UNIVERSITY OF IOANNINA

MASTER THESIS

CKM corrections in the Standard Model Effective Field Theory

Submitted by:
Dimitrios Beis

Supervisor:
Dr Athanasios Dedes

Division of Theoritical Physics
Physics Department

May 10, 2023

Abstract

CKM corrections in the Standard Model Effective Field Theory

by Dimitrios BEIS

In this thesis, we investigate the impact of New Physics (NP) on the global CKM fit and propose a straightforward approach to constrain the CKM matrix. The theoretical framework used in analyzing flavor data is the Standard Model Effective Field Theory (SMEFT). SMEFT also contains the leading NP effects from the six-dimensional Wilson coefficients and parameters present in the SM Lagrangian. In addition, SMEFT can be used to account for correlations between different observables, such as Electroweak precision measurements, leptonic processes, and quark-flavor transitions.

Our approach uses a set of input observables and express the CKM parameters in terms of Wilson coefficients that can be produced from these observables. We work with this framework to match the LEFT (Low Energy Effective Field Theory)'s Wilson coefficients with those of SMEFT. Additionally, we use experimental data on the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix to verify its contribution to the parameters of the CKM matrix.The resulting combinations of Wilson coefficients define the corrected matrix elements of the CKM matrix and we apply these new parameters in order to set limits on NP processes.

Acknowledgements

I would like to express my gratitude to the following individuals who have helped me throughout my thesis:

- My Supervisor, Athanasios Dedes, for his guidance and support during this thesis. I also thank him for providing us with an office space, which has greatly facilitated our research.
- Phd Student Kostas Mantzaropoulos and Lambros Tryffilis for their valuable comments and discussions about physics.
- My friends and colleagues Theodoros Katsoulas and Vaggelis Karantanis for their company during my master's thesis.
- Last but not least i would like to thank my family for their support and encouragement throughout my academic journey.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 The Standard Model of Particles physics

The Standard Model (SM) is a theoretical framework in particle physics that describes the fundamental particles and their interactions. Several textbooks are available that cover different aspects of SM, such as:[9],[41],[30]. To construct this theory, a mathematical framework known as Quantum Field Theory (QFT) is required. QFT is a physical theory that combines Quantum Mechanics and Special Relativity into a single theory and plays a crucial role in describing the physics of elementary particle. Some of the most well-known textbooks regarding quantum field theory are the following:[36],[38],[40]. Standard Model is a non-abelian gauge theory and the corresponding group is:

$$\boxed{G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y}, \tag{1.1}$$

where C stands for color, Y for hypercharge and L for Left.The Higgs Mechanism is the way in which particles acquire mass in the Standard Model. The general mathematical concept of the Higgs mechanism can be found in [29] . Using Higgs mechanism we break the Electroweak Symmetry in order to give masses to the Gauge bosons. We can interpret the breaking symmetry as:

$$\boxed{SU_L(2) \times U_Y(1) \to U_{QED}(1)}. \tag{1.2}$$

In the following sections we analyze in detail the Electroweak and Fermion Sector.

1.2 Electroweak Sector

The Electroweak theory unifies electromagnetic and weak interactions and was established by Glashow, Weinberg, and Salam [42],[25]. The Kinetic term of the Lagrangian that governed by the Electroweak Symmetry: $SU_L(2) \times U(1)_Y$ is:

$$\mathcal{L}_{kin} = -\frac{1}{4} W_a^{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \left| D_\mu \phi \right|^2, \tag{1.3}$$

where $W^{a\mu\nu}$ $(a = 1, 2, 3)$ and $B^{\mu\nu}$ are the field strength tensors for the weak isospin and weak hypercharge gauge fields. The covariant derivative of the Higgs doublet, ϕ, with hypercharge $Y_\phi = \frac{1}{2}$, is given by:

$$D_\mu \phi = (\partial_\mu - i g A_\mu^a \tau^a - \frac{i g'}{2} B_\mu) \phi, \tag{1.4}$$

where $A^{a\mu}$ and B^{μ} are the $SU(2)$ and $U(1)$ gauge bosons respectively while $\tau^a = \frac{\sigma^a}{2}$. where σ^a are the Pauli matrices. We define g and g' to be the coupling constants of $SU(2)$ and $U(1)$ respectively. The relevant potential of the EW theory is:

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda\phi^4. \tag{1.5}$$

Therefore the field ϕ acquires acquires a vacuum expectation value(Vev):

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \tag{1.6}$$

where $v = \sqrt{\frac{\mu^2}{\lambda}}$.

We identify the mass terms of the weak gauge bosons in the term $\mathcal{L}_\phi = |D_\mu\phi|^2$ which after the SSB is:

$$\mathcal{L}_\phi = \frac{1}{2}\begin{pmatrix}0 & v\end{pmatrix}\left(gA^a_\mu\tau^a + \frac{1}{2}g'B_\mu\right)(gA^{b\mu}\tau^b + \frac{1}{2}g'B^\mu)\begin{pmatrix}0\\ v,\end{pmatrix} \tag{1.7}$$

after some straightforward calculations we find:

$$\mathcal{L}_\phi = \frac{v^2}{8}\left(g^2(A^{1\mu})^2 + g^2(A^{2\mu})^2 + \left(g'B^\mu - gA^{3\mu}\right)^2\right). \tag{1.8}$$

From equation 1.8 one can identify the masses of the following gauge bosons:

$$\boxed{W^\pm_\mu = \frac{1}{\sqrt{2}}\left(A^1_\mu + iA^2_\mu\right)} \quad with \;\; mass \;\; M_W = \frac{gv}{2}$$

$$\boxed{Z^0_\mu = \frac{g'B^\mu - gA^{3\mu}}{\sqrt{g^2+g'^2}}} \quad with \;\; mass \;\; M_Z = \frac{v\sqrt{g^2+g'^2}}{2}$$

There is also a fourth massless vector field, orthogonal to Z^0_μ:

$$\boxed{A_\mu = \frac{g'A^3_\mu + gB_\mu}{\sqrt{g^2+g'^2}}} \quad with \;\; mass \;\; M_A = 0. \tag{1.9}$$

Since we identify the gauge bosons we write the covariant derivative in terms of these fields as follows:

$$\begin{aligned} D_\mu =& \partial_\mu - igA^a_\mu T^a - ig'YB_\mu = \\ &= \partial_\mu - i\frac{g}{\sqrt{2}}\left(W^+_\mu T^+ + W^-_\mu T^-\right) - \frac{iZ_\mu}{\sqrt{g^2+g'^2}}\left(g^2T^3 - g'^2Y\right) - \\ &- \frac{igg'}{\sqrt{g^2+g'^2}}A_\mu\left(T^3+Y\right), \end{aligned} \tag{1.10}$$

we denote with $T^{\pm} = (T^1 \pm iT^2)$, where $T^a = \frac{\sigma^a}{2}$, for $a = \{1,2\}$. We identify the electric charge in the last term of eq.(2.10) to be:

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} \tag{1.11}$$

and the electric charge number:

$$Q = T^3 + Y. \tag{1.12}$$

In order to simplify the covariant derivative we define the weak mixing angle θ_w in order to rotate the fields as:

$$\begin{pmatrix} Z_0 \\ A \end{pmatrix} = \begin{pmatrix} cos\theta_w & -sin\theta_w \\ sin\theta_w & cos\theta_w \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix}. \tag{1.13}$$

From equation (2.12) we extract the relations:

$$sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}, \qquad cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}}. \tag{1.14}$$

We can write now the covariant derivative as:

$$\boxed{D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) - \frac{ig}{cos\theta_w} Z_\mu \left(T^3 - sin^2\theta_w Q\right) - ieQA_\mu}, \tag{1.15}$$

where we used the relation $g = \frac{e}{sin\theta_w}$. We notice that the mass of gauge bosons are not independent since the relation holds:

$$M_Z = \frac{M_W}{cos\theta_w}. \tag{1.16}$$

Therefore all interactions of W and Z boson can be written in terms of the parameters $\{\theta_w, v, M_W\}$. Experimental results for W and Z boson masses can be found in [35].

1.3 Higgs Sector

The Higgs boson's mass and its interactions with the W and Z gauge fields arise from the Higgs field's coupling to these fields in the Lagrangian:

$$\mathcal{L}_h = \left|D_\mu \phi\right|^2 - V(\phi), \tag{1.17}$$

which after the SSB($\phi \rightarrow \frac{h(x)+v}{\sqrt{2}}$) can be writen as:

$$\begin{aligned} \mathcal{L}_h =& \frac{1}{2}\left(\partial_\mu h\right)^2 + \left[M_W^2 W^{\mu+} W_{\mu-} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu\right]\left(1 + \frac{h}{v}\right)^2 - \\ & - \mu^2 h^2 - \lambda v h^3 - \frac{\lambda h^4}{4} + \frac{\mu^4}{4\lambda}. \end{aligned} \tag{1.18}$$

By using the relation $v = \frac{\sqrt{\mu^2}}{\sqrt{\lambda}}$, we can identify the mass of the Higgs boson to be:

$$m_h = \sqrt{2}\mu = \sqrt{2\lambda}v. \tag{1.19}$$

The experimental value of Higgs boson is [35]:

$$m_h = 125.18(16)GeV. \tag{1.20}$$

1.4 Fermion sector

We continue with the issue of constructing mass terms for quarks and leptons. It is worth noting that one cannot put ordinary mass terms into the Lagrangian. This is because the left and right-handed components of the fermion fields have distinct gauge quantum numbers, therefore single mass terms would violate gauge invariance. In order to give masses to quarks and leptons, we must use the mechanism of spontaneous symmetry breaking. The left-handed fermion fields are represented as doublets in the fundamental representation of $SU(2)$ while the right-handed fields are represented as singlets in the same group. We denote for the left-handed side:

$$l^j_{Lp} = \begin{pmatrix} \nu_{Lp} \\ e_{Lp} \end{pmatrix} \tag{1.21}$$

and

$$q^{aj}_{Lp} = \begin{pmatrix} u^a_{Lp} \\ d^a_{Lp} \end{pmatrix}. \tag{1.22}$$

The indices $j = \{1,2\}$, $a = \{1,2,3\}$ and $p = \{1,2,3\}$ correspond to isospin, color and generation respectively. We note that for the right-handed fermion fields, i.e. e_R, we have $T^3 = 0$, since they are singlets under $SU(2)$. Therefore, according to equation 1.12, it can be seen that the hypercharges of the right-fermion fields are the same as their electric charges. For the left-handed fermion fields, we have to specify the value of T^3 to compute the electric charge. Since we have defined left and right-fermion fields, we can write the kinetic terms in the following Lagrangian:

$$\mathcal{L} = \bar{l}_L i \not{D} l + \bar{e}_R i \not{D} e_R + \bar{q}_L i \not{D} q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R, \tag{1.23}$$

where for simplicity, we ignore the indices j, a, and p.We continue by expanding the covariant derivative in the form of equation 1.15. Therefore the Lagrangian transforms to:

$$\begin{aligned} \mathcal{L} =& \bar{l}_L i \not{\partial} l + \bar{e}_R i \not{\partial} e_R + \bar{q}_L i \not{\partial} \bar{q}_L + \bar{u}_R i \not{\partial} u_R + \bar{d}_R i \not{\partial} d_R \\ &+ g(W^+_\mu J^{\mu +}_W + W^-_\mu J^{\mu -}_W + Z^0_\mu J^\mu_Z) + e A_\mu J^\mu_{EM}, \end{aligned} \tag{1.24}$$

where we define the currents:

$$
\begin{aligned}
J_W^{\mu+} =& \frac{1}{\sqrt{2}} (\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L) \\
J_W^{\mu-} =& \frac{1}{\sqrt{2}} (\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L) \\
J_\mu^Z =& \frac{1}{cos\theta_w} (\bar{\nu}_L \gamma^\mu \left(\frac{1}{2}\right) \nu_L + \bar{e}_L \gamma^\mu \left(-\frac{1}{2} + sin^2(\theta_w)\right) e_L + \\
& \bar{u}_L \gamma^\mu \left(\frac{1}{2} - \frac{2}{3} sin^2(\theta_w)\right) u_L + \bar{d}_L \gamma^\mu \left(-\frac{1}{2} + \frac{1}{3} sin^2(\theta_w)\right) d_L \\
& \bar{e}_R \gamma^\mu e_R \sin^2\theta_w - \frac{2}{3} \sin^2\theta_w \bar{u}_R \gamma^\mu u_R + \frac{\sin^2\theta_w}{3} \bar{d}_R \gamma^\mu d_R) \\
J_\mu^{EW} =& (-\sum_{i=L,R} \bar{e}_i \gamma^\mu e_i + \frac{2}{3} \bar{u}_i \gamma^\mu u_i - \frac{1}{3} \bar{d}_i \gamma^\mu d_i),
\end{aligned}
\tag{1.25}
$$

where we used that $Y = -1/2$ for l_L^j and $Y = +1/6$ for q_L^j, while $Y = +2/3$ for the right-handed u and $Y = -1$ for e_R.

Next we generalize the Lagrangian of equation 1.23, writing a Lagrangian invariant under $SU(2)$:

$$
\mathcal{L}_{mass} = -\lambda_d^{ij} \bar{l}_L^i \cdot \phi d_R^j - \lambda_u^{ij} \epsilon^{ab} \bar{l}_{La}^i \cdot \phi_b^\dagger u_R^j + h.c. \tag{1.26}
$$

Where λ_d and λ_u general complex-valued matrices. To obtain the physical masses of the fields we will perform a rotation of the terms in the mass basis. To do this we define unitary matrices $U_u$and K_u for which, the following relation holds:

$$
\lambda_u = U_u M_u K_u^\dagger \tag{1.27}
$$

From equation 1.27 can be proved that:

$$
\begin{aligned}
\lambda_u \lambda_u^\dagger &= U_u M_u^2 U_u^\dagger, \\
\lambda_u^\dagger \lambda_u &= K_u M_u^2 K_u^\dagger,
\end{aligned}
\tag{1.28}
$$

Where:

$$
M_u = U_u^\dagger \lambda_u U_u, \tag{1.29}
$$

is a diagonal matrix with real elements. In a similar way we define unitary matrices U_d and K_d for which the relation holds:

$$
\lambda_d = U_d M_d K_d^\dagger, \tag{1.30}
$$

where:

$$
M_d = U_d^\dagger \lambda_d U_d. \tag{1.31}
$$

is a diagonal matrix with real eigenvalues. Next we make the following change of variables in order to simplify the Lagrangian of equation 1.26:

$$
\begin{aligned}
u_L &\to U_u u_L, \qquad & u_R &\to K_u u_R, \\
d_L &\to U_d d_L \qquad & d_R &\to K_d d_R.
\end{aligned}
\tag{1.32}
$$

Under these transformations we rewrite the Lagrangian of equation 1.26 as follows(after SSB of the theory, where $\phi \to \frac{h+v}{\sqrt{2}}$):

$$\mathcal{L}_{mass} = -m_d^i \bar{d}_L^i d_R^i \left(1 + \frac{h}{v}\right) - m_u^i \bar{u}_L^i u_R^i (1 + \frac{h}{v}) + h.c. \tag{1.33}$$

Where we redefine the masses of quarks to be:

$$m_u^i = \frac{vM_u^{ii}}{\sqrt{2}}, \qquad m_d^i = \frac{vM_d^{ii}}{\sqrt{2}}. \tag{1.34}$$

From relation 1.33, we can extract the usual Higgs couplings to quarks and the Higgs mass. From the rescaling formulation in equation 1.32, we can find the transformation of the currents that couple to the charged $W^{\pm}$ gauge boson, which is:

$$J^{\mu +} = \frac{\bar{u}_L^i \gamma^\mu u_L^i}{\sqrt{2}} \to \frac{\bar{u}_L^i (V)_{ij} d_L^j}{\sqrt{2}}. \tag{1.35}$$

Where we define as $V = U_u^\dagger U_d$ the Cabibbo-Kobayashi-Matrix (CKM)[6]. In an equivalent manner, we can express the general coupling of three-generation neutrinos with a Higgs boson as follows:

$$\mathcal{L}_{mass} = -\lambda_l^{ij} l_L^i \cdot \phi e_R^j + h.c. \tag{1.36}$$

We make the redfinition of the coupling:

$$\lambda_l = U_l M_l K_l^\dagger. \tag{1.37}$$

Therefore we can eliminate the diagonal matrices U_l and K_l by rotating the fields as follows:

$$e_L \to U_l e_L, \qquad \nu_l \to U_l \nu_l, \qquad e_R \to K_l e_R. \tag{1.38}$$

The introduction of this new set of variables for the fermion fields does not result in any mixing between generations. This is due to the fact that the matrices in equation 1.38, which represent the change of variables commute with the $SU(2)$ interactions in the covariant derivative, and therefore cancel out from the theory. It should be noted that this only holds true when neutrinos are massless in our theory.

Finally, we denote the particles that are contained in the SM, which describe their behavior and interactions in Figure(1.1).

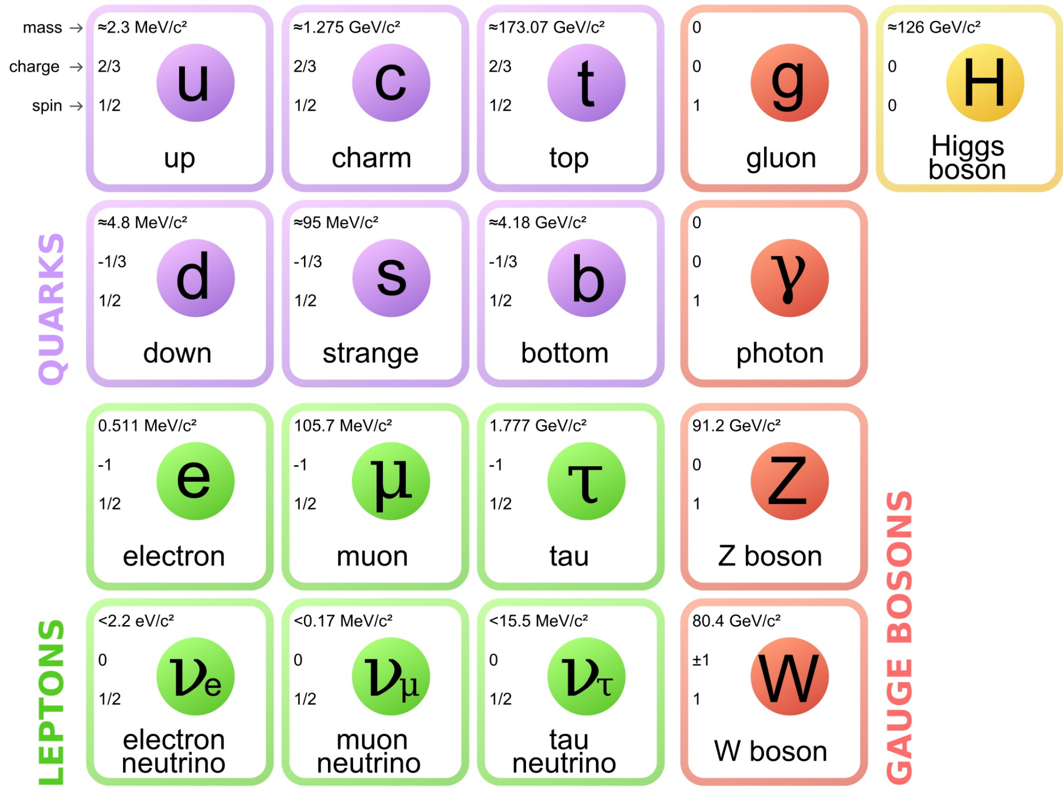


FIGURE 1.1: The particle spectrum of the Standard Model.

Chapter 2

Effective Field Theories

2.1 Introduction

The concept of Effective Field Theory (EFT) is discussed in many lecture notes, such as [33],[14] , and for the sake of simplicity, we will limit ourselves to some basic concepts that are useful for understanding its potential. It is important to note that an EFT is a quantum theory, and like any other quantum field theory, it requires a regularization and renormalization scheme to obtain finite matrix elements. S-matrix elements in an EFT can be computed from the EFT Lagrangian in the same way as in QED, starting from a QED Lagrangian.

The basic idea behind EFTs is that physics at different energy scales can be described by different theories. This principle can be used to study low-energy phenomena without the need for the full theory. In this context, we will procceed to two common approaches which can be used for the study of EFTs.

- **Top-down Construction of an EFT**: In this approach we have the knowledge of the full theory. First we identify the relevant light fields and their symmetries that belong in the physical domain that we want to study. The purpose of this approach is to exclude the fields that are heavier under this consideration. For this we can choose a cutt-off energy level and integrate out field modes with momenta above this level of cut-off point. To integrate out these heavy fields we have to write down a general Lagragnian for the light fields, as a sum of all allowed operators. The operators that we need to keep can determined through power counting. The resulting effective Lagrangian describes the low-energy dynamics of the light fields, taking into account the effects of the heavy fields that have been integrated out.

- **Bottom-up Construction of an EFT**: In this case, the UV theory is unknown, so we have to work the other way around. We need to identify the degrees of freedom and symmetries of the system at the low-energy level and then extend the theory. All allowed terms in the Lagrangian can be written with no limitations on the dimension of the operators. The coefficients in front of the operators need to be determined by experiments. A bottom up theory is the Standard Model Effective Field Theory (SMEFT) which is an EFT approach that we will focus on in the rest of this thesis.

In order to combine the two different approaches, we define the effective Lagrangian that describes the low-energy regime as follows:

$$\mathcal{L}_{effective} = \mathcal{L}_{d\leq 4} + \sum_i \frac{\mathcal{O}_i}{\Lambda^{d_i-4}}. \tag{2.1}$$

Λ is the cutoff energy scale and $\mathcal{O}_i$ are higher dimensional operators that arise from the removal of heavy degrees of freedom. Although an infinite number of higher dimensional operators can be generated, each of them is suppressed by a power of Λ proportional to its dimension d_i. Therefore their contributions to calculations will also be suppressed by high powers of Λ. Introducing the concept of power counting becomes crucial for developing the theory, based on this observation.

2.1.1 Power Counting

In order to avoid having to consider an infinite number of operators, a working criterion is needed to determine which terms can be safely neglected before any calculations are performed. Power counting methods provide a way to achieve this. We start by considering the EFT functional integral to be:

$$Z = \int \mathcal{D}\phi e^{iS}. \tag{2.2}$$

Where we assumed units: $\hbar = 1, c = 1$. Therefore the action S must be dimensionless: $[S] = 0$. The EFT action S is the integral of the local Lagragnian density:

$$S = \int d^d x \mathcal{L}(x). \tag{2.3}$$

If we assume that the dimensionality of the spacetime is d we have that: $[d^d x] = -d$ therefore the Lagragnian density must have mass dimension d:

$$[\mathcal{L}(x)] = d, \tag{2.4}$$

and can be described by the sum of local, gauge and Lorentz invariant operators $\mathcal{O}_i$ with coefficients c_i:

$$\mathcal{L}(x) = \sum_i c_i \mathcal{O}_i(x). \tag{2.5}$$

Therefore if we denote the dimension of the operators as $[D]$, the dimension of the coefficient must be: $[c_i] = d - D$. For example, if we assume a generic scalar field ϕ and a spinor field ψ we can find the dimensionality to be:

$$[\psi] = \frac{1}{2}(d-1), \quad [\phi] = \frac{1}{2}(d-2). \tag{2.6}$$

In the special case where $d = 4$ we have:

$$[\psi] = \frac{3}{2}, \quad [\phi] = 1, \quad [F_{\mu\nu}] = 2, \quad [D_\mu] = 1 \tag{2.7}$$

Where $F_{\mu\nu}$ is the field strength tensor and $D_\mu = \partial_\mu + igA_\mu$ the covariant derivative. In the following work we will try to find all the gauge invariant operators in 4 spacetime dimension: $d = 4$ up to operator dimension: $\mathcal{D} = 6$. We can write the general form of an operator in $d = 4$ as:

$$\mathcal{O} = (\bar{\psi}\psi)^{n_\psi}(F^{\mu\nu})^{n_F}(D_\mu)^{n_D}\phi^{n_\phi}, \tag{2.8}$$

from which the following equation arises:

$$\boxed{\mathcal{D} = n_\phi + \frac{3}{2}n_\psi + 2n_F + n_D}, \tag{2.9}$$

where we denote with n_i, $i = \{\phi, F, \psi, D\}$ the number of fields we must use in order to construct Lorentz invariant 6-dimension operators. We can see that the left hand side of equation 2.9 is integer: $\mathcal{D} = 6$ therefore the number of fermion fields must be even. The possible values that n_ψ can take are the following: $n_\psi = \{0, 2, 4\}$. For $n_\psi = 4$ we have that: $n_\phi = n_F = n_D = 0$. Therefore the only $\mathcal{D} = 6$ operator is:

$$\boxed{(\bar{\psi}\psi)^2}. \tag{2.10}$$

For the case: $n_\psi = 2$, we have the equation:

$$n_\phi + 2n_F + n_D = 3. \tag{2.11}$$

For $n_F = 1$ we have that: $(n_\phi, n_D) = \{(1, 0), (0, 1)\}$, which gives the operator:

$$\boxed{i\bar{\psi}\sigma^{\mu\nu}\psi X_{\mu\nu}\phi}. \tag{2.12}$$

For the case: $n_F = 0$ we have the possible values: $(n_D, n_\phi) = \{(0, 3), (1, 2), (2, 1), (3, 0)\}$ which gives the possible $\mathcal{D} = 6$ operators:

$$\boxed{\bar{\psi}\psi(\phi)^3, (\bar{\psi}i\not{D}\psi)\phi^2, (\bar{\psi}i\not{D}^2)\psi\phi, \bar{\psi}\not{D}\psi D^2}. \tag{2.13}$$

For the subcase: $n_F \geq 2$ we do not have a possible solution. We continue with the case: $n_\psi = 0$. Equation 2.9 gives:

$$6 = 2n_F + n_\phi + n_D. \tag{2.14}$$

The n_F take values: $n_F \leq 3$. The case $n_F = 3$ can be rejected since we can not find a Lorentz invariant operator. Therefore: $n_F < 3$. For $n_F = 2$ we have the possible values: $(n_\phi, n_D) = \{(2, 0), (1, 1), (0, 2)\}$ which gives the two $\mathcal{D} = 6$ operators:

$$\boxed{F^2\phi^2, D^2F^2}. \tag{2.15}$$

For the subcase $n_F = 1$ the possible numbers of the operators are:
$(n_\phi, n_D) = \{(4, 0), (3, 1), (2, 2), (1, 3), (0, 4)\}$. The only Lorentz invariant operator is:

$$\boxed{D^2 D_\mu D_\nu F^{\mu\nu}}. \tag{2.16}$$

Finally for the subcase: $n_F = 0$, we have that: $(n_\phi, n_D) = \{(0, 6), (2, 4), (4, 2)\}$ (the number of n_D must be even since we want to construct Lorentz invariant operators).

$\mathcal{D}$	Operators
1	ϕ
2	ϕ^2
3	$\phi^3, \bar{\psi}\psi, D^2\phi$
4	$\phi^4, F^2, (\bar{\psi}\psi)\phi, D^2\phi^2, i\bar{\psi}\not{D}\psi$
5	$\phi^5, (\bar{\psi}\psi)\phi^2, F^2\phi, D^2\phi^3, D^4\phi, \bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}, D_\mu\bar{\psi}D^\mu\psi, \phi\bar{\psi}\not{D}\psi, F^{\mu\nu}D_\mu D_\nu\phi$
6	$\phi^6, (\bar{\psi}\psi)\phi^3, F^2\phi^2, (\bar{\psi}\psi)^2, D^2\phi^4, D^4\phi^2, D^2F^2, \bar{\psi}D^3\psi,$ $\phi^2\bar{\psi}i\not{D}\psi, F_{\mu\nu}D^\mu D^\nu\phi^2, \phi\bar{\psi}\not{D}^2\psi, i\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}\phi$

TABLE 2.1: Operator classes in $d = 4$ dimension up to $\mathcal{D} = 6$.

The possible operators are three:

$$\boxed{D^4\phi^2, D^2\phi^4, \phi^6}. \tag{2.17}$$

We can work similarly for the less complex case: $\mathcal{D} = 5$. We start with the equation:

$$5 = n_\phi + \frac{3}{2}n_\psi + 2n_F + n_D. \tag{2.18}$$

The n_ψ can take the possible values $(0, 2)$. For $n_\psi = 2$, n_F can take the value 0 or 1. For $n_F = 0$ we have the possible pairs: $(n_F, n_D) = \{(0, 2), (2, 0), (1, 1)\}$ from which we construct the Lorentz invariant operators:

$$\boxed{(\bar{\psi}\not{D}\psi)\phi, D_\mu\bar{\psi}D^\mu\psi, \bar{\psi}\psi\phi^2}. \tag{2.19}$$

For $n_F = 1$ we have the $\mathcal{D} = 5$ operator:

$$\boxed{\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}}. \tag{2.20}$$

For $n_\psi = 0$, the possible values of $n_F = \{0, 1, 2\}$. For $n_F = 0$ we find the possible pairs: $(n_D, n_F) = \{(0, 5), (2, 3), (4, 1)\}$ since n_D must be even. In this subcase we construct the $\mathcal{D} = 5$ operators:

$$\boxed{\phi^5, D^4\phi, D^2\phi^3}. \tag{2.21}$$

For $n_F = 1$ there is only one possibility: $n_D = 2, n_\phi = 1$ and only one $\mathcal{D} = 5$ operator:

$$\boxed{F^{\mu\nu}D_\mu D_\nu\phi} \tag{2.22}$$

For $n_F = 2$ we can construct only one $\mathcal{D} = 5$ operator:

$$\boxed{F^2\phi}. \tag{2.23}$$

Working equivalent for the less complex cases: $\mathcal{D} = \{1,2,3,4\}$ we can construct all the operators in $d = 4$ dimensions up to operator dimension $\mathcal{D} = 6$. The operators are given in the table (2.1). Using the same method, we could construct operators of higher dimension $\mathcal{D} > 6$.

2.1.2 The Fermi Theory as an EFT approach

In order to understand better the two different approaches (bottom-up and top-down) we can work the following example: A muon decaying into an electron and a pair of neutrinos. Since the muon mass is much lower than the weak vector boson masses ($m_{\mu^-} \approx 105MeV$), we can work with the bottom-up approach. With the knowledge of the experimental results we can make the following hypothesis: We add in the QED Lagragnian a term that contains interaction between four fermions, supressed by an energy scale Λ:

$$\mathcal{L} = -\frac{c_W}{\Lambda^2}(\bar{\nu}_\mu \gamma^\alpha P_L \mu)(\bar{e}\gamma_\alpha P_L \nu_e) + h.c., \tag{2.24}$$

where we call c_W a Wilson coefficient. We can calculate the decay rate of the process:

$$\mu^- \to e^- \bar{\nu}_e \nu_\mu, \tag{2.25}$$

to be:

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = (\frac{c_W^2}{\Lambda^2})^2 \frac{m_\mu^5}{1536\pi^3}. \tag{2.26}$$

Therefore from the definition of the Fermi constant(in SM):

$$\frac{1}{\tau_\mu} = G_F^2 \frac{m_\mu^5}{192\pi^3}(1 + \Delta q), \tag{2.27}$$

where τ_μ the muon life-time, G_F the Fermi constant and Δq the quantity that includes the phase space, QED and radiative corrections (which are relatively small). From equations 2.27 and 2.26 we find that:

$$\frac{c_W}{\Lambda^2} = \frac{4}{\sqrt{2}} G_F. \tag{2.28}$$

Where the numerical value of G_F is: $G_F = 1.1663787(6) \times 10^{-5} GeV^{-2}$[8]. Therefore from this approach (bottom-up) one can relate low-energy phenomena with higher energy theories. Someone could work in exactly the opposite way: The top-down approach. First we compute the muon decay amplitude in the SM, where a muon decays into an electron and two neutrinos through the exchange of a W boson. We find the amplitude to be:

$$M = \frac{g^2}{2(q^2 - M_W^2)}[\bar{u}(p_1)\gamma^\alpha P_L v(p_2)][\bar{u}(p_3)\gamma_\alpha P_L u(p_4)], \tag{2.29}$$

where p_1, p_2, p_3, p_4 are the 4-momentum of e^-, ν_e, ν_μ, μ respectively and q is the momentum of the W-propagator.

In the limit of low energies we can make the following approximation:

$$M = -\frac{g^2}{2M_W^2}[\bar{u}(p_1)\gamma^\alpha P_L v(p_2)][\bar{u}(p_3)\gamma_\alpha P_L u(p_4)]. \tag{2.30}$$

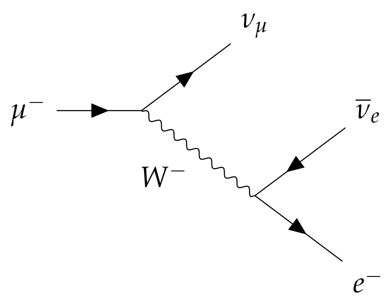


FIGURE 2.1: Tree level process of: $\mu^- \to \nu_\mu e \nu_e$

The top down approach imposes that in the limit of low energies the matrix elements of the different theories must be the same. Therefore from equations 2.30 and 2.26 we have the matching condition:

$$\frac{c_W}{\Lambda^2} = \frac{g^2}{2M_W^2}. \tag{2.31}$$

With this process we found the relation between the factor $\frac{c_W}{\Lambda^2}$ and the UV parameters g, M_W of the SM. We can see that the estimation of the mass of W boson is around 80GeV, where we used the experimental result: $G_F = 1.1663787(6) \times 10^{-5} GeV^{-2}$ for the Fermi constant. Therefore the prediction for the Fermi theory is valid. The top-down approach is a technique which we will follow in the present work, in order to find the matching conditions for the Wilsons coefficients, since the calculations in the low energies are simpler than the high energy theory.

2.2 EFT Lagrangian

In general a Lagrangian in EFT can be written in the following form:

$$\mathcal{L}_{EFT} = \sum_{\mathcal{D}\geq 0,i} \frac{c_i^{(\mathcal{D})} \mathcal{O}_i^{(\mathcal{D})}}{\Lambda^{\mathcal{D}-d}}, \tag{2.32}$$

where $\mathcal{O}_i^{(\mathcal{D})}$ represents the Lorentz and gauge invariant operators of dimension $0 < \mathcal{D} < d$ for the renormalizable part of operators while $\mathcal{D} > d$ for the non-renormalizable part of operators. The scale Λ has been introduced to make the coefficients $c_i^{(\mathcal{D})}$ dimensionless. A secondary use of Λ is to characterise the short-distance scale at which new physics occur. For example in $d = 4$ space-time dimensions we can write the general Lagrangian:

$$\mathcal{L}_{EFT} = \mathcal{L}_{\mathcal{D}<5} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + ..., \tag{2.33}$$

where all operators of dimension $\mathcal{D}$ are combined into the Lagrangian $\mathcal{L}_{\mathcal{D}}$. Therefore the Lagrangian $\mathcal{L}_{EFT}$ has to be treated as an expansion of $\frac{1}{\Lambda^n}$, where n is an integer. We should note that we cannot sum terms to all orders since, we would violate the EFT power counting rule and the EFT would break down.

If we consider a scattering amplitude $\mathcal{A}$ in $d = 4$ dimensions, normalized to have mass dimension zero, for a momentum scale p, the contribution to the amplitude order will be (by dimensional analysis):

$$\mathcal{A} \sim (\frac{p}{\Lambda})^{D-4}, \tag{2.34}$$

where we introduce the Λ, the characteristic scale of the system. Therefore the operator has a coefficient of mass dimension $\frac{1}{\Lambda^{D-4}}$. From this relation it is possible to divide into three categories the operators:

- **Irrelevant**: Operators with dimension $D > 4$.
- **Relevant**: Operators with dimension $D < 4$.
- **Marginal**: Operators with dimension $D = 4$.

For the case of Irrelevant operators, the operators are non-renormalizabe. From equation 2.34 we see that the influence of these type of operators are less important the bigger the dimension of the operator is. Relevant and Marginal operators constitute the SM, and they are renormalizable operators. Equation 2.34 corresponds to a single insertion of an operator of dimension $\mathcal{D} > 4$. An insertion of a set of higher operators in $d < \mathcal{D}$ dimensions leads to an amplitude:

$$\mathcal{A} \sim \left(\frac{p}{\Lambda}\right)^{n}, \tag{2.35}$$

where:

$$\boxed{n = \sum_{i}^{k}(\mathcal{D}_i - d)}, \tag{2.36}$$

where k the number of operators and $\mathcal{D}_i$ corresponds to the mass dimension of the i-th operator that we conclude in our theory. Equation 2.36 is the power counting formula of EFT and it holds for any graph and not just tree graphs. From equation 2.35 we can understand the difference between any renormalizable theories and EFT's. For example we can assume one insertion of a $\mathcal{D} = 5$ operator in $d = 4$ spacetime dimensions. This insertion will give a correction of the form:

$$\mathcal{A} \sim \frac{p}{\Lambda}. \tag{2.37}$$

With a single insertion of a $\mathcal{D} = 6$ operator we obtain corrections $\frac{p^2}{\Lambda^2}$. This case is similar to the introduction of two $\mathcal{D} = 5$ operators. If we have a loop graph, with the insertion of the $\mathcal{D} = 6$ operator, will be divergent. In order to eliminate the divergence we will need a counterterm from $\mathcal{L}_6$. in order to cancel the divergence from four $\mathcal{D} = 5$ operators or two insertions of $\mathcal{D} = 6$ operators, we will need a counterterm which is an $\mathcal{L}_8$ operator. Continuing in this way we can generate operators of arbitrarily high dimension, in order to eliminate the UV divergences for the case: $\mathcal{D} > d$. Thus the main difference between renormalizable theories and EFT's becomes apparent. In the case of renormalizable theories: $0 \leq \mathcal{D} \leq d$ ($\mathcal{L}_{\mathcal{D}<5}$) we generate operators with $\mathcal{D} < 5$ which we have already included in $\mathcal{L}_{\mathcal{D}<5}$ in order to eliminate the UV divergences. Therefore the necessary counterterms have already been included in the Lagrangian. In order to make EFT a renormalizable theory we will need an infinite number of higher dimension operators.Nevertheless, we can avoid this by choosing a maximum value of n in order to obtain the correction of the amplitude of a process. In this case, only a finite number of operators contribute.

2.3 LSZ Reduction Formula

In QFT, the LSZ formula provides a relation between correlation functions and S-matrix elements. Therefore, it is a useful tool for calculating scattering amplitudes using Feynman diagrams in field theory. In momentum space, correlation or Green's functions are defined by the relation:

$$\begin{aligned} G(p_1,...p_m;q_1,...q_n) =& \prod_{i=1}^{n}\int d^4y_i e^{ip_i\cdot y_i}\prod_{j=1}^{n} d^4x_j e^{-iq_j\cdot x_j} \\ &\times \langle 0| T\{\phi(y_1)...\phi(y_m)\phi(x_1)...\phi(x_n)\} |0\rangle . \end{aligned} \tag{2.38}$$

where p_i and q_i corresponds to the outgoing and incoming momenta respectively. If we assume a special case where $p_1 = q_1 = p$ then we have the ϕ propagator

$$G(p) = \int d^4x e^{ip\cdot x} \langle 0| T\{\phi(x)\phi(0)\} |0\rangle . \tag{2.39}$$

Therefore if the field $\phi(x)$ has the property to create a single particle state with invariant mass from vacuum then the propagator has a pole at $p^2 = m^2$:

$$D(p) \sim \frac{i\sqrt{Z}}{p^2 - m^2 + i\epsilon}, \tag{2.40}$$

where Z is the renormalization factor. From Eq. 2.40 we can extract that the renormalization factor is finite, since $D(P)$ is also a finite quantity. Generalizing this result for n incoming momenta and m outgoing momenta we take the result:

$$\begin{aligned} &\prod_{i=1}^{m}(i\sqrt{Z_i})\prod_{j=1}^{n}(i\sqrt{Z_j}) \langle p_1...p_m| S |q_1...q_n\rangle = \\ &\lim_{q_i^2\to m^2}\lim_{p_j^2\to m^2}\prod_{i=1}^{m}(p_i^2-m^2)\prod_{j=1}^{n}(q_j^2-m^2)G(p_1,...p_m;q_1,...q_n). \end{aligned}$$

This result can be modified as:

$$\begin{aligned} \langle p_1...p_m| S |q_1...q_n\rangle =& i^{n+m}\prod_{i=1}^{n}\int d^4x_i e^{ip_i\cdot x_i}(-\partial_i^2+m^2)\prod_{j=1}^{m}\int d^4y_j e^{-iq_j\cdot y_j}(-\partial_j^2+m^2) \\ &\times \langle 0| T\{\phi(y_1)...\phi(y_m)\phi(x_1)...\phi(x_n)\} |0\rangle . \end{aligned} \tag{2.41}$$

From equation 2.41 we can conclude that correlation functions have a complete separation from physical particle states that enter the S-matrix. Additionally, equation 2.41 holds for composite operators, i.e., those that appear in EFT.

2.3.1 Field Redefinitions

To demonstrate that fields redefinitions leave the S-matrix unchanged, we will prove that the latter is not affected by the choice of field. To do so, we will begin with the definition of the functional integral in the presence of a source $J(x)$ as follows:

$$Z[J] = \int \mathcal{D}\phi e^{i\int L[\phi]+J\phi}. \tag{2.42}$$

The definition of the Green's functions, in the path integral formulation is given by:

$$\langle 0| T\{\phi(x_1)...\phi(x_m)\} |0\rangle = \frac{\int \mathcal{D}\phi\phi(x_1)\phi(x_2)...\phi(x_m)e^{iS(\phi)}}{\int \mathcal{D}\phi e^{iS(\phi)}}. \tag{2.43}$$

We can rewrite equation 2.43 in the form:

$$\langle 0| T\{\phi(x_1)...\phi(x_m)\} |0\rangle = \frac{1}{Z[J]}\frac{1}{i^m}\frac{\delta}{\delta J(x_1)}...\frac{\delta}{\delta J(x_m)}Z[J]\big|_{J=0}. \tag{2.44}$$

If we assume a local field redefinition of $\phi(x)$:

$$\phi(x) = F[\phi'(x)], \tag{2.45}$$

i.e. a transformation of the form:

$$\phi(x) = \sum_{n=0}^{N_1} a_n\phi'(x)^n + \sum_{n=0}^{N_2} b_n\partial^n\phi'(x), \tag{2.46}$$

where N_1, N_2 are finite, then the new Lagrangian defined by:

$$L[\phi(x)] = L[F[\phi'(x)]] = L'[\phi'(x)]. \tag{2.47}$$

We define a new functional integral with the field $\phi'(x)$ and the new Lagrangian L' as

$$Z'[J] = \int \mathcal{D}\phi' e^{L'[\phi']+J\phi'} = \int \mathcal{D}\phi e^{i\int L'[\phi]+J\phi}. \tag{2.48}$$

If we apply the same transformation to the functional integral that we started with, we obtain:

$$Z[J] = \int \mathcal{D}\phi \left|\frac{\delta F}{\delta\phi'}\right| e^{i\int L'[\phi']+JF[\phi']}. \tag{2.49}$$

Therefore the functional Z produce Green's functions of $\phi(x)$ computed using the Lagrangian $L[\phi(x)]$ or Green's functions of $F[\phi(x)]$ computed using the Lagrangian $L'[\phi(x)]$. The functional Z' produce Green's functions of $\phi(x)$ using the Lagrangian $L'[\phi(x)]$. However, the S-matrix element is invariant under field redefinition as long as:

$$\langle p| F[\phi] |0\rangle \neq 0, \tag{2.50}$$

since the LSZ formula is not affected by the choice of the field redefinition. In conclusion in an EFT we have much more freedom to make field redefinition from renormalizable theories(where only linear redefinitions are allowed) provided that we respect the EFT power counting.

2.3.2 Equations of Motion

A common problem we face when working with the bottom-up up approach is how many operators to consider at a scale $1/\Lambda$. To minimize the number of operators needed, we will use the equation of motion in the path integral approach, as shown above. Let $E[\phi]$ be the classical equation of motion:

$$E[\phi] = \frac{\delta S}{\delta \phi}. \tag{2.51}$$

Also we define $\theta[\phi]$ to be an operator with a factor of the classical equation of motion:

$$\theta[\phi] = F[\phi]E[\phi] = F[\phi]\frac{\delta S}{\delta \phi} \tag{2.52}$$

and the functional integral of the form:

$$Z[J,\tilde{J}] = \int \mathcal{D}\phi e^{i\int L[\phi]+J\phi+\tilde{J}\theta[\phi]}. \tag{2.53}$$

The correlation function $\langle 0| T\{\phi(x_1)...\phi(x_m)\theta(x)\} |0\rangle$, with one insertion of the operator θ is given by the relation:

$$\langle 0| T\{\phi(x_1)...\phi(x_m)\theta(x)\} |0\rangle = Z[J,\tilde{J}]^{-1}\frac{1}{i^m}\frac{\delta}{\delta J(x_1)}...\frac{\delta}{\delta J(x_m)}\frac{\delta}{\delta \tilde{J}(x)}|_{J=\tilde{J}=0}. \tag{2.54}$$

The change of variables:

$$\phi \to \phi' - \tilde{J}F[\phi'] \tag{2.55}$$

give us the transformed functional integral:

$$Z[J,\tilde{J}] = \int \mathcal{D}\phi' \left|\frac{\delta\phi}{\delta\phi'}\right| e^{L[\phi']+J\phi'-J\tilde{J}F[\phi']+\mathcal{O}(\tilde{J}^2)}. \tag{2.56}$$

Where we used the Taylor expansion:

$$\theta[\phi' - \tilde{J}F[\phi']] = \theta[\phi'] - \tilde{J}F[\phi']\theta(\phi') + \mathcal{O}\left(\tilde{J}^2\right). \tag{2.57}$$

Using that the jacobian:

$$\left|\frac{\delta\phi(x)}{\delta\phi'(x')}\right| = det\left[\delta(x-y) - \tilde{J}\frac{\delta F[\phi'(x)]}{\delta\phi'(x')}\right] \tag{2.58}$$

is unity in DR we rewrite the functional integral as:

$$Z[J,\tilde{J}] = \int \mathcal{D}\phi e^{i\int L[\phi]+J\phi-J\tilde{J}F[\phi]+\mathcal{O}(\tilde{J})^2}, \tag{2.59}$$

where we make the replacement: $\phi' \to \phi$. Taking the derivative with respect to $\tilde{J}$, and setting $\tilde{J} = 0$ after the calculation we take:

$$\int \mathcal{D}\phi(x)\theta(x)e^{i\int L[\phi]+J\phi} = -\int \mathcal{D}\phi(x)J(x)F[\phi(x)]e^{i\int L[\phi]+J\phi}. \tag{2.60}$$

We can prove by induction (or by differentiating n-times equation 2.60) the relation:

$$\langle 0| T\{\phi(x_1)...\phi(x_n)\theta(x)\} |0\rangle = i\sum_r \delta(x-x_r) \langle 0| T\{\phi(x_1)...\phi(x_r)...\phi(x_n)\theta(x)\} |0\rangle . \tag{2.61}$$

The S-matrix element with an insertion of θ vanishes since if we work in momentum space, the right-hand side of eq. 2.61 has no pole in p_r at the r^{th} place. Therefore the RHS of 2.61 vanishes:

$$\boxed{\langle q_1...q_k| \theta |q_{k+1}...q_n\rangle = 0}. \tag{2.62}$$

Equation 2.62 implies that the classical equations of motion can be dropped. This means that we can shift the Lagrangian by equation of motion terms. We can work more practically in the following example. We assume an EFT Lagragnian of the form:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{3!}g\phi^3 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda}\phi^2\partial^2\phi + \frac{c_2}{\Lambda}\phi^5, \tag{2.63}$$

where ϕ a real scalar. We find the equation of motion:

$$E[\phi] = -\partial^2\phi - m^2\phi - \frac{1}{2}g\phi^2 - \frac{1}{3!}\lambda\phi^3 + \mathcal{O}(\frac{1}{\Lambda}), \tag{2.64}$$

where terms of $\frac{1}{\Lambda}$ are dropped since they produce higher order operators. We redefine the field ϕ as:

$$\phi \to \phi + \frac{c_1}{\Lambda}\phi^2. \tag{2.65}$$

After the substitution, we take the new Lagrangian:

$$\mathcal{L}' = \mathcal{L} + \frac{c_1}{\Lambda}E[\phi] \tag{2.66}$$

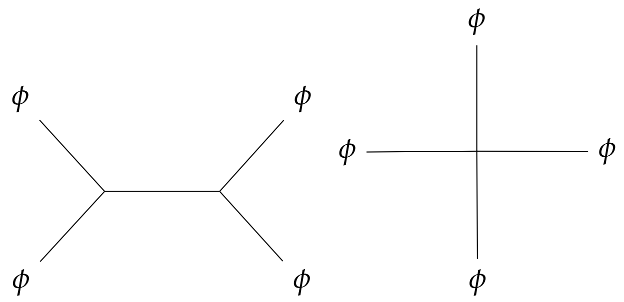
which can be written in terms of ϕ:

$$\mathcal{L}' = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{3!}\tilde{g}\phi^3 - \frac{1}{4}\tilde{\lambda}\phi^4 + \frac{\tilde{c}_2}{\Lambda}\phi^5, \tag{2.67}$$

where we make the substitution:

$$\begin{aligned}\frac{\tilde{g}}{3!} &= \frac{g}{3!} + m^2\frac{c_1}{\Lambda},\\ \frac{\tilde{\lambda}}{4!} &= \frac{\lambda}{4!} + \frac{gc_1}{2\Lambda}m^2,\\ \frac{\tilde{c}_2}{\Lambda} &= \frac{\tilde{c}_2}{\Lambda} - \frac{\lambda c_1}{3!\Lambda}.\end{aligned}$$

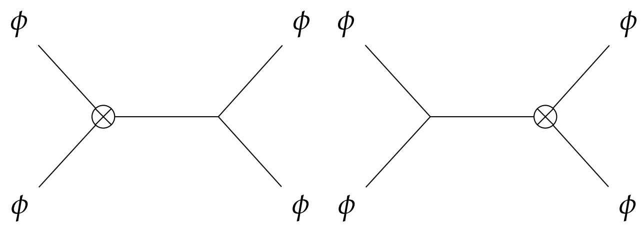
Therefore, by using equation 2.67, we arrive at a simpler theory where the factor $\phi^2\partial^2\phi$ is extracted. Despite this difference, the two Lagrangian describe the same physics. For example, if we consider the amplitude of the process $\phi\phi \to \phi\phi$, in the case of the second Lagrangian, the diagrams that contribute to the process are as follows:



We calculate the amplitude to be:

$$i\mathcal{M}_{\phi\phi\to\phi\phi} = i\tilde{\lambda} - i\frac{\tilde{g}^2}{q^2 - m^2}, \tag{2.68}$$

where q the 4-momentum of ϕ propagator. The Feynman diagrams that contribute to the same process are different in the case of the first Lagrangian as it includes two additional diagrams. These are shown below:



where the crossed dot corresponds to the vertex from the term $\phi^2\partial^2\phi$. Therefore the total amplitude is:

$$iM'_{\phi\phi\to\phi\phi} = i\lambda - \frac{ig^2}{q^2 - m^2} - 12\frac{c_1 g}{\Lambda}\frac{q^2}{q^2 - m^2}, \tag{2.69}$$

which after some algebraic manipulation gives:

$$iM'_{\phi\phi\to\phi\phi} = iM_{\phi\phi\to\phi\phi} = i\tilde{\lambda} - i\frac{\tilde{g}^2}{q^2 - m^2} \tag{2.70}$$

Therefore we found the same amplitude we obtain from the previous Lagrangian. We can use the same approach for more complicated theories where calculations become difficult due to the presence of derivatives. We note that the use of equations of motion and field redefinitions of operators at higher orders can be found in [4], [17] .

2.3.3 The SMEFT Lagrangian

The SMEFT which stands for Standard Model Effective Field Theory, is an EFT that is built from the fields of the SM. It is commonly utilized in computations for Beyond

Standard Model (BSM) scenarios. We can write the full gauge invariant Lagrangian of SMEFT up to $\mathcal{O}(\Lambda^{-3})$ in the following form:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + C^{\nu\nu} Q_{\nu\nu}^{(5)} + \sum_X C^X Q_X^{(6)} + \sum_f C'^f Q_f^{(6)} + \mathcal{O}(\frac{1}{\Lambda^3}). \tag{2.71}$$

Where $\mathcal{L}_{SM}^{(4)}$ is the Standard Model Lagrangian and is renormalisable since it contains only two and four dimensional operators .We denote with $Q_X^{(6)}$ the 6-dimensional operators that do not contain fermion fields, and with $Q_f^{(6)}$ the 6-dimension operators that contain fermion fields. The only 5-dimension operator that contribute to the SMEFT Lagrangian is $Q_{\nu\nu}^{(5)}$ which is a neutrino mass term, the lepton -number violating operator (Weibner operator) and violates the lepton number: $\Delta L = 2$. In equation 2.71 we made the following rescaling for the Wilson coefficients: $C^{\nu\nu} \to \frac{C^{\nu\nu}}{\Lambda}$, and $(C^X, C'^f) \to (\frac{C^X}{\Lambda^2}, \frac{C'^f}{\Lambda^2})$. Since we have the definition of the Lagrangian of the theory we can proceed to the spontaneous breaking of the gauge symmetry and after the appropriate rotations of the fields, we can derive a physical mass basis of the SMEFT. The SSB of the theory, the quantization in the SMEFT in the Warsaw basis and the Feynman rules that we are going to use for the calculations was given in:[18].

Chapter 3

Constraining the CKM Matrix in the Presence of New Physics

In this thesis, we aim to investigate the impact of New Physics (NP) on the global CKM fit and determine an effective and straightforward approach to constrain the CKM matrix while incorporating the corrections arising from BSM physics. Our theoretical framework, in order to analyze flavour data, is the Standard Model Effective Field Theory(SMEFT). By performing this analysis within the SMEFT, one can establish the relevant Wilson coefficients, which can be applied to a wide range of New Physics models. Additionally, SMEFT allows the accounting of complex correlations between different observables, such as EW precision measurements, leptonic processes, quark-flavour transitions. As we refer, SMEFT is a extension of the SM Lagrangian with higher dimensional operators composed of SM fields. The leading NP effects are encoded in the six-dimension Wilson coefficients and in the parameters that are already present in the SM Lagrangian as the gauge and Yukawa couplings and vacuum expectation value. Therefore in a consistent analysis we should consider the presence of those NP contribution that affect the input observables used to extract the SM parameters. In contrast Standard Model CKM fits necessitate multiple distinct observables which are measured using experimental set-ups and hadronic inputs with big complexity. This results in the use of more complex theoretical approaches that may not rely solely on the SM. Due to this factor, the CKM fitting in a global scale requires a general BSM framework such as SMEFT.

In the following work, we will apply a framework to select a set of input observables and use them to express the CKM parameters in terms of Wilson coefficients that can be deduced from these observable. Therefore, the new combinations of Wilson coefficients will define the corrected matrix elements of the CKM matrix. The tilde Wolfenstein parameters can be utilized to analyze various flavor processes in a coherent manner and to establish constraints on the behaviour of the new physics.

3.1 Fermion sector and Definition of the CKM Beyond SM

In the current thesis we will make use of the fact that we rotate the fermion sector from the flavour to the mass basis. More specifically in order to diagonalize the mass matrices:

$$\begin{aligned} M'_\nu &= -v^2 C'^{\nu\nu}, \qquad & M'_e &= \frac{v}{\sqrt{2}}\left(\Gamma_e - C'^{e\phi}\frac{v^2}{2}\right) \\ M'_u &= \frac{v}{\sqrt{2}}\left(\Gamma_u - C'^{u\phi}\frac{v^2}{2}\right), \qquad & M'_d &= \frac{v}{\sqrt{2}}\left(\Gamma_d - C'^{d\phi}\frac{v^2}{2}\right), \end{aligned} \tag{3.1}$$

which arise after the SSB, we perform the rotation of the fields by the unitary matrices:

$$\psi'_X = U_{\psi X}\psi_X, \tag{3.2}$$

where $\psi = \{\nu, e, u, d\}$ and $X = L, R$ are the indices for the chirality. The CKM matrix on SMEFT and the PMNS matrix are defined as:

$$V = U^\dagger_{uL} U_{dL}, \qquad U_{PMNS} = U^\dagger_{eL} U_{\nu L}. \tag{3.3}$$

We note that the matrices that we used to rotate the fermionic fields are "absorbed" into the redefinition of the Wilson coefficients, which can be found in [18] (in addition the final Feynman rules have been written in terms of V and U_{PMNS} matrices).

Therefore we can define the diagonalizable mass matrices as follows:

$$\begin{aligned} M_e &= U^\dagger_{eL} M'_e U_{eR} = diag(m_e, m_\mu, m_\tau), \\ M_u &= U^\dagger_{eL} M'_u U_{uR} = diag(m_u, m_c, m_t), \\ M_d &= U^\dagger_{eL} M'_d U_{dR} = diag(m_d, m_s, m_b), \\ M_\nu &= U^T_{\nu L} M'_\nu U_{\nu R} = diag(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \end{aligned} \tag{3.4}$$

Since we have defined the basis on which we will work, we define the Wolfenstein parameterization of the CKM matrix in the SM:

$$\begin{aligned} V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \\ \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(1+\frac{1}{2}\lambda^2)(\bar\rho - i\bar\eta) \\ -\lambda + A^2\lambda^5\left(\frac{1}{2}-\bar\rho - i\bar\eta\right) & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\bar\rho - i\bar\eta) & -A\lambda^2 + A\lambda^4(\frac{1}{2}-\bar\rho - i\bar\eta) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} \\ + \mathcal{O}(\lambda^6). \end{aligned} \tag{3.5}$$

The parameters $W_i = \{\lambda, A, \bar\rho, \bar\eta\}$ are the Wolfenstein parameters in SM, while we redefine the parameters W_i as $\tilde W_i$ in the SMEFT and we refer to these as tilde Wolfenstein parameters. The parameters in SMEFT must contain the contribution of the Wilson coefficients(NP contributions). We also define the tilde CKM matrix elements in SMEFT to be: $\tilde{V}_{ud} = 1 - \frac{1}{2}\tilde{\lambda}^2 - \frac{1}{8}\tilde{\lambda}^4,\ \tilde{V}_{us} = \tilde{\lambda}$ etc. In the following analysis, we will calculate the contributions to SMEFT from flavor observables.

.

3.2 LEFT Operators

In this thesis we will work at low energy level, bellow the EW scale, where we integrate out the particles in the EW theory which are heavy. To do this we will work with the low energy field theory lagragnian (LEFT):

$$\mathcal{L}_{LEFT} = \mathcal{L}_{QED+QCD} + \sum_i L_i \mathcal{O}_i^{(5,6)} + \mathcal{O}(\Lambda^{-4}), \tag{3.6}$$

where the Wilson coefficients, L_i of 6-dimension operators in the LEFT lagragnian can be related through a tree-level matching at the EW scale, with the Wilson coefficients of the SMEFT. In order to fix the CKM matrix we will consider the operators that affect semileptonic and $\Delta F = 2$ transitions. The relevant $\Delta F = 2$ operators are as follow:

$$\begin{aligned}
[Q_{dd}^{VLL}]_{ijij} =& (\bar{d}_{L,i}\gamma^\mu d_{L,j})(\bar{d}_{L,i}\gamma^\mu d_{L,j}) \\
[Q_{dd}^{VRR}]_{ijij} =& (\bar{d}_{R,i}\gamma^\mu d_{R,j})(\bar{d}_{R,i}\gamma^\mu d_{R,j}) \\
[Q_{dd}^{V1LR}]_{ijij} =& (\bar{d}_{L,i}\gamma^\mu d_{L,j})(\bar{d}_{R,i}\gamma^\mu d_{R,j}) \\
[Q_{dd}^{V8LR}]_{ijij} =& (\bar{d}_{L,i}\gamma^\mu T^a d_{L,j})(\bar{d}_{R,i}\gamma^\mu T^a d_{R,j}) \\
[Q_{dd}^{S1RR}]_{ijij} =& (\bar{d}_{L,i} d_{R,j})(\bar{d}_{L,i} d_{R,j}) \\
[Q_{dd}^{S8RR}]_{ijij} =& (\bar{d}_{L,i} T^a d_{R,j})(\bar{d}_{L,i} T^a d_{R,j}),
\end{aligned} \tag{3.7}$$

where T^a are the $SU(3)_c$ colour generators of the fundamental representation. The semileptonic operators are the following:

$$\begin{aligned}
[Q_{vedu}^{VLL}]_{iijk} =& (\bar{\nu}_{L,i}\gamma^\mu e_{L,i})(\bar{d}_{L,j}\gamma^\mu u_{L,k}) \\
[Q_{vedu}^{VLR}]_{iijk} =& (\bar{\nu}_{L,i}\gamma^\mu e_{L,i})(\bar{d}_{R,j}\gamma^\mu u_{R,k}) \\
[Q_{vedu}^{SRR}]_{iijk} =& (\bar{\nu}_{L,i}\gamma^\mu e_{R,j})(\bar{d}_{L,i}\gamma^\mu u_{R,j}) \\
[Q_{vedu}^{TRR}]_{iijk} =& (\bar{\nu}_{L,i}\gamma^\mu T^a e_{R,j})(\bar{d}_{L,i}\gamma^\mu T^a u_{R,j}) \\
[Q_{vedu}^{SRL}]_{iijk} =& (\bar{\nu}_{L,i} e_{R,i})(\bar{d}_{R,j} u_{L,k}).
\end{aligned} \tag{3.8}$$

As we will see, we focus on these two cases since our choice of observables to calculate the corrections of the CKM matrix are processes that contain semileptonic operators that contribute to the transitions $d \to u\mu^-\nu_\mu$, $s \to u\mu^-\nu_\mu$, $b \to u\tau^-\nu_\tau$, and $\Delta F = 2$ operators that contribute to the mass differences ΔM_s and ΔM_d. We denote the full tree-level matching condition for the LEFT Wilson coefficients with the SMEFT Wilson coefficients in appendix D. The general relation after the tree-level matching can be written in the following form :

$$L_i = f^{SM}(g_i, m_i) + \sum_X f_X(g_i, m_i) C^{(X,6)}, \tag{3.9}$$

where $f^{SM}(g_i, m_i)$ are function of the mass and couplings and they contribute at the tree-level matching of the SM. With the function $f_X(g_i, m_i)$ we denote the contribution of SMEFT's 6-dimension Wilson coefficients $C^{(X,6)}$. We note that in order to find the matching of coefficients at the electroweak scale, one needs to use renormalization group equations (RGEs). The RG evolution between the 6-dimensional coefficients and the Low Energy Effective Field Theory (LEFT) Wilson coefficients can be written as:

$$L_i(\mu_1) = \sum_j \eta(\mu_1, \mu_2)_{j,X} C^{(X,6)}(\mu_2), \tag{3.10}$$

where $\eta(\mu_1, \mu_2)$ can be obtained using QCD+QED running. One-loop QED and QCD running are known for the full set of LEFT operators [28],[1].

3.3 Flavour observables for the Extraction of the CKM in SMEFT

For the extraction of the numerical value of the CKM matrix in the SM (more specifically, for the extraction of the numerical value of the non-tilde Wolfenstein parameters: $W_i = \lambda, A, \bar{\rho}, \eta$), the following experimental observables have been used (which are described in detail in ref.[22]): leptonic decays and semileptonic decays ($\Delta F = 1$ branching ratios), CP-violating observables($\Delta F = 1$), and neutral-meson mixing ($\Delta F = 2$ observables). For completeness, we present them in the following table:

Observables	SM-processes
Leptonic-decays (ΔF=1)	$\pi \to \mu\nu, K \to e\nu, K \to \mu\nu, \tau \to K\nu, \tau \to \pi\nu,$ $D \to \mu\nu, D_s \to \mu\nu, D_s \to \tau\nu, B \to \tau\nu$
Semileptonic-decays(ΔF=1)	$K \to \pi e\nu, D \to \pi e\nu, D \to K e\nu, B \to \pi e\nu,$ $B \to D e\nu, B \to D^* e\nu$
CP-asymmetries	$B \to \pi\pi, \rho\pi, \rho\rho, B \to J/\psi K^{(*)}, (c\bar{c})K,$ $B \to D^{(*)}K^{(*)}, B_s \to J/\psi\phi, \psi(2S)\phi$
Neutral-meson mixing(ΔF=2)	$\epsilon_K(K\bar{K}), \Delta M_d(B_d\bar{B}_d), \Delta M_s(B_s\bar{B}_s)$

In contrast for the extraction of the CKM matrix in SMEFT we will need a shorter list of experimental observables. More specifically we will choose the observables in order to satisfy the following conditions:

- The set of observable must have a good sensitivity to all four Wolfenstein parameters,
- The set of observable must contain the minimum number of the Wilson coefficients in order to minimize the number of correlated observables.
- Each observable must have the minimum experimental uncertainty and the theoretical framework of these observables must provide clear results.

From these three rules, we can reject many observables for extracting the CKM matrix in the Standard Model (SM). For instance, the second rule leads us to exclude transitions such as $b \to cl\nu_l$, where $l = e, \mu$, since there is no clear evidence that new physics (NP) could be responsible for the deviation of these decays. Thus, we cannot establish a clear relation between the CKM parameters and the NP contribution (Wilson coefficients). Additionally, based on the second condition, we can exclude D and D_s meson decays compared to the $\Delta F = 2$ decays. According to the third rule, non-leptonic decays of ref.[22], which were used to extract the CKM matrix in the SM, cannot be used for calculating the CKM Beyond the Standard Model, as SMEFT introduces hadronic matrix elements that cannot be calculated or whose connection with the hadronic elements is unknown. Moreover, the third condition suggests that semileptonic decays are often more sensitive to a broader set of Beyond the Standard Model (BSM) operators than leptonic decays. Hence, semileptonic decays may be disfavored.

One observable that is sensitive to the λ parameter of the CKM matrix is K decays. However, it is worth noting that the form factor f_K from the decay $K^- \to \mu^- \nu_\mu$ has a large experimental uncertainty. If f_K does not rely in the experimental result of $\pi \to \mu\nu$, we cannot use K decays to calculate the CKM parameters in BSM. On the other hand, if we assume that the decay constant f_K relies in the experimental result of f_π, the calculation of the ratio $\frac{f_K}{f_\pi}$ is much more accurate than in the former case. Therefore, instead of using $\Gamma(K \to \mu\nu)$, we will use the ratio $\frac{\Gamma(K\to\mu\nu)}{\Gamma(\pi\to\mu\nu)}$ to calculate $\frac{|\tilde{V}us|}{|\tilde{V}ud|}$. Finally, decays that are sensitive to the remaining Wolfenstein parameters are ΔM_d, ΔM_s, and $B \to \tau\nu$ for the calculation of V_{td}, V_{ts}, and V_{ub}, respectively. Therefore, the flavor observables that we will use for extracting the CKM matrix are as follows:

$$\begin{aligned} &\Gamma(K \to \mu\nu)/\Gamma(\pi \to \mu\nu), \\ &\Gamma(B \to \tau\nu), \quad \Delta M_d, \quad \Delta M_s. \end{aligned} \tag{3.11}$$

We can observe the absence of the top quark from the choice of observables, althought this is usual in the case of B-physics where we include leptons and quarks exept the top quark, in contrast for EFT in lower energies we integrate out particles as b quark.

3.4 General Strategy for the Extraction of the CKM

In the SM theory in order to calculate the value of VEV, we have to measure the value of the Fermi constant: G_F in μ decay: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$. More specifically in the effective theory, in a scale: $\mu \sim m_\mu$, we can define, similar to 2.24 the addition to the QED lagragnian which describe the interaction between 4-fermions as the effective lagragnian:

$$\mathcal{L}_{eff} = -2\sqrt{2}G_F(\bar{\nu}_\mu \gamma^\alpha \mu_L)(\bar{e}_L \gamma_\alpha \nu_e) + h.c. \tag{3.12}$$

Therefore, by integrating out the heavy W field at tree level, we find the relation $G_F = \frac{1}{\sqrt{2}v^2}$. The corresponding numerical value of G_F in the Standard Model is $1.1663787 \times 10^{-5} GeV^{-2}$[8], using this we can find the numerical value of the Higgs Vev to be $v = 246.21965(6) GeV$. However, in the case where we assume the SMEFT as the high energy theory, we have to add the contribution of the LEFT Lagrangian, which contributes to muon decay at the scale of the muon mass $\mu \sim m_\mu$.

$$\mathcal{L} = L^{VLL}(\bar{\nu}_{L\mu}\gamma^{\alpha}\nu_{Le})(\bar{e}_L\gamma_{\alpha}\mu_L) + L^{VLR}(\bar{\nu}_{L\mu}\gamma^{\mu}\nu_{Le})(\bar{e}_R\gamma_{\mu}\mu_R), \tag{3.13}$$

where for simplicity we used the following symbolism: $[L^{VLL}_{\nu e}]_{\mu e e \mu} \to L^{VLL}$. We proceed to the computation of the muon decay rate and we find terms proportional to $|L^{VLL}|^2$, $|L^{VLR}|^2$ and a term proportional of both the LEFT coefficients: $Re(L^{VLL}L^{VLR*})$. Since the SM has only left-handed charged currents the $|L^{VLR}|^2$ term, is of the order $\frac{v^2}{\Lambda^2}$ and can be dropped, while the third term is suppressed by a term of the order $\frac{m_e}{m_\mu} \times \frac{v^2}{\Lambda^2} \approx \frac{1}{200}\frac{v^2}{\Lambda^2}$, and can be dropped. Therefore the only contribution in the μ decay comes from the L^{VLL}. Using the Fermi theory as above we find:

$$G_F = -\frac{\sqrt{2}L^{VLL}}{4}. \tag{3.14}$$

We proceed for the calculation of the tree level matching of the LEFT coefficients with the Wilson coefficients (SMEFT). We find for L^{VLL}:

$$L^{VLL} = -\frac{2}{v^2} + C^{ll}_{\mu e e \mu} + C^{ll}_{e \mu \mu e} - 2C^{\phi l(3)}_{\mu\mu} - 2C^{\phi l(3)}_{ee}. \tag{3.15}$$

The Fermi constant can be written as:

$$G_F = \frac{1}{\sqrt{2}v^2}(1 + \frac{\delta G_F}{G_F}), \tag{3.16}$$

where:

$$\frac{\delta G_F}{G_F} = -v^2(\frac{1}{2}C^{ll}_{\mu e e \mu} + \frac{1}{2}C^{ll}_{e \mu \mu e} - C^{\phi l(3)}_{\mu\mu} - C^{\phi l(3)}_{ee}) + \mathcal{O}(\Lambda^{-4}), \tag{3.17}$$

therefore the definition of the tilde VEV which contains SMEFT correction can be written as:

$$\tilde{v} = \frac{v}{\sqrt{1 + \frac{\delta G_F}{G_F}}} = v(1 + \frac{\delta v}{v}), \tag{3.18}$$

where: $\frac{\delta v}{v} = -\frac{1}{2}\frac{\delta G_F}{G_F}$. From the definition 3.18 we can give $\tilde{v}$ the value of the VEV of the SM: $\tilde{v} = 246.21965(6)GeV$. We will use this redefinition of the Higgs VEV in order to express the initial SMEFT parameters in terms of the "indirect" Wilson coefficients of equation 3.18.

In section 3.1 we defined the tilde parameters of the CKM matrix in SMEFT as $\tilde{W}_i$. The relation between the parameters in SM can be written: $\tilde{W}_i = W_i + \delta W_i$, where δW_i are the linearized contribution from LEFT or SMEFT Wilson coefficient. In order to constraining NP in SMEFT we will work as follows:

If we denote with $\mathcal{O}_a$ where $a = \{1, 2, 3, 4\}$ the flavour observables of equation 3.11, we can expand it in terms of the NP contribution as:

$$\mathcal{O}_a = \mathcal{O}_{SM,a}(W_i)(1 + \sum a_i L_i) = \mathcal{O}_{SM,a}(W_i)(1 + \sum b_i C^{(6)}_i) \tag{3.19}$$

,where L_i are the LEFT wilson coefficients and $C^{(6)}_i$ the SMEFT 6-dimension operators while a_i and b_i are some complex numbers. From the mapping of the LEFT to SMEFT we can find the relation between the parameters a_i, b_i. From the definition of the $\tilde{W}_i$

parameters we have that the flavour observables can be written in a form dependent exclusively from the $\tilde{W}_i$ in a form similar to the SM:

$$\mathcal{O}_a(W_i) = \mathcal{O}_{SM,a}(\tilde{W}_i). \tag{3.20}$$

Using equation 3.20 and the experimental inputs, we can extract numerical values for the $\tilde{W}_i$. With these numerical values, we can obtain the numerical result of the CKM matrix in BSM. Once we have found the numerical result for the CKM matrix, we can proceed to expanding the elements of the CKM matrix in terms of the linearized Wilson coefficients. We can write a general form for the expansion of any flavor measurement O_i in terms of NP contributions as follows

$$\mathcal{O}_a = \mathcal{O}_{a,SM}(\tilde{W}_i) + \delta O_{a,NP}^{indirect} + \delta \mathcal{O}_{a,NP}^{direct}, \tag{3.21}$$

where the term $\mathcal{O}_{a,NP}^{direct}$ denotes the Wilson coefficients that contribute directly to the observable, while the indirect contribution is:

$$\delta O_{a,NP}^{indirect} = -\frac{\partial \mathcal{O}_{a,SM}}{\partial W_i}\delta W_i + \frac{\tilde{v}}{2}\frac{\delta G_F}{G_F}\frac{\partial \mathcal{O}_{a,SM}}{\partial \tilde{v}} + \mathcal{O}(\Lambda^{-4}), \tag{3.22}$$

where the second term is the indirect part from the redefinition of the Higgs VEV.

We note that the CKM matrix $\tilde{V} = \tilde{V}(\tilde{W}_i)$ is unitary by construction. This does not result in any loss of generality since we do not define the nine different elements of $\tilde{V}$ as the elements extracted from nine different observables. In this approach, we only need to "sacrifice" four measurements to fix the four elements of the CKM matrix. Therefore, any additional observable can then serve as a probe of new physics. In the next sections, we will apply this algorithm (equations 3.20-3.22) in order to find the corrections in CKM for BSM.

3.5 P_{l2} Decays

The decays $P \to l\nu$, where P can be π, K, or B, and $l = \tau^-, \mu^-$, can provide accurate data in hadronic weak decays and give information to test the CKM (Cabibbo-Kobayashi-Maskawa) matrix in the Standard Model, as well as detect possible new physics (NP) corrections. We note that a similar analysis for these types of processes has been conducted in [26]. The decay rate for the process $P^- \to l^-\bar{\nu}l$ can be calculated as:

$$\Gamma(P^- \to l^-\bar{\nu}_l) = \sum_a |U_{l\nu_a}|^2 |V_{uq}|^2 \frac{f_{P^\pm}^2 m_{P^\pm} m_l^2}{16\pi\tilde{v}^4}\left(1 - \frac{m_l^2}{m_{P^\pm}^2}\right)^2 (1+\delta_{Pl})(1+\Delta_{Pl_2}), \tag{3.23}$$

where $q = d, s$ for $P = \pi, K$, respectively. The factor $\sum_a |U_{l\nu_a}|^2$ comes from the sum over all possible neutrino flavors, where U is the PMNS mixing matrix. The values of U have been taken from [43].

$$U_{PMNS} = \begin{pmatrix} 0.822 \pm 0.010 & 0.547 \pm 0.015 & 0.155 \pm 0.008 \\ 0.451 \pm 0.014 & 0.648 \pm 0.013 & 0.614 \pm 0.018 \\ 0.347 \pm 0.015 & 0.529 \pm 0.014 & 0.774 \pm 0.013 \end{pmatrix}, \tag{3.24}$$

where the PMNS matrix has been calculated with great accuracy. We note that equation 3.24 corresponds to the magnitude of the elements of the U_{PMNS} matrix. We define $f_{P^{\pm}}$ to be the QCD semileptonic decay constant of P through the relation:

$\langle 0| \bar{q}\gamma^{\alpha}\gamma_5 u \,|P^+(q)\rangle = iq^{\mu} f_{P^{\pm}}$. The factor δ_{Pl} corresponds to the electromagnetic corrections in the SM and are given [37]:

$$1 + \delta_{Pl} = S_{ew}\left[1 + \frac{\alpha}{\pi}\left(F(m_l^2/m_P^2) + \frac{3}{2}\log\frac{m_P}{m_\rho} - c_1^P\right)\right] + \mathcal{O}(e^2p^4), \tag{3.25}$$

where α is the structure constant, $S_{ew} = 1.0232(3)$[34] encodes universal short distance distance corrections to the semileptonic transitions in the SM at $\mu = m_\rho$. The function $F(x)$ describes the leading universal long-distance radiative corrections to a point-like meson [39]. The constant c_1^P encodes hadronic structure effects that can be calculated in Chiral Perturbation Theory [21]. We note that the EM corrections are estimated to be between $1-3\%$, with an uncertainty smaller than the current uncertainty of f_P. We define Δp_{l2} to be the linearized NP contribution and they are given by the relation:

$$\begin{aligned}\Delta p_{l2} = \left(\frac{\tilde{v}}{v}\right)^4 \left|1 + \epsilon_A^{luq} - \frac{m_P^2}{(m_u+m_q)m_l}\right|^2 - 1 \approx \\ 2Re(\epsilon_A^{luq}) - \frac{2m_P^2}{(m_u+m_q)m_l}Re(\epsilon_P^{luq}) + 4\frac{\delta v}{v} + \mathcal{O}(\Lambda^{-4}).\end{aligned} \tag{3.26}$$

Where we define ϵ_A^{luq} and ϵ_P^{luq} to be linearized in terms of the SMEFT's Wilson coefficients, or the LEFT coefficients by the relations:

$$\begin{aligned}\epsilon_A^{luq} &= -1 - \frac{v^2}{2V_{uq}}\left([L_{vedu}^{VLL}(\mu_q)]^*_{llqu} - [L_{vedu}^{VLR}(\mu_q)]^*_{llqu}\right), \\ \epsilon_P^{luq} &= -\frac{v^2}{2V_{uq}}\left([L_{vedu}^{SRR}(\mu_q)]^*_{llqu} - [L_{vedu}^{SRL}(\mu_q)]^*_{llqu} - [L_{vedu}^{TRR}(\mu_q)]^*_{llqu}\right),\end{aligned} \tag{3.27}$$

at hadronic scale where: $\mu_q = \{2, 2, 4.3\}GeV$ for $q = \{d, s, b\}$ correspondingly. In the case where we approach the EW scale, in order to relate the coefficients at this scales with the matching conditions at the EW scale one needs to use RGEs. Using three-loop plus one-loop QED running[2], [27], the parameters $\epsilon_{A,P}^{luq}$ of equation (7.16) can be correlated with the LEFT operators as follows:

$$\begin{aligned}
\epsilon_A^{\mu ud} &= -1.0094 - \frac{v^2}{2V_{ud}}\sum_a U_{\mu a}\left(1.0094[L^{VLL}_{\nu edu}(\mu_{EW})]^*_{\mu a du} - 1.0047[L^{VLR}_{\nu edu}(\mu_{EW})]^*_{\mu a du}\right),\\
\epsilon_P^{\mu ud} &= -\frac{v^2}{2V_{ud}}\sum_a U_{\mu a}\left(1.73[L^{SRR}_{\nu edu}(\mu_q)]^*_{\mu a du} - 1.73[L^{SRL}_{\nu edu}(\mu_q)]^*_{\mu a du} - 0.0024[L^{TRR}_{\nu edu}(\mu_q)]^*_{\mu a du}\right),\\
\epsilon_A^{\mu us} &= -1.0094 - \frac{v^2}{2V_{us}}\sum_a U_{\mu a}\left(1.0094[L^{VLL}_{\nu edu}(\mu_{EW})]^*_{\mu a su} - 1.0047[L^{VLR}_{\nu edu}(\mu_{EW})]^*_{\mu a su}\right),\\
\epsilon_P^{\mu us} &= -\frac{v^2}{2V_{us}}\sum_a U_{\mu a}\left(1.73[L^{SRR}_{\nu edu}(\mu_s)]^*_{\mu a su} - 1.73[L^{SRL}_{\nu edu}(\mu_q)]^*_{\mu a su} - 0.0024[L^{TRR}_{\nu edu}(\mu_q)]^*_{\mu a su}\right)\\
\epsilon_A^{\tau ub} &= 1.0075 - \frac{v^2}{2V_{ub}}\sum_a U_{\tau a}\left(-1.0075[L^{VLL}_{\nu edu}(\mu_{EW})]^*_{\tau a bu} - 1.0038[L^{VLR}_{\nu edu}(\mu_{EW})]^*_{\tau a bu}\right),\\
\epsilon_P^{\tau ub} &= -\frac{v^2}{2V_{ub}}\sum_a U_{\tau a}\left(1.45[L^{SRR}_{\nu edu}(\tau_q)]^*_{\tau a bu} - 1.45[L^{SRL}_{\nu edu}(\tau_q)]^*_{\tau a bu} - 0.0024[L^{TRR}_{\nu edu}(\tau_q)]^*_{\tau a bu}\right),
\end{aligned} \tag{3.28}$$

where a is the coefficient corresponding to a neutrino, namely $a = \{\nu_e, \nu_\mu, \nu_\tau\}$ since we have summed over all possible neutrinos to compute the decay rate. The LEFT Wilson coefficients of Equation 3.28 are related to the SMEFT Wilson coefficients in appendix D. We will use the results of Equation 3.28 to calculate the corrections of CKM elements in terms of the SMEFT Wilson coefficients.

3.6 Mass Differences

The Mass Differences ΔM_d and ΔM_s of neutral mesons B_q, where: $q = \{d, s\}$ are given by the relation:

$$\Delta M_q = |V_{tb}V_{tq}|^2(1+\Delta_{\Delta M_q})\frac{m_{B_q} f^2_{B_q} m^2_W}{12\pi\tilde{v}^2} B_1^q S_1(m_b), \tag{3.29}$$

which is in agreement with [24] and [5],where B_i^q are the so-called bag-parameters, and they are defined as the matrix elements: $\langle \bar{B}^0_q | Q_i | B^0_q \rangle$ up to a normalization factor. We denote the parity-even components of operators in the SUSY basis as Q_i (appendix D). The numerical values of the bag parameters are given in [15] . We denote with $\Delta_{\Delta M_q}$ the quantity:

$$\begin{aligned}
1+\Delta_{\Delta M_q} &= \frac{\tilde{v}^4}{v^4}\left|\frac{C_1^{(q)}+\tilde{C}_1^{(q)}}{C_{1,SM}^{(q)}} + R_{B_q}\left[\sum_{i=2}^{5}\frac{C_i B_i^{(q)}}{B_1^{(q)}}\frac{C_i^{(q)}}{C_{1,SM}^{(q)}} + \sum_{i=2,3}\frac{C_i B_i^{(q)}}{B_1^{(q)}}\frac{\tilde{C}_i^{(q)}}{C_{1,SM}^{(q)}}\right]\right|^2 =\\
1+4\frac{\delta v}{v} &+ Re\left[\frac{C_{1,NP}^{(q)}+\tilde{C}_1^{(q)}}{C_{1,SM}^{(q)}} + R_{B_q}\left[\sum_{i=2}^{5}\frac{C_i B_i^{(q)}}{B_1^{(q)}}\frac{C_i^{(q)}}{C_{1,SM}^{(q)}} + \sum_{i=2,3}\frac{C_i B_i^{(q)}}{B_1^{(q)}}\frac{\tilde{C}_i^{(q)}}{C_{1,SM}^{(q)}}\right]\right] +\\
\mathcal{O}(\frac{1}{\Lambda^4})
\end{aligned} \tag{3.30}$$

where the factors $\mathcal{C}_i = \{1, \frac{-5}{8}, \frac{1}{8}, \frac{3}{4}, \frac{1}{4}\}$ and $R_{B_q} = \frac{m_{B_q}}{(m_b+m_q)^2}$ while: $C_{1,NP} = C_1^{(q)} - C_{1,SM}^{(q)}$. We note that the mass of b-quark mass had been taken at the scale of the b-quark mass itself, $\mu_b \sim 4.3GeV$. Therefore the mass of b-quark in $\overline{MS}$ scheme had been calculated:

$$m_b(m_b, \overline{MS}) = 4.29(12)GeV. \tag{3.31}$$

The parameters C_i and $\tilde{C}i$ express the contributions of the operators from the SUSY basis as denoted in Appendix D. The parameter $C_{1,SM}$ is the exact calculation of the two-loop contribution in the SM, as defined in the appendix. We define the ξ parameter to be:

$$\xi = \frac{f_{B_s}}{f_{B_d}}\sqrt{\frac{B_1^s}{B_1^d}}. \tag{3.32}$$

This parameter can be calculated more explicitly in the case where we calculate the $SU(3)$-breaking ratio $B_1^{(d/s)} = \frac{B_1^d}{B_1^s}$ in the MS scheme instead of calculating B_1^s or B_1^q separately. This occurs due to the correlations between the parameters. To take into account these correlations, we rewrite equation 3.30 in terms of the parameter ξ in the case where $q = d$ as follows:

$$\Delta M_d = |V_{tb}V_{td}|^2(1+\Delta_{\Delta M_q})\frac{m_{B_d} f_{B_s}^2 B_1^s m_W^2}{12\pi\tilde{v}^2\xi^2} S_1(m_b). \tag{3.33}$$

Numerical values for the ξ parameter can be found in [31], [3]. In order to express the NP contribution in terms of the SMEFT's Wilson coefficients, we performed the tree-level matching between the LEFT Wilson coefficients of the process and the SMEFT's Wilson coefficients in Appendix D. Additionally, to find the relation between the coefficients C_i and $\tilde{C}i$ with the LEFT Wilson coefficients at the electroweak scale, one should make use of the NLO evolution matrix [13] . At a scale of $\mu_{EW} = M_Z$, the relations are as follows[1]:

$$\begin{aligned} C_1^{(q)} &= 0.858[L_{dd}^{VLL}(\mu_{EW})]_{qbqb}, \\ C_4^{(q)} &= -0.755[L_{dd}^{V1LR}(\mu_{EW})]_{qbqb} - 1.940[L_{dd}^{V8LR}(\mu_{EW})]_{qbqb}, \\ C_5^{(q)} &= -1.856[L_{dd}^{V1LR}(\mu_{EW})]_{qbqb} + 0.237[L_{dd}^{V8LR}(\mu_{EW})]_{qbqb}, \\ \tilde{C}_1^{(q)} &= 0.858[L_{dd}^{VRR}(\mu_{EW})]_{qbqb} \end{aligned} \tag{3.34}$$

Using the relations of equation 3.34 and (D.2) one can find the matching conditions of C_i and $\tilde{C}_i$ coefficients with the SMEFT Wilson coefficients. We note that a matching at higher order, $\mathcal{O}(1/\Lambda^4)$ between the LEFT and SMEFT coefficients has been in ref.[10].

3.7 Numerical Values of Observables

We will use two of the four input observables: $\Gamma(B^- \to \sum_a \tau^-\nu_a)$ and $\frac{\Gamma(K^- \to \sum_a \mu^-\nu_a)}{\Gamma(\pi^- \to \sum_a \tau^-\nu_a)}$, in order to find the numerical value of the CKM matrix element $|\tilde{V}_{ub}|^2$ and the ratio $\frac{|\tilde{V}_{us}|^2}{|V_{ud}|^2}$. We define $\tilde{V}_{ij}$ as the CKM elements that absorb the NP contribution. For the decay

[1]We notice that we only took into account the coefficients C_i and $\tilde{C}_i$ up to order $\mathcal{O}(\frac{1}{\Lambda^2})$

$\Gamma(B^- \to \sum_a \tau^- \nu_a)$, we define:

$$|\tilde{V}_{ub}|^2 \equiv |V_{ub}|^2(1 + \Delta_{B\tau_2}). \tag{3.35}$$

To calculate the numerical value of $|V_{ub}|^2$, we will use the input values in Table 2 of [23]. Using these values, we obtain:

$$\boxed{|\tilde{V}_{ub}| = 0.00425 \pm 0.00049}, \tag{3.36}$$

where we neglected electromagnetic corrections since they are practically negligible compared to the experimental sensitivity.[2] Additionally, we note that the factor $\sum_a |U_{\tau^-\nu_a}|^2 = 0.999326 \approx 1$, which does not affect the result of equation 3.36. We proceed to calculate the ratio $\frac{|\tilde{V}us|^2}{|Vud|^2}$. One could determine the CKM matrix elements $|V_{us}|$ and $|V_{ud}|$ separately, but in these cases the element $W_1 = \tilde{\lambda}$ would have large uncertainty, since the form factor for Kaon, f_K, does not rely on f_π and has been calculated with large uncertainty [7]. Therefore, to compute the W_1 parameter, we will use the ratio $\frac{\Gamma(K^- \to \sum_a \mu^- \nu_a)}{\Gamma(\pi^- \to \mu^- \sum_a \nu_a)}$, which is calculated to be:

$$\frac{\Gamma(K^- \to \mu^- \sum_a \nu_a)}{\Gamma(\pi^- \to \mu^- \sum_a \nu_a)} = \frac{\left|\tilde{V}_{us}\right|^2}{\left|\tilde{V}_{ud}\right|^2} \frac{f_K^2}{f_\pi^2} \frac{m_K}{m_\pi} \frac{\left(1 - \frac{m_\mu^2}{m_K^2}\right)^2}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} (1 + \delta_{K/\pi}), \tag{3.37}$$

We notice that the value of $\frac{f_K}{f_\pi}$ had been taken from FLAG [3], which combines several lattice determinations for this ratio of decay constants without introducing any uncontrollable dependence on NP via the pion leptonic width. The NP contribution is encoded in the ratio:

$$\frac{\left|\tilde{V}_{us}\right|}{\left|\tilde{V}_{ud}\right|} = \frac{|V_{us}|}{|V_{ud}|}(1 + \Delta_{K/\pi}), \tag{3.38}$$

where we define:

$$1 + \Delta_{K/\pi} = \frac{1 + \Delta_{K\mu_2}}{1 + \Delta_{\pi\mu_2}}, \tag{3.39}$$

which can be calculated up to order $\mathcal{O}(\Lambda^{-4})$:

$$\begin{aligned} 1 + \Delta_{K/\pi} =& \frac{1 + \Delta_{K\mu_2}}{1 + \Delta_{\pi\mu_2}} \approx \\ & (1 + \Delta_{K\mu_2})(1 - \Delta_{\pi\mu_2}) = \\ & 2Re(\epsilon_A^{\mu us} - \epsilon_A^{\mu ud}) - \frac{2}{m_{\mu^-}} \left(\frac{m_K^2 Re(\epsilon_P^{\mu us})}{m_u + m_s} - \frac{m_\pi^2 Re(\epsilon_P^{\mu ud})}{m_u + m_d} \right) + \mathcal{O}(\Lambda^{-4}). \end{aligned} \tag{3.40}$$

[2]We note that the decay constant $fB^\pm$ is independent of the experimental value of the QCD form factor f_π.

Using the numerical inputs of Table 2 of [23]we find the ratio to be:

$$\boxed{\frac{|\tilde{V}_{us}|}{|\tilde{V}_{ud}|} = 0.23131 \pm 0.00051}. \tag{3.41}$$

The numerical error is completely dominated by the uncertainty of $f_{K/\pi}$. Continuing with the mass-difference transitions, we use Equation 3.33 to calculate the following quantities:

$$|\tilde{V}_{tb}\tilde{V}_{tq}|^2 = |V_{tb}V_{td}|^2(1 + \Delta_{\Delta M_q}) \tag{3.42}$$

Referring back to Table 2 in [23], we find:

$$\boxed{|\tilde{V}_{tb}\tilde{V}_{td}| = 0.00851(25), \quad |\tilde{V}_{tb}\tilde{V}_{ts}| = 0.0414(10)} \tag{3.43}$$

3.8 Numerical Results of CKM

We summarize the results from the above analysis to be:

$$\begin{aligned} \frac{|\tilde{V}_{us}|}{|\tilde{V}_{ud}|} &= 0.23131 \pm 0.000050, \qquad & |\tilde{V}_{ub}| &= 0.00426 \pm 0.00048 \\ |\tilde{V}_{tb}\tilde{V}_{td}| &= 0.00851 \pm 0.00026 & |\tilde{V}_{tb}\tilde{V}_{ts}| &= 0.0414 \pm 0.0010. \end{aligned} \tag{3.44}$$

We found identical results compared to those of ref.[23]. In the following work, we will use the numerical results of equation 3.44 to calculate the value of the tilde Wolfenstein parameters. We start by writting the observable quantities as functions of the Wolfenstein parameters as follows:

$$\begin{aligned} \frac{|\tilde{V}_{us}|}{|\tilde{V}_{ud}|} &= \tilde{\lambda} + \frac{1}{2}\tilde{\lambda}^3 + \frac{3}{8}\tilde{\lambda}^5 + \mathcal{O}(\tilde{\lambda}^6) \\ |\tilde{V}_{ub}| &= \left(\tilde{\lambda}^3 + \frac{1}{2}\tilde{\lambda}^5 + \mathcal{O}(\tilde{\lambda}^6)\right)\tilde{A}\sqrt{\tilde{\rho}^2 + \tilde{\eta}^2} \\ |\tilde{V}_{tb}\tilde{V}_{td}| &= \tilde{\lambda}^3\tilde{A}\sqrt{(1-\tilde{\rho})^2 + \tilde{\eta}^2} + \mathcal{O}(\tilde{\lambda}^6) \\ |\tilde{V}_{tb}\tilde{V}_{ts}| &= \tilde{\lambda}^2\tilde{A} - \frac{1}{2}\tilde{\lambda}^4\tilde{A}(1-2\tilde{\rho}) + \mathcal{O}(\tilde{\lambda}^6). \end{aligned} \tag{3.45}$$

The other set of equations to calculate the numerical errors of the observable quantities are the following:

$$\delta\mathcal{K}_a = \sqrt{\left(\frac{\partial\mathcal{K}_a}{\partial\tilde{\rho}}\delta\tilde{\rho}\right)^2 + \left(\frac{\partial\mathcal{K}_a}{\partial\tilde{\eta}}\delta\tilde{\eta}\right)^2 + \left(\frac{\partial\mathcal{K}_a}{\partial\tilde{A}}\delta\tilde{A}\right)^2 + \left(\frac{\partial\mathcal{K}_a}{\partial\tilde{\lambda}}\delta\tilde{\lambda}\right)^2}, \tag{3.46}$$

where $\mathcal{K}_a = \{\frac{|\tilde{V}_{us}|}{|\tilde{V}_{ud}|}, |\tilde{V}_{ub}|, |\tilde{V}_{tb}\tilde{V}_{td}|, |\tilde{V}_{tb}\tilde{V}_{ts}|\}$ and $\delta\mathcal{K}_a$ corresponds to the numerical errors of the observables quantities for $a = \{1, 2, 3, 4\}$. Solving these sets of equations, we find the numerical values of tilde Wolfenstein parameters:

$$\begin{pmatrix} \tilde{\lambda} = \lambda + \delta\lambda \\ \tilde{A} = A + \delta A \\ \tilde{\rho} = \bar{\rho} + \delta\bar{\rho} \\ \tilde{\eta} = \bar{\eta} + \delta\bar{\eta} \end{pmatrix} = \begin{pmatrix} 0.22537 \pm 0.00046 \\ 0.828 \pm 0.020 \\ 0.194 \pm 0.024 \\ 0.391 \pm 0.056. \end{pmatrix}. \tag{3.47}$$

From our results, we can see that the smaller numerical error comes from the uncertainty of $\tilde{\lambda}$. Therefore, our choice of keeping terms up to order $\mathcal{O}(\tilde{\lambda}^6)$ was appropriate. Secondly, we can tell that our decision to use a small number of observable quantities to extract the CKM matrix results in a loss of accuracy in the limit where the coefficients of NP are zero, in comparison to SM fits that use significantly larger sets of observables[16] . In addition, we have to remark that there is a third possible set of solutions $(\lambda, A, \rho, -\eta)$, obtained by replacing η with $-\eta$. This symmetry of equation 3.47 with respect to η results in "mirror solutions" to the global fits. Although the CKM parameters in these solutions may differ significantly from those in the SM fit, the total shift will be cancelled out by a large number of Wilson coefficients. In the present thesis we will not examine the case of "mirror solution" any further.

Since we found numerical values for the Wolfenstein parameters we can proceed with calculating the CKM matrix for BSM. In the tilde Wolfenstein corrections $\delta W = \{\delta\tilde{\lambda}, \delta\tilde{\rho}, \delta\tilde{\eta}, \delta\tilde{A}\}$ there are encoded the NP shifts. In order to relate the Wolfenstein corrections with the Wilson Coefficients we have to solve a complicated non-linear equation that relates δW and $\delta\mathcal{O}_a = \{\Delta_{K/\pi}, \Delta_{B\tau_2}, \Delta_{\Delta M_d}, \Delta_{\Delta M_s}\}$. However we can simplify this equation by working in the case where $\delta\mathcal{O}_a$ are relatively small, therefore we can keep terms up to order $\mathcal{O}(\frac{1}{\Lambda^2})$(linear terms of Wilson coefficients). The relation of those quantities can be written:

$$\begin{pmatrix} \delta\lambda \\ \delta A \\ \delta\bar{\rho} \\ \delta\bar{\eta} \end{pmatrix} = M(\tilde{\lambda}, \tilde{A}, \tilde{\eta}, \tilde{\rho}) \begin{pmatrix} \Delta_{K/\pi} \\ \Delta_{B\tau 2} \\ \Delta_{\Delta M_d} \\ \Delta_{\Delta M_s}. \end{pmatrix}. \tag{3.48}$$

The matrix $M(\tilde{\lambda}, \tilde{A}, \tilde{\eta}, \tilde{\rho})$ is defined throught the relation:

$$M(\tilde{\lambda}, \tilde{A}, \tilde{\eta}, \tilde{\rho}) \equiv (O')^{-1}\mathcal{O}, \tag{3.49}$$

where we define the diagonal matrix:

$$\mathcal{O} = diag\left(\frac{\Gamma(K \to \mu\nu)}{\Gamma(\pi \to \mu\nu)}, \Gamma(B \to \tau\nu), \Delta M_d, \Delta M_s\right), \tag{3.50}$$

whoose elements are the SM expressions of the input observables. The elements of $\mathcal{O}'$ are defined by the relation:

$$\mathcal{O}'_{ij} \equiv \frac{\partial \mathcal{O}_{ii}}{\partial W_j}. \tag{3.51}$$

We can express the matrix M in terms of the tilde Wolfenstein parameters up to order $\tilde{\lambda}^6$ as follows:

$$M(\tilde{\lambda},\tilde{A},\tilde{\eta},\tilde{\rho}) = \begin{pmatrix} \frac{1}{2}\tilde{\lambda}-\frac{1}{2}\tilde{\lambda}^3 & 0 & 0 & 0 \\ -\tilde{A}+\tilde{A}\tilde{\lambda}^2+c\tilde{A}\tilde{\lambda}^4 & -ce\tilde{A} & be\tilde{A} & \frac{1}{2}\tilde{A}-ae\tilde{A} \\ a-b\tilde{\lambda}^2+c\tilde{A}\tilde{\lambda}^4 & c(1-2ae) & -b(1-2ae) & a(1-2ae) \\ \frac{d}{2\tilde{\eta}}+\frac{b\tilde{\rho}}{\tilde{\eta}}\tilde{\lambda}^2-\frac{c(2d+3(\tilde{\rho}-1))}{2\tilde{\eta}} & \frac{c}{\tilde{\eta}}(1-\tilde{\rho}+de) & \frac{b}{\tilde{\eta}}(\tilde{\rho}-de) & -\frac{d}{2\tilde{\eta}}(1-2ae) \end{pmatrix} \tag{3.52}$$

, where we defined the quantities:

$$a=\frac{1-2\tilde{\rho}}{2}, \quad b=\frac{\tilde{\eta}^2+(1-\tilde{\rho})^2}{2}, \quad c=\frac{\tilde{\eta}^2+\tilde{\rho}^2}{2}, \\ d=\tilde{\eta}^2-\tilde{\rho}^2+\tilde{\rho}, \quad e=\tilde{\lambda}^2(1-a\tilde{\lambda}^2) \tag{3.53}$$

Using Eq 3.47 we calculate the numerical value of M matrix to be:

$$M(\tilde{\lambda},\tilde{A},\tilde{\eta},\tilde{\rho}) = \begin{pmatrix} 0.1070 & 0 & 0 & 0 \\ -0.786 & -0.0039 & 0.0166 & 0.401 \\ 0.286 & 0.093 & -0.390 & 0.298 \\ -0.384 & 0.200 & 0.182 & -0.382 \end{pmatrix} \tag{3.54}$$

.

Since we have the numerical value of M, we can easily express the Wolfenstein corrections in terms of the linearized Wilson coefficients. Continuing, we use equation 3.47 to calculate the numerical value of the tilde CKM matrix elements. The numerical value of the CKM matrix, including the contributions of the Wilson coefficient, is:

$$\tilde{V} = \begin{pmatrix} 0.97428(11) & 0.22537(46) & 0.00189(29)-i0.0038(66) \\ -0.22524(46)-i0.000156(31) & 0.97340(15) & 0.0421(11) \\ 0.00764(3)-i0.00370(74) & -0.0414(10)-i0.000083(13) & 0.999115(49) \end{pmatrix}. \tag{3.55}$$

The NP contributions are encoded in the numerical error of $\tilde{V}$.

Using equation 3.48 and the numerical value of $\tilde{V}$, we expressed the corrections δV_{ij} in terms of the Wilson coefficients of the SMEFT. We note that δV_{ij} symbolizes the linear combination of 6-dimensional Wilson coefficients. We assumed $U_{ij}=\delta_{ij}$ for the PMNS matrix at the beginning, and the results in this case, where $U_{ij}=\delta_{ij}$, are provided in the Appendix F. Subsequently, we negated our initial condition and considered the general PMNS matrix, which can be expressed in SMEFT as:

$$U_{ij} = U_{PMNS} + \sum_i k_i C_i, \tag{3.56}$$

where k_i are some complex numbers and C_i are Wilson coefficients. The values of U_{PMNS} corresponds to the experimental results of [43]. Since δV_{ij} had been calculated up to order $\mathcal{O}(\Lambda^{-2})$, it is efficient to neglect the second term in equation 3.56. Therefore, only the U_{PMNS} part was considered for the calculations. Comparing the two computations leads us to conclude that when we assume $U_{ij}=\delta_{ij}$, the contribution of SMEFT Wilson coefficients is identical to that when we assume $U_{ij}=U_{PMNS}$. This can be shown by expanding the sum over the Wilson coefficients algebraically in equation 3.28 (Appendix

E). However, in the latter case, where $U_{ij} = U_{PMNS}$, additional components of SMEFT Wilson coefficients appear, which may play a crucial role in determining the corrections to the CKM matrix. We denote the contribution of those components as δU_{ij}. The corrections of the CKM elements, $\delta \tilde{V}_{ij}$, where:

$$\delta \tilde{V}_{ij} = \delta V_{ij} + \delta U_{ij}, \tag{3.57}$$

have been computed in the Appendix F [3].

In conclusion, we have computed the numerical values of $\delta W_i = \{\delta\lambda, \delta A, \delta\rho, \delta\eta\}$ with the assumption that we have concluded the PMNS matrix in our calculation (Equation 3.47). The numerical errors of the CKM matrix elements can be expressed as linear combinations of δW_i. Therefore, assuming $U_{ij} = \delta_{ij}$ would result in a loss of generality since the linear combination δV_{ij} would not include non-diagonal Wilson coefficients. It is possible that the linear combination $\delta \tilde{V}_{ij}$ of 6-dimensional Wilson coefficients provides a better fit for the numerical results of the corrections in Equation 3.55, considering that it includes non-diagonal Wilson coefficients. Although it should be noted that the numbers that multiply the Wilson coefficients in δU_{ij} are typically comparable to the multiplication factors in the diagonal case. This implies that the contribution of the off-diagonal Wilson coefficients would generally be small in comparison to the numerical values of the correction in the diagonal case, in order for the numerical values of the correction to be the same in both cases. We note that we cross-checked our numerical results for the CKM corrections with those in [19].

[3] The numerical results of δU_{ij} correspond to a zero CP violation phase.

Chapter 4

Applications

In this section we will use the tilde Wolfenstein parameters to investigate various flavour processes in a systematic manner. We will go over a few examples (in tree-level) to illustrate their application and also demonstrate how they can be used to place limits in new physics.

4.1 Leptonic Decays

First we will assume the process: $K^- \to \mu\nu$. By comparing the branching fraction, which has been measured with great accuracy, to the predictions of the Standard Model, we aim to restrict the influence of effective interactions that exist beyond the SM. The branching ratio of the process is proportional to $|V_{us}|$ which obtained from the SM-fits. Therefore to overcome the fact that these analyses may be influenced by NP, which could have the same effective operators we will write the branching fraction in terms of the tilde Wolfenstein parameters. Therefore the decay rate of the process can be written as:

$$\begin{aligned}
\Gamma(K^- \to \mu^- \bar{\nu}_l) =& |V_{us}|^2 \frac{f_{K^\pm}^2 m_{K^\pm} m_\mu^2}{16\pi\tilde{v}^4}\left(1-\frac{m_\mu^2}{m_{K^\pm}^2}\right)^2 (1+\delta_{K\mu})(1+\Delta_{K\mu_2}) = \\
& |\tilde{V}_{us}|^2 \frac{|V_{us}|^2}{|\tilde{V}_{us}|^2} \frac{f_{K^\pm}^2 m_{K^\pm} m_\mu^2}{16\pi\tilde{v}^4}\left(1-\frac{m_\mu^2}{m_{K^\pm}^2}\right)^2 (1+\delta_{K\mu})(1+\Delta_{K\mu_2}) = \\
& |\tilde{\lambda}|^2 \frac{|\lambda|^2}{|\lambda+\delta\lambda|^2} \frac{f_{K^\pm}^2 m_{K^\pm} m_\mu^2}{16\pi\tilde{v}^4}\left(1-\frac{m_\mu^2}{m_{K^\pm}^2}\right)^2 (1+\delta_{K\mu})(1+\Delta_{K\mu_2}) = \\
& |\tilde{\lambda}|^2 \frac{1}{|1+\frac{\delta\lambda}{\lambda}|^2} \frac{f_{K^\pm}^2 m_{K^\pm} m_\mu^2}{16\pi\tilde{v}^4}\left(1-\frac{m_\mu^2}{m_{K^\pm}^2}\right)^2 (1+\delta_{K\mu})(1+\Delta_{K\mu_2}) \approx \\
& |\tilde{\lambda}|^2 \frac{f_{K^\pm}^2 m_{K^\pm} m_\mu^2}{16\pi\tilde{v}^4}\left(1-\frac{m_\mu^2}{m_{K^\pm}^2}\right)^2 (1+\delta_{K\mu})(1-\frac{2\delta\lambda}{\tilde{\lambda}}+\Delta_{K\mu_2}),
\end{aligned} \tag{4.1}$$

where we used that $\left|1+\frac{\delta\lambda}{\lambda}\right|^2 \approx 1-2\frac{\delta\lambda}{\tilde{\lambda}}$. In addition we define:

$$\begin{aligned}
&\tilde{\Delta}_{K\mu_2} = \\
&\quad 2Re\left(\epsilon_A^{\mu us}\right) - \frac{2m_{K^\pm}^2}{(m_u+m_s)m_\mu} Re\left(\epsilon_P^{\mu us}\right) + 4\frac{\delta v}{v} - 2\frac{\delta\lambda}{\tilde{\lambda}} + \mathcal{O}(\Lambda^{-4}).
\end{aligned} \tag{4.2}$$

We note that in quantities $\epsilon_A^{\mu us}$, $\epsilon_P^{\mu us}$, $\delta\lambda$ are encoded NP terms as defined in equations 3.28, 3.47 correspondingly. Using equation 3.47 we can rewrite equation 4.2 as:

$$\begin{aligned}\tilde{\Delta}_{K\mu_2} =& 0.1 Re\left(\epsilon_A^{\mu us}\right) - 0.1\frac{m_{K^\pm}^2}{(m_u+m_s)m_\mu}Re(\epsilon_P^{\mu us}) + \\ & 1.9 Re(\epsilon_A^{\mu ud}) - 1.9\frac{m_\pi^2}{m_\mu(m_u+m_d)}Re(\epsilon_P^{\mu ud}) + \frac{4\delta v}{v}.\end{aligned} \tag{4.3}$$

It is clear from Equation 4.3 that the insertion of $\delta\lambda$ results in the correlation of the process $\pi \to \mu\nu$, as the terms $\epsilon_{A,P}^{\mu ud}$ appear. We will use the value of $f_{K^\pm} = 155.62(44)$ MeV from FLAG and the numerical value of $\mathcal{B}(K \to \mu\nu)$ from the Particle Data Group to obtain the following constraints for Equation 4.3:

$$\boxed{\tilde{\Delta}_{K\mu_2} = 0.0091 \pm 0.0002}. \tag{4.4}$$

We assume that the electromagnetic radiative corrections for Kaon are negligible since they are of the order: $\delta_{K_\mu} \sim 10^{-2}$. The numerical error is dominated by the uncertainty of $f_{K^\pm}$. Another constraint has been calculated for the process $\pi \to \mu\nu$, in which we found, using an equivalent method, that:

$$\boxed{\tilde{\Delta}_{\pi\mu_2} = 0.004 \pm 0.013}, \tag{4.5}$$

where:

$$\tilde{\Delta}_{\pi\mu_2} = 2Re(\epsilon_A^{\mu ud}) - \frac{2m_{\pi^\pm}^2}{(m_u+m_d)m_\mu}Re(\epsilon_P^{\mu ud}) + 4\frac{\delta v}{v} + 2\tilde{\lambda}(1+\frac{1}{2}\tilde{\lambda}^2)\delta\lambda + \mathcal{O}(\Lambda^{-4}) \tag{4.6}$$

From equations 4.6 and 4.5 we can extract a third condition, for the quantity:

$$\begin{aligned}\mathcal{R}+1 =& \frac{1+\tilde{\Delta}_{K\mu 2}}{1+\tilde{\Delta}_{\pi\mu 2}} \approx \\ & (1+\tilde{\Delta}_{K\mu 2})(1-\tilde{\Delta}_{K\mu 2}) = 1+\tilde{\Delta}_{K\mu 2} - \tilde{\Delta}_{\pi\mu 2} = \\ & 1+\Delta_{K\mu 2} - \Delta_{\pi\mu 2} - 2\frac{\delta\lambda}{\tilde{\lambda}} - 2\tilde{\lambda}(1+\frac{1}{2}\tilde{\lambda}^2)\delta\lambda = \\ & 1+\Delta_{K/\pi} - \frac{2\delta\lambda}{\tilde{\lambda}} - 2\tilde{\lambda}(1+\frac{1}{2}\tilde{\lambda}^2)\delta\lambda.\end{aligned} \tag{4.7}$$

where the numerical value of $\mathcal{R}$ is:

$$\boxed{\mathcal{R} = 0.0050 \pm 0.0129}. \tag{4.8}$$

Another possible process in order to isolate $\delta\lambda$ in order to create a constraint would be the following ratio of decays rates:

$$\frac{\Gamma(D_s \to l\nu)}{\Gamma_{SM}(t \to Wb)} = \frac{|V_{cs}|^2 \frac{f_{D_s}^2 m_{D_s} m_l^2}{16\pi\tilde{v}^4}\left(1 - \frac{m_l^2}{m_{K^\pm}^2}\right)^2 (1+\delta_{D_s l})(1+\Delta_{D_s l_2})}{\frac{\sqrt{2}G_F |V_{tb}|^2 m_t^3 \left(1-\frac{m_w^2}{m_t^2}\right)^2\left(1+\frac{2m_w^2}{m_t^2}\right)}{16\pi} f(a_s)}, \tag{4.9}$$

where we consider the decay width of the process $t \to Wb$ in the SM. We denote with:

$$f(a_s) = 1 - \frac{2a_s}{3\pi}\left(\frac{2\pi^2}{3} - \frac{5}{2}\right). \tag{4.10}$$

For a value of $m_t \approx 173.3GeV$ and $a_s \approx 0.118$ we find the decay Width of SM to be 1.35 GeV. Using that:

$$\left|\frac{V_{cs}}{V_{tb}}\right| = \left|\frac{1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1+4A^2)}{1 - \frac{1}{2}A^2\lambda^4}\right| \approx \left|1 + \cancel{\frac{1}{2}A^2\lambda^4} - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1+\cancel{4A^2})\right| = \left|1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4\right|. \tag{4.11}$$

Therefore the Ratio of equation 4.11 can be written as:

$$\frac{\Gamma(D_s \to l\nu)}{\Gamma_{SM}(t \to Wb)} = \frac{|1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4|^2 \frac{f_{D_s}^2 m_{D_s} m_\mu^2}{16\pi\tilde{v}^4}\left(1 - \frac{m_\mu^2}{m_{K^\pm}^2}\right)^2 (1+\delta_{D_s l})(1+\tilde{\Delta}_{D_s l_2})}{\frac{\sqrt{2}G_F m_t^3 \left(1-\frac{m_w^2}{m_t^2}\right)^2\left(1+\frac{2m_w^2}{m_t^2}\right)}{16\pi} f(a_s)}, \tag{4.12}$$

where:

$$\boxed{\tilde{\Delta}_{D_s l_2} = 2Re(\epsilon_A^{lcs}) - \frac{2m_{D_s}^2}{(m_c+m_s)m_l} Re(\epsilon_P^{lcs}) + 4\frac{\delta v}{v} + 2\tilde{\lambda}(1+\frac{1}{2}\tilde{\lambda}^2)\delta\lambda + \mathcal{O}(\Lambda^{-4})}. \tag{4.13}$$

The last Leptonic decay that we will consider in order to set contraints for all the corrections δW_i is: $B^- \to \tau^- \nu$. We can rewrite the decay rate in the form:

$$\Gamma(B^- \to \tau^-\nu) = |A\lambda^3\left(1 + \frac{1}{2}\lambda^2\right)(\rho - i\bar{\eta})|^2 \frac{f_{B^\pm}^2 m_{B^\pm} m_\tau^2}{16\pi\tilde{v}^4}\left(1 - \frac{m_\tau^2}{m_{B^\pm}^2}\right)^2 \cdot (1+\delta_{B\tau})(1+\tilde{\Delta}_{B\tau_2}), \tag{4.14}$$

where we define:

$$\tilde{\Delta}_{B\tau 2} = 2Re(\epsilon_A^{\tau ub}) - \frac{2m_B^2}{(m_u+m_b)m_\tau} Re(\epsilon_P^{\tau ub}) + 4\frac{\delta v}{v} - \delta V_{ub} + \mathcal{O}(\Lambda^{-4}), \tag{4.15}$$

where:

$$\delta V_{ub} = \delta A\lambda^3 \left(1+\frac{1}{2}\lambda^2\right)(\bar{\rho}-i\bar{\eta}) + A\delta\lambda\left(3\lambda^2+\frac{5}{2}\lambda^4\right) + A\lambda^3\left(\delta\rho - i\delta\eta\right). \tag{4.16}$$

Using the experimental value of the decay rate $\Gamma(B^- \to \tau^-\nu) = 4.38(5)\cdot 10^{-8} eV$ [8] and $f_{B^\pm} = 184(4) MeV$ we get the constrain for $\tilde{\Delta}_{B\tau 2}$ to be:

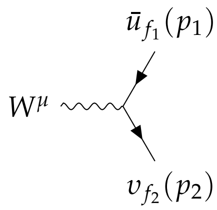
$$\boxed{\tilde{\Delta}_{B\tau 2} = -0.555 \pm 0.009}. \tag{4.17}$$

The error is totally dominated by the lattice uncertainty of $f_{B^\pm}$.

4.2 W Decays

We will consider the general process: $W \to u_{f_1} d_{f_2}$,where $u_{f_1} = \{c, u\}$ and $d_{f_2} = \{d, s, b\}$. The amplitude of this process can be written as follows:

$$\begin{aligned} i\mathcal{M}^{\mu}_{W\to ud} =& \bar{u}(p_1)\Big(\frac{-i\bar{g}}{\sqrt{2}} V_{f_1 f_2}\gamma^\mu P_L - 2vq_\nu V_{f_1 g_1} C^{dW}_{g_1 f_2}\sigma^{\mu\nu} P_R - \frac{i\bar{g}v^2}{\sqrt{2}} V_{f_1 g_1} C^{\phi q(3)}{}_{g_1 f_2}{}^{\gamma^\mu P_L} - \\ & \frac{i\bar{g}v^2}{2\sqrt{2}} C^{\phi ud}_{f_1 f_2}\gamma^\mu P_R - 2vq_\nu V_{g_1 f_2}\sigma^{\mu\nu} P_L C^{uW*}_{g_1 f_1}\Big) v(p_2), \end{aligned} \tag{4.18}$$



$$= i\mathcal{M}_{W\to ud} \tag{4.19}$$

, where $\bar{u}(p_1)$ and $v(p_2)$ the spinors of d_k and u_j respectively(additionally we consider $\sum_{polarization} \epsilon_\mu \epsilon_\nu \to -g_{\mu\nu}$, where ϵ_μ the polarization vector of W gauge boson). The purpose of this application is to compare the decay width of the process with exclusive decay widths predicted in the SM. These exclusive decays predicted within the Standard Model depend on the CKM matrix $V_{f_1 f_2}$, which may contain corrections from new physics. Therefore, in order to understand how the tilde CKM matrix elements affect the decay rate, we will consider the ratio:

$$\boxed{\frac{\Gamma(W\to ud)}{\Gamma_{SM}(W\to ud)} \approx 1 + Re\Big(\frac{\delta g}{\tilde{V}_{f_1 f_2}} - \frac{2\delta V_{f_1 f_2}}{\tilde{V}_{f_1 f_2}}\Big)}, \tag{4.20}$$

where $\Gamma_{SM}(W \to ud)$ represents the SM prediction obtained by using the numerical values of the tilde CKM elements and:

$$\delta g = -2\frac{\delta v}{v}\tilde{V}_{f_1 f_2} - 2V_{f_1 g_1} C^{\phi q(3)}_{g_1 f_2} + \frac{3\sqrt{2}m_u}{M_W} C^{uW*}_{g_1 f_2} V_{g_1 f_2} - \frac{3\sqrt{2}m_d}{M_W} K_{f_1 g_1} C^{dW}_{g_1 f_2}. \tag{4.21}$$

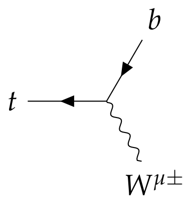
is a linear combination in terms of Wilson coefficients. Therefore the total shift of the specific W decays is $\delta g - 2\delta V_{f_1 f_2}$,where the terms $\delta V_{f_1 f_2}$, up to $\mathcal{O}(\lambda^4)$are as follows:

$$\begin{aligned}\delta V_{ud} =&\delta V_{cs} - \tilde{\lambda}\delta\lambda \\ &\delta V_{us} = \delta\lambda \\ &\delta V_{ud} = -\delta\lambda \\ &\delta V_{ub} = \delta A\tilde{\lambda}^3\left(1+\frac{1}{2}\tilde{\lambda}^2\right)(\bar{\rho}-i\bar{\eta}) + \tilde{A}\delta\tilde{\lambda}\left(3\tilde{\lambda}^2+\frac{5}{2}\tilde{\lambda}^4\right) + A\lambda^3\left(\delta\rho - i\delta\eta\right) \\ &\delta V_{cb} = 2\tilde{A}\tilde{\lambda} + \delta A\tilde{\lambda}^2.\end{aligned} \tag{4.22}$$

One can use equation 3.48 to express $\delta V_{f_1 f_2}$ in terms of linearized Wilson coefficients and related quantities such as $\Delta_{K/\pi}$, for example: $\delta V_{ud} \approx -0.0247\Delta_{K/\pi}$, $\delta V_{us} = 0.107\Delta_{K/\pi}$, etc.

4.3 Top-quark Decay

We will consider the top quark decay to a b quark and a $W-boson$, in this case the leading contribution comes from:



$$= i\mathcal{M}_{t\to bW}. \tag{4.23}$$

The decay rate of the process up to order $\mathcal{O}\left(\frac{m_b^2}{m_t^2}\right)$ is calculated to be:

$$\Gamma(t\to bW) = |V_{tb}|^2\frac{\bar{g}(f_W^2-1)}{64\pi f_W^2}\left((f_W^2-1)\bar{g}(1+2f_W^2)+\frac{\delta C}{V_{tb}}\right)+\mathcal{O}(\frac{m_b^2}{m_t^2}), \tag{4.24}$$

where:

$$\delta C = (f_W^2-1)f_W^2 12\sqrt{2}C_{g_1 b}^{dW}V_{tg_1}(m_t v) + 6\frac{m_b}{m_t}f_W^2 v^2\bar{g}C_{tb}^{\phi ud} + 12\sqrt{2}\frac{m_b}{m_t}(m_t v)(1+f_W^2)C_{g_1 b}^{uW*}V_{g_1 t}, \tag{4.25}$$

where we denote with $f_W = \frac{M_W}{m_t}$. Working equivalent as equation 4.20 we find:

$$\boxed{\frac{\Gamma(t\to Wb)}{\Gamma_{SM}(t\to Wb)} \approx 1 + Re\left(\frac{\delta C}{\tilde{V}_{tb}} - \frac{2\delta V_{tb}}{\tilde{V}_{tb}}\right)}, \tag{4.26}$$

therefore the contribution of NP in $t \to bW$ decay has as a result the shift of the SM value $\delta h = \delta C - 2\delta V_{tb}$, where:

$$\delta V_{tb} = -\tilde{A}\delta A\lambda^4 - 2\tilde{\lambda}^3\tilde{A}^2\delta\lambda. \tag{4.27}$$

4.4 Setting bounds in Leptoquark Model

Our approach in this application involves using the results obtained in F to set constraints on specific Wilson coefficients that are commonly found in both ref.[20] and the CKM matrix corrections. In ref.[20], a comprehensive matching onto the SMEFT of a model with two scalar leptoquarks, weak isospin, and a doublet has been performed. Our focus is on the tree-level matching scenario, and the following B-conserving Wilson coefficients are of particular interest, as they also appear in the CKM matrix(equations (3.13-3.14) in ref.[20]):

$$\begin{aligned} [C^{lq(3)}]^{(0)}_{prst} &= -\frac{(\lambda^{1L}_{sp})^*(\lambda^{1L}_{tr})}{4M_1^2} \\ [C^{lequ(1)}]^{(0)}_{prst} &= \frac{(\lambda^{1L}_{sp})^*(\lambda^{1R}_{tr})}{2M_1^2} \\ [C^{lequ(3)}]^{(0)}_{prst} &= -\frac{(\lambda^{1L}_{sp})^*(\lambda^{1R}_{tr})}{8M_1^2}. \end{aligned} \tag{4.28}$$

Where M_1 represents the mass of the Leptoquark in ref.[20]. We would like to emphasize that, when going from the Green basis to the Warsaw basis, the Wilson coefficients remain the same for the case presented in 4.28. We proceed to set constraints for the 3×3 matrices $\{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3\} = \{\frac{(\lambda^{1L}_{sp})^*(\lambda^{1L}_{tr})}{M_1^2}, \frac{(\lambda^{1L}_{sp})^*(\lambda^{1R}_{tr})}{M_1^2}, \frac{(\lambda^{1L}_{sp})^*(\lambda^{1R}_{tr})}{M_1^2}\}$(we will set the indices p and r to be fixed). Our initial approach to setting constraints on the matrices in equation 4.28 will be based on the results presented in F. Specifically, we will impose the constraint that every element of the corrections δV_{ij} has an upper bound of approximately $\Pi = 20\%$ of the corresponding CKM elements. The following constraints are our results for specific choices of the p and r indices($p = r = 2$):

$$(\mathcal{M}_1)_{22st} < \begin{pmatrix} 10^{-5}, 6\cdot 10^{-5}, * \\ 3\cdot 10^{-6}, 10^{-5}, * \\ *, \quad *, \quad * \end{pmatrix}_{st}. \tag{4.29}$$

We utilized the linear combination of Wilson coefficients of δV_{us} to establish bounds in 4.29. In addition, we use the symbol $*$ to denote cases in which we cannot establish strict bounds or for which there are no such components. An example of this notation is $C^{lq(3)}_{2231}$, where it should be noted that there is no corresponding Wilson coefficient in CKM corrections. Working equivalently with 4.29, we obtain the following results for $C^{lequ(1)}_{2211}$:

$$\begin{aligned} (\mathcal{M}_2)_{2211} &= \frac{(\lambda^{1L}_{12})^*(\lambda^{1R}_{12})}{M_1^2} < 10^{-6}, \\ (\mathcal{M}_2)_{2221} &= \frac{(\lambda^{1L}_{22})^*(\lambda^{1R}_{12})}{M_1^2} < 7\cdot 10^{-8}. \end{aligned} \tag{4.30}$$

Working equivalently for $[C^{lequ(3)}]^{(0)}_{prst}$ we find:

$$\begin{aligned}(\mathcal{M}_3)_{2211} =& \frac{(\lambda^{1L}_{12})^*(\lambda^{1R}_{12})}{M_1^2} < 2 \cdot 10^{-5},\\ (\mathcal{M}_3)_{2221} =& \frac{(\lambda^{1L}_{22})^*(\lambda^{1R}_{12})}{M_1^2} < 5 \cdot 10^{-5}.\end{aligned} \tag{4.31}$$

It is worth noting that the same approach used in setting bounds for equation 4.28 can be employed for the case where $p = r = 3$.

Chapter 5

Conclusions

In this thesis, we have examined how the CKM matrix contributes to the search for NP in the SMEFT. The dimension-6 operators of SMEFT can affect the determination of the CKM parameters, and therefore we have limited the number of observables to the minimum possible(3.11). It is important to note that the results from global fits, which combine all available observations in the SM, cannot be directly applied to explore additional flavor constraints related to the CKM matrix. Additionally, we have calculated the NP corrections for these observables in LEFT at the weak scale, using a tree-level matching process. Next, we used these observables to determine the numerical values of the Wolfenstein parameters, $\tilde{W}_j$ (3.47), in order to derive the numerical result of the CKM matrix in the context of SMEFT. Furthermore, we expanded all nine elements of the CKM matrix in terms of linearized 6-dimensional Wilson coefficients, up to order $\mathcal{O}(1/\Lambda^2)$. This expansion was cross-checked with [19]. Additionally, we defined the corrections to the CKM matrix, $\delta\tilde{V}_{ij}$, under the assumption that the PMNS matrix is non-diagonal. In this case, we found an additional contribution, δU_{ij}(3.57), caused by the PMNS matrix, expressed in terms of the linearized Wilson coefficients. The contribution of these Wilson coefficients is presented in Appendix F. Our conclusion regarding the Wilson coefficient that appears in δU_{ij} is that it must generally be small in order to not significantly affect the numerical values of the CKM corrections.

We have also discussed applications concerning tree-level diagrams to see how the CKM matrix in SMEFT affects those processes and how the matrix can be used to place limits on new physics. In addition, we established bounds on some of the Wilson coefficients that appear in the tree-level matching of the Leptoquark model in Ref. [20].

The method for determining CKM parameters in SMEFT analyses in this thesis considers only a subset of possible flavor observables. While our current selection of input observables may be considered valid, it is possible that this may change in the future with advancements in theory or experimental measurements. In cases where the fit includes all the measurements that are most sensitive to the CKM parameters, no additional assumptions are needed for the selection of observables. This approach has been demonstrated in studies of NP by UTfit[11] and CKMfitter[16], but only in simple case of NP scenario. In the full SMEFT case this is not currently possible, since global SMEFT analysis do not account for NP corrections that affect their extraction. Therefore our current work provides an appropriate framework to consistently include such NP effects and the uncertainty of the CKM parameters. Althought a global SMEFT determination of CKM parameters is crucial because it can offer valuable informations about the nature of physics BSM at high energies.

Appendix A

Notation and Conventions

We work in the natural (Planck) units where:

$$\hbar = c = 1, \tag{A.1}$$

where in this system:

$$[length] = [time] = [energy]^{-1} = [mass]^{-1} = GeV^{-1} \tag{A.2}$$

or:

$$[x] = [t] = -1, \tag{A.3}$$
$$[p_\mu] = [\partial_\mu] = 1. \tag{A.4}$$

Therefore we can express every physical quantity in terms of the mass or energy dimension. For example the mass of electron can be written:

$$m_e = 0.511eV = (3.862 \cdot 10^{-11})^{-1} cm^{-1}. \tag{A.5}$$

Our convention for the metric is the following:

$$(g_{\mu\nu}) = (g^{\mu\nu}) = diag(1, -1, -1, -1), \tag{A.6}$$

therefore a massive particle has: $p_\mu p^\mu = p^2 = E^2 - |\boldsymbol{p}|^2 = m^2$. The displacement vector x^μ is "naturally raised", while the derivative operator:

$$\partial_\mu = \frac{\partial}{\partial_\mu} = \left(\frac{\partial}{\partial x^0}, \nabla\right), \tag{A.7}$$

is "naturally lowered". The convention for the 4-momentum follow the Shrödinger wavefunction of single quantum-mechanics particle:

$$p_\mu = i\partial_\mu. \tag{A.8}$$

The gamma matrices satisfy the Dirac algebra:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}. \tag{A.9}$$

We also define:

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]. \tag{A.10}$$

The γ^5 matrix:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{-i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma, \tag{A.11}$$

which anti-commutes with a single γ^μ matrix:

$$\{\gamma^\mu, \gamma^5\} = 0. \tag{A.12}$$

We also use the projection operators:

$$P_L = \frac{1-\gamma^5}{2}, \qquad P_R = \frac{1+\gamma^5}{2}, \tag{A.13}$$

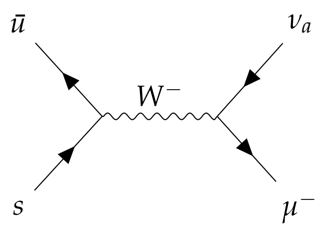
Finally the Feynman's slash notation is:

$$\not{a} = \gamma^\mu a_\mu. \tag{A.14}$$

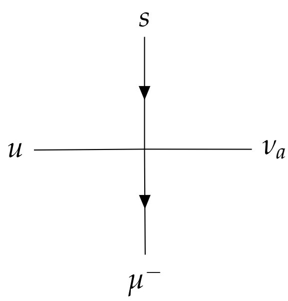
Appendix B

Computation of the Decay Rate for $P_{l2}^{\pm}$

For the computation of the decay rate we will consider the case where: $l^- = K^-$(Equation 3.23), therefore the diagrams that contribute to the process are the following:



(1)

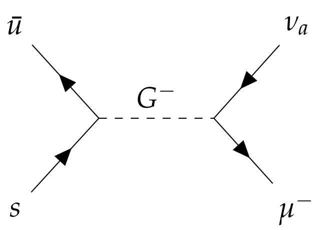


(2) (B.1)

The amplitude for the first diagram can be written (in R_ξ gauge) as:

$$\begin{aligned} iM^{(1)} =& q^\mu f_\pi \left[\frac{i\bar{g}}{\sqrt{2}} V_{12} + \frac{i\bar{g}}{\sqrt{2}} V_{1g_1} C^{\phi q(3)}_{g_1 2} - \frac{i\bar{g}v^2}{2\sqrt{2}} C^{\phi ud}_{12}\right] \times \\ & \left(\frac{-i}{q^2 - M_W^2}\left(\eta_{\mu\nu} - (1-\xi_W)\frac{q_\mu q_\nu}{q^2 - \xi_W M_W^2}\right)\right) \times \\ & \bar{u}(3)\left(\frac{-i\bar{g}}{\sqrt{2}} U_{2a}\gamma^\nu P_L - \frac{i\bar{g}v^2}{\sqrt{2}} U_{g_1 a} C^{\phi l(3)*}_{2g_1}\gamma^\nu P_L\right) v(2) \end{aligned} \tag{B.2}$$

where $u(3)$ and $v(2)$ are the spinors for the muon and the neutrino. We neglected terms of $\sigma_{\mu\nu}$ since they are $\mathcal{O}(\frac{1}{M_W})$. The contribution from the third diagram which contains a goldstone boson is:



(3)

$$\begin{aligned} iM^{(3)} =& q^2 f_\pi \left[\frac{i\sqrt{2}}{v} V_{12} + i\sqrt{2} v V_{1g_1} C^{\phi q(3)}_{g_1 2} - \frac{iv}{\sqrt{2}} C^{\phi ud}_{12} \right] \times \\ & \frac{i}{q^2 - \xi_W M_W^2} \left[\bar{u}(3) \left(-\frac{i\sqrt{2}}{v} U_{2a} m_a P_L + i\sqrt{2} v \not{q} U_{g_1 a} C^{\psi l(3)*}_{2 g_1} \right) \right], \end{aligned} \tag{B.3}$$

where we used the relation:

$$\langle 0 | \bar{u}\gamma^5 s \, | P^+(q) \rangle = -\frac{q^2}{m_u + m_s} f_\pi, \tag{B.4}$$

where m_u, m_s, m_μ are the masses of up ,strange quarks and muon while u and s are the spinors of up and strange quark respectively. If we denote the propagator of W particle as $D_{\mu\nu}$ in R_ξ gauge we can rewrite it as the sum of two terms:

$$D_{\mu\nu} = R_{\mu\nu} + K_{\mu\nu}. \tag{B.5}$$

The first term is independent of ξ_W:

$$R_{\mu\nu} = \frac{-i\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{q^2 - M_W^2}, \tag{B.6}$$

and the second term is:

$$K_{\mu\nu} = \frac{i q_\mu q_\nu}{(q^2 - \xi_W M_W^2) M_W^2}. \tag{B.7}$$

If we add the two amplitudes we have:

$$\begin{aligned} iM^{(1)} + iM^{(3)} =& q^\mu f_\pi \left[\frac{i\bar{g}}{\sqrt{2}} V_{12} + \frac{i\bar{g}}{\sqrt{2}} V_{1g_1} C^{\phi q(3)}_{g_1 2} - \frac{i\bar{g} v^2}{2\sqrt{2}} C^{\phi ud}_{12} \right] \cdot \\ & \frac{-i\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{q^2 - M_W^2} \cdot \\ & \bar{u}(3) \left(\frac{-i\bar{g}}{\sqrt{2}} U_{2a} \gamma^\nu P_L - \frac{i\bar{g} v^2}{\sqrt{2}} U_{g_1 a} C^{\phi l(3)*}_{2 g_1} \gamma^\nu P_L \right) v(2). \end{aligned} \tag{B.8}$$

Therefore we found the amplitude to be independent of ξ_W. In practise q^2 is much smaller than M_W^2 therefore we can safely write in the limit: $q^2 << M_W^2$, that:

$$\frac{-i\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{q^2 - M_W^2} \to \frac{i g_{\mu\nu}}{M_W^2}. \tag{B.9}$$

adding the contribution of the 4-vertex we have the total amplitude of the process:

$$\begin{aligned}
iM^{tot} =& iM^{(1)} + iM^2 + iM^{(3)} = \\
& \frac{iq^\mu f_\pi}{M_W^2}\left(\frac{\bar{g}^2}{2}V_{12}U_{2a} + \frac{\bar{g}^2v^2}{2}V_{1g_1}U_{g_1a}C^{\phi l(3)*}_{2g_1} + \frac{\bar{g}^2v^2}{2}V_{1g_1}U_{2a}C^{\phi q(3)}_{g_12} - \frac{\bar{g}^2v^2}{4}U_{2\alpha}C^{\phi ud*}_{12}\right)\cdot \\
& [\bar{u}(3)\gamma_\mu P_L v(2)] - \\
& - 2iq^\mu f_\pi V_{1g_1}U_{g_2a}C^{lq(3)}_{g_222g_1}[\bar{u}(3)\gamma_\mu P_L v(2)] - \\
& \frac{iq^2 f_\pi}{m_u+m_s}V_{1g_1}U_{g_2a}C^{ledq*}_{g_222g_1}[\bar{u}(3)P_L v(2)] - \\
& + \frac{iq^2}{m_u+m_s}U_{g_1a}C^{lequ(1)*}_{g_1221}[\bar{u}(3)P_L v(2)] - \\
& \frac{iq^2 f_\pi}{4(q\cdot p_2)(m_u+m_s)}U_{g_1a}C^{lequ(3)*}_{g_1221}(q^\mu p_2^\nu - q^\nu p_2^\mu)\cdot[\bar{u}(3)[\gamma_\mu,\gamma_\nu]P_L v(2)],
\end{aligned} \tag{B.10}$$

where we used: $M_W = \frac{\bar{g}v}{2}$ and the relation:

$$\langle 0|\,\bar{u}\sigma^{\mu\nu}\gamma^5 s\,|P^+(q)\rangle = \frac{-i}{2}\cdot\frac{q^2 f_\pi(q^\mu p_2^\nu - q^\nu p_2^\mu)}{(q\cdot p_2)(m_u+m_s)}. \tag{B.11}$$

This result arises as follows: Let $A(q^2)$ be a scalar quantity for which:

$$\langle 0|\,\bar{u}\gamma^\mu\gamma^\nu\gamma^5 s\,|P^+(q)\rangle = A(q^2)q^\mu p_2^\nu. \tag{B.12}$$

This is the only possible choice in order for equation B.11 not to be zeroed. Using the identities:

$$q = p_1 + p_4, \qquad \bar{u}\not{p}_1 = \bar{u}m_u, \qquad \not{p}_4 s = m_s s, \tag{B.13}$$

where p_1, p_4 are the 4-momentum of up quark and s quark, respectively, we found that:

$$A(q^2) = \frac{-q^2 f_\pi}{(q\cdot p_2)(m_u+m_s)}, \tag{B.14}$$

from which equation B.11 follows.

Once we found the amplitude we can proceed to calculate the decay rate by working at the C.M.(center of mass) as follows:

$$\boxed{\Gamma = \frac{|p_2|}{8\pi m_K^2} < |M^{tot}|^2 >}, \tag{B.15}$$

where: $|p_2| = \frac{m_K^2 - m_\mu^2}{2m_K}$ and $< |M^{tot}|^2 >= \frac{1}{4}\sum_{spins}|M^{tot}|^2$. Using the following equations for the 4-momentum:

$$q = p_2 + p_3, \qquad q\cdot p_2 = p_2\cdot p_3, \qquad q\cdot p_3 = m_{\mu^-}^2 + p_2\cdot p_3, \tag{B.16}$$

we find the decay rate to be:

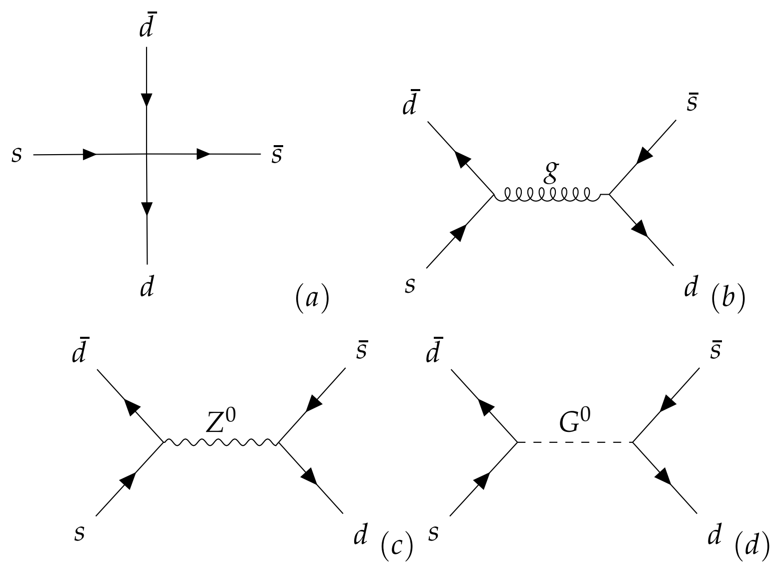
$$\begin{aligned}\Gamma(K^- \to \mu^- \nu_a) =& \frac{f_K^2 m_\mu^2 m_K}{16\pi\tilde{v}^4}\left(1-\frac{m_\mu^2}{m_K^2}\right)^2 \sum_a |U_{2\nu_a}|^2 |K_{12}|^2 \\ & (1+\frac{4\delta\tilde{v}}{v} + 2Re(-1-\frac{v^2}{2V_{12}^*}(-\frac{2}{v^2}V_{12}^* + 2V_{1g_1}^*\sum_a \frac{U_{g_2a}^* U_{2a}}{\sum_a |U_{2\nu_a}|^2} C_{g_2 2 2 g_1}^{lq(3)} - \\ & 2V_{12}^* \sum_a \frac{U_{g_1a}^* U_{2a} C_{g_1 2}^{\phi l(3)}}{\sum_a |U_{2\nu_a}|^2} - \frac{2V_{1g_1} C_{g_1 2}^{\phi q(3)*}}{\sum_a |U_{2\nu_a}|^2} + C_{12}^{\phi ud*})) + \\ & \frac{2m_K^2}{(m_u+m_s)m_\mu} Re(\frac{v^2}{2V_{13}^* \sum_a |U_{2\nu_a}|^2}(\sum_a U_{g_1a}^* U_{2a} C_{g_1 221}^{lequ(1)} - \\ & V_{1g_1}^* \sum_a U_{g_1a}^* U_{2a} C_{g_2 2 2 g_1}^{ledq} - \sum_a U_{g_1a}^* U_{2a} C_{g_1 221}^{lequ(3)})))\end{aligned} \tag{B.17}$$

The corresponding decay rate for the case where: $l^- = \pi^-$ is a completely propotional expression, with the substitution: $2 \to 1$ in the Wilson Coefficients.

Appendix C

Mass Differences

If we consider the mass differences of neutral B_q mesons ($q = \{d, s\}$) in SMEFT, there are the following Feynman diagrams that contribute (in tree-level) to the transition:



(C.1)

The diagram (c) vanishes due to the GIM (or Glashow-Iliopoulos-Maiani) mechanism, as described in ref. [32]. The diagrams (d) and (b) do not contribute to the process. For example, if we consider the amplitude of diagram (d):

$$\begin{aligned} iM_d =& i\bar{d}(p_1)\left(\frac{1}{v}\delta_{sd}m_d\gamma^5 - \frac{v}{4}\delta_{sd}C^{\phi D}m_d\gamma^5 - v\not{q}P_L C_{ds}^{\phi q(1)}\right)s(p_2) \\ &\times \frac{1}{q^2 - \xi_W M_W^2}\bar{s}(p_3)\left(\frac{1}{v}\delta_{sd}m_s\gamma^5 - \frac{v}{4}\delta_{sd}C^{\phi D}m_s\gamma^5 - v\not{q}P_L C_{sd}^{\phi q(1)}\right)d(p_4). \end{aligned} \tag{C.2}$$

The δ_{sd} stands for delta kronecker, therefore is zero. It is straightforward to see that the amplitude is of the order $iM_d \sim \mathcal{O}(\frac{1}{\Lambda^4})$ and it can be dropped . Same result holds for diagrams (b) therefore:

$$\boxed{iM_d = iM_b = iM_c = 0}. \tag{C.3}$$

The diagram (a) gives the contribution:

$$
\begin{aligned}
iM_b = & -2iC^{qq(1)}_{dbdb}[\bar{d}(p_1)(\gamma^\mu P_L)b(p_2)][\bar{d}(p_3)(\gamma_\mu P_L)b(p_4)]- \\
& 2iC^{qq(3)}_{dbdb}[\bar{d}(p_1)\gamma^\mu P_L b(p_2)][\bar{d}(p_3)\gamma_\mu P_L b(p_4)]- \\
& 2iC^{dd}_{dbdb}[\bar{d}(p_1)(\gamma^\mu P_R)b(p_2)][\bar{d}(p_3))\gamma_\mu P_R b(p_4)]- \\
& 2iC^{qd(1)}_{dbdb}[\bar{d}(p_1)(\gamma^\mu P_L)b(p_2)][\bar{d}(p_3)(\gamma_\mu P_L)b(p_4)]+ \\
& \frac{4i}{3}C^{qd(8)}_{dbdb}[\bar{d}(p_1)(\gamma^\mu P_L)b(p_2)][\bar{d}(p_3)(\gamma_\mu P_R)b(p_4)]
\end{aligned}
\tag{C.4}
$$

Appendix D

Matching the LEFT to the SMEFT

The terms in the LEFT Lagrangian that contribute to semileptonic charged-current transitions at the EW scale are:

$$\begin{aligned}\mathcal{L} =&[L^{VLL}_{\nu edu}(\mu_{EW})]_{iixk}(\bar{\nu}_{L,i}\gamma^\mu e_{L,i})(\bar{d}_{L,x}\gamma^\mu u_{L,k})+\\ &[L^{SRR}_{\nu edu}(\mu_{EW})]_{iixk}(\bar{\nu}_{L,i}\gamma^\mu e_{R,x})(\bar{d}_{L,i}\gamma^\mu u_{R,j}) + [L^{VLR}_{\nu edu}(\mu_{EW})]_{iixk}(\bar{\nu}_{L,i}\gamma^\mu e_{L,i})(\bar{d}_{R,x}\gamma^\mu u_{R,k})+\\ &[L^{TRR}_{\nu edu}(\mu_{EW})]_{iixk}(\bar{\nu}_{L,i}\gamma^\mu T^a e_{R,x})(\bar{d}_{L,i}\gamma^\mu T^a u_{R,k}) + [L^{SRL}_{\nu edu}(\mu_{EW})]_{iixk}(\bar{\nu}_{L,i} e_{R,i})(\bar{d}_{R,x} u_{L,k})\end{aligned} \tag{D.1}$$

We consider the matching conditions for the LEFT operators with the SMEFT operators in the Mass basis as follows:

$$\begin{aligned}&[L^{VLL}_{\nu edu}(\mu_{EW})]_{iaxk} = -\frac{2}{v^2}V^*_{kx}U_{ia} + 2V^*_{kg_1}U^*_{g_2a}C^{lq(3)}_{g_2ixg_1} - 2V^*_{kx}U^*_{g_1a}C^{\phi l(3)}_{g_1i} - 2V_{kg_1}C^{\phi q(3)*}_{g_1x},\\ &[L^{SRR}_{\nu edu}(\mu_{EW})]_{iaxk} = U^*_{g_1a}C^{lequ(1)}_{g_1ixk}, \qquad [L^{VLR}_{\nu edu}(\mu_{EW})]_{iaxk} = -C^{\phi ud*}_{kx}\\ &[L^{TRR}_{\nu edu}]_{iaxk} = U^*_{g_1a}C^{lequ(3)}_{g_1ixk}, \qquad [L^{SRL}_{\nu edu}]_{iaxk} = V^*_{1g_1}U^*_{g_2a}C^{ledq}_{g_2ixg_1}.\end{aligned} \tag{D.2}$$

These matching conditions refer to the coefficients of the LEFT and SMEFT operators, in the mass basis. We denote: $x = \{d, s, b\}$, $i = \{e, \mu, \tau\}$, $a = \{\nu_e, \nu_\mu, \nu_\tau\}$, and $k = \{u, c\}$.

We continue with the matching conditions for the LEFT operators of $\bar{B}_q - B_q$ mixing. The tree-level matching conditions to the SMEFT are the following:

$$\begin{aligned}&[L^{VLL}_{dd}(\mu_{EW})]_{dbdb} = [L^{VLL}_{dd}]^{SM}_{dbdb} + C^{qq(1)}_{dbdb} + C^{qq(3)}_{dbdb},\\ &[L^{VRR}_{dd}(\mu_{EW})] = C^{dd}_{dbdb}, \quad [L^{V1LR}_{dd}(\mu_{EW})]_{dbdb} = C^{qd(1)}_{dbdb},\\ &[L^{V8LR}_{dd}(\mu_{EW})] = C^{qd(8)}_{dbdb},\\ &[L^{S1RR}_{dd}(\mu_{EW})]_{dbdb} = [L^{S1RR}_{dd}(\mu_{EW})]_{bdbd} = [L^{S8RR}_{dd}(\mu_{EW})]_{dbdb} = [L^{S8RR}_{dd}(\mu_{EW})]_{dbdb} = 0.\end{aligned} \tag{D.3}$$

The same relation holds for $\overline{B}s - B_s$, with the replacement of d with s. The term $\left[L^{VLL}_{dd}(\mu_{\rm EW})\right]^{\rm SM}_{dbdb}$ corresponds to the contribution at one-loop level in the SM-limit. This term is equal to:

$$C^{(q)}_{1,SM} = [L^{VLL}_{dd}(\mu_{EW})]^{SM}_{dbdb} = \frac{-M^2_w}{32\pi^2 v^4}\left(V_{tq}V^*_{tb}\right)^2 S_1(\mu_{EW}), \tag{D.4}$$

where the function $S_1(\mu_{EW}) \approx 2.3124$, and contains the NLO(two loop) QCD corrections to the SM matching [12] at an energy scale $\mu_{EW} = M_Z$. We note that we keep terms up to order $\mathcal{O}(\Lambda^{-2})$, therefore the latter LEFT operators of equation D.3 can be dropped. Our choice of basis in order to calculate the Mass-Differences is the SUSY basis for $\Delta F = 2$. This basis consists of the following operators:

$$\begin{aligned} Q_1 &= (\bar{d}_L^\alpha \gamma^\mu b_L^\alpha)(\bar{d}_L^\beta \gamma_\mu b_L^\beta), \\ Q_2 &= (\bar{d}_R^\alpha b_L^\alpha)(\bar{d}_R^\beta b_L^\beta), \\ Q_3 &= (\bar{d}_R^\alpha b_L^\beta)(\bar{d}_R^\beta b_L^\alpha), \\ Q_4 &= (\bar{d}_R^\alpha b_L^\alpha)(\bar{d}_L^\beta b_R^\beta) \\ Q_5 &= (\bar{d}_R^\alpha b_L^\beta)(\bar{d}_L^\beta b_R^\alpha). \end{aligned} \tag{D.5}$$

Additionally, we have the $\tilde{Q}_{1,2,3}$ operators, which are derived from the $Q_{1,2,3}$ operators by substituting L with R and R with L. We notice that α, β are colour indices. We notice that the parity-even parts of the operators Q_i and $\tilde{Q}_i$ are identical. Therefore due to parity conservation, for the study of $\bar{B}_q - B_q$ we will consider only the hadronic matrix elements: $\langle \bar{B}_q | \mathcal{Q}_i | B_q \rangle$, where $i = \{1, 2, 3, 4, 5\}$. The matrix elements are defined from the relations:

$$\begin{aligned} \langle \bar{B}_q | \mathcal{Q}_1 | B_q \rangle &= \mathcal{C}_1 B_1^q(\mu_{EW}) m_{B_q}^2 f_{B_q}^2 \\ \langle \bar{B}_q | \mathcal{Q}_i | B_q \rangle &= \mathcal{C}_i B_i^{(q)}(\mu_{EW}) \left(\frac{m_{B_q}^2 f_{B_q}}{m_b + m_q} \right)^2, \qquad i = \{2, 3, 4, 5\}, \end{aligned} \tag{D.6}$$

where $\mathcal{C}_i = \{8/3, -5/3, 1/3, 2, 2/3\}$ and $B_i^{(q)}$ are the bag-parameters. The relation between the LEFT operators and operators in SUSY basis, in ref.[24], are given from the following conditions:

$$\begin{aligned} &[\mathcal{O}_{dd}^{VLL}]_{dbdb} = Q_1 \qquad [\mathcal{O}_{dd}^{V1LR}]_{dbdb} = -2Q_5 \\ &[\mathcal{O}_{dd}^{S1RR}]_{dbdb} = \tilde{Q}_2 \qquad [\mathcal{O}_{dd}^{S8RR}] = -\frac{\tilde{Q}_2}{2N_c} + \frac{\tilde{Q}_3}{2} \\ &[\mathcal{O}_{dd}^{VRR}] = \tilde{Q}_1 \qquad [\mathcal{O}_{dd}^{V8LR}]_{dbdb} = -Q_4 + \frac{Q_5}{N_c} \\ &[\mathcal{O}_{dd}^{S1RR}]^\dagger_{bdbd} = Q_2 \qquad [\mathcal{O}_{dd}^{S8RR}]^\dagger_{bdbd} = \frac{-Q_2}{2N_c} + \frac{Q_3}{2}, \end{aligned} \tag{D.7}$$

where we used Fierz's identities and the relation:

$$T^A_{\alpha\gamma} T^A_{\delta\kappa} = \frac{\delta_{\alpha\kappa}\delta_{\gamma\delta}}{2} - \frac{\delta_{\alpha\gamma}\delta_{\delta\kappa}}{2N_c} \tag{D.8}$$

for the Gellman matrices.

Appendix E

Corrections in terms of SMEFT's Wilson Coefficients

In the following work, we will expand the quantities in Equation 3.28 in terms of 6-dimensional Wilson coefficients, as follows:

$$\begin{aligned}
\epsilon_A^{\mu ud*} =& \frac{v^2}{2V_{11}^*}(2V_{1g_1}^* \sum_a U_{g_2a}^* U_{2a} C_{g_2 21 g_1}^{lq(3)} - 2V_{11}^* \sum_a U_{g_1a}^* U_{2a} C_{g_1 2}^{\phi l(3)} - 2V_{1g_1}^* C_{g_1 1}^{\phi q(3)*} + C_{11}^{\phi ud*}) = \\
& \frac{v^2}{2V_{11}^*}(2V_{1g_1}^* \sum_a U_{2a}^* U_{2a} C_{221 g_1}^{lq(3)} - 2V_{11}^* \sum_a U_{2a}^* U_{2a} C_{22}^{\phi l(3)} - 2V_{1g_1}^* C_{g_1 1}^{\phi q(3)*} + C_{11}^{\phi ud*}) + \\
& \frac{v^2}{2V_{11}^*}(2V_{1g_1}^* \sum_{a, g_2 \neq 2} U_{g_2a}^* U_{2a} C_{g_2 21 g_1}^{lq(3)} - 2V_{11}^* \sum_{a, g_1 \neq 2} U_{g_1a}^* U_{2a} C_{g_1 2}^{\phi l(3)}) = \\
& \frac{v^2}{2V_{11}^*}(2V_{1g_1}^* C_{221 g_1}^{lq(3)} - 2V_{11}^* C_{22}^{\phi l(3)} - 2V_{1g_1}^* C_{g_1 1}^{\phi q(3)*} + C_{11}^{\phi ud*}) + \\
& \frac{v^2}{2V_{11}^*}(2V_{1g_1}^* \sum_{a, g_2 \neq 2} U_{g_2a}^* U_{2a} C_{g_2 21 g_1}^{lq(3)} - 2V_{11}^* \sum_{a, g_1 \neq 2} U_{g_1a}^* U_{2a} C_{g_1 2}^{\phi l(3)}),
\end{aligned} \tag{E.1}$$

where we used that $\sum_a U_{2a}^* U_{2a} \approx 1$(for simplicity, we will ignore the multiplication factors from the three-loop plus one-loop QED running at the moment, although they are needed to calculate the CKM corrections). Similarly, for $\epsilon_P^{\mu ud}$:

$$\begin{aligned}
\epsilon_P^{\mu ud*} =& \frac{v^2}{2V_{11}^*}(C_{2211}^{lequ(1)} - V_{1g_1}^* C_{221 g_1}^{ledq} - C_{2211}^{lequ(3)}) + \\
& \frac{v^2}{2V_{11}^*}(\sum_{a, g_1 \neq 2} U_{2a} U_{g_1a}^* C_{g_1 211}^{lequ(1)} - \sum_{a, g_2 \neq 2} U_{g_2a}^* U_{2a} V_{1g_1}^* C_{g_2 21 g_1}^{ledq} - \sum_{a, g_1 \neq 2} U_{g_1a}^* U_{2a} C_{g_1 211}^{lequ(3)}).
\end{aligned} \tag{E.2}$$

Similar relations hold for the set of parameters $\{\epsilon_A^{\mu us}, \epsilon_P^{\mu us}, \epsilon_A^{\tau ub}, \epsilon_P^{\tau ub}\}$. Continuing, we can write the corrections $\Delta_{K/\pi}, \Delta B_{\tau 2}$ as:

$$\begin{aligned}
\Delta_{K/\pi} =& F(V) + G(U, V), \\
\Delta B_{\tau 2} =& H(V) + K(U, V).
\end{aligned} \tag{E.3}$$

Here, F and H are independent of the U matrix, while G and K depend on the U matrix. The analytical expressions for F, G, H, and K are as follows:

$$
\begin{aligned}
F\,(V) =& 2Re(\frac{v^2}{2V_{12}^*}(2V_{1g_1}^*C_{222g_1}^{lq(3)} - 2V_{12}^*C_{22}^{\phi l(3)} - 2V_{1g_1}^*C_{g_12}^{\phi q(3)*} + \\
& C_{12}^{\phi ud*}) - \frac{v^2}{2V_{11}^*}(2V_{1g_1}^*C_{221g_1}^{lq(3)} - 2V_{11}^*C_{22}^{\phi l(3)} - 2V_{1g_1}^*C_{g_11}^{\phi q(3)*} + C_{11}^{\phi ud*}) \\
& - \frac{2}{m_\mu}(\frac{m_K^2}{(m_u+m_s)}Re(\frac{v^2}{2V_{12}^*}(C_{2221}^{lequ(1)} - \\
& V_{1g_1}^*C_{222g_1}^{ledq} - C_{2221}^{lequ(3)}) - \frac{m_\pi^2}{(m_u+m_d)}Re(\frac{v^2}{2V_{11}^*}(C_{2211}^{lequ(1)} - V_{1g_1}^*C_{221g_1}^{ledq} - C_{2211}^{lequ(3)}))), \\
G(U,V) =& 2Re(\frac{v^2}{2V_{12}^*}(2V_{1g_1}^*\sum_{a,g_2\neq 2}U_{g_2a}^*U_{2a}C_{g_222g_1}^{lq(3)} - 2V_{12}^*\sum_{a,g_1\neq 2}U_{g_1a}^*U_{2a}C_{g_12}^{\phi l(3)}) - \\
& \frac{v^2}{2V_{11}^*}(2V_{1g_1}^*\sum_{a,g_2\neq 2}U_{g_2a}^*U_{2a}C_{g_221g_1}^{lq(3)} - 2V_{11}^*\sum_{a,g_1\neq 2}U_{g_1a}^*U_{2a}C_{g_12}^{\phi l(3)})) - \\
& \frac{2}{m_\mu}(\frac{m_K^2}{(m_u+m_s)}Re(\frac{v^2}{2V_{12}^*}(\sum_{a,g_1\neq 2}U_{2a}U_{g_1a}^*C_{g_1221}^{lequ(1)} - \sum_{a,g_2\neq 2}U_{g_2a}^*U_{2a}V_{1g_1}^*C_{g_222g_1}^{ledq} - \\
& \sum_{a,g_1\neq 2}U_{g_1a}^*U_{2a}C_{g_1221}^{lequ(3)})) - \frac{m_\pi^2}{(m_u+m_d)}Re(\frac{v^2}{2V_{11}^*}(\sum_{a,g_1\neq 2}U_{2a}U_{g_1a}^*C_{g_1221}^{lequ(1)} - \\
& \sum_{a,g_2\neq 2}U_{g_2a}^*U_{2a}V_{1g_1}^*C_{g_221g_1}^{ledq} - \sum_{a,g_1\neq 2}U_{g_1a}^*U_{2a}C_{g_1211}^{lequ(3)}))), \\
H(V) =& 2Re(\frac{v^2}{2V_{13}^*}(2V_{1g_1}^*C_{333g_1}^{lq(3)} - 2V_{13}^*C_{33}^{\phi l(3)} - 2V_{1g_1}^*C_{g_12}^{\phi q(3)*} + \\
& C_{13}^{\phi ud*}) - 2\frac{m_B^2}{(m_\tau(m_u+m_b))}Re(\frac{v^2}{2V_{13}^*}(C_{3331}^{lequ(1)} - V_{1g_1}^*C_{333g_1}^{ledq} - C_{3331}^{lequ(3)})), \\
K(U,V) =& 2Re(\frac{v^2}{2V_{13}^*}(2V_{1g_1}^*\sum_{a,g_2\neq 3}U_{g_2a}^*U_{3a}C_{g_233g_1}^{lq(3)} - 2V_{11}^*\sum_{a,g_1\neq 3}U_{g_1a}^*U_{3a}C_{g_13}^{\phi l(3)})) - \\
& \frac{2m_B^2}{(m_u+m_b)m_\tau}Re(\frac{v^2}{2V_{13}^*}(\sum_{a,g_1\neq 3}U_{3a}U_{g_1a}^*C_{g_1331}^{lequ(1)} - \\
& \sum_{a,g_2\neq 3}U_{g_2a}^*U_{3a}V_{1g_1}^*C_{g_233g_1}^{ledq} - \sum_{a,g_1\neq 3}U_{g_1a}^*U_{3a}C_{g_1331}^{lequ(3)})).
\end{aligned}
\tag{E.4}
$$

We can see that $F(V)$ and $H(V)$ correspond to the contribution, in the corrections of CKM, when we set $U_{ij} = \delta_{ij}$, while $K(U,V)$ and $G(U,V)$ correspond to the contribution when the PMNS matrix is non-diagonal.

Appendix F

Numerical Results

F.1 Corrections of CKM with $U_{ij} = \delta_{ij}$

$$
\begin{aligned}
\delta V_{cb} =& 6.263813592Re(C^{lq(3)}_{2221}) + 0.326538864Re(C^{lq(3)}_{2212}) + Re(C^{lq(3)}_{2213}(0.0027385 - 0.0055059i)) - \\
& 1.4116721472Re(C^{\phi q(3)*}_{11}) - 0.326538864Re(C^{\phi q(3)*}_{21}) - Re(C^{\phi q(3)*}_{31}(0.0027385 - 0.0055059i)) - \\
& 1.4116721472 * Re(C^{\phi l(3)}_{21}) + 0.7210960848Re(C^{\phi ud*}_{11}) - 127.2509149824Re(C^{lequ(1)}_{2221}) + \\
& 123.978017448Re(C^{ledq}_{2221}) + 28.6785323712Re(C^{ledq}_{2222}) + 1.4116721472Re(C^{lq(3)}_{2222}) + \\
& Re(C^{ledq}_{2223}(0.2405038 + 0.4835451i)) + 1.7653283856Re(C^{lequ(3)}_{2222}) + 31.373106624Re(C^{lequ(1)}_{2211}) - \\
& 30.5661946032Re(C^{ledq}_{2211}) - 7.0705525968Re(C^{ledq}_{2212}) - \\
& Re(C^{ledq}_{2213}(0.0592868 - 0.119208i)) - 0.7529540976Re(C^{lequ(3)}_{2212}) + \\
& Re(C^{lq(3)}_{2223}(0.0126417 - 0.0237955i)) - 6.1027010928Re(C^{\phi q(3)*}_{12}) - \\
& 1.4116721472Re(C^{\phi q(3)*}_{22}) - Re(C^{\phi q(3)*}_{32}(0.0118343 - 0.0237955i)) - \\
& 1.4116721472Re(C^{\phi l(3)}_{22}) + 3.1173215472Re(C^{\phi ud*}_{12}) + 1.4489397936Re(C^{lq(3)}_{2211}) - \\
& 0.0002031Re(C^{lq(3)}_{3331}(-12511840 + 25156081i)) + \\
& 0.0002031Re(C^{ledq}_{3331}(33699787 - 67756187i)) + \\
& 0.0002031Re(C^{ledq}_{3332}(7795419 - 15673329i)) + 66.9506964Re(C^{ledq}_{3333}) + \\
& 0.0002031Re(C^{lequ(3)}_{3331}(3458942 - 69544881i)) + \\
& 0.0002031Re(C^{lq(3)}_{3332}(2894233 - 5819093i)) + \\
& 24.8570028Re(C^{lq(3)}_{3333}) - 0.0002031Re(C^{\phi q(3)*}_{13} * (12511840 - 25156081i)) - \\
& 0.0002031Re(C^{\phi q(3)*}_{23}(2894233 - 5819093i)) - \\
& 24.8570028Re(C^{\phi q(3)*}_{33}) - 24.8570028Re(C^{\phi l(3)}_{33}) + \\
& 0.0002031Re(C^{\phi ud*}_{13}(6391172 - 12849975i)) - \\
& 0.0002031Re(C^{lequ(1)}_{3331}(34589427 - 69544881i)) + \\
& 0.0008482Re(C^{qd(1)}_{1313}(1528461088022 + 1934061094379i)) + \\
& 0.0008482Re(C^{qd(8)}_{1313}(1622594546434 + \\
& 2053174273656i)) + 0.020418Re(C^{qd(1)}_{2323}(104200229183 - 4179756811i)) + \\
& 0.020418Re(C^{qd(8)}_{2323}(104618244701 - 4196524559i)) + \\
& 0.0008482Re((-574033456415 - 726361818188i)(C^{qq(1)}_{1313} + C^{qq(3)}_{1313} + C^{dd}_{1313})) + \\
& 0.020418Re((-38870709427 + 1559210702i)(C^{qq(1)}_{2323} + C^{qq(3)}_{2323} + C^{dd}_{2323})) + 1276.45\frac{\delta\tilde{v}}{v},
\end{aligned}
\tag{F.1}
$$

$$
\begin{aligned}
\delta V_{td} =&(0.0038505 - 0.0018187i)Re[C^{qd(1)}_{1313}(1528461088022 + 1934061094379i)+\\
&C^{qd(8)}_{1313}(1622594546434 + 2053174273656i)+\\
&(-574033456415 - 726361818188i)(C^{qq(1)}_{1313} + C^{qq(3)}_{1313} + C^{dd}_{1313})]+\\
&(0.000903 + 0.00184i)Re[(C^{qd(1)}_{2323}(104200229183 - 4179756811i)+\\
&C^{qd(8)}_{2323}(104618244701 - 4196524559i)+\\
&(-38870709427 + 1559210702i)(C^{qq(1)}_{2323} + C^{qq(3)}_{2323} + C^{dd}_{2323})]-\\
&(0.000927 + 0.001877i)Re[C^{lq(3)}_{3331}(-12511840 + 25156081i)-\\
&C^{ledq}_{3331}(33699787 - 67756187i) - C^{ledq}_{3332}(7795419 - 15673329i)-\\
&329644C^{ledq}_{3333} - C^{lequ(3)}_{3331}(3458942 - 69544881i)-\\
&C^{lq(3)}_{3332}(2894233 - 5819093i) - 122388C^{lq(3)}_{3333}+\\
&C^{\phi q(3)*}_{13}(12511840 - 25156081i) + C^{\phi q(3)*}_{23}(2894233 - 5819093i)+\\
&122388 * C^{\phi q(3)*}_{33} + 122388C^{\phi l(3)}_{33}-\\
&C^{\phi ud*}_{13}(6391172 - 12849975i) + C^{lequ(1)}_{3331}(34589427 - 69544881i)]+\\
&(0.0009183 + 0.001888i)Re[543055C^{lq(3)}_{2221} + 28310C^{lq(3)}_{2212}+\\
&C^{lq(3)}_{2213}(237.42 - 477.35i)-\\
&122388C^{\phi q(3)*}_{11} - 28310C^{\phi q(3)*}_{21} - C^{\phi q(3)*}_{31}(237.42 - 477.35i)-\\
&122388C^{\phi l(3)}_{21}+\\
&62517C^{\phi ud*}_{11} - 11032296C^{lequ(1)}_{2221}+\\
&10748545C^{ledq}_{2221} + 2486348C^{ledq}_{2222} + 122388C^{lq(3)}_{2222}+\\
&C^{ledq}_{2223}(20851 + 41922i) + 153049C^{lequ(3)}_{2222}+\\
&2719960C^{lequ(1)}_{2211} - 2650003C^{ledq}_{2211} - 612997C^{ledq}_{2212}-\\
&C^{ledq}_{2213}(5140 - 10335i) - 65279C^{lequ(3)}_{2212}+\\
&C^{lq(3)}_{2223}(1096 - 2063i) - 529087C^{\phi q(3)*}_{12} - 122388 * C^{\phi q(3)*}_{22}-\\
&C^{\phi q(3)*}_{32}(1026 - 2063i) - 122388C^{\phi l(3)}_{22} + 270263C^{\phi ud*}_{12} + 125619C^{lq(3)}_{2211}]+\\
&(231.94 - 112.52i)\frac{\delta\tilde{v}}{v},
\end{aligned}
\tag{F.2}
$$

$$
\begin{aligned}
\delta V_{tb} =& -0.263428494022234 Re(C^{lq(3)}_{2221}) - 0.0137327907224304 Re(C^{lq(3)}_{2212}) - \\
& 0.0593687315767154 Re(C^{\phi q(3)*}_{11}) + 0.0137327907224304 Re(C^{\phi q(3)*}_{21}) + \\
& 0.0593687315767154 Re(C^{\phi l(3)}_{21}) - 0.0303261348496708 Re(C^{\phi ud*}_{11}) + \\
& 5.35161469996136 Re(C^{lequ(1)}_{2221}) - 5.21397100161164 Re(C^{ledq}_{2221}) - \\
& 1.20609313836571 Re(C^{ledq}_{2222}) - 0.0593687315767154 Re(C^{lq(3)}_{2222}) - \\
& 0.0742419599885994 Re(C^{lequ(3)}_{2222}) - 1.31941509902444 Re(C^{lequ(1)}_{2211}) + \\
& 1.28547992274153 Re(C^{ledq}_{2211}) + 0.297356394011927 Re(C^{ledq}_{2212}) + \\
& 0.0316659429731379 Re(C^{lequ(3)}_{2212}) + \\
& 0.256652809783064 Re(C^{\phi q(3)*}_{12}) + 0.0593687315767154 Re(C^{\phi q(3)*}_{22}) + \\
& 0.0593687315767154 Re(C^{\phi l(3)}_{22}) - 0.131100855493331 Re(C^{\phi ud*}_{12}) - \\
& 0.0609360451346162 Re(C^{lq(3)}_{2211}) + \\
& Re(C^{lq(3)}_{3331}(-106.9044858 + 214.9402409i)) - \\
& Re(C^{ledq}_{3331}(287.9399353 - 578.9268669i)) - \\
& Re(C^{ledq}_{3332}(66.6061315 - 133.9170879i)) - \\
& 2.81656593400983 Re(C^{ledq}_{3333}) - Re(C^{lequ(3)}_{3331} * (29.5541196 - 594.2099438i)) - \\
& Re(C^{lq(3)}_{3332}(24.7290959 - 49.7198769i)) - 1.04571559479801 Re(C^{lq(3)}_{3333}) + \\
& Re(C^{\phi q(3)*}_{13}(106.9044858 - 214.9402409i)) + \\
& Re(C^{\phi q(3)*}_{23}(24.7290959 - 49.7198769i)) + \\
& 1.04571559479801 Re(C^{\phi q(3)*}_{33}) + 1.04571559479801 Re(C^{\phi l(3)}_{33}) - \\
& Re((54.6078719 - 109.7936011i)) + \\
& Re(C^{lequ(1)}_{3331}(295.5412559 - 594.2099438i)) - \\
& 3.56723094444038 \times 10^{-5} Re[C^{qd(1)}_{1313}(1528461088022 + 1934061094379i) - \\
& C^{qd(8)}_{1313}(1622594546434 + 2053174273656i)] - \\
& 0.000858698706386247 * Re[C^{qd(1)}_{2323} * (104200229183 - 4179756811i) - \\
& C^{qd(8)}_{2323}(104618244701 - 4196524559i)] - \\
& Re[(-20477099.088677 - 25911003.547002i)(C^{qq(1)}_{1313} + C^{qq(3)}_{1313} + C^{dd}_{1313})] - \\
& Re[(-33378227.901281 + 1338892.212791i)(C^{qq(1)}_{2323} + C^{qq(3)}_{2323} + C^{dd}_{2323})] - \\
& 53.66 \frac{\delta \tilde{v}}{v},
\end{aligned}
\tag{F.3}
$$

$$
\begin{aligned}
\delta V_{cs} = &- Re[13428.3842775327C^{lq(3)}_{2221} - 700.035095702922C^{lq(3)}_{2212} - \\
&0.0247274848358503C^{lq(3)}_{2213}(237.42 - 477.35i)) + 3026.34741409005C^{\phi q(3)*}_{11} + \\
&700.035095702922C^{\phi q(3)*}_{21} + 0.0247274848358503C^{\phi q(3)*}_{31}(237.42 - 477.35i) + \\
&3026.34741409005C^{\phi l(3)}_{21} - 1545.88816948285C^{\phi ud*}_{11} + \\
&272800.932044612C^{lequ(1)}_{2221} - 265784.483494955C^{ledq}_{2221} - \\
&61481.1324666467C^{ledq}_{2222} - 3026.34741409005C^{lq(3)}_{2222} - \\
&0.0247274848358503C^{ledq}_{2223}(20851 + 41922i) - 3784.51682664205C^{lequ(3)}_{2222} - \\
&67257.7696541194C^{lequ(1)}_{2211} + 65527.9089974578C^{ledq}_{2211} + \\
&15157.8740219217C^{ledq}_{2212} + 0.0247274848358503C^{ledq}_{2213}(5140 - 10335i) + \\
&1614.18548259947C^{lequ(3)}_{2212} - 0.0247274848358503C^{lq(3)}_{2223}(1096 - 2063i) + \\
&13082.9907693455C^{\phi q(3)*}_{12} + 3026.34741409005C^{\phi q(3)*}_{22} + \\
&0.0247274848358503C^{\phi q(3)*}_{32}(1026 - 2063i) + 3026.34741409005C^{\phi l(3)}_{22} - \\
&6682.92423419141C^{\phi ud*}_{12} - 3106.24191759468C^{lq(3)}_{2211} + \\
&8.54426573518653 \times 10^{-6}C^{lq(3)}_{3331}(-12511840 + 25156081i) - \\
&8.54426573518653 \times 10^{-6}C^{ledq}_{3331}(33699787 - 67756187i) - \\
&8.54426573518653 \times 10^{-6}C^{ledq}_{3332}(7795419 - 15673329i) - \\
&2.81656593400983C^{ledq}_{3333} - \\
&8.54426573518653 \times 10^{-6}C^{lequ(3)}_{3331}(3458942 - 69544881 * I) - \\
&8.54426573518653 \times 10^{-6}C^{lq(3)}_{3332}(2894233 - 5819093i) - \\
&1.04571559479801C^{lq(3)}_{3333} + \\
&8.54426573518653 \times 10^{-6}C^{\phi q(3)*}_{13}(12511840 - 25156081i) + \\
&8.54426573518653 \times 10^{-6}C^{\phi q(3)*}_{23}(2894233 - 5819093i) + \\
&1.04571559479801C^{\phi q(3)*}_{33} + 1.04571559479801C^{\phi l(3)}_{22} - \\
&8.54426573518653 \times 10^{-6}C^{\phi ud*}_{13}(6391172 - 12849975i) + \\
&8.54426573518653 \times 10^{-6}C^{lequ(1)}_{3331}(34589427 - 69544881i) - \\
&3.56723094444038 \times 10^{-5}C^{qd(1)}_{1313}(1528461088022 + 1934061094379i) - \\
&3.56723094444038 \times 10^{-5}C^{qd(8)}_{1313}(1622594546434 + 2053174273656i) - \\
&0.000858698706386247C^{qd(1)}_{2323}(104200229183 - 4179756811i) - \\
&0.000858698706386247C^{qd(8)}_{2323}(104618244701 - 4196524559i) - \\
&3.56723094444038 \times 10^{-5}(-574033456415 - 726361818188i)(C^{qq(1)}_{1313} + C^{qq(3)}_{1313} + C^{dd}_{1313}) - \\
&0.000858698706386247(-38870709427 + 1559210702i)(C^{qq(1)}_{2323} + C^{qq(3)}_{2323} + C^{dd}_{2323})] + 53.7\frac{\delta\tilde{v}}{v},
\end{aligned}
\tag{F.4}
$$

$$\begin{aligned}
\delta V_{us} = Re[&58106.885C_{2221}^{lq(3)} + 3029.17C_{2212}^{lq(3)} + \\
&0.107 * C_{2213}^{lq(3)}(237.42 - 477.35 * I) - 13095.516 * C_{11}^{\phi q(3)*} - \\
&3029.17 * C_{21}^{\phi q(3)*} - 0.107C_{31}^{\phi q(3)*}(237.42 - 477.35i) - \\
&13095.516C_{21}^{\phi l(3)} + 6689.319C_{11}^{\phi ud*} - \\
&1180455.672C_{2221}^{lequ(1)} + 1150094.315C_{2221}^{ledq} + \\
&266039.236C_{2222}^{ledq} + 13095.516C_{2222}^{lq(3)} + \\
&0.107C_{2223}^{ledq}(20851 + 41922 * I) + 16376.243C_{2222}^{lequ(3)} + \\
&291035.72C_{2211}^{lequ(1)} - 283550.321 * C_{2211}^{ledq} - \\
&65590.679C_{2212}^{ledq} - 0.107C_{2213}^{ledq}(5140 - 10335i) - \\
&6984.853C_{2212}^{lequ(3)} + 0.107C_{2223}^{lq(3)}(1096 - 2063i) - \\
&56612.309C_{12}^{\phi q(3)} - 13095.516C_{22}^{\phi q(3)} - \\
&0.107 * C_{32}^{\phi q(3)*} * (1026 - 2063i) - 13095.516C_{22}^{\phi l(3)} + \\
&28918.141C_{12}^{\phi ud*} + 13441.233C_{2211}^{lq(3)}],
\end{aligned} \tag{F.5}$$

$$\begin{aligned}
\delta V_{ud} =Re[&-13359.153C_{2221}^{lq(3)} - 696.426C_{2212}^{lq(3)} - \\
&0.0246C_{2213}^{lq(3)}(237.42 - 477.35i) + 3010.7448C_{11}^{\phi q(3)*} + \\
&696.426C_{21}^{\phi q(3)*} + 0.0246C_{31}^{\phi q(3)*}(237.42 - 477.35i) + \\
&3010.7448C_{21}^{\phi l(3)} - 1537.9182C_{11}^{\phi ud*} + 271394.4816C_{2221}^{lequ(1)} - \\
&264414.207C_{2221}^{ledq} - 61164.1608A_{19} - 3010.7448C_{2222}^{lq(3)} - \\
&0.0246C_{2223}^{ledq}(20851 + 41922i) - 3765.0054C_{2222}^{lequ(3)} - \\
&66911.016C_{2211}^{lequ(1)} + 65190.0738C_{2211}^{ledq} + 15079.7262C_{2212}^{ledq} + \\
&0.123C_{2213}^{ledq}(1028 - 2067i) + 1605.8634C_{2212}^{lequ(3)} - \\
&0.0246C_{2223}^{lq(3)}(1096 - 2063i) + 13015.5402C_{12}^{\phi q(3)*} + \\
&3010.7448C_{22}^{\phi q(3)*} + 0.0246C_{32}^{\phi q(3)*}(1026 - 2063i) + \\
&3010.7448C_{22}^{\phi l(3)} - 6648.4698C_{12}^{\phi ud*} - 3090.2274C_{2211}^{lq(3)}],
\end{aligned} \tag{F.6}$$

$$
\begin{aligned}
\delta V_{ub} = & -(0.003752 + 0.001864i)Re[(C^{qd(1)}_{1313}(1528461088022 + 1934061094379i)+ \\
& C^{qd(8)}_{1313}(1622594546434 + 2053174273656i)+ \\
& (-574033456415 - 726361818188i)(C^{qq(1)}_{1313} + C^{qq(3)}_{1313} + C^{dd}_{1313}))]+ \\
& (0.003792 + 0.001887i)Re[(C^{qd(1)}_{2323}(104200229183 - 4179756811i)+ \\
& C^{qd(8)}_{2323}(104618244701 - 4196524559i)+ \\
& (-38870709427 + 1559210702i)(C^{qq(1)}_{2323} + C^{qq(3)}_{2323} + C^{dd}_{2323}))]+ \\
& (0.0009044 - 0.0019253i)Re[(C^{lq(3)}_{3331}(-12511840 + 25156081i)- \\
& C^{ledq}_{3331}(33699787 - 67756187i) - C^{ledq}_{3332}(7795419 - 15673329i)- \\
& 329644C^{ledq}_{3333} - C^{lequ(3)}_{3331}(3458942 - 69544881i)- \\
& C^{lq(3)}_{3332}(2894233 - 5819093i) - 122388C^{lq(3)}_{3333}+ \\
& C^{\phi q(3)*}_{13}(12511840 - 25156081i) + C^{\phi q(3)*}_{23}(2894233 - 5819093i)+ \\
& 122388 * C^{\phi q(3)*}_{33} + 122388 * C^{\phi l(3)}_{33} - C^{\phi ud*}_{13}(6391172 - 12849975i)+ \\
& C^{lequ(1)}_{3331}(34589427 - 69544881i))]+ \\
& (0.003719 + 0.001847i)(543055C^{lq(3)}_{2221} + 28310C^{lq(3)}_{2212}+ \\
& C^{lq(3)}_{2213}(237.42 - 477.35i) - 122388C^{\phi q(3)*}_{11}- \\
& 28310C^{\phi q(3)*}_{21} - C^{\phi q(3)*}_{31}(237.42 - 477.35i)- \\
& 122388C^{\phi l(3)}_{21} + 62517C^{\phi ud*}_{11} - 11032296C^{lequ(1)}_{2221}+ \\
& 10748545C^{ledq}_{2221} + 2486348C^{ledq}_{2222} + 122388C^{lq(3)}_{2222}+ \\
& C^{ledq}_{2223}(20851 + 41922i) + 153049C^{lequ(3)}_{2222} + 2719960 * C^{lequ(1)}_{2211}- \\
& 2650003C^{ledq}_{2211} - 612997C^{ledq}_{2212} - C^{ledq}_{2213}(5140 - 10335i)- \\
& 65279C^{lequ(3)}_{2212} + C^{lq(3)}_{2223}(1096 - 2063i)- \\
& 529087C^{\phi q(3)*}_{12} - 122388C^{\phi q(3)*}_{22} - C^{\phi q(3)*}_{32}(1026 - 2063i)- \\
& 122388C^{\phi l(3)}_{22} + 270263C^{\phi ud*}_{12} + 125619C^{lq(3)}_{2211})+ \\
& (57.25 - 115.32i)\frac{\delta\tilde{v}}{v},
\end{aligned}
\tag{F.7}
$$

$$\begin{aligned}
\delta V_{ts} =&(-0.0206874+0.0004114i)Re[(C^{qd(1)}_{1313}(1528461088022+1934061094379i)+\\
&C^{qd(8)}_{1313}*(1622594546434+2053174273656*I)+\\
&(-574033456415-726361818188*I)*(C^{qq(1)}_{1313}+C^{qq(3)}_{1313}+C^{dd}_{1313}))]+\\
&(3.029039\cdot 10^{-6}-0.0004055i)*Re[(C^{qd(1)}_{2323}(104200229183-4179756811i)+\\
&C^{qd(8)}_{2323}(104618244701-4196524559i)+\\
&(-38870709427+1559210702i)(C^{qq(1)}_{2323}+C^{qq(3)}_{2323}+C^{dd}_{2323}))]-\\
&(3.6455162\cdot 10^{-6}+0.0004232i)Re[(C^{lq(3)}_{3331}*(-12511840+25156081i)-\\
&C^{ledq}_{3331}(33699787-67756187i)-C^{ledq}_{3332}(7795419-15673329i)-\\
&329644C^{ledq}_{3333}-C^{lequ(3)}_{3331}(3458942-69544881i)-\\
&C^{lq(3)}_{3332}(2894233-5819093i)-122388C^{lq(3)}_{3333}+\\
&C^{\phi q(3)*}_{13}(12511840-25156081i)+C^{\phi q(3)*}_{23}(2894233-5819093i)+\\
&122388*C^{\phi q(3)*}_{33}+122388C^{\phi l(3)}_{33}-C^{\phi ud*}_{13}(6391172-12849975i)+\\
&C^{lequ(1)}_{3331}(34589427-69544881i))]+\\
&(-1.610031\cdot 10^{-6}+2.695471\cdot 10^{-5}i)Re[(543055C^{lq(3)}_{2221}+28310C^{lq(3)}_{2212}+\\
&C^{lq(3)}_{2213}(237.42-477.35i)-122388C^{\phi q(3)*}_{11}-28310C^{\phi q(3)*}_{21}-\\
&C^{\phi q(3)*}_{31}(237.42-477.35i)-122388C^{\phi l(3)}_{21}+\\
&62517C^{\phi ud*}_{11}-11032296C^{lequ(1)}_{2221}+10748545C^{ledq}_{2221}+\\
&2486348C^{ledq}_{2222}+122388C^{lq(3)}_{2222}+C^{ledq}_{2223}(20851+41922i)+\\
&153049C^{lequ(3)}_{2222}+2719960C^{lequ(1)}_{2211}-2650003C^{ledq}_{2211}-\\
&612997C^{ledq}_{2212}-C^{ledq}_{2213}(5140-10335i)-\\
&65279C^{lequ(3)}_{2212}+C^{lq(3)}_{2223}(1096-2063i)-529087C^{\phi q(3)*}_{12}-\\
&122388C^{\phi q(3)*}_{22}-C^{\phi q(3)*}_{32}(1026-2063i)-\\
&122388C^{\phi l(3)}_{22}+270263C^{\phi ud*}_{12}+125619C^{lq(3)}_{2211})]
\end{aligned} \tag{F.8}$$

$$
\begin{aligned}
\delta V_{cd} =&(0.0001603 - 7.9629 \times 10^{-5} i) Re[(C^{qd(1)}_{1313}(1528461088022 + 1934061094379i)+\\
&C^{qd(8)}_{1313} * (1622594546434 + 2053174273656 * I)+\\
&(-574033456415 - 726361818188 * I) * (C^{qq(1)}_{1313} + C^{qq(3)}_{1313} + C^{dd}_{1313}))]+\\
&(4.5059 \times 10^{-7} + 1.7272 \times 10^{-6} * I) * Re[(C^{qd(1)}_{2323}(104200229183 - 4179756811i)+\\
&C^{qd(8)}_{2323}(104618244701 - 4196524559i)+\\
&(-38870709427 + 1559210702i)(C^{qq(1)}_{2323} + C^{qq(3)}_{2323} + C^{dd}_{2323}))]-\\
&(3.8647 \times 10^{-5} + 7.8214 \times 10^{-5} i) Re[(C^{lq(3)}_{3331} * (-12511840 + 25156081i)-\\
&C^{ledq}_{3331}(33699787 - 67756187i) - C^{ledq}_{3332}(7795419 - 15673329i)-\\
&329644 C^{ledq}_{3333} - C^{lequ(3)}_{3331}(3458942 - 69544881i)-\\
&C^{lq(3)}_{3332}(2894233 - 5819093i) - 122388 C^{lq(3)}_{3333}+\\
&C^{\phi q(3)*}_{13}(12511840 - 25156081i) + C^{\phi q(3)*}_{23}(2894233 - 5819093i)+\\
&122388 * C^{\phi q(3)*}_{33} + 122388 C^{\phi l(3)}_{33} - C^{\phi ud*}_{13}(6391172 - 12849975i)+\\
&C^{lequ(1)}_{3331}(34589427 - 69544881i))]+\\
&(-0.10705 + 7.9381 \times 10^{-5} i) Re[(543055 C^{lq(3)}_{2221} + 28310 C^{lq(3)}_{2212}+\\
&C^{lq(3)}_{2213}(237.42 - 477.35i) - 122388 C^{\phi q(3)*}_{11} - 28310 C^{\phi q(3)*}_{21}-\\
&C^{\phi q(3)*}_{31}(237.42 - 477.35i) - 122388 C^{\phi l(3)}_{21}+\\
&62517 C^{\phi ud*}_{11} - 11032296 C^{lequ(1)}_{2221} + 10748545 C^{ledq}_{2221}+\\
&2486348 C^{ledq}_{2222} + 122388 C^{lq(3)}_{2222} + C^{ledq}_{2223}(20851 + 41922i)+\\
&153049 C^{lequ(3)}_{2222} + 2719960 C^{lequ(1)}_{2211} - 2650003 C^{ledq}_{2211}-\\
&612997 C^{ledq}_{2212} - C^{ledq}_{2213}(5140 - 10335i)-\\
&65279 C^{lequ(3)}_{2212} + C^{lq(3)}_{2223}(1096 - 2063i) - 529087 C^{\phi q(3)*}_{12}-\\
&122388 C^{\phi q(3)*}_{22} - C^{\phi q(3)*}_{32}(1026 - 2063i)-\\
&122388 C^{\phi l(3)}_{22} + 270263 C^{\phi ud*}_{12} + 125619 C^{lq(3)}_{2211})]+\\
&(0.082 - 3.34i)\frac{\delta \tilde{v}}{v},
\end{aligned}
\tag{F.9}
$$

where:

$$
\frac{\delta \tilde{v}}{v} = -C^{ll}_{2112} + C^{\phi l(3)}_{22}. \tag{F.10}
$$

F.2 Corrections of CKM with $U_{ij} = U_{PMNS}$

In the case of non-diagonal PMNS matrix the corrections can be written as:

$$
\delta \tilde{V}_{ij} = \delta V_{ij} + \delta U_{ij}, \tag{F.11}
$$

where the corrections δU_{ij} are as follows:

$$\begin{aligned}\delta U_{ud} =&Re(-2482.46219446478C^{lq(3)}_{1211} + 2948.80599164299C^{lq(3)}_{3221}+\\
&0.0247269997496362C^{lq(3)}_{3223}(999 + 2008.9i)+\\
&58940.1000123245C^{lequ(1)}_{1211} + 70020.5405971337C^{lequ(1)}_{3211} - 57424.7453958676C^{ledq}_{1211}-\\
&13278.3939201547C^{ledq}_{1212} - 0.0247269997496362C^{ledq}_{1213}(4504.1 + 9008.4i)-\\
&68217.0348344945C^{ledq}_{3211} - 15778.8722066371C^{ledq}_{3212}-\\
&0.0247269997496362C^{ledq}_{3213}(5346.7 + 10751.6i) - 574.022462987955C^{lq(3)}_{1212}-\\
&817.67737312092C^{lequ(3)}_{1211} - 971.382876264633C^{lequ(3)}_{3211}-\\
&228806.327001709C^{lequ(3)}_{1221} - 271817.340756617C^{lequ(1)}_{3221} + 222920.902956599C^{ledq}_{1221}+\\
&51546.2745830879C^{ledq}_{1222}+\\
&0.0247269997496362C^{ledq}_{1223}(17484.6 + 34269.2i) + 264816.194919605C^{ledq}_{3221}+\\
&61253.0413146057C^{ledq}_{3222} + 0.0247269997496362C^{ledq}_{3223}(20755.9 + 41737.4i)-\\
&0.0247269997496362C^{lq(3)}_{1213}(194.7 + 38.9i) + 3174.19259436092C^{lequ(3)}_{2221}+\\
&3770.87735261942C^{lequ(3)}_{3221} - 2949.01369844089C^{lq(3)}_{3211} - 682.116542393489C^{lq(3)}_{3212}-\\
&0.0247269997496362C^{lq(3)}_{3213}(231.1 + 464.7i) + 10731.7429070398C^{lq(3)}_{1221}+\\
&0.0247269997496362C^{lq(3)}_{1223}(841.6 + 168.1i) + 12748.6575137183C^{lq(3)}_{1223}+\\
&4965.217290C^{\phi l(3)}_{12} + 5786.8966307C^{\phi l(3)}_{32})\end{aligned} \tag{F.12}$$

$$\begin{aligned}\delta U_{us} =&Re(10742.2436C^{lq(3)}_{1211} - 12760.2315C^{lq(3)}_{3221} - 0.107C^{lq(3)}_{3223}(999 + 2008.9i)-\\
&255048.7631C^{lequ(1)}_{1211} - 302996.6401C^{lequ(1)}_{3211} + 248491.4393C^{ledq}_{1212}+\\
&57458.9786C^{ledq}_{1213} + 0.107C^{ledq}_{1213}(4504.1 + 9008.4i) + 295192.4132C^{ledq}_{3211}+\\
&68279.1824C^{ledq}_{3212} + 0.107C^{ledq}_{3212}(5346.7 + 10751.6i)+\\
&2483.9408C^{lq(3)}_{1212} + 3538.2974C^{lequ(3)}_{1211} + 4203.4201C^{lequ(3)}_{3211}+\\
&990103.0144C^{lequ(3)}_{1221} + 1176222.5808C^{lequ(1)}_{3221} - 964635.2917C^{ledq}_{1221} - 223053.805C^{ledq}_{1222}-\\
&0.107C^{ledq}_{1223}(17484.6 + 34269.2i) - 1145926.8469C^{ledq}_{3221}-\\
&265057.4468C^{ledq}_{3222} - 0.107C^{ledq}_{3223}(20755.9 + 41737.4i)+\\
&0.107C^{lq(3)}_{1213}(194.7 + 38.9i) - 13735.5365C^{lequ(3)}_{2221} - 16317.5428C^{lequ(3)}_{3221} + 12761.1303C^{lq(3)}_{3211}+\\
&2951.6913C^{lq(3)}3212 + 0.107C^{lq(3)}_{3213}(231.1 + 464.7I) - 46438.9737C^{lq(3)}_{1221}-\\
&0.107C^{lq(3)}_{1223}(841.6 + 168.1I) - 55166.6748C^{lq(3)}_{1223} - 21485.754657C^{\phi l(3)}_{12} - 25041.3695860C^{\phi l(3)}_{32})\end{aligned} \tag{F.13}$$

$$
\begin{aligned}
\delta U_{cb} =& Re(1.157552044 \cdot C^{lq(3)}_{1211} - 1.375004385 \cdot C^{lq(3)}_{3221} - \\
& 1.153 \cdot 10^{-5} \cdot C^{lq(3)}_{3223} \cdot (999 + 2008.9i) - 27.483291949 \cdot C^{lequ(1)}_{1211} - \\
& 32.650011779 \cdot C^{lequ(1)} 3211 + 26.776694347 \cdot C^{ledq}_{1211} + \\
& 6.191607694 \cdot C^{ledq}_{1212} + \\
& 1.153 \cdot 10^{-5} \cdot C^{ledq}_{1213} \cdot (4504.1 + 9008.4i) + \\
& 31.809051628 \cdot C^{ledq}_{3211} + 7.357560496 \cdot C^{ledq}_{3212} + \\
& 1.153 \cdot 10^{-5} \cdot C^{ledq}_{3212} \cdot (5346.7 + 10751.6i) + \\
& 0.267662032 \cdot C^{lq(3)}_{1212} + 0.381276346 \cdot C^{lequ(3)}_{1211} + \\
& 0.452947979 \cdot C^{lequ(3)}_{3211} + 106.690539776 \cdot C^{lequ(3)}_{1221} + \\
& 126.746227632 \cdot C^{lequ(1)} 3221 - 103.946214143 \cdot C^{ledq}_{1221} - \\
& 24.03561095 \cdot C^{ledq}_{1222} - 1.153 \cdot 10^{-5} \cdot C^{ledq}_{1223} \cdot (17484.6 + 34269.2i) - \\
& 123.481649951 \cdot C^{ledq}_{3221} - 28.561797772 \cdot C^{ledq}_{3222} - \\
& 1.153 \cdot 10^{-5} \cdot C^{ledq}_{3223} \cdot (20755.9 + 41737.4i) + 1.153 \cdot 10^{-5} \cdot C^{lq(3)}_{1213} \cdot (194.7 + 38.9i) - \\
& 1.480100335 \cdot C^{lequ(3)}_{2221} - 1.758329612 \cdot C^{lequ(3)}_{3221} + \\
& 1.375101237 \cdot C^{lq(3)} 3211 + 0.318065427 \cdot C^{lq(3)}_{3212} + \\
& 1.153 \cdot 10^{-5} \cdot C^{lq(3)}_{3213} \cdot (231.1 + 464.7i) - \\
& 5.004124923 \cdot C^{lq(3)}_{1221} - 1.153 \cdot 10^{-5} \cdot C^{lq(3)}_{1223} \cdot (841.6 + 168.1i) - 5.944595892 \cdot C^{lq(3)}_{1223}) - \\
& 0.000203 Re[C^{lq(3)}_{1331}(-8673938.8 + i17439668.3) + \\
& C^{lq(3)}_{1332}(-2006395.1 + i4034022.7) + C^{lq(3)}_{1333}(-84845.9 + i0.0057) + \\
& C^{lq(3)}_{2331}(-12170139.7 + i24469068.3) + C^{lq(3)}_{2332}(-2815080.2 + i5669950.6) + \\
& C^{lq(3)}_{2333}(-119045.3 + i0.0080) + C^{lequ(1)}_{1331}(24028352.5 - i48310973) + \\
& C^{lequ(1)}_{2331}(33713588.5 - i67783934.3) - C^{ledq}_{1331}(23410553.7 - i47068837.9) - \\
& C^{ledq}_{1333}(228995.2 + i0.0005) - C^{ledq}_{2331}(32846636.1 - i66040855.2) - \\
& C^{ledq}_{2332}(7597769.4 - i15275938.5) - C^{ledq}_{2333}(321297.6 + i0.00075) - \\
& C^{ledq}_{1332}(5415166.4 - i10887636.2) - C^{lequ(3)}_{1331}(298282.9 - i599722.4) - \\
& C^{lequ(3)}_{2331}(418513.5 - i841455.7)] + Re[-2.316131 C^{\phi l(3)}_{12} + \\
& C^{\phi l(3)}_{1} 3(-16.7260 + 6.75597i) + C^{\phi l(3)}_{23}(-23.1793 - 5.404782i) - 2.699421 C^{\phi l(3)}_{32}]
\end{aligned} \tag{F.14}
$$

$$
\begin{aligned}
\delta U_{tb} =& Re(-0.048691478 \cdot C^{lq(3)}_{1211} + 0.0578384325 \cdot C^{lq(3)}_{3221} + \\
& 4.85 \cdot 10^{-7} \cdot C^{lq(3)}_{3223} \cdot (999 + 2008.9i) + 1.1560621505 \cdot C^{lequ(1)}_{1211} + \\
& 1.3733959855 \cdot C^{lequ(1)}_{3211} - 1.1263397015 \cdot C^{ledq}_{1211} - \\
& 0.260444903 \cdot C^{ledq}_{1212} - 4.85 \cdot 10^{-7} \cdot C^{ledq}_{1213} \cdot (4504.1 + 9008.4i) - \\
& 1.338021686 \cdot C^{ledq}_{3211} - 0.309489752 \cdot C^{ledq}_{3212} - \\
& 4.85 \cdot 10^{-7} \cdot C^{ledq}_{3212} \cdot (5346.7 + 10751.6i) - 0.011258984 \cdot C^{lq(3)}_{1212} - \\
& 0.016038077 \cdot C^{lequ(3)}_{1211} - 0.0190528855 \cdot C^{lequ(3)}_{3211} - \\
& 4.487850112 \cdot C^{lequ(3)}_{1221} - 5.331476184 \cdot C^{lequ(1)}_{3221} + \\
& 4.3724123035 \cdot C^{ledq}_{1221} + 1.011038275 \cdot C^{ledq}_{1222} + \\
& 4.85 \cdot 10^{-7} \cdot C^{ledq}_{1223} \cdot (17484.6 + 34269.2i) + \\
& 5.1941543995 \cdot C^{ledq}_{3221} + 1.201428614 \cdot C^{ledq}_{3222} + \\
& 4.85 \cdot 10^{-7} \cdot C^{ledq}_{3223} \cdot (20755.9 + 41737.4i) - \\
& 4.85 \cdot 10^{-7} \cdot C^{lq(3)}_{1213} \cdot (194.7 + 38.9i) + 0.0622592075 \cdot C^{lequ(3)}_{2221} + \\
& 0.073962694 \cdot C^{lequ(3)}_{3221} - 0.0578425065 \cdot C^{lq(3)}_{3211} - \\
& 0.0133791615 \cdot C^{lq(3)}_{3212} - 4.85 \cdot 10^{-7} \cdot C^{lq(3)}_{3213} \cdot (231.1 + 464.7i) + \\
& 0.2104944135 \cdot C^{lq(3)}_{1221} + 4.85 \cdot 10^{-7} \cdot C^{lq(3)}_{1223} \cdot (841.6 + 168.1i) + 0.250054554 \cdot C^{lq(3)}_{1223}) + \\
& 8.544 \cdot 10^{-6} Re[C^{lq(3)}_{1331}(-8673938.8 + i17439668.3) + \\
& C^{lq(3)}_{1332}(-2006395.1 + i4034022.7) + C^{lq(3)}_{1333}(-84845.9 + i0.0057) + \\
& C^{lq(3)}_{2331}(-12170139.7 + i24469068.3) + C^{lq(3)}_{2332}(-2815080.2 + i5669950.6) + \\
& C^{lq(3)}_{2333}(-119045.3 + i0.0080) + C^{lequ(1)}_{1331}(24028352.5 - i48310973) + \\
& C^{lequ(1)}_{2331}(33713588.5 - i67783934.3) - C^{ledq}_{1331}(23410553.7 - i47068837.9) - \\
& C^{ledq}_{1333}(228995.2 + i0.0005) - C^{ledq}_{2331}(32846636.1 - i66040855.2) - \\
& C^{ledq}_{2332}(7597769.4 - i15275938.5) - C^{ledq}_{2333}(321297.6 + i0.00075) - \\
& C^{ledq}_{1332}(5415166.4 - i10887636.2) - C^{lequ(3)}_{1331}(298282.9 - i599722.4) - \\
& C^{lequ(3)}_{2331}(418513.5 - i841455.7)] + \\
& Re[337.33004 C^{\phi l(3)}_{12} + C^{\phi l(3)}_{13}(0.70342 - 2.84125i) + \\
& C^{\phi l(3)}_{23}(0.97481 + i2.27300) + 393.1538 C^{\phi l(3)}_{32}]
\end{aligned}
\tag{F.15}
$$

$$
\begin{aligned}
\delta U_{cs} = & -2479.75156C^{lq(3)}_{1211} + 2945.58615C^{lq(3)}_{3221} + \\
& 0.0247C^{lq(3)}_{3223}(999 + 2008.9I) + 58875.74251C^{lequ(1)}_{1211} + \\
& 69944.08421C^{lequ(1)}_{3211} - 57362.04253C^{ledq}_{1211} - 13263.89506C^{ledq}_{1212} - \\
& 0.0247C^{ledq}_{1213}(4504.1 + 9008.4I) - 68142.54772C^{ledq}_{1221} - \\
& 15761.64304C^{ledq}_{3212} - 0.0247C^{ledq}_{3213}(5346.7 + 10751.6I) - \\
& 573.39568C^{lq(3)}_{1212} - 816.78454C^{lequ(3)}_{1211} - 970.32221C^{lequ(3)}3211 - \\
& 228556.49024C^{lequ(3)}_{1221} - 271520.53968C^{lequ(1)}_{3221} + \\
& 222677.49257C^{ledq}_{1221} + 51489.9905C^{ledq}_{1222} + \\
& 0.0247C^{ledq}_{1223}(17484.6 + 34269.2I) + 264527.03849C^{ledq}_{3221} + \\
& 61186.15828C^{ledq}_{3222} + 0.0247C^{ledq}_{3223}(20755.9 + 41737.4I) - 0.0247C^{lq(3)}_{1213}(194.7 + 38.9I) + \\
& 3170.72665C^{lequ(3)}_{2221} + 3766.75988C^{lequ(3)}_{3221} - 2945.79363C^{lq(3)}_{3211} - \\
& 681.37173C^{lq(3)}_{3212} - 0.0247C^{lq(3)}_{3213}(231.1 + 464.7I) + \\
& 10720.02477C^{lq(3)}_{1221} + 0.0247C^{lq(3)}_{1223}(841.6 + 168.1I) + \\
& 12734.73708C^{lq(3)}_{1223} + 8.544 \cdot 10^{-6} Re[C^{lq(3)}_{1331}(-8673938.8 + i17439668.3) + \\
& C^{lq(3)}_{1332}(-2006395.1 + i4034022.7) + C^{lq(3)}_{1333}(-84845.9 + i0.0057) + \\
& C^{lq(3)}_{2331}(-12170139.7 + i24469068.3) + C^{lq(3)}_{2332}(-2815080.2 + i5669950.6) + \\
& C^{lq(3)}_{2333}(-119045.3 + i0.0080) + C^{lequ(1)}_{1331}(24028352.5 - i48310973) + \\
& C^{lequ(1)}_{2331}(33713588.5 - i67783934.3) - C^{ledq}_{1331}(23410553.7 - i47068837.9) - \\
& C^{ledq}_{1333}(228995.2 + i0.0005) - C^{ledq}_{2331}(32846636.1 - i66040855.2) - \\
& C^{ledq}_{2332}(7597769.4 - i15275938.5) - C^{ledq}_{2333}(321297.6 + i0.00075) - \\
& C^{ledq}_{1332}(5415166.4 - i10887636.2) - \\
& C^{lequ(3)}_{1331}(298282.9 - i599722.4) - C^{lequ(3)}_{2331}(418513.5 - i841455.7)] + \\
& Re[4965.31469C^{\phi l(3)}_{12} + C^{\phi l(3)}_{13}(0.7034 - 2.84 \cdot 10^{-17} i) + \\
& C^{\phi l(3)}_{23}(0.974818 + 2.273 \cdot 10^{-17} i + 5787C^{\phi l(3)}_{32}]
\end{aligned}
\tag{F.16}
$$

$$
\begin{aligned}
\delta U_{td} =&(0.000918 + 0.001888i)Re[(100394.8C^{lq(3)}_{1211} - 119254.5C^{lq(3)}_{3221} - \\
&C^{lq(3)}_{3223}(999 + 2008.9i) - 2383633.3C^{lequ(1)}_{1211} - 2831744.3C^{lequ(1)}_{3211} + 2322349.9C^{ledq}_{1211} + \\
&536999.8C^{ledq}_{1212} + C^{ledq}_{1213}(4504.1 + 9008.4i) + \\
&2758807.6C^{ledq}_{3211} + 638123.2C^{ledq}_{3212} + C^{ledq}_{3212}(5346.7 + 10751.6I) + \\
&23214.4C^{lq(3)}_{1212} + 33068.2C^{lequ(3)}_{1211} + 39284.3C^{lequ(3)}_{3211} + \\
&9253299.2C^{lequ(3)}_{1221} + 10992734.4C^{lequ(1)}_{3221} - 9015283.1C^{ledq}_{1221} - \\
&2084615.0C^{ledq}_{1222} - C^{ledq}_{1223}(17484.6 + 34269.2I) - \\
&10709596.7C^{ledq}_{3221} - 2477172.4C^{ledq}3222 - C^{ledq}_{3223}(20755.9 + \\
&41737.4I) + C^{lq(3)}_{1213}(194.7 + 38.9I) - 128369.5C^{lequ(3)}_{2221} - \\
&152500.4C^{lequ(3)}_{3221} + 119262.9C^{lq(3)}_{3211} + 27585.9C^{lq(3)}_{3212} + \\
&C^{lq(3)}3213_(231.1 + 464.7I) - 434009.1C^{lq(3)}_{1221} - C^{lq(3)}_{1223}(841.6 + 168.1i) - 515576.4C^{lq(3)}_{1223})] - \\
&(0.000927 + 0.00187i)Re[C^{lq(3)}_{1331}(-8673938.8 + i17439668.3) + \\
&C^{lq(3)}_{1332}(-2006395.1 + i4034022.7) + C^{lq(3)}_{1333}(-84845.9 + i0.0057) + \\
&C^{lq(3)}_{2331}(-12170139.7 + i24469068.3) + C^{lq(3)}_{2332}(-2815080.2 + i5669950.6) + \\
&C^{lq(3)}_{2333}(-119045.3 + i0.0080) + C^{lequ(1)}_{1331}(24028352.5 - i48310973) + \\
&C^{lequ(1)}_{2331}(33713588.5 - i67783934.3) - C^{ledq}_{1331}(23410553.7 - i47068837.9) - \\
&C^{ledq}_{1333}(228995.2 + i0.0005) - C^{ledq}_{2331}(32846636.1 - i66040855.2) - \\
&C^{ledq}_{2332}(7597769.4 - i15275938.5) - C^{ledq}_{2333}(321297.6 + i0.00075) - \\
&C^{ledq}_{1332}(5415166.4 - i10887636.2) - C^{lequ(3)}_{1331}(298282.9 - i599722.4) - \\
&C^{lequ(3)}_{2331}(418513.5 - i841455.7)]
\end{aligned}
\tag{F.17}
$$

$$\begin{aligned}
\delta U_{ts} =&(1.15 \cdot 10^{-6} + 2.69 \cdot 10^{-5} i) Re[(100394.8 C^{lq(3)}_{1211} - 119254.5 C^{lq(3)}_{3221} - \\
& C^{lq(3)}_{3223}(999 + 2008.9i) - 2383633.3 C^{lequ(1)}_{1211} - 2831744.3 C^{lequ(1)}_{3211} + 2322349.9 C^{ledq}_{1211} + \\
& 536999.8 C^{ledq}_{1212} + C^{ledq}_{1213}(4504.1 + 9008.4i) + \\
& 2758807.6 C^{ledq}_{3211} + 638123.2 C^{ledq}_{3212} + C^{ledq}_{3212}(5346.7 + 10751.6I) + \\
& 23214.4 C^{lq(3)}_{1212} + 33068.2 C^{lequ(3)}_{1211} + 39284.3 C^{lequ(3)}_{3211} + \\
& 9253299.2 C^{lequ(3)}_{1221} + 10992734.4 C^{lequ(1)}_{3221} - 9015283.1 C^{ledq}_{1221} - \\
& 2084615.0 * C^{ledq}_{1222} - C^{ledq}_{1223}(17484.6 + 34269.2I) - \\
& 10709596.7 C^{ledq}_{3221} - 2477172.4 C^{ledq}_{3222} - C^{ledq}_{3223}(20755.9 + \\
& 41737.4I) + C^{lq(3)}_{1213}(194.7 + 38.9I) - 128369.5 C^{lequ(3)}_{2221} - \\
& 152500.4 C^{lequ(3)}_{3221} + 119262.9 C^{lq(3)}_{3211} + 27585.9 C^{lq(3)}_{3212} + \\
& C^{lq(3)} 3213_{(} 231.1 + 464.7I) - 434009.1 C^{lq(3)}_{1221} - C^{lq(3)}_{1223}(841.6 + 168.1i) - 515576.4 C^{lq(3)}_{1223})] - \\
& (3.64 \cdot 10^{-6} + 0.0004i) Re[C^{lq(3)}_{1331}(-8673938.8 + i17439668.3) + \\
& C^{lq(3)}_{1332}(-2006395.1 + i4034022.7) + C^{lq(3)}_{1333}(-84845.9 + i0.0057) + \\
& C^{lq(3)}_{2331}(-12170139.7 + i24469068.3) + C^{lq(3)}_{2332}(-2815080.2 + i5669950.6) + \\
& C^{lq(3)}_{2333}(-119045.3 + i0.0080) + C^{lequ(1)}_{1331}(24028352.5 - i48310973) + \\
& C^{lequ(1)}_{2331}(33713588.5 - i67783934.3) - C^{ledq}_{1331}(23410553.7 - i47068837.9) - \\
& C^{ledq}_{1333}(228995.2 + i0.0005) - C^{ledq}_{2331}(32846636.1 - i66040855.2) - \\
& C^{ledq}_{2332}(7597769.4 - i15275938.5) - C^{ledq}_{2333}(321297.6 + i0.00075) - \\
& C^{ledq}_{1332}(5415166.4 - i10887636.2) - C^{lequ(3)}_{1331}(298282.9 - i599722.4) - \\
& C^{lequ(3)}_{2331}(418513.5 - i841455.7)]
\end{aligned} \tag{F.18}$$

$$
\begin{aligned}
\delta U_{cd} =&(-0.1070 + 7.9381 \cdot 10^{-5} i) Re[(100394.8 C^{lq(3)}_{1211} - 119254.5 C^{lq(3)}_{3221} - \\
& C^{lq(3)}_{3223}(999 + 2008.9i) - 2383633.3 C^{lequ(1)}_{1211} - 2831744.3 C^{lequ(1)}_{3211} + 2322349.9 C^{ledq}_{1211} + \\
& 536999.8 C^{ledq}_{1212} + C^{ledq}_{1213}(4504.1 + 9008.4i) + \\
& 2758807.6 C^{ledq}_{3211} + 638123.2 C^{ledq}_{3212} + C^{ledq}_{3212}(5346.7 + 10751.6I) + \\
& 23214.4 C^{lq(3)}_{1212} + 33068.2 C^{lequ(3)}_{1211} + 39284.3 C^{lequ(3)}_{3211} + \\
& 9253299.2 * C^{lequ(3)}_{1221} + 10992734.4 C^{lequ(1)}_{3221} - 9015283.1 C^{ledq}_{1221} - \\
& 2084615.0 * C^{ledq}_{1222} - C^{ledq}_{1223}(17484.6 + 34269.2I) - \\
& 10709596.7 C^{ledq}_{3221} - 2477172.4 C^{ledq}_{3222} - C^{ledq}_{3223}(20755.9 + \\
& 41737.4I) + C^{lq(3)}_{1213}(194.7 + 38.9I) - 128369.5 C^{lequ(3)}_{2221} - \\
& 152500.4 C^{lequ(3)}_{3221} + 119262.9 C^{lq(3)}_{3211} + 27585.9 C^{lq(3)}_{3212} + \\
& C^{lq(3)} 3213_{(} 231.1 + 464.7I) - 434009.1 C^{lq(3)}_{1221} - C^{lq(3)}_{1223}(841.6 + 168.1i) - 515576.4 C^{lq(3)}_{1223})] - \\
& (3.864 \cdot 10^{-5} + 7.821 \cdot 10^{-5} i) Re[C^{lq(3)}_{1331}(-8673938.8 + i17439668.3) + \\
& C^{lq(3)}_{1332}(-2006395.1 + i4034022.7) + C^{lq(3)}_{1333}(-84845.9 + i0.0057) + \\
& C^{lq(3)}_{2331}(-12170139.7 + i24469068.3) + C^{lq(3)}_{2332}(-2815080.2 + i5669950.6) + \\
& C^{lq(3)}_{2333}(-119045.3 + i0.0080) + C^{lequ(1)}_{1331}(24028352.5 - i48310973) + \\
& C^{lequ(1)}_{2331}(33713588.5 - i67783934.3) - C^{ledq}_{1331}(23410553.7 - i47068837.9) - \\
& C^{ledq}_{1333}(228995.2 + i0.0005) - C^{ledq}_{2331}(32846636.1 - i66040855.2) - \\
& C^{ledq}_{2332}(7597769.4 - i15275938.5) - C^{ledq}_{2333}(321297.6 + i0.00075) - \\
& C^{ledq}_{1332}(5415166.4 - i10887636.2) - C^{lequ(3)}_{1331}(298282.9 - i599722.4) - \\
& C^{lequ(3)}_{2331}(418513.5 - i841455.7) + C^{\phi l(3)}_{13} 84437.537 + C^{\phi l(3)}_{23} 117015.7457]
\end{aligned}
\tag{F.19}
$$

$$
\begin{aligned}
\delta U_{ub} =&(0.00371 + 0.00184i)Re[(100394.8C^{lq(3)}_{1211} - 119254.5C^{lq(3)}_{3221} - \\
& C^{lq(3)}_{3223}(999 + 2008.9i) - 2383633.3C^{lequ(1)}_{1211} - 2831744.3C^{lequ(1)}_{3211} + 2322349.9C^{ledq}_{1211} + \\
& 536999.8C^{ledq}_{1212} + C^{ledq}_{1213}(4504.1 + 9008.4i) + \\
& 2758807.6C^{ledq}_{3211} + 638123.2C^{ledq}_{3212} + C^{ledq}_{3212}(5346.7 + 10751.6I) + \\
& 23214.4C^{lq(3)}_{1212} + 33068.2C^{lequ(3)}_{1211} + 39284.3C^{lequ(3)}_{3211} + \\
& 9253299.2C^{lequ(3)}_{1221} + 10992734.4C^{lequ(1)}_{3221} - 9015283.1C^{ledq}_{1221} - \\
& 2084615.0C^{ledq}_{1222} - C^{ledq}_{1223}(17484.6 + 34269.2I) - \\
& 10709596.7C^{ledq}_{3221} - 2477172.4C^{ledq}_{3222} - C^{ledq}_{3223}(20755.9 + \\
& 41737.4I) + C^{lq(3)}_{1213}(194.7 + 38.9I) - 128369.5C^{lequ(3)}_{2221} - \\
& 152500.4C^{lequ(3)}_{3221} + 119262.9C^{lq(3)}_{3211} + 27585.9C^{lq(3)}_{3212} + \\
& C^{lq(3)}_{3213}(231.1 + 464.7I) - 434009.1C^{lq(3)}_{1221} - C^{lq(3)}_{1223}(841.6 + 168.1i) - 515576.4C^{lq(3)}_{1223})] + \\
& (0.000904 - i0.00192)Re[C^{lq(3)}_{1331}(-8673938.8 + i17439668.3) + \\
& C^{lq(3)}_{1332}(-2006395.1 + i4034022.7) + \\
& C^{lq(3)}_{1333}(-84845.9 + i0.0057) + C^{lq(3)}_{2331}(-12170139.7 + i24469068.3) + \\
& C^{lq(3)}_{2332}(-2815080.2 + i5669950.6) + C^{lq(3)}_{2333}(-119045.3 + i0.0080) + \\
& C^{lequ(1)}_{1331}(24028352.5 - I48310973) + C^{lequ(1)}_{2331}(33713588.5 - i67783934.3) - \\
& C^{ledq}_{1331}(23410553.7 - i47068837.9) - C^{ledq}_{1333}(228995.2 + i0.0005) - \\
& C^{ledq}_{2331}(32846636.1 - i66040855.2) - C^{ledq}_{2332}(7597769.4 - i15275938.5) - \\
& C^{ledq}_{2333}(321297.6 + i0.00075) - C^{ledq}_{1332}(5415166.4 - i10887636.2) - \\
& C^{lequ(3)}_{1331}(298282.9 - i599722.4) - C^{lequ(3)}2331(418513.5 - i841455.7)] + \\
& (0.003719 + 0.0018374i)Re[-200801.445C^{\phi l(3)}_{12} - 234031.491C^{\phi l(3)}_{13}] + \\
& (0.0008949 - 0.001925i)Re[84437.537C^{\phi l(3)}_{13} + 117015.745C^{\phi l(3)}_{23}]
\end{aligned}
\tag{F.20}
$$

Bibliography

[1] Jason Aebischer, Jacky Kumar, and David M Straub. ": a Python package for the running and matching of Wilson coefficients above and below the electroweak scale". In: *The European Physical Journal C* 78 (2018), pp. 1–8.

[2] Jason Aebischer et al. "B physics beyond the standard model at one loop: complete renormalization group evolution below the electroweak scale". In: *Journal of High Energy Physics* 2017.9 (2017), pp. 1–36.

[3] S Aoki et al. "FLAG review 2019: flavour lattice averaging group (FLAG)". In: *The European Physical Journal C* 80.2 (2020), p. 113.

[4] Christopher Arzt. "Reduced effective lagrangians". In: *Physics Letters B* 342.1-4 (1995), pp. 189–195.

[5] Jonathan A Bagger, Konstantin T Matchev, and Ren-Jie Zhang. "QCD corrections to flavor-changing neutral currents in the supersymmetric standard model". In: *Physics Letters B* 412.1-2 (1997), pp. 77–85.

[6] Marco Battaglia et al. "The CKM matrix and the unitarity triangle". In: *arXiv preprint hep-ph/0304132* (2003).

[7] Alexei Bazavov et al. "Nonperturbative QCD simulations with 2+ 1 flavors of improved staggered quarks". In: *Reviews of modern physics* 82.2 (2010), p. 1349.

[8] Juerg Beringer et al. "Particle data group". In: *Phys. Rev. D* 86.010001 (2012).

[9] Alessandro Bettini. *Introduction to elementary particle physics*. Cambridge University Press, 2014.

[10] Christoph Bobeth et al. "Yukawa enhancement of Z-mediated new physics in S= 2 and B= 2 processes". In: *Journal of High Energy Physics* 2017.7 (2017), pp. 1–39.

[11] M Bona et al. "Model-independent constraints on ΔF= 2 operators and the scale of new physics". In: *Journal of High Energy Physics* 2008.03 (2008), p. 049.

[12] Andrzej J Buras, Matthias Jamin, and Peter H Weisz. "Leading and next-to-leading QCD corrections to -parameter and B0- B
0 mixing in the presence of a heavy top quark". In: *Nuclear Physics B* 347.3 (1990), pp. 491–536.

[13] Andrzej J Buras, Mikoaj Misiak, and Jörg Urban. "Two-loop QCD anomalous dimensions of flavour-changing four-quark operators within and beyond the standard model". In: *Nuclear Physics B* 586.1-2 (2000), pp. 397–426.

[14] Cliff Peter Burgess. *Introduction to effective field theory*. Cambridge University Press, 2020.

[15] Nuria Carrasco Vela et al. "B-physics from Nf= 2 tmQCD: the Standard Model and beyond". In: *Journal of High Energy Physics, 2014, vol. 2014, num. 3, p. 016* (2014).

[16] Jérôme Charles et al. “CP violation and the CKM matrix: Assessing the impact of the asymmetric B factories”. In: *The European Physical Journal C-Particles and Fields* 41.1 (2005), pp. 1–131.

[17] Juan Carlos Criado and Manuel Perez-Victoria. “Field redefinitions in effective theories at higher orders”. In: *Journal of High Energy Physics* 2019.3 (2019), pp. 1–41.

[18] A Dedes et al. “Feynman rules for the Standard Model Effective Field Theory in R ξ-gauges”. In: *Journal of High Energy Physics* 2017.6 (2017), pp. 1–105.

[19] A. Dedes et al. “SmeftFR v3 – Feynman rules generator for the Standard Model Effective Field Theory”. In: (Feb. 2023). arXiv: 2302.01353 [hep-ph].

[20] Athanasios Dedes and Kostas Mantzaropoulos. “Universal scalar leptoquark action for matching”. In: *Journal of High Energy Physics* 2021.11 (2021), pp. 1–65.

[21] Sebastien Descotes-Genon and Bachir Moussallam. “Radiative corrections in weak semi-leptonic processes at low energy: a two-step matching determination”. In: *The European Physical Journal C-Particles and Fields* 42.4 (2005), pp. 403–417.

[22] Sébastien Descotes-Genon and Patrick Koppenburg. “The CKM parameters”. In: *Annual Review of Nuclear and Particle Science* 67 (2017), pp. 97–127.

[23] Sébastien Descotes-Genon et al. “The CKM parameters in the SMEFT”. In: *Journal of High Energy Physics* 2019.5 (2019), pp. 1–35.

[24] F Gabbiani et al. “A complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model”. In: *Nuclear Physics B* 477.2 (1996), pp. 321–352.

[25] Sheldon L Glashow. “Partial-symmetries of weak interactions”. In: *Nuclear physics* 22.4 (1961), pp. 579–588.

[26] Martn González-Alonso and Jorge Martin Camalich. “Global Effective-Field-Theory analysis of New-Physics effects in (semi) leptonic kaon decays”. In: *Journal of High Energy Physics* 2016.12 (2016), pp. 1–38.

[27] Martn González-Alonso, Jorge Martin Camalich, and Kin Mimouni. “Renormalization-group evolution of new physics contributions to (semi) leptonic meson decays”. In: *Physics Letters B* 772 (2017), pp. 777–785.

[28] Elizabeth E Jenkins, Aneesh V Manohar, and Peter Stoffer. “Low-energy effective field theory below the electroweak scale: anomalous dimensions”. In: *Journal of High Energy Physics* 2018.1 (2018).

[29] Tom WB Kibble. “Englert–brout–higgs–guralnik–hagen–kibble mechanism (history)”. In: *International Journal of Modern Physics A* 24.32 (2009), pp. 6001–6009.

[30] Paul Langacker. *The standard model and beyond*. Taylor & Francis, 2017.

[31] Fermilab Lattice et al. “B (s) 0-mixing matrix elements from lattice QCD for the Standard Model and beyond”. In: *Physical Review D* 93.11 (2016), p. 113016.

[32] Luciano Maiani. “The GIM Mechanism: origin, predictions and recent uses”. In: *arXiv preprint arXiv:1303.6154* (2013).

[33] Aneesh V Manohar. “Introduction to effective field theories”. In: (2018).

[34] William J Marciano and A Sirlin. "Radiative corrections to π l 2 decays". In: *Physical Review Letters* 71.22 (1993), p. 3629.

[35] M PDG2019. "Tanabashi et al., Review of Particle Physics". In: *Phys. Rev. D* 98 (2019), p. 030001.

[36] Michael Peskin. *An introduction to quantum field theory*. CRC press, 2018.

[37] Jonathan L Rosner, Sheldon Stone, and Ruth S Van de Water. "Leptonic decays of charged pseudoscalar mesons-2015". In: *arXiv preprint arXiv:1509.02220* (2015).

[38] Matthew D Schwartz. *Quantum field theory and the standard model*. Cambridge university press, 2014.

[39] A Sirlin. "Large mW, mZ behaviour of the O (α) corrections to semileptonic processes mediated by W". In: *Nuclear Physics B* 196.1 (1982), pp. 83–92.

[40] Mark Srednicki. *Quantum field theory*. Cambridge University Press, 2007.

[41] Mark Thomson. *Modern particle physics*. Cambridge University Press, 2013.

[42] Steven Weinberg. "A model of leptons". In: *Physical review letters* 19.21 (1967), p. 1264.

[43] Yijia Zhang, Xinyi Zhang, and Bo-Qiang Ma. "Quark-lepton complementarity and self-complementarity in different schemes". In: *Physical Review D* 86.9 (2012), p. 093019.