

Contour Plot

$(w_{20}*)(frr_3)-(w_{20}*)(frr_0)$

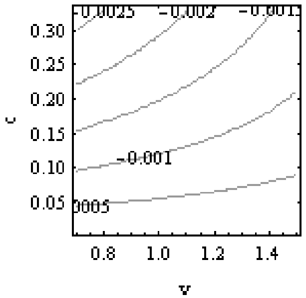


Figure 5.B3.8c |g=0.2

Contour Plot

$(w_{10}*-w_{20}*)(frr_3)-(w_{10}*-w_{20}*)(frr_0)$

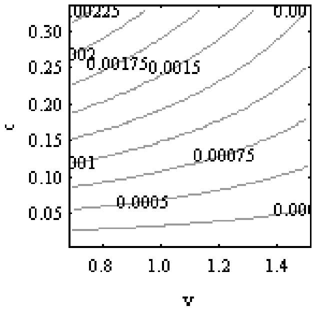


Figure 5.B3.8d |g=0.2

Contour Plot

$(w_{10}*-w_{20}*)(frr_0)$

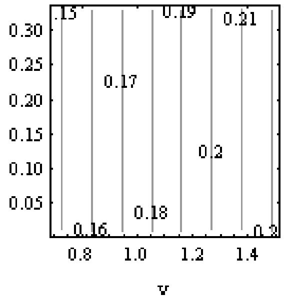


Figure 5.B3.8: $(w_{10}*)(frr_3)<(w_{10}*)(frr_0)$; $(w_{20}*)(frr_3)<(w_{20}*)(frr_0)$; $(w_{10}*-w_{20}*)(frr_3)>(w_{10}*-w_{20}*)(frr_0)$; $(w_{10}*-w_{20}*)(frr_0)>0$

Figure 5.B3.9a |g=0.2
Contour Plot
$(q_{11}^*-q_{10}^*)$ (frr_3)-$(q_{11}^*-q_{10}^*)$ (frr_0)

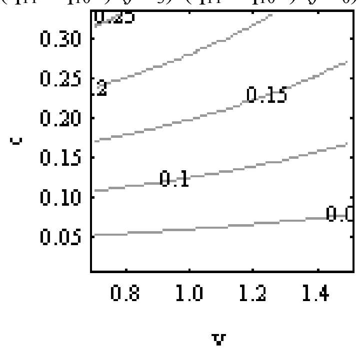


Figure 5.B3.9b |g=0.2
Contour Plot
$(q_{21}^*-q_{20}^*)$ (frr_3)-$(q_{21}^*-q_{20}^*)$ (frr_0)

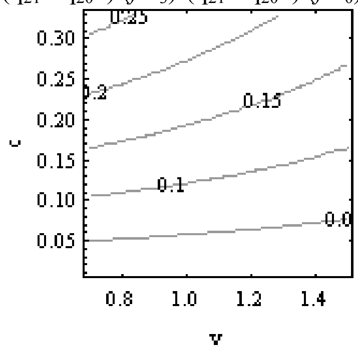


Figure 5.B3.9c |g=0.2
Contour Plot
$(q_{10}\text{-}^*q_{20}^*)$ (frr_3)-$(q_{10}^*-q_{20}^*)$ (frr_0)

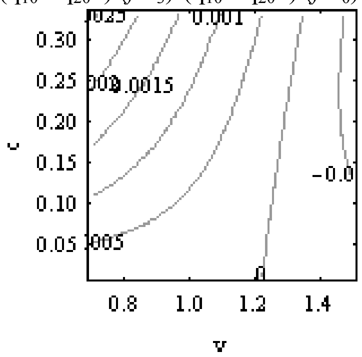


Figure 5.B3.9d |g=0.2
Contour Plot
$(q_{11}^*-q_{21}^*)$ (frr_3)-$(q_{11}^*-q_{21}^*)$ (frr_0)

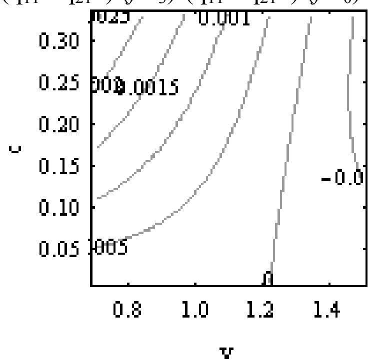


Figure 5.B3.9: $(q_{11}^*-q_{10}^*)$ (frr_3)>$(q_{11}^*-q_{10}^*)$ (frr_0) ; $(q_{21}^*-q_{20}^*)$ (frr_3)>$(q_{21}^*-q_{20}^*)$ (frr_0); $(q_{10}\text{-}^*q_{20}^*)$ (frr_3)>$(q_{10}^*-q_{20}^*)$ (frr_0) ; $(q_{11}^*-q_{21}^*)$ (frr_3)>$(q_{11}^*-q_{21}^*)$ (frr_0)

Figure 5.B3.10a |g=0.2
Contour Plot
tq_1^* (frr_3)-tq_1^* (frr_0)

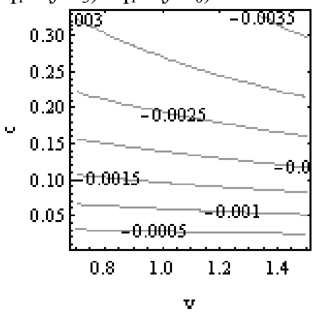


Figure 5.B3.10b |g=0.2
Contour Plot
tq_2^* (frr_3)-tq_2^* (frr_0)

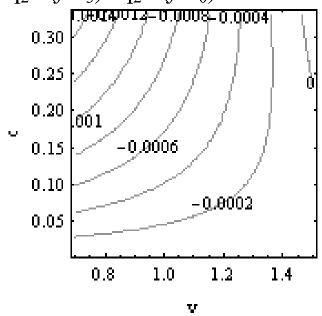


Figure 5.B3.10c |g=0.2
Contour Plot
Tq_A^* (frr_3)-Tq_A^* (frr_0)

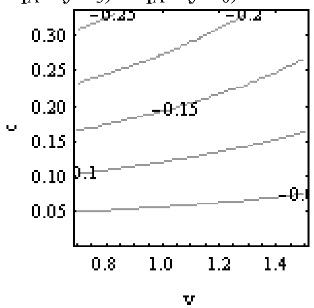


Figure 5.B3.10d |g=0.2
Contour Plot
Tq_B^* (frr_3)-Tq_B^* (frr_0)

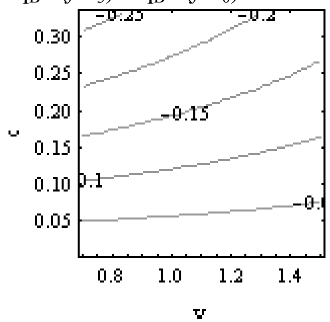


Figure 5.B3.10e |g=0.2
Contour Plot
Tq^* (frr_3)-Tq^* (frr_0)

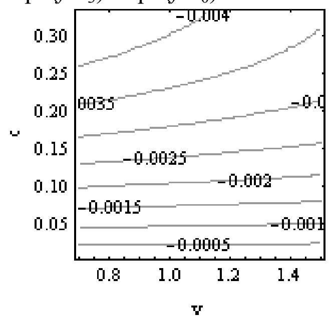


Figure 5.B3.10: $tq_1^*=q_{10}^*+q_{11}^*$ and $tq_2^*= q_{20}^*+q_{21}^*$; $tq_1^*(frr_3)<tq_1^*(frr_0)$; and $tq_2^*(frr_3)<tq_2^*(frr_0)$; $Tq_A^*=q_{10}^*+q_{20}^*$ and $Tq_B^*=q_{11}^*+q_{21}^*$; Tq_A^* (frr_3)<Tq_A^* (frr_0) and Tq_B^* (frr_3)<Tq_B^* (frr_0); Tq^* (frr_3)<Tq^* (frr_0)

Figure 5.B3.11a |g=0.2

Contour Plot $\Pi_2(frr_3)$-$\Pi_2(frr_0)$

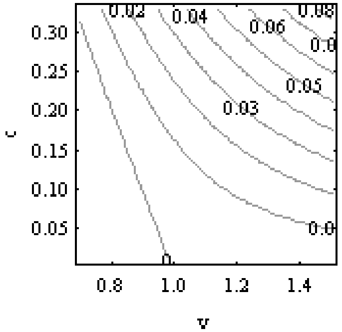


Figure 5.B3.11b |g=0.2

Contour Plot $U_1(frr_3)$-$U_1(frr_0)$

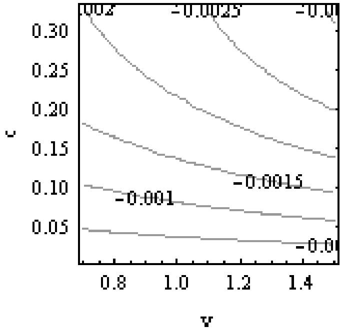


Figure 5.B3.11c |g=0.2

Contour Plot $U_2(frr_3)$-$U_2(frr_0)$

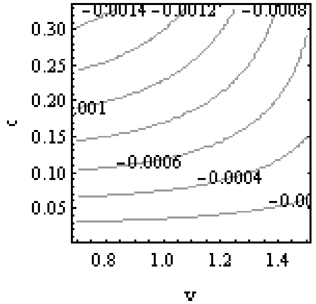


Figure 5.B3.11: $\Pi_2(frr_3)>\Pi_2(frr_0)$ v>1; $U_1(frr_3)<U_1(frr_0)$; $U_2(frr_3)<U_2(frr_0)$

Figure 5.B3.12a |g=0.2

Contour Plot

$\Pi_1(frr_3)$-$\Pi_1(frr_0)$

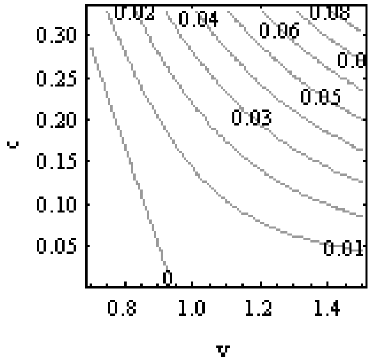


Figure 5.B3.12b |g=0.2

Contour Plot

$CS(frr_3)$-$CS(frr_0)$

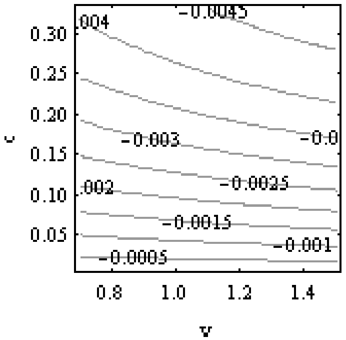


Figure 5.B3.12c |g=0.2

Contour Plot

$EV_1(frr_3)$-$EV_1(frr_0)$

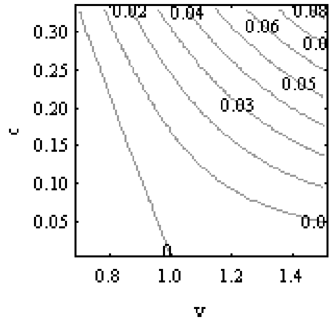


Figure 5.B3.12d |g=0.2

Contour Plot $\frac{dEV_1}{dc}(frr_3)$

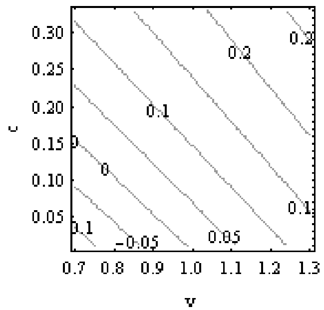


Figure 5.B3.12: $\Pi_1(frr_3)>\Pi_1(frr_0)$; $CS(frr_3)<CS(frr_0)$; $EV_1(frr_3)>EV_1(frr_0)$ v>1; EV_1 increasing with *c*, if c>0.15

Figure 5.B3.13a |g=0.2

Contour Plot $\frac{d\Pi_1}{dc}$ (frr_3)

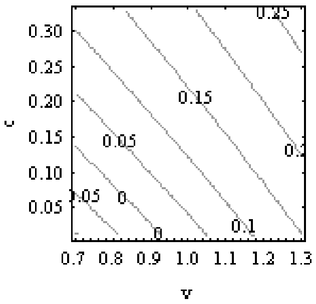


Figure 5.B3.13b |g=0.2

Contour Plot $\frac{d\Pi_2}{dc}$ (frr_3)

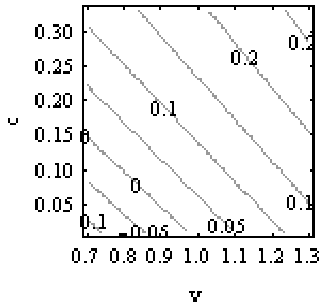


Figure 5.B3.13c |g=0.2

Contour Plot $\frac{dU_1}{dc}$ (frr_3)

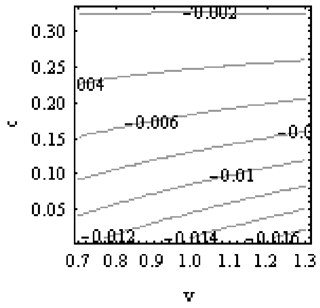


Figure 5.B3.13d |g=0.2

Contour Plot $\frac{dU_2}{dc}$ (frr_3)

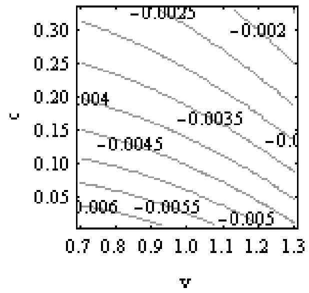


Figure 5.B3.13e |g=0.2

Contour Plot $\frac{dCS}{dc}$ (frr_3)

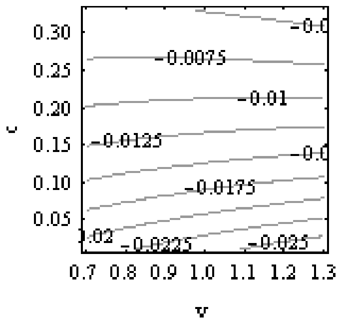


Figure 5.B3.13: Π_1 and Π_2 are increasing with c provided that $c \in [0.15, 0.33]$