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Theoretical and Phenomenological Aspects of the Standard Model Effective Field Theory

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Dedication

"Life before death.
Strength before weakness.
Journey before destination."

— Brandon Sanderson,
The Stormlight Archive

. . . to Athanasia,
my beloved partner in life,
who always helped me to find the much needed strength
to go through with this long journey.

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Abstract

In this thesis, we study the extension of the Standard Model (SM) of elementary particle physics in the framework of the Effective Field Theory (EFT) description. This modern-era approach aims to augment the well-established theory of the SM in a way that is, under mild assumptions, as generic as possible. The resulting theory, abbreviated as SMEFT, can be utilised to improve the theoretical predictions of the SM in a systematic manner, without specifying the new physics that will appear at higher energy scales (the bottom-up EFT approach), or to simplify the description of a more complete theory by matching it to the SM (the top-down EFT approach). In this thesis we are mainly concerned with the bottom-up SMEFT. In particular, we focus on the theoretical and phenomenological aspects of the leading non-trivial EFT order by working out in detail two processes of high complexity: the Higgs di-photon decay and the Higgs decay to a Z boson and a photon, both at one-loop order in the $\hbar$ expansion. These calculations serve as a benchmark for resolving technical issues in loop EFT calculations, such as consistency with gauge invariance and renormalisation of the amplitudes, as well as providing bounds for the unspecified parameters of the model in phenomenological analyses. Additional technical details about the calculations are collected in the appendices to be useful for future reference. Furthermore, due to recent developments in the literature, we are also concerned with the extension of the SMEFT formalism to higher orders in the EFT expansion, providing the relevant analytic formulae up to any possible EFT order. Finally, because of the high-complexity of the calculations in the SMEFT, we also focus our attention on the development of efficient computer codes that will hopefully serve the physics community by providing a platform capable of performing consistent (semi-)automatised calculations up-to the next-to-leading EFT order.

Περίληψη
(Abstract in Greek)

Η παρούσα διδακτορική διατριβή πραγματεύεται την επέκταση του Καθιερωμένου Προτύπου (Standard Model ή SM) των στοιχειωδών σωματιδίων στα πλαίσια της περιγραφής του ως μία Ενεργή Θεωρία Πεδίου (Effective Field Theory ή EFT). Η σύγχρονη αυτή προσέγγιση στοχεύει στην γενίκευση της καλά εδραιωμένης θεωρίας του SM με έναν τρόπο ο οποίος είναι, υπό ήπιες προϋποθέσεις, όσο το δυνατόν γενικότερος. Η θεωρία αυτή, που εν συντομία καλείται SMEFT, μπορεί να χρησιμοποιηθεί για την συστηματική βελτίωση των θεωρητικών προβλέψεων του SM χωρίς να προσδιορίζεται η νέα φυσική που εμφανίζεται στις υψηλότερες ενέργειες (η EFT αυτού του τύπου καλείται bottom-up), ή ώστε να απλοποιηθεί η περιγραφή μίας πληρέστερης θεωρίας αντιπαραβάλλοντάς την με το SM (η EFT αυτού του τύπου καλείται top-down). Σε αυτή τη διατριβή θα ασχοληθούμε κυρίως με την bottom-up SMEFT. Συγκεκριμένα, θα εστιάσουμε στις θεωρητικές και φαινομενολογικές πτυχές των διορθώσεων της πρώτης μη-τετριμμένης τάξης στο ανάπτυγμα της EFT, αναλύοντας σε βάθος δύο πολύπλοκες φυσικές διεργασίες: την διάσπαση του μποζονίου Higgs σε δύο φωτόνια και την διάσπασή του σε ένα φωτόνιο και ένα μποζόνιο Z σε επίπεδο ενός βρόχου. Οι υπολογισμοί αυτοί θα αποτελέσουν σημεία αναφοράς για την επίλυση τεχνικών ζητημάτων που αφορούν υπολογισμούς επιπέδου βρόχου στα πλαίσια μίας EFT, όπως είναι η συνέπεια των αποτελεσμάτων με τη συμμετρία βαθμίδας και η επανακανονικοποίηση των στοιχείων μήτρας, και θα χρησιμοποιηθούν επίσης για την επιβολή αριθμητικών περιορισμών στις ελεύθερες παραμέτρους του μοντέλου μας σε φαινομενολογικές αναλύσεις. Περαιτέρω χρήσιμες τεχνικές λεπτομέρειες των υπολογισμών έχουν συλλεχθεί στα παραρτήματα για μελλοντική αναφορά. Επιπλέον, λόγω πρόσφατων εξελίξεων στη διεθνή βιβλιογραφία, θα ασχοληθούμε με την ανάπτυξη του φορμαλισμού της SMEFT σε υψηλότερες τάξεις του αναπτύγματος της EFT, παρέχοντας τις σχετικές εκφράσεις σε αναλυτική μορφή για κάθε πιθανή τάξη του αναπτύγματος. Τέλος, εξαιτίας της μεγάλης πολυπλοκότητας των υπολογισμών στην SMEFT, θα εστιάσουμε την προσοχή μας στην ανάπτυξη ισχυρών πακέτων λογισμικού, τα οποία ευελπιστούμε πως θα αποτελέσουν μία πλατφόρμα ικανή να συνεισφέρει στην προσπάθεια της ερευνητικής κοινότητας για (ημι-)αυτοματοποίηση των υπολογισμών στην SMEFT έως και στην δεύτερη τάξη του αναπτύγματος στην EFT.

Contents

Preface

The question *"What are the building blocks of our Universe, and how do they interact?"*, seems to have puzzled the greatest minds of human history from the ancient times through present. Nowadays, we call these building blocks elementary particles, and we have reasons to believe that our understanding about elementary particles and their interactions has reached a remarkable level, since the physicists have devised a theory, called the Standard Model (SM) of elementary particle physics, which explains and predicts physical phenomena with (most of the time) very good experimental accuracy. Of course, SM is not a final theory of nature. There are numerous open problems that may be solved when we finally come up with a more general theory which gives us back the SM as a limiting case.

The elementary particle physics community is therefore assigned a new task: we have to construct new, more sophisticated theories, able to describe nature even more accurately and provide solutions to (some of) the open problems of the scientific field of high energy physics. These physical theories are generally known as Beyond the Standard Model (BSM) theories. It seems that the SM works almost perfectly in the energy scales reached by the modern experiments, so a BSM theory should probably be a physical theory that is defined in a higher energy scale (usually referred to as the ultraviolet (UV) theory), and when we take the low energy limit the SM should be the lowest-order approximation. Furthermore, a fundamental theory that would describe nature as a whole, including the gravitational force, generally known as the Theory of Everything, is the Holy Grail of Theoretical Physics (though, it is questionable how would we know that a theory is the final step in our journey of understanding the universe as a whole; such a pursuit should, therefore, be mostly viewed as an overarching ideal in our ongoing attempts to mathematically model and understand nature more accurately).

Each BSM theory can be defined by devising a self-consistent mathematical framework which is required to respect some postulates dictated by experimental and observational data. But there is a problem in this simple statement: even if we have some clues pointing towards the right direction of what the UV theory should look like, the paths that may lead there can in practice be numerous. Therefore, it requires excellent skill and intuition for one to not get lost on the endless possibilities and stay on the right track. One way out of that problem is to avoid specifying a high energy theory and instead use a general model which introduces a set of corrections to the SM. This approach of constructing an Effective Field Theory (EFT) is justified by a very important feature of nature, which is the non-interference of the physics at different energy scales. To elaborate, physical phenomena at a given energy scale do not have a direct effect on the physics at much lower energy scales, in the sense that we can model the low energy physics and make predictions without needing to resort to the higher energy physics. The advantage with this approach is that we can construct a theory even without explicit knowledge of the underlying UV physics, and then use this theory in order to systematically improve our theoretical predictions. The price we have to pay, however, is that we introduce a lot of undetermined parameters in our model.

This is indeed the main idea behind the theoretical structure known as the Standard Model Effective Field Theory (SMEFT). We start by considering the well-known SM spectrum and gauge symmetry as the underpinning upon which we wish to build a more capable theory for the description of elementary particle physics. To accomplish this, we construct all possible gauge invariant operators that are omitted in the SM framework and then we add them to the SM Lagrangian. These operators have dimension higher than 4 and each one is accompanied by an undetermined coefficient. The effective operators are suppressed by inverse powers of a single UV energy scale and therefore act as small corrections to the SM Lagrangian. By categorising the EFT corrections according to their UV scale dependence, we are able to construct a power series expansion of the SMEFT Lagrangian where the SM is recovered as the zeroth-order term and the contribution from each new expansion order is less dominant than from the previous one. Therefore, one can improve the accuracy of the theoretical calculations in a systematic manner.

For the first part of this thesis, starting with the leading non-trivial order EFT corrections, we consider two technically challenging processes of great physical interest: the Higgs boson decay to two photons and its decay to a photon and a Z-boson, both at one-loop order in the $\hbar$ expansion. These highly non-trivial calculations are presented in detail, and we focus our attention on several technical issues of interest for the theoretical and physical consistency of the calculation. Specifically, we prove analytically the gauge invariance of our results, we develop a concise renormalisation framework for the EFT amplitudes and, consequently, we prove the cancellation of the infinities in the physical results. Finally, we are also concerned with the phenomenological analysis of the results, by using the experimental data provided by the Large Hadron Collider (LHC) in order to place bounds in the unspecified parameters of the model.

As emphasised above, calculations within the SMEFT framework tend to be remarkably lengthy and demanding, firstly because of the mere number of the added effective operators and secondly because of the added complexity of these operators in comparison to the SM ones. Furthermore, as the recent literature starts delving into the next-to-leading EFT order, this problem will only grow exponentially in the future. The above indicate the increasing demand for software tools in order to minimise the physical labour and to increase the efficiency of calculations within the SMEFT. For the second part of this thesis we, therefore, focus our attention on the development of a code that provides the user with the manipulations necessary to derive the physical basis of the SMEFT for a given set of effective operators and produces the full set of Feynman rules with outputs that can be used as inputs in other high-energy physics software for analytic and/or numeric calculations. This code will hopefully extend the range of much-needed tools in our effort to perform consistent (semi-)automatised calculations up-to the next-to-leading EFT order. Additionally, in the appendices we provide an extension of the SMEFT formalism to any possible order in the bosonic EFT expansion, explaining in detail the methodology of obtaining the results. These results are presented in compact analytic formulae and can be readily used as a basis for future SMEFT studies in which the theoretical calculations will need to be of even higher accuracy to match the new and improved experimental data of the LHC and future colliders.

Πρόλογος
(Preface in Greek)

Η ερώτηση *"Ποιοί είναι οι θεμέλιοι λίθοι του σύμπαντος και με ποιόν τρόπο αλληλεπιδρούν;"*, φαίνεται πως έχει προβληματίσει κάποια από τα σπουδαιότερα άτομα της παγκόσμιας διανόησης ήδη από τους αρχαίους χρόνους. Στην σημερινή εποχή καλούμε αυτούς τους δομικούς λίθους στοιχειώδη σωμάτια και έχουμε καλούς λόγους να θεωρούμε πως η κατανόηση μας για τα στοιχειώδη σωματίδια και τις αλληλεπιδράσεις τους έχει φτάσει σε ένα αξιοσέβαστο επίπεδο, διότι οι φυσικοί έχουν κατασκευάσει μία θεωρία, την οποία καλούμε το Καθιερωμένο Πρότυπο (Standard Model ή SM) των στοιχειωδών σωματιδίων, το οποίο εξηγεί και προβλέπει τα φυσικά φαινόμενα με πολύ καλή (τις περισσότερες φορές) συμφωνία με τα πειραματικά δεδομένα. Παρόλα αυτά, το SM δεν είναι η τελική θεωρία που περιγράφει τη φύση. Υπάρχουν πολλά ανοιχτά προβλήματα που πιθανώς να λυθούν όταν κατασκευάσουμε μία πιο γενική θεωρία η οποία θα μας επιστρέφει το SM ως μία οριακή περίπτωση.

Επομένως, οι φυσικοί των στοιχειωδών σωματιδίων θα πρέπει να κατασκευάσουν νέες, πιο πλήρεις θεωρίες, ικανές να περιγράψουν τη φύση με ακόμη μεγαλύτερη ακρίβεια και να παρέχουν λύση σε κάποια από τα ανοιχτά προβλήματα του πεδίου της φυσικής των υψηλών ενεργειών. Οι φυσικές αυτές θεωρίες είναι εν γένει γνωστές ως θεωρίες πέραν του Καθιερωμένου Προτύπου (Beyond the SM ή BSM θεωρίες). Το SM φαίνεται πως συμπεριφέρεται σχεδόν άψογα στις ενεργειακές κλίμακες που αγγίζουμε με τα σύγχρονα πειράματα, συνεπώς μία BSM θεωρία θα πρέπει κατά πάσα πιθανότητα να είναι μία φυσική θεωρία ορισμένη σε μία υψηλότερη ενεργειακή κλίμακα — θα αναφερόμαστε σε αυτήν ως την υπεριώδη (ultraviolet ή UV) θεωρία — και όταν παίρνουμε το όριο της UV θεωρίας στις χαμηλές ενέργειες το SM θα πρέπει να προκύπτει ως η προσέγγιση χαμηλότερης τάξης. Επιπλέον, μία θεμελιώδης θεωρία που θα περιγράφει τη φύση στο σύνολό της, η Θεωρία των Πάντων, αποτελεί το Ιερό Δισκοπότηρο της θεωρητικής φυσικής (βέβαια, η ερώτηση για το εάν θα μπορούσαμε να γνωρίζουμε ότι μία θεωρία αποτελεί τον τελικό σταθμό στο ταξίδι μας για την κατανόηση του σύμπαντος στο σύνολό του δεν έχει σίγουρη απάντηση· η αναζήτησή μας, λοιπόν, για μία τελική θεωρία θα πρέπει να τεθεί υπό το πρίσμα της συνεχούς προσπάθειας να βελτιώσουμε τα μαθηματική μοντελοποίηση της φύσης και την κατανόησή μας για αυτήν).

Κάθε BSM θεωρία μπορεί να περιγραφεί μέσα από ένα αυτοσυνεπές μαθηματικό πλαίσιο, υπό την απαίτηση ότι το τελευταίο θα σέβεται κάποια αξιώματα που προκύπτουν από τα πειραματικά και παρατηρησιακά δεδομένα. Υπάρχει όμως ένα πρόβλημα με αυτόν τον τρόπο σκέψης: ακόμη και εάν έχουμε κάποια στοιχεία για την μορφή της UV θεωρίας, πιθανώς να υπάρχει μεγάλο πλήθος από μονοπάτια που οδηγούν σε αυτή. Συνεπώς, απαιτείται ένα υψηλό επίπεδο τεχνικής κατάρτισης και φυσικής διαίσθησης έτσι ώστε να μην παρεκκλίνουμε από την σωστή κατεύθυνση. Ένας τρόπος για να διαφύγουμε από αυτό το πρόβλημα είναι να αποφύγουμε τεχνηέντως τον ακριβή προσδιορισμό της UV θεωρίας και αντ' αυτού να χρησιμοποιήσουμε ένα γενικό μοντέλο το οποίο θα εισαγάγει ένα σύνολο από διορθώσεις στο SM.

Η προσέγγιση αυτή, που εμπλέκει την κατασκευή μίας ενεργού θεωρίας πεδίου (Effective Field Theory ή EFT), στηρίζεται σε μία πολύ σημαντική ιδιότητα της φύσης, τον διαχωρισμό της φυσικής σε διαφορετικές ενεργειακές κλίμακες. Πιο συγκεκριμένα, φυσικά φαινόμενα που λαμβάνουν χώρα σε μια ορισμένη ενεργειακή κλίμακα δεν έχουν άμεση επίδραση στη φυσική που περιγράφει πολύ χαμηλότερες ενεργειακές κλίμακες, υπό την έννοια ότι μπορούμε να μοντελοποιήσουμε την φυσική στις χαμηλές ενέργειες και να κάνουμε προβλέψεις χωρίς να χρειαζόμαστε γνώση για την φυσική που λαμβάνει χώρα στις υψηλές ενέργειες. Το όφελος με αυτή την προσέγγιση είναι ότι μπορούμε να κατασκευάσουμε μία θεωρία χωρίς πλήρη γνώση της πιο θεμελιώδους UV θεωρίας και έπειτα να χρησιμοποιήσουμε τη θεωρία αυτή ώστε συστηματικά να βελτιώσουμε τις θεωρητικές μας προβλέψεις. Το κόστος που καλούμαστε να πληρώσουμε, ωστόσο, είναι ότι εισαγάγουμε ένα μεγάλο πλήθος απροσδιόριστων παραμέτρων στο μοντέλο μας.

Αυτή είναι η κύρια ιδέα πίσω από το θεωρητικό οικοδόμημα γνωστό ως Standard Model Effective Field Theory ή, εν συντομία, SMEFT. Ως πρώτο βήμα, αντιμετωπίζουμε την ευρέως γνωστή συμμετρία βαθμίδας και το σωματιδιακό φάσμα του SM ως τα θεμέλια πάνω στα οποία προσδοκούμε να χτίσουμε μία θεωρία πιο ικανή στην περιγραφή της φυσικής των στοιχειωδών σωματιδίων. Για να το πετύχουμε αυτό, κατασκευάζουμε όλους τους δυνατούς τελεστές οι οποίοι είναι αναλλοίωτοι κάτω από τη συμμετρία βαθμίδας και δεν εμφανίζονται στα πλαίσια του SM, και έπειτα τους προσθέτουμε στην Λαγκρανζιανή του SM. Οι τελεστές αυτοί έχουν διάσταση μεγαλύτερη του 4 και κάθε ένας συνοδεύεται από έναν απροσδιόριστο συντελεστή. Οι ενεργοί αυτοί τελεστές είναι διαιρεμένοι με δυνάμεις ενέργειας μίας κοινής UV κλίμακας και συνεπώς αποτελούν μικρές διορθώσεις στην Λαγκρανζιανή του SM. Κατηγοριοποιώντας τις ενεργές διορθώσεις ανάλογα με την εξάρτησή τους από την UV κλίμακα, είμαστε σε θέση να δημιουργήσουμε μία δυναμοσειρά για την Λαγκρανζιανή της SMEFT, από όπου μπορούμε να εκμαιεύσουμε το SM ως την διόρθωση μηδενικής τάξης, με κάθε επόμενη τάξη να έχει όλο και πιο μικρή συνεισφορά. Συνεπώς, είμαστε σε θέση να βελτιώσουμε συστηματικά την ακρίβεια των θεωρητικών μας υπολογισμών.

Στο πρώτο μέρος της διπλωματικής αυτής εργασίας, ξεκινώντας από την πρώτη μη-τετριμμένη τάξη του αναπτύγματος της EFT, θεωρούμε δύο φυσικές διεργασίες που παρουσιάζουν μεγάλο φυσικό ενδιαφέρον και έχουν υψηλή τεχνική δυσκολία: την διάσπαση του μποζονίου Higgs σε δύο φωτόνια και την διάσπασή του σε ενα φωτόνιο και ένα μποζόνιο Z. Οι μη-τετριμμένοι αυτοί υπολογισμοί παρουσιάζονται με πλήρη λεπτομέρεια και εστιάζουμε την προσοχή μας σε διάφορα τεχνικά ζητήματα που παρουσιάζουν ενδιαφέρον για την θεωρητική και φυσική συνέπεια των υπολογισμών. Συγκεκριμένα, αποδεικνύουμε αναλυτικά την αναλλοιότητα των αποτελεσμάτων μας κάτω από τη συμμετρία βαθμίδας, αναπτύσσουμε ένα απλό και περιεκτικό σχήμα επανακανονικοποίησης για τα ενεργά στοιχεία μήτρας και αποδεικνύουμε τον μηδενισμό των όρων που εμπλέκουν απειρισμό στα φυσικά αποτελέσματα. Τέλος, ασχολούμαστε επίσης με την φαινομενολογική ανάλυση των αποτελεσμάτων, χρησιμοποιώντας πειραματικά δεδομένα από τον Μεγάλο Αδρονικό Επιταχυντή (Large Hadron Collider ή LHC) του CERN έτσι ώστε να θέσουμε περιορισμούς στις απροσδιόριστες παραμέτρους του μοντέλου.

Όπως τονίστηκε προηγουμένως, οι υπολογισμοί εντός του πλαισίου της SMEFT τείνουν να είναι αξιοσημείωτα μακροσκελείς και απαιτητικοί, πρώτον λόγω του μεγάλου αριθμού των πρόσθετων ενεργών τελεστών και δεύτερον λόγω της πολυπλοκότητας που αυτοί παρουσιάζουν σε σχέση με τους τελεστές του SM. Επιπρόσθετα, καθώς η σύγχρονη βιβλιογραφία κινείται προς την εξερεύνηση της επόμενης τάξης του αναπτύγματος στην EFT, το πρόβλημα αυτό αναμένεται να αυξηθεί εκθετικά στο εγγύς μέλλον. Τα παραπάνω αναδεικνύουν την αυξανόμενη ανάγκη για χρήση πακέτων λογισμικού, έτσι ώστε να ελαχιστοποιηθεί ο φόρτος εργασίας και να αυξηθεί η αποδοτικότητα στους υπολογισμούς εντός της SMEFT. Στο δεύτερο μέρος της διπλωματικής αυτής εργασίας εστιάζουμε τις προσπάθειές μας στην ανάπτυξη ενός υπολογιστικού πακέτου που εφοδιάζει τον χρήστη με τις απαραίτητες διεργασίες για την εξαγωγή της φυσικής βάσης της SMEFT για ένα δοθέν σύνολο ενεργών τελεστών και στη συνέχεια κατασκευάζει το πλήρες σύνολο των κανόνων Feynman, παράγοντας αρχεία που μπορούν έπειτα να εισαχθούν σε άλλα λογισμικά πακέτα για την τέλεση αναλυτικών ή αριθμητικών υπολογισμών. Ευελπιστούμε πως η προσθήκη αυτού του λογισμικού πακέτου στα υπολογιστικά εργαλεία της φυσικής υψηλών ενεργειών θα συνεισφέρει στην προσπάθεια μας για την τέλεση συνεπών (ημι-)αυτοματοποιημένων υπολογισμών έως και την δεύτερη μη-τετριμμένη τάξη του αναπτύγματος της EFT. Επιπλέον, στα παραρτήματα έχουμε συλλέξει την ανάπτυξη του φορμαλισμού της SMEFT σε οποιαδήποτε τάξη του αναπτύγματος του μποζονικού μέρους του EFT αναπτύγματος, αναλύοντας σε βάθος την μεθοδολογία που χρησιμοποιούμε για να παράγουμε τα αποτελέσματα. Τα αποτελέσματα παρουσιάζονται σε συμπαγείς αναλυτικές εκφράσεις που μπορούν να χρησιμοποιηθούν απευθείας σε μελλοντικές εργασίες πάνω στην SMEFT, όπου οι θεωρητικοί υπολογισμοί θα πρέπει να είναι ακόμη πιο ακριβείς ώστε να αγγίξουν την ακρίβεια των νέων πειραματικών δεδομένων του LHC και των μελλοντικών επιταχυντών σωματιδίων.

Chapter 1

Introduction

1.1 Units, Notation and Conventions

The fundamental physical constants of a theory that respects Quantum Mechanics and Special Relativity would be Planck's constant, $\hbar$, and the speed of light in vacuum, c, respectively. These constants have experimental values (all experimental data used in this thesis are taken from the Particle Data group [1])

$$\hbar = 6.582 \times 10^{-22}\,\text{MeV s}\,, \tag{1.1}$$

$$c = 2.998 \times 10^{8}\,\text{m s}^{-1}\,. \tag{1.2}$$

Through this thesis we are going to use natural units, in which

$$\hbar = c = 1\,, \tag{1.3}$$

with the electron's absolute charge defined as in the Heaviside-Lorentz system of units,

$$e = \sqrt{4\pi\alpha}\,, \tag{1.4}$$

where $\alpha = 1/137.036$ is the fine-structure constant[1]. One can always restore the missing factors of $\hbar$ and c in any formula be using dimensional analysis. A convenient unit of energy in elementary particle physics is the giga-electronvolt, GeV (roughly equal to the proton's rest energy). Having fixed $\hbar = c = 1$, we can express every quantity in terms of units of energy, as follows:

$$[\text{energy}] = [\text{mass}] = [\text{momentum}] = [\text{time}]^{-1} = [\text{lenght}]^{-1} = \text{GeV}\,. \tag{1.5}$$

[1]Evaluated at $Q^2 = 0$.

The Minkowski metric we are going to adopt here is the "mostly-minus" one, i.e.

$$(g_{\mu\nu}) = \text{diag}(1, -1, -1, -1)\,. \tag{1.6}$$

All four vectors will be symbolised with lower-case Latin letters, and we will often use index free notation, e.g. $x_\mu y^\mu = x \cdot y$, $x_\mu x^\mu = x \cdot x = x^2$, etc.

When calculating Feynman diagrams we will be referring to one-particle-irreducible (1PI) diagrams, which are graphs that stay connected if an internal line is removed by them. A connected diagram is a graph that one can trace completely by continuously following its lines. For a scattering amplitude, $\mathcal{M}$, we define

$$i\mathcal{M} = \text{sum of the connected Feynman diagrams}\,. \tag{1.7}$$

For loop Feynman diagram calculations we will use Dimensional Regularisation (DR) and Passarino-Veltman (PV) functions (for our definitions of PV functions and further discussion, see appendix B). The space-time dimensionality in which the loop-integrals are calculated is symbolised by d, and at the end of the calculations we take the formal limit $\varepsilon \to 0$, where $\varepsilon = d - 4$.

The gamma-matrices are 4×4 matrices defined by the Dirac-Clifford algebra

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}\,. \tag{1.8}$$

The γ^5 matrix anti-commutes with every γ^μ matrix, i.e.

$$\left\{\gamma^\mu, \gamma^5\right\} = 0\,. \tag{1.9}$$

We also use Feynman's slash notation, where

$$\not{\alpha} = \alpha_\mu \gamma^\mu\,. \tag{1.10}$$

1.2 Elementary Particle Physics

An elementary particle is loosely defined as a fundamental physical particle (i.e. not having a substructure, in which case it would be a composite particle). Of course, we don't always know with certainty if a particle is composite or elementary — in fact the history of high energy physics has examples where particles that once physicists believed to be elementary turned out to be composite (e.g. the atoms and later the nuclei). What physicists *can* do, however, is to expand the region in which the particles that we consider to be elementary don't appear to have any substructure, by means of lower length bounds that we get from experimental data, and making the experimental measurements increasingly more accurate.

There are four known forces in nature, the electromagnetic force, the weak and strong nuclear forces and the gravitational force. At the microscopic level the gravitational force is much weaker than the three other forces (by an enormous factor of around 10^{40}). Therefore, it is clear that at a very good approximation one can completely ignore the gravitational interactions when trying to construct a theory capable of describing elementary particles and their interactions at the energies reached in the modern era particle colliders. On the other hand the gravitational force, which dominates at very large length scales, is described very accurately by Einstein's theory of General Relativity, and in this case it suffices to completely ignore the microscopic structure of the objects. It is at very small length scales, comparable to the Planck length $l_P = \sqrt{\hbar G/c^3} \sim 10^{-35}$ m, with G being Newton's gravitational constant, that the quantum fluctuations of space-time are expected to be important. At such scales General Relativity will have to be replaced with a quantum description of gravity.

As of today we know of a collection of particles which don't have a composite structure (or don't appear to in the energy scales reached by our experiments). These particles synthesise the *spectrum* of the Standard Model (SM). We may divide the spectrum into "matter" particles (charged leptons, neutrinos and quarks) and mediators of forces (photons, gluons, the $W^{\pm}$ and Z^0 bosons and the Higgs boson). More on that on section 1.4. For some pedagogical textbooks on elementary particle physics, see refs. [2–4].

1.3 The Mathematical Framework: Quantum Field Theory

To be successful in our study of Elementary Particle physics, abandoning all biases infused to us by our everyday life experience (the physical laws of which fall into the territory of Classical Mechanics) is a critical step. There are two reasons for that. An elementary particle is, by definition, a microscopic object, and therefore we need to use Quantum Mechanics (QM) to describe its dynamics. As a further complication, elementary particles often travel with speeds comparable to the speed of light, and therefore their kinematics are to be described in terms of Einstein's Special theory of Relativity (SR). Both theories are well-known, but simply trying to fuse them together gives rise to a plethora of serious problems. The mathematical framework that combines the two theories with success is known as Quantum Field Theory, and in this section we will briefly review some of its features.

1.3.1 General aspects of a QFT

A detailed analysis of the mathematical framework needed to describe elementary particles lies far beyond the scope of this introduction, but we are going to give an intuitive hint towards the right direction. As we know, the basic geometric idea behind SR is that space and time are not to be seen as different entities, but rather as components of a more general entity, the space-time four-vector $x^\mu = (t, \mathbf{x})$. Here lies the problem: in QM, space is promoted to an operator acting on an abstract Hilbert space of state vectors, while time remains simply a label on the Hamiltonian. Clearly, this distinction between time and space is not how a relativistic theory should behave. Therefore, we have to find a way to modify our description so that time and space are on equal footing.

One solution would be to promote time to an operator and, in turn, describe the space-time four-vector by an operator as well. It turns out that this approach is technically much more difficult than the second one which we will consider here. The second solution is to demote the space three-vector from an operator to just a label (as it originally was in Classical Mechanics). Then we could follow the SR paradigm and construct the space-time four-vector, use that covariant label as an argument on a function, say $\phi(x)$, and use functions of that type to construct our action. These functions are classical fields spanning all space-time, and the final step is to quantise these classical fields to make the transition to a quantum theory.

That is the starting point of the mathematical theory we use to describe the physics of elementary particles, which is known as Quantum Field Theory (QFT). Some of the most well-known QFT textbooks can be found at refs. [5–13].

1.3.2 Formulating a QFT

As in Classical Field Theory, the starting point of formulating our model is to postulate an action functional,

$$S = \int dt\, L = \int d^4x\, \mathcal{L}\,, \tag{1.11}$$

where L is the Lagrangian and $\mathcal{L}$ is the Lagrangian density, which is a function of the fields and their space-time derivatives.[2] That functional is still written in terms of classical fields. Then, Hamilton's principle states that the action should be stationary. This principle, also known as the *principal of stationary action*, can be elegantly expressed as

$$\delta S = 0\,. \tag{1.12}$$

[2]In the rest of this thesis we are only going to use Lagrangian densities, and, as is customary in the literature, we are going to drop the prefix 'density'.

Focusing on a Lagrangian that depends on a field ϕ and its first space-time derivative,

$$\mathcal{L} = \mathcal{L}(\phi, \partial\phi)\,, \tag{1.13}$$

we apply the stationary-action principle for small variations of the field ϕ in a closed region, and by assuming that the variation of the field vanishes in the boundary of the region we end up with the Euler-Lagrange equations for the field:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = 0\,. \tag{1.14}$$

This is the Equation of Motion (EoM) for the field ϕ. As an example, for the Klein-Gordon Lagrangian for a real scalar field ϕ,

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \tfrac{1}{2} m^2 \phi^2\,, \tag{1.15}$$

the EoM turns out to be

$$(\partial^2 + m^2)\phi = 0\,. \tag{1.16}$$

In the models used in this thesis we are going to assume that the fields vanish at spatial infinity, and therefore surface terms in the action (or, equivalently, total derivatives in the Lagrangian) can be dropped. In our Klein-Gordon example we can for instance integrate by parts in the kinetic term and write our new Lagrangian as:

$$\mathcal{L} = -\tfrac{1}{2}\phi\big(\partial^2 + m^2\big)\phi\,. \tag{1.17}$$

We now have introduced higher derivative terms and the Euler-Lagrange equations should be modified accordingly. The generalisation is simple:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) + \partial_\mu \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \phi)} \right) - \ldots = 0\,, \tag{1.18}$$

where the dots represent the terms that would correspond to higher orders of derivatives in our Lagrangian. It is clear that the equation of motion for our field remains the same, as expected.

Using methods similar to the ones used to derive the Euler-Lagrange equations, let us study what happens when a set of continuous transformations $\delta_i \phi$ leave the Lagrangian of the system invariant. In this case, using the equations of motion derived above, we are left with

$$\partial_\mu j^\mu = 0\,, \quad \text{where} \quad j^\mu = \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta_i \phi \right). \tag{1.19}$$

The above result is known as Noether's theorem [14] and it states that continuous symmetries of the theory lead to conserved quantities. The final step to go from the classical to the quantum theory, is to quantise our fields, which is done either by canonical quantisation or by using the path integral formulation. This analysis if far beyond the scope of this introduction; the reader is referred to the QFT textbooks [5–10, 12, 13].

The action that serves as a postulation of a QFT should be the most general functional with the following properties: it is invariant under Poincare transformations (or under a local Lorentz symmetry if want to incorporate gravity as well, since the translation invariance included in the Poincare group is a special feature of the Minkowski space-time), it is real (so probabilities are conserved in the quantum level), it comes from a Lagrangian that is local and at most bilinear in the derivatives of the fields (for EoMs to be at most second order differential equations), and finally it is left invariant under all transformations which are symmetries of the physical system at hand (the internal symmetries of the system).

Speaking of internal symmetries, we take the opportunity to discuss the consequences of a specific type of internal symmetry which plays a crucial role in the field theories that describe elementary particles. The consequences of this symmetry — though it isn't a physical symmetry as we are going to discuss below — play a crucial role in the algebraic manipulations of amplitude calculations, and will be thoroughly examined in part I of this thesis which is dedicated to the theoretical and phenomenological analysis of the Higgs boson decays to two photons (chapter 2) and to one photon and a Z boson (chapter 3). This internal symmetry is non other than *gauge invariance*.

1.3.3 Gauge symmetries and gauge fixing

To construct a QFT able to describe a system of elementary particles, we usually have to impose in the action a continuous symmetry, described by a Lie group, with the purpose of reflecting the actual symmetries of the physical system under consideration. That is what we call a *global* symmetry. Then, using only the *matter* fields (i.e. fermions and scalar bosons) that we want to be part of the spectrum of the theory, we write down the most general Lagrangian that respects that global symmetry. As an example in this subsection we study the theory of Quantum Electrodynamics (QED), the theory that describes the electromagnetic interactions. The starting point is the Dirac Lagrangian,

$$\mathcal{L}_{\text{Dirac}} = \overline{\psi}\big(i\partial\!\!\!/ - m_\psi\big)\psi\,, \tag{1.20}$$

where ψ is a Dirac field describing a charged fermion with mass m_ψ, $\partial\!\!\!/ \equiv \gamma_\mu \partial^\mu$ and $\overline{\psi} \equiv \psi^\dagger \gamma^0$. The Lagrangian (1.20), which describes a free fermion field, is invariant under a *global* U(1) symmetry,

$$\psi \to e^{i\alpha}\psi\,, \qquad \psi^\dagger \to e^{-i\alpha}\psi^\dagger\,, \tag{1.21}$$

where α is a real constant.

The next step is to promote the global symmetry to a *local* symmetry, i.e. a symmetry which depends on the space-time argument, x. In the literature local symmetries are more usually called *gauge* symmetries. Then, we demand invariance of the Lagrangian under the gauge symmetry. Here is where a remarkable thing happens: for the Lagrangian to be invariant under the local symmetry, we need to introduce some vector fields with specific transformation properties, and these fields turn out to be the mediators of forces of our theory! That is the elegance of a gauge theory: the symmetries of the system give us the forces without having to introduce them by hand. For the Dirac Lagrangian, we replace $\alpha \to e\,f(x)$, where e is a real constant, so we make the global U(1) symmetry a *gauge* symmetry. For the Lagrangian (1.20) to be invariant under the gauge symmetry, we have to substitute the ordinary derivative with a *gauge covariant* derivative,

$$D_\mu = \partial_\mu - ieA_\mu\,, \tag{1.22}$$

where the gauge field A_μ should transform as

$$A_\mu \to A_\mu - \partial_\mu f\,. \tag{1.23}$$

Finally, we need to add a gauge invariant kinetic term for that gauge field. Defining the field strength tensor as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the kinetic term turns out to be equal to $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. Finally, the complete QED Lagrangian (still at the classical level) becomes

$$\mathcal{L}_{\rm QED} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\not{D} - m_\psi)\psi\,. \tag{1.24}$$

The massless gauge field A_μ is identified with the photon field, and the real constant e plays the role of the electric charge.

When we take the path integral to quantise a gauge theory (we use the path integral language for convenience; the same things happen with every quantisation method), it is going to sum over all gauge field configurations, even the ones that are connected by the gauge symmetry. This is a bit problematic. We are going to get multiple copies of the physical result, so the integration will left as with a non-enumerable infinite constant to multiply our result, the volume of the total gauge group. In fact that problem is not a serious one, since we can absorb that infinite multiplication constant in the normalisation of the path integral measure. But there is another far more troublesome complication when we want to use perturbative methods with our theory. Due to the gauge symmetry, quadratic terms of the gauge field acquire *zero modes*, thus we cannot inverse these expressions to get the propagators of the theory.

A solution to that problem is to integrate over only the physically distinct gauge field configurations (that is, the ones that are not connected via gauge transformations). We glimpse the details and we just declare that this particular solution can be achieved by adding to the Lagrangian a *gauge fixing term*. For QED the gauge fixing term is

$$\mathcal{L}_{\rm GF} = -\tfrac{1}{2\xi}(\partial_\mu A^\mu)^2\,, \tag{1.25}$$

and it should be added in the QED Lagrangian (1.24).

Notice the parameter ξ in eq. (1.25). The *gauge invariance* of the physical results dictate that this parameter is *not* a physical one; it is just a real constant appearing on the gauge fixing term. Therefore, one can fix that parameter to any value she finds convenient; this is equivalent to choosing a gauge. Due to ξ, these gauges are called linear R_ξ-gauges. Some of the most popular choices for ξ include:

- $\xi = 1$, the ‘t Hooft-Feynman gauge,
- $\xi = 0$, the Landau gauge,
- $\xi \to \infty$, the unitary gauge.

The unitary gauge is somewhat special, in the sense that for spontaneously broken theories *only* the physical particles and the physical polarisations of the gauge bosons appear in that gauge. There is an interesting implication of that: the purely ξ-dependent part of a calculation, which we define to be equal to the full R_ξ result minus the unitary gauge result, doesn't contain any physical information and should therefore add to zero. This independence of the physical results on the gauge fixing parameter ξ, which is a consequence of gauge invariance, can serve as a non-trivial check for theoretical calculations. For example, one may check that the results obtained with different ξs are equivalent. But the most strict check is to calculate everything leaving ξ as an undetermined parameter and show that, at the end of the calculation, all ξ-dependent terms exactly cancel. That is precisely what we are going to do for the calculations presented in part I thesis.

1.4 The Standard Model of Elementary Particle Physics

As a model describing elementary particle physics, SM is embodied in the mathematical framework of Quantum Field Theory. From that viewpoint, particles are the excitations of quantum fields that exist in space-time. The SM, also known as the Glashow-Weinberg-Salam model [15–17], reflects for almost half a century now humanity's best understanding of high energy physics. A historical time-line, both from an experimental and theoretical point of view, of the fascinating story of how the SM was established, can be found in most elementary particle textbooks. Here we are going to adopt a modern, mostly axiomatic, viewpoint to briefly review its basic concepts.

SM is based on a non-abelian gauge theory. More precisely, the corresponding gauge group is

$$\mathrm{G_{SM}} = \mathrm{SU(3)_C} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}\,, \tag{1.26}$$

where indices C, L, and Y stand for *colour*, *left* and *hypercharge*, respectively. Fermion fields are represented by left-handed Weyl fields in the representation (see ref. [18] for a review article about constructing fermionic theories)

$$\left(1,2,-\tfrac{1}{2}\right) \oplus (1,1,-1) \oplus \left(3,2,+\tfrac{1}{6}\right) \oplus \left(\bar{3},1,+\tfrac{2}{3}\right) \oplus \left(\bar{3},1,-\tfrac{1}{3}\right) \tag{1.27}$$

of $\mathrm{G_{SM}}$, where the fields are (from left to right): a lepton $\mathrm{SU(2)_L}$ doublet (containing a left-handed neutrino and a left-handed electron), a right-handed electron, a quark $\mathrm{SU(2)_L}$ doublet (containing a left-handed up-type and a left-handed down-type quark), an up-type right-handed quark and a down-type right-handed quark (there are three generations of fermions, each of which is assigned to a distinct copy of the above representation).

In order to introduce masses for the various SM massive particles in a gauge invariant way, one has to make use of the *Higgs mechanism* [19–26]. The $\mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$ part of eq. (1.26) (the *electroweak* part of the SM) is considered as an exact symmetry of nature at very high energies. We introduce a scalar field φ in the $\left(1,2,+\tfrac{1}{2}\right)$ representation of the gauge group $\mathrm{G_{SM}}$. That field is known as the *Higgs field*, and it acquires a *non*-zero vacuum expectation value (VEV) at around 246 GeV. The VEV triggers the *spontaneous symmetry breaking* of the electroweak sector of the SM:

$$\mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y} \to \mathrm{U(1)_{QED}}\,. \tag{1.28}$$

What we are left with is the gauge symmetry of Quantum Electrodynamics, which remains as an exact symmetry. A prediction of the SM is that a component of the Higgs doublet remains as a massive, neutral particle, known as the *Higgs boson*. Its discovery [27, 28] is one of the greatest indications in favour of the SM. A great survey for the SM Higgs boson can be found at refs. [29, 30]. In figure 1.1 we present the spectrum of the SM. For a modern summary of the SM as a gauge theory accompanied by the complete set of Feynman rules for it, see ref. [31].

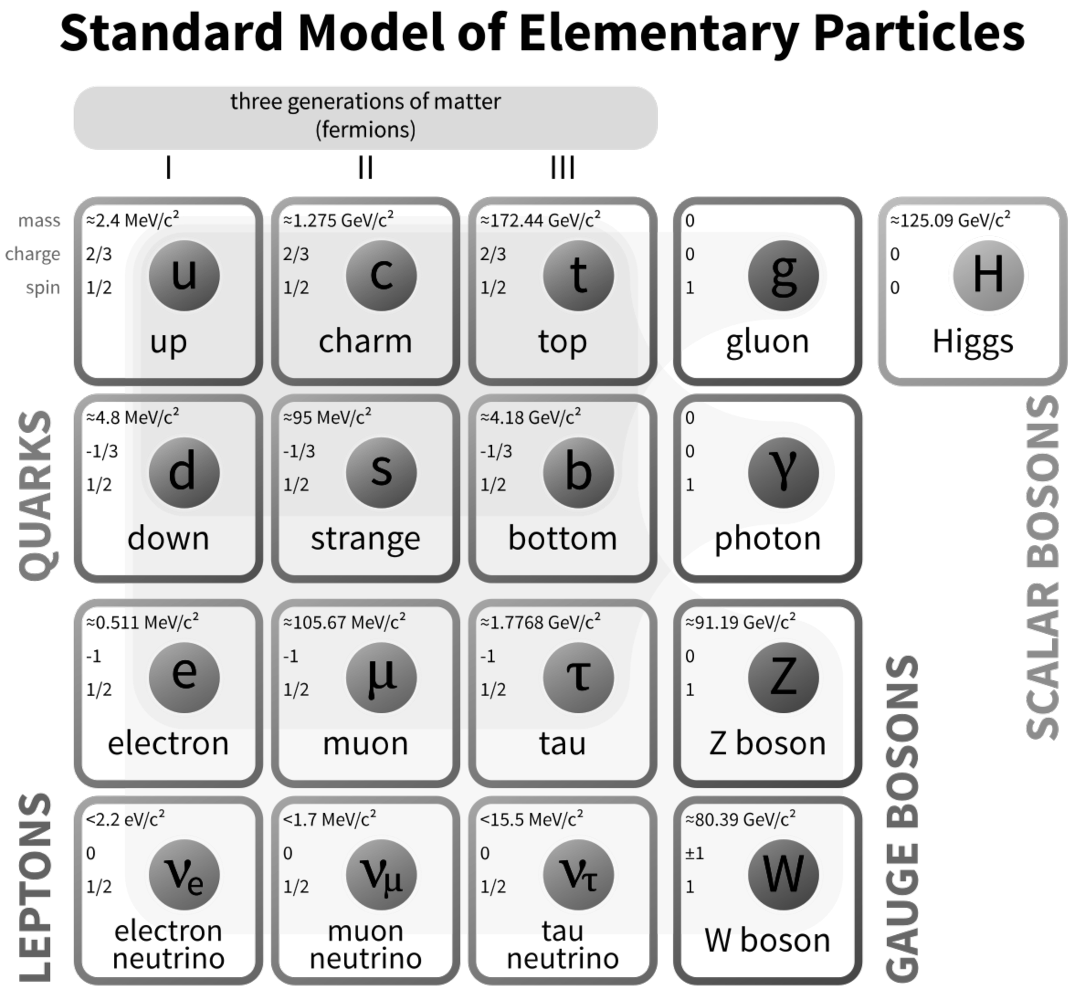


Figure 1.1: Particle spectrum of the Standard Model of elementary particle physics.

As we pointed out many times thus far, SM is clearly *not* a final theory of nature and therefore we have to come up with ways to generalise it. The rest of that introduction is dedicated to discuss the theory that we are going to use in this thesis, and which tries to shed light on what lies beyond the SM.

1.5 Effective Field Theories

A very important feature of Nature is the fact that the physics at a given energy scale aren't directly affected by the physics at much higher energy scales. This statement becomes obvious by simply examining well-established physical theories. For example, Classical

Mechanics is a theory perfectly capable of making predictions about everyday life phenomena without any reference to quantum physics or relativistic effects. That is not to say that the underlying physics is completely irrelevant. For example, we may be perfectly satisfied with just measuring the temperature of an object, but that doesn't change the fact that the physical quantity known as temperature is actually an aftermath of the motion of the particles that compose the object. In this example a macroscopic phenomenon, described by Thermodynamics, is clearly the *remnant* of a microscopic phenomenon, which is described by Statistical Mechanics, but the important thing is that one can actually use the laws of thermodynamics and make accurate predictions about the temperature of this object practically without taking into account all these microscopic effects.

It should come as no surprise that this very powerful feature of Nature is already utilised in the study of elementary particle physics. The classic example here is Fermi's theory [32, 33], which was originally proposed as an explanation for the beta decay. This simple QFT model assumes contact 4-fermion interactions through which processes like the beta decay or the muon decay are possible. Each vertex should be multiplied with an unknown dimensionful coefficient, but the simplicity of this theory makes it very easy to make some interesting predictions. Using the muon decay, $\mu \to e\,\nu_\mu\,\bar{\nu}_e$, as our example, one can compute the decay width of the muon and compare with the experimental data in order to estimate that the scale of the new physics is of order 100 GeV. From the modern viewpoint, Fermi's theory for the muon decay can be considered as an approximation of the SM where the mediator in the muon decay diagram, the charged electroweak gauge boson, has been integrated out. Since the mass of this heavy degree of freedom is about 80 GeV, we can see that the prediction of Fermi's theory is remarkably successful.

To generalise the above, let us assume that we are running an experiment at an energy scale E_{low}. When we try to construct a theory that will be able to explain the results of the experiment, we are usually not interested on what the physics look like on energy scales E_{high}, where $E_{\text{high}} \gg E_{\text{low}}$, but instead we make use of a more suitable 'effective' description, i.e. an approximation that neglects effects of the order $\mathcal{O}(E_{\text{low}}/E_{\text{high}})$. This is the essence of an effective theory. In this thesis we are interested in the physics of elementary particles, and since the use of field theory dominates this sub-area of physics, we are going to specialise in the effective theory description of a field theory, which is known in the literature as an Effective Field Theory (EFT) description. There are two general approaches on EFTs: the top-down and the bottom-up approach. They differ on many aspects, so let us briefly discuss them in turns.

1.5.1 Top-down EFTs

Most of the times in physics it is much simpler to use convenient approximations when we try to perform explicit calculations. For example, even if we have a well-constructed theory for the microscopic regime, in that case QM, we are not going to use it to calculate quantities in the macroscopic region, where Classical Mechanics is accurate. The same principle is applied in elementary particle physics: when we want to describe a low-energy phenomenon, even if we have a high-energy theory that works perfectly fine it is sometimes more convenient to simplify our analysis by considering limiting of the high-energy theory before we proceed to the actual calculation.

The example used above, Fermi's theory, is nowadays used in a top-down fashion. The SM, which is the UV completion of Fermi's theory, may be too complicated to use for some complex low-energy fermionic processes, for example in nuclear physics. Therefore, we integrate out the heavy degrees of freedom from the SM spectrum and we are left with simple effective 4-fermion contact terms for the diagrammatic calculations. By performing the *matching* of the UV theory to the EFT, the parameters of the UV theory are used to derive the coefficients of the effective operators (the Wilson coefficients). In this case the Wilsons aren't undetermined parameters, since they are known functions of the UV parameters.

The top-down EFT approach and the matching is beyond the scope of this thesis. For the interested reader, we are just going to refer here to the leptoquark extension of the SM [34]. Since the matching of this physically interesting UV extension to the SMEFT has been performed in the literature using traditional Feynman diagrammatic techniques [35] and by using modern functional matching [36], it may serve as a showcase for the technical details.

1.5.2 Bottom-up EFTs

This thesis is focused around the second type of EFTs mentioned above, the bottom-up EFTs. Whilst in the top-down EFT approach we have knowledge of the high-energy theory and we just use convenient approximations to simplify our calculational tasks, the bottom-up EFT approach works the opposite way: it serves as an educated guess, by using a known low-energy theory as a stable foundation, and then extending it in a systematic manner to accommodate the finer details of the physics that the model describes. These small corrections enter this low-energy regime as the remnants of an (unknown) UV theory.

Lets say that we are ignorant about the high-energy theory, but at lower energies, where our experiments take place, we have a theory that works pretty well at explaining and predicting the physical phenomena. In fact, this is exactly the situation that we experience today: on one hand, the SM works very well at the energy scales that we can currently reach

Dimensionality	Renormalisation	Relevance
$D < d$	super-renormalisable	relevant
$D = d$	renormalisable	marginal
$D > d$	non-renormalisable	irrelevant

Table 1.1: Categorising operators according to their dimensionality, D, for a d-dimensional space-time.

with the Large Hadron Collider (LHC), the state of the art experiment for modern high energy physics,[3] but on the other hand, the SM lies far from being a candidate for the final theory of nature. The bottom-up approach starts from the low-energy theory and extents it by simply adding new terms as a power series on a small parameter, say ϵ. For example, it is common to use the ratio of the high- and low-energy scales, $E_{\mathrm{low}}/E_{\mathrm{high}}$, as this small parameter. The very important aspect of an EFT is that we can expand in the parameter ϵ systematically,

Of course the extra terms add to the calculational work, making it even more laborious than before, but now we get to see how these new terms, which interpret new physics effects, affect our calculations. Furthermore, we can cross-check our theoretical results against the experimental ones, and that (hopefully) will give us some hints about the new physics, extending our understanding about the high energy regime. For the rest of this introduction, we are going to focus on the EFT of the SM, known as the SMEFT, which will be the model under consideration for the rest of this thesis. We have already discussed the SM at section 1.4, but now we will get to see its extension under the prism of a bottom-up EFT.

1.6 The Standard Model Effective Field Theory

The SM is a gauge QFT, with the gauge group $\mathrm{G_{SM}} = \mathrm{SU(3)_C} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$ dictating the symmetries and interactions of the model (see section 1.4). As a QFT, the SM is postulated by defining its Lagrangian, which should be taken to be the most general one with the requirement that its operators are invariant under the gauge group $\mathrm{G_{SM}}$. There is one additional assumption when constructing the SM Lagrangian, however. We further restrict the allowed operators that are to be added to the Lagrangian by demanding them to have mass-dimension (we will usually call it simply dimension in the rest of the text) less

[3]With the current LHC run-2 data we are pretty positive that, excluding the possibility of particles that may be extremely weekly coupled, there are no new particles up-to around 800 GeV.

than or equal to 4. Before moving further, let us pose here to briefly explain what this new nomenclature means, and also introduce some more terms we will need for our discussion.

In units where $\hbar = c = 1$ all physical units can be re-expressed as units of mass to some power (see section 1.1), e.g. a particle's mass, energy and momentum all have mass-dimension $+1$, $[m] = [E] = [p] = 1$, its wavelength has mass-dimension -1, $[l] = -1$, etc. Since $\hbar = 1$, the action functional is dimensionless, $[S] = 0$. Assuming that d is the dimensionality of space-time, and since $[d^d x] = -d$, the Lagrangian must have dimension $[\mathcal{L}] = d$ for the action to be dimensionless. Every Lagrangian term consists of the product of an operator and a coefficient. With arguments similar to the above we can find the dimensionality of any operator in the Lagrangian. For example, considering the mass term for a real scalar field, $\frac{1}{2}m^2\phi^2$, it's trivial to conclude that $[\phi] = \frac{d-2}{2}$. Repeating this procedure, we find the dimensions for a generic scalar field ϕ, a generic fermion field ψ and a field strength tensor F, to be

$$[\phi] = \tfrac{d-2}{2}\,, \quad [\psi] = \tfrac{d-1}{2}\,, \quad [F] = \tfrac{d}{2}\,. \tag{1.29}$$

With this information we are able to calculate the dimension of every operator, Q, in the Lagrangian and, since $[\mathcal{L}] = d$, the dimension of the operators' accompanying coefficient, C, will be $[C] = d - [Q]$.

The above leads us to the important conclusion that the operators with dimension greater than d must be multiplied with coefficients with negative mass-dimension. These dimensionful coefficients can be re-expressed as dimensionless ones divided by appropriate powers of an energy scale Λ. It is convenient to identify this energy scale with a physical quantity, the energy scale where new physics effects take place. We generally refer to that as the UV energy scale. Therefore, these operators are suppressed by a UV energy scale and thus have increasingly less impact as we decrease the energy scales we examine (we say that operators of that type are *irrelevant* at the macroscopic (i.e. low-energy) regime). The opposite is true for operators with dimension less than d (so these are *relevant* operators), whilst operators of dimension d need further examination to reveal their behaviour (we call them *marginal* operators, for living on the border of the two regions, and under the circumstances they can be either marginally relevant or marginally irrelevant). To sum up, there are three different types of operators:

- Relevant operators, with dimension $< d$,
- Marginal operators, with dimension $= d$,
- Irrelevant operators, with dimension $> d$.

Irrelevant operators are usually called non-renormalisable in the literature. In table 1.1 we present a synopsis of the categorisation of the operators according to their relevance and

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^*\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu^I\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu^I\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

Table 1.2: Dimension 6 operators in Warsaw basis other than the four-fermion ones [37].

their renormalisation properties as a function of their dimensionality. The SM, containing only relevant and marginal operators (in four space-time dimensions), consists what in the literature is called a renormalisable theory (for a review of renormalisation in the SM, see ref. [38]). This categorisation of operators based on their 'renormalisability' may be somewhat misleading. Let us see where this term come from. To renormalise a theory that consists only of relevant and marginal operators, we can define a finite number of counterterms and then use only this set of counterterms to do the renormalisation of the theory to each order in the loop-expansion. The higher-dimension operators complicate this procedure. Even if we include only one effective operator in our renormalisable theory, with a coefficient C such that $[C] = \Lambda^{-1}$, it is easy to see the problem that arises. Using a double insertion of this operator in a divergent diagram, we need a counterterm of dimension Λ^{-2} to cancel the infinities. By adding more and more insertions we need higher and higher orders of counterterms to make the theory finite. These counterterms indicate that one cannot omit the higher-dimensional operators by which they are derived. We are therefore facing a disaster here: if we try to renormalise our theory, the insertion of a single effective operator generates an infinite number of new operators and counterterms.

There is a trivial solution to this problem. As we discussed above, the effective operators are *suppressed* by the UV energy scale. Therefore, the inverse of this scale serves as a

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jn}\epsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$		

Table 1.3: Four-fermion dimension 6 operators in Warsaw basis [37].

perturbation parameter in our model and, as such, it can be truncated at some order that we choose. This order is usually chosen to be the one which allows for the theoretical calculations to reach the accuracy of the experimental results we want to compare with. By restricting ourselves to keeping only terms up to a maximum order, say Λ^{-n}, we achieve to stay within a finite set of new operators and counterterms that have to be added in our model, and the new extended model can be proven to be as renormalisable as its non-effective starting point (see sections 2.3 and 3.3 for an in-depth analysis of applying renormalisation in the SMEFT). This is, in a nutshell, the correct way to approach an EFT: an EFT is just a regular QFT, further equipped with an expansion parameter. Every renormalisable QFT ever used is just a special case of its EFT counterpart, where the expansion parameter is taken to infinity. Since we have reasons to believe that all QFTs we have constructed as of today aren't candidates for a final theory of Nature up to the Planck scale, and therefore there is a UV scale on which the predictions of these QFTs will break down, we could go as far as to say that a bottom-up EFT is not a generalisation of its corresponding QFT, but it is actually the correct model from which we can extract the 'renormalisable' QFT as the zeroth order approximation. We will therefore try to restrain ourselves from using the misleading terminology of 'non-renormalisable' operators, and we are going to refer to these operators appropriately as higher-dimension and/or effective operators.

Let us finally focus on the definition and the construction of the SMEFT. If we assume that new physics lies not too far from the electroweak (EW) scale, to be capable of meaningfully affecting the lower energy physics, we could write an effective Lagrangian of the SM as

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{p=1}^{\infty} \sum_{i} \frac{c_{p,i}}{\Lambda^p} Q_i^{(4+p)} \,. \tag{1.30}$$

In this formal expression, $\mathcal{L}_{\text{SM}}$ is the usual SM Lagrangian (which is renormalisable in the sense described above), Λ is a UV energy scale, the symbols $c_{p,i}$ denote the Wilson coefficients of the effective higher-dimensional operators and the sum over i runs over all possible operators of dimension $4+p$, $Q_i^{(4+p)}$. The dimensionality of the operator and the power of the scale Λ in its coefficient is in one-to-one correspondence, and for the rest of this section we are going to absorb the powers of Λ to new capitalised Wilson coefficients, $c_{p,i}\Lambda^{-p} \to C_{p,i}$ to simplify the notation. We are going to use this shorthand notation also in other parts of this thesis, and we are not going to bother with using lower-case Wilsons to distinguish them since it is clear by the context if powers of Λ are absorbed or not.

Clearly, having an infinite power series is not that practical for explicit calculations. As explained above, to define an EFT we also need to appropriately truncate the power series to a desired order. Guided by the experiment, we are going to make the reasonable assumption that new physics also lies not *too* close to the EW scale. Therefore, keeping only the first few terms of the power series in eq. (1.30) should make for a good approximation. For example, assuming (conservatively) that the UV scale is around 1 TeV, and keeping in mind that the SM energy scale, as dictated by its VEV, is around 250 GeV, we have an EFT expansion parameter of order $\epsilon = E_{\text{SM}}/E_{\text{UV}} \approx 1/4$.[4] This is of order of the electron charge $e \approx 1/3$ in natural units, which is regarded as a very good expansion parameter in QED. The similarity is even more striking if we consider that the first non-trivial order in the SMEFT expansion is, as we are going to discuss below, the ϵ^2 order, which is the lowest order which affects the bosonic sector of the theory. This is usually considered to be the leading SMEFT order, and in this thesis we are often going to use this nomenclature by calling the ϵ^2 SMEFT as the leading order, and the next order that affects the bosonic sector, i.e. the ϵ^4 SMEFT, as the next-to-leading order.

As should be apparent from our discussion in the previous paragraph, the genuine first order corrections in the SMEFT, i.e. the dimension 5 operators, will only affect the fermionic sector of the theory (not only pure fermion interactions, but at least two fermions should appear in the operators). Actually, due to the hypercharge U(1) symmetry of the SM, there

[4] Of course the SMEFT expansion parameter may be driven by the low-energy momentum transfer, i.e. $\epsilon = p/E_{\text{UV}}$. In this case one should stay within the EFT validity region, where the momentum should be much smaller than the UV scale.

is only one way to get an odd-dimension operator at 4 space-time dimensions, and this is to introduce pairs of fermion fields. Therefore, every odd-dimension expansion order will be tied to the fermionic sector, and we'll need to consider the next order to affect the pure bosonic sectors again. There is only one effective operator at dimension 5, the Weinberg operator

$$\mathcal{L}^{(5)}_{\text{SMEFT}} = C^{\nu\nu} Q^{(5)}_{\nu\nu} \,, \tag{1.31}$$

which is simply a neutrino mass term (absent in the SM). This operator won't be of much interest in this thesis, so we move on to discuss the dimension 6 operators.

As the first non-trivial SMEFT order, the dimension 6 operators are expected to bring interesting changes in the SM. This order is also already complicated enough in the construction of the operators. Surely it is easy enough to write down a gauge invariant operator using the SM fields, and make sure that its ingredients make it a dimension 6 operator. But to construct a Lagrangian one has to make sure that the operators used are independent, i.e. they create a basis. This, in a QFT, means that they should obviously be linearly independent, and that they are not equivalent to each other when using field redefinitions and integration by parts. This is a very technical discussion, and the reader is referred to appendix G for a detailed technical analysis about the construction of bases of operators in EFTs. Also, in appendix F, where we are attempting to construct the SMEFT up to any arbitrary order in the EFT expansion, we provide some insights about the construction of bases in the bosonic SMEFT.

There is a plethora of dimension 6 operators, which we formally depict here as

$$\mathcal{L}^{(6)}_{\text{SMEFT}} = \sum_X C^X Q^{(6)}_X + \sum_f C^f Q^{(6)}_f \,, \tag{1.32}$$

where $Q^{(6)}_X$ denotes the dimension 6 operators that do *not* involve fermion fields, and those that involve fermions are written as $Q^{(6)}_f$. The full list of the dimension 6 operators was first given in ref. [39], but many of the operators presented there were redundant. A complete set of all the *inequivalent* dimension 6 operators is given in ref. [37]; this complete set is known as the *Warsaw basis*. There are 59 baryon conserving operators and 4 baryon violating ones (not counting different flavour structures and Hermitian conjugations). In table 1.2 we give the full list of the dimension 6 operators, except for the four-fermion ones which are presented separately in table 1.3; both tables are taken from ref. [37].

The next step is to go to the broken phase of the theory, after the spontaneous breaking of the gauge symmetry, and make the necessary transformations to derive a physical mass basis of the SMEFT. After that, one has to properly quantise the theory by introducing the gauge-fixing terms in the Lagrangian, as explained in section 1.3.3. Delving deep into the algebraic manipulations of going to the physical mass basis in the SMEFT is beyond the scope of this introduction, partly to avoid getting too technical, and also to avoid a big overlap with the analysis of appendix F. There, the whole procedure is presented in great detail, and also it is generalised to account for any arbitrary order in the bosonic SMEFT expansion. We are instead going to conclude this section by presenting a number of useful references. The quantisation of the SMEFT in the Warsaw basis and the complete set of Feynman rules in linear R_ξ-gauges was given in ref. [40]. We are going to use these Feynman rules for all of our calculations in part I of in this thesis. These calculations are restricted in the dimension 6 SMEFT. For a discussion about higher (SM)EFT orders we point the reader to the appendices F and G. For discussions about higher dimensional operators in the literature, see: refs. [41–43] for dimension 7 operators, refs. [44, 45] for dimension 8 operators, and ref. [46] for a discussion of operators up to dimension 12. A series of pedagogical lectures on the SMEFT can be found in ref. [47], and a recent review can be found in ref. [48].

Phenomenology

Chapter **2**

The decay $h \to \gamma\gamma$ in the Standard Model Effective Field Theory

Assuming that new physics effects are parameterised by the Standard Model Effective Field Theory (SMEFT) written in a complete basis of up to dimension 6 operators, in this chapter we calculate the CP-conserving one-loop amplitude for the decay $h \to \gamma\gamma$ in general R_ξ-gauges. We employ a simple renormalisation scheme that is a hybrid between on-shell SM-like renormalised parameters and running $\overline{\text{MS}}$ Wilson coefficients. The resulting amplitude is then finite, renormalisation scale invariant, independent of the gauge choice and respects the SM Ward identities. Remarkably, the S-matrix amplitude calculation resembles very closely the one usually known from renormalisable theories and can be automatised to a high degree. We use this gauge invariant amplitude and recent LHC data to check upon sensitivity to various Wilson coefficients entering from a more complete theory at the matching energy scale. We present a closed expression for the ratio of the Beyond the SM versus the SM contributions, $\mathcal{R}_{h\to\gamma\gamma}$, as appeared in the LHC searches for the Higgs di-photon decay. The most important contributions arise at tree-level from the operators $Q_{\varphi B}$, $Q_{\varphi W}$ and $Q_{\varphi WB}$, and at one-loop level from the dipole operators Q_{uB} and Q_{uW}. Our calculation shows also that, for operators that appear at tree-level in SMEFT, one-loop corrections can modify their contributions by less than 10%. Wilson coefficients corresponding to these five operators are bounded from current LHC $h \to \gamma\gamma$ data with the bounds being, in some cases, an order of magnitude stronger than from other searches. This chapter is based on ref. [49].

2.1 Introduction

The discovery of the Higgs boson [19, 22, 23] in year 2012 was made possible mainly because of its decay into two photons [27, 28]. The current outcome for this decay channel from LHC (Run-2) with centre-of-mass energy $\sqrt{s} = 13\,\text{TeV}$, integrated luminosity of $36.1\,\text{fb}^{-1}$ and Higgs boson mass $m_h = 125.09 \pm 0.24\,\text{GeV}$, is summarised as the ratio between the experimentally measured value (which may include contributions from new physics scenarios) relative to the Standard Model (SM) predicted value [50, 51]

$$\mathcal{R}_{h\to\gamma\gamma} = \frac{\Gamma(\text{EXP}, h \to \gamma\gamma)}{\Gamma(\text{SM}, h \to \gamma\gamma)}\,. \tag{2.1}$$

The most recent measurements are presented by ATLAS [52] and CMS [53] experiments of LHC,

$$\begin{aligned} \text{ATLAS:} \qquad & \mathcal{R}_{h\to\gamma\gamma} = 0.99^{+0.15}_{-0.14}\,, \\ \text{CMS:} \qquad & \mathcal{R}_{h\to\gamma\gamma} = 1.18^{+0.17}_{-0.14}\,, \end{aligned} \tag{2.2}$$

and are consistent with the SM prediction, with the error margin expected to be reduced in the near future.

If we consider the SM as a complete theory of electroweak (EW) and strong interactions up to the Planck scale, with no other scale involved in between, then the decay amplitude $h \to \gamma\gamma$ arises purely from dimension $d \leq 4$ (renormalisable) interactions. In this case the amplitude is finite, calculable and, since all relevant parameters are experimentally known, it is a certain prediction of the SM. It is this prediction entering the denominator in eq. (2.1). If however, there is New Physics beyond the SM already at a scale Λ which is above, but not far from, the EW scale, say $\Lambda \sim \mathcal{O}(1-10)\,\text{TeV}$, then its effects can be parameterised by the presence of effective operators with dimension $d > 4$ at scale Λ. These operators together with various parameters (or Wilson coefficients) will then run down to the EW scale and feed the on-shell scattering S-matrix amplitude together with $d \leq 4$ interactions.

All dimension $d \leq 6$ effective operators among SM particles that obey the SM gauge symmetry have been classified in refs. [37, 39]. The SM augmented with these effective operators — remnants of the unknown heavy particles' decoupling [54] — is called the SM Effective Field Theory (or SMEFT for short). The quantisation of SMEFT has recently been undertaken in ref. [40] in linear R_ξ-gauges with explicit proof of BRST symmetry and where all relevant primitive interaction vertices have been collected.

Within SM, numerous calculations for the $h \to \gamma\gamma$ amplitude exist. The first calculation was performed in ref. [50] in the limit of light Higgs mass ($m_h \ll m_W$), using dimensional regularisation in the 't Hooft-Feynman gauge. Since then, there are other works completing this calculation in linear and non-linear gauges [51, 55, 56], with different regularisation schemes [57–63]. To our knowledge the complete SM one-loop $h \to \gamma\gamma$ amplitude in linear R_ξ-gauges is performed in ref. [64].

In SMEFT[1] there is already a number of papers that calculate the $h \to \gamma\gamma$ amplitude [65–68].[2,3] The current, state of the art calculation, has been presented by Hartmann and Trott in refs. [72, 73]. The analysis was carried out using the Background Field Method (BFM) [74][4] consistent with minimal subtraction renormalisation scheme ($\overline{\text{MS}}$) and included all relevant (CP-conserving) dimension $d \leq 6$ operators in calculating finite, non-log parts of the diagrams. Our work here is complementary but incorporates some additional features of importance:

- a simple calculational treatment in linear R_ξ-gauges based on Feynman rules of ref. [40],
- an analytical proof of gauge invariance (independence on the gauge choice parameters ξ) of the S-matrix element,
- a simple renormalisation framework which leads to a finite and renormalisation scale invariant amplitude,
- a compact semi-analytical expression highlighting the effect of new operators in the ratio $\mathcal{R}_{h\to\gamma\gamma}$ and corresponding bounds on Wilson coefficients,
- a field content of simple, perturbative, high energy models valid at the energy scale Λ, which, under gentle assumptions, can affect the ratio $\mathcal{R}_{h\to\gamma\gamma}$.

There are quite a few papers addressing a global fit to the Higgs data from LHC Run-1 and Run-2 in the SMEFT framework [76–78]. Our work provides a simple semi-analytic one-loop formula for the ratio $\mathcal{R}_{h\to\gamma\gamma}$ in eq. (2.1) that can be used by these (usually tree-level) fits or by analogous experimental analysis at LHC for Higgs boson searches.

This chapter is organised as follows. In section 2.2 we list operators contributing to the decay $h \to \gamma\gamma$ in SMEFT. Next, in section 2.3 we develop, in a pedagogical fashion, the renormalisation scheme for calculating the $h \to \gamma\gamma$ amplitude. In section 2.4 we give analytical expressions for all types of SM and SMEFT contributions to the $h \to \gamma\gamma$ amplitude and to the ratio $\mathcal{R}_{h\to\gamma\gamma}$. A semi-analytical formula for $\mathcal{R}_{h\to\gamma\gamma}$, depending on the running Wilson coefficients and renormalisation scale μ, is presented in section 2.5, and supplied

[1]For a recent review see ref. [48] and for pedagogical lectures see ref. [47].
[2]For earlier attempts see refs. [69, 70].
[3]Also, recently, the one-loop calculation for $h \to ZZ$ and $h \to Z\gamma$ decay in SMEFT has appeared in ref. [71].
[4]For a more recent approach on BFM-SMEFT see ref.[75].

with a discussion on numerical constraints of these coefficients. We conclude in section 2.6. Finally, in appendix 2.A we collect analytical expressions for the relevant one-loop self-energies and, relevant to $h \to \gamma\gamma$, three-point one-loop corrections in general R_ξ-gauges.

2.2 Relevant Operators

In EFT, an effect from the decoupling of heavy particles with masses of order Λ is captured by the running parameters of the low energy theory influenced by higher dimensional operators added to SM renormalisable Lagrangian $\mathcal{L}_{\rm SM}^{(4)}$. The full effective Lagrangian we consider here can be expressed as

$$\mathcal{L} = \mathcal{L}_{\rm SM}^{(4)} + \sum_X C^X Q_X^{(6)} + \sum_f C'^f Q_f^{(6)} \,, \tag{2.3}$$

where $Q_X^{(6)}$ denotes dimension 6 operators that do not involve fermion fields, while $Q_f^{(6)}$ denotes operators that contain fermion fields. All Wilson coefficients should be rescaled by Λ^2, for example $C^X \to C^X/\Lambda^2$. In this chapter we shall restore $1/\Lambda^2$ only in section 2.4 and thereafter. The prime in C'^f, denotes a coefficient in the flavour basis of ref. [37] (known as the Warsaw basis) while we use unprimed coefficients in fermion mass basis defined in ref. [40].

The operators involved in the calculation of decay $h \to \gamma\gamma$ are collected in table 2.1. They can easily be identified when drawing the Feynman diagrams for $h \to \gamma\gamma$ looking at the primitive vertices listed in ref. [40]. There are 8 classes of such operators X^3, φ^6, $\varphi^4 D^2$, $\psi^2\varphi^3$, $X^2\varphi^2$, $\psi^2 X\varphi, \psi^2\varphi^2 D, \psi^4$ where X represents a gauge field strength tensor, φ the Higgs doublet, D a covariant derivative and ψ a generic fermion field. Not counting flavour multiplicities and hermitian conjugation, in general, there are 16+2 CP-conserving operators.[5] Actually, *not all* operators in table 2.1 contribute in the final result for the $h \to \gamma\gamma$ amplitude. The operator Q_φ cancels out completely after adding all contributions. This leaves 17 CP-conserving operators (or Wilson coefficients) relevant to the $h \to \gamma\gamma$ amplitude.

Another classification of various $d = 6$ operators can be devised alongside with their strength [79, 80]. The division is between operators that are potentially tree-level generated (PTG operators) and those that are loop generated (LG operators) by the more fundamental theory at high energies (UV theory) under the assumption that the latter is *perturbatively*

[5]Incorporating the CP-violating operators will not create any problem in the procedure of renormalisation or elsewhere in our analysis (however, these operators are usually strongly suppressed by CP-violating type of observables such as particles' Electric Dipole Moments). At the dimension 6 SMEFT considered here, however, the CP violating part cancels out since the SM is CP-even.

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}'_p e'_r \varphi)$
		$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}'_p u'_r \widetilde{\varphi})$
		$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^* \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}'_p d'_r \varphi)$
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{eW}	$(\bar{l}'_p\sigma^{\mu\nu}e'_r)\tau^I\varphi W^I_{\mu\nu}$	$Q^{(3)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar{l}'_p\tau^I\gamma^\mu l'_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu}W^{I\mu\nu}$	Q_{eB}	$(\bar{l}'_p\sigma^{\mu\nu}e'_r)\varphi B_{\mu\nu}$		
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu}B^{\mu\nu}$	Q_{uW}	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\tau^I\widetilde{\varphi}\, W^I_{\mu\nu}$		
		Q_{uB}	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\widetilde{\varphi}\, B_{\mu\nu}$		
		Q_{dW}	$(\bar{q}'_p\sigma^{\mu\nu}d'_r)\tau^I\varphi\, W^I_{\mu\nu}$		
		Q_{dB}	$(\bar{q}'_p\sigma^{\mu\nu}d'_r)\varphi\, B_{\mu\nu}$		
		ψ^4			
		Q_{ll}	$(\bar{l}'_p\gamma_\mu l'_r)(\bar{l}'_s\gamma^\mu l'_t)$		

Table 2.1: A set of $d = 6$ operators in Warsaw basis that contribute to the $h \to \gamma\gamma$ decay amplitude, directly or indirectly, in R_ξ-gauges. We consider only CP-conserving operators in our analysis. The operator Q_φ cancels out completely in the $h \to \gamma\gamma$ amplitude. The operators Q_{ll} and $Q^{(3)}_{\varphi l}$ present themselves indirectly through the translation of the renormalised vacuum expectation value (vev) into the well measured Fermi coupling constant, cf. eq. (2.8). The notation is the same as in refs. [37, 40]. For brevity we suppress fermion chiral indices L, R.

PTG	LG
φ^6 and $\varphi^4 D^2$	X^3
$\psi^2\varphi^3$	$X^2\varphi^2$
$\psi^2\varphi^2 D$	$\psi^2 X\varphi$
ψ^4	

Table 2.2: PTG and LG classes of operators shown in table 2.1.

decoupled. Under this classification, operators relevant for $h \to \gamma\gamma$ amplitude are arranged as follows:

LG operators are suppressed by $1/(4\pi)^2$ factors for each loop and may be thought to be sub-dominant corrections with respect to PTG operators. In table 2.2 we list the PTG and LG classes of operators relevant to the $h \to \gamma\gamma$ decay. On the other hand, a perturbative decoupling of the UV theory may not necessarily be the case that Nature has chosen. In this work, although we do not *assume* any distinction amongst the $d = 6$ operators involved in $h \to \gamma\gamma$ amplitude, we shall be referring to table 2.2 as our analysis progresses.

2.3 Renormalisation

2.3.1 Parameter initialisation in SMEFT

There is a set of very well measured quantities, to which we rely upon, in relating our calculation for $\mathcal{R}_{h\to\gamma\gamma}$ to the LHC data. This set of experimental values is [81]

$$\begin{aligned} G_F &= 1.1663787(6) \times 10^{-5}\,\mathrm{GeV}^{-2}\,, \\ \alpha_{em} &= 1/137.035999139(31), \quad \text{at } Q^2 = 0\,, \\ m_W &= 80.385(15)\,\mathrm{GeV}\,, \\ m_Z &= 91.1876(21)\,\mathrm{GeV}\,, \\ m_h &= 125.09 \pm 0.24\,\mathrm{GeV}\,, \\ m_t &= 173.1 \pm 0.6\,\mathrm{GeV}\,. \end{aligned} \tag{2.4}$$

We identify these input values with the ones obtained in SMEFT consistent with the given accuracy of up to $1/\Lambda^2$ expansion terms. Consequently, following ref. [40] for the gauge and Higgs boson masses at tree-level, it is enough to set m_W, m_Z and m_h, respectively, equal to

$$\begin{aligned} m_W &= \frac{1}{2}\bar{g}v\,, \\ m_Z &= \frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}\,v\left(1 + \frac{\bar{g}\bar{g}' C^{\varphi WB} v^2}{\bar{g}^2+\bar{g}'^2} + \frac{1}{4}C^{\varphi D}v^2\right), \\ m_h^2 &= \lambda v^2 - \left(3C^{\varphi} - 2\lambda C^{\varphi\Box} + \frac{\lambda}{2}C^{\varphi D}\right)v^4\,, \end{aligned} \tag{2.5}$$

where λ is the Higgs quartic coupling, $\bar{g}', \bar{g}$ are, respectively, the $U(1)_Y$ and $SU(2)_L$ gauge couplings (redefined to obtain canonical form of the gauge kinetic terms, see ref. [40]) and the C-coefficients correspond to operators defined in table 2.1. Moreover, the fine-structure constant is identified through the Thomson limit ($Q^2 = 0$) as $\alpha_{em} = \bar{e}^2/4\pi$ where $\bar{e}$ is given at tree-level by

$$\bar{e} = \frac{\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left(1 - \frac{\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}\,C^{\varphi WB}\,v^2\right). \tag{2.6}$$

Similarly, the experimental values for lepton and quark masses, taken as pole masses from ref. [81], are equal to eqs. (3.27) and (3.29) of ref. [40].

The Fermi coupling constant, G_F, is identified through the muon decay process. In addition to the W-boson exchange which is modified in SMEFT by the PMNS matrix that is now a *non-unitary* matrix containing the coefficient $C^{(3)}_{\varphi l}$, G_F is also affected by dipole operators like Q_{eW} or by new diagrams with Z- or Higgs-boson exchange. However, the

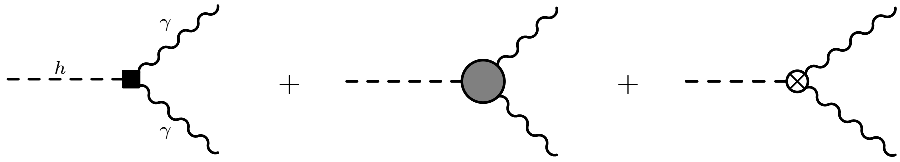


Figure 2.1: The sum of three types of diagrams (left to right): the SMEFT tree-level contribution, the 1PI vertex corrections from all operators (effective or not) and the vertex counterterms containing δC and δv. These corrections should be self-explained in eq. (2.16).

expression for G_F is simplified by making the approximation of zero neutrino masses and also by *assuming* that

$$C_1\, v^2 \gg C_2\, v\, m_l\,, \tag{2.7}$$

for any generic C_1 and C_2 coefficients entering the muon-decay amplitude and m_l being a charged lepton mass. Only then we identify the Fermi coupling constant of eq. (2.4), within *tree-level* in SMEFT, as

$$\frac{G_F}{\sqrt{2}} = \frac{\bar{G}_F}{\sqrt{2}}\Big[1 + v^2(C^{\varphi l(3)}_{11} + C^{\varphi l(3)}_{22}) - v^2 C^{ll}_{1221}\Big]\,, \quad \text{with} \quad \frac{\bar{G}_F}{\sqrt{2}} \equiv \frac{\bar{g}^2}{8m_W^2} = \frac{1}{2v^2}\,. \tag{2.8}$$

All Wilson coefficients entering in eq. (2.8) are real since they are diagonal elements of Hermitian matrices. In fact, and as a side test of the approximations assumed in eq. (2.7), we have checked that, at tree-level in SMEFT, the full S-matrix element for the process $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ is gauge invariant independently of lepton-number conservation. The formula (2.8) agrees with the corresponding one from refs. [48, 71].

2.3.2 Renormalisation framework

We ultimately want to bring the expression for the amplitude $\mathcal{M}(h \to \gamma\gamma)$ into a form that contains only renormalised parameters that are most closely related to observable quantities, the relevant ones given in eq. (2.4). At tree-level in SMEFT, the $h\gamma\gamma$-vertex appears only in association with the unrenormalised (bare) Wilson coefficients, $C_0^{\varphi B}, C_0^{\varphi W}$ and $C_0^{\varphi WB}$ and these are multiplied by the bare vev parameter v_0 (in what follows bare parameters are always denoted with a subscript zero). In order to set the stage, let us for example consider from table 2.1 the $d = 6$, CP-invariant operator of the form $X^2\varphi^2$,

$$C_0^{\varphi B}\, \varphi^\dagger \varphi\, B_{\mu\nu}\, B^{\mu\nu}\,, \tag{2.9}$$

where φ is the scalar Higgs doublet and $B_{\mu\nu}$ is the $U(1)_Y$ hypercharge gauge field strength tensor. All fields and coupling constants are unrenormalised quantities in this expression. In what follows, and in order to keep the expressions as simple as possible, we keep working with unrenormalised fields, i.e. no usual field redefinition is performed. This is justified, because we are interested in calculating only an on-shell S-matrix amplitude rather than a Green function.[6]

After Spontaneous Symmetry Breaking (SSB) in SMEFT (see ref. [40] for details), the expression in eq. (2.9) contains the following term describing the interaction of the Higgs field and two "photons",

$$C_0^{\varphi B}\, v_0\, h\, B_{\mu\nu}\, B^{\mu\nu}\,, \tag{2.10}$$

where h is the Higgs field. We split these bare quantities into renormalised parameters $v, C^{\varphi B}$ and counterterms, $\delta v, \delta C^{\varphi B}$ respectively, as

$$v_0 = v - \delta v\,, \qquad C_0^{\varphi B} = C^{\varphi B} - \delta C^{\varphi B}\,. \tag{2.11}$$

We follow the steps of a simple on-shell renormalisation scheme, first described in SM by Sirlin [85], and introduce new unrenormalised fields A_μ and Z_μ through the linear combinations

$$B_\mu = cA_\mu - sZ_\mu\,, \tag{2.12}$$
$$W^3_\mu = sA_\mu + cZ_\mu\,, \tag{2.13}$$

with $c \equiv \cos\theta_W$ and $s \equiv \sin\theta_W$ defined as a ratio of the physical masses of W and Z bosons, like

$$c^2 \equiv \cos^2\theta_W = \frac{m_W^2}{m_Z^2}\,. \tag{2.14}$$

Therefore, the Lagrangian term for the considered operator, $Q_{\varphi B}$, describing (part of) the $h\gamma\gamma$ interaction, reads,

$$c^2\, v\, C^{\varphi B}\left[1 - \frac{\delta C^{\varphi B}}{C^{\varphi B}} - \frac{\delta v}{v}\right] h\, F_{\mu\nu} F^{\mu\nu}\,. \tag{2.15}$$

Note that the vev counterterm arises from pure SM contributions because it multiplies $C^{\varphi B}$, while $\delta C^{\varphi B}$ cancels infinities that arise only from pure SMEFT diagrams i.e. in general, diagrams proportional to other C-coefficients, not necessarily only $C^{\varphi B}$.

Besides operator $Q_{\varphi B}$, counterterms for operators $Q_{\varphi W}$ and $Q_{\varphi WB}$ need to be added, too. Because all these three operators are proportional to the Higgs bilinear combination,

[6]This is more important than, as it sounds, just a calculational scheme. Certain operators vanish when using equations of motion. Green functions are affected by these operators whereas their S-matrix elements vanish [82–84].

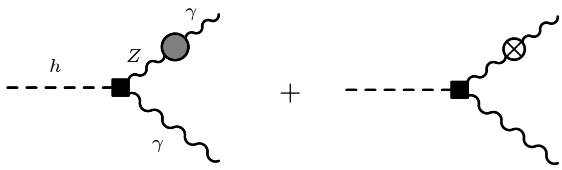


Figure 2.2: $Z\gamma$ mixing contributions with their associated counterterms. Cross denotes the SM $\delta m^2_{Z\gamma}$ counterterm and the black boxes indicate pure $d = 6$ operator insertions. The contributions to the other external photon leg contribute an overall factor of 2 in these diagrams.

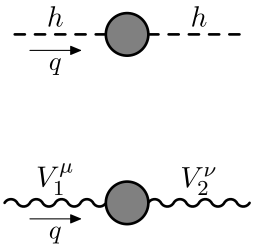
$\varphi^\dagger\varphi$, they all contain the vev counterterm as a universal contribution to $h \to \gamma\gamma$ amplitude. The contributions discussed so far are depicted and explained in figure 2.1. By making use of the Feynman rules of ref. [40], their sum is written in momentum space, as

$$\begin{aligned} 4i\,[\,p_1^\nu\,p_2^\mu - (p_1\cdot p_2)\,g^{\mu\nu}\,]\Bigg\{ & c^2\,v\,C^{\varphi B}\left[1+\Gamma^{\varphi B}-\frac{\delta C^{\varphi B}}{C^{\varphi B}}-\frac{\delta v}{v}\right] \\ & + s^2\,v\,C^{\varphi W}\left[1+\Gamma^{\varphi W}-\frac{\delta C^{\varphi W}}{C^{\varphi W}}-\frac{\delta v}{v}\right] \\ & - sc\,v\,C^{\varphi WB}\left[1+\Gamma^{\varphi WB}-\frac{\delta C^{\varphi WB}}{C^{\varphi WB}}-\frac{\delta v}{v}\right] \\ & + \frac{1}{m_W}\overline{\Gamma}^{\rm SM} + \sum_{X\neq\varphi B,\varphi W,\varphi WB} v\,C^X\,\Gamma^X\Bigg\}. \end{aligned} \tag{2.16}$$

One-loop, 1PI vertex contributions proportional to $C^{\varphi B}, C^{\varphi W}$ and $C^{\varphi WB}$ are denoted (up to pre-factors) with $\Gamma^{\varphi B}, \Gamma^{\varphi W}$ and $\Gamma^{\varphi WB}$ in the first three lines of the above equation. The SM contribution, $\overline{\Gamma}^{\rm SM}$, is just the SM-famous result of ref. [50] but with the SM parameters replaced by the SMEFT ones (that is why we use a bar over Γ), taken from refs. [40, 86]. Furthermore, there are additional one-loop corrections, Γ^X, proportional to Wilson coefficients C^X, like for instance C^W, which are collected in the last line, last term of eq. (2.16).

There are additional diagrams participating in the $h \to \gamma\gamma$ amputated amplitude. These are shown in figure 2.2.[7] The two diagrams in figure 2.2 represent the $Z\gamma$-self energy at $q^2 = 0$, $A_{Z\gamma}(0)$, plus its counterterm, $\delta m^2_{Z\gamma}$. The expression for the counterterm $\delta m^2_{Z\gamma}$ (given below) is gauge invariant independently of the renormalisation condition for the Higgs

[7] We omit the Higgs tadpole and its counterterm contribution diagrams since, following the renormalisation scheme of ref. [87], the Higgs tadpole counterterm is adjusted to cancel the 1PI Higgs tadpole diagrams. This guarantees that the vev is unchanged to one-loop order.



$$= -i\Pi_{hh}(q^2)\,,$$

$$= i\Pi^{\mu\nu}_{V_1V_2}(q^2) = iA_{V_1V_2}(q^2)g^{\mu\nu} + iB_{V_1V_2}(q^2)q^\mu q^\nu\,,$$

Figure 2.3: Definitions for the 1PI Higgs self-energy and vector boson ($V = \gamma, Z, W$) self-energies and mixing.

tadpole. This is practically very useful for proving the gauge invariance of the $h \to \gamma\gamma$ amplitude.

Finally, as usual, by multiplying the amputated graph with the LSZ-factors [88] (see for instance section 7.2 of textbook [7]) for the external Higgs and photon fields,

$$\sqrt{Z_{hh}}\, Z_{\gamma\gamma} = 1 + \frac{1}{2}\Pi'_{hh}(m_h^2) - \Pi_{\gamma\gamma}(0)\,, \tag{2.17}$$

the reduced S-matrix amplitude for our process,

$$\langle\gamma(p_1),\, \gamma(p_2)|\, S\, |h(q)\rangle = (2\pi)^4\delta^{(4)}(q - p_1 - p_2)\, [i\mathcal{M}^{\mu\nu}(h \to \gamma\gamma)]\, \epsilon^*_\mu(p_1)\epsilon^*_\nu(p_2)\,. \tag{2.18}$$

can be written as:

$$
\begin{aligned}
i\mathcal{M}^{\mu\nu}(h \to \gamma\gamma) =& 4i\,[\,p_1^{\nu}\,p_2^{\mu} - (p_1 \cdot p_2)\, g^{\mu\nu}\,]\times \\
&\Bigg\{ c^2\, v\, C^{\varphi B}\left[1 + \mathcal{X}^{\varphi B} + 2\tan\theta_W \frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{m_Z^2}\right] \\
&+ s^2\, v\, C^{\varphi W}\left[1 + \mathcal{X}^{\varphi W} - \frac{2}{\tan\theta_W}\frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{m_Z^2}\right] \\
&- sc\, v\, C^{\varphi WB}\left[1 + \mathcal{X}^{\varphi WB} - \frac{2}{\tan 2\theta_W}\frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{m_Z^2}\right] \\
&+ \frac{1}{m_W}\overline{\Gamma}^{\rm SM} + \sum_{X \neq \varphi B, \varphi W, \varphi WB} v\, C^X\, \Gamma^X \Bigg\}\,. \qquad (2.19)
\end{aligned}
$$

Eq. (2.19) is our master formula for the renormalised amplitude $\mathcal{M}^{\mu\nu}(h \to \gamma\gamma)$. For brevity, we have defined the quantity

$$
\mathcal{X}^i \equiv \Gamma^i - \frac{\delta C^i}{C^i} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{hh}(m_h^2) - \Pi_{\gamma\gamma}(0)\,, \qquad (2.20)
$$

where $i = \varphi B, \varphi W, \varphi WB$. The definitions for the various self-energies[8] are stated in figure 2.3 and

$$
\Pi'_{hh}(m_h^2) \equiv \left.\frac{\partial \Pi_{hh}(q^2)}{\partial q^2}\right|_{q^2 = m_h^2}\,, \quad A_{\gamma\gamma}(q^2) = -q^2\,\Pi_{\gamma\gamma}(q^2) + \mathcal{O}(\alpha_{em}^2)\,, \qquad (2.21)
$$

where $\Pi_{\gamma\gamma}(q^2)$ is regular at $q^2 = 0$. All self-energies in eq. (2.19) should arise purely from SM diagrams because we are including terms up to $1/\Lambda^2$ in SMEFT. As noted earlier, the SM counterterm, $\delta m^2_{Z\gamma}$, is *gauge invariant* and is given by [85]:

$$
\frac{\delta m^2_{Z\gamma}}{m_Z^2} = \frac{1}{2\tan\theta_W}\,{\rm Re}\left[\frac{A_{ZZ}(m_Z^2)}{m_Z^2} - \frac{A_{WW}(m_W^2)}{m_W^2}\right]. \qquad (2.22)
$$

The quantity $\delta v/v$ is not gauge invariant. Following standard on-shell renormalisation conditions of refs. [85, 87], we write

$$
\frac{\delta v}{v} = {\rm Re}\left[\frac{A_{WW}(m_W^2)}{2m_W^2}\right] - \frac{\delta g}{g}\,, \qquad (2.23)
$$

[8] We follow closely the notation of ref. [85].

where the counterterm δg of the $SU(2)_L$ gauge coupling is *gauge invariant* and reads as

$$\frac{\delta g}{g} = \frac{\delta e}{e} - \frac{1}{\tan\theta_W} \frac{\delta m^2_{Z\gamma}}{m^2_Z} . \tag{2.24}$$

Here δe is the electromagnetic charge renormalisation counterterm which is also *gauge invariant*. This is given by eq. (26) of ref. [85]

$$\frac{\delta e}{e} = -\frac{1}{2}\Pi^{\rm lept}_{\gamma\gamma}(0) - \frac{1}{2}\Pi^{\rm had}_{\gamma\gamma}(0) + \frac{7e^2}{32\pi^2}\left[\left(\frac{2}{\epsilon} - \gamma + \log 4\pi\right) - \log\frac{m^2_W}{\mu^2} + \frac{2}{21}\right] , \tag{2.25}$$

where μ is the renormalisation scale parameter and $\epsilon \equiv 4 - d$. Leptonic and hadronic contributions, $\Pi^{\rm lept}_{\gamma\gamma}(0)$ and $\Pi^{\rm had}_{\gamma\gamma}(0)$, to the photon vacuum polarisation are gauge invariant and the infinite part in the squared brackets should be gauge invariant too. The hadronic contribution from light quarks, $\Pi^{\rm had}_{\gamma\gamma}(0)$, is in principle non-calculable due to strong interaction at zero momenta. A dispersive or other non-perturbative methods should be in order. There is no such problem of course with $\Pi^{\rm lept}_{\gamma\gamma}(0)$.

SM vector boson self-energy contributions can be found in ref. [89]. The Higgs self-energy contribution can be found in refs. [73, 87]. These results have been obtained in the particular case of the 't Hooft-Feynman gauge where $\xi = 1$. Thanks to the set of SMEFT Feynman rules in general R_ξ-gauges [40], we present in appendix 2.A all contributions needed in eq. (2.19) with the explicit ξ-dependence. This is necessary for checking the gauge invariance of the amplitude. Finally, the counterterms $\delta C^{\varphi B}, \delta C^{\varphi W}$ and $\delta C^{\varphi WB}$ can be read from refs. [66, 72, 73, 86, 90, 91] where they have been calculated again in 't Hooft-Feynman ($\xi = 1$) gauge. However, in $\overline{\rm MS}$ renormalisation scheme and at one-loop, cancellation of infinities should be independent on the gauge choice as we confirm below.

2.3.3 ξ-independence

Knowing the gauge invariant and non-invariant parts of various contributions, as described above, is particularly useful for proving the ξ-independence of the amplitude. We first prove gauge invariance by means of ξ-independence for the infinite parts proportional to ξ_W or ξ_Z. We find that the combination of $\delta v/v$ and $\Pi'_{hh}(m^2_h)$ in eq. (2.19) is ξ-independent. For the $C^{\varphi B}$ contribution in eq. (2.19), the ξ_W-dependent terms inside $\Pi_{\gamma\gamma}(0)$ and $A_{Z\gamma}(0)$ cancel among each other, as they should since the infinite part of $\Gamma^{\varphi B}$ is ξ-independent by itself. For contributions proportional to $C^{\varphi W}$ ($C^{\varphi WB}$), the ξ_W cancellations take place throughout the self-energy contributions and $\Gamma^{\varphi W}$ ($\Gamma^{\varphi WB}$). Furthermore, diagrams proportional to C^X with $X \neq \varphi B, \varphi W, \varphi WB$, contributing to the last term of eq. (2.19), are gauge invariant on their own. Of course $\overline{\Gamma}^{\rm SM}$ is finite and gauge invariant as it is known from a direct calculation in R_ξ-gauges with dimensional regularisation [64].[9]

We then prove analytically the cancellation of all ξ-dependent finite parts. This was done by first performing a maximal reduction on the related Passarino-Veltman functions [92] and then analytically checking for ξ-dependence among the parametric integrals. This is a highly non-trivial check of the validity of our calculation because the gauge parameter ξ appears everywhere in *both* the SM and SMEFT contributions which are directly related to the $h \to \gamma\gamma$ amplitude. Moreover, this should be also considered as a direct proof for the validity of the expressions for vertices given in ref. [40] in general R_ξ-gauges. Most importantly, the ξ-cancellation shows that the amplitude $\mathcal{M}^{\mu\nu}(h \to \gamma\gamma)$ given in eq. (2.19) is *gauge invariant*, as it should be. Needless to say, this is a very encouraging indication towards the correctness of our final result.

As an additional non-trivial check of our calculation, we have also proved gauge invariance for our amplitude before adopting any renormalisation scheme. We confirm that the regularised but yet unrenormalised S-matrix amplitude for $h \to \gamma\gamma$, written in terms of bare parameters, is *gauge invariant*.

2.3.4 $\overline{\text{MS}}$ scheme for Wilson coefficients

All renormalised coefficients, say C, and the counterterms, δC, in eq. (2.19), can be readily written in terms of the $\overline{\text{MS}}$-scheme running C-coefficients as

$$C - \delta C = \bar{C}(\mu) - \delta\bar{C}\,, \tag{2.26}$$

where μ is the renormalisation (or subtraction) scale that lays somewhere between the EW scale and the scale Λ, while $\delta\bar{C}$ is a counterterm that subtracts only terms proportional to

$$E \equiv \frac{2}{\epsilon} - \gamma + \log 4\pi\,, \quad \text{with} \quad \epsilon \equiv 4 - d\,, \tag{2.27}$$

in the loop corrections for the Wilson C-coefficients. In $\overline{\text{MS}}$ scheme and at one-loop, these counterterms are independent of the choice of the gauge fixing and can be read directly from refs. [86, 90, 91] to be

$$\begin{aligned}\delta\bar{C}^{\varphi B} =& \frac{E}{16\pi^2}\Bigg\{\left(-3\lambda - Y + \frac{9}{4}\bar{g}^2 - \frac{85}{12}\bar{g}'^2\right)C^{\varphi B} - \frac{3}{2}\bar{g}\bar{g}'C^{\varphi WB} \\ & - \left[\frac{3}{2}\bar{g}'\,\mathrm{Tr}(C'^{eB}\Gamma_e^\dagger) - \frac{5}{6}\bar{g}'N_c\,\mathrm{Tr}(C'^{uB}\Gamma_u^\dagger) + \frac{1}{6}\bar{g}'N_c\,\mathrm{Tr}(C'^{dB}\Gamma_d^\dagger) + \text{H.c.}\right]\Bigg\}\,,\end{aligned} \tag{2.28}$$

[9] For a strict four-dimensional calculation in *unitary* gauge, see ref. [61].

$$\delta\bar{C}^{\varphi W} = \frac{E}{16\pi^2}\Bigg\{\left(-3\lambda - Y + \frac{53}{12}\bar{g}^2 + \frac{3}{4}\bar{g}'^2\right)C^{\varphi W} - \frac{1}{2}\bar{g}\bar{g}' C^{\varphi WB} + \frac{15}{2}\bar{g}^3 C^W$$
$$+ \left[\frac{1}{2}\bar{g}\ \mathrm{Tr}(C'^{eW}\Gamma_e^\dagger) + \frac{1}{2}\bar{g}\, N_c\, \mathrm{Tr}(C'^{uW}\Gamma_u^\dagger) + \frac{1}{2}\bar{g}\, N_c\, \mathrm{Tr}(C'^{dW}\Gamma_d^\dagger) + \mathrm{H.c.}\right]\Bigg\}, \quad (2.29)$$

$$\delta\bar{C}^{\varphi WB} = \frac{E}{16\pi^2}\Bigg\{\left(-\lambda - Y - \frac{2}{3}\bar{g}^2 - \frac{19}{6}\bar{g}'^2\right)C^{\varphi WB} - \bar{g}\bar{g}'(C^{\varphi B} + C^{\varphi W}) - \frac{3}{2}\bar{g}'\bar{g}^2 C^W$$
$$+ \Bigg[\frac{1}{2}\bar{g}\ \mathrm{Tr}(C'^{eB}\Gamma_e^\dagger) - \frac{1}{2}\bar{g}\, N_c\, \mathrm{Tr}(C'^{uB}\Gamma_u^\dagger) + \frac{1}{2}\bar{g}\, N_c\, \mathrm{Tr}(C'^{dB}\Gamma_d^\dagger)$$
$$-\frac{3}{2}\bar{g}'\ \mathrm{Tr}(C'^{eW}\Gamma_e^\dagger) - \frac{5}{6}\bar{g}' N_c\, \mathrm{Tr}(C'^{uW}\Gamma_u^\dagger) - \frac{1}{6}\bar{g}' N_c\, \mathrm{Tr}(C'^{dW}\Gamma_d^\dagger) + \mathrm{H.c.}\Bigg]\Bigg\}, \quad (2.30)$$

where $\Gamma_{u,d,e}$ is our notation [37, 40] for the usual Yukawa couplings in SM, and using table 4 from ref. [40], the coefficients C'^f are rotated to the fermion mass basis (denoted now as unprimed ones), and

$$Y \equiv \frac{2}{v^2}\sum_{i=1}^{3}(m_{e_i}^2 + N_c m_{d_i}^2 + N_c m_{u_i}^2)\,, \qquad \mathrm{Tr}(C'^{eB}\Gamma_e^\dagger) = \frac{\sqrt{2}}{v} C_{ii}^{eB} m_{e_i}\,, \quad \text{etc.} \quad (2.31)$$

$N_c = 3$ is the number of colours and m_{f_i} a mass of the SM fermion belonging to the i-th generation. All C-coefficients have been taken real. We have checked explicitly and analytically that the counterterms of eqs. (2.28), (2.29) and (2.30) render the amplitude for $h \to \gamma\gamma$ of eq. (2.19) *finite*, at one-loop and up to $1/\Lambda^2$ in EFT expansion.

2.3.5 The amplitude

The remaining part of $\mathcal{M}^{\mu\nu}(h \to \gamma\gamma)$ in eq. (2.19) is, at one-loop and up to $1/\Lambda^2$ terms, renormalisation scale invariant: the renormalisation group running of $\bar{C}(\mu)$ coefficients cancels the explicit μ-dependence within various contributions in the RHS of eq. (2.19). Therefore, the amplitude, to be squared in finding the $h \to \gamma\gamma$ decay width, is

$$i\mathcal{M}^{\mu\nu}(h \to \gamma\gamma) = 4i[\, p_1^\nu\, p_2^\mu - (p_1 \cdot p_2)\, g^{\mu\nu}]\, \mathcal{M}_{h\to\gamma\gamma}\,, \quad (2.32)$$

where

$$\mathcal{M}_{h\to\gamma\gamma} = \Bigg\{ c^2\, v\, \bar{C}^{\varphi B}(\mu) \left[1 + \Gamma^{\varphi B} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{hh}(m_h^2) - \Pi_{\gamma\gamma}(0) + 2\tan\theta_W \frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{m_Z^2} \right]$$

$$+ s^2\, v\, \bar{C}^{\varphi W}(\mu) \left[1 + \Gamma^{\varphi W} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{hh}(m_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan\theta_W} \frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{m_Z^2} \right]$$

$$- sc\, v\, \bar{C}^{\varphi WB}(\mu) \left[1 + \Gamma^{\varphi WB} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{hh}(m_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan 2\theta_W} \frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{m_Z^2} \right]$$

$$+ \frac{1}{m_W}\overline{\Gamma}^{\rm SM} + \sum_{X\neq \varphi B, \varphi W, \varphi WB} v\, C^X(\mu)\, \Gamma^X \Bigg\}_{\rm finite} . \tag{2.33}$$

The subscript "finite" in the final parenthesis means that infinities proportional to E have been subtracted from all contributions in eq. (2.33) such as Γ, Π'_{hh}, Π_{VV}, A_{VV}, etc. The $\mathcal{M}_{h\to\gamma\gamma}$ in eq. (2.33) is finite, gauge and renormalisation scale invariant[10] as a physical amplitude must be. In eq. (2.33), $\Gamma^{\varphi B}$, $\Gamma^{\varphi W}$ and $\Gamma^{\varphi WB}$ are given in appendix 2.A in eqs. (2.69), (2.70) and (2.71). The quantities $\delta v/v$ and $\delta m^2_{Z\gamma}/m_Z^2$ are presented in eqs. (2.23) and (2.22), respectively. All vector boson self-energies in general R_ξ-gauges as well as the quantity $\Pi'_{hh}(m_h^2)$ are also given in appendix 2.A.

Although all $\bar{C}(\mu)$ coefficients in eq. (2.33) are $\overline{\rm MS}$ parameters, the weak mixing angle θ_W and the vev v that appear explicitly to multiply Wilson coefficients are defined in terms of physical quantities through eqs. (2.14) and (2.8) [see also eq. (2.50) below]. This is a virtue of our hybrid renormalisation scheme: SM on-shell parameters appear together with $\overline{\rm MS}$ SMEFT parameters (Wilson coefficients) in the renormalised amplitude. This scheme can easily be applied to every process at one-loop in SMEFT.

From now on, all Wilson coefficients should be considered as running $\overline{\rm MS}$ quantities, $C \equiv \bar{C}(\mu)$. We remove the "bar" over the $\overline{\rm MS}$-coefficients letting the argument to denote, or to implicitly imply, the difference.

2.4 Anatomy of the effective amplitude

In this section we present explicit expressions for the SM contribution, and, contributions proportional to all Wilson coefficients entering the $h \to \gamma\gamma$ amplitude in eq. (2.33), and in table 2.1. These coefficients are taken to be real. For clarity, we reinstate explicitly $1/\Lambda^2$ factors in the expressions appeared in this and subsequent sections, so they are no longer incorporated into the definition of C's. Our EFT expansion stops at the order $1/\Lambda^2$ and is

[10] In the sense that $\frac{d}{d\mu}\mathcal{M}_{h\to\gamma\gamma}(\mu) = 0$.

one-loop at the $\hbar$-expansion. In our conventions, we denote electromagnetic fermion charges and the third component of the weak isospin, respectively, as

$$Q_f = \begin{cases} 0, & \text{for} \quad f = \nu_e, \nu_\mu, \nu_\tau \\ -1, & \text{for} \quad f = e, \mu, \tau \\ 2/3, & \text{for} \quad f = u, c, t \\ -1/3, & \text{for} \quad f = d, s, b \end{cases} \tag{2.34}$$

and

$$T_f^3 = \begin{cases} 1/2, & \text{for} \quad f = \nu_e, \nu_\mu, \nu_\tau, u, c, t \\ -1/2, & \text{for} \quad f = e, \mu, \tau, d, s, b \end{cases}. \tag{2.35}$$

The colour factors are $N_{c,e} = 1$ and $N_{c,u} = N_{c,d} = 3$. It is useful to note, when reading the expressions below, that the actual dimensionless EFT expansion parameter is $\frac{1}{G_F \Lambda^2}$. To get a quantitative feeling of its numerical magnitude and to compare with standard loop expansion in the EW gauge couplings, we simply note that it is $\frac{1}{G_F m_W^2} \sim 4\pi$, while for $\Lambda = 1\,\text{TeV}$ one has $\frac{1}{G_F \Lambda^2} \sim \frac{1}{4\pi}$, for $\Lambda = 10\,\text{TeV}$ one has $\frac{1}{G_F \Lambda^2} \sim \frac{\alpha_{em}}{4\pi}$ and, finally, for $\Lambda = 100\,\text{TeV}$ one has $\frac{1}{G_F \Lambda^2} \sim \frac{\alpha_{em}^2}{\pi^2}$.

2.4.1 SM and $C^{\varphi WB}$, $C^{\varphi l(3)}$, C^{ll}

The famous "SM" contributions from W and fermion triangle loops are represented by the penultimate term in eq. (2.33). This is

$$\frac{\overline{\Gamma}^{\text{SM}}}{m_W} = \frac{1}{64\pi^2} \frac{\bar{g}^2 \bar{g}'^2}{(\bar{g}^2 + \bar{g}'^2)} \frac{\bar{g}}{m_W} I_{\gamma\gamma}\,, \tag{2.36}$$

with

$$I_{\gamma\gamma} \equiv I_{\gamma\gamma}(r_f, r_W) = \sum_f Q_f^2 N_{c,f} A_{1/2}(r_f) - A_1(r_W)\,, \tag{2.37}$$

and

$$A_{1/2}(r_f) = 2r_f[1 + (1 - r_f) f(r_f)]\,, \tag{2.38}$$

$$A_1(r_W) = 2 + 3r_W[1 + (2 - r_W) f(r_W)]\,. \tag{2.39}$$

Here Q_f and m_f are the fermion charge (in the units of proton charge), and mass, respectively, $N_{c,f}$ is the colour factor for fermions (3 for quarks, 1 for leptons) and

$$r_f \equiv \frac{4m_f^2}{m_h^2}\,, \qquad r_W \equiv \frac{4m_W^2}{m_h^2}\,. \tag{2.40}$$

The result is of course finite and is governed by a single function $f(r)$, which reads

$$f(r) = \begin{cases} \arcsin^2\left(\frac{1}{\sqrt{r}}\right), & r \geq 1\,, \\ -\frac{1}{4}\left[\log\left(\frac{1+\sqrt{1-r}}{1-\sqrt{1-r}}\right) - i\pi\right]^2, & r \leq 1\,. \end{cases} \tag{2.41}$$

It is useful for order of magnitude calculations to state that $A_1(r_W) \approx 8.33$, $A_{1/2}(r_t) \approx 1.38$ and $I_{\gamma\gamma} \approx -6.56$ with a negligible imaginary part.

The expression given in eq. (2.36) is *not* exactly the SM contribution for it is written in terms of SMEFT parameters and not in terms of measurable quantities like those listed in eq. (2.4). We therefore rewrite eq. (2.36) in terms of physical quantities using the expression for $\bar{e}$ from eq. (2.6) and G_F from eq. (2.8) that bring in the new coefficients $C^{\varphi WB}$ and $C_{11}^{\varphi l(3)}, C_{22}^{\varphi l(3)}, C_{1221}^{ll}$, respectively,

$$\frac{\overline{\Gamma}^{\rm SM}}{m_W} = \frac{\alpha_{em}}{16\pi}\left(\frac{8G_F}{\sqrt{2}}\right)^{1/2} I_{\gamma\gamma}\left[1 + 2sc\,\frac{v^2}{\Lambda^2}C^{\varphi WB} - \frac{v^2}{2\Lambda^2}(C_{11}^{\varphi l(3)} + C_{22}^{\varphi l(3)}) + \frac{v^2}{2\Lambda^2}C_{1221}^{ll}\right]. \tag{2.42}$$

Note that the piece before the square brackets on the RHS is the SM contribution to amplitude [up to a Lorentz factor in eq. (2.32)], as it would be calculated in the absence of any higher order operators. Inside the square brackets there are contributions from SMEFT i.e. running Wilson coefficients evaluated at a scale μ. Hence, the precise determination of the $\mathcal{R}_{h\to\gamma\gamma}$ in eq. (2.1) is

$$\mathcal{R}_{h\to\gamma\gamma} = \frac{\Gamma(\text{SMEFT}, h\to\gamma\gamma)}{\Gamma(\text{SM}, h\to\gamma\gamma)} \equiv 1 + \delta\mathcal{R}_{h\to\gamma\gamma}\,, \tag{2.43}$$

where the SM decay width reads, in accordance with standard refs. [29, 30, 64], as

$$\Gamma(\text{SM}, h\to\gamma\gamma) = \frac{G_F\,\alpha_{em}^2\,m_h^3}{128\sqrt{2}\pi^3}\,|I_{\gamma\gamma}|^2\,, \tag{2.44}$$

with $I_{\gamma\gamma}$ given in eq. (2.37). The SMEFT contributions of eq. (2.42) are encoded in a part of $\delta\mathcal{R}_{h\to\gamma\gamma}$ of eq. (2.43), in terms of measurable quantities s, c and G_F, as

$$\delta\mathcal{R}_{h\to\gamma\gamma}^{(1)} \simeq \frac{4sc}{\sqrt{2}}\frac{1}{G_F\Lambda^2}C^{\varphi WB} - \frac{1}{\sqrt{2}}\frac{1}{G_F\Lambda^2}(C_{11}^{\varphi l(3)} + C_{22}^{\varphi l(3)}) + \frac{1}{\sqrt{2}}\frac{1}{G_F\Lambda^2}C_{1221}^{ll}\,, \tag{2.45}$$

where $c^2 = 1 - s^2 = m_W^2/m_Z^2$. Following our EFT expansion assumption, in obtaining eq. (2.45), corrections of $\mathcal{O}(1/\Lambda^4)$ have been consistently ignored.

2.4.2 $C^{\varphi D}$, $C^{\varphi\Box}$, C^{φ}

A direct calculation shows that the contribution from operators $C^{\varphi\Box}$ and $C^{\varphi D}$ is simply

$$\left(1 + \frac{v^2}{\Lambda^2}C^{\varphi\Box} - \frac{v^2}{4\Lambda^2}C^{\varphi D}\right)(i\mathcal{M}^{\text{SM}}) \equiv Z_h^{-1}(i\mathcal{M}^{\text{SM}})\,, \tag{2.46}$$

where Z_h is the field redefinition factor for making the kinetic term of the Higgs field canonical in going from SM to SMEFT (see eq. (3.5) of ref. [40]) and $i\mathcal{M}^{\text{SM}}$ is the full SM contribution to $h \to \gamma\gamma$ amplitude. There is an explanation for this result based on the quantisation of SMEFT presented in ref. [40]. In unitary gauge these operators appear in Higgs boson vertices (hWW and hff) with exactly the same Lorentz structure as in the corresponding SM vertices. On the other hand, in "renormalisable" gauges these operators appear in a complicated way, e.g. there are contributions from Goldstone bosons hG^0G^0 that have a non-trivial, non-SM Lorentz structure [40] and eq. (2.46) is not easily seen without performing the actual calculation. However, the result should be independent on the gauge choice as we explicitly confirm. We can view eq. (2.46) in a different way starting from the SM amplitude and perform the redefinition $H = Z_h^{-1}h$ on the single external Higgs boson leg.

As we already mentioned in section 2.2, the coefficient C^{φ} does not contribute explicitly to the $h \to \gamma\gamma$ amplitude in unitary gauge. Although there are apparent non-trivial contributions from it to vertices in R_ξ-gauges, once again, gauge invariance implies that the amplitude is explicitly independent of C^{φ}. Again, we explicitly verify this situation as well.

In summary, the contribution of operators discussed in this subsection to the ratio (2.43) reads trivially, up to $\sim 1/\Lambda^2$ terms, as

$$\delta\mathcal{R}^{(2)}_{h\to\gamma\gamma} \simeq \sqrt{2}\frac{1}{G_F\Lambda^2}C^{\varphi\Box} - \frac{\sqrt{2}}{4}\frac{1}{G_F\Lambda^2}C^{\varphi D}\,. \tag{2.47}$$

2.4.3 $C^{e\varphi}$, $C^{u\varphi}$, $C^{d\varphi}$

The relevant diagrams for these operators contain a fermion circulating in the loop. They contribute a ξ-independent piece in the last term of eq. (2.33) which takes the form

$$\Gamma_i^{f\varphi} = -\frac{1}{4\pi^2}\frac{\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}N_{c,f}Q_f^2\frac{vm_{f_i}}{\sqrt{2}m_h^2}[1 + (1 - r_{f_i})f(r_{f_i})]\,. \tag{2.48}$$

The contribution runs over all charged fermions $f = e, u, d$ with their generation flavours denoted as $i = 1, 2, 3$, i.e. $u_1 = u, u_2 = c, u_3 = t$ etc. The electromagnetic charges Q_f and colour factors $N_{c,f}$, are given in and below eq. (2.35). The function $f(r)$ is defined in eq. (2.41). Turning all parameters into measurable ones in eq. (2.48) we obtain for the $\mathcal{R}_{h\to\gamma\gamma}$ ratio of eq. (2.43)

$$\delta\mathcal{R}^{(3)}_{h\to\gamma\gamma} \simeq -\frac{2^{3/4}}{(G_F m_h^2)^{1/2}} \sum_{f=e,u,d} N_{c,f} Q_f^2 \sum_{i=1}^{3} \mathrm{Re}\left[\frac{A_{1/2}(r_{f_i})}{I_{\gamma\gamma}\, r_{f_i}^{1/2}}\right] \frac{1}{G_F \Lambda^2}\, C^{f\varphi}_{ii}\,, \tag{2.49}$$

with $A_{1/2}(r)$ being a function defined in eq. (2.38) and $I_{\gamma\gamma}$ defined in eq. (2.37). The function inside the square parenthesis peaks at the charm mass and as we shall see below (cf. eq. (2.62)) this is the most important contribution in $\delta\mathcal{R}^{(3)}_{h\to\gamma\gamma}$.

All operators we have examined thus far are of PTG type. These operators create *only* finite contributions in the $h \to \gamma\gamma$ amplitude. On contrary, operators that will be examined next will need to be renormalised.

2.4.4 $C^{\varphi B}$, $C^{\varphi W}$, $C^{\varphi WB}$

The amplitude in eq. (2.33) contains contributions from $Q_{\varphi B}, Q_{\varphi W}$, $Q_{\varphi WB}$ operators[11] appearing already at tree-level in SMEFT. These are collected in the first three lines of eq. (2.33), but still contain the renormalised vev v. This parameter needs to be turned into Fermi coupling constant, G_F, that is a measurable quantity with experimental value given in eq. (2.4). We only need the SM one loop corrections to Δr, which appear through the expression

$$\frac{\bar{G}_F}{\sqrt{2}} = \frac{1}{2v^2} \frac{1}{(1-\Delta r)}\,. \tag{2.50}$$

Note that Δr is a gauge invariant quantity and its form can be found in ref. [85]. This is consistent with our remark in section 2.3 that the pre-factors of $C^{\varphi B}, C^{\varphi W}$, $C^{\varphi WB}$ in eq. (2.33) are respectively gauge invariant quantities and therefore the whole amplitude is gauge invariant. We then use eq. (2.8) to order $1/\Lambda^2$ i.e. set $\bar{G}_F \to G_F$ in eq. (2.50) and apply the result in eq. (2.33). We find that Δr nicely cancels out when using an alternative expression for $\delta v/v$ derived in ref. [87] in Feynman gauge $\xi = 1$,

$$\frac{\delta v}{v} = \frac{1}{2}\left[\frac{A_{WW}(0)}{m_W^2} + \Delta r - \widetilde{E}\right]_{\xi=1}\,, \tag{2.51}$$

[11]There is an additional contribution from the operator $Q_{\varphi WB}$, arising from eq. (2.42), which must be added in the final amplitude, cf. eq. (2.62).

where the parameter $\widetilde{E}$ is given in ref. [87]

$$\widetilde{E}_{\xi=1} = \frac{\alpha_{em}}{2\pi s^2}\left[2E - 2\log\frac{m_Z^2}{\mu^2} + \frac{\log c^2}{s^2}\left(\frac{7}{4} - 3s^2\right) + 3\right]. \tag{2.52}$$

The quantity $A_{WW}(0)$ is presented in ref. [89] in 't Hooft-Feynman gauge and is recalculated here for completeness in eq. (2.80). By putting eqs. (2.50) and (2.51) in eq. (2.33), the relevant finite contributions from operators $Q_{\varphi B}$, $Q_{\varphi W}$ and $Q_{\varphi WB}$ to the physical amplitude $\mathcal{M}_{h\to\gamma\gamma}$ read:

$$\begin{aligned}
&\frac{c^2\, C^{\varphi B}(\mu)}{(\sqrt{2}G_F)^{1/2}\Lambda^2}\left[1 + \Gamma^{\varphi B} - \frac{A_{WW}(0)}{2m_W^2} + \frac{\widetilde{E}}{2}\right.\\
&\qquad\qquad\left. + \frac{1}{2}\Pi'_{hh}(m_h^2) - \Pi_{\gamma\gamma}(0) + 2\tan\theta_W \frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{m_Z^2}\right]_{\text{finite}}\\
&+\frac{s^2\, C^{\varphi W}(\mu)}{(\sqrt{2}G_F)^{1/2}\Lambda^2}\left[1 + \Gamma^{\varphi W} - \frac{A_{WW}(0)}{2m_W^2} + \frac{\widetilde{E}}{2}\right.\\
&\qquad\qquad\left. + \frac{1}{2}\Pi'_{hh}(m_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan\theta_W} \frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{m_Z^2}\right]_{\text{finite}}\\
&-\frac{sc\, C^{\varphi WB}(\mu)}{(\sqrt{2}G_F)^{1/2}\Lambda^2}\left[1 + \Gamma^{\varphi WB} - \frac{A_{WW}(0)}{2m_W^2} + \frac{\widetilde{E}}{2}\right.\\
&\qquad\qquad\left. + \frac{1}{2}\Pi'_{hh}(m_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan 2\theta_W} \frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{m_Z^2}\right]_{\text{finite}}.
\end{aligned} \tag{2.53}$$

This expression takes this particular form *only* in $\xi = 1$ gauge and replaces the first three lines in eq. (2.33). It is important for the reader to notice, that numerically big corrections from Δr have been cancelled out in eq. (2.53). The quantities $\Gamma^{\varphi V}, V = B, W, WB$ are fairly lengthy and are given in the appendix 2.A together with the self-energies, all in general R_ξ-gauges. Nevertheless, following our tactic here, we can write down a clear formula for the relevant corrections to the ratio $\mathcal{R}^{(4)}_{h\to\gamma\gamma}$ in eq. (2.43), as (recall that $\tan\theta_W = s/c = \bar{g}'/\bar{g}$)

$$\begin{aligned}
\delta\mathcal{R}^{(4)}_{h\to\gamma\gamma} \simeq \frac{8\pi^2}{G_F m_W^2 \tan^2\theta_W}&\left[\frac{C^{\varphi B}}{G_F\Lambda^2}\text{Re}\left(\frac{I_{\varphi B}}{I_{\gamma\gamma}}\right) + \tan^2\theta_W \frac{C^{\varphi W}}{G_F\Lambda^2}\text{Re}\left(\frac{I_{\varphi W}}{I_{\gamma\gamma}}\right)\right.\\
&\left. - \tan\theta_W \frac{C^{\varphi WB}}{G_F\Lambda^2}\text{Re}\left(\frac{I_{\varphi WB}}{I_{\gamma\gamma}}\right)\right]_{\text{finite}},
\end{aligned} \tag{2.54}$$

where $I_{\varphi B}$, $I_{\varphi W}$, $I_{\varphi WB}$ represent the expressions in corresponding squared brackets of eq. (2.53).

As we already mentioned in the discussion below eq. (2.25), the photon self-energy, $\Pi_{\gamma\gamma}(0)$, contains hadronic contributions from five light quarks, that is all quarks but the top quark. Therefore, for the related part, $\Pi_{\gamma\gamma}^{\rm had}(0)$, the perturbative formula (2.72) is not reliable. We use instead,

$$\Pi_{\gamma\gamma}^{\rm had}(0) = -\Delta\alpha_{\rm had}^{(5)}(m_Z^2) + \Pi_{\gamma\gamma}^{\rm had}(m_Z^2)\,, \tag{2.55}$$

where now, thanks to asymptotic freedom, $\Pi_{\gamma\gamma}^{\rm had}(m_Z^2)$ is a reliable perturbative one-loop calculation for the light quark contributions (see (2.82)) while $\Delta\alpha_{\rm had}^{(5)}(m_Z^2) = \Pi_{\gamma\gamma}^{\rm had}(m_Z^2) - \Pi_{\gamma\gamma}^{\rm had}(0)$ is finite and is computed via a dispersion relation that involves experimental data for the ratio $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. A recent analysis [81] gives $\Delta\alpha_{\rm had}^{(5)}(m_Z^2) = 0.02764 \pm 0.00013$.

The form for $\delta\mathcal{R}_{h\to\gamma\gamma}^{(4)}$ in eq. (2.54) is given semi-analytically below (cf. eq. (2.62)). Since these corrections appear at tree-level in SMEFT they are generically the biggest ones from all operators involved in $h \to \gamma\gamma$ amplitude.

2.4.5 C^W

The contribution from W-loops gives rise to terms proportional to C^W in eq. (2.33). The relevant expression is ξ-independent, and is written as

$$\Gamma^W = \frac{3}{16\pi^2}\frac{\bar{g}^3\bar{g}'^2}{(\bar{g}^2+\bar{g}'^2)}[3E + B]\,, \tag{2.56}$$

where E is the infinite piece [see eq. (2.27)] formed as usual in dimensional regularisation, of course removed from eq. (2.33). The integral function B is

$$B \equiv B(r_W) = 2 - r_W f(r_W) + 2J_2(r_W) - 3\log\frac{m_W^2}{\mu^2}\,, \tag{2.57}$$

where the functions $f(r), J_2(r)$ are given in eqs. (2.41) and (2.78), respectively, and μ is the renormalisation scale. The contribution from the operator Q_W in the ratio (2.43) is

$$\delta\mathcal{R}_{h\to\gamma\gamma}^{(5)} \simeq 24\sqrt{\frac{G_F m_W^2}{\sqrt{2}}}\,\text{Re}\left[\frac{B(r_W)}{I_{\gamma\gamma}}\right]\frac{1}{G_F\Lambda^2}\,C^W\,, \tag{2.58}$$

with $I_{\gamma\gamma}$ defined in eq. (2.37).

2.4.6 C^{eB}, C^{eW}, C^{uB}, C^{uW}, C^{dB}, C^{dW}

These are again contributions from operators affecting fermion loops and, as such, they are ξ-independent. They are, however, infinite since they involve dipole operators (as one can easily see from ref. [40] there is an extra momentum in the numerator of their corresponding Feynman rules expressions). We obtain the following contribution in the last term of eq. (2.33):

$$\Gamma_i^{fB} = \frac{1}{4\pi^2} \frac{\bar{g}^2 \bar{g}'}{\bar{g}^2 + \bar{g}'^2} N_{c,f} Q_f \frac{m_{f_i}}{\sqrt{2} v} [2E + D(r_{f_i})] ,$$
$$\Gamma_i^{fW} = 2T_f^3 \frac{\bar{g}'}{\bar{g}} \Gamma_i^{fB} , \tag{2.59}$$

where the function $D(r_{f_i})$ is defined as

$$D(r_{f_i}) \equiv -2 \log \frac{m_{f_i}^2}{\mu^2} + 1 - r_{f_i} f(r_{f_i}) + J_2(r_{f_i}) . \tag{2.60}$$

Here again f stands for a fermion type, $f = e, u, d$, and $i = 1, 2, 3$ runs over its flavour eigenstates. The relevant contribution from the operators Q_{fB} and Q_{fW} to the ratio $\mathcal{R}_{h\to\gamma\gamma}$ of eq. (2.43) is

$$\delta\mathcal{R}^{(6)}_{h\to\gamma\gamma} \simeq \frac{2m_h}{m_W \tan\theta_W} \sum_{f=e,u,d} N_{c,f} Q_f$$
$$\times \sum_{i=1}^{3} \mathrm{Re} \left[\frac{r_{f_i}^{1/2} D(r_{f_i})}{I_{\gamma\gamma}} \right] \frac{1}{G_F \Lambda^2} (C_{ii}^{fB} + 2T_f^3 \tan\theta_W C_{ii}^{fW}) . \tag{2.61}$$

Functions $I_{\gamma\gamma}, f(r)$ and $J_2(r)$ are defined in eqs. (2.37), (2.41) and (2.78), respectively.

The expression $\delta\mathcal{R}^{(6)}_{h\to\gamma\gamma}$ in eq. (2.61) has few interesting features. It is proportional to the mass of the fermion circulated in the loop and also proportional to $\mathcal{O}(1)$ loop functions ratio. Comparing $\delta\mathcal{R}^{(6)}_{h\to\gamma\gamma}$, which arises from LG operators, with, for example, $\delta\mathcal{R}^{(3)}_{h\to\gamma\gamma}$ of eq. (2.49) which arises from PTG operators and recall table 2.2, we see that there is a huge enhancement of the former by a factor of $\mathcal{O}(10)$ in particular for the top-quark. Hence, for the top quark in the loop and for $\mu = m_W$, this is the biggest correction from all one-loop contributions in SMEFT as we shall see shortly in section 2.5.

2.5 Results

2.5.1 Semi-numerical expression for the ratio $\mathcal{R}_{h\to\gamma\gamma}$

In this section, we sum all contributions to $\mathcal{R}_{h\to\gamma\gamma}$ found in section 2.4, leaving as unknowns, the renormalisation group running Wilson coefficients, $C = C(\mu)$, the renormalisation scale μ divided by the W-boson mass and the energy scale Λ. Everything we have discussed so far is within the perturbative renormalisation framework explained in section 2.3. For EFT expansion to be valid, this means that the maximum value of a generic coefficient,

C/Λ^2, is at most $\mathcal{O}(1)$. Experimentally, it is suggested from eq. (2.2) that the corrections to $\delta\mathcal{R}_{h\to\gamma\gamma}$ should be at most 15%. Being conservative, and in order to display all (potentially) important contributions from operators in $\delta\mathcal{R}_{h\to\gamma\gamma}$, we present below semi-numerical results for $\delta\mathcal{R}_{h\to\gamma\gamma}$ that are up to $1\% \times C/\Lambda^2$.

With the energy scale Λ written in TeV units, we obtain (in Warsaw basis):[12]

$$\begin{aligned}
\delta\mathcal{R}_{h\to\gamma\gamma} = \sum_{i=1}^{6} \delta\mathcal{R}^{(i)}_{h\to\gamma\gamma} \simeq\ & 0.06\left(\frac{C^{ll}_{1221} - C^{\varphi l(3)}_{11} - C^{\varphi l(3)}_{22}}{\Lambda^2}\right) + 0.12\left(\frac{C^{\varphi\Box} - \frac{1}{4}C^{\varphi D}}{\Lambda^2}\right) \\
& - 0.01\left(\frac{C^{e\varphi}_{22} + 4C^{e\varphi}_{33} + 5C^{u\varphi}_{22} + 2C^{d\varphi}_{33} - 3C^{u\varphi}_{33}}{\Lambda^2}\right) \\
& - \left[48.04 - 1.07\log\frac{\mu^2}{m_W^2}\right]\frac{C^{\varphi B}}{\Lambda^2} - \left[14.29 - 0.12\log\frac{\mu^2}{m_W^2}\right]\frac{C^{\varphi W}}{\Lambda^2} \\
& + \left[26.62 - 0.52\log\frac{\mu^2}{m_W^2}\right]\frac{C^{\varphi WB}}{\Lambda^2} \\
& + \left[0.16 - 0.22\log\frac{\mu^2}{m_W^2}\right]\frac{C^{W}}{\Lambda^2} \\
& + \left[2.11 - 0.84\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uB}_{33}}{\Lambda^2} + \left[1.13 - 0.45\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uW}_{33}}{\Lambda^2} \\
& - \left[0.03 + 0.01\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uB}_{22}}{\Lambda^2} - \left[0.01 + 0.00\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uW}_{22}}{\Lambda^2} \\
& + \left[0.03 + 0.01\log\frac{\mu^2}{m_W^2}\right]\frac{C^{dB}_{33}}{\Lambda^2} - \left[0.02 + 0.01\log\frac{\mu^2}{m_W^2}\right]\frac{C^{dW}_{33}}{\Lambda^2} \\
& + \left[0.02 + 0.00\log\frac{\mu^2}{m_W^2}\right]\frac{C^{eB}_{33}}{\Lambda^2} - \left[0.01 + 0.00\log\frac{\mu^2}{m_W^2}\right]\frac{C^{eW}_{33}}{\Lambda^2} + \dots\,, \qquad (2.62)
\end{aligned}$$

where the ellipses denote contributions from the operators Q in table 2.1 that are less than $1\% \times C/\Lambda^2$. Terms in the first three parentheses arise from finite loop contributions, $\delta\mathcal{R}^{(1,2,3)}_{h\to\gamma\gamma}$ in eqs. (2.45), (2.47) and (2.49), while all the rest arise from "infinite" diagrams; for these the renormalisation scale μ appears explicitly. All coefficients are running quantities, $C = C(\mu)$, and $\delta\mathcal{R}_{h\to\gamma\gamma}$ should be RGE invariant up to one-loop and up to $1/\Lambda^2$ expansion terms. This can be checked numerically already from the explicit μ-dependence in eq. (2.62) and the β-functions for the C-coefficients calculated in refs. [86, 90, 91].[13] Furthermore, we remark that in eq. (2.62) and for $\mu = 1\,\text{TeV}$, the logarithmic parts are of the same order of magnitude as the finite, constant, parts. Interestingly, for the coefficients in the last three

[12] Unlike refs. [72, 73] we have made no rescaling of Wilson coefficients with gauge couplings. Of course, the coefficients-$C^{fB,fW}$ are the rotated coefficients in the quark or lepton mass basis adopted in ref. [40] as already noted in section 2.2.

[13] For this purpose, one can use the numerical codes of refs. [93, 94] or can exploit analytic techniques appeared recently in ref. [95].

lines of eq. (2.62), the two parts constructively interfere, while for the rest of coefficients they partially cancel.

At the end of the day, only five operators in eq. (2.62) can be bounded by the LHC experimental measurement (2.2) of the ratio $R_{h\to\gamma\gamma}$. Taking $\mu = m_W$, we find

$$\frac{|C^{\varphi B}|}{\Lambda^2} \lesssim \frac{0.002}{(1\,\text{TeV})^2}\,, \qquad \frac{|C^{\varphi W}|}{\Lambda^2} \lesssim \frac{0.007}{(1\,\text{TeV})^2}\,, \qquad \frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim \frac{0.004}{(1\,\text{TeV})^2}\,,$$
$$\frac{|C^{uB}_{33}|}{\Lambda^2} \lesssim \frac{0.047}{(1\,\text{TeV})^2}\,, \qquad \frac{|C^{uW}_{33}|}{\Lambda^2} \lesssim \frac{0.088}{(1\,\text{TeV})^2}\,. \tag{2.63}$$

All bounded coefficients above are associated with LG operators in table 2.2 in a perturbative decoupled UV theory. Eq. (2.63) seems to be consistent with this observation and $\Lambda \approx 1\,\text{TeV}$. On the other hand, assuming $|C^{\varphi V}|$ ($|C^{uB,uW}_{33}|$) $\simeq 1$ we obtain $\Lambda \gtrsim 10\,(3)\,\text{TeV}$, outside but close to the near-future LHC region. Other operators in eq. (2.62) may contribute at most 15% only when $C = 1$ and $\Lambda = 1\,\text{TeV}$ so their effects are less likely to be observed at present in LHC searches for the $h \to \gamma\gamma$ process.

Operators $Q_{\varphi B}$, $Q_{\varphi W}$ and $Q_{\varphi WB}$ contribute already at tree-level in SMEFT and this explains the large value of their coefficients in eq. (2.62). As our calculation shows, taking also into account one-loop corrections, modify their respective tree-level contributions to the ratio $\delta R_{h\to\gamma\gamma}$ by 1.3% for $C^{\varphi B}$, by 7.5% for $C^{\varphi WB}$ and by 8.7% for $C^{\varphi W}$ at the renormalisation scale $\mu = m_W$, in agreement with the commonly expected magnitude of the SM-like electroweak one-loop corrections. What is surprising however, is the large loop contribution of dipole operators $Q^{33}_{uB,uW}$. This is basically due to the largeness of the top-quark mass and other features already noted in the discussion below eq. (2.60).

2.5.2 Other constraints

In the section above, we found that the dominant coefficients in $\mathcal{R}_{h\to\gamma\gamma}$ are those given in eq. (2.63). These coefficients maybe also bounded by observables other than $h \to \gamma\gamma$. It has been noted in refs. [96, 97] that the coefficient $C^{\varphi WB}$ contributes directly to the electroweak S-parameter, one of the parameters that fits Z-pole observables. Its contribution reads

$$\frac{C^{\varphi WB}}{\Lambda^2} = \frac{G_F\,\alpha_{em}}{2\sqrt{2}sc}\Delta S\,. \tag{2.64}$$

With $\Delta S \in [-0.06, 0.07]$ [78] we obtain $\frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim 0.005\,\text{TeV}^{-2}$ which is of the same order of magnitude as the upper bound we find here in eq. (2.63) from $h \to \gamma\gamma$ measurement. The coefficients $C^{\varphi W}$ and $C^{\varphi B}$ are constrained by LHC Higgs data (giving upper limits on deviations from the SM predictions) or electroweak fits to EW observables. The respective bounds, as they read from refs. [78, 98], are also about the same order of magnitude as in eq. (2.63).

The other two operators in eq. (2.63), Q_{uB}^{33} and Q_{uW}^{33}, are constrained from the $\bar{t}tZ$ production and the latter also by the single top production measurements at LHC. Bounds quoted in ref. [99] are $|C_{33}^{uB}|/\Lambda^2 \lesssim 7.1\,\mathrm{TeV}^{-2}$ and $|C_{33}^{uW}|/\Lambda^2 \lesssim 2.5\,\mathrm{TeV}^{-2}$. Here, bounds from $h \to \gamma\gamma$ derived in eq. (2.63) are *more than an order of magnitude stronger*.

Restrictions to all other coefficients appeared in eq. (2.62) can be found in various articles in the literature. For example, following ref. [78], $Q_{\varphi D}$ contributes to the T-electroweak parameter and the corresponding bound is, $|C^{\varphi D}|/\Lambda^2 \lesssim 0.03\,\mathrm{TeV}^{-2}$. This makes its contribution in $h \to \gamma\gamma$ negligible. However, the coefficients $C^{\varphi\Box}$ and C^W are not really constrained by fitting the LHC Higgs data. It is obvious from eq. (2.62) that these two coefficients can give $\mathcal{O}(10)\%$ contributions to $\mathcal{R}_{h\to\gamma\gamma}$ only when one is in the vicinity of EFT validity.

2.5.3 $h \to \gamma\gamma$ relevant UV-models

The question we want to address here is related to possible UV-field theories connected with the Wilson coefficients of eq. (2.62) contributing to the $h \to \gamma\gamma$ amplitude. A possible UV-theory, which could be a renormalizable theory or yet another EFT, is considered to be valid in and above the neighborhood of the energy scale Λ and contains heavy (w.r.t. the EW scale) fields. When these fields are integrated out a subset of SMEFT operators appears in the low-energy theory. In a recent analysis [100], based on power counting rules it has been shown that in UV-completions of SMEFT the heavy fields are restricted to have definite quantum numbers and spins 0, 1/2, 1. This result, which we will follow here, assumes that the candidate UV-theory is invariant under the linearly realised SM-gauge group, that it is chirally non-anomalous, and that it contains a multiplet with the SM Higgs field in the representation $(SU(3)_C, SU(2)_L)_{U(1)_Y} = (1,2)_{\frac{1}{2}}$.

We divide the Wilson coefficients appeared in eq. (2.62) into PTG and LG operators [79] as in Table 2.2. Then, following the tables in Appendix C of ref. [100], we check which coefficients can originate from integrating out fields with certain quantum numbers. Our results are shown in Tables 2.3 and 2.4. There are 5 spin-0 scalars, 13 Weyl fermions with vector-like masses, and 5 spin-1 gauge bosons, that can possibly appear in a UV-theory and affect the $h \to \gamma\gamma$ amplitude through eq. (2.62). Remarkably, the LG coefficients in Table 2.4 are only a small subset of the PTG ones shown in Table 2.3. In addition, the C^W-coefficient is absent from both Tables 2.3 and 2.4.

Tables 2.3 and 2.4, which in connection with Appendix D of ref. [100] relate the Wilson coefficient to the actual couplings of heavy fields, can be used to put bounds on the latter.

Potentially Tree Generated (PTG) Operators involved in $h \to \gamma\gamma$								
Spin	Field	$C^{\ell\ell}$	$C^{\varphi\ell(3)}$	$C^{\varphi\Box}$	$C^{\varphi D}$	$C^{u\varphi}$	$C^{d\varphi}$	$C^{e\varphi}$
Spin-0	$\mathcal{S}(1,1)_0$			✓		✓	✓	✓
	$\mathcal{S}(1,1)_1$	✓						
	$\phi(1,2)_{\frac{1}{2}}$					✓	✓	✓
	$\Xi(1,3)_0$			✓	✓	✓	✓	✓
	$\Xi_1(1,3)_1$	✓		✓	✓	✓	✓	✓
Spin-1/2	$N(1,1)_0$		✓					
	$E(1,1)_{-1}$		✓					✓
	$\Delta_1(1,2)_{-\frac{1}{2}}$							✓
	$\Delta_3(1,2)_{-\frac{3}{2}}$							✓
	$\Sigma(1,3)_0$		✓					✓
	$\Sigma_1(1,3)_{-1}$		✓					✓
	$U(3,1)_{\frac{2}{3}}$					✓		
	$D(3,1)_{-\frac{1}{3}}$						✓	
	$Q_1(3,2)_{\frac{1}{6}}$					✓	✓	
	$Q_5(3,2)_{-\frac{5}{6}}$						✓	
	$Q_7(3,2)_{\frac{7}{6}}$					✓		
	$T_1(3,3)_{-\frac{1}{3}}$					✓	✓	
	$T_2(3,3)_{\frac{2}{3}}$					✓	✓	
Spin-1	$\mathcal{B}(1,1)_0$	✓		✓	✓	✓	✓	✓
	$\mathcal{B}_1(1,1)_1$			✓	✓	✓	✓	✓
	$\mathcal{W}(1,3)_0$	✓	✓	✓	✓	✓	✓	✓
	$\mathcal{W}_1(1,3)_1$			✓	✓	✓	✓	✓
	$\mathcal{L}_1(1,2)_{\frac{1}{2}}$		✓	✓	✓	✓	✓	✓

Table 2.3: Dictionary for possible UV-completions with fields that, upon their "integration out", lead to PTG operators affecting the $h \to \gamma\gamma$ amplitude in eq. (2.62). Flavour indices are suppressed. The field notation follows ref. [100].

We illustrate it by presenting an example. Imagine a triplet scalar, $\Xi(1,3)_0$, that is directly found or implied by an experiment with mass M in the TeV-range. According to Tables 2.3 and 2.4, at low energies there are contributions from "integrating out" Ξ in PTG coefficients $C^{\varphi\Box}, C^{\varphi D}, C^{u\varphi}, C^{d\varphi}, C^{e\varphi}$ and in a LG coefficient $C^{\varphi WB}$. From eq. (2.62) we obtain that $C^{\varphi\Box}$ and $C^{\varphi WB}$ are multiplied by the biggest pre-factors and therefore play more important

Loop Generated (LG) Operators involved in $h \to \gamma\gamma$											
Spin	Field	$C^{\varphi B}$	$C^{\varphi W}$	$C^{\varphi WB}$	C^{W}	C^{uB}	C^{uW}	C^{dB}	C^{dW}	C^{eB}	C^{eW}
Spin-0	$\mathcal{S}(1,1)_0$	✓	✓								
	$\Xi(1,3)_0$			✓							
Spin-1/2	$E(1,1)_{-1}$									✓	
	$\Delta_1(1,2)_{-\frac{1}{2}}$									✓	✓
	$\Sigma_1(1,3)_{-1}$										✓
	$U(3,1)_{\frac{2}{3}}$					✓					
	$D(3,1)_{-\frac{1}{3}}$							✓			
	$Q_1(3,2)_{\frac{1}{6}}$					✓	✓	✓	✓		
	$T_1(3,3)_{-\frac{1}{3}}$								✓		
	$T_2(3,3)_{\frac{2}{3}}$						✓				
Spin-1	$\mathcal{L}_1(1,2)_{\frac{1}{2}}$	✓	✓	✓		✓	✓	✓	✓	✓	✓

Table 2.4: Dictionary for possible UV-completions with fields that, upon their "integration out", lead to LG operators affecting the $h \to \gamma\gamma$ amplitude in eq. (2.62). Again, flavour indices are suppressed. The field notation follows ref. [100].

role in $h \to \gamma\gamma$ amplitude. The UV-Lagrangian,[14] which originates these coefficients, is [100]

$$\mathcal{L} = \mathcal{L}_{\rm SM} + (D_\mu \Xi^I)^\dagger (D^\mu \Xi^I) - M^2 \Xi^{I\,\dagger} \Xi^I - \kappa \varphi^\dagger \Xi^I \tau^I \varphi - \frac{1}{f} \tilde{\kappa}\, \Xi^I W^I_{\mu\nu} B^{\mu\nu} \,, \tag{2.65}$$

where D_μ is the covariant derivative acting on the triplet Ξ, τ^I are Pauli matrices and f is an energy scale with $\Lambda \lesssim 4\pi f$. Upon integrating out the field Ξ, or simply reading from the Appendix D of ref. [100] we identify (at tree level for the UV-theory)

$$\frac{C^{\varphi\Box}}{\Lambda^2} \to \frac{\kappa^2}{2M^4} \,, \qquad \frac{C^{\varphi WB}}{\Lambda^2} \to \frac{1}{f} \frac{\kappa\tilde{\kappa}}{M^2} \,, \tag{2.66}$$

[14] This could be any Lagrangian that consists of fields arranged in Tables 2.3 and 2.4 portal to SM and partly responsible for Dark Mattter or other phenomena beyond the SM.

and using our eq. (2.62) we arrive very easily at the bounds

$$\kappa \lesssim 1.6 \frac{M^2}{1\ \text{TeV}}\,, \qquad \frac{\kappa\tilde{\kappa}}{f} \lesssim 0.06 \left(\frac{M}{1\ \text{TeV}}\right)^2 . \tag{2.67}$$

If κ takes on its maximal value then $\tilde{\kappa}/f \lesssim 0.004\ \text{TeV}^{-1}$. Of course one can advance a similar analysis in every case of an observable, not necessarily $h \to \gamma\gamma$, that is needed to be explained by a subset of fields affecting eq. (2.62).

We note in passing that Tables 2.3 and 2.4 do not include operators that are induced at one-loop in the UV-theory. Trivial examples comprise of heavy electromagnetically charged fermions that obtain part of their masses through the SM Higgs field or heavy charged scalars that are coupled to it. A nice and non-trivial example illustrating this case can be found in ref. [101].

2.5.4 Comparison with literature

As we mentioned in the introduction, the calculation for $h \to \gamma\gamma$ in SMEFT was first performed several years ago in refs. [72, 73] and to our knowledge these are the only complete studies prior to ours here. Our check shows that there are two, numerically important differences. First, all corresponding $\delta\mathcal{R}_{h\to\gamma\gamma}$ in ref. [72] are smaller by exactly a factor of four. We think that this is due to a mistake in eq. (26) of ref. [72][arXiv v3]. Second, our eq. (2.49) is *not* in agreement with the corresponding expression of ref. [72]. We believe there is a Yukawa coupling missing for each generation and flavour in the corresponding expression of ref. [72]. Up to the aforementioned differences, we found agreement with $\delta\mathcal{R}^{(1,2,3,5,6)}_{h\to\gamma\gamma}$. As far as $\delta\mathcal{R}^{(4)}_{h\to\gamma\gamma}$ is concerned, a direct comparison of our formulae in eq. (2.53) with the corresponding one in ref. [73] is very difficult. Checking individually quantities appearing in both works, for example, $\delta v/v$ or Π'_{hh}, is meaningless since the calculations in refs. [72, 73] were performed in background field gauges while ours in linear R_ξ-gauges. Comparing numerically the correction, $\delta\mathcal{R}^{(4)}_{h\to\gamma\gamma}$, appearing in our eq. (2.62) with a corresponding ratio based on refs. [72, 73], we find, upon fixing the factor of four mentioned above, a maximal difference of 5% for $\mu = m_W$, originating from what multiplies the coefficient $C^{\varphi B}$.

2.6 Conclusions

In our analysis we have calculated the one-loop decay width of the $h \to \gamma\gamma$ process in the SM extended by all CP-conserving gauge invariant operators up to dimension 6 in Warsaw basis. We performed the calculations using the general R_ξ-gauges and a hybrid renormalisation scheme, where we assumed the on-shell conditions for the SM parameters and $\overline{\mathrm{MS}}$ subtraction for the running Wilson coefficients of the higher order operators. We explicitly checked the gauge ξ-parameter cancellation, which provides the very strict test of correctness of our calculations. In addition, we also explicitly proven that at the one-loop and $1/\Lambda^2$ order, the calculated amplitude is independent of the renormalisation scale μ. Our work is complementary to previous analyses [72, 73] of this process using the Background Field Method and comparisons of our results with theirs were made whenever possible. Our master formula for the S-matrix amplitude is given by eqs. (2.32) and (2.33).

We give a complete set of analytical formulae for all classes of SM and SMEFT contributions to $h \to \gamma\gamma$ decay rate, normalised to the SM result as in published LHC searches [see eq. (2.43)]. We also present them in a form of simple and compact semi-analytical expressions depending only on running Wilson coefficients and renormalisation scale μ. Eq. (2.62) summarises all dominant contributions. Such formula can be readily used as additional constraint in experimental or theoretical analyses considering other observables in SMEFT.

We show that numerically largest corrections to the SM prediction can arise from $Q_{\varphi B}$, $Q_{\varphi W}$ and $Q_{\varphi WB}$ operators, contributing already at the tree-level, and from Q_{uB}^{33}, Q_{uW}^{33} operators arising at the loop level. Only Wilson coefficients of these operators can be meaningfully constrained using the current precision of the LHC measurements for the $h \to \gamma\gamma$ decay width. In some cases, like C_{33}^{uB} and C_{33}^{uW}, such constraints are already stronger than those from other measurements, in this case for instance from top-quark LHC-physics.

It would be useful to connect our main outcome, the expression eq. (2.62), with a particular UV-model. One may follow ref. [100] in integrating out heavy fields, which under reasonable assumptions but limited to perturbative decoupling at tree-level, results in a subset of operators arranged in table 2.1. Interestingly, one can arrange a finite number of heavy fields with renormalisable (or not) interactions that affect both PTG and LG operators in table 2.2. Another possibility may be a direct model like the one of ref. [101] where the operators, $Q_{\varphi B}, Q_{\varphi W}$ and $Q_{\varphi WB}$, are generated. In general however, it is quite difficult, if possible in any way, to find a model with appreciable, $\mathcal{O}(1)$, coefficients for these operators. Possibly, some examples will be found in the future.

A general look of our SMEFT calculational framework does not differ from common frameworks calculating electroweak one-loop corrections, like in the renormalisable SM for example. Our work can easily be automatised although we performed as many manual calculations we could for comparisons and cross-checks. For example, one can use the SMEFT Feynman rules, given also in a `Mathematica` code, from ref. [40], and existed codes to calculate Feynman diagrams, employ a "traditional" renormalisation prescription from 80's described also here and, checking gauge invariance at every step, present a concise form of an amplitude in a useful semi-numeric form, as in eq. (2.62). It is worth for pursuing this SMEFT framework further.

2.A SMEFT amplitudes and SM self-energies in R_ξ-gauges

We append here the one-loop corrections in general renormalisable gauges for the three-point 1PI functions, $\Gamma^{\varphi B}$, $\Gamma^{\varphi W}$ and $\Gamma^{\varphi WB}$, as well as for the SM vector boson self-energies that are needed for eqs. (2.33) and (2.53). The first, ξ-independent, terms of the equations below refer always to a part in unitary gauge. The `Mathematica` package `FeynCalc` [102, 103] was used for most of our Feynman diagram calculations. To bring Feynman integrals into analytic forms we used the `Mathematica` package `Package-X` [104, 105]. In what follows, we use the mass-ratios

$$r_X \equiv \frac{4m_X^2}{m_h^2} \qquad \text{and} \qquad r_{XY} \equiv \frac{4m_X^2}{m_Y^2}\,. \tag{2.68}$$

For the SMEFT one-loop corrections we have

$$\begin{aligned}\Gamma^{\varphi B} = \frac{-\lambda}{32\pi^2}\Bigg\{&3\left(E+2-\frac{\pi}{\sqrt{3}}-\log\frac{m_h^2}{\mu^2}\right)+2\left(E+2-\log\frac{m_W^2}{\mu^2}-\log\xi_W+J_2(\xi_W r_W)\right)\\ &+E+2-\log\frac{m_Z^2}{\mu^2}-\log\xi_Z+J_2(\xi_Z r_Z)\Bigg\}\,,\end{aligned} \tag{2.69}$$

$$\begin{aligned}\Gamma^{\varphi W} = \frac{-1}{32\pi^2}\Bigg\{&3\lambda\left(E+2-\frac{\pi}{\sqrt{3}}-\log\frac{m_h^2}{\mu^2}\right)+\bar{g}^2[6r_W(1-r_W f(r_W))-16(1-r_W)f(r_W)]\\ &+2\big(\lambda-\bar{g}^2(\xi_W+3)\big)\left(E-\log\frac{m_W^2}{\mu^2}-\log\xi_W\right)\\ &+4\lambda-\bar{g}^2(\xi_W+5)+\frac{6\bar{g}^2}{\xi_W-1}\log\xi_W+2\lambda J_2(\xi_W r_W)\\ &+\lambda\left(E+2-\log\frac{m_Z^2}{\mu^2}-\log\xi_Z+J_2(\xi_Z r_Z)\right)\Bigg\}\,,\end{aligned} \tag{2.70}$$

$$\begin{aligned}
\Gamma^{\varphi WB} = \frac{-1}{32\pi^2}\Bigg\{ & -\lambda\left(E+2+\sqrt{3}\pi-\log\frac{m_W^2}{\mu^2}\right)+6\bar{g}^2\left(E-\log\frac{m_W^2}{\mu^2}\right)+\frac{2\bar{g}^2\bar{g}'^2\big(3\bar{g}^2+2\lambda\big)}{\lambda(\bar{g}^2+\bar{g}'^2)} \\
& -3\lambda\log\frac{m_h^2}{m_W^2}-\frac{2\bar{g}^2(3\bar{g}^2\bar{g}'^2+2\lambda\bar{g}^2-4\lambda\bar{g}'^2)}{\lambda(\bar{g}^2+\bar{g}'^2)}r_W f(r_W)+2(\bar{g}^2-2\lambda)J_2(r_W) \\
& -\frac{16}{m_h^2}\frac{\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\sum_f m_f^2 Q_f^2 N_{c,f}[1+(1-r_f)f(r_f)] \\
& +\lambda\left(E+2-\log\frac{m_Z^2}{\mu^2}-\log\xi_Z+J_2(\xi_Z r_Z)\right) \\
& +\big(2\lambda-\bar{g}^2(\xi_W+3)\big)\left(E-\log\frac{m_W^2}{\mu^2}-\log\xi_W\right) \\
& +4\lambda-\frac{\bar{g}^2}{2}(\xi_W+5)+\frac{3\bar{g}^2}{\xi_W-1}\log\xi_W+2\lambda J_2(\xi_W r_W)\Bigg\}\,. \qquad (2.71)
\end{aligned}$$

The SM self-energies are presented (to our knowledge for the first time) also in ref. [106], for general renormalisable gauges, and in ref. [89] for $\xi = 1$. We have recalculated them here for consistency. The results are:

$$\begin{aligned}
\Pi_{\gamma\gamma}(0) = & -\frac{1}{48\pi^2}\frac{\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left[21\left(E-\log\frac{m_W^2}{\mu^2}\right)+2-4\sum_f N_{c,f}Q_f^2\left(E-\log\frac{m_f^2}{\mu^2}\right)\right] \\
& +\frac{1}{32\pi^2}\frac{\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left[2(\xi_W+3)\left(E-\log\frac{m_W^2}{\mu^2}\right)+\xi_W+5+\frac{2\xi_W(\xi_W+2)}{1-\xi_W}\log\xi_W\right], \qquad (2.72)
\end{aligned}$$

$$A_{Z\gamma}(0) = \frac{\bar{g}^3\bar{g}'v^2}{(16\pi)^2}\left[2(\xi_W+3)\left(E-\log\frac{m_W^2}{\mu^2}\right)+\xi_W+5+\frac{2\xi_W(\xi_W+2)}{1-\xi_W}\log\xi_W\right], \qquad (2.73)$$

$$
\begin{aligned}
A_{ZZ}(m_Z^2) = \frac{v^2}{768\pi^2}\Bigg\{ & \big(59\bar{g}^4 - 36\bar{g}^2\bar{g}'^2 - 11\bar{g}'^4\big)E \\
& + \frac{2\big(278\bar{g}^6 + 29\bar{g}^4\bar{g}'^2 - 140\bar{g}^2\bar{g}'^4 - 24\lambda^2(\bar{g}^2+\bar{g}'^2) + 36\lambda(\bar{g}^2+\bar{g}'^2)^2 - 35\bar{g}'^6\big)}{3(\bar{g}^2+\bar{g}'^2)} \\
& + \lambda\left(\frac{32\lambda^2}{\bar{g}^2+\bar{g}'^2} - 48\lambda + 36(\bar{g}^2+\bar{g}'^2)\right)\log\frac{m_h^2}{\mu^2} \\
& + 2\left(\frac{-16\lambda^3}{\bar{g}^2+\bar{g}'^2} + 24\lambda^2 - 18\lambda(\bar{g}^2+\bar{g}'^2) + 5(\bar{g}^2+\bar{g}'^2)^2\right)\log\frac{m_Z^2}{\mu^2} \\
& + \big(-69\bar{g}^4 + 16\bar{g}^2\bar{g}'^2 + \bar{g}'^4\big)\log\frac{m_W^2}{\mu^2} \\
& + \frac{(3\bar{g}^2-\bar{g}'^2)(33\bar{g}^4 + 22\bar{g}^2\bar{g}'^2 + \bar{g}'^4)}{\bar{g}^2+\bar{g}'^2}J_2(r_{WZ}) \\
& - 16\big[4\lambda^2 - 4\lambda(\bar{g}^2+\bar{g}'^2) + 3(\bar{g}^2+\bar{g}'^2)^2\big]J_1(r_Z) \\
& + 16(\bar{g}^2+\bar{g}'^2)^2\sum_f N_{c,f} \\
& \quad\times\Bigg\{g_{A,f}^2\left[\left(\frac{3}{2}r_{fZ}-1\right)\left(E - \log\frac{m_f^2}{\mu^2}\right) + 2r_{fZ} - \frac{5}{3} + (r_{fZ}-1)J_2(r_{fZ})\right] \\
& \qquad - g_{V,f}^2\left[E - \log\frac{m_f^2}{\mu^2} + r_{fZ} + \frac{5}{3} + \left(\frac{1}{2}r_{fZ}+1\right)J_2(r_{fZ})\right]\Bigg\} \\
& - 6\xi_W\bar{g}^2(\bar{g}^2+\bar{g}'^2)\left(E + 1 - \log\xi_W - \log\frac{m_W^2}{\mu^2}\right) \\
& - 3\xi_Z(\bar{g}^2+\bar{g}'^2)^2\left(E + 1 - \log\xi_Z - \log\frac{m_Z^2}{\mu^2}\right)\Bigg\}\,,
\end{aligned}
\tag{2.74}
$$

where the axial-vector and vector couplings are defined as $g_{A,f} = \frac{1}{2}T_f^3$ and $g_{V,f} = \frac{1}{2}T_f^3 - \sin^2\theta_W Q_f$, respectively. The neutrino term in $A_{ZZ}(m_Z^2)$ is contained in the fermionic part, and can readily be obtained by taking the limit $m_f \to 0$.

$$
\begin{aligned}
A_{WW}(m_W^2) = \frac{v^2}{768\pi^2}\Bigg\{ & \bar{g}^2\big(59\bar{g}^2 - 9\bar{g}'^2\big)E + \frac{1}{3}\big(556\bar{g}^4 - 75\bar{g}^2\bar{g}'^2 - 3\bar{g}'^4 + 72\lambda\bar{g}^2 - 48\lambda^2\big) \\
& + \frac{4\lambda}{\bar{g}^2}\big(8\lambda^2 - 12\lambda\bar{g}^2 + 9\bar{g}^4\big)\log\frac{m_H^2}{\mu^2} \\
& + \frac{1}{2\bar{g}^2}\big(-69\bar{g}^6 - 53\bar{g}^4\bar{g}'^2 + 17\bar{g}^2\bar{g}'^4 + \bar{g}'^6\big)\log\frac{m_Z^2}{\mu^2} \\
& - \frac{1}{2\bar{g}^2}\big[49\bar{g}^6 + \bar{g}^4(72\lambda - 71\bar{g}'^2) + \bar{g}^2(17\bar{g}'^4 - 96\lambda^2) + \bar{g}'^6 + 64\lambda^3\big]\log\frac{m_W^2}{\mu^2} \\
& - 16\big(3\bar{g}^4 - 4\bar{g}^2\lambda + 4\lambda^2\big)J_1(r_W) + \frac{4(99\bar{g}^6 + 33\bar{g}^4\bar{g}'^2 - 19\bar{g}^2\bar{g}'^4 - \bar{g}'^6)}{\bar{g}^2 + \bar{g}'^2}J_1(r_{WZ}) \\
& + 2\bar{g}^4\sum_{\ell=e,\mu,\tau}\Bigg\{\left(\frac{3}{4}r_{\ell W} - 2\right)\left(E - \log\frac{m_\ell^2}{\mu^2}\right) + \frac{r_{\ell W}^2}{16} + \frac{1}{2}r_{\ell W} \\
& \qquad - \frac{10}{3} + \left(\frac{r_{\ell W}^3}{64} - \frac{3}{4}r_{\ell W} + 2\right)\log\left(1 - \frac{m_W^2}{m_\ell^2}\right)\Bigg\} \\
& + \frac{8\bar{g}^2N_c}{v^2}\sum_{\alpha,\beta}|K_{\alpha\beta}|^2\Bigg\{\left(3m_{d_\beta}^2 + 3m_{u_\alpha}^2 - 2m_W^2\right)E \\
& \qquad + \frac{(m_{d_\beta}^2 - m_{u_\alpha}^2)^2}{m_W^2} + 2(m_{d_\beta}^2 + m_{u_\alpha}^2) - \frac{10}{3}m_W^2 \\
& \qquad + \left[\frac{(m_{d_\beta}^2 - m_{u_\alpha}^2)^3}{2m_W^4} - \frac{3}{2}(m_{d_\beta}^2 + m_{u_\alpha}^2) + m_W^2\right]\log\frac{m_{u_\alpha}^2}{\mu^2} \\
& \qquad + \left[\frac{(m_{u_\alpha}^2 - m_{d_\beta}^2)^3}{2m_W^4} - \frac{3}{2}(m_{d_\beta}^2 + m_{u_\alpha}^2) + m_W^2\right]\log\frac{m_{d_\beta}^2}{\mu^2} \\
& \qquad + \left[\frac{(m_{d_\beta}^2 - m_{u_\alpha}^2)^2}{m_W^4} + \frac{(m_{d_\beta}^2 + m_{u_\alpha}^2)}{m_W^2} - 2\right]J_3(m_{u_\alpha}, m_{d_\beta})\Bigg\} \\
& - 6\xi_W\bar{g}^4\left(E + 1 - \log\xi_W - \log\frac{m_W^2}{\mu^2}\right) \\
& - 3\xi_Z\bar{g}^2(\bar{g}^2 + \bar{g}'^2)\left(E + 1 - \log\xi_Z - \log\frac{m_Z^2}{\mu^2}\right)\Bigg\}, \qquad (2.75)
\end{aligned}
$$

where

$$m_u = \mathrm{diag}(m_u, m_c, m_t), \qquad m_d = \mathrm{diag}(m_d, m_s, m_b), \qquad (2.76)$$

$K_{\alpha\beta}$ is the CKM matrix, and the summation indices in the hadronic contribution run over all the quark generations. The infinite quantity E is given by eq. (2.27), and the functions

$J_1(x), J_2(x)$ and $J_3(x)$ are defined through

$$J_1(x) \equiv \begin{cases} \frac{\sqrt{1-x}}{x} \log\Big(\frac{1+\sqrt{1-x}}{\sqrt{x}}\Big)\,, & 0 < x \leq 1\,, \\ -2\frac{\sqrt{x-1}}{x} \arctan\Big(\frac{\sqrt{x-1}}{1+\sqrt{x}}\Big)\,, & x \geq 1\,, \end{cases} \tag{2.77}$$

$$J_2(x) \equiv \begin{cases} \sqrt{1-x}\Big[\log\Big(\frac{2-x-2\sqrt{1-x}}{x}\Big) + i\pi\Big]\,, & 0 < x \leq 1\,, \\ -2\sqrt{x-1}\arctan\Big(\frac{1}{\sqrt{x-1}}\Big)\,, & x \geq 1\,, \end{cases} \tag{2.78}$$

and

$$\begin{aligned} J_3(m_u, m_d) \equiv & \sqrt{\left[(m_d - m_u)^2 - m_W^2\right]\left[(m_d + m_u)^2 - m_W^2\right]} \\ & \times \log\left[\frac{(m_d^2 + m_u^2 - m_W^2) + \sqrt{\left[(m_d - m_u)^2 - m_W^2\right]\left[(m_d + m_u)^2 - m_W^2\right]}}{2m_d m_u}\right]. \end{aligned} \tag{2.79}$$

For completeness we also add here the W-boson one-loop self-energy at zero external momentum, evaluated in Feynman gauge, needed in the master formula (2.53). It reads

$$\begin{aligned} A_{WW}(0) = & \frac{\bar{g}^4 v^2}{64\pi^2}\Bigg\{\left(1 - \frac{\bar{g}'^2}{\bar{g}^2}\right)E + \frac{\lambda}{2\bar{g}^2} - \frac{7\bar{g}'^2}{8\bar{g}^2} + \frac{27}{8} - \frac{3\lambda}{(\bar{g}^2 - 4\lambda)}\log\frac{m_h^2}{\mu^2} \\ & \quad + \left(\frac{17\bar{g}^2}{4\bar{g}'^2} + \frac{3\bar{g}^2}{4(\bar{g}^2 - 4\lambda)} - \frac{1}{2}\right)\log\frac{m_W^2}{\mu^2} - \left(\frac{17\bar{g}^2}{4\bar{g}'^2} - \frac{\bar{g}'^2}{\bar{g}^2} + \frac{5}{4}\right)\log\frac{m_Z^2}{\mu^2}\Bigg\} \\ & + \frac{\bar{g}^2 N_c}{32\pi^2}\sum_{\alpha,\beta}|K_{\alpha\beta}|^2\Bigg[\left(m_{u_\alpha}^2 + m_{d_\beta}^2\right)\left(E - \log\frac{m_{d_\beta}^2}{\mu^2}\right) \\ & \qquad + \frac{m_{u_\alpha}^2 + m_{d_\beta}^2}{2} + \frac{m_{u_\alpha}^4}{m_{u_\alpha}^2 - m_{d_\beta}^2}\log\frac{m_{d_\beta}^2}{m_{u_\alpha}^2}\Bigg] \\ & + \frac{\bar{g}^2}{32\pi^2}\sum_{\ell=e,\mu,\tau} m_\ell^2\left[\left(E - \log\frac{m_\ell^2}{\mu^2}\right) + \frac{1}{2}\right]. \end{aligned} \tag{2.80}$$

Moreover, the derivative of the Higgs self-energy reads:

$$\begin{aligned}
\Pi'_{hh}(m_h^2) = \frac{1}{128\pi^2}\Bigg\{&(12\bar{g}^2-16\lambda)\left(E-\log\frac{m_W^2}{\mu^2}\right)+\frac{6}{\lambda}(\bar{g}^4+2\bar{g}^2\lambda-4\lambda^2)\\
&+\frac{16\lambda^3-20\bar{g}^2\lambda^2+4\bar{g}^4\lambda+3\bar{g}^6}{\lambda(\bar{g}^2-\lambda)}J_2(r_W)\\
&+\left[6(\bar{g}^2+\bar{g}'^2)-8\lambda\right]\left(E-\log\frac{m_Z^2}{\mu^2}\right)+\frac{3}{\lambda}\left[(\bar{g}^2+\bar{g}'^2)^2+2\lambda(\bar{g}^2+\bar{g}'^2)-4\lambda^2\right]\\
&+\frac{16\lambda^3-20\lambda^2(\bar{g}^2+\bar{g}'^2)+4\lambda(\bar{g}^2+\bar{g}'^2)^2+3(\bar{g}^2+\bar{g}'^2)^3}{2\lambda(\bar{g}^2+\bar{g}'^2-\lambda)}J_2(r_Z)+4\lambda(9-2\sqrt{3}\pi)\\
&-16\sum_f N_{c,f}\left(\frac{m_f}{v}\right)^2\left[E-\log\frac{m_f^2}{\mu^2}+1+r_f+\left(1+\frac{r_f}{2}\right)J_2(r_f)\right]\\
&+4\left(4\lambda-\bar{g}^2\xi_W\right)\left(E-\log\frac{m_W^2}{\mu^2}-\log\xi_W\right)+4\left(8\lambda-\bar{g}^2\xi_W\right)+16\lambda J_2(\xi_W r_W)\\
&+2\left[4\lambda-(\bar{g}^2+\bar{g}'^2)\xi_Z\right]\left(E-\log\frac{m_Z^2}{\mu^2}-\log\xi_Z\right)+2\left[8\lambda-(\bar{g}^2+\bar{g}'^2)\xi_Z\right]+8\lambda J_2(\xi_Z r_Z)\Bigg\},
\end{aligned} \tag{2.81}$$

and the light quark contribution needed in eq. (2.55) is

$$\Pi_{\gamma\gamma}^{\text{had}}(m_Z^2) = \frac{\bar{g}^2 g'^2}{12\pi^2(\bar{g}^2+\bar{g}'^2)}\sum_q N_c Q_q^2\left[E-\log\frac{m_q^2}{\mu^2}+\left(1+\frac{r_{qZ}}{2}\right)J_2(r_{qZ})+r_{qZ}+\frac{5}{3}\right]. \tag{2.82}$$

Chapter 3

The decay $h \to Z\gamma$ in the Standard Model Effective Field Theory

In this chapter we calculate the S-matrix element for the Higgs boson decay to a Z-boson and a photon, $h \to Z\gamma$, at one-loop in the Standard Model Effective Field Theory (SMEFT) framework and in linear R_ξ-gauges. Our SMEFT expansion includes all relevant operators up to dimension 6 considered in Warsaw basis without resorting to any flavour or CP-conservation assumptions. Within this approximation there are 23 dimension 6 operators affecting the amplitude, not including flavour and hermitian conjugation. The result for the on-shell $h \to Z\gamma$ amplitude is gauge invariant, renormalisation-scale invariant and gauge-fixing parameter independent. The calculated ratio of the SMEFT versus the SM expectation for the $h \to Z\gamma$ decay width is then written in a semi-numerical form which is useful for further comparisons with related processes. For example, the $h \to Z\gamma$ amplitude contains 16 operators in common with the $h \to \gamma\gamma$ amplitude and one can draw useful results about its feasibility at current and future LHC data. This chapter is based on ref. [107].

3.1 Introduction

The Higgs boson decay processes $h \to \gamma\gamma$ and $h \to Z\gamma$ are extremely important probes for physics beyond the Standard Model (SM) and are under intensive research ever since the Higgs boson discovery at LHC [27, 28]. Experimental bounds for both $h \to \gamma\gamma$ and $h \to Z\gamma$ decays were set by the CMS and ATLAS collaborations at LHC [108–111]. Although the $h \to \gamma\gamma$ decay width has been observed to within 15% w.r.t. the SM prediction, the situation is not the same for $h \to Z\gamma$. An upper bound for $h \to Z\gamma$ given by ATLAS [111], with centre-of-mass energy $\sqrt{s} = 13\,\text{TeV}$ proton-proton collisions, integrated luminosity 36.1 fb^{-1}, and Higgs boson mass $m_h = 125.09\,\text{GeV}$, finds that $\sigma(pp \to h) \times B(h \to Z\gamma)$ is 6.6 times the SM prediction with 95% confidence level. More specifically, it is

$$\mu_{h\to Z\gamma} = \frac{\sigma(pp \to h) \times \text{Br}(h \to Z\gamma)}{\sigma(pp \to h)_{\text{SM}} \times \text{Br}(h \to Z\gamma)_{\text{SM}}} \lesssim 6.6\,. \tag{3.1}$$

If physics beyond the SM does not affect the Higgs production,[1] which mainly goes via the gluon fusion process, $gg \to h$, then the bound of (3.1) is translated to a bound on a ratio

$$\mathcal{R}_{h\to Z\gamma} = \frac{\Gamma(\text{EXP}, h \to Z\gamma)}{\Gamma(\text{SM}, h \to Z\gamma)}\,. \tag{3.2}$$

The decay $h \to Z\gamma$ has been calculated for the first time in the SM in refs. [112–114]. To our knowledge, in the Standard Model Effective Field Theory (SMEFT) this process has been studied using a partial list of $d = 6$ operators in refs. [67, 68, 115], while an analysis with a complete set of $d = 6$ operators has recently been performed in ref. [71]. Here, we advance the current status of the SMEFT one-loop calculation for $\mathcal{R}_{h\to Z\gamma}$ in eq. (3.2) by presenting

- a clear and concise renormalisation framework in general R_ξ-gauges,
- a gauge invariant master formula for the amplitude which self-explains several issues even for the SM-amplitude,
- a semi-analytic formula for $\mathcal{R}_{h\to Z\gamma}$,
- correlations between the ratios $\mathcal{R}_{h\to Z\gamma}$ and $\mathcal{R}_{h\to\gamma\gamma}$.

Obviously there are many similarities in the calculation with the $h \to \gamma\gamma$ decay worked out at one loop in SMEFT in ref. [49][2] and we follow faithfully the renormalisation framework and the results found in there. We shall only focus on technical aspects that arise strictly in calculating the $h \to Z\gamma$ amplitude. This involves some subtle issues regarding gauge

[1] We shall comment upon this issue at the end of section 3.4.
[2] For similar studies see also refs. [72, 73, 116].

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}'_p e'_r \varphi)$
		$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^* \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}'_p u'_r \widetilde{\varphi})$
				$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}'_p d'_r \varphi)$
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{eW}	$(\bar{l}'_p\sigma^{\mu\nu}e'_r)\tau^I\varphi W^I_{\mu\nu}$	$Q^{(1)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{l}'_p\gamma^\mu l'_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{eB}	$(\bar{l}'_p\sigma^{\mu\nu}e'_r)\varphi B_{\mu\nu}$	$Q^{(3)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar{l}'_p\tau^I\gamma^\mu l'_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{uW}	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\tau^I\widetilde{\varphi}\, W^I_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{e}'_p\gamma^\mu e'_r)$
		Q_{uB}	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q^{(1)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{q}'_p\gamma^\mu q'_r)$
		Q_{dW}	$(\bar{q}'_p\sigma^{\mu\nu}d'_r)\tau^I\varphi\, W^I_{\mu\nu}$	$Q^{(3)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar{q}'_p\tau^I\gamma^\mu q'_r)$
		Q_{dB}	$(\bar{q}'_p\sigma^{\mu\nu}d'_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{u}'_p\gamma^\mu u'_r)$
				$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{d}'_p\gamma^\mu d'_r)$
		ψ^4			
		Q_{ll}	$(\bar{l}'_p\gamma_\mu l'_r)(\bar{l}'_s\gamma^\mu l'_t)$		

Table 3.1: Dimension 6 operators contributing to $h \to Z\gamma$ decay. For brevity we suppress fermion chiral indices L, R. We follow here the notation of refs. [37, 40]. The operator class $\psi^2\phi^2 D$ does not enter the $h \to \gamma\gamma$ amplitude.

invariance which we address in section 3.3. The operators relevant for $h \to Z\gamma$ are discussed in section 3.2 and their effects in $\mathcal{R}_{h\to Z\gamma}$ in section 3.4. We conclude in section 3.5.

3.2 Operators

Let the lightest of the heavy-particle masses be of order Λ. Following the decoupling theorem [54], their effects at low energies can be encoded in the renormalisation group running of the SM parameters in addition to the appearance of local higher-dimensional operators. The later are parameterised at low energies by a SMEFT Lagrangian, which takes the form

$$\mathcal{L} = \mathcal{L}^{(4)}_{\rm SM} + \sum_X C^X Q^{(6)}_X + \sum_f C'^f Q^{(6)}_f\,. \tag{3.3}$$

Eq. (3.3) contains the, renormalisable, SM Lagrangian $\mathcal{L}^{(4)}_{\rm SM}$, the dimension 6 operators $Q^{(6)}_X$ that do not involve fermion fields, and the dimension 6 operators $Q^{(6)}_f$ which are operators that contain fermion fields.[3] All Wilson coefficients should be rescaled by Λ^2, for instance $C^X \to C^X/\Lambda^2$. We shall restore $1/\Lambda^2$ explicitly in section 3.4 later on. The

[3]The single $d = 5$ lepton number violating operator does not affect $h \to Z\gamma$ at one-loop.

primed coefficients C'^f are written in the gauge invariant Warsaw basis of ref. [37], while the unprimed coefficients C^f in fermion mass basis are defined in ref. [40].

The operators contributing to the $h \to Z\gamma$ decay are collected in table 3.1. They are classified into 8 different classes according to the notation of ref. [37]. There are in total 23 relevant operators, not counting flavour structure and Hermitian conjugation. In unitary gauge, the coefficient C^{φ} associated with the operator $Q_{\varphi} = (\varphi^{\dagger}\varphi)^3$ does *not* appear in the calculation at $\mathcal{O}(\Lambda^{-2})$ and therefore does not contribute in the final amplitude.[4] The four-fermion operator Q_{ll} enters indirectly into the calculation through the relation between the vacuum expectation value (VEV) and the Fermi coupling constant G_F. There are no contributions from CP-violating operators up to $1/\Lambda^2$ terms in the EFT expansion. This is based upon the fact that the SM amplitude is CP-invariant (symmetric in particle momenta interchange) and all interference terms with CP-violating coefficients (antisymmetric in particle momenta interchange) of $\mathcal{O}(1/\Lambda^2)$, vanish identically.

The 16 out of 23 operators affecting $h \to Z\gamma$ are identical with those affecting the $h \to \gamma\gamma$ amplitude.[5] The 7 operators that appear only in $h \to Z\gamma$ (those belonging to category $\psi^2\varphi^2 D$ of table 3.1) may provide assistance in disentangling models for new physics in case of a $h \to Z\gamma$ experimental discovery. This is interesting because, if perturbative decoupling of the UV theory is assumed, the operators in $\psi^2\varphi^2 D$ category are potentially tree-level generated [79, 80]. If the two amplitudes, $h \to Z\gamma$ and $h \to \gamma\gamma$, are calculated in the same renormalisation input scheme, we can compare the relative strengths of the various contributions assuming dominance of one operator at a time. Within EFT we should be able to pose predictions on possible sensitivity at LHC and future colliders for the $h \to Z\gamma$ decay rate.

3.3 Renormalisation of the $h \to Z\gamma$ Amplitude

Our renormalisation procedure follows an old but clear description invented by A. Sirlin [85]. This procedure has already been applied successfully in a SMEFT calculation for $h \to \gamma\gamma$ in ref. [49] and is quickly repeated here for completeness before applying it to the calculation of the $h \to Z\gamma$ on-shell matrix element.

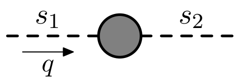
[4] On the contrary, in the R_ξ-gauges C^{φ} enters in individual diagrams, but it cancels out completely in the final sum. This adds to a list of several checks we performed in the final amplitude (cf. eq. (3.17)).

[5] The operator $Q_{\varphi l}^{(3)}$ does in fact enters in the $h \to \gamma\gamma$ amplitude, as well as in $h \to Z\gamma$, but only through the Fermi coupling constant redefinition, and not directly to $h \to \gamma\gamma$ one-loop amplitude.

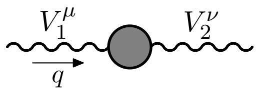
3.3.1 Counterterms

We first start with the part of SMEFT Lagrangian bilinear in gauge fields in gauge basis given in eq. (3.14) of ref. [40], and write all bare parameters as differences between renormalised parameters and corresponding counterterms, for example $g_0 = g - \delta g$. Then, mass diagonalisation for vector fields is performed by the matrix $\mathbb{X}$ given in eq. (3.19) of ref. [40]. As we are only interested in an S-matrix element, we keep all fields unrenormalised but multiplying the $h \to Z\gamma$ one-particle irreducible (1PI) amplitude by proper LSZ constants [88] for the external fields of h, γ and Z. In this way and after some algebra, counterterms are generated and connected to self-energy corrections for vector bosons. We work at one-loop in $\hbar$-expansion, and at $1/\Lambda^2$ in EFT expansion according to our discussion below eq. (3.3).

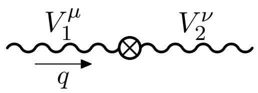
The definition of 2- and 3-point 1PI correlation functions contains all information we need to calculate the amplitude. Our definitions and conventions follow directly those of refs. [85] and [49]. We introduce the unrenormalised (but regularised) self-energies, that is 1PI diagrams for scalars $s_{1,2} = h$, and vector bosons $V_{1,2} = W^{\pm}, Z, \gamma$,



$$= -i\Pi_{s_1 s_2}(q^2)\,, \tag{3.4}$$



$$= i\Pi^{\mu\nu}_{V_1 V_2}(q^2) = iA_{V_1 V_2}(q^2) g^{\mu\nu} + iB_{V_1 V_2}(q^2) q^\mu q^\nu\,, \tag{3.5}$$



$$= ig^{\mu\nu}\delta m^2_{V_1 V_2} + iq^\mu q^\nu \delta^{(q)} m^2_{V_1 V_2}\,. \tag{3.6}$$

We also include the definition (3.6) for the vector boson counterterms since these are needed in the final amplitude. Physical masses for vector bosons, m_W and m_Z, are defined to keep their tree-level form in SMEFT, (cf. the first two lines of eq. (2.5) in chapter 2) by choosing the corresponding counterterms such that

$$\delta m_W^2 = \mathrm{Re}\, A_{WW}(m_W^2)\,, \quad \text{and} \quad \delta m_Z^2 = \mathrm{Re}\, A_{ZZ}(m_Z^2)\,. \tag{3.7}$$

The physical masses m_W and m_Z for the $W^\pm$ and Z vector bosons are inputs in our calculation. In general, tadpole and tadpole-counterterm diagrams also appear in the right-hand side of (3.7). However, one can arrange a renormalisation condition where the tree-level

VEV, v, is the exact one up to one-loop order or beyond. Such a condition implies that tadpole plus tadpole-counterterm diagrams vanish identically [87]. In addition, we define the weak mixing angle, θ_W, through

$$c^2 \equiv \cos^2\theta_W = \frac{m_W^2}{m_Z^2}\,, \qquad s^2 \equiv 1 - c^2\,, \qquad t \equiv \frac{s}{c}\,. \tag{3.8}$$

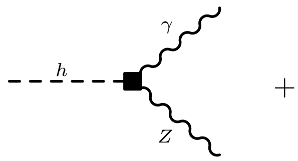
3.3.2 The Amplitude

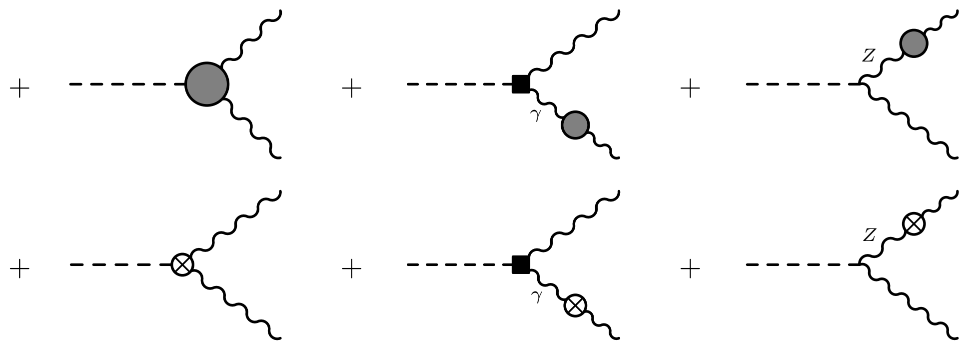
The on-shell S-matrix element for the $h \to Z\gamma$ amplitude can be written as

$$\langle \gamma(p_1),\, Z(p_2)|\, S\, |h(q)\rangle = \sqrt{Z_h}\sqrt{Z_\gamma}\sqrt{Z_Z}\,[i\mathcal{A}^{\mu\nu}(h \to Z\gamma)]\,\epsilon^*_\mu(p_1)\,\epsilon^*_\nu(p_2)\,, \tag{3.9}$$

where $q = p_1 + p_2$ is the incoming Higgs boson momentum, and p_1 (p_2) is the outgoing four-momentum of photon (Z-boson) along with the polarisation four-vector $\epsilon(p_1)$ ($\epsilon(p_2)$). Similar to the mass counterterms δm_V^2 of (3.7), the LSZ factors Z_h, Z_γ and Z_Z are calculated by the requirement for the full propagators to look like those of free particle states asymptotically. Diagrammatically, the amputated diagrams needed to sum up in eq. (3.9) are given in terms of 2- and 3-point 1PI Feynman diagrams calculated on the mass shell, $p_1^2 = 0$, $p_2^2 = m_Z^2$ and $p_1 \cdot p_2 = (m_h^2 - m_Z^2)/2$,

$i\mathcal{A}^{\mu\nu}(h \to Z\gamma)\epsilon^*_\mu(p_1)\,\epsilon^*_\nu(p_2) \;=$





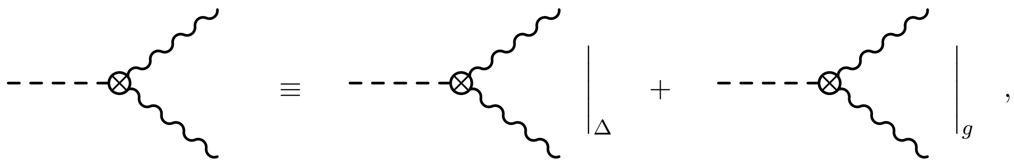
. (3.10)

A square ("■") in a vertex stands for a vertex generated by *only* $d = 6$ operators. Shaded blobs in the second line denote, as before, 1PI 3-point $hZ\gamma$-vertex and 2-point $Z\gamma$- or γZ-mixing at one-loop, while diagrams with "$\otimes$" symbol denote counterterms generated following the procedure described above.

Before deriving the master formula for the $h \to Z\gamma$ decay amplitude, it is worth noting a cancellation between some gauge non-invariant parts of the counterterms. For this reason, let us focus on the third line of the diagrams in (3.10) and collect the terms of the diagrams proportional to the gauge invariant quantity

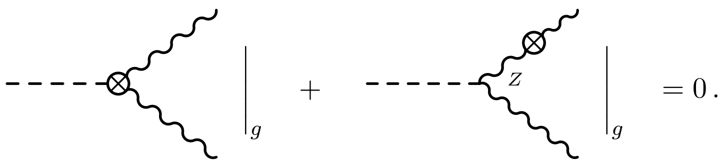
$$\Delta^{\mu\nu}(p_1, p_2) = p_1^\nu \, p_2^\mu - (p_1 \cdot p_2) g^{\mu\nu} \, . \tag{3.11}$$

Then, the gauge non-invariant leftovers are proportional to $g^{\mu\nu}$ ("pure-metric" terms). For example, the $hZ\gamma$-vertex counterterm expands diagrammatically as,



(3.12)

and similarly for the diagram containing the $Z\gamma$-mixing counterterm. We can then prove that the sum of the "pure-metric" contributions from the first and the third diagram of (3.10) vanishes:[6]



(3.13)

As a result, only the gauge invariant parts of these two counterterm diagrams make it into the master formula for the amplitude below. Note that these counterterm contributions exist even in the pure SM amplitude but usually not discussed in the literature. One can of course exploit gauge invariance to start with, as it was done for example in the first $h \to Z\gamma$ complete calculation of ref. [113], but it is really a nice cross-check of the calculation to see how contributions turn out to be gauge-invariant, respecting the usual Ward identities. Finally, note that the second diagram in the third line of (3.10) is gauge invariant by itself.

We are now ready to present the on-shell reduced matrix element defined as

$$\langle \gamma(p_1), \, Z(p_2) | \, S \, | h(q) \rangle = (2\pi)^4 \delta^{(4)}(q - p_1 - p_2) \, [i\mathcal{M}^{\mu\nu}(h \to Z\gamma)] \, \epsilon_\mu^*(p_1) \epsilon_\nu^*(p_2) \, . \tag{3.14}$$

[6]We remark here that the counterterm for the $h\gamma\gamma$-vertex is gauge-invariant by itself and, of course, zero in the SM.

Adding the diagrams in (3.10) together and by comparing eqs. (3.9) and (3.14) we obtain

$$
\begin{aligned}
i\mathcal{M}^{\mu\nu}(h \to Z\gamma) &= 4i\,\Delta^{\mu\nu}(p_1,p_2) \\
&\quad \times \Bigg\{ -sc\,v\,C^{\varphi B}\left[1 + \mathcal{X}^{\varphi B} - \frac{1}{t}\frac{A_{Z\gamma}(m_Z^2) + \delta m_{Z\gamma}^2}{m_Z^2} + t\frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{m_Z^2}\right] \\
&\quad + sc\,v\,C^{\varphi W}\left[1 + \mathcal{X}^{\varphi W} + t\frac{A_{Z\gamma}(m_Z^2) + \delta m_{Z\gamma}^2}{m_Z^2} - \frac{1}{t}\frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{m_Z^2}\right] \\
&\quad + \frac{s^2 - c^2}{2}\,v\,C^{\varphi WB}\left[1 + \mathcal{X}^{\varphi WB} - \frac{2sc}{s^2 - c^2}\frac{A_{Z\gamma}(m_Z^2) + A_{Z\gamma}(0) + 2\delta m_{Z\gamma}^2}{m_Z^2}\right] \\
&\quad + \frac{1}{m_W}\overline{\Gamma}^{\rm SM} + \sum_{i \neq \varphi B, \varphi W, \varphi WB} v\,C^i\,\Gamma^i \Bigg\} \\
&\quad - 4ig^{\mu\nu}\tfrac{1}{8}v(\bar{g}^2 + \bar{g}'^2)\big[1 + v^2 C^{\varphi\Box} + \tfrac{3}{4}v^2 C^{\varphi D} + 2sc\,v^2 C^{\varphi WB}\big]\frac{A_{Z\gamma}(0)}{m_Z^2} \\
&\quad - 4i(p_1 \cdot p_2)g^{\mu\nu}\left[\frac{1}{m_W}\overline{\Gamma}_g^{\rm SM} + \sum_i vC^i\,\Gamma_g^i\right]. \qquad (3.15)
\end{aligned}
$$

Eq. (3.15) is the master formula for the $h \to Z\gamma$ on-shell amplitude. The gauge-invariant quantity $\Delta^{\mu\nu}(p_1,p_2)$ has been defined in (3.11) while the self-energies and the counterterm $\delta m_{Z\gamma}^2$ in eqs. (3.5) and (3.6), respectively. Moreover, in (3.15) and for brevity, we defined the quantity

$$
\mathcal{X}^i \equiv \Gamma^i - \frac{\delta C^i}{C^i} - \frac{\delta v}{v} + \tfrac{1}{2}\Pi'_{hh}(m_h^2) + \tfrac{1}{2}A'_{ZZ}(m_Z^2) + \tfrac{1}{2}A'_{\gamma\gamma}(0)\,, \qquad (3.16)
$$

where $i = \varphi B, \varphi W, \varphi WB$. In (3.16), Γ^i stands for 1PI contributions from the first diagram in the 2nd line of (3.10). $\overline{\Gamma}^{\rm SM}$ is the SM contribution from triangle diagrams with W-bosons and fermions. In addition, δC^i and δv are counterterms for the Wilson coefficients with $i = \varphi B, \varphi W, \varphi WB$ and the VEV, respectively. The $\delta v/v$ counterterm is specified in eqs. (3.18)–(3.20) of ref. [49] after following the renormalisation scheme of refs. [85, 87]. The coefficients C^i (and in fact all Wilson coefficients in (3.3)) can be readily transformed in $\overline{\rm MS}$-scheme, $C - \delta C \to C(\mu) - \delta C$. As usual, in this scheme the counterterms δC^i subtract infinite parts proportional to $(\frac{2}{4-d} - \gamma + \log 4\pi)$ and can be read directly from eqs. (3.23)–(3.25) of ref. [49] as they have been adapted from refs. [86, 90, 91]. We confirm, even analytically, that these counterterms are capable of subtracting all infinities arising from the one-loop diagrams. The last three terms in the right-hand side arise from the product of the square roots of the LSZ factors in (3.9) where the prime denotes derivative with respect to q^2, for example $\Pi'_{hh}(q^2) = d\Pi_{hh}(q^2)/dq^2$. Finally, note that the hadronic contributions from light quarks in $A'_{\gamma\gamma}(0)$, as given in eq. (4.21) of ref. [49], have been taken into account since they have an important contribution (one order of magnitude) in the non-logarithmic parts of the one-loop amplitude.

Note that eq. (3.15) is divided in two parts: the first part is proportional to the gauge-invariant quantity $\Delta^{\mu\nu}$, while the second part (last two lines of eq. (3.15)) is proportional to $g^{\mu\nu}$ and, therefore, is *not* gauge-invariant and violates the Ward-identity for charge conservation. We have proved that for every gauge-fixing choice these contributions vanish. To be more specific, we have checked explicitly that in unitary gauge $A_{Z\gamma}(0) = 0$ and that there are no leftover corrections proportional to $g^{\mu\nu}$, i.e. $\overline{\Gamma}_g^{\text{SM}} = \Gamma_g^i = 0$. What happens in R_ξ-gauges is discussed at the end of subsection 3.3.3.

We are now ready to write the $h \to Z\gamma$ amplitude at one-loop and at $1/\Lambda^2$ in EFT expansion. After removing the last two lines in (3.15) and checking that infinities cancel when applying the counterterms δC^i, we arrive at the matrix element

$$\begin{aligned} i\mathcal{M}^{\mu\nu}(h \to Z\gamma) = 4i\,\Delta^{\mu\nu}(p_1, p_2) & \\ \times \Bigg\{ - sc\,v\,C^{\varphi B} & \left[1 + \mathcal{X}^{\varphi B} - \frac{1}{t}\frac{A_{Z\gamma}(m_Z^2) + \delta m_{Z\gamma}^2}{m_Z^2} + t\frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{m_Z^2} \right] \\ + sc\,v\,C^{\varphi W} & \left[1 + \mathcal{X}^{\varphi W} + t\frac{A_{Z\gamma}(m_Z^2) + \delta m_{Z\gamma}^2}{m_Z^2} - \frac{1}{t}\frac{A_{Z\gamma}(0) + \delta m_{Z\gamma}^2}{m_Z^2} \right] \\ + \frac{s^2 - c^2}{2}\,v\,C^{\varphi WB} & \left[1 + \mathcal{X}^{\varphi WB} - \frac{2sc}{s^2 - c^2}\frac{A_{Z\gamma}(m_Z^2) + A_{Z\gamma}(0) + 2\delta m_{Z\gamma}^2}{m_Z^2} \right] \\ + \frac{1}{m_W}\overline{\Gamma}^{\text{SM}} + & \sum_{i \neq \varphi B, \varphi W, \varphi WB} v\,C^i\,\Gamma^i \Bigg\}_{\text{finite}}, \end{aligned} \tag{3.17}$$

which is gauge invariant and renormalisation scale μ-independent, in a sense that $\mu\, d\mathcal{M}^{\mu\nu}/d\mu = 0$. The subscript "finite" means that infinities proportional to $(\frac{2}{4-d} - \gamma + \log 4\pi)$ have been removed from expressions such as A_{VV}, A'_{VV}, Γ^i, etc, with counterterms δC^i removed from the quantity $\mathcal{X}^i$ of (3.16) as well. All self-energies but $A_{Z\gamma}(m_Z^2)$ and $A'_{ZZ}(m_Z^2)$ appearing in (3.17) are given analytically in general R_ξ-gauges, in appendix A of ref. [49] (see also [89] for formulae in $\xi = 1$). It is obvious from (3.17) that self-energies for the Higgs or vector bosons should be calculated *only* in the SM, not (necessarily) in SMEFT. The three-point vertex functions Γ^i are in general too lengthy and is not really illuminating to be given here.

Although we leave the expression (3.17) for the matrix element in a slightly involved form, it can be reduced further by noting the following. As in the case of the $h \to \gamma\gamma$ amplitude, there is a remarkable relation between factors multiplying the coefficients $C^{\varphi B}$

and $C^{\varphi W}$ when replacing

$$\tan\theta_W \to -\frac{1}{\tan\theta_W}\,, \tag{3.18}$$

while on the other hand, factors multiplying $C^{\varphi WB}$ in (3.17) remain invariant. In addition, elementary trigonometric relations may reduce eq. (3.17) further. For example, by using $\tan\theta_W - 1/\tan\theta_W = -2\cot^2(2\theta_W)$ one may factor out $\delta m^2_{Z\gamma}/m_Z^2$ terms. We believe, however, that eq. (3.17) is more transparent and easily understood when read in conjunction with the list of diagrams and counterterms of eq. (3.10).

Finally, some words about calculating the diagrams appearing in the shaded blobs of (3.10). We used the Feynman Rules of ref. [40], given in general R_ξ-gauges, and passed them manually to the `Mathematica` package `FeynCalc` [102, 103]. The Feynman integrals are regulated with dimensional regularisation [117] with the Dirac algebra performed in d-dimensions. The result is reduced to basic Passarino-Veltman functions [92]. We then checked expressions for analytic functions, some of them presented in ref. [49], against the numerical library `LoopTools` [118, 119]. The most crucial (and time consuming) test is the gauge-fixing parameter independence of the amplitude (3.15).

3.3.3 Gauge-fixing parameters cancellation

Since the cancellation in the amplitude of the gauge-fixing parameters, collectively denoted as ξ, is a very involved and important cross-check of the validity of our calculation, let us give here some insight on this particular computational task. In general, there are two different ways of how ξ-dependent contributions arise in SMEFT. Let us call the result one finds by subtracting the unitary gauge result from the full result in R_ξ-gauges the ξ-dependent result. For an operator C^i there are *explicit* ξ-dependent contributions, coming from the ξ-dependent result which is proportional to the Wilson coefficient C^i. There are also *implicit* contributions, coming from the ξ-dependent SM-like result by Taylor-expanding the masses with C^i as an expansion parameter.

In the $h \to Z\gamma$ process there are two independent gauge-fixing parameters, ξ_W and ξ_Z. We therefore prove the ξ-cancellation in the amplitude for each of these parameters *independently*. Interchanging between gauge-fixing parameters is a great advantage of the Feynman rules written in general R_ξ-gauges in ref. [40]. We also checked gauge-invariance without any renormalisation scheme. In this case, one has to add a Higgs tadpole diagram in the "hhAZ" vertex. As explained in eq. (3.15), the last two lines do not appear in the unitary gauge at all. On the other hand, each of the terms in these lines contributes in the ξ-dependent part, so one has to prove that they add to zero. Note that there are explicit contribution from the SMEFT Γs in the last line as well as from the vertex and the

$Z\gamma$-mixing in the penultimate line, and also implicit contributions from the Z-boson mass and the $Z\gamma$-mixing in the penultimate line and the ξ-dependent SM result in the last line.

It is important to stress here the analytic result of the $Z\gamma$-mixing in SMEFT with $d = 6$ operators. One can prove that the result is simply given by

$$A_{Z\gamma}^{\text{SMEFT}}(0) = \big(1 + \tfrac{1}{2}v^2 C^{\varphi D}\big) A_{Z\gamma}^{\text{SM}}(0)\,, \tag{3.19}$$

where $A_{Z\gamma}^{\text{SM}}(0)$ is the SM-like value at $q^2 = 0$, given analytically in eq. (2.73) of chapter 2. Note that $A_{Z\gamma}^{\text{SM}}(0)$ is a function of the SMEFT couplings, the VEV and the W boson mass. Therefore, the SM-like and the SM values coincide. We believe that (3.19) has interesting consequences in the general SMEFT renormalisation program.

Each coefficient has its own unique way of how the ξ-cancellation occurs. As an example, let us discuss here the $C^{\varphi WB}$ coefficient. Since $A_{Z\gamma}^{\text{SMEFT}}$ doesn't depend on this coefficient (either explicitly or implicitly) and the vertex contribution cancels that of the Z-boson mass, $C^{\varphi WB}$ cancels trivially in the penultimate line. Therefore, the implicit and explicit contributions from the last line should cancel among each other, which we have proved that this is exactly the case.

3.4 Results

3.4.1 $h \to Z\gamma$ in the Standard Model and the input parameters scheme

As it is well known, the $h \to Z\gamma$ SM contribution, $\overline{\Gamma}^{\text{SM}}/m_W$, in eq. (3.17) is a sum of one-loop diagrams with only $W^{\pm}$ bosons and charged fermions, f, circulating in the loop. In terms of the SMEFT parameters $\{\bar{g}, \bar{g}', v\}$, defined in ref. [40], we find that

$$\frac{\overline{\Gamma}^{\text{SM}}}{m_W} = \frac{\bar{g}\bar{g}'}{16\pi^2 v}\left[\sum_f N_{c,f} Q_f \left(T_f^3 - 2Q_f \frac{\bar{g}'^2}{\bar{g}^2 + \bar{g}'^2}\right) I_f + \frac{\bar{g}^2}{2(\bar{g}^2 + \bar{g}'^2)} I_W\right], \tag{3.20}$$

where the electromagnetic fermion charges and the third component of the weak isospin are given, respectively, by

$$Q_f = \begin{cases} 0, & \text{for} \quad f = \nu_e, \nu_\mu, \nu_\tau \\ -1, & \text{for} \quad f = e, \mu, \tau \\ 2/3, & \text{for} \quad f = u, c, t \\ -1/3, & \text{for} \quad f = d, s, b \end{cases} \tag{3.21}$$

and

$$T_f^3 = \begin{cases} 1/2, & \text{for} \quad f = \nu_e, \nu_\mu, \nu_\tau, u, c, t \\ -1/2, & \text{for} \quad f = e, \mu, \tau, d, s, b \end{cases} . \tag{3.22}$$

The colour factor $N_{c,f}$ is equal to 1 for leptons and 3 for quarks. In (3.20) I_f and I_W contain the contribution from the fermionic and the bosonic sector respectively. Explicitly, these quantities are given in terms of PV functions as:[7]

$$\begin{aligned} I_f = \frac{m_f^2}{(m_h^2 - m_Z^2)^2} \Big\{ & 2m_Z^2 \left[B_0(m_h^2, m_f^2, m_f^2) - B_0(m_Z^2, m_f^2, m_f^2) \right] \\ & - (m_h^2 - m_Z^2) \left[(m_h^2 - m_Z^2 - 4m_f^2) C_0(0, m_h^2, m_Z^2, m_f^2, m_f^2, m_f^2) - 2 \right] \Big\} , \end{aligned} \tag{3.23}$$

and

$$\begin{aligned} I_W = \frac{1}{(m_h^2 - m_Z^2)^2} \Big\{ & \frac{m_Z^2}{m_W^2} \left[m_h^2 (m_Z^2 - 2m_W^2) + 2m_W^2 (m_Z^2 - 6m_W^2) \right] \\ & \times \left[B_0(m_h^2, m_W^2, m_W^2) - B_0(m_Z^2, m_W^2, m_W^2) \right] \\ & + \frac{m_h^2 - m_Z^2}{m_W^2} \Big[m_h^2 (m_Z^2 - 2m_W^2) + 2m_W^2 (m_Z^2 - 6m_W^2) \\ & + 2m_W^2 \left[m_h^2 (6m_W^2 - m_Z^2) - 12m_W^4 - 6m_W^2 m_Z^2 + 2m_Z^4 \right] \\ & \times C_0(0, m_h^2, m_Z^2, m_W^2, m_W^2, m_W^2) \Big] \Big\} . \end{aligned} \tag{3.24}$$

We have proved explicitly that the SM matrix element is finite, gauge invariant and gauge-fixing parameter independent.

We can express the SM-like result of eq. (3.20), or for that matter any other contribution in (3.17), in terms of well-measured quantities that will be taken as inputs in evaluating the $h \to Z\gamma$ amplitude. The set of well-measured quantities we have chosen, contains G_F, m_W and m_Z. The relevant formulae for expressing the parameters $\bar{g}'$, $\bar{g}$ and v as functions of the Fermi coupling constant G_F, the physical W-boson mass, m_W, and the physical Z-boson

[7] Our notation for PV-functions is identical to those of `LoopTools` in ref. [118].

mass, m_Z, are given in appendix E, section E.1. We repeat them here for convenience:

$$\bar{g}' = 2^{5/4}\sqrt{m_Z^2 - m_W^2}\sqrt{G_F}\left[1 - \frac{1}{2\sqrt{2}G_F}\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2}\right)\right.$$
$$\left. - \frac{m_Z^2}{4\sqrt{2}G_F(m_Z^2 - m_W^2)}\left(\frac{C^{\varphi D}}{\Lambda^2} + 4\frac{m_W}{m_Z}\sqrt{1 - \frac{m_W^2}{m_Z^2}}\frac{C^{\varphi WB}}{\Lambda^2}\right)\right], \quad (3.25)$$

$$\bar{g} = 2^{5/4} m_W\sqrt{G_F}\left[1 - \frac{1}{2\sqrt{2}G_F}\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2}\right)\right], \quad (3.26)$$

$$v = \frac{1}{2^{1/4}\sqrt{G_F}}\left[1 + \frac{1}{2\sqrt{2}G_F}\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2}\right)\right]. \quad (3.27)$$

Finally we can express the parameters in eq. (3.20) as a function of the experimental quantities G_F, m_W and m_Z taken from PDG [120]. The reason for choosing the input scheme $\{G_F, m_W, m_Z\}$ is twofold: first, it has natural implementation[8] into our renormalisation prescription discussed already in section 3.3 and especially into the simple definition of the weak mixing angle in eq. (3.8)[9] and second it is a scheme that is becoming increasingly popular after refs. [71, 116] with whom we would like to compare our results. Other advantages of this scheme have also been put forward by ref. [121].

After replacing $\bar{g}'$, $\bar{g}$ and v in eq. (3.20) with eqs. (3.25)–(3.27), it is rather more instructive to present also the numerical result here. This reads

$$\begin{aligned}\frac{\overline{\Gamma}^{\rm SM}}{m_W} =& -(1.43\times 10^{-5} - 1.11\times 10^{-8} i)\\ &+ (1.07 + 1.38\times 10^{-4} i)\frac{C^{\varphi WB}}{\Lambda^2} + (0.64 + 8.28\times 10^{-5} i)\frac{C^{\varphi D}}{\Lambda^2}\\ &+ (1.30 - 1.00\times 10^{-3} i)\left[\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2}\right]. \end{aligned} \quad (3.28)$$

As one can see, the imaginary part of the SM-like amplitude is more than three orders of magnitude smaller than the real part and can be safely ignored in the following. Our result agrees with ref. [30] and partially with refs. [29, 113].[10] The pure SM contribution,

[8] At least more natural than the scheme with the input set $\{\alpha_{em}, G_F, m_Z\}$.

[9] Note that the expression for $\bar{g}'$ in eq. (3.25) becomes much simpler upon substitution of the weak mixing angle definition of (3.8). Then the second line of (3.25) reads:

$$-\frac{1}{4\sqrt{2}\, s^2 G_F}\left(\frac{C^{\varphi D}}{\Lambda^2} + 4sc\frac{C^{\varphi WB}}{\Lambda^2}\right).$$

[10] We have a minus sign difference in the term before the last parenthesis of eq. (4) of ref. [113]. Furthermore,

$\overline{\Gamma}^{\rm SM}/m_W$, can be factored out in the amplitude of (3.17) and after squaring and integrating over the phase space of the final state particles, γ and Z, one can easily find the decay rate for $h \to Z\gamma$ in the SM and in SMEFT. It is then useful to express our results in terms of the quantity

$$\mathcal{R}_{h\to Z\gamma} = \frac{\Gamma(\text{SMEFT}, h \to Z\gamma)}{\Gamma(\text{SM}, h \to Z\gamma)} \equiv 1 + \delta\mathcal{R}_{h\to Z\gamma}, \tag{3.29}$$

and compare with the experimental bound of eq. (3.2). In the next subsection we present corrections for $\delta\mathcal{R}_{h\to Z\gamma}$ from new physics in the form of running Wilson coefficients of the operators listed in table 3.1. In addition, we search for correlations with an analogous expression arising from the $h \to \gamma\gamma$ decay.

3.4.2 Semi-numerical expression for the ratio $\mathcal{R}_{h\to Z\gamma}$

In this section we finally present our results for $\delta\mathcal{R}_{h\to Z\gamma}$. As in ref. [49], we shall separate constant and renormalisation scale μ-dependent logarithmic parts which multiply RGE running Wilson coefficients, $C(\mu)$. In "Warsaw" mass-basis of ref. [40], by exploiting the input parameters scheme $\{G_F, m_W, m_Z\}$ with the new-physics scale Λ written in TeV units, we find:[11]

$$\begin{aligned}
\delta\mathcal{R}_{h\to Z\gamma} \simeq\; & 0.18\frac{C^{ll}_{1221} - C^{\varphi l(3)}_{11} - C^{\varphi l(3)}_{22}}{\Lambda^2} + 0.12\frac{C^{\varphi\Box} - C^{\varphi D}}{\Lambda^2} \\
& - 0.01\frac{C^{d\varphi}_{33} - C^{u\varphi}_{33}}{\Lambda^2} + 0.02\frac{C^{\varphi u}_{33} + C^{\varphi q(1)}_{33} - C^{\varphi q(3)}_{33}}{\Lambda^2} \\
& + \left[14.99 - 0.35\log\frac{\mu^2}{m_W^2}\right]\frac{C^{\varphi B}}{\Lambda^2} - \left[14.88 - 0.15\log\frac{\mu^2}{m_W^2}\right]\frac{C^{\varphi W}}{\Lambda^2} \\
& + \left[9.44 - 0.26\log\frac{\mu^2}{m_W^2}\right]\frac{C^{\varphi WB}}{\Lambda^2} + \left[0.10 - 0.20\log\frac{\mu^2}{m_W^2}\right]\frac{C^{W}}{\Lambda^2} \\
& - \left[0.11 - 0.04\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uB}_{33}}{\Lambda^2} + \left[0.71 - 0.28\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uW}_{33}}{\Lambda^2} \\
& - \left[0.01 + 0.00\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uW}_{22}}{\Lambda^2} - \left[0.01 + 0.00\log\frac{\mu^2}{m_W^2}\right]\frac{C^{dW}_{33}}{\Lambda^2} + \ldots,
\end{aligned} \tag{3.30}$$

where the ellipses denote contributions from operators that are less than $0.01 \times C/\Lambda^2$. Note that the VEV appearing at tree-level introduces one-loop corrections when exchanged for the Fermi constant through $\overline{G}_F = 1/[\sqrt{2}v^2(1-\Delta r)]$. We follow here the same procedure as

our SM result agrees with ref. [29] only if the branches of the piecewise function $g(\tau)$ in eq. (2.56) are reversed.

[11] Our result is in agreement with the revised (arXiv v3) version of ref. [71].

in ref. [49] below eq. (4.16). Formula (3.30) should be renormalisation scale (μ) independent at one-loop and up to terms with $1/\Lambda^2$ in EFT expansion. Assuming that Higgs boson production is not affected by the operators listed in table 3.1, the current experimental bound of (3.1) sets rather weak constraints on tree-level SMEFT Wilson coefficients. As an example, for $\mu = m_W$ we obtain

$$\frac{|C^{\varphi B}|}{\Lambda^2} \lesssim \frac{0.4}{(1\,\text{TeV})^2}\,, \qquad \frac{|C^{\varphi W}|}{\Lambda^2} \lesssim \frac{0.4}{(1\,\text{TeV})^2}\,, \qquad \frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim \frac{0.7}{(1\,\text{TeV})^2}\,. \tag{3.31}$$

For loop-induced operators, the logarithmic part is of the same order of magnitude as of the constant part. Contributions in the first and second line of (3.30) arise from finite fermionic triangle diagrams that just rescale the SM result. Wilson coefficients $C^{\varphi u}_{33}$, $C^{\varphi q(1)}_{33}$, $C^{\varphi q(3)}_{33}$ are the new operators appearing now in $h \to Z\gamma$ decay relative to $h \to \gamma\gamma$ (see table 3.1). Interestingly, out of many operators only three made a contribution for more than 1% and in fact they are just barely pass that threshold![12]

How to use eq. (3.30)? First, decouple heavy particles from a more fundamental theory. Match to Warsaw-basis operators relevant for $h \to Z\gamma$, listed in table 3.1. Set the coefficients, $C(\mu)$ at a scale $\mu = \Lambda$. Use RGEs to run the parameters down to the Higgs mass scale — one could use dedicated codes for this purpose like those in refs. [93, 94]. Plug in the results for $C(\mu = m_h)$ coefficients in eq. (3.30) and obtain $\delta\mathcal{R}_{h\to Z\gamma}$. As long as discussing the same physical process in the same input parameter scheme, the result should be unambiguous.

Keeping in mind the current experimental sensitivity for $h \to Z\gamma$, eq. (3.30) is not of much use. It is however, quite interesting to check for a $h \to Z\gamma$ projective reach by comparing $\delta\mathcal{R}_{h\to Z\gamma}$ of eq. (3.30) with $\delta\mathcal{R}_{h\to\gamma\gamma}$ taken from ref. [49] but translated into the

[12]We consider 1% of corrections as an indicative limit that LHC can reach for $\delta\mathcal{R}_{h\to Z\gamma}$ at later stages of its run.

$\{G_F, m_W, m_Z\}$ input scheme. We have,

$$
\begin{aligned}
\delta\mathcal{R}_{h\to\gamma\gamma} \simeq\; & 0.18\frac{C^{ll}_{1221} - C^{\varphi l(3)}_{11} - C^{\varphi l(3)}_{22}}{\Lambda^2} + 0.12\frac{C^{\varphi\Box} - 2C^{\varphi D}}{\Lambda^2} \\
& - 0.01\frac{C^{e\varphi}_{22} + 4C^{e\varphi}_{33} + 5C^{u\varphi}_{22} + 2C^{d\varphi}_{33} - 3C^{u\varphi}_{33}}{\Lambda^2} \\
& - \left[48.04 - 1.07\log\frac{\mu^2}{m_W^2}\right]\frac{C^{\varphi B}}{\Lambda^2} - \left[14.29 - 0.12\log\frac{\mu^2}{m_W^2}\right]\frac{C^{\varphi W}}{\Lambda^2} \\
& + \left[26.17 - 0.52\log\frac{\mu^2}{m_W^2}\right]\frac{C^{\varphi WB}}{\Lambda^2} + \left[0.16 - 0.22\log\frac{\mu^2}{m_W^2}\right]\frac{C^{W}}{\Lambda^2} \\
& + \left[2.11 - 0.84\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uB}_{33}}{\Lambda^2} + \left[1.13 - 0.45\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uW}_{33}}{\Lambda^2} \\
& - \left[0.03 + 0.01\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uB}_{22}}{\Lambda^2} - \left[0.01 + 0.00\log\frac{\mu^2}{m_W^2}\right]\frac{C^{uW}_{22}}{\Lambda^2} \\
& + \left[0.03 + 0.01\log\frac{\mu^2}{m_W^2}\right]\frac{C^{dB}_{33}}{\Lambda^2} - \left[0.02 + 0.01\log\frac{\mu^2}{m_W^2}\right]\frac{C^{dW}_{33}}{\Lambda^2} \\
& + \left[0.02 + 0.00\log\frac{\mu^2}{m_W^2}\right]\frac{C^{eB}_{33}}{\Lambda^2} - \left[0.01 + 0.00\log\frac{\mu^2}{m_W^2}\right]\frac{C^{eW}_{33}}{\Lambda^2} + \dots . \qquad (3.32)
\end{aligned}
$$

One can draw interesting remarks by comparing eqs. (3.30) and (3.32). Wilson coefficients in the first line of both equations are dominated from input scheme dependencies.[13] The only "real" difference is a factor of 2 enhancement in front of the coefficient $C^{\varphi D}$ in the case of $h \to \gamma\gamma$. Another issue is the surprisingly large loop enhancement of the C^{uB}_{33} coefficient (top-quark inside the loop) in $\delta\mathcal{R}_{h\to\gamma\gamma}$ as shown and discussed in ref. [49]. This enhancement has been reduced by a factor of 20 in $\delta\mathcal{R}_{h\to Z\gamma}$ in (3.30). The reason seems to be an accidental cancellation. In the $h \to \gamma\gamma$ case we have an overall factor $16sc^2 \approx 6$, while in the $h \to Z\gamma$ case we have an overall factor $3c^3 - 13cs^2 \approx -0.5$. It is this factor of (-10) that gives such a big difference in the relevant results. This suppression may be used to disentangle new-physics effects between the two observables. Interestingly, however, the coefficient C^{uW}_{33} does not suffer by similar accidental suppression.

In comparing eqs. (3.30) and (3.32), even the dominant contributions from the operators $C^{\varphi B}$ and $C^{\varphi WB}$ are smaller in $h \to Z\gamma$ by factor of 3 and only the coefficient of $C^{\varphi W}$ is similar in both $\delta\mathcal{R}_{h\to Z\gamma}$ and $\delta\mathcal{R}_{h\to\gamma\gamma}$. This is very interesting for disentangling among the three operators in case new physics enters through those. For example, one may envisage a

[13]The large scheme dependence can be understood by comparing (3.32) with the first line of eq. (5.1) of ref. [49].

new-physics scenario, like the one of ref. [122], with a heavy hypercharged $SU(2)_L$-singlet scalar, which is decoupled from the theory at the TeV scale. Since this will only make $C^{\varphi B}$ non-zero, and say positive in $h \to \gamma\gamma$, it will only make a suppressed reduction in case of $h \to Z\gamma$. However, there are Wilson coefficients like the prefactor of C^W that are similar in both cases.

The real power, however, of EFT, is when using experimental data to constrain Wilson coefficients of various operators and therefore making estimates for projective reach of observables. For example, bounds have been set in some of the coefficients appearing in $\delta\mathcal{R}_{h\to\gamma\gamma}$ in refs. [49, 116]. We easily see that, if we consider *one coupling at a time*, bounds from $h \to \gamma\gamma$ on these coefficients kill any possible excess arising from these operators in the $h \to Z\gamma$ process. In addition to the already mentioned cancellation in top-quark loop for $h \to Z\gamma$, the relevant operators bounded from $h \to \gamma\gamma$ are now numerically completely irrelevant for $h \to Z\gamma$.

That is quite a lot one can infer by just comparing only two observables! One may use best fit values to EW observables to check upon other coefficients, such as C^W, C^{ll}_{1221}, $C^{\varphi l(3)}$, $C^{\varphi u}$, $C^{\varphi q(1,3)}$, $C^{\varphi D}$ that enter similarly in $\delta\mathcal{R}_{h\to Z\gamma}$ and $\delta\mathcal{R}_{h\to\gamma\gamma}$ of eqs. (3.30) and (3.32), respectively. By taking, for instance, the best fit values from the 4th column of table 6 in ref. [121][14] we obtain that it is unlikely to discover any possible new-physics effect through $h \to Z\gamma$ decay in current LHC data before seeing a $h \to \gamma\gamma$ anomaly. Of course this statement weakens if one allows for more operators to be present at the same time.

As we already mentioned in Introduction, below eq. (3.1), in deriving bounds from $\delta\mathcal{R}_{h\to Z\gamma}$ we implicitly assumed that at least the dominant Higgs-boson production mechanism (gluon fusion) is not affected by the operators involved in the $h \to Z\gamma$ decay. Indeed, for the same reason we explained in section 3.2, only CP-invariant operators contribute to $gg \to h$ process. The main, i.e. tree-level in SMEFT, gluon-Higgs operator, $Q_{\varphi G}$, as well as the ones affecting the one-loop diagrams, Q_{uG}, Q_{dG}, do not interfere with the list of operators in table 3.1 relevant to $h \to Z\gamma$. However, operators $Q_{u\varphi}$ and $Q_{d\varphi}$ enter in both $h \to Z\gamma$ and $h \to gg$ but their associated Wilson coefficients are multiplied by small numbers in $\delta\mathcal{R}_{h\to Z\gamma}$ of eq. (3.30). Finally, the combination $(C^{\varphi\Box} + 1/4\, C^{\varphi D})$ enters only multiplicative in all three observables, $h \to gg$, $h \to Z\gamma$ and $h \to \gamma\gamma$ which is just a rescale effect. From these two coefficients, $C^{\varphi D}$ is a custodial violating parameter and therefore highly suppressed. Of course the safest is to calculate $h \to gg$ at one-loop in SMEFT. The reader is referred to refs. [115, 123, 124]

What about future $h \to Z\gamma$ sensitivity? Only at later stages of high luminosity of 3000 fb^{-1} at LHC, ATLAS will have enough significance ($\sim 5\sigma$) for the $h \to Z\gamma$ mode [125]. Assuming SM Higgs production and decay, the signal strength is expected to be measured

[14] Similar results one can draw from other tree-level studies, see refs. [76, 78].

with $\delta\mathcal{R}_{h\to Z\gamma} \approx \pm 0.24$ uncertainty. On the other hand for $h \to \gamma\gamma$, and the same projective reach, ATLAS expects $\delta\mathcal{R}_{h\to\gamma\gamma} \approx \pm 0.04$ for a SM Higgs boson produced from gluon fusion process and decaying dominantly to $b\bar{b}$. By comparing, our EFT calculations for $\delta\mathcal{R}_{h\to Z\gamma}$ and $\delta\mathcal{R}_{h\to\gamma\gamma}$ in eqs. (3.30) and (3.32), we obtain that any new-physics signal for $h \to Z\gamma$ is unlikely to be seen at near future LHC upgrades without seeing new physics first at $h \to \gamma\gamma$ data.

3.5 Conclusions

We have performed a one-loop calculation for the Higgs-boson decay to a Z-boson and a photon, $h \to Z\gamma$, in SMEFT with $d = 6$ operators written in Warsaw basis. We find a general formula for the amplitude (3.17) which is finite, it respects the Ward-identities, and is gauge-fixing parameter independent. We present our result in terms of the ratio $\delta\mathcal{R}_{h\to Z\gamma}$ in eq. (3.30) and compare this with the previously calculated ratio $\delta\mathcal{R}_{h\to\gamma\gamma}$. We find that, for most Wilson-coefficients, $\delta\mathcal{R}_{h\to Z\gamma}$ is less sensitive to new physics than $\delta\mathcal{R}_{h\to\gamma\gamma}$. Some of the operators entering in $h \to Z\gamma$, but *not* in $h \to \gamma\gamma$, can modify $\delta\mathcal{R}_{h\to Z\gamma}$ at a rate hardly noticeable, currently or in the near future, at the LHC.

Code

Chapter **4**

SmeftFR – Feynman rules generator for the Standard Model Effective Field Theory

In this chapter we present the Mathematica package SmeftFR, a software tool dedicated to the generation of the Feynman rules for the Standard Model Effective Field Theory (SMEFT). In its current development version, the SmeftFR code includes the complete set of the dimension 5 and 6 SMEFT operators in Warsaw basis, as well as the complete bosonic subset of the dimension 8 operators in a basis that extends Warsaw basis. The package can be used to produce the Feynman rules for the SMEFT consistently up to $1/\Lambda^4$ order in the EFT expansion, including the interference terms, for any chosen subset of the effective operators. The Feynman rules, generated through the Mathematica package FeynRules, are produced in the physical mass basis for all the fields. For versatility, the Feynman rules can be produced in either unitary or linear R_ξ-gauges. Additionally, the user is given the choice of producing the result in terms of the SMEFT couplings or in terms of physical parameters, in which case two different convenient physical input schemes are provided. The mass basis Lagrangian produced by SmeftFR can be exported in various formats supported by FeynRules, such as UFO, FeynArts, etc, while a dedicated LaTeX generator is used to print the results up to $1/\Lambda^2$ order in human-readable format. The numerical initialisation for the Wilson coefficients is interfaced to WCxf format. The open source code can be downloaded from the address www.fuw.edu.pl/smeft. This chapter is based on refs. [126, 127].

4.1 Introduction

During the last years, the extension of the Standard Model (SM) of particle physics [15–17] within an Effective Field Theory (EFT) framework [128–130] has become increasingly popular. The resulting theory, abbreviated as SMEFT,[1] parameterises the, beyond the SM, New Physics (NP) effects by extending the SM with a complete set of gauge invariant operators, constructed out of the SM fields spectrum. These NP effects are assumed to take place at and above an indicative energy scale Λ, which is considered to be large with the respect to the masses of the SM particles. The new effective operators can be categorised according to their mass dimension, with their accompaning couplings, the Wilson coefficients, being suppressed by suitable inverse powers of the scale Λ.

The lowest SMEFT order contains only one dimension 5 operator, known as the Weinberg operator, which can be interpreted as a Majorana neutrino mass term. The first non-trivial SMEFT order is consisted of the operators at dimension 6. A complete set of the gauge invariant operators up to dimension 6 was first presented in ref. [39], and more recently put in a non-redundant form in ref. [37]. This independent, non-redundant basis, is referred to the literature as the Warsaw basis. Suppressing the flavour indices of the fields and not counting hermitian conjugated operators, Warsaw basis contains 59 + 1 baryon-number conserving and 4 baryon-number violating operators.

Beyond the dimension 6, explicit operator bases where constructed for the dimension 7 operators in ref. [41, 43]. Dimension 8 bosonic operators basese where given in refs. [131–133] and, more recently, the complete dimension 8 basis was presented in ref. [45], using a minimal-derivative basis, similar to the Warsaw basis.

The SMEFT can be in general a very complex model, simply due to the mere number of the effective operators that are included. Even at dimension 6, without applying any restrictions regarding CP-conservation, flavour-violation etc, the Wilson coefficients add up to 2499. In addition to the large number of free parameters, the complicated structure of the effective operators results in even more complicated interaction terms. Therefore, theoretical calculations of physical processes within the SMEFT can be very challenging — it is enough to notice that the number of primary vertices when SMEFT is quantised in R_ξ-gauges, printed for the first time in ref. [40], is almost 400 without counting the hermitian conjugates. As the recent direction of the literature flows towards the use of the SMEFT beyond the linear approximation, with the inclusion of interference of mulitple $d = 6$ terms and/or the inclusion of higher-dimensional operators, the calculational complexity makes old-fashioned, hand-made calculations almost not realistic.

It is, therefore, important to develop technical methods and tools facilitating such

[1] For a recent review and pedagogical lectures, see refs. [47, 48], respectively.

calculations, starting from developing the universal set of the Feynman rules for propagators and vertices for physical fields, after spontaneous symmetry breaking (SSB) of the full effective theory. The initial version of relevant package, `SmeftFR` v1.0, was announced and briefly described for the first time in appendix B of ref. [40] and later expanded and refined in ref. [126]. Here we present the current development version v3.0 [127] of the code `SmeftFR`, a `Mathematica` symbolic language package generating Feynman rules in several formats. Let us briefly list here some of the main features of the package:

- `SmeftFR` is able to generate interactions in the most general form of the SMEFT Lagrangian, without any restrictions on the structure of flavour violating terms and on CP-, lepton- or baryon-number conservation.[2] Feynman rules are expressed in terms of physical SM fields and canonically normalised Goldstone and ghost fields. Expressions for interaction vertices are analytically expanded in powers of inverse New Physics scale $1/\Lambda$, with the option to truncate the EFT series to the chosen order, up-to $1/\Lambda^4$.

- `SmeftFR` is written as an overlay to `FeynRules` package [134], used as the engine to generate Feynman rules.

- Including the full set of SMEFT parameters in model files for `FeynRules` may lead to very slow computations. `SmeftFR` can generate `FeynRules` model files dynamically, including only the user defined subset of higher dimension operators. It significantly speeds up the calculations and produces simpler final result, containing only the Wilson coefficients relevant for a process chosen to analyse.

- Feynman rules can be generated in the unitary or in linear R_ξ-gauges by exploiting four different gauge-fixing parameters $\xi_\gamma, \xi_Z, \xi_W, \xi_G$ for thorough amplitude checks. In the latter case also all relevant ghost vertices are obtained.

- Feynman rules are calculated first in `Mathematica/FeynRules` format. They can be further exported in other formats: `UFO` [135] (importable to Monte Carlo generators like `MadGraph5_aMC@NLO 5` [136], `Sherpa` [137], `CalcHEP` [138], `Whizard` [139, 140]), `FeynArts` [141] which generates inputs for loop amplitude calculators like `FeynCalc` [103], or `FormCalc` [142], and others output types supported by `FeynRules`.

- `SmeftFR` provides a dedicated Latex generator, allowing to display vertices and analytical expressions for Feynman rules in clear human readable form, best suited for hand-made calculations.

[2]However, we do restrict ourselves to linear realisations of the SSB.

- `SmeftFR` is interfaced to the WCxf format [143] of Wilson coefficients. Numerical values of SMEFT parameters in model files can be read from WCxf JSON-type input produced by other computer packages written for SMEFT. Alternatively, `SmeftFR` can translate `FeynRules` model files to the WCxf format.

- Further package options allow to treat neutrino fields as massless Weyl or (in the case of non-vanishing dimension 5 operator) massive Majorana fermions, to correct signs in 4-fermion interactions not yet fully supported by `FeynRules` and to perform some additional operations as described later in this manual.

Feynman rules derived in ref. [40] using the `SmeftFR` package have been used successfully in many articles including refs. [49, 68, 71, 107, 116, 144–149] and have passed certain non-trivial tests, such as gauge-fixing parameter independence of the S-matrix elements, validity of Ward identities, cancellation of infinities in loop calculations, etc.

We note here in passing, that there is a growing number of publicly available codes performing computations related to SMEFT. These include, `Wilson` [94], `DSixTools` [93], `MatchingTools` [150], which are codes for running and matching Wilson coefficients, `SMEFTsim` [151], a package for calculating tree-level observables, `CoDEx` [152] or a version of `SARAH` code [153], that calculate Wilson Coefficients after the decoupling of a more fundamental theory, and finally, `DirectDM` [154], a code for dark matter EFT. To a degree, these codes (especially the ones supporting WCxf format) can be used in conjunction with `SmeftFR`. For example, some of them can provide the numerical input for Wilson coefficients of higher dimensional operators at scale Λ, while others, the running of these coefficients from that scale down to the EW one. Alternatively, Feynman rules evaluated by `SmeftFR` can be used with Monte Carlo generators to test the predictions of other packages.

This chapter is organised as follows. After this Introduction, in section 4.3 we define the notation and conventions, listing for reference the operator set in Warsaw basis and the formulae for transition to the mass basis. In section 4.2 we provide a brief chronology of the previous versions of the code. In section 4.4, we present the structure of the code, installation procedure and available functions.

4.2 Chronology of `SmeftFR` versions

Before delving deeper into the details of the most current version of the code, let us present here in brief the chronology of the previous versions of the `SmeftFR` package. The initial development phase of `SmeftFR` took place alongside ref. [40]. There, the package was used by the authors to produce the full set of Feynman rules for the dimension 6 SMEFT, and the LaTeX generator of the code provided the printed version of the, rather lengthy, list of the Feynman rules in clear human-readable format. This list can be found in appendix A of ref. [40]. A short synopsis of how to install and run this legacy v1.0 version of the code was presented in appendix B of the same paper, but an extended manual wasn't provided at the time.

The first fully-fledged version of the code for the dimension 6 SMEFT was presented some years later in ref. [126], expanding on the original v1.0 version. This v2.0 version augmented the previous one by providing many user options and integration with other software packages, as well as CPU time optimisation. In a nutshell, WCxf format [143] was used for the numerical initialisation of the Wilson coefficients, and the model files in `FeynRules` were generated dynamically for any chosen subset of the dimension 6 operators. The output of `SmeftFR` could be interfaced to other dedicated software tools like `UFO` and `FeynArts`, which greatly extended the usefulness of the code in practical calculations.

The output of v1.0 and v2.0 versions of `SmeftFR` was tested in many physical calculations, for example in the Higgs decay calculations [49] and [107] which we presented in part I, in refs. [68, 71, 116, 144–149] and even in an extension of the code with a subset of dimension 8 operators relevant for vector boson scattering in the SMEFT [155]. The current development version of the code, v3.0, expands upon the polished v2.0 version, by including also the full bosonic subset of the dimension 8 SMEFT operators from ref. [45]. The computations can be performed consistently up to $1/\Lambda^4$ order in the EFT expansion, including any interference terms from the dimension 6 operators. The user has the option to reduce the EFT expansion order to $1/\Lambda^2$, which in essence reproduces the linearly expanded dimension 6 SMEFT, or even reproduce the SM by setting the EFT expansion order to $1/\Lambda^0$, while still taking advantage of the newly added features. The model files are dynamically generated for any chosen subset of the included operators, and further CPU optimisation has been provided to reduce computation times. New in this version is also the addition of two convenient physical input parameter schemes, which can be chosen instead of the standard SMEFT couplings for the expression of the results in the output files.

4.3 SMEFT Lagrangian in Warsaw and mass basis

The first step of defining an EFT is the classification of the higher order effective operators. These operators are the ones that are constructed out of the spectrum of the model under consideration, with the additional requirement of being invariant under the internal symmetries of said model. In the case of the SMEFT, one should include every possible operator constructed out of the SM fields that is invariant under the Lorentz group and the SM gauge group. The initial construction of the operators is performed, as usual, in the electroweak (flavour) basis, before the SSB. We call this generic set of operators the Green basis. There is a caveat, however: many of these new effective operators will be connected with other operators of the Green basis by field redefinitions and/or integration by parts. Operators connected in this manner would therefore represent the same physical effect in S-matrix elements, and are therefore considered redundant. A subset of the Green basis which contains a full set of independent (non-redundant) operators is therefore a suitable basis for an EFT. There may be, of course, many possible non-redundant bases, and all can be connected to each other by the use of field redefinitions and integration by parts. Here we will use a specific basis which eliminates as many higher-derivative operators as possible. We will call this basis "Warsaw basis", which is the commonly used name in the literature for this specific dimension 6 basis [37], even when using the dimension 8 extension of the basis [45].

Starting from the lowest possible SMEFT order, the dimension 5, there is only a single lepton flavour violating operator:

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m(l'^k_{Lp})^T\,\mathbb{C}\,l'^n_{Lr} \equiv (\widetilde{\varphi}^\dagger l'_{Lp})^T\,\mathbb{C}\,(\widetilde{\varphi}^\dagger l'_{Lr})\,, \tag{4.1}$$

where $\mathbb{C}$ is the charge conjugation matrix. This operator, known as the Weinberg operator, can be interpreted as a Majorana neutrino mass term.

The first non-trivial SMEFT order emerges at dimension 6. The non-redundant basis which we use here is, as mentioned, the very-well known Warsaw basis of ref. [37]. The full list of the independent dimension 6 SMEFT operators in Warsaw basis was presented in ref. [37], and is reproduced here for convenience in table 4.1.[3] In this work, we also consider the complete bosonic subset of the dimension 8 operators (all operators that do not contain fermionic fields). The dimension 8 bosonic SMEFT operators are presented in tables 4.2, 4.3 and 4.4. All three tables follow the ones from the latest version (arXiv v6 as of this writting) of ref. [45], and are reproduced here for convenience and easy or reference. They are also slightly modified here to reflect our notation. Before moving further, let us make some additional comments about the content of each table, and explain possible changes in notation with respect to ref. [45].

Table 4.2 collects the pure Higgs operators, i.e. operators constructed only out of the Higgs doublet, φ, and covariant derivatives. There, we performed a change of basis in the operators of the $\varphi^6 D^2$ class so that they have immediate connection with the Warsaw basis. The original operators where defined in [45] as

$$\begin{aligned} Q^{(1)}_{\varphi^6} &= (\varphi^\dagger\varphi)^2(D_\mu\varphi^\dagger D^\mu\varphi)\,, \\ Q^{(2)}_{\varphi^6} &= (\varphi^\dagger\varphi)(\varphi^\dagger\tau^I\varphi)(D_\mu\varphi^\dagger\tau^I D^\mu\varphi)\,, \end{aligned} \tag{4.2}$$

[3]We do not list here all details of conventions used — they are identical to these listed in refs. [37, 40].

and here we use instead the set

$$Q_{\varphi^6\Box} = (\varphi^\dagger\varphi)^2\Box(\varphi^\dagger\varphi)\,,$$
$$Q_{\varphi^6 D^2} = (\varphi^\dagger\varphi)(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi)\,, \tag{4.3}$$

which naturally extends the definition of the dimension 6 operators $Q_{\varphi\Box}$ and $Q_{\varphi D}$ from table 4.1. This change of basis is consistent with the rest of the basis from ref. [45]. A proof of this result can be found in section F.3 of appendix F for any order in the EFT expansion. Additionally, we added the number of covariant derivatives in the naming of the operators that belong in the third class, $\varphi^4 D^4$, to avoid confusion with the SM quartic Higgs operator, φ^4.

Table 4.3 collects the operators that are constructed purely from gauge field strengths. Therefore, each operator there contains exactly four field strengths, and the operator classes are further divided as X^4, where only one of the field strengths of the B, W or G gauge fields appears in the operator, X^3X', where the G field strength appears thrice together with a B field strength in the operator, and finally $X^2X'^2$, where the operators are consisted of two pairs of different field strengths. The notation in this table follows exactly ref. [45]. Finally, table 4.4 collects the operators that are constructed from a combination of Higgs doublets, φ, and gauge field strengths.

Having defined the operators that we wish to include in our analysis, we are in place to construct the SMEFT Lagrangian. We organise the terms by their EFT order, as

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}C^{\nu\nu}Q^{(5)}_{\nu\nu} + \frac{1}{\Lambda^2}\sum_f C'^f Q^{(6)}_f + \frac{1}{\Lambda^2}\sum_X C^X_{(8)}Q^{(6)}_X + \frac{1}{\Lambda^4}\sum_X C^X_{(8)}Q^{(8)}_X\,, \tag{4.4}$$

where we have included, from left to right, the SM terms, the dimension 5 Weinberg operator, the fermionic and bosonic dimension 6 terms and, finally, the bosonic dimension 8 terms. This Lagrangian, as mentioned above, is written in the electroweak (flavour) basis, before the SSB. To re-express the Lagrangian in terms of the physical fields we have to rotate everything in the mass basis, after the SSB mechanism takes place. To achieve this, we extend the prescription of ref. [40], by generalising the results up to $\mathcal{O}(1/\Lambda^4)$ to incorporate the effects of the dimension 8 operators and dual insertion of dimension 6 terms. In the gauge and Higgs sectors physical and Goldstone fields (h, G^0, $G^\pm$, $W^\pm_\mu$, Z^0_μ, A_μ) are related to the initial Warsaw basis fields (φ, W^i_μ, B_μ, G^A_μ) by introducing normalisation constants, as follows:

$$
\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\, Z_{G^+}^{-1} G^+ \\ v + Z_h^{-1} h + i Z_{G^0}^{-1} G^0 \end{pmatrix},
$$
$$
\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = Z_{AZ} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix},
$$
$$
W_\mu^1 = \frac{Z_W^{-1}}{\sqrt{2}} \left(W_\mu^+ + W_\mu^- \right),
$$
$$
W_\mu^2 = \frac{i Z_W^{-1}}{\sqrt{2}} \left(W_\mu^+ - W_\mu^- \right),
$$
$$
G_\mu^A = Z_G^{-1} g_\mu^A \,. \tag{4.5}
$$

In addition, the Feynman rules for the physical fields are expressed in terms of effective gauge couplings, defined by

$$
g = Z_g \bar{g} \,, \qquad g' = Z_{g'} \bar{g}' \,, \qquad g_s = Z_{g_s} \bar{g}_s \,. \tag{4.6}
$$

In dimension 6 SMEFT, the $SU(2)$ and $SU(3)$ gauge field and gauge normalisation constants are equal, $Z_g = Z_W$, $Z_{g_s} = Z_G$. We will keep the definitions from eq. (4.6) also for the dimension 8 SMEFT. A genuine new type of contribution to the bilinears which arises at dimension 8 comes from the operator $Q^{(3)}_{W^2\varphi^4} = (\varphi^\dagger \tau^I \varphi)(\varphi^\dagger \tau^J \varphi) W^I_{\mu\nu} W^{J\mu\nu}$. In the broken phase of the theory this operator introduces an asymmetry between the W^3 gauge field and the W^1 and W^2 fields. By setting $Z_g = Z_W$, which is also the definition used in ref. [45], all the information from this new contribution is absorbed the rotation and rescaling matrix Z_{AZ}, and therefore the notation resembles closely the one developed for the dimension 6 in ref. [40].

The charged Goldstone boson normalisation in our Warsaw-like basis for the dimension 8 operators is the same as in the dimension 6 case, $Z_{G^+} = 1$. The complete expressions for all the field normalisation constants, Z_X, for the corrected Higgs field VEV, v, and for the gauge and Higgs boson masses, m_Z, m_W and m_h, can be extracted from appendix F. These expressions are dynamically computed by `SmeftFR`, with only the subset of the dimension 6 and 8 Wilson coefficients that are chosen by the user being taken into account, as described in section 4.4.

Since we don't include any fermionic operators beyond the dimension 6 SMEFT, the rotation of the fermionic sector from the flavour to the mass basis follows the prescription of refs. [40] and [126]. In particular, we perform the following unitary rotation in the flavour space,

$$\psi'_X = U_{\psi_X} \psi_X \,, \tag{4.7}$$

where $\psi = \nu, e, u, d$ and $X = L, R$ denotes the chirality. The unitary rotations are chosen in such a way that the mass eigenstates ψ_X correspond to real, non-negative eigenvalues of the 3×3 fermion mass matrices:

$$\begin{aligned} M'_\nu &= -v^2 C'^{\nu\nu} \,, & M'_e &= \tfrac{v}{\sqrt{2}} \left(\Gamma_e - \tfrac{v^2}{2} C'^{e\varphi} \right), \\ M'_u &= \tfrac{v}{\sqrt{2}} \left(\Gamma_u - \tfrac{v^2}{2} C'^{u\varphi} \right), & M'_d &= \tfrac{v}{\sqrt{2}} \left(\Gamma_d - \tfrac{v^2}{2} C'^{d\varphi} \right). \end{aligned} \tag{4.8}$$

The fermion flavour rotations can be adsorbed in redefinitions of the Wilson coefficients. The CKM and PMNS matrices, denoted by K and U, respectively, will appear in Feynman rules, are defined here as:

$$K = U^\dagger_{u_L} U_{d_L} \,, \qquad U = U^\dagger_{e_L} U_{\nu_L} \,. \tag{4.9}$$

The complete list of the redefinitions of the flavour-dependent Wilson coefficients can be found in table 4 of ref. [40]. After rotations, they are defined in the Warsaw mass basis (as also described in WCxf [143]). The `SmeftFR` package assumes that the numerical values of Wilson coefficients are given in this particular basis.

The Feynman rules that are generated by the `SmeftFR` package, describe interactions of the physical SMEFT fields in the mass basis, with numerical values of Wilson coefficients defined within the same (Warsaw) mass basis. It is also important to stress that in the general case of lepton number flavour violation, with non-vanishing Weinberg operator of eq. (4.1), neutrinos are described by massive Majorana spinors, whereas under the assumption of L-conservation they can be regarded as massless Weyl spinors. `SmeftFR` provides the choice of selecting between the two cases before generating the Feynman rules for the neutrino interactions. One should take into consideration that the treatment of neutrinos as Majorana particles requires a special set of rules for propagators, vertices and diagram combinatorics. We follow here the treatment of refs. [40, 156–158].

4.4 Deriving SMEFT Feynman rules with `SmeftFR` package

4.4.1 Installation

`SmeftFR` package works using the `FeynRules` system, so both need to be properly installed first. A recent version and installation instructions for the `FeynRules` package can be downloaded from the address:

`https://feynrules.irmp.ucl.ac.be`

`SmeftFR` has been tested with `FeynRules` version 2.3.

Standard `FeynRules` installation assumes that the new models description is put into `Model` subdirectory of its main tree. We follow this convention, so that `SmeftFR` archive should be unpacked into

`Models/SMEFT_N_NN`

catalogue, where `N_NN` denotes the package version.

Before running the package, one needs to set properly the main `FeynRules` installation directory, defining the `$FeynRulesPath` variable at the beginning of `smeft_init.m` and `smeft_outputs.m` files. For non-standard installations (not advised!), also the variable `SMEFT$Path` has to be updated accordingly.

4.4.2 Code structure

The most general version of SMEFT, including all possible flavour violating couplings, is very complicated. Symbolic operations on the full SMEFT Lagrangian, including complete set of dimension 5 and 6 and bosonic set of dimension 8 operators and with numerical values of all Wilson coefficients assigned are time consuming and can take hours or even days on a standard personal computer. For most of the physical applications it is sufficient to derive interactions only for a subset of operators.[4]

To speed up the calculations, `SmeftFR` can evaluate Feynman rules for a chosen subset of operators only, generating dynamically the proper `FeynRules` "model files". The calculations are divided in two stages, as illustrated in flowchart of figure 4.1. First, the SMEFT Lagrangian is initialised in Warsaw basis and transformed to mass eigenstates basis analytically, truncating all terms higher than the chosen EFT order, which can be set to $\frac{1}{\Lambda^2}$ or $\frac{1}{\Lambda^4}$.To speed up the program, at this stage all flavour parameters are considered to be tensors with indices without assigned numerical values (they are "Internal" parameters in `FeynRules` notation). The resulting mass basis Lagrangian and Feynman rules written in `Mathematica` format are stored on disk. In the second stage, the previously generated output can be used together with new "model file", this time containing numerical values of ("External") parameters, to export mass basis SMEFT interactions in various commonly used external formats such as Latex, WCxf and standard `FeynRules` supported interfaces – UFO, `FeynArts` and others.

[4] Eventually, operators must be selected with care as in general they may mix under renormalisation [86, 90, 91].

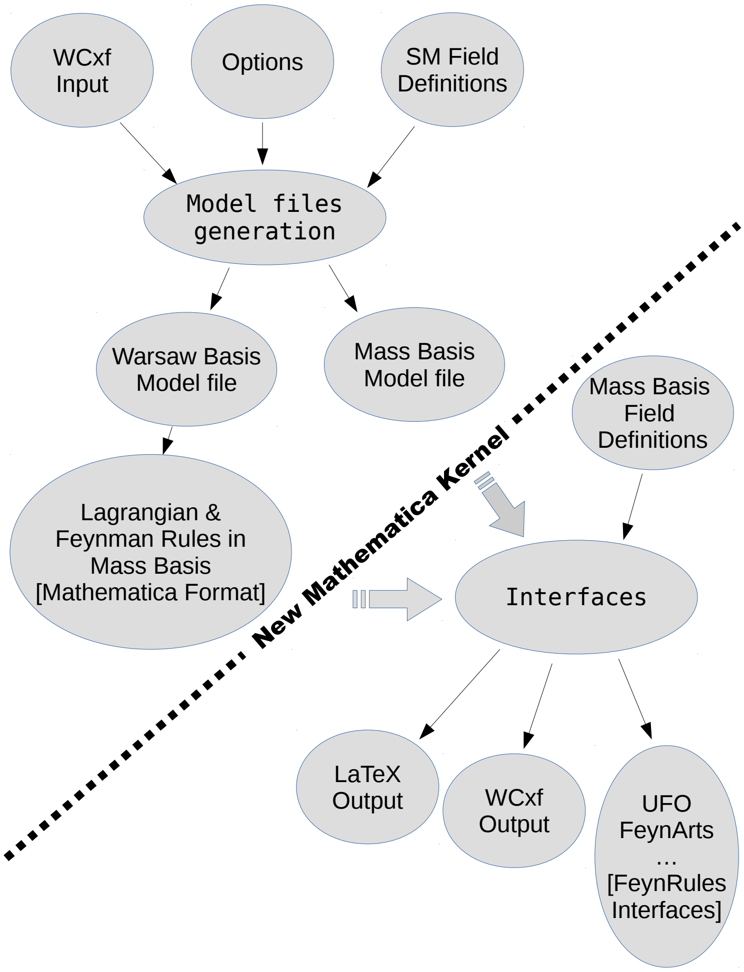


Figure 4.1: Structure of the `SmeftFR` code [126].

4.4.3 Model initialisation

In the first step, the relevant `FeynRules` model files must be generated. This is done by calling the function:

`SMEFTInitializeModel`[*Option1* → *Value1*, *Option2* → *Value2*, ...]

with the allowed options listed in table 4.5.

Names of operators used in `SmeftFR` are derived from the subscript indices of operators listed in table 4.1, with obvious transcriptions of "tilde" symbol and Greek letters to Latin alphabet.

SmeftFR is fully integrated with the WCxf standard. Apart from numerically editing Wilson coefficients in FeynRules model files, reading them from the WCxf input is the only way of automatic initialisation of their numerical values. Such an input format is exchangeable between a larger set of SMEFT-related public packages [143] and may help to compare their results.

An additional advantage of using WCxf input format comes in the flavour sector of the theory. Here, Wilson coefficients are in general tensors with flavour indices, in many cases symmetric under various permutations. WCxf input requires initialisation of only the minimal set of flavour dependent Wilson coefficients, those which could be derived by permutations are also automatically properly set.[5]

Further comments concern MajoranaNeutrino and Correct4Fermion options. They are used to modify the analytical expressions only for the Feynman rules, not at the level of the mass basis Lagrangian from which the rules are derived. This is because some FeynRules interfaces, like UFO, intentionally leave the relative sign of 4-fermion interactions uncorrected[6], as it is later changed by Monte Carlo generators like MadGraph5. Correcting the sign before generating UFO output would therefore lead to wrong final result. Similarly, treatment of neutrinos as Majorana fields could not be compatible with hard coded quantum number definitions in various packages. On the other hand, in the manual or symbolic computations it is convenient to have from the start the correct form of Feynman rules, as done by SmeftFR when both options are set to their default values.

SMEFTInitializeModel routine does not require prior loading of FeynRules package. After execution, it creates in the output subdirectory three model files listed in table 4.6. Parameter files generated by SMEFTInitializeModel contain also definitions of SM parameters, copied from templates smeft_par_head_WB.fr and smeft_par_head_MB.fr located in definitions subdirectory. The values of SM parameters can be best updated directly by editing the template files and the header of the code/smeft_variables.m file, otherwise they will be overwritten in each rerun of SmeftFR initialisation routines.

As mentioned above, in all analytical calculations performed by SmeftFR, terms that are of higher order than the chosen EFT order are always truncated. Therefore, the resulting Feynman rules can be consistently used to calculate physical observables, symbolically or numerically by Monte Carlo generators, up to the linear order in dimension 6 operators, or up to quadratic order in the dimension 6 operators and linear order in the dimension 8 operators operators operators operators operators operators operators operators operators. This information is encoded in FeynRules SMEFT model files by assigning the "interaction

[5] We would like to thank D. Straub for supplying us with a code for symmetrisation of flavour-dependent Wilson coefficients.

[6] B. Fuks, private communication.

order" parameter `NP=1` (`NP=2`) to each dimension 6 (8) Wilson coefficients and setting in `smeft_field_WB.fr` and `smeft_field_MB.fr` the limits:

An additional remark concerns the value of neutrino masses. In mass basis, the neutrino masses are equal to $-v^2 C_{\nu\nu}^{II}$ [see eq. (4.8)]. Thus, the numerical values of $C_{\nu\nu}^{II}$ coefficients should be real and negative. If positive or complex values of $C_{\nu\nu}^{II}$ are given in the WCxf input file, then the `SMEFTInitializeModel` routine evaluates neutrino masses as $M_{\nu^I} = v^2 |C_{\nu\nu}^{II}|$.

4.4.4 Calculation of mass basis Lagrangian and Feynman rules

By loading the `FeynRules` model files the derivation of SMEFT Lagrangian in mass basis is performed by calling the following sequence of routines:

`SMEFTLoadModel[ ]`	Loads `output/smeft_par_WB.par` model file and calculates SMEFT Lagrangian in Warsaw basis for chosen subset of operators
`SMEFTFindMassBasis[ ]`	Finds field bilinears and analytical transformations diagonalizing mass matrices up to the chosen EFT order
`SMEFTFeynmanRules[ ]`	Evaluates analytically SMEFT Lagrangian and Feynman rules in the mass basis, again truncating consistently all terms higher than the chosen EFT order.

The calculation time may vary considerably depending on the choice of operator (sub-)set and gauge fixing conditions chosen. For example, the full list of SMEFT $d = 5$ and $d = 6$ operators and in R_ξ-gauges, one can expect CPU time necessary to evaluate all Feynman rules, from about an hour to many hours on a typical personal computer, depending on its speed capabilities.

One should note that when neutrinos are treated as Majorana particles, (as necessary in case of non-vanishing Wilson coefficient of $d = 5$ Weinberg operator), their interactions involve lepton number non-conservation. When `FeynRules` is dealing with them it produces warnings of the form:

QN::NonConserv: Warning: non quantum number conserving vertex encountered!

Quantum number LeptonNumber not conserved in vertex ...

Obviously such warnings should be ignored.

Evaluation of Feynman rules for vertices involving more than two fermions is not fully implemented yet in `FeynRules`. To our experience, apart from the issue of relative sign of four fermion diagrams mentioned earlier, particularly problematic was the correct automatic derivation of quartic interactions with four Majorana neutrinos and similar vertices which violate B- and L-quantum numbers. For these special cases, `SmeftFR` overwrites the `FeynRules` result with manually calculated formulae encoded in `Mathematica` format.

Another remark concerns the hermicity property of the SMEFT Lagrangian. For some types of interactions, e.g. four-fermion vertices involving two-quarks and two-leptons, the function `CheckHermicity` provided by `FeynRules` reports non-Hermitian terms in the Lagrangian. However, such terms are actually Hermitian if permutation symmetries of indices of relevant Wilson coefficients are taken into account. Such symmetries are automatically imposed if numerical values of Wilson coefficients are initialized with the use of `SMEFTInitializeMB` or `SMEFTToWCXF` routines (see sections 4.4.5 and 4.4.5).

Results of the calculations are collected in file `output/smeft_feynman_rules.m`. The Feynman rules and pieces of the mass basis Lagrangian for various classes of interactions are stored in the variables with self-explanatory names listed in table 4.7.

File `output/smeft_feynman_rules.m` contains also expressions for the normalisation factors relating Higgs and gauge fields and couplings in the Warsaw and mass basis. Namely, variables `Hnorm, G0norm, GPnorm, AZnorm[i,j], Wnorm, Gnorm`, correspond to, respectively, Z_h^{-1}, $Z_{G^0}^{-1}$, $Z_{G^+}^{-1}$ Z_{AZ}^{-1}, Z_W^{-1} and Z_G^{-1} in eq. (4.5). In addition, formulae for tree-level corrections to SM mass parameters and Yukawa couplings are stored in variables `SMEFT$vev`, `SMEFT$MH2`, `SMEFT$MW2`, `SMEFT$MZ2`, `SMEFT$YL[i,j]`, `SMEFT$YD[i,j]` and `SMEFT$YU[i,j]`.

It is important to note that although at this point the Feynman rules for the mass basis Lagrangian are already calculated, definitions for fields and parameters used to initialise the SMEFT model in `FeynRules` are still given in Warsaw basis. To avoid inconsistencies, it is strongly advised to quit the current `Mathematica` kernel and start new one reloading the mass basis Lagrangian together with the compatible model files with fields defined also in mass basis, as described next in section 4.4.5. All further calculations should be performed within this new kernel.

4.4.5 Interfaces

`SmeftFR` output in some of portable formats must be generated from the SMEFT Lagrangian transformed to mass basis, with all numerical values of parameters initialised. As `FeynRules` does not allow for two different model files loaded within a single `Mathematica` session, one needs to quit the kernel used to run routines necessary to obtain Feynman rules and, as described in previous section, start a new `Mathematica` kernel. Within it, the user must reload `FeynRules` and `SmeftFR` packages and call the following routine:

```
SMEFTInitializeMB[ Options ]
```

Allowed options are given in table 4.8. After call to `SMEFTInitializeMB`, mass basis model files are read and the mass basis Lagrangian is stored in a global variable `SMEFTMBLagrangian` for further use by interface routines.

WCxf input and output

Translation between `FeynRules` model files and WCxf format is done by the functions `SMEFTToWCXF` and `WCXFToSMEFT`. They can be used standalone and do not require loading `FeynRules` and calling first `SMEFTInitializeMB` routine to work properly.

Exporting numerical values of Wilson coefficients of operators in the WCxf format is done by the function:

```
SMEFTToWCXF[ SMEFT_Parameter_File, WCXF_File ]
```

where the arguments `SMEFT_Parameter_File, WCXF_File` define the input model parameter file in the `FeynRules` format and the output file in the WCxf JSON format, respectively. The created JSON file can be used to transfer numerical values of Wilson coefficients to other codes supporting WCxf format. Note that in general, the `FeynRules` model files may contain different classes of parameters, according to the `Value` property defined to be a number (real or complex), a formula or even not defined at all. Only the Wilson coefficients with `Value` defined to be a number are transferred to the output file in WCxf format.

Conversely, files in WCxf format can be translated to `FeynRules` parameter files using:

```
WCXFToSMEFT[ WCXF_File, SMEFT_Parameter_File Options]
```

with the allowed options defined in table 4.9.

Latex output

`SmeftFR` provides a dedicated Latex generator (not using the generic `FeynRules` Latex export routine). Its output has the following structure:

- For each interaction vertex, the diagram is drawn, using the `axodraw` style [159]. Expressions for Feynman rules are displayed next to corresponding diagrams.
- In analytical expressions, all terms multiplying a given Wilson coefficient are collected together and simplified.
- Long analytical expressions are automatically broken into many lines using `breakn` style (this does not always work perfectly but the printout is sufficiently readable).

Latex output is generated by the function:

```
SMEFTToLatex[ Options ]
```

with the allowed options listed in table 4.10. The function `SMEFTToLatex` assumes that the variables listed in table 4.7 are initialised. It can be called either after executing relevant commands, described in section 4.4.4, or after reloading the mass basis Lagrangian with the `SMEFTInitializeMB` routine, see section 4.4.5.

Latex output is stored in `output/latex subdirectory`, split into smaller files each containing one primary vertex. The main file is named `smeft_feynman_rules.tex`. The style files necessary to compile Latex output are supplied with the `SmeftFR` distribution.

Note that the correct compilation of documents using "axodraw.sty" style requires creating intermediate Postscript file. Programs like *pdflatex* producing directly PDF output will not work properly. One should instead use e.g.:

```
latex smeft_feynman_rules.tex
dvips smeft_feynman_rules.dvi
ps2pdf smeft_feynman_rules.ps
```

The `smeft_feynman_rules.tex` does not contain analytical expressions for five and six gluon vertices. Such formulae are very long (multiple pages, hard to even compile properly) and not useful for hand-made calculations. If such vertices are needed, they should be rather directly exported in some other formats as described in the next subsection.

Other details not printed in the Latex output, such as, the form of field propagators, conventions for parameters and momenta flow in vertices (always incoming), manipulation of four-fermion vertices with Majorana fermions etc, are explained thoroughly in the appendices A1–A3 of ref. [40].

Standard `FeynRules` interfaces

After calling the initialisation routine `SMEFTInitializeMB`, the output to UFO, `FeynArts` and other formats supported by `FeynRules` interfaces, can be generated using `FeynRules` commands and options from the mass basis Lagrangian stored in the `SMEFTMBLagrangian` variable. For instance, one could call:

```
WriteUFO[ SMEFTMBLagrangian, Output → "output/UFO", AddDecays → False, ...]

WriteFeynArtsOutput[ SMEFTMBLagrangian, Output → "output/FeynArts", ...]
```

and similarly for other formats.

It is important to note that FeynRules interfaces like UFO or FeynArts generate their output starting from the level of SMEFT mass basis Lagrangian. Thus, options of the function SMEFTInitializeModel, like MajoranaNeutrino and Correct4Fermion (see table 4.5), have no effect on output generated by the interface routines. As explained in section 4.4.3 they affect only the expressions for Feynman rules.

If four-fermion vertices are included in SMEFT Lagrangian, UFO produces warning messages of the form:

Warning: Multi-Fermion operators are not yet fully supported!

Therefore, the output for four-fermion interactions in UFO or other formats must be treated with care and limited trust — performing appropriate checks are left to users' responsibility. To our experience, implementation in FeynRules of baryon and lepton number violating four-fermion interactions, with charge conjugation matrix appearing explicitly in vertices, is even more problematic. Thus, for safety in current SmeftFR version (2.00) such terms are never included in SMEFTMBLagrangian variable, eventually they can be passed to interface routines separately via the BLViolatingLagrangian variable.

Exporting to UFO or other formats can take a long time, even several hours for R_ξ-gauges and complete SMEFT Lagrangian with fully general flavour structure and all numerical values of parameters initialised. Finally, it is important to stress here that our Feynman rules communicate properly with MadGraph5 and FeynArts. In particular, we ran without errors test simulations in MadGraph5 using UFO model files produced by SmeftFR v3.0. Similar tests were performed with amplitude generation for sample processes using SmeftFR v3.0 FeynArts output.

4.5 Summary

The high-complexity of the calculations within the SMEFT framework creates a need for computer software dedicated to the task. This need grews even stronger when considering EFT analyses beyond the leading non-trivial order. Aiming in this direction, we present here the Mathematica package SmeftFR, a code dedicated to the generation of the Feynman rules in SMEFT, after the computation of the mass basis Lagrangian. The code of SmeftFR is written as an overlay upon the Mathematica package FeynRules. In its current version, the code includes the effects of the full set of dimension 5 and 6 operators given in the Warsaw basis of ref. [37], as well as the complete bosonic subset of the bosonic dimension 8 operators written in a Warsaw-like basis, that follows ref. [45]. No restrictions are applied about the flavour structure, or about CP-, B- and L-number conservation. The quantisation of SMEFT with this set of operators was performed in unitary and R_ξ-gauges by generalising the procedure followed in ref. [40].

In this chapter, we described the general use the `SmeftFR` package, in order to produce the Feynman rules for a chosen subset of operators of interest. The package output can be interfaced to other software tools supported by `FeynRules`, such as `UFO` (which can then be imported to Monte Carlo generators), `FeynArts` (which can then be used for tree and loop calculations with packages like `FeynCalc` and `FormCalc`), etc. A dedicated LaTeX generator is also provided for printing the output in human-readable form, which can be used for handmade calculations. The numerical initialisation of the Wilson coefficients is interfaced to WCxf format. Additionally, the Feynman rules can be generated in unitary gauge or in linear R_ξ-gauges, and the SMEFT parameters can be exchanged with physical, well-measured, input parameters. The most recent public version of the `SmeftFR` code, together with the most up-to-date user manual, can be downloaded from the webpage `www.fuw.edu.pl/smeft`.

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^* \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I\widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I\varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde W B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu \tau^I q_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu \tau^I l_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\epsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\epsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jn}\epsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\epsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

Table 4.1: The full set of dimension 6 operators in Warsaw basis [37]. The subtables in the two upper rows collect all operators except the four-fermion ones, which are collected separately in the subtables of the two bottom rows.

φ^8		$\varphi^6 D^2$		$\varphi^4 D^4$	
Q_{φ^8}	$(\varphi^\dagger\varphi)^4$	$Q_{\varphi^6\Box}$	$(\varphi^\dagger\varphi)^2\Box(\varphi^\dagger\varphi)$	$Q^{(1)}_{\varphi^4D^4}$	$(D_\mu\varphi^\dagger D_\nu\varphi)(D^\nu\varphi^\dagger D^\mu\varphi)$
		$Q_{\varphi^6D^2}$	$(\varphi^\dagger\varphi)(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi)$	$Q^{(2)}_{\varphi^4D^4}$	$(D_\mu\varphi^\dagger D_\nu\varphi)(D^\mu\varphi^\dagger D^\nu\varphi)$
				$Q^{(3)}_{\varphi^4D^4}$	$(D_\mu\varphi^\dagger D^\mu\varphi)(D_\nu\varphi^\dagger D^\nu\varphi)$

Table 4.2: Dimension 8 operators containing only the Higgs field. Table taken (and modified according to our notation) from ref. [45].

$X^4,\ X^3X'$		$X^2X'^2$	
$Q^{(1)}_{G^4}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{B\rho\sigma})$	$Q^{(1)}_{G^2W^2}$	$(W^I_{\mu\nu}W^{I\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(2)}_{G^4}$	$(G^A_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$	$Q^{(2)}_{G^2W^2}$	$(W^I_{\mu\nu}\widetilde{W}^{I\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(3)}_{G^4}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}G^{B\rho\sigma})$	$Q^{(3)}_{G^2W^2}$	$(W^I_{\mu\nu}G^{A\mu\nu})(W^I_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(4)}_{G^4}$	$(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$	$Q^{(4)}_{G^2W^2}$	$(W^I_{\mu\nu}\widetilde{G}^{A\mu\nu})(W^I_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(5)}_{G^4}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$	$Q^{(5)}_{G^2W^2}$	$(W^I_{\mu\nu}\widetilde{W}^{I\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(6)}_{G^4}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$	$Q^{(6)}_{G^2W^2}$	$(W^I_{\mu\nu}W^{I\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(7)}_{G^4}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}G^{D\rho\sigma})$	$Q^{(7)}_{G^2W^2}$	$(W^I_{\mu\nu}G^{A\mu\nu})(W^I_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(8)}_{G^4}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^C_{\rho\sigma}\widetilde{G}^{D\rho\sigma})$	$Q^{(1)}_{G^2B^2}$	$(B_{\mu\nu}B^{\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(9)}_{G^4}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}\widetilde{G}^{D\rho\sigma})$	$Q^{(2)}_{G^2B^2}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(1)}_{W^4}$	$(W^I_{\mu\nu}W^{I\mu\nu})(W^J_{\rho\sigma}W^{J\rho\sigma})$	$Q^{(3)}_{G^2B^2}$	$(B_{\mu\nu}G^{A\mu\nu})(B_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(2)}_{W^4}$	$(W^I_{\mu\nu}\widetilde{W}^{I\mu\nu})(W^J_{\rho\sigma}\widetilde{W}^{J\rho\sigma})$	$Q^{(4)}_{G^2B^2}$	$(B_{\mu\nu}\widetilde{G}^{A\mu\nu})(B_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(3)}_{W^4}$	$(W^I_{\mu\nu}W^{J\mu\nu})(W^I_{\rho\sigma}W^{J\rho\sigma})$	$Q^{(5)}_{G^2B^2}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$
$Q^{(4)}_{W^4}$	$(W^I_{\mu\nu}\widetilde{W}^{J\mu\nu})(W^I_{\rho\sigma}\widetilde{W}^{J\rho\sigma})$	$Q^{(6)}_{G^2B^2}$	$(B_{\mu\nu}B^{\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(5)}_{W^4}$	$(W^I_{\mu\nu}W^{I\mu\nu})(W^J_{\rho\sigma}\widetilde{W}^{J\rho\sigma})$	$Q^{(7)}_{G^2B^2}$	$(B_{\mu\nu}G^{A\mu\nu})(B_{\rho\sigma}\widetilde{G}^{A\rho\sigma})$
$Q^{(6)}_{W^4}$	$(W^I_{\mu\nu}W^{J\mu\nu})(W^I_{\rho\sigma}\widetilde{W}^{J\rho\sigma})$	$Q^{(1)}_{W^2B^2}$	$(B_{\mu\nu}B^{\mu\nu})(W^I_{\rho\sigma}W^{I\rho\sigma})$
$Q^{(1)}_{B^4}$	$(B_{\mu\nu}B^{\mu\nu})(B_{\rho\sigma}B^{\rho\sigma})$	$Q^{(2)}_{W^2B^2}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(W^I_{\rho\sigma}\widetilde{W}^{I\rho\sigma})$
$Q^{(2)}_{B^4}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(B_{\rho\sigma}\widetilde{B}^{\rho\sigma})$	$Q^{(3)}_{W^2B^2}$	$(B_{\mu\nu}W^{I\mu\nu})(B_{\rho\sigma}W^{I\rho\sigma})$
$Q^{(3)}_{B^4}$	$(B_{\mu\nu}B^{\mu\nu})(B_{\rho\sigma}\widetilde{B}^{\rho\sigma})$	$Q^{(4)}_{W^2B^2}$	$(B_{\mu\nu}\widetilde{W}^{I\mu\nu})(B_{\rho\sigma}\widetilde{W}^{I\rho\sigma})$
$Q^{(1)}_{G^3B}$	$d^{ABC}(B_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{C\rho\sigma})$	$Q^{(5)}_{W^2B^2}$	$(B_{\mu\nu}\widetilde{B}^{\mu\nu})(W^I_{\rho\sigma}W^{I\rho\sigma})$
$Q^{(2)}_{G^3B}$	$d^{ABC}(B_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{C\rho\sigma})$	$Q^{(6)}_{W^2B^2}$	$(B_{\mu\nu}B^{\mu\nu})(W^I_{\rho\sigma}\widetilde{W}^{I\rho\sigma})$
$Q^{(3)}_{G^3B}$	$d^{ABC}(B_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}G^{C\rho\sigma})$	$Q^{(7)}_{W^2B^2}$	$(B_{\mu\nu}W^{I\mu\nu})(B_{\rho\sigma}\widetilde{W}^{I\rho\sigma})$
$Q^{(4)}_{G^3B}$	$d^{ABC}(B_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{C\rho\sigma})$		

Table 4.3: Dimension 8 operators containing only gauge field strengths. Table taken from ref. [45].

$X^3\varphi^2$		$X^2\varphi^4$	
$Q_{G^3\varphi^2}^{(1)}$	$f^{ABC}(\varphi^\dagger\varphi)G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{G^2\varphi^4}^{(1)}$	$(\varphi^\dagger\varphi)^2 G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{G^3\varphi^2}^{(2)}$	$f^{ABC}(\varphi^\dagger\varphi)G_\mu^{A\nu}G_\nu^{B\rho}\widetilde{G}_\rho^{C\mu}$	$Q_{G^2\varphi^4}^{(2)}$	$(\varphi^\dagger\varphi)^2 \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
$Q_{W^3\varphi^2}^{(1)}$	$\epsilon^{IJK}(\varphi^\dagger\varphi)W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{W^2\varphi^4}^{(1)}$	$(\varphi^\dagger\varphi)^2 W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{W^3\varphi^2}^{(2)}$	$\epsilon^{IJK}(\varphi^\dagger\varphi)W_\mu^{I\nu}W_\nu^{J\rho}\widetilde{W}_\rho^{K\mu}$	$Q_{W^2\varphi^4}^{(2)}$	$(\varphi^\dagger\varphi)^2 \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
$Q_{W^2B\varphi^2}^{(1)}$	$\epsilon^{IJK}(\varphi^\dagger\tau^I\varphi)B_\mu^{\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{W^2\varphi^4}^{(3)}$	$(\varphi^\dagger\tau^I\varphi)(\varphi^\dagger\tau^J\varphi)W_{\mu\nu}^I W^{J\mu\nu}$
$Q_{W^2B\varphi^2}^{(2)}$	$\epsilon^{IJK}(\varphi^\dagger\tau^I\varphi)(\widetilde{B}^{\mu\nu}W_{\nu\rho}^J W_\mu^{K\rho} + B^{\mu\nu}W_{\nu\rho}^J\widetilde{W}_\mu^{K\rho})$	$Q_{W^2\varphi^4}^{(4)}$	$(\varphi^\dagger\tau^I\varphi)(\varphi^\dagger\tau^J\varphi)\widetilde{W}_{\mu\nu}^I W^{J\mu\nu}$
		$Q_{WB\varphi^4}^{(1)}$	$(\varphi^\dagger\varphi)(\varphi^\dagger\tau^I\varphi)W_{\mu\nu}^I B^{\mu\nu}$
		$Q_{WB\varphi^4}^{(2)}$	$(\varphi^\dagger\varphi)(\varphi^\dagger\tau^I\varphi)\widetilde{W}_{\mu\nu}^I B^{\mu\nu}$
		$Q_{B^2\varphi^4}^{(1)}$	$(\varphi^\dagger\varphi)^2 B_{\mu\nu}B^{\mu\nu}$
		$Q_{B^2\varphi^4}^{(2)}$	$(\varphi^\dagger\varphi)^2 \widetilde{B}_{\mu\nu}B^{\mu\nu}$

$X^2\varphi^2D^2$		$X\varphi^4D^2$	
$Q_{G^2\varphi^2D^2}^{(1)}$	$(D^\mu\varphi^\dagger D^\nu\varphi)G_{\mu\rho}^A G_\nu^{A\rho}$	$Q_{W\varphi^4D^2}^{(1)}$	$(\varphi^\dagger\varphi)(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)W_{\mu\nu}^I$
$Q_{G^2\varphi^2D^2}^{(2)}$	$(D^\mu\varphi^\dagger D_\mu\varphi)G_{\nu\rho}^A G^{A\nu\rho}$	$Q_{W\varphi^4D^2}^{(2)}$	$(\varphi^\dagger\varphi)(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)\widetilde{W}_{\mu\nu}^I$
$Q_{G^2\varphi^2D^2}^{(3)}$	$(D^\mu\varphi^\dagger D_\mu\varphi)G_{\nu\rho}^A \widetilde{G}^{A\nu\rho}$	$Q_{W\varphi^4D^2}^{(3)}$	$\epsilon^{IJK}(\varphi^\dagger\tau^I\varphi)(D^\mu\varphi^\dagger\tau^J D^\nu\varphi)W_{\mu\nu}^K$
$Q_{W^2\varphi^2D^2}^{(1)}$	$(D^\mu\varphi^\dagger D^\nu\varphi)W_{\mu\rho}^I W_\nu^{I\rho}$	$Q_{W\varphi^4D^2}^{(4)}$	$\epsilon^{IJK}(\varphi^\dagger\tau^I\varphi)(D^\mu\varphi^\dagger\tau^J D^\nu\varphi)\widetilde{W}_{\mu\nu}^K$
$Q_{W^2\varphi^2D^2}^{(2)}$	$(D^\mu\varphi^\dagger D_\mu\varphi)W_{\nu\rho}^I W^{I\nu\rho}$	$Q_{B\varphi^4D^2}^{(1)}$	$(\varphi^\dagger\varphi)(D^\mu\varphi^\dagger D^\nu\varphi)B_{\mu\nu}$
$Q_{W^2\varphi^2D^2}^{(3)}$	$(D^\mu\varphi^\dagger D_\mu\varphi)W_{\nu\rho}^I \widetilde{W}^{I\nu\rho}$	$Q_{B\varphi^4D^2}^{(2)}$	$(\varphi^\dagger\varphi)(D^\mu\varphi^\dagger D^\nu\varphi)\widetilde{B}_{\mu\nu}$
$Q_{W^2\varphi^2D^2}^{(4)}$	$i\epsilon^{IJK}(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)W_{\mu\rho}^J W_\nu^{K\rho}$		
$Q_{W^2\varphi^2D^2}^{(5)}$	$\epsilon^{IJK}(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(W_{\mu\rho}^J\widetilde{W}_\nu^{K\rho} - \widetilde{W}_{\mu\rho}^J W_\nu^{K\rho})$		
$Q_{W^2\varphi^2D^2}^{(6)}$	$i\epsilon^{IJK}(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(W_{\mu\rho}^J\widetilde{W}_\nu^{K\rho} + \widetilde{W}_{\mu\rho}^J W_\nu^{K\rho})$		
$Q_{WB\varphi^2D^2}^{(1)}$	$(D^\mu\varphi^\dagger\tau^I D_\mu\varphi)B_{\nu\rho}W^{I\nu\rho}$		
$Q_{WB\varphi^2D^2}^{(2)}$	$(D^\mu\varphi^\dagger\tau^I D_\mu\varphi)B_{\nu\rho}\widetilde{W}^{I\nu\rho}$		
$Q_{WB\varphi^2D^2}^{(3)}$	$i(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(B_{\mu\rho}W_\nu^{I\rho} - B_{\nu\rho}W_\mu^{I\rho})$		
$Q_{WB\varphi^2D^2}^{(4)}$	$(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(B_{\mu\rho}W_\nu^{I\rho} + B_{\nu\rho}W_\mu^{I\rho})$		
$Q_{WB\varphi^2D^2}^{(5)}$	$i(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(B_{\mu\rho}\widetilde{W}_\nu^{I}{}^{\rho} - B_{\nu\rho}\widetilde{W}_\mu^{I}{}^{\rho})$		
$Q_{WB\varphi^2D^2}^{(6)}$	$(D^\mu\varphi^\dagger\tau^I D^\nu\varphi)(B_{\mu\rho}\widetilde{W}_\nu^{I}{}^{\rho} + B_{\nu\rho}\widetilde{W}_\mu^{I}{}^{\rho})$		
$Q_{B^2\varphi^2D^2}^{(1)}$	$(D^\mu\varphi^\dagger D^\nu\varphi)B_{\mu\rho}B_\nu{}^\rho$		
$Q_{B^2\varphi^2D^2}^{(2)}$	$(D^\mu\varphi^\dagger D_\mu\varphi)B_{\nu\rho}B^{\nu\rho}$		
$Q_{B^2\varphi^2D^2}^{(3)}$	$(D^\mu\varphi^\dagger D_\mu\varphi)B_{\nu\rho}\widetilde{B}^{\nu\rho}$		

Table 4.4: Dimension 8 operators containing both gauge field strengths and the Higgs field. Table taken (and modified according to our notation) from ref. [45].

Option	Allowed values	Description
Operators	default: all operators	List with subset of SMEFT operators included in calculations.
Gauge	**Unitary**, Rxi	Choice of gauge fixing conditions
WCXFInitFile	””	Name of file with numerical values of Wilson coefficients in the WCxf format. If this option is not set or the file does not exist, all Wilson coefficients are set to 0.
MajoranaNeutrino	**False**, True	Neutrino fields are treated as Majorana spinors if $Q_{\nu\nu}$ is included in the operator list, massless Weyl spinors otherwise. Setting this option to **True** allows one to use Majorana spinors also in the massless case.
Correct4Fermion	False, **True**	Corrects relative sign of some 4-fermion interactions, fixing results produced by `FeynRules`.
WBFirstLetter	**”c”**	Customisable first letter of Wilson coefficient names in Warsaw basis (default $c_G, \ldots$). Can be used to avoid convention clashes when comparing with other SMEFT bases.
MBFirstLetter	**”C”**	Customisable first letter of Wilson coefficient names in mass basis (default $C_G, \ldots$).

Table 4.5: The allowed options of `SMEFTInitializeModel` routine. If an option is not specified, the default value (marked above in boldface) is assumed.

`smeft_par_WB.par`	SMEFT parameter file with Wilson coefficients in Warsaw basis (defined as “Internal”, with no numerical values assigned).
`smeft_par_MB.par`	SMEFT parameter file with Wilson coefficients in mass basis (defined as “External”, numerical values imported from the input file in WCxf format).
`smeft_par_MB_real.par`	as `smeft_par_MB.par`, but only real values of Wilson coefficients given in WCxf file are included in SMEFT parameter file, as required by many event generators.

Table 4.6: Model files generated by the `SMEFTInitializeModel` routine.

`LeptonGaugeVertices`	`QuarkGaugeVertices`
`LeptonHiggsGaugeVertices`	`QuarkHiggsGaugeVertices`
`QuarkGluonVertices`	
`GaugeSelfVertices`	`GaugeHiggsVertices`
`GluonSelfVertices`	`GluonHiggsVertices`
`GhostVertices`	
`FourLeptonVertices`	`FourQuarkVertices`
`TwoQuarkTwoLeptonVertices`	
`DeltaLTwoVertices`	`BLViolatingVertices`

Table 4.7: Names of variables defined in the file `output/smeft_feynman_rules.m` containing expressions for Feynman rules. Parts of mass basis Lagrangian are stored in equivalent set of variables, with "`Vertices`" replaced by "`Lagrangian`" in part of their names (i.e. `LeptonGaugeVertices` → `LeptonGaugeLagrangian`, etc.).

Option	Allowed values	Description
RealParameters	**False**, True	Default initialisation is done using `output/smeft_par_MB.par` file, which may contain complex parameters, not compatible with matrix element generators. Setting *RealParameters* → *True* forces loading of `output/smeft_par_MB_real.par` file where imaginary parts of all Wilson coefficients are set to 0. Imaginary phases of CKM and PMNS matrices, if present, are also set to zero after loading this file.
Include4Fermion	**False**, True	4-fermion vertices are not fully implemented in `FeynRules` and by default not included in SMEFT interactions. Set this option to True to include such terms.

Table 4.8: Options of `SMEFTInitializeMB` routine, with default values marked in boldface.

Option	Allowed values	Description
Operators	default: all	List with subset of Wilson coefficients to be included in the SMEFT parameter file
RealParameters	False, **True**	Decides if only real values of Wilson coefficients given in WCxf file are included in SMEFT parameter file
OverwriteTarget	**False**, True	If set to True, target file is overwritten without warning
Silent	**False**, True	Debug option, suppresses screen comments
FirstLetter	**"C"**	Customisable first letter of Wilson coefficient names in mass basis (default $C_G, \ldots$).

Table 4.9: Options of `WCXFToSMEFT` routine. Default values are marked in boldface.

Option name	Allowed values	Description
FullDocument	False, **True**	By default a complete document is generated, with all headers necessary for compilation. If set to False, headers are stripped off and the output file can, without modifications, be included into other Latex documents.
ScreenOutput	**False**, True	For debugging purposes, if set to True the Latex output is printed also to the screen.

Table 4.10: Options of `SMEFTToLatex` routine, with default values marked in boldface.

Epilogue

Chapter 5

Conclusions and future directions

The discovery of the Higgs boson at the LHC in 2012 indicated the end of an era for the elementary particle physics community. Four decades after the independent efforts of Glashow, Weinberg and Salam (GWS) to construct a theory in order to describe the known subatomic particles and their interactions, this discovery served as the verification of a main prediction of the theory. The Higgs mechanism, which plays a central role in the GWS theory, predicts the existence of the Higgs boson, a massive scalar particle with zero electric charge. In fact, with the exception of the quarks which appear only in bound states, it was the only part of the spectrum left to be directly observed in an experiment. The remarkable agreement with the experimental data, however, had led the GWS theory to be known in the literature as the Standard Model (SM) of elementary particle physics long before the actual discovery of the Higgs boson. Putting this final piece together completed the puzzle, and served as an even stronger indication in favour of the SM.

In the meantime, while the theoretical predictions of the SM were thoroughly studied, many extensions of the theory were proposed in an attempt to fill the gaps still left unexplained by the SM. Most of these beyond the SM (BSM) theories predicted new particles whose discovery would strongly suggest their validity. For better or worse, the search for a direct discovery of new subatomic particles didn't prove to be fruitful as of today. In addition, the ongoing efforts of explaining the discrepancies between the theory predictions and the experimental data through the indirect effects of the proposed BSM theories at lower energies haven't led to any strong conclusions in favour of one of the BSM theories. Therefore, we find ourselves in a situation where our benchmark theory, the SM, is studied ad nauseam, with the theoretical SM predictions reaching a great level of accuracy, while the existing anomalies in the SM predictions cannot be dealt with by using the existing theoretical framework or a well-defined UV extension of it. Therefore, using an approach that avoids hard assumptions about the specifics of the BSM physics at the energy scale we wish to examine, while at the same time allowing for the effects of the unknown new physics to trickle down and affect our predictions, is a very promising alternative. The best way to achieve this generic approach is to embed the SM into an Effective Field Theory (EFT) framework.

In this thesis we utilised this generic method, by extending the SM inside the framework of a bottom-up EFT. There, the new physics effects are methodically incorporated in our effective Lagrangian as terms of a power series. The expansion parameter of this series is the inverse of an indicative energy scale, Λ, in which the BSM physics is assumed to take place. The very important advantage of this EFT approach is that it not only captures the remnants of the new physics effects, but we can now expand in the small parameter $1/\Lambda$ systematically, until our theoretical calculations reach the level of accuracy needed to be on equal footing with the accuracy achieved experimentally. Since this EFT extension of the SM, abbreviated as SMEFT, is a relatively new framework, there are still many open questions about the computational details and techniques when using it in involved calculations, even at the leading EFT order. Furthermore, the pressing need for even more accurate and divergent theoretical results creates the need for more SMEFT calculations, sometimes even beyond the leading EFT order. Therefore, automatising the computations at a high degree is a necessity in order to minimise potential human errors and to reduce the physical labour involved. These are the issues which we addressed in this study.

For the first part of this thesis we focused our efforts on the analysis of the calculational challenges and the phenomenological implications of two important physical processes involving the Higgs boson. The processes we chose had to do with the decay modes of the Higgs boson into a pair of photons and into a photon and a massive Z-boson, within the leading non-trivial EFT order (with operators up to dimension 6) and at one-loop in the $\hbar$ expansion. These computations, being highly non-trivial, served as test-cases for technical issues regarding the validity of loop EFT calculations. We presented every aspect of the calculations in detail, and we focused our efforts in shedding some light on these technical issues. In particular, special emphasis was given in the detailed construction of a simple renormalisation framework for the EFT amplitudes. By taking into account the running of the Wilson coefficients, which were treated as $\overline{\text{MS}}$ parameters in our renormalisation scheme, we proved the cancellation of the infinities in the physical amplitude, and that the later is also independent of the renormalisation scale μ. All calculations were performed in linear R_ξ-gauges, with independent ξ parameters. Therefore, we were able to prove analytically the gauge invariance of our results, before and after the application of our renormalisation framework. Finally, we also performed a phenomenological analysis of our results, by placing bounds in the unspecified Wilson coefficients of the model using the most recent experimental data provided by the LHC.

The second part of this thesis was dedicated to the effort of streamlining the diagrammatic computations in the SMEFT, while at the same time addressing the need of developing the theory beyond the dimension 6 order. It should be clear that the main disadvantage of using such a generic theoretical framework is the introduction of a huge set of unknown parameters, in the form of Wilson coefficients, to our model. In addition, the new effective operators add heavily to the complexity of the Feynman diagrammatic computations. These complications make calculations within the SMEFT quite lengthy and technically demanding even at the leading EFT order. As a further complication, the SMEFT community is interested in formulating the model beyond the leading EFT order and applying it for the calculation of physical observables, making handmade calculations virtually impossible. The above indicates the increasing demand for the introduction of powerful software tools in order to minimise the physical labour and to increase the efficiency of calculations in the SMEFT. We addressed these issues by the development of software tools specifically designed to handle the construction of the SMEFT Feynman rules in the physical mass basis and to provide extensive integration with existing software for further automatising the diagrammatic amplitude calculations. The software package `SmeftFR`, a Feynman rules generator for the SMEFT written in `Mathematica`, was initially developed to include the full dimension 6 SMEFT and very recently extended to include the complete bosonic sector at dimension 8, to consistently expand the EFT up to $1/\Lambda^4$ order, and to provide the user with the ability to express the final results in a convenient physical input scheme.

The theoretical development of the theory beyond the leading order was of special interest. We systematically included all possible effects from the bosonic sector of the model up to any arbitrary EFT expansion order, and explained in detail the methodology of deriving the physical mass basis of the theory. This reformulation of the SMEFT to any order was presented in compact analytic formulae in the appendices, using a concise formalism. The results there can be readily used in future works that would involve calculations of even higher SMEFT orders, or in the construction of new and more sophisticated computer software. This covers the potential need of further advancing the accuracy of the theoretical calculations in order to interpret the experimental data of the next LHC runs and of future colliders. Summarising our conclusions, let us state here that especially in this era of heavy automatisation, the SMEFT approach goes hand-in-hand with the needs of the physics community for theories that are able to provide state of the art predictions, able to catch up with the accuracy of the current and future experiments which are driven by the rapid technological advancement. The self-consistent and well-defined theoretical framework of the SMEFT is a modern and, in our opinion, future-proof way of approaching the study of the quantum microcosmos of the elementary particle physics.

Appendices

Appendix **A**

Calculating Loop Diagrams Step-by-Step

In this appendix we will present the steps one should follow to calculate loop diagrams *without* making use of the automated procedure of Passarino-Veltman functions, which is the topic of appendix B. We will post the steps of the algorithmic procedure and give all needed formulae in their most general form.

A.1 Dimensional Regularisation

Feynman diagrams that contain closed loops produce, due to the *superposition principle* of QM, integrals over the momentum running in the loop (the loop momentum). There are many different methods for formally calculating such integrals, but in this thesis we use the method of Dimensional Regularisation (DR), introduced in refs. [160, 161]. DR is known to respect the gauge and Lorentz invariance of a theory. Regularisation of the integrals is needed since most of the times we face ultraviolet divergences when calculating the diagrams (see the comment below eq. (A.20)).

DR consists on evaluating the corresponding integrals in d space-time dimensions and, at the end of the calculation, one should take the formal limit $d \to 4$. To do so, we define an infinitesimal number ϵ such that

$$\epsilon = 4 - d\,, \tag{A.1}$$

and, therefore, infinities will appear as $\frac{1}{\epsilon}$. DR respects the following three postulates:

1. Linearity,

$$\int d^d k\,(a\,f(k) + b\,g(k)) = a \int d^d k\,f(k) + b \int d^d k\,g(k)\,;$$

2. Scaling,
$$\int d^d k\, f(ak) = a^{-d} \int d^d k\, f(k)\,;$$

3. Translation invariance,
$$\int d^d k\, f(k+q) = \int d^d k\, f(k)\,.$$

In the above set of axioms k and q are momenta, f and g are functions of the momenta and a and b are complex numbers.

A.2 Feynman Parameterisation

To begin with, we are going to present Feynman's trick for simplifying the denominator of the loop-momentum integrals. This is accomplished by making use of *Feynman parameters*. The scope of this trick is to make the d-momentum integral spherically symmetric, to be easier to manipulate. The general Feynman parameterisation formula reads:

$$\frac{1}{D_1 \cdots D_n} = \int dF_n \big(x_1 D_1 + \cdots + x_n D_n\big)^{-n}\,, \tag{A.2}$$

where we defined the *Feynman integration measure* to be

$$\int dF_n = (n-1)! \left(\prod_{i=1}^{n} \int_0^1 dx_i \right) \delta(x_1 + \cdots + x_n - 1)\,. \tag{A.3}$$

That way, Feynman's measure is normalised to unit, i.e.

$$\int dF_n \cdot 1 = 1\,. \tag{A.4}$$

In fact, let us give Feynman's formula in its most general form, for the denominators to be raised in different powers. Then,

$$\frac{1}{D_1^{\alpha_1} \cdots D_n^{\alpha_n}} = \frac{\Gamma(\sum_i \alpha_i)}{\prod_i \Gamma(\alpha_i)} \frac{1}{\Gamma(n)} \int dF_n\, \frac{\prod_i x_i^{\alpha_i - 1}}{(\sum_i x_i D_i)^{\sum_i \alpha_i}}\,, \quad i = 1, \ldots, n\,, \tag{A.5}$$

where with $\Gamma(x)$ we symbolise the Euler gamma function. The definition and some useful identities of the gamma function are given at the end of section A.4. One could also make use of the Dirac delta function to write

$$\begin{aligned}\int dF_n f(x) =&(n-1)! \int_0^1 dx_1 \cdots \int_0^1 dx_n\, \delta(x_1+\cdots+x_n-1) f(x)\\ =&(n-1)! \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \cdots\\ &\cdots \int_0^{1-x_1-\cdots-x_{n-2}} dx_{n-1}\, f(x)\Big|_{x_n=1-x_1-\cdots-x_{n-1}} .\end{aligned} \tag{A.6}$$

After Feynman parameterisation, each diagram can be written schematically as

$$i\mathcal{M} = \int \frac{d^d k}{(2\pi)^d} \int dF_n \frac{\mathcal{N}}{\mathcal{D}}\,, \tag{A.7}$$

where $\mathcal{N}$ is the numerator, $\mathcal{D}$ the denominator and $d = 4-\epsilon$ the dimensionality of space-time.

Now that we established notation, let us be more concrete and give specific formulae in a convenient form. We start with a general amplitude

$$i\mathcal{M} = \int \frac{d^d k}{(2\pi)^d} \frac{\mathcal{N}(k,p)}{D_0\, D_1 \cdots D_{N-1}}\,, \tag{A.8}$$

where $\mathcal{N}(k,p)$ is a function of k and the p_is, and

$$D_0 = k^2 - m_0^2\,; \qquad D_i = (k+p_i)^2 - m_i^2\,, \quad i = 1,\ldots,N-1\,. \tag{A.9}$$

For N denominators we introduce N Feynman parameters, x_i, and write

$$\begin{aligned}\frac{1}{D_0\, D_1 \cdots D_{N-1}} &= \frac{1}{\big(k^2-m_0^2\big)\big((k+p_1)^2-m_1^2\big)\cdots\big((k+p_{N-1})^2-m_{N-1}^2\big)}\\ &= \int dF_N \left[x_N\big(k^2-m_0^2\big) + \sum_{i=1}^{N-1} x_i\big((k+p_i)^2-m_i^2\big)\right]^{-N}\,,\end{aligned} \tag{A.10}$$

where

$$dF_N = \Gamma(N) \int_0^1 dx_1 \cdots \int_0^1 dx_N\, \delta(x_1+\cdots+x_N-1) \tag{A.11}$$

is the Feynman integration measure and $\Gamma(N) = (N-1)!$ since N is a non-negative integer. Now we use the delta function to replace x_N with $1-x_1-\cdots-x_{N-1}$ in the integrand, and then by defining a *reduced* Feynman integration measure, $d\widetilde{F}_N$, such that

$$\int d\widetilde{F}_N\, f(x_i) = \Gamma(N) \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \cdots \\ \cdots \int_0^{1-x_1-\cdots-x_{N-2}} dx_{N-1}\, f(x_i)\Big|_{x_N=1-x_1-\cdots-x_{N-1}}, \tag{A.12}$$

we have

$$\frac{1}{D_0\, D_1 \cdots D_{N-1}} = \int d\widetilde{F}_N \big(l_N^2 - \Delta_N\big)^{-N}. \tag{A.13}$$

In the last equation we defined a *shifted momentum*

$$l_N = k + \sum_{i=1}^{N-1} x_i\, p_i \tag{A.14}$$

and a function

$$\Delta_N = -\sum_{i=1}^{N-1} x_i(1-x_i)\, p_i^2 + \sum_{j\neq i} \sum_{i=1}^{N-1} (x_i x_j\, p_i \cdot p_j) \\ + m_0^2 \left(1 - \sum_{i=1}^{N-1} x_i\right) + \sum_{i=1}^{N-1} x_i\, m_i^2\,. \tag{A.15}$$

Thus far we brought the denominator in a spherically symmetric form and we know the appropriate shifting in the integration variable needed to do so. The next thing to do is to apply the momentum shifting in the numerator and, by taking advantage of the symmetric form of the integral, simplify it as much as possible.

A.3 Numerator Simplification

To simplify the numerator, we shift the integration variable in it according to eq. (A.14) and make use of the spherical symmetry of the momentum integrals. This symmetry implies that products with an odd number of l vanish and the even products can be re-expressed as

$$l^\mu l^\nu \to \frac{l^2}{d} g^{\mu\nu}\,, \tag{A.16}$$

$$l^\mu l^\nu l^\rho l^\sigma \to \frac{\left(l^2\right)^2}{d(d+2)} \Big(g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}\Big)\,, \tag{A.17}$$

or in general, for n pairs of momenta l,

$$l^{\mu_1} l^{\nu_1} \cdots l^{\mu_n} l^{\nu_n} \to \frac{(l^2)^n}{d(d+2)\cdots(d+2(n-1))} \times \\ \left(g^{\mu_1\nu_1} \cdots g^{\mu_n\nu_n} + \text{non-redundant permutations}\right), \tag{A.18}$$

where as redundant permutations we consider these that give equal terms after using the symmetry properties of the metric.

After these relations are applied, we expect the numerator $\mathcal{N}$ to be a polynomial in l^2, namely

$$\mathcal{N} = \sum_{i=0} c_i (l^2)^i . \tag{A.19}$$

The upper limit of the summation is different for each application.

A.4 Loop Momentum Integrals

The momentum integrals that will occur after all these steps are concluded behave differently for different powers of the shifted momentum l in the numerator. The general formula for these loop-momentum integrals is

$$\int \frac{d^d l}{(2\pi)^d} \frac{(l^2)^a}{(l^2-\Delta)^b} = i(-1)^{b-a} \frac{\Gamma\left(b-a-\frac{1}{2}d\right)\Gamma\left(a+\frac{1}{2}d\right)}{(4\pi)^{d/2}\Gamma(b)\Gamma\left(\frac{1}{2}d\right)\Delta^{b-a-d/2}} . \tag{A.20}$$

By naive power counting, when $a - b \geq -d/2$ the integral will diverge.

The Euler gamma function appeared many times thus far in our discussion, so we should give some basic formulae needed in loop calculations. The gamma function has many equivalent definitions. It can be defined by Euler's integral,

$$\Gamma(z) = \int_0^\infty dt\, e^{-t} t^{z-1} = 2\int_0^\infty dt\, e^{-t^2} t^{2z-1} , \quad Re(z) > 0 , \tag{A.21}$$

or by

$$\Gamma(z) = \int_0^1 dt \left[\log\left(\tfrac{1}{t}\right)\right]^{z-1} , \quad Re(z) > 0 , \tag{A.22}$$

etc. In our calculations we are going to need the following identities for the Euler gamma function:

$$\Gamma(n+1) = n!\,, \tag{A.23}$$

$$\Gamma(n+\tfrac{1}{2}) = \frac{(2n)!}{n!2^n}\sqrt{\pi}\,, \tag{A.24}$$

$$\Gamma(\epsilon - n) = \frac{(-1)^n}{n!}\left(\frac{1}{\epsilon} - \gamma + \sum_{k=1}^{n}\frac{1}{k} + \mathcal{O}(\epsilon)\right), \tag{A.25}$$

where n is a non-negative integer, $\epsilon \ll 1$, and γ is the Euler-Mascheroni constant, defined by

$$\gamma = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k} - \log n\right) \approx 0.577\,215\,665\,. \tag{A.26}$$

Notice that in DR, where all loop integrals are given by eq. (A.20), divergences make their appearance only through identity (A.25). This means that every diagram in DR can be split into a finite and an infinite part, where the infinite part is proportional to $\frac{1}{\epsilon}$, and this makes tracking of infinities in calculations using the DR scheme much simpler.

Appendix **B**

Passarino-Veltman Functions

In this appendix we will present a general formula for translating each *scalar* Passarino-Veltman function (PV function for short) into an integral over Feynman parameters (see appendix A). PV functions were first defined in ref. [92]. For the methodology of PV reduction, see ref. [162]. See also refs. [163, 164] for some interesting recursive relations for the PV functions. We consider only scalar integrals since modern computer algorithms can automatically reduce all PV functions into scalar ones (when, of course, this reduction is permissible). Therefore, the final result will most likely contain only scalar PV functions, so it will prove useful to have exact integral expressions for clarity or in order to solve these integrals in terms of analytic functions, if this is possible.

B.1 General definitions

We start with the general definition of the PV integral:

$$\mathcal{T}^N_{\mu_1\ldots\mu_p}(p_1,\ldots,p_{N-1};m_0,\ldots,m_{N-1}) = \frac{(2\pi\mu)^{4-d}}{i\pi^2}\int d^dq\,\frac{q_{\mu_1}\cdots q_{\mu_p}}{D_0\,D_1\cdots D_{N-1}}\,, \tag{B.1}$$

where d is the dimension of space-time, μ is a parameter with mass dimension $+1$, known as ‘t Hooft's renormalisation scale, and

$$D_0 = q^2 - m_0^2\,; \qquad D_i = (q+p_i)^2 - m_i^2\,, \quad i = 1,\ldots,N-1\,. \tag{B.2}$$

In these equations the index N stands for an N-point integral, i.e. the momentum integral coming from a Feynman loop diagram with N vertices attached to the loop (and therefore with N propagators in the loop). The nomenclature is as follows: starting with $N=1$, $\mathcal{T}^N$ stands for the Nth letter of the alphabet, e.g. $\mathcal{T}^1 = A$, $\mathcal{T}^2 = B$, and so on.

Now for the scalar integrals, where the integrand's numerator is equal to one, the definition (B.1) is simplified to

$$\mathcal{T}_0^N = \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int d^d q \frac{1}{D_0\, D_1 \cdots D_{N-1}}\,. \tag{B.3}$$

Using the formulae and algorithmic procedure of appendix A, we write the denominator as

$$\frac{1}{D_0\, D_1 \cdots D_{N-1}} = \int d\widetilde{F}_N \big(l_N^2 - \Delta_N\big)^{-N}\,, \tag{B.4}$$

where

$$l_N = q + \sum_{i=1}^{N-1} x_i\, p_i\,, \tag{B.5}$$

$$\begin{aligned} \Delta_N = &- \sum_{i=1}^{N-1} x_i(1-x_i)\, p_i^2 + \sum_{j\neq i} \sum_{i=1}^{N-1} (x_i x_j\, p_i \cdot p_j) \\ &+ m_0^2 \left(1 - \sum_{i=1}^{N-1} x_i \right) + \sum_{i=1}^{N-1} x_i\, m_i^2\,, \end{aligned} \tag{B.6}$$

and the reduced Feynman integration measure is defined by

$$\begin{aligned} \int d\widetilde{F}_N\, f(x_i) = \Gamma(N) \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \cdots \\ \cdots \int_0^{1-x_1-\cdots-x_{N-2}} dx_{N-1}\, f(x_i)\, \big|_{x_N = 1-x_1-\cdots-x_{N-1}}\,. \end{aligned} \tag{B.7}$$

The general formula for the loop-momentum integrals *without* momentum in the numerator simplifies to

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2-\Delta)^N} = i(-1)^N \frac{\Gamma(N-\frac{1}{2}d)}{(4\pi)^{d/2}\Gamma(N)} \left(\frac{1}{\Delta}\right)^{N-d/2}. \tag{B.8}$$

Using all of the above, we conclude that

$$\mathcal{T}_0^N = (-1)^N \big(4\pi\mu^2\big)^{\epsilon/2} \frac{\Gamma(N-\frac{1}{2}d)}{\Gamma(N)} \int d\widetilde{F}_N \left(\frac{1}{\Delta_N}\right)^{N-d/2}. \tag{B.9}$$

It is clear that infinities come only from the first two scalar PV functions, namely A_0 and B_0. From the identity

$$\Gamma(\epsilon/2 - n) = \frac{(-1)^n}{n!} \left(\frac{2}{\epsilon} - \gamma + \sum_{k=1}^{n} \frac{1}{k} + \mathcal{O}(\epsilon) \right), \quad \epsilon \ll 1\,, \tag{B.10}$$

where γ is the Euler-Mascheroni constant, defined in eq. (A.26), we have that

$$B_0\big(q^2, m_1^2, m_2^2\big)\Big|_{\text{infinite}} = \frac{2}{\epsilon}\,, \tag{B.11}$$

$$A_0\big(m^2\big)\Big|_{\text{infinite}} = \frac{2}{\epsilon} m^2\,. \tag{B.12}$$

In fact, A_0 can always be re-expressed in terms of the B_0 function, as

$$A_0\big(m^2\big) = m^2\big[B_0\big(0, m^2, m^2\big) + 1\big]\,. \tag{B.13}$$

The infinite parts are always accompanied by the same constants, so it is sometimes useful to define a quantity

$$E \equiv \frac{2}{\epsilon} - \gamma + \log(4\pi)\,, \tag{B.14}$$

as we did in the calculation in chapter 2.

B.2 Useful formulae

Let us present here two PV reduction formulae that proved useful (in addition to the standard reduction of tensor to scalar integrals) in our attempts to analytically demonstrate the gauge invariance of the $h \to \gamma\gamma$ and $h \to Z\gamma$ matrix elements in chapters 2 and 3, respectively. The first reduces the B_1 function with light-like external momenta, $q^2 = 0$, to the scalar B function and its derivative:

$$B_1(0, a, b) = \tfrac{1}{2}(b-a)B_0'(0, a, b) - \tfrac{1}{2}B_0(0, a, b)\,, \tag{B.15}$$

where $B_0'(x, a, b) \equiv (\partial/\partial t)B_0(t, a, b)\big|_{t=x}$, and the other identity 'symmetrises' the arguments on the B_0 function with light-like external momenta, $q^2 = 0$,

$$B_0(0, a, b) = 1 + \frac{1}{b-a}\left[bB_0(0, b, b) - aB_0(0, a, a)\right]. \tag{B.16}$$

These two identities for the special values $a = m_W^2$, $b = \xi m_W^2$ are used in our calculations in chapters 2 and 3.

To conclude this appendix, we give some formulae that express scalar PV functions to analytic functions. We give a detailed example of how to solve such integrals in appendix C, were we solve a frequently encountered C_0 function in terms of analytic functions. For the

B_0 function, some useful identities are

$$B_0(0,a,a) = E - \log\bigl(a/\mu^2\bigr)\,, \tag{B.17}$$
$$B_0(a,a,a) = E + 2 - \frac{\pi}{\sqrt{3}} - \log\bigl(a/\mu^2\bigr)\,, \tag{B.18}$$
$$B_0(0,a,b) = E + 1 - \frac{1}{a-b}\bigl[a\log\bigl(a/\mu^2\bigr) - b\log\bigl(b/\mu^2\bigr)\bigr]\,, \tag{B.19}$$
$$B_0(a,b,b) = E + 2 - \log\bigl(b/\mu^2\bigr) - 2\sqrt{r-1}\,\arctan\Bigl(\tfrac{1}{\sqrt{r-1}}\Bigr)\,, \quad r \equiv \tfrac{4b}{a}\,, \tag{B.20}$$
$$B_0'(0,a,a) = \frac{1}{6a}\,, \tag{B.21}$$
$$B_0'(a,a,a) = \frac{1}{a}\left(\frac{2\pi}{3\sqrt{3}} - 1\right), \tag{B.22}$$
$$B_0'(0,a,b) = \frac{1}{(a-b)^3}\left[\frac{1}{2}\bigl(a^2-b^2\bigr) - ab\log\Bigl(\frac{a}{b}\Bigr)\right], \tag{B.23}$$
$$B_0'(a,b,b) = \frac{1}{a}\left[\frac{r}{\sqrt{r-1}}\arctan\Bigl(\tfrac{1}{\sqrt{r-1}}\Bigr) - 1\right], \quad r \equiv \tfrac{4b}{a}\,. \tag{B.24}$$

Finally, let us give here the result for a commonly encountered C_0 PV function which also appears in the $h \to \gamma\gamma$ decay (chapter 2), namely

$$C_0(0,0,a;b,b,b) = -\frac{2}{a}f(r)\,, \tag{B.25}$$

where $r = \frac{4b}{a}$, and $f(r)$ is given by

$$f(r) = \begin{cases} \arcsin^2\Bigl(\frac{1}{\sqrt{r}}\Bigr)\,, & r \geq 1\,, \\ -\frac{1}{4}\Bigl[\log\Bigl(\frac{1+\sqrt{1-r}}{1-\sqrt{1-r}}\Bigr) - i\pi\Bigr]^2\,, & r \leq 1\,. \end{cases} \tag{B.26}$$

For details about solving this non-trivial PV function in terms of analytic functions, see appendix C.

Appendix C

Dilogarithms and Analytic Functions

In this appendix, we give a detailed example of how to solve the integrals appearing in a diagrammatic calculation, using a non-trivial scalar PV function as a showcase for the analysis.[1,2] The result for $h \to \gamma\gamma$ in chapter 2 contains the following linear combination of polylogarithms of order 2 (also known as dilogarithms or Spence's functions):

$$f(x) = \tfrac{1}{2}\Big[\mathrm{Li}_2\Big(\tfrac{2}{1+\sqrt{1-x}}\Big) + \mathrm{Li}_2\Big(\tfrac{2}{1-\sqrt{1-x}}\Big)\Big]\,. \tag{C.1}$$

The function $f(x)$ enters in our calculation through the scalar PV function

$$C_0(0,0,a;b,b,b) = -\frac{2}{a} f(x)\,, \tag{C.2}$$

where $x = 4b/a$.

A dilogarithm is defined by the integral

$$\mathrm{Li}_2(x) = \int_x^0 dt \frac{\log(1-t)}{t}\,, \tag{C.3}$$

or, by re-scaling $t \to xt$ and flipping the limits of integration,

$$\mathrm{Li}_2(x) = -\int_0^1 dt \frac{\log(1-xt)}{t}\,. \tag{C.4}$$

[1]The calculations in this appendix were performed in collaboration with Kristaq Suxho.
[2]In practice, calculating PV functions in terms of analytic functions can be a very laborious task (if such an analytic function even exists). The reader is referred to `Package-X` [104, 105], a `Mathematica` package which contains an extensive library dedicated to the translation of PV functions in terms of analytic functions.

Now, using the definition (C.4) we write $f(x)$ as

$$\begin{aligned} f(x) &= \tfrac{1}{2}\Big[\mathrm{Li}_2\Big(\tfrac{2}{1+\sqrt{1-x}}\Big) + \mathrm{Li}_2\Big(\tfrac{2}{1-\sqrt{1-x}}\Big)\Big] \\ &= -\int_0^1 \frac{dt}{2t}\Big[\log\Big(1-\tfrac{2t}{1+\sqrt{1-x}}\Big) + \log\Big(1-\tfrac{2t}{1-\sqrt{1-x}}\Big)\Big] \\ &= -\int_0^1 \frac{dt}{2t}\log\Big(1-\tfrac{4t(1-t)}{x}\Big)\,. \end{aligned} \tag{C.5}$$

In this appendix we show how to calculate the integral (C.5) by following the methods of ref. [165]. We start by differentiating $f(x)$ with respect to x. We have:

$$\begin{aligned} f'(x) &= \frac{df(x)}{dx} \\ &= \frac{-1}{2x}\int_0^1 dt\frac{1-t}{t^2-t+\frac{x}{4}} \\ &= \frac{-1}{4x}\int_{-1/2}^{1/2} dy\frac{1-2y}{y^2+\frac{x-1}{4}}\,. \end{aligned} \tag{C.6}$$

To get the last line we defined a new variable $y = t - 1/2$. Now the integral (C.6) can be further simplified by making use of its reflection symmetry $y \to -y$, to

$$f'(x) = \frac{-1}{4x}\int_{-1/2}^{1/2} dy\frac{1}{y^2+\frac{x-1}{4}}\,. \tag{C.7}$$

The integral (C.7) can be evaluated in terms of analytic functions.

We begin by considering the region $x > 1$. There we can define a real parameter A as

$$A = \tfrac{1}{2}\sqrt{x-1}\,, \quad \text{for} \quad x > 1\,. \tag{C.8}$$

The integral that occurs is well-known:

$$\begin{aligned} f'(x) &= \frac{-1}{4x}\int_{-1/2}^{1/2}\frac{dy}{y^2+A^2} \\ &= \frac{-1}{4xA}\arctan\big(\tfrac{y}{A}\big)\Big|_{y=-1/2}^{1/2} \\ &= \frac{-1}{x\sqrt{x-1}}\arctan\Big(\tfrac{1}{\sqrt{x-1}}\Big) \\ &= \frac{-1}{x\sqrt{x-1}}\arcsin\Big(\tfrac{1}{\sqrt{x}}\Big)\,. \end{aligned} \tag{C.9}$$

Now let

$$u = \arcsin\Big(\tfrac{1}{\sqrt{x}}\Big)\,, \tag{C.10}$$

$$du = \frac{-dx}{2x\sqrt{x-1}}\,. \tag{C.11}$$

Then,

$$f(x) = \int dx\, f'(x) = u^2 + C = \arcsin^2\Big(\tfrac{1}{\sqrt{x}}\Big) + C\,, \tag{C.12}$$

where C is the constant of integration. Since $x > 1$, we fix C by taking the limit $x \to \infty$, or

$$\lim_{x\to\infty} \arcsin^2\Big(\tfrac{1}{\sqrt{x}}\Big) + C = \lim_{x\to\infty} \frac{-1}{2}\int_0^1 \frac{dt}{t}\log\Big(1 - \tfrac{4t(1-t)}{x}\Big)\,,$$

which results in

$$C = 0\,. \tag{C.13}$$

Consider now the region $x < 1$. Here we define a real parameter B as

$$B = \tfrac{1}{2}\sqrt{1-x}\,, \quad \text{for} \quad x < 1\,, \tag{C.14}$$

and, once again, the integral is well-known:

$$\begin{aligned} f'(x) &= \frac{-1}{4x}\int_{-1/2}^{1/2} \frac{dy}{y^2 - B^2} \\ &= \frac{-1}{8xB}\big[\log\big(1 - \tfrac{y}{B}\big) - \log\big(1 + \tfrac{y}{B}\big)\big]\Big|_{y=-1/2}^{1/2} \\ &= \frac{-1}{4xB}\Big[\log\Big(1 - \tfrac{1}{\sqrt{1-x}}\Big) - \log\Big(1 + \tfrac{1}{\sqrt{1-x}}\Big)\Big] \\ &= \frac{1}{2x\sqrt{1-x}}\Big[-\log\Big(\tfrac{1-\sqrt{1-x}}{\sqrt{1-x}}\Big) + \log\Big(\tfrac{1+\sqrt{1-x}}{\sqrt{1-x}}\Big) - i\pi\Big] \\ &= \frac{1}{2x\sqrt{1-x}}\Big[\log\Big(\tfrac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\Big) - i\pi\Big]\,, \end{aligned} \tag{C.15}$$

where we used the identity

$$\log(a) = \log(|a|) + i\arg(a)\,, \tag{C.16}$$

where by log and arg we denote the principal value and the principal argument of the complex logarithm, respectively, in the term

$$\log\Big(1 - \tfrac{1}{\sqrt{1-x}}\Big) = \log\Big(\tfrac{1}{\sqrt{1-x}} - 1\Big) - i\pi\,, \tag{C.17}$$

which is true for $0 < x < 1$. Note that for $x < 0$ all logarithms have positive arguments, and the integral takes the form:

$$f'(x) = \frac{1}{2x\sqrt{1-x}} \log\left(\tfrac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right) , \quad \text{for} \quad x < 0 \,. \tag{C.18}$$

That region can be combined with the $0 < x < 1$ region in the final expression.

Now, for $0 < x < 1$, let

$$u = \log\left(\tfrac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right) - i\pi \,, \tag{C.19}$$

$$du = \frac{-dx}{x\sqrt{1-x}} \,. \tag{C.20}$$

Then,

$$f(x) = \int dx\, f'(x) = -\tfrac{u^2}{4} + C = -\tfrac{1}{4}\left[\log\left(\tfrac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right) - i\pi\right]^2 + C \,, \tag{C.21}$$

where C is the constant of integration. We fix C by requiring $f(x)$ to be a continuous function at $x = 1$, which results in

$$C = 0 \,. \tag{C.22}$$

Therefore, we conclude that

$$f(x) = \begin{cases} \arcsin^2\left(\tfrac{1}{\sqrt{x}}\right) , & x \geq 1 \,, \\ -\tfrac{1}{4}\left[\log\left(\tfrac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right) - i\pi\right]^2 , & x \leq 1 \,. \end{cases} \tag{C.23}$$

The interested reader is referred to appendix D of ref. [166], where the authors provide a plethora of loop integrals with three propagators expressed in terms of analytic functions.

APPENDIX D

Global fits in SMEFT

The correct approach to place bounds on the Wilson coefficients of the SMEFT is to make a global fit of various processes (for global fit analyses in the leading order SMEFT see for example refs. [76–78, 167–169]). A major problem that can arise in global fit scenarios is the occurrence of flat directions in the space of the Wilson coefficients, leaving the constraints along these directions lacking. A recent paper, ref. [170], describes a method used to solve this issue through the use of principle-component analysis. Though a global fit analysis is beyond the scope of this thesis, we use this machinery to perform the global fit for a small subset of the results derived in this work. The reader can refer to ref. [170] for the technical details of the method. In particular, we use our results for the Higgs decays derived in part I of this thesis together with the formula for the tree-level Peskin-Takeuchi S parameter [97] at the dimension 6 SMEFT.[1]

D.1 Relevant formulae

For the analysis in this appendix we use our result for the $h \to \gamma\gamma$ and $h \to Z\gamma$ decays at one-loop for the dimension 6 SMEFT. For simplicity, we use only the dominant Wilson coefficients in these processes that appear already at the tree-level, namely the $C^{\varphi W}$, $C^{\varphi B}$ and $C^{\varphi WB}$ Wilson coefficients. Setting the EFT scale at $\Lambda = 1\,\text{TeV}$ and the renormalisation scale at the W boson mass, $\mu = m_W$, the numerical ratios of the SMEFT vs the SM contributions become:

$$\begin{aligned} \delta R_{h\to\gamma\gamma} &= -48.04\, C^{\varphi B} - 14.29\, C^{\varphi W} + 26.17\, C^{\varphi WB}\,, \\ \delta R_{h\to Z\gamma} &= +14.99\, C^{\varphi B} - 14.88\, C^{\varphi W} + 9.44\, C^{\varphi WB}\,. \end{aligned} \tag{D.1}$$

[1]This analysis was performed in collaboration with Konstantinos Mantzaropoulos.

The above quantities are calculated in the $\{G_F, m_W, m_Z\}$ input scheme. The experimental values for this input scheme are given by

$$\begin{aligned} G_F &= 1.1663787(6) \times 10^{-5}\,\text{GeV}^{-2}\,, \\ m_W &= 80.385(15)\,\text{GeV}\,, \\ m_Z &= 91.1876(21)\,\text{GeV}\,. \end{aligned} \tag{D.2}$$

We also use the tree-level result for the EFT contributions to the Peskin-Takeuchi parameter S at dimension 6, which are simply given by

$$\Delta S = 2\sqrt{2}\frac{s\,c}{G_F\,a_{em}}\,C^{\varphi WB}\,, \tag{D.3}$$

where, since we work at dimension 6 accuracy, $c = m_W/m_Z$, $s = \sqrt{1-c^2}$ and $a_{em} = e^2/(4\pi)$ with $e = gg'/\sqrt{g^2+g'^2}$. Expressed numerically in the $\{G_F, m_W, m_Z\}$ scheme[2] with $\Lambda = 1\,\text{TeV}$, the parameter ΔS reads:

$$\Delta S = 13.35\,C^{\varphi WB}\,. \tag{D.4}$$

We collect all of the above results together in the vector

$$O_{\text{SMEFT}} = (1+\delta R_{h\to\gamma\gamma},\, 1+\delta R_{h\to Z\gamma},\, \Delta S)\,. \tag{D.5}$$

The vector containing the experimental results used in this analysis for the three observables $1+\delta R_{h\to\gamma\gamma}$, $1+\delta R_{h\to Z\gamma}$ and ΔS, respectively, is given by

$$O_{\text{exp}} = (1.10,\, 2.05,\, 0.02)\,, \tag{D.6}$$

together with the uncertainties

$$\sigma_{\text{exp}} = (0.10,\, 0.95,\, 0.10)\,. \tag{D.7}$$

We take the observables to be uncorrelated, that is

$$\rho_{\text{exp}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\,, \tag{D.8}$$

[2]For the relation between couplings and the input parameters, see appendix E.

and then the $\hat{\sigma}^2$ quantity from ref. [170] reads:

$$\hat{\sigma}^2 = \begin{pmatrix} 0.01 & 0 & 0 \\ 0 & 0.90 & 0 \\ 0 & 0 & 0.01 \end{pmatrix} . \tag{D.9}$$

Finally, we are able to derive the function χ^2, whose minimisation will give as the best fit for the Wilsons. This function is defined as

$$\chi^2 = (O_{\text{SMEFT}} - O_{\text{exp}}) \cdot \left(\hat{\sigma}^2\right)^{-1} \cdot (O_{\text{SMEFT}} - O_{\text{exp}})^{\text{T}} . \tag{D.10}$$

D.2 Results

After performing the principal component analysis, we find that the best fit Wilsons, together with the uncertainties, are given by

$$\begin{aligned} C^{\varphi B} &= 0.015 \pm 0.030 \,, \\ C^{\varphi W} &= -0.055 \pm 0.100 \,, \\ C^{\varphi WB} &= 0.001 \pm 0.015 \,. \end{aligned} \tag{D.11}$$

Comparing with the upper bounds from chapter 2 for the ratio $R_{h\to\gamma\gamma}$,

$$\frac{|C^{\varphi B}|}{\Lambda^2} \lesssim \frac{0.002}{(1\,\text{TeV})^2} \,, \qquad \frac{|C^{\varphi W}|}{\Lambda^2} \lesssim \frac{0.007}{(1\,\text{TeV})^2} \,, \qquad \frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim \frac{0.004}{(1\,\text{TeV})^2} \,, \tag{D.12}$$

and the (updated for the experimental value used here) bounds for the ratio $R_{h\to Z\gamma}$ from chapter 3,

$$\frac{|C^{\varphi B}|}{\Lambda^2} \lesssim \frac{0.137}{(1\,\text{TeV})^2} \,, \qquad \frac{|C^{\varphi W}|}{\Lambda^2} \lesssim \frac{0.138}{(1\,\text{TeV})^2} \,, \qquad \frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim \frac{0.217}{(1\,\text{TeV})^2} \,, \tag{D.13}$$

which were placed by considering only one Wilson at a time, we see that in the first case for $h \to \gamma\gamma$ the bounds are sometimes even stricter than the optimal values suggested by the global fit, while in the case of $h \to Z\gamma$ the bounds are much weaker than the optimal values by one or even two orders or magnitude.

This change in the coefficient values is also driven by the fact that the Wilson coefficients in this simple example are highly correlated. To get a quantitative view on this phenomenon, we have derived the correlation matrix for the vector $\{C^{\varphi B}, C^{\varphi W}, C^{\varphi WB}\}$ of the Wilson

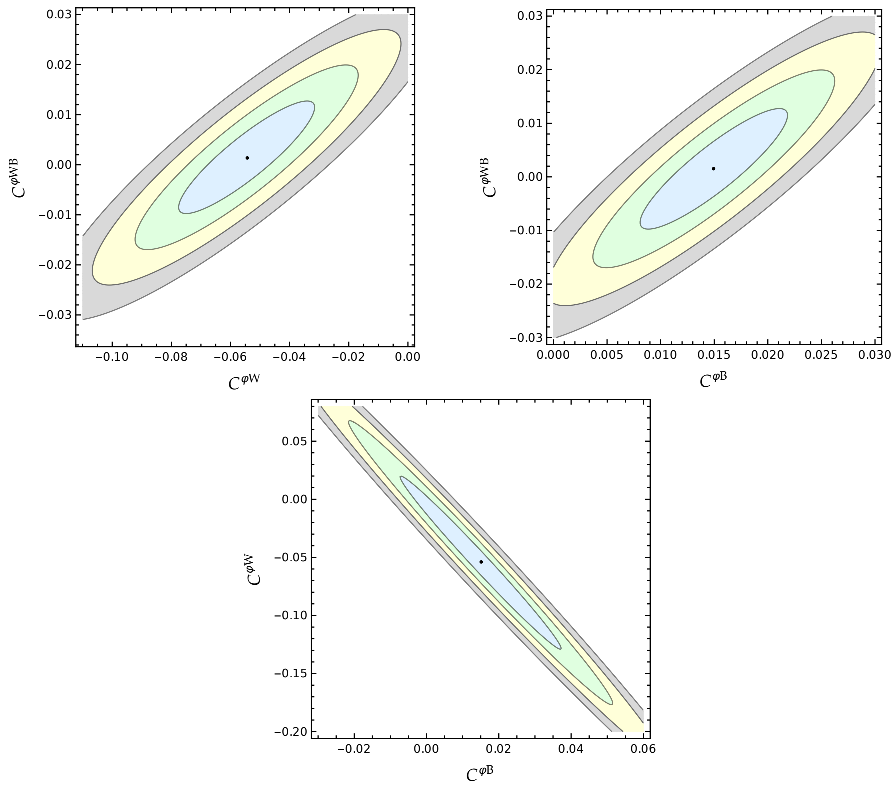


Figure D.1: From top-left to bottom: contour plots in the ($C^{\varphi W}$–$C^{\varphi WB}$), ($C^{\varphi B}$–$C^{\varphi WB}$) and ($C^{\varphi B}$–$C^{\varphi W}$) plane, with $C^{\varphi B}$, $C^{\varphi W}$ and $C^{\varphi WB}$, respectively, set to the best fit value. The dot depicts the minimising value for the χ^2 function and the ellipses correspond to the 1σ–4σ regions.

coefficients, which is given by

$$\rho_{\text{coeff}} = \begin{pmatrix} 1.00 & -0.95 & 0.14 \\ -0.95 & 1.00 & 0.14 \\ 0.14 & 0.14 & 1.00 \end{pmatrix} . \tag{D.14}$$

This matrix suggests a very strong anti-correlation between the Wilsons $C^{\varphi B}$ and $C^{\varphi W}$ is clearly radically different than the unit matrix, in which case a one-at-a-time fit for the Wilson coefficients would be valid.

For completeness, we present also the contour plots seen in figure D.1. For these plots, we keep one Wilson coefficient set to the best fit value, given in eqs. (D.11), and we plot the sigma contours with respect to the two remaining Wilson coefficients.

Appendix **E**

Input schemes in SMEFT

In this appendix we present the expressions for the couplings of the gauge sector of the SMEFT in terms of measurable quantities. Several choices for the input parameters are considered and we briefly comment on the use of the input schemes in real-world applications.[1] The results are given explicitly for the dimension 6 SMEFT and we comment on higher-order generalisations. In what follows, the Higgs mass is always considered as an input parameter and therefore we won't add it explicitly in the input parameters list to simplify the notation.[2]

E.1 The $\{G_F, m_W, m_Z\}$ scheme

This scheme is used in our analysis of the $h \to Z\gamma$ decay in chapter 3 and is also used to re-express the results for the $h \to \gamma\gamma$ calculation in the same chapter. Starting with the VEV, which is defined through muon decay (see chapter 2 for details), we solve for $\bar{g}$ from the m_W definition. We have:

$$v = \frac{1}{2^{1/4}\sqrt{G_F}}\left[1 + \frac{1}{2\sqrt{2}G_F}\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2}\right)\right], \tag{E.1}$$

$$\bar{g} = 2^{5/4} M_W \sqrt{G_F}\left[1 - \frac{1}{2\sqrt{2}G_F}\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2}\right)\right]. \tag{E.2}$$

[1] See also ref. [121] for additional details about SMEFT input schemes.
[2] The calculations in this appendix were performed in collaboration with Kristaq Suxho.

After that we use the m_Z definition and solve for $\bar{g}'$ to get

$$\bar{g}' = 2^{5/4}\sqrt{M_Z^2 - M_W^2}\sqrt{G_F}\Bigg[1 - \frac{1}{2\sqrt{2}G_F}\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2}\right) - \frac{M_Z^2}{4\sqrt{2}G_F(M_Z^2 - M_W^2)}\left(\frac{C^{\varphi D}}{\Lambda^2} + 4\frac{M_W}{M_Z}\sqrt{1 - \frac{M_W^2}{M_Z^2}}\frac{C^{\varphi WB}}{\Lambda^2}\right)\Bigg]. \tag{E.3}$$

Finally, starting from the definition of the Higgs mass and using the formula we derived for $v = f(G_F)$ we have:

$$\lambda = \sqrt{2}G_F m_h^2\Bigg\{1 - \frac{1}{\sqrt{2}G_F}\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2}\right) + \frac{1}{2G_F^2 m_h^2}\left[3\frac{C^{\varphi}}{\Lambda^2} - 2\sqrt{2}G_F m_h^2\left(\frac{C^{\varphi\Box}}{\Lambda^2} - \frac{1}{4}\frac{C^{\varphi D}}{\Lambda^2}\right)\right]\Bigg\}. \tag{E.4}$$

Since λ isn't needed in any other derivation, we won't mention it again until the last section where we need to redefine it since we also redefine the VEV there as well.

E.2 The $\{G_F, m_W, \alpha_{em}\}$ scheme

From the previous scheme we already have the expressions for

$$v = f(G_F)\,, \qquad \bar{g} = f(m_W, G_F)\,. \tag{E.5}$$

After that, we use the definition of $\bar{e} = \sqrt{4\pi\alpha_{em}}$ and we solve for $\bar{g}'$. We obtain:

$$\bar{g}' = \frac{2^{5/4}m_W\sqrt{\pi\alpha_{em}G_F}}{\sqrt{2^{1/2}G_F m_W^2 - \pi\alpha_{em}}}\Bigg\{1 + \frac{1}{2G_F\left(\sqrt{2}G_F m_W^2 - \pi\alpha_{em}\right)}\times \left[\frac{\pi\alpha_{em}}{\sqrt{2}}\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2} + \frac{C_{22}^{\varphi l(3)}}{\Lambda^2} - \frac{C_{1221}^{ll}}{\Lambda^2}\right) + \sqrt{2\pi\alpha_{em}}\sqrt{2^{1/2}G_F m_W^2 - \pi\alpha_{em}}\frac{C^{\varphi WB}}{\Lambda^2}\right]\Bigg\}. \tag{E.6}$$

E.3 The $\{G_F, m_Z, \alpha_{em}\}$ scheme

The difference here is that we *don't* make use of the m_W definition. Instead we try to solve $\bar{g}$ and $\bar{g}'$ in terms of G_F and α_{em}. The VEV is once more defined through G_F. For $\bar{g}$ and $\bar{g}'$

we have:

$$\begin{aligned}\bar{g} =&2^{3/4} m_Z \sqrt{G_F} \sqrt{1+\sqrt{1-\frac{4\pi\alpha_{em}}{\sqrt{2}G_F m_Z^2}}} \times \\ &\Bigg\{1 - \frac{1}{4\sqrt{2}G_F}\left[1+\frac{1}{\sqrt{1-\frac{4\pi\alpha_{em}}{\sqrt{2}G_F m_Z^2}}}\right]\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2}+\frac{C_{22}^{\varphi l(3)}}{\Lambda^2}-\frac{C_{1221}^{ll}}{\Lambda^2}\right) \\ &- \frac{1}{8\sqrt{2}G_F}\left[1+\frac{1}{\sqrt{1-\frac{4\pi\alpha_{em}}{\sqrt{2}G_F m_Z^2}}}\right]\frac{C^{\varphi D}}{\Lambda^2} - \frac{1}{2^{3/4}G_F^{3/2}m_Z}\sqrt{\frac{\pi\alpha_{em}}{1-\frac{4\pi\alpha_{em}}{\sqrt{2}G_F m_Z^2}}}\frac{C^{\varphi WB}}{\Lambda^2}\Bigg\}, \end{aligned} \tag{E.7}$$

$$\begin{aligned}\bar{g}' =&2^{3/4} m_Z \sqrt{G_F} \sqrt{1-\sqrt{1-\frac{4\pi\alpha_{em}}{\sqrt{2}G_F m_Z^2}}} \times \\ &\Bigg\{1 - \frac{1}{4\sqrt{2}G_F}\left[1-\frac{1}{\sqrt{1-\frac{4\pi\alpha_{em}}{\sqrt{2}G_F m_Z^2}}}\right]\left(\frac{C_{11}^{\varphi l(3)}}{\Lambda^2}+\frac{C_{22}^{\varphi l(3)}}{\Lambda^2}-\frac{C_{1221}^{ll}}{\Lambda^2}\right) \\ &- \frac{1}{8\sqrt{2}G_F}\left[1-\frac{1}{\sqrt{1-\frac{4\pi\alpha_{em}}{\sqrt{2}G_F m_Z^2}}}\right]\frac{C^{\varphi D}}{\Lambda^2} + \frac{1}{2^{3/4}G_F^{3/2}m_Z}\sqrt{\frac{\pi\alpha_{em}}{1-\frac{4\pi\alpha_{em}}{\sqrt{2}G_F m_Z^2}}}\frac{C^{\varphi WB}}{\Lambda^2}\Bigg\}. \end{aligned} \tag{E.8}$$

E.4 The $\{\alpha_{em}, m_W, m_Z\}$ scheme

Until now we used the Fermi constant to express the VEV. In this scheme we will not make use of G_F and, therefore, we will redefine both the VEV and the quartic Higgs coupling λ. The results are:

$$v = \frac{m_W}{m_Z}\sqrt{\frac{m_Z^2-m_W^2}{\pi\alpha_{em}}}\left\{1-\frac{1}{4\pi\alpha_{em}}\frac{m_W^4}{m_Z^2}\frac{C^{\varphi D}}{\Lambda^2}-\frac{1}{\pi\alpha_{em}}\frac{m_W^3}{m_Z^2}\sqrt{m_Z^2-m_W^2}\frac{C^{\varphi WB}}{\Lambda^2}\right\}, \tag{E.9}$$

$$\bar{g} = 2m_Z\sqrt{\frac{\pi\alpha_{em}}{m_Z^2-m_W^2}}\left\{1+\frac{1}{4\pi\alpha_{em}}\frac{m_W^4}{m_Z^2}\frac{C^{\varphi D}}{\Lambda^2}+\frac{1}{\pi\alpha_{em}}\frac{m_W^3}{m_Z^2}\sqrt{m_Z^2-m_W^2}\frac{C^{\varphi WB}}{\Lambda^2}\right\}, \tag{E.10}$$

$$\bar{g}' = \frac{2m_Z}{m_W}\sqrt{\pi\alpha_{em}}\left\{1-\frac{1}{4\pi\alpha_{em}}\frac{m_W^2}{m_Z^2}\left(m_Z^2-m_W^2\right)\frac{C^{\varphi D}}{\Lambda^2}\right\}, \tag{E.11}$$

$$\lambda = \frac{\pi\alpha_{em} m_h^2 m_Z^2}{m_W^2(m_Z^2 - m_W^2)} \left\{ 1 + \frac{3m_W^4\left(m_Z^2 - m_W^2\right)^2}{\pi^2\alpha_{em}^2 m_h^2 m_Z^4} \frac{C^{\varphi}}{\Lambda^2} - \frac{2}{\pi\alpha_{em}} \frac{m_W^2}{m_Z^2}\left(m_Z^2 - m_W^2\right) \frac{C^{\varphi\Box}}{\Lambda^2} \right.$$
$$\left. + \frac{m_W^2}{2\pi\alpha_{em}} \frac{C^{\varphi D}}{\Lambda^2} + \frac{2}{\pi\alpha_{em}} \frac{m_W^3}{m_Z^2} \sqrt{m_Z^2 - m_W^2} \frac{C^{\varphi WB}}{\Lambda^2} \right\}. \tag{E.12}$$

As expected, each choice for the input scheme is valid and the physics isn't affected by this choice. In practice, though, in the SMEFT framework different Wilson coefficients will be introduced through different schemes, and it is useful to have the EFT definitions for the masses identified with physical parameters, making the schemes $\{G_F, m_W, m_Z\}$ and $\{a_{em}, m_W, m_Z\}$ more convenient. By identifying the masses as input parameters, we avoid introducing explicit Wilson coefficients in the denominators of Feynman propagators (or, equivalently, as arguments in the Passarino-Veltman functions), which would complicate the calculations further since then we would have to expand these expressions to the relevant EFT order.[3]

E.5 Input schemes at higher orders

The same methodology can be used to derive the input schemes up to any desired order in the EFT expansion. The expressions for the translation between couplings and measurable quantities in this case will, of course, be very lengthy. We have derived the expressions for the two most useful input schemes, namely the $\{G_F, m_W, m_Z\}$ and $\{a_{em}, m_W, m_Z\}$ schemes, for the dimension 8 case (including the interference of dual dimension 6 insertions). These expressions are used in the new iteration of the `smeftFR` code [127] and are included in the open source code.

[3]There are exceptions to this rule, however. For example, in the $h \to \gamma\gamma$ amplitude in chapter 2, the Z boson mass appears only through the $Z\gamma$-mixing contributions. This mixing enters the amplitude multiplied overall by the effective $hZ\gamma$-vertex. Therefore, at the dimension 6 SMEFT considered there, the $Z\gamma$-mixing should be calculated in the SM and there are no EFT corrections in the Z mass. That made it possible to express the $h \to \gamma\gamma$ amplitude in chapter 2 in the $\{a_{em}, G_F, m_W\}$ scheme without unnecessary complications. This amplitude was also re-expressed in the $\{G_F, m_W, m_Z\}$ scheme in chapter 3.

Appendix F

Exact Reformulation of SMEFT

In this appendix we construct the effective Lagrangian for the Standard Model (SM) up to an arbitrary (but fixed) order in the effective field theory (EFT) expansion. We limit ourselves in the bosonic sector of the theory which presents a high level of complexity due to the mixing of the neutral electroweak gauge bosons. After constructing a non-redundant basis for every distinct order of the SMEFT, one should make the necessary manipulations to derive the physical mass basis of the theory after the spontaneous electroweak symmetry breaking. The most important operators for this analysis are those that affect the bilinears of the theory. In this appendix we show exactly which operators are capable of affecting the bilinears, and then we proceed to systematically demonstrate the derivation of the mass basis of the theory and to derive all the results up-to any order in the EFT expansion.

F.1 Introduction

The analysis of a generic EFT in R_ξ-gauges up to any arbitrary fixed order in the EFT expansion has already been accomplished in ref. [171].[1] There, the bilinears were derived and emphasis was given in the gauge-fixing of the theory. Here, we perform a similar analysis with the difference that we focus solely on the SMEFT and we derive the exact expressions for the field redefinitions, masses, diagonalisation of the EW gauge sector etc up to any possible order in the SMEFT expansion, creating a framework by which on can derive the Feynman rules of the theory [127].

Following the analysis of section 2 of ref. [171] we arrive to the conclusion that the only operators that affect the bilinears up to an arbitrary but fixed order N in the EFT expansion,

[1]The reader is also referred to the GeoSMEFT [172] formulation, where similar conclusions are derived.

i.e. up to order $1/\Lambda^N$, are operators of the type:

$$\varphi^n, \quad \varphi^n D^2, \quad \varphi^n X^2 \tag{F.1}$$

where φ is the Higgs field, D is the covariant derivative and X stands for a field strength tensor.[2]

F.2 Scalar potential and the Higgs mass

Due to hypercharge restrictions the operators should have the same number of φ and $\varphi^\dagger$ fields. Furthermore, using the Pauli matrices completeness relation,

$$\tau^I_{ab}\tau^I_{cd} = 2\delta_{ad}\delta_{bc} - \delta_{ab}\delta_{cd}, \tag{F.2}$$

the only way to contract the Higgs fields is like $(\varphi^\dagger\varphi)$ (also, attempts to contract triplets anti-symmetrically will give vanishing results since the objects we contract are identical). Thus, only operators of the type $Q^{\varphi,n} \equiv (\varphi^\dagger\varphi)^{n+3}$ can appear in the Higgs potential, where $n = -2$ and $n = -1$ correspond to the SM-like Higgs mass term and quartic interaction, respectively, and values $n \in \mathbb{N}_0$ correspond to the effective operators.

To fix our notation, the Higgs potential reads

$$-V(\varphi) = m^2(\varphi^\dagger\varphi) - \frac{\lambda}{2}(\varphi^\dagger\varphi)^2 + \sum_{n=0} C^{\varphi,n}(\varphi^\dagger\varphi)^{n+3}. \tag{F.3}$$

Here and in what follows we extend the Warsaw basis notation for the Wilson coefficients and effective operators, with the understanding that $D = 6$ Wilsons/operators correspond to the value $n = 0$, i.e. $C^{\varphi,0} = C^\varphi$ etc. We leave the upper limit of the summation symbol empty, implying that all our results are valid up-to an arbitrary but fixed order N in the EFT expansion. We also absorb the EFT scale Λ in the Wilson coefficients in order to unclatter our notation; to bring back the scale Λ the Wilson coefficient $C^{x,n}$ should be replaced by $C^{x,n}/\Lambda^{2n+2}$. Minimising the potential and setting the Higgs field to a non-vanishing expectation value (VEV) as $\varphi^\dagger\varphi \to v^2/2$ we get

$$m^2 = \frac{\lambda v^2}{2} - \sum_{n=0}(n+3)C^{\varphi,n}\left(\frac{v^2}{2}\right)^{n+2}. \tag{F.4}$$

[2] Let us clarify here why the dual tensors do not appear in the bilinears as stated in ref. [171]: for two dual tensors the operator reduces to the one without duals after contractions when bilinears are concerned; for one dual tensor, and again focusing only on the bilinears, many simplifications happen due to the antisymmetry of Levi-Civita, and we are only left with a total derivative which we neglect.

Let us now collect the Lagrangian terms proportional to H^2. From $(\varphi^\dagger\varphi)^n$ we could have n terms containing a single H^2 insertion and $n(n-1)/2$ terms containing twice a $2vH$ insertion. Thus:

$$\mathcal{L} \supset -\frac{1}{2}H^2\left[\frac{3}{2}\lambda v^2 - m^2 - \sum_{n=0}(n+3)(2n+5)C^{\varphi,n}\left(\frac{v^2}{2}\right)^{n+2}\right]. \tag{F.5}$$

The physical Higgs mass, after canonicalisation of the kinetic term through $h = Z_h H$ (see section F.3), reads:

$$m_h^2 = \frac{1}{Z_h^2}\left[\lambda v^2 - 2\sum_{n=0}(n+2)(n+3)C^{\varphi,n}\left(\frac{v^2}{2}\right)^{n+2}\right]. \tag{F.6}$$

Eq. (F.6) allows us to exchange the parameter λ for the physical Higgs mass and Wilson coefficients to any order in the EFT expansion.

F.3 Scalar sector kinetic bilinears

In this section we are interested in operators that may affect the kinetic terms and/or mixing of the Higgs and the would-be Goldstone bosons. Therefore, we consider only operators of the class $\varphi^{2(n+2)}D^2$, with $n \in \mathbb{N}_0$ for the effective operators and $n = -1$ for the SM. Due to hypercharge restrictions the operators will contain exactly $(n+2)$ of each φ and $\varphi^\dagger$ fields. Also, using integration by parts and ignoring terms that can be re-expressed in terms of lower derivative operators by using equations of motion (EoMs),[3] we consider only the case were one derivative acts on φ and the other acts on $\varphi^\dagger$.[4] For $n = -1$ we find the SM kinetic term, $(D_\mu\varphi)^\dagger(D^\mu\varphi)$.[5] Moving up to dimension six ($n = 0$) there are two possible ways to contract the weak isospin indices: using the triplet or the singlet representation (anti-symmetric contraction will always involve at least two identical objects, and will therefore vanish). The operators are:

$$Q^{\varphi D(3)} = \left(\varphi^\dagger\tau^I\varphi\right)\left[(D_\mu\varphi)^\dagger\tau^I(D^\mu\varphi)\right], \tag{F.7}$$

$$Q^{\varphi D(1)} = \left(\varphi^\dagger\varphi\right)\left[(D_\mu\varphi)^\dagger(D^\mu\varphi)\right]. \tag{F.8}$$

Following the Warsaw basis construction [37] we use the completeness relation for the

[3] More correctly, we have to use field redefinitions (cf. appendix G), and operators of this type (belonging to the more general Green basis) will affect the Higgs mass parameter in the bilinears.

[4] This can be seen qualitatively the group theory relation for the su2 algebra, $2 \otimes 2 = 1 \oplus 3$

[5] Each (covariant) derivative acts only to the first object to the right.

Pauli matrices (F.2) to re-express the triplet operator as

$$\begin{aligned} Q^{\varphi D(3)} &= 2\Big(\varphi^\dagger D^\mu \varphi\Big)\Big((D_\mu \varphi)^\dagger \varphi\Big) - \Big(\varphi^\dagger \varphi\Big)\Big[(D_\mu \varphi)^\dagger (D^\mu \varphi)\Big] \\ &= 2Q^{\varphi D} - Q^{\varphi D(1)}, \end{aligned} \tag{F.9}$$

where $Q^{\varphi D} \equiv \left|\varphi^\dagger D_\mu \varphi\right|^2$.

To simplify the singlet operator we utilise integration by parts (IBP) and the Leibniz rule for the covariant derivative. To simplify the notation we use the symbol $\sim$ for the steps we use IBP and simultaneously we drop total derivative terms and all terms that can be cast into lower-derivative operators using the EoMs. Defining $Q^{\varphi\Box} \equiv (\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$ we have:

$$\begin{aligned} Q^{\varphi D(1)} &\sim -\partial^\mu\Big(\varphi^\dagger\varphi\Big)\Big[(D_\mu\varphi)^\dagger\varphi\Big] \\ &\sim \Big(\varphi^\dagger\varphi\Big)\partial^2\Big(\varphi^\dagger\varphi\Big) + \partial^\mu\Big(\varphi^\dagger\varphi\Big)\Big(\varphi^\dagger D_\mu\varphi\Big) \\ &= Q^{\varphi\Box} + \Big(\varphi^\dagger D_\mu\varphi\Big)^2 + \Big(\varphi^\dagger D^\mu\varphi\Big)\Big((D_\mu\varphi^\dagger)\varphi\Big) \\ &\sim Q^{\varphi\Box} + \Big(\varphi^\dagger D_\mu\varphi\Big)^2 - Q^{\varphi D(1)} - \Big(\varphi^\dagger D_\mu\varphi\Big)^2 \\ &= Q^{\varphi\Box} - Q^{\varphi D(1)}, \end{aligned} \tag{F.10}$$

and therefore $Q^{\varphi D(1)} \sim 1/2\, Q^{\varphi\Box}$. Thus, we can use the operators $Q^{\varphi\Box}$ and $Q^{\varphi D}$ instead of $Q^{\varphi D(1)}$ and $Q^{\varphi D(3)}$.

This identity can be easily generalised up to any order in the EFT expansion. Starting from the operators

$$Q^{\varphi D(3),n} = \Big(\varphi^\dagger\varphi\Big)^n\Big(\varphi^\dagger\tau^I\varphi\Big)\Big[(D_\mu\varphi)^\dagger\tau^I(D^\mu\varphi)\Big], \tag{F.11}$$

$$Q^{\varphi D(1),n} = \Big(\varphi^\dagger\varphi\Big)^n\Big(\varphi^\dagger\varphi\Big)\Big[(D_\mu\varphi)^\dagger(D^\mu\varphi)\Big], \tag{F.12}$$

we re-write the triplet using the Pauli matrices completeness relation (F.2) as

$$Q^{\varphi D(3),n} = 2Q^{\varphi D,n} - Q^{\varphi D(1),n}, \tag{F.13}$$

where $Q^{\varphi D,n} \equiv \big(\varphi^\dagger\varphi\big)^n\big|\varphi^\dagger D_\mu\varphi\big|^2$. To simplify the singlet we define $Q^{\varphi\Box,n} \equiv \big(\varphi^\dagger\varphi\big)^{n+1}\Box\big(\varphi^\dagger\varphi\big)$

and we have:

$$\begin{aligned}
Q^{\varphi D(1),n} &\sim -(n+1)\Big(\varphi^\dagger\varphi\Big)^n\partial^\mu\Big(\varphi^\dagger\varphi\Big)\Big[(D_\mu\varphi)^\dagger\varphi\Big] \\
&\sim (n+1)\Big(\varphi^\dagger\varphi\Big)^n\partial^\mu\Big(\varphi^\dagger\varphi\Big)\Big(\varphi^\dagger D_\mu\varphi\Big) + (n+1)Q^{\varphi\Box,n} \\
&\quad + n(n+1)\Big(\varphi^\dagger\varphi\Big)^n\Big(\partial_\mu(\varphi^\dagger\varphi)\Big)^2 \\
&= (n+1)Q^{\varphi\Box,n} + n(n+1)\Big(\varphi^\dagger\varphi\Big)^n\Big(\partial_\mu(\varphi^\dagger\varphi)\Big)^2 \\
&\quad + (n+1)\Big(\varphi^\dagger\varphi\Big)^n\Big[\Big(\varphi^\dagger D_\mu\varphi\Big)^2 + \Big(\varphi^\dagger D^\mu\varphi\Big)\Big((D_\mu\varphi^\dagger)\varphi\Big)\Big] \\
&\sim (n+1)\Big[Q^{\varphi\Box,n} - Q^{\varphi D(1),n}\Big] + n(n+1)K, \qquad\qquad (F.14)
\end{aligned}$$

where K appears for $D \geq 8$ operators and can be simplified as

$$\begin{aligned}
K &= \Big(\varphi^\dagger\varphi\Big)^n\Big[\Big(\partial_\mu(\varphi^\dagger\varphi)\Big)^2 - \Big(\varphi^\dagger D_\mu\varphi\Big)\partial^\mu(\varphi^\dagger\varphi)\Big] \\
&\sim Q^{\varphi D(1),n} - Q^{\varphi\Box,n} - nK.
\end{aligned}$$

Therefore,

$$Q^{\varphi D(1),n} \sim 1/2\; Q^{\varphi\Box,n}, \quad \forall\, n \in \mathbb{N}_0. \qquad (F.15)$$

and we can use the generalisation of the Warsaw basis operators $Q^{\varphi\Box,n}$ and $Q^{\varphi D,n}$ instead of $Q^{\varphi D(1),n}$ and $Q^{\varphi D(3),n}$. Note that this means that the charged Goldstone bosons are already canonically normalised, and therefore $Z_{G^\pm} = 1$ up to any order.[6] We list here the rescaling for the fields of the scalar sector:

$$h = Z_h H, \qquad G^0 = Z_{G^0}\Phi^0, \qquad G^\pm = Z_{G^\pm}\Phi^\pm, \qquad (F.16)$$

where the rescaling factors are given by

$$\begin{aligned}
Z_{G^\pm}^2 &= 1, \\
Z_{G^0}^2 &= 1 + \sum_{n=0}\left(\frac{v^2}{2}\right)^{n+1} C^{\varphi D,n}, \\
Z_h^2 &= 1 + \sum_{n=0}\left(\frac{v^2}{2}\right)^{n+1}\big[C^{\varphi D,n} - 4C^{\varphi\Box,n}\big]. \qquad (F.17)
\end{aligned}$$

[6] Since $\Box = \partial^2$ only $C^{\varphi D,n}$ from the operator class $\varphi^{2(n+2)}D^2$ can affect the gauge bilinears, not $C^{\varphi\Box,n}$. The same is true for the bilinear gauge-Goldstone mixing terms, which will affect the gauge fixing procedure. This is an advantage of choosing $Q^{\varphi\Box,n}$ and $Q^{\varphi D,n}$ instead of $Q^{\varphi D(1),n}$ and $Q^{\varphi D(3),n}$ for the Warsaw basis.

F.4 Gauge sector kinetic bilinears

In this section we are interested in operators that may affect the kinetic terms and/or mixing of the gauge bosons. Therefore, we consider only operators of the class $\varphi^{2(n+1)}X^2$, with $n \in \mathbb{N}_0$ for the effective operators and $n = -1$ for the SM. X here stands for a field strength tensor (the arguments we are going to use are valid also for the dual tensors). Hypercharge restricts the operators to contain exactly $(n+1)$ of each φ and $\varphi^\dagger$ fields. For $n = -1$ we find the usual SM gauge boson kinetic terms.

We can only create scalar products of the Higgs fields using the singlet and triplet representations, i.e. $\varphi^\dagger\varphi$ and $\varphi^\dagger\tau^I\varphi$, and we contract the two field strength tensors like $X_{\mu\nu}Y^{\mu\nu}$. We discuss each case in turn, by counting the weak isospin $SU(2)_w$ indices in the X^2 product. Note that we cannot contract the $SU(2)_w$ indices anti-symmetrically, since at least two objects in the product would have to be identical. We also make use of the Pauli matrices completeness relation, eq. (F.2), to reduce products of two scalar-field triplets into singlets.

- 0 indices, i.e. $B_{\mu\nu}B^{\mu\nu}$. Only combination is $Q^{\varphi B,n} \equiv (\varphi^\dagger\varphi)^{n+1}B_{\mu\nu}B^{\mu\nu}$.
- 1 index, i.e. $W^I_{\mu\nu}B^{\mu\nu}$. Only combination is $Q^{\varphi WB,n} \equiv (\varphi^\dagger\varphi)^n(\varphi^\dagger\tau^I\varphi)W^I_{\mu\nu}B^{\mu\nu}$, which is the gauge kinetic term mixing that appears for the first time at $D = 6$ SMEFT.
- 2 indices, i.e. $W^I_{\mu\nu}W^{J,\mu\nu}$.

  One possibility is to contract with δ_{IJ}, resulting in the operator $Q^{\varphi W,n} \equiv (\varphi^\dagger\varphi)^{n+1}W^I_{\mu\nu}W^{I,\mu\nu}$. For $D \geq 8$ we have another possibility, and that is to contract each W^I with a triplet of the Higgs fields, resulting in $Q^{\varphi W(3),n} \equiv (\varphi^\dagger\varphi)^{n-1}(\varphi^\dagger\tau^I\varphi)(\varphi^\dagger\tau^J\varphi)W^I_{\mu\nu}W^{J,\mu\nu}$. We will discuss the new effects arising from this operator below.

This concludes the possible bilinears for the gauge sector.

Let us now focus on the operator $Q^{\varphi W(3),n}$, appearing for the first time at dimension 8 (see ref. [132] for a discussion of dimension 8 (D8) operators, where the effects of this particular operator are also discussed). To contribute to the bilinears we reduce the Higgs fields to VEVs by $\varphi \to v/\sqrt{2}\,(0,1)^T$. This configuration for the VEV allows only the $I = 3$ Pauli matrix in $(\varphi^\dagger\tau^I\varphi)$ to contribute, and therefore these operators only affect the kinetic term of the W^3 boson.

Therefore, the re-canonicalisation of B_μ and the gluons G^A_μ (nothing changes for the gluons other than the addition of $Q^{\varphi G,n} \equiv (\varphi^\dagger\varphi)^{n+1}G^A_{\mu\nu}G^{A,\mu\nu}$ operators) is simply given by

$$\begin{aligned} \bar{B}_\mu &= Z_{g'}B_\mu, \qquad & \bar{g}' &= Z^{-1}_{g'}g', \\ \bar{G}^A_\mu &= Z_{g_s}G^A_\mu, \qquad & \bar{g}_s &= Z^{-1}_{g_s}g_s, \end{aligned} \tag{F.18}$$

with the rescaling factors

$$Z_{g'}^2 = 1 - 4\sum_{n=0}\left(\frac{v^2}{2}\right)^{n+1} C^{\varphi B,n},$$
$$Z_{g_s}^2 = 1 - 4\sum_{n=0}\left(\frac{v^2}{2}\right)^{n+1} C^{\varphi G,n}. \tag{F.19}$$

For the $SU(2)_w$ gauge fields W^I the rescaling now reads:

$$\bar{W}_\mu^I \equiv Z_g^I W_\mu^I, \tag{F.20}$$

where the rescaling factors are compactly written as

$$(Z_g^I)^2 = (Z_g)^2 - 4\delta^{I3}\sum_{n=1}\left(\frac{v^2}{2}\right)^{n+1} C^{\varphi W(3),n}, \tag{F.21}$$

so that

$$(Z_g)^2 = 1 - 4\sum_{n=0}\left(\frac{v^2}{2}\right)^{n+1} C^{\varphi W,n} \tag{F.22}$$

is the rescaling factor for the $W^{1,2}$ fields, i.e. $Z_g^{I=1,2} = Z_g$. From now on we will also define $Z_g^{I=3} = Z_3$. We choose *not* to include the extra Z_3 contributions in the normalisation of the $SU(2)_w$ coupling in accordance with ref. [132]:

$$\bar{g} \equiv Z_g^{-1} g. \tag{F.23}$$

This coupling non-universality after the canonical normalisation of the kinetics terms has an important consequence: the covariant derivative in the "barred" basis isn't equal to the "unbarred" one, as was the case in the dimension 6 (D6) SMEFT. Now it reads:

$$D_\mu = \bar{D}_\mu + i\bar{g}Z_g(Z_3^{-1} - Z_g^{-1})\bar{W}_\mu^3 T^3. \tag{F.24}$$

An immediate effect of this extra term is that the Higgs kinetic term will now introduce extra contributions to the EW sector mass matrix. Defining $\bar{\bar{g}} \equiv Z_g(Z_3^{-1} - Z_g^{-1})\bar{g}$ we have:

$$(D_\mu\varphi)^\dagger(D^\mu\varphi) = |\bar{D}_\mu\varphi|^2 + \bar{\bar{g}}^2|T^3\bar{W}_\mu^3\varphi|^2$$
$$+ i\bar{\bar{g}}(\bar{D}^\mu\varphi)^\dagger(T^3\bar{W}_\mu^3\varphi) - i\bar{\bar{g}}(T^3\bar{W}_\mu^3\varphi)^\dagger(\bar{D}^\mu\varphi). \tag{F.25}$$

Setting both of the Higgs fields to the VEV we get extra contributions to the gauge sector bilinears as follows:

$$|D_\mu\varphi|^2 - |\bar{D}_\mu\varphi|^2 \supset \frac{v^2}{8}\bar{g}(\bar{g}+2\bar{g})\bar{W}^{3,\mu}\bar{W}^3_\mu - \frac{v^2}{4}\bar{g}\bar{g}'\bar{B}^\mu\bar{W}^3_\mu. \tag{F.26}$$

Wilsons $C^{\varphi D,n}$ also contribute in the same manner, therefore they just sum with the SM-like contributions to give $Z^2_{G^0}$, exactly like in the D6 case. The Higgs kinetic term also affects the gauge-Goldstone mixing in the theory, which is related to our choice of gauge fixing. We discuss this in section F.6.

F.5 EW bilinear Lagrangian

We are now in position to write down the bilinear Lagrangian for SMEFT up to any order in the EFT expansion. It is a simple generalisation of eq. (3.14) of ref. [40]:[7]

$$\begin{aligned}
\mathcal{L}^{\text{Bilinear}}_{EW} &= -\frac{1}{4}(\bar{W}^1_{\mu\nu}\bar{W}^{1\mu\nu} + \bar{W}^2_{\mu\nu}\bar{W}^{2\mu\nu}) \\
&- \frac{1}{4}\begin{pmatrix} \bar{W}^3_{\mu\nu} \\ \bar{B}_{\mu\nu} \end{pmatrix}^\top \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \begin{pmatrix} \bar{W}^{3\mu\nu} \\ \bar{B}^{\mu\nu} \end{pmatrix} \\
&+ \frac{\bar{g}^2 v^2}{8}(\bar{W}^1_\mu \bar{W}^{1\mu} + \bar{W}^2_\mu \bar{W}^{2\mu}) \\
&+ \frac{v^2}{8} Z^2_{G^0} \begin{pmatrix} \bar{W}^3_\mu \\ \bar{B}_\mu \end{pmatrix}^\top \begin{pmatrix} \tilde{g}^2 & -\tilde{g}\bar{g}' \\ -\tilde{g}\bar{g}' & \bar{g}'^2 \end{pmatrix} \begin{pmatrix} \bar{W}^{3\mu} \\ \bar{B}^\mu \end{pmatrix},
\end{aligned} \tag{F.27}$$

where we defined

$$\tilde{g} \equiv Z_3^{-1} Z_g \bar{g} \tag{F.28}$$

and

$$\epsilon \equiv \frac{v^2}{Z_3 Z_{g'}} \sum_{n=0} \left(\frac{v^2}{2}\right)^n C^{\varphi WB,n}. \tag{F.29}$$

Note that the EW bilinear Lagrangian has the same form as the D6 one, with the difference that $\bar{g}$ appears in the $W^\pm$ mass but now $\tilde{g} = Z_3^{-1} Z_g \bar{g}$ appears in the W^3–B mass matrix. This effect appears for the first time at D8, and *no new types of effects appear at higher orders.* Every higher order effect is just an existing one augmented by powers of v^2, and can be 'resumed' in Z factors.

[7] Note that the strength-energy tensors here denote only the abelian part in order to create bilinears.

Let us define a matrix

$$\mathbb{X} = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \begin{pmatrix} \cos\bar{\theta} & \sin\bar{\theta} \\ -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix} = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} \tag{F.30}$$

such that the fields Z and A,

$$\begin{pmatrix} \bar{W}^3_\mu \\ \bar{B}_\mu \end{pmatrix} = \mathbb{X} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \tag{F.31}$$

are in the mass-basis and canonically normalised. To achieve that we choose the parameters to be

$$\begin{aligned} a &= \frac{\epsilon}{\sqrt{2}\sqrt{1-\epsilon^2}\sqrt{1-\sqrt{1-\epsilon^2}}}, \\ b &= -\frac{\sqrt{1-\sqrt{1-\epsilon^2}}}{\sqrt{2}\sqrt{1-\epsilon^2}}, \end{aligned} \tag{F.32}$$

and the mixing angle is defined by

$$\tan\bar{\theta} = \frac{\tilde{g}^2 + 2\epsilon\tilde{g}\bar{g}' + \bar{g}'^2 - \sqrt{1-\epsilon^2}(\tilde{g}^2 - \bar{g}'^2)}{2\tilde{g}\bar{g}' + \epsilon(\tilde{g}^2 + \bar{g}'^2)} \tag{F.33}$$

After diagonalisation, the masses of the Z and A gauge bosons are given by

$$m_Z = \frac{v}{2} Z_{G^0} \sqrt{\frac{\tilde{g}^2 + 2\epsilon\tilde{g}\bar{g}' + \bar{g}'^2}{1-\epsilon^2}}, \tag{F.34}$$

$$m_A = 0. \tag{F.35}$$

F.6 Gauge fixing

F.6.1 Gauge-Goldstone mixing

Since the covariant derivative only redefines the $\bar{W}^3$ coupling from $\bar{g}$ to $\tilde{g}$ with respect to the D6 case, it is trivial to prove that eq. (3.25) from ref. [40] still holds in the same form if we use our new definition for the Z mass, i.e.

$$\mathcal{L}_{G-EW} = i m_W (W^+_\mu \partial^\mu G^- - W^-_\mu \partial^\mu G^+) - m_Z Z_\mu \partial^\mu G^0. \tag{F.36}$$

F.6.2 Gauge fixing

The D6 SMEFT has the special feature that changing naively every coupling and field from the Warsaw (unbarred) basis to the canonically normalised bar basis doesn't affect the form of the Lagrangian; barring everything acts like a re-parameterisation in the Lagrangian. Therefore, the gauge-fixing of the theory could be performed in the unbar or the bar basis. The BRST transformations can be taken from the SM case, by simply putting bars in the fields and couplings if one chooses to start from the bar basis. This was the method applied in ref. [40].

When adding D8 or higher operators, this useful feature of the effective Lagrangian isn't true any more, since the W_μ^I fields may not scale like the inverse of the gauge coupling g. In our formalism we choose W_μ^3 to scale differently. Therefore, the easiest approach to gauge fixing the theory is to start from the Warsaw basis Lagrangian, the unbar basis, and use the SM BRST transformations. Later, we simply use the appropriate rescaling factors to translate everything to our bar basis.

F.7 EW couplings

Let us define here the EW Lagrangian using the electric charge and the Z coupling

$$\begin{aligned}\bar{e} &= \frac{\tilde{g}\bar{g}'}{\sqrt{\tilde{g}^2 + 2\epsilon\tilde{g}\bar{g}' + \bar{g}'^2}},\\ \bar{g}_Z &= \sqrt{\frac{\tilde{g}^2 + 2\epsilon\tilde{g}\bar{g}' + \bar{g}'^2}{1-\epsilon^2}}.\end{aligned} \tag{F.37}$$

We could also define the photon and Z couplings by

$$\begin{aligned}\bar{g}_A &= \tilde{g}X_{12} - \bar{g}'X_{22} = 0,\\ \bar{g}_Z &= \tilde{g}X_{11} - \bar{g}'X_{21}.\end{aligned} \tag{F.38}$$

F.8 Utilising the Warsaw basis

In this section we describe how one can go directly from the Warsaw (unbar) basis to the physical basis, without making use of the intermediate "bar" basis were the fields are canonically normalised. We use the normalising Z factors that we derived before. We draw the attention of the reader to the fact *our notation* about the $\mathbb{X}$ matrix *in this section is different than the rest of the appendix.* This section introduces the notation that's in accordance with the gauge fixing procedure in the new version of the `smeftFR` code [127].

Let us introduce a new $\mathbb{X}$ matrix, such that

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \mathbb{X} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \tag{F.39}$$

were the fields Z and A are the mass basis fields. This $\mathbb{X}$ matrix is connected to the old one (which didn't include the normalisation factors) via

$$\mathbb{X} = \begin{pmatrix} X^{\text{old}}_{11}/Z_3 & X^{\text{old}}_{12}/Z_3 \\ X^{\text{old}}_{21}/Z_{g'} & X^{\text{old}}_{22}/Z_{g'} \end{pmatrix}. \tag{F.40}$$

Of course the charged gauge fields in the mass basis are defined as

$$W^{\pm}_\mu = \frac{Z_g}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu). \tag{F.41}$$

With this new $\mathbb{X}$ matrix the couplings proportional to the Z and A mass read simply: $\bar{g}_Z = gX_{11} - g'X_{21}$ and $\bar{g}_A = gX_{12} - g'X_{22} = 0$.

The unwanted gauge-Goldstone mixing terms, written in terms of the physical masses and fields in eq. (3.24) of ref. [40] is still exact. We now try and generalise the gauge fixing procedure to cancel this unwanted term. We follow section 5 of ref. [40], but we now start from the Warsaw basis D4 Lagrangian.

The gauge fixing Lagrangian is still defined as

$$\mathcal{L}_{GF} = -\frac{1}{2}\boldsymbol{F}^\top \hat{\boldsymbol{\xi}}^{\boldsymbol{-1}} \boldsymbol{F}, \tag{F.42}$$

where the gauge fixing functionals are now defined in terms of unbared couplings and fields:

$$\boldsymbol{F} = \begin{pmatrix} F^1 \\ F^2 \\ F^3 \\ F^0 \end{pmatrix} = \begin{pmatrix} \partial_\mu W^{1\mu} \\ \partial_\mu W^{2\mu} \\ \partial_\mu W^{3\mu} \\ \partial_\mu B^\mu \end{pmatrix} - \frac{v\hat{\boldsymbol{\xi}}}{2} \begin{pmatrix} -ig\frac{\Phi^+ - \Phi^-}{\sqrt{2}} \\ g\frac{\Phi^+ + \Phi^-}{\sqrt{2}} \\ -gZ^2_{G^0}\Phi_0 \\ g'Z^2_{G^0}\Phi_0 \end{pmatrix}. \tag{F.43}$$

To compensate for this change, we have to redefine the symmetric 4×4 matrix $\hat{\boldsymbol{\xi}}$ as follows:

$$\hat{\boldsymbol{\xi}} = \begin{pmatrix} \xi_W / Z_g^2 \, \mathbb{1}_{2\times 2} & \mathbb{O}_{2\times 2} \\ \mathbb{O}_{2\times 2} & \mathbb{X} \begin{pmatrix} \xi_Z & \\ & \xi_A \end{pmatrix} \mathbb{X}^\top \end{pmatrix}. \tag{F.44}$$

(Note that, equivalently, we could define a 4×4 matrix, say $\mathbb{X}_4$, that would include also the factor Z_g^{-1}, and then $\hat{\boldsymbol{\xi}} = \mathbb{X}_4 \, \mathrm{diag}(\xi_W, \xi_W, \xi_Z, \xi_A) \, \mathbb{X}_4^\top$. This is the procedure followed in ref. [171]. Here we choose to follow a formalism more easily adapted in the smeftFR code.)

Using these adaptations, we can prove that the unwanted gauge-Goldstone mixing terms cancel.

We now move on to the ghost Lagrangian. Like before, the definition of the ghost Lagrangian remains the same,

$$\mathcal{L}_{FP} = \overline{\boldsymbol{N}}^\top \hat{\boldsymbol{E}} \left(\hat{\boldsymbol{M}}_{\boldsymbol{F}} \boldsymbol{N} \right) \tag{F.45}$$

and the symmetric 4×4 matrix $\hat{\boldsymbol{E}}$ is redefined:

$$\hat{\boldsymbol{E}} = \begin{pmatrix} Z_g^2 \, \mathbb{1}_{2\times 2} & \mathbb{O}_{2\times 2} \\ \mathbb{O}_{2\times 2} & \left(\mathbb{X}^\top\right)^{-1} \mathbb{X}^{-1} \end{pmatrix}. \tag{F.46}$$

Using these new definitions, and using the SM BRST transformations (taken from eq. (5.9) of [40] by unbarring fields and couplings), we can prove that eq. (5.10) of [40] has the same form and the only change is that *we have to unbar every field and coupling* there.

Like we did in the charged gauge sector, the charged ghosts should explicitly contain the normalisation factor Z_g for their kinetic terms to be canonically normalised:

$$\begin{aligned} \eta^\pm &= \frac{Z_g}{\sqrt{2}} (N^1 \mp i N^2), \\ \bar{\eta}^\pm &= \frac{Z_g}{\sqrt{2}} (\bar{N}^1 \pm i \bar{N}^2). \end{aligned} \tag{F.47}$$

while the definitions for the Z and A ghosts remain unaffected if we use the new $\mathbb{X}$ matrix.

The last change in section 5 of [40] is that the N^I ghost BRST transformations in eq. (5.13) are now proportional to the unbarred coupling g in order to satisfy the nilpotency of the BRST transformation in $\boldsymbol{s}^2 F_i$, and everything else in this section remains the same.

F.9 Comparisons

Let us collect here some reference formulae for comparison with the results of ref. [171] that relate to the SMEFT (see appendix D of ref. [171]). The J matrix elements used in that paper are given by

$$\begin{aligned} J_+ &= Z_g^2 - 1, \\ J_1 &= Z_3^2 - 1, \\ J_2 &= Z_{g'}^2 - 1, \\ J_3 &= \epsilon Z_3 Z_{g'}, \end{aligned} \tag{F.48}$$

and for the K matrix of [171] we have

$$\begin{aligned} K_+ &= Z_{G^\pm}^2 - 1, \\ K_1 &= Z_{G^0}^2 - 1, \\ K_2 &= Z_h^2 - 1, \\ K_3 &= Z_{G^0/h}^2. \end{aligned} \tag{F.49}$$

Using these expressions and by direct comparison we find agreement in the gauge fixing procedure and in the final results for the gauge boson masses presented in appendix D of ref. [171].

In this appendix we have proven that there is no kinetic mixing between the H and Φ^0 components of the Higgs doublet to all orders in the EFT expansion. Furthermore, we proved that the charged component is already canonically normalised up to any order. This means that the first equation in (D.13) of ref. [171], namely

$$K_+ = K_3 = 0, \tag{F.50}$$

should be true *to all orders in SMEFT*.

Appendix G

Field redefinitions in effective field theories

In effective field theories (EFTs) it is common practice to use the equations of motion (EoMs) to reduce operators containing higher derivatives. This procedure, however, is erroneous if one is interested in EFT terms beyond the leading order. The correct approach is to make use of local field redefinitions. A detailed analysis of this approach is outlined in ref. [173]. In this appendix we give the field redefinitions that are necessary in order to get rid of the terms containing higher derivatives in two simple toy model EFTs and discuss the complications related with higher order EFT corrections.

G.1 Introduction

Let us formally define the EFT action as a power series

$$S[\phi] = \sum_{i=0}^{N} \varepsilon^i S_i[\phi] \,, \tag{G.1}$$

where ϕ is a placeholder for various quantum fields, N is an arbitrary but finite integer number, ε is a dimensionful expansion parameter, usually the inverse of a UV scale (for example in our toy models here, as well as in the bosonic SMEFT, we have that $\varepsilon^{-1} = \Lambda^2$, with Λ representing the unknown scale of the UV physics). Then one can use a local perturbative field redefinition of the form

$$\phi \to \phi' = \phi + \sum_{i=1}^{N} \varepsilon^i F_i(\phi) \,, \tag{G.2}$$

and this will affect the EFT action as follows:

$$\begin{aligned} S'[\phi] &\equiv S[\phi'] \\ &= S_0[\phi] + \varepsilon\left[S_1[\phi] + F_1^\alpha(\phi)\frac{\delta S_0}{\delta\phi^\alpha}\right] \\ &\quad + \varepsilon^2\left[S_2[\phi] + F_1^\alpha(\phi)\frac{\delta S_1}{\delta\phi^\alpha} + F_2^\alpha(\phi)\frac{\delta S_0}{\delta\phi^\alpha} + \frac{1}{2}F_1^\alpha(\phi)F_1^\beta(\phi)\frac{\delta^2 S_0}{\delta\phi^\alpha\delta\phi^\beta}\right] + \mathcal{O}(\varepsilon^3)\,, \end{aligned} \tag{G.3}$$

where repeated indices denote a sum over the various fields ϕ_i and the corresponding functions F_i and an integral over the space-time argument.

The functional derivatives of the action can be evaluated straightforwardly using the definition of the functional derivative:

$$\int dx\,\rho(x)\frac{\delta S[f(x)]}{\delta f(y)} = \lim_{\epsilon\to 0}\frac{S[f(y)+\epsilon\rho(y)] - S[f(y)]}{\epsilon}\,. \tag{G.4}$$

The single functional derivative of the renormalisable action S_0 is by definition equivalent to the lowest EFT order classical EoM. For example, using an action

$$S_0[\phi] = \int d^4x\left[\tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}\lambda\phi^4\right]\,, \tag{G.5}$$

we find that the first functional derivative gives

$$\frac{\delta S_0[\phi]}{\delta\phi} = -\partial^2\phi - \lambda\phi^3 \equiv -E(\phi)\,, \tag{G.6}$$

where $E(\phi) = 0$ is the classical EoM for this Lagrangian,the second functional derivative of $S_0[\phi]$ gives

$$\frac{\delta^2 S_0[\phi]}{\delta\phi(x)\delta\phi(y)} = \left[-\partial^2 - 3\lambda\phi^2\right]\delta^4(x-y)\,, \tag{G.7}$$

and so on.

G.2 Real scalar field

G.2.1 Leading EFT order

Formal derivation

This is the simplest toy model possible, containing only a single real scalar field.[1] For simplicity we ignore the mass term and we assume that our theory is invariant under $\phi \to -\phi$.

[1] We follow the example given in page 458 of ref. [11] (the reader should be aware of a typo in the quartic coupling there).

Then, the most general renormalisable Lagrangian (we are assuming a 4-dimensional space-time, so that means that we only include operators up to dimension 4) is given by:

$$\mathcal{L}_4 = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}\lambda\phi^4\,. \tag{G.8}$$

The classical EoM for this Lagrangian, which is also the lowest order classical EoM in the EFT expansion, can be written as $E(\phi) = 0$, where we defined

$$E(\phi) = \partial^2\phi + \lambda\phi^3\,. \tag{G.9}$$

After using integration by parts (IBP) and assuming that the surface terms in the action vanish, we find that the most general Lagrangian at dimension 6 is given by

$$\mathcal{L}_6 = \tfrac{1}{\Lambda^2}\big[a_6\phi^6 + b_6\phi^3\partial^2\phi + c_6(\partial^2\phi)^2\big]\,. \tag{G.10}$$

These operators consist, as is often called in the literature, the Green basis at dimension 6.

For completeness, we list here the IBP relations of the operators that don't appear in our chosen Green basis. For the operator class $\phi^2\partial^4$ each IBP trivially introduces an overall minus sign since we only have two fields. For the operator class $\phi^4\partial^2$ there are two choices for the placement of the derivatives. We use the symbol "$\approx$" to indicate that the equality is up-to surface terms in the action and we find

$$\phi^2(\partial\phi)^2 \approx -\tfrac{1}{3}\phi^3\partial^2\phi\,. \tag{G.11}$$

Let us now perform a perturbative field redefinition of the form

$$\phi \to \phi + \tfrac{1}{\Lambda^2}F(\phi)\,, \tag{G.12}$$

which takes $\mathcal{L}_4 \to \mathcal{L}_4 + \delta\mathcal{L}_6 + \cdots$, where the ellipses stand for higher order terms and the leading order correction is

$$\delta\mathcal{L}_6 = -\tfrac{1}{\Lambda^2}F(\phi)\big[\partial^2\phi + \lambda\phi^3\big] = -\tfrac{1}{\Lambda^2}F(\phi)E(\phi)\,. \tag{G.13}$$

Notice that the full contribution at dimension 6 (D6) comes from the renormalisable Lagrangian and is proportional to the lowest order classical EoM. Beyond the leading order, however, this simple behaviour ceases to exist; see for example the second functional derivative given in eq. (G.7). Using the local[2] function

$$F(\phi) = p_6 \phi^3 + q_6 \partial^2 \phi \,, \tag{G.14}$$

we see that the choice $q_6 = c_6$ cancels the $(\partial^2\phi)^2$ and then the choice $p_6 = b_6 - \lambda c_6$ cancels the $\phi^3\partial^2\phi$ term. The effect of this field redefinition at D6 is that the Wilson coefficient of the term ϕ^6 is modified like $a_6 \to a_6' = a_6 - \lambda b_6 + \lambda^2 c_6$. The higher order terms of the theory would be modified as well.

Every result we derive by using the field redefinitions explicitly in the Lagrangian, as we did in this section and in the rest of the text, can be reproduced by using the Taylor series expansion of the action (G.3) and calculating the functional derivatives as explained in section G.1. Therefore, from now on we are not going to mention this fact explicitly and we are just going to use the redefinitions in the level of the Lagrangian.

Using naively the EoMs

One could reproduce this result by using naively the EoM for the scalar field in the lowest order in the EFT expansion. Using the fact that insertions of EoM in the D6 operators won't affect the S-matrix to this order (a fact that can be deduced from eq. (G.13)), we proceed to replace the terms $\partial^2\phi$ with $-\lambda\phi^3$ in the D6 terms. This has the effect that the only D6 operator left is ϕ^6, with its new Wilson coefficient reading

$$a_6' = a_6 - \lambda b_6 + \lambda^2 c_6 \,, \tag{G.15}$$

which is the same result as using the field redefinition, even for the double insertion of $\partial^2\phi$.

Therefore, the leading order EFT Lagrangian (which in this case is D6) after IBP and EoM reduction reads

$$\mathcal{L}_6 = \tfrac{1}{\Lambda^2} a_6' \phi^6 \,. \tag{G.16}$$

We call this basis of operators Warsaw basis to connect our findings with the SMEFT case. Of course both the Green and the Warsaw basis aren't always unique, since we may have the freedom to perform IBP and different field redefinitions to re-shuffle the terms. This Lagrangian, combined with the renormalisable Lagrangian (G.8), contains the full information of the theory up-to D6 without containing any redundant operator when we are interested in on-shell results. This is because field redefinitions affect the Green's functions but don't affect the S-matrix.

[2] A local function of the field ϕ is defined formally as $F(\phi) = f(\phi, \partial\phi, \ldots, \partial^N\phi)$, where N is an arbitrary but finite non-negative integer. If N is infinite then this would be a shifting operator acting on the field ϕ, resulting in a non-local term.

G.2.2 Next-to-Leading EFT order

At dimension 8 the Green basis Lagrangian is found to be (again, we use IBP to arrive in the Green basis):

$$\begin{aligned}\mathcal{L}_8 = \tfrac{1}{\Lambda^4}\big[&a_8\phi^8 + b_8\phi^5\partial^2\phi + c_8(\partial^2\phi)(\partial^4\phi) \\ &+ d_8(\partial\phi)^4 + e_8\phi(\partial^2\phi)(\partial\phi)^2 + f_8\phi^2(\partial^2\phi)^2\big]\,. \end{aligned}\tag{G.17}$$

As before, we list here the IBP relations of the operators that don't appear in our chosen Green basis. For the operator class $\phi^2\partial^6$ each IBP trivially introduces an overall minus sign since we only have two fields. For the other operators we use the symbol "$\approx$" to indicate that the equality is up-to surface terms in the action and we find

$$\begin{aligned}\phi^4(\partial\phi)^2 &\approx -\tfrac{1}{5}\phi^5\partial^2\phi\,,\\ \phi^3\partial^4\phi &\approx 3\phi^2(\partial^2\phi)^2 + 6\phi(\partial^2\phi)(\partial\phi)^2\,,\\ \phi^2(\partial_\mu\partial_\nu\phi)(\partial^\mu\partial^\nu\phi) &\approx (\partial\phi)^4 + \phi^2(\partial^2\phi)^2 + 3\phi(\partial^2\phi)(\partial\phi)^2\,,\\ \phi(\partial_\mu\phi)(\partial_\nu\phi)(\partial^\mu\partial^\nu\phi) &\approx -\tfrac{1}{2}(\partial\phi)^4 - \tfrac{1}{2}\phi(\partial^2\phi)(\partial\phi)^2\,,\\ \phi^2(\partial_\mu\phi)(\partial^\mu\partial^2\phi) &\approx -\phi^2(\partial^2\phi)^2 - 2\phi(\partial^2\phi)(\partial\phi)^2\,. \end{aligned}\tag{G.18}$$

We should now follow the same procedure as we did for the D6 Lagrangian and perform a field redefinition

$$\phi \to \phi + \tfrac{1}{\Lambda^4}G(\phi)\,. \tag{G.19}$$

This redefinition will modify the dimension 8 (D8) and higher order terms in the Lagrangian. Like before, the leading order contribution (which now is D8) comes solely from the renormalisable part of the Lagrangian. For our field redefinition we use the local function

$$G(\phi) = p_8\phi^5 + q_8\partial^4\phi + r_8\phi(\partial\phi)^2 + s_8\phi^2\partial^2\phi\,. \tag{G.20}$$

We then try to cancel the *contracted* higher derivative terms one by one (see section G.2.3 for our definition of "contracted" higher derivative terms). Choosing $q_8 = c_8$ we cancel the $(\partial^2\phi)(\partial^4\phi)$ term and also get the modifications $f_8 \to f_8 - 3\lambda q_8$ and $e_8 \to e_8 - 6\lambda q_8$. Then, choosing $s_8 = f_8 - 3\lambda q_8$ we cancel the $\phi^2(\partial^2\phi)^2$ term and modify $b_8 \to b_8 - \lambda s_8$. Then, choosing $r_8 = e_8 - 6\lambda q_8$ we cancel the $\phi(\partial^2\phi)(\partial\phi)^2$ term and get $b_8 - \lambda s_8 \to b_8 - \lambda s_8 + \lambda r_8/5$. Finally, setting $p_8 = b_8 - \lambda s_8 + \lambda r_8/5$ cancels the $\phi^5\partial^2\phi$ term and modifies

$$a_8 \to a_8 - \lambda p_8\,. \tag{G.21}$$

Again, it is trivial to check that one could derive the same result by using naively the lowest order classical EoM, $E(\phi) = 0$. Therefore in a purely bottom-up EFT approach, where the Wilsons are unknown parameters, it is enough to use the classical EoM to zeroth order in the EFT expansion (i.e. the classical EoMs for the renormalisable Lagrangian) to cancel the contracted higher derivative terms. But if one wants to use the EFT in a top-down fashion, e.g. matching a UV model in the EFT, then the modifications in the parameters $a_{6,8}$ that we derived so far are important, since parameters of the theory that produce operators which are later simplified using the EoM may still be relevant for the determination of Wilsons of other operators that survive. For the leading term, having specified a_6' is enough, but beyond the leading term there are complications that cannot be fixed in the framework of "EoM redundancy".

If we want to work consistently up to D8 we need to take into account also the double insertions at D6; we will call these contributions D6^2. These contributions will appear from double insertion of $F(\phi)$ in the renormalisable Lagrangian and single insertion in the D6 Lagrangian. Therefore the D8 Wilsons we posted in eq. (G.17) should be modified to include these D6^2 contributions. Of course, the D6 field redefinitions will produce D8 operators of the D8 Green basis (after utilising IBPs as explained above), since the redefinitions respect the symmetries of the theory.

These expressions are in general complicated linear combinations of the Wilsons, even for this very simple toy model. We find,

$$\begin{aligned}
a_8 &\to a_8 + 6a_6b_6 - \left(6a_6c_6 + \tfrac{3}{2}b_6^2\right)\lambda + 3b_6c_6\lambda^2 - \tfrac{3}{2}c_6^2\lambda^3\,,\\
b_8 &\to b_8 + \tfrac{39}{10}b_6^2 + 6a_6c_6 - 6b_6c_6\lambda + \tfrac{21}{10}c_6^2\lambda^2\,,\\
c_8 &\to c_8 + \tfrac{3}{2}c_6^2\,,\\
d_8 &\to d_8\,,\\
e_8 &\to e_8 + 12b_6c_6 - 6c_6^2\lambda\,,\\
f_8 &\to f_8 + 9b_6c_6 - \tfrac{9}{2}c_6^2\lambda\,.
\end{aligned} \tag{G.22}$$

Now these expressions should be used to redefine the Wilsons *in addition* to the replacements that we derive when we reduce the Green basis at D8.

Using primes to denote Warsaw basis Wilson coefficients we conclude that the D8 Lagrangian in Warsaw basis can be written as

$$\mathcal{L}_8 = \tfrac{1}{\Lambda^4}\left[a_8'\phi^8 + d_8'(\partial\phi)^4\right]\,, \tag{G.23}$$

with the Wilson coefficients related to the ones in the Green basis like $d'_8 = d_8$ and

$$a'_8 = a_8 + 6a_6 b_6 - \left(b_8 + 12 a_6 c_6 + \tfrac{27}{5} b_6^2\right)\lambda \\ + \left(f_8 - \tfrac{1}{5} e_8 + \tfrac{78}{5} b_6 c_6\right)\lambda^2 - \tfrac{1}{5}\left(9c_8 + 48 c_6^2\right)\lambda^3 \,. \qquad \text{(G.24)}$$

G.2.3 Choices for the non-redundant basis

Let us comment here that the most useful thing about having the freedom to use field redefinitions in EFTs is that one can get rid of *contracted* higher derivative terms. Here by "contracted" we mean the terms with derivatives that appear in the lowest order classical EoM, like $\partial^2\phi$ for scalar fields, $\partial_\mu W^{\mu\nu}$ for gauge fields and $\not{\partial}\psi$ for fermionic fields. This is very useful for avoiding consistency problems in the EFT, like Ostrogradsky instabilities [174], that happen because of the higher derivative terms in the EoMs.[3] Getting rid of these operators allows us to write the propagators in the same form as in the renormalisable theory. As we discussed, in a bottom-up EFT approach where we treat the Wilsons as independent numbers one could use the lowest order classical EoM to cancel these problematic operators recursively to every EFT order.

Let us try here to see formally what freedom we are given when we are constructing a non-redundant basis. We revisit the real scalar field example at D6, and once again perform the field redefinition

$$\phi \to \phi + \tfrac{1}{\Lambda^2} F(\phi) \,. \qquad \text{(G.25)}$$

Since we restrict ourselves in the subset of perturbative field redefinitions where the EFT expansion parameter plays the role of the perturbation parameter, it is trivial to see that the mass dimension of the function $F(\phi)$ is 3. Therefore, the most general Lorentz invariant form of F that includes only the scalar field ϕ of our spectrum and derivatives of that field reads

$$F(\phi) = p_6 \phi^3 + q_6 \partial^2 \phi \,, \qquad \text{(G.26)}$$

where p_6 and q_6 are free parameters. This means that in our toy model, where the Green basis contains three independent parameters, we are free to apply two constraints when using field redefinitions. Therefore, the non-redundant basis contains exactly one free parameter.

We present here the three different but equivalent[4] non-redundant Lagrangians one can

[3] Ostrogradsky's theorem states that, in classical mechanics, a non-degenerate Lagrangian which contains time derivatives higher than the first, produces a Hamiltonian which is unbounded from below.

[4] In classical mechanics the term "equivalent Lagrangians" is used for two Lagrangians that produce the same EoM. Here each Lagrangian produces a qualitatively different EoM since the number of derivatives acting on ϕ changes. In EFTs we use this term loosely for Lagrangians that are built from non-redundant bases and give us on-shell equivalence.

derive:

$$\begin{aligned} F^{(1)}(\phi) &= (b_6 - \lambda c_6)\phi^3 + c_6\partial^2\phi\,, \\ \mathcal{L}_6^{(1)} &= \tfrac{1}{\Lambda^2}\big(a_6 - \lambda b_6 + \lambda^2 c_6\big)\phi^6\,, \end{aligned} \tag{G.27}$$

$$\begin{aligned} F^{(2)}(\phi) &= \tfrac{1}{\lambda}a_6\phi^3 + c_6\partial^2\phi\,, \\ \mathcal{L}_6^{(2)} &= \tfrac{1}{\Lambda^2}\big(b_6 - \tfrac{1}{\lambda}a_6 - \lambda c_6\big)\phi^3\partial^2\phi\,, \end{aligned} \tag{G.28}$$

$$\begin{aligned} F^{(3)}(\phi) &= \tfrac{1}{\lambda}a_6\phi^3 + (\tfrac{1}{\lambda}b_6 - \tfrac{1}{\lambda^2}a_6)\partial^2\phi\,, \\ \mathcal{L}_6^{(3)} &= \tfrac{1}{\Lambda^2}\big(c_6 - \tfrac{1}{\lambda}b_6 + \tfrac{1}{\lambda^2}a_6\big)(\partial^2\phi)^2\,. \end{aligned} \tag{G.29}$$

G.3 Complex scalar field

G.3.1 Green basis

Let us now repeat the procedure in the case of a complex scalar field, with an action invariant under the global symmetry $\phi \to e^{i\alpha}\phi$. Keeping the mass term this time, the renormalisable Lagrangian reads:

$$\mathcal{L}_4 = |\partial\phi|^2 - m^2|\phi|^2 - \tfrac{1}{2}\lambda|\phi|^4\,. \tag{G.30}$$

The classical EoM for the field ϕ is given by $E(\phi) = 0$, where

$$E(\phi) = \partial^2\phi + m^2\phi + \lambda|\phi|^2\phi\,, \tag{G.31}$$

and similarly for ϕ^*. Using IBP to reduce the number of operators, our choice for the Green basis Lagrangian at dimension 6 reads:

$$\mathcal{L}_6 = \tfrac{1}{\Lambda^2}\Big[a_6|\phi|^6 + b_6|\phi|^2|\partial\phi|^2 + c_6\big|\partial^2\phi\big|^2 + (d_6|\phi|^2\phi^*\partial^2\phi + \text{c.c.})\Big]\,, \tag{G.32}$$

where c.c. stands for the complex conjugate term and a_6, b_6 and c_6 are real numbers. To derive this particular Green basis we used the fact that the operator class $\phi^2\partial^4$ has trivial IBP relations, and within the class $\phi^4\partial^2$ we have omitted the operator

$$(\phi^*)^2(\partial\phi)^2 \approx -2|\phi|^2|\partial\phi|^2 - |\phi|^2\phi^*\partial^2\phi \tag{G.33}$$

and its complex conjugate.

Let us also derive the Green basis at D8. For the operator class $\phi^6\partial^2$ we use the IBP relation

$$\phi(\phi^*)^3(\partial\phi)^2 \approx -\tfrac{1}{2}|\phi|^4\phi^*\partial^2\phi - \tfrac{1}{2}|\phi|^4|\partial\phi|^2 \tag{G.34}$$

and its complex conjugate. The operator class $\phi^4\partial^4$ is much more complicated. We introduce a compact notation for this class, where we use indices that denote the number of derivatives acting on each field, e.g. $Q_{(30)(10)} \equiv (\partial_\mu\partial^2\phi)\phi(\partial^\mu\phi^*)\phi^*$. To avoid double counting we place the unstarred fields to the left of the starred ones, and we move the higher derivative terms to the left. When there is ambiguity about the contractions of the Lorentz indices we use a hat symbol to indicate contraction inside a "2" subscript index or inside two "1" indices that belong to the same subscript parenthesis. For example $Q_{(\hat{2}1)(10)} \equiv (\partial^2\phi)(\partial^\mu\phi)(\partial_\mu\phi^*)\phi^*$ and $Q_{(\hat{1}1)(\hat{1}1)} \equiv (\partial_\mu\phi)(\partial^\mu\phi)(\partial_\nu\phi^*)(\partial^\nu\phi^*)$. If no hat is used the contractions take place between a starred and an unstarred field or between two different "2" indices. When there is no ambiguity hats are omitted to unclatter the notation. The IBP relations can then be written as:

$$\begin{aligned}
Q_{(40)(00)} &\approx +Q_{(\hat{2}\hat{2})(00)} + 2Q_{(\hat{2}0)(\hat{2}0)} + 4Q_{(\hat{2}1)(10)} + 2Q_{(\hat{2}0)(11)}\,,\\
Q_{(31)(00)} &\approx -Q_{(\hat{2}\hat{2})(00)} - 2Q_{(\hat{2}1)(10)}\,,\\
Q_{(30)(10)} &\approx -Q_{(\hat{2}0)(\hat{2}0)} - Q_{(\hat{2}0)(11)} - Q_{(\hat{2}1)(10)}\,,\\
Q_{(22)(00)} &\approx +Q_{(\hat{2}\hat{2})(00)} + 2Q_{(\hat{2}1)(10)} + Q_{(\hat{1}1)(\hat{1}1)} + Q_{(11)(\hat{2}0)}\,,\\
Q_{(20)(11)} &\approx +\tfrac{1}{2}Q_{(\hat{2}0)(11)} + \tfrac{1}{2}Q_{(\hat{1}1)(\hat{1}1)} - Q_{(11)(11)} - Q_{(10)(\hat{2}1)}\,,\\
Q_{(21)(10)} &\approx -\tfrac{1}{2}Q_{(\hat{1}1)(\hat{1}1)} - \tfrac{1}{2}Q_{(11)(\hat{2}0)}\,,\\
Q_{(20)(20)} &\approx +Q_{(\hat{2}0)(\hat{2}0)} + Q_{(\hat{2}1)(10)} + Q_{(10)(\hat{2}1)}\\
&\qquad + \tfrac{1}{2}Q_{(\hat{2}0)(11)} + \tfrac{1}{2}Q_{(11)(\hat{2}0)} + Q_{(11)(11)}\,.
\end{aligned} \tag{G.35}$$

The corresponding identities for the conjugated operators can be derived by simply swapping the parentheses in every term.

For the operator class $\phi^2\partial^6$ IBP identities are trivial, and, for our convenience later on, we choose to keep the combination of operators

$$j_8\big[(\partial^2\phi)(\partial^4\phi^*) + (\partial^4\phi)(\partial^2\phi^*)\big]\,, \tag{G.36}$$

where the common Wilson coefficient, j_8, should be a real number.

Therefore, the Green basis Lagrangian at D8 is given by

$$\begin{aligned}\mathcal{L}_8 = a_8|\phi|^8 + b_8|\phi|^4|\partial\phi|^2 + c_8(|\partial\phi|^2)^2 + d_8(\partial\phi)^2(\partial\phi^*)^2 + e_8|\phi|^2\left|\partial^2\phi\right|^2 \\ +\big[f_8\phi^*|\phi|^4(\partial^2\phi) + g_8\phi^*|\partial\phi|^2(\partial^2\phi) + h_8\phi(\partial\phi^*)^2(\partial^2\phi) \\ + i_8(\phi^*)^2(\partial^2\phi)^2 + j_8(\partial^2\phi)(\partial^4\phi^*) + \text{c.c.}\big]\,,\end{aligned} \tag{G.37}$$

with the Wilsons in the first line being real numbers and those inside the square parentheses (second and third line) being complex numbers (with the exception of j_8, as discussed above). It is interesting to notice that every operator inside the square parentheses (and the last operator from the first line) is “EoM redundant”. To be more accurate in our terminology, these operators are going to be cancelled after our choice of field redefinitions — their Wilson coefficients, however, will intermingle with those of the first four operators that will end up being our Warsaw basis. Since these Warsaw basis operators are real, each complex Wilson C from the second line will contribute like $C + C^* = 2\,Re\,C$.

G.3.2 Field redefinitions

Let us now derive the field redefinitions to derive a Warsaw basis out of the Green basis at D6 and D8. As before, we are going to use perturbative field redefinitions with the perturbation parameter being the EFT expansion parameter $1/\Lambda$.

For the D6 field redefinitions we use the local function

$$F(\phi) = p_6|\phi|^2\phi^* + q_6\partial^2\phi^*\,, \tag{G.38}$$

and the renormalisable Lagrangian will contribute at D6

$$\delta\mathcal{L}_6 = -\tfrac{1}{\Lambda^2}[F(\phi)E(\phi) + \text{c.c.}]\,, \tag{G.39}$$

where

$$E(\phi) = \partial^2\phi + m^2\phi + \lambda|\phi|^2\phi\,, \tag{G.40}$$

and setting $E(\phi) = 0$ gives us the classical EoM at lowest order for the field ϕ.

In order to cancel the term $\left|\partial^2\phi\right|^2$ we have to set $Re\,q_6 = c_6/2$. In order to cancel the term $|\phi|^2\phi^*\partial^2\phi$ and its charge conjugate we have to set $Re\,p_6 = Re\,d_6 - \lambda c_6/2$ and $Im\,p_6 = Im\,d_6 + \lambda\,Im\,q_6$. For simplicity, we define our parameters to be

$$\begin{aligned} q_6 &= \tfrac{1}{2}c_6\,, \\ p_6 &= d_6 - \tfrac{1}{2}\lambda c_6\,. \end{aligned} \tag{G.41}$$

This field redefinition will affect the Wilson coefficient of the sextic scalar interaction term like

$$a_6' = a_6 - 2\lambda\, Re\, d_6 + \lambda^2 c_6\,, \tag{G.42}$$

whilst $b_6' = b_6$.

In this example we kept the mass term in the model, and the dimensionful parameter m messes with our power counting since it effectively reduces the dimensionality of our operators in the field redefinition by a factor of 2. Therefore, our D6 perturbative field redefinitions will now produce terms of dimension 4, and specifically:

$$\delta\mathcal{L}_6 \supset -\tfrac{1}{\Lambda^2}\left[m^2(2\, Re\, d_6 - \lambda c_6)|\phi|^4 - m^2 c_6 |\partial\phi|^2\right]. \tag{G.43}$$

Of course this is formally consistent with the EFT expansion, we just have to keep track of these contributions to the renormalisable couplings along the way, but the conclusion is that *mass terms mess up with the naive EFT power counting.*

This time the counting of the free parameters in the redefinition is a bit more complicated since we now have to take into account the complex conjugate term in $\delta\mathcal{L}_6$. The function $F(\phi)$ should be constructed in such a way that (i) is Lorentz invariant, (ii) it has mass dimension 3, and (iii) when multiplied with $E(\phi)$ produces terms invariant under the symmetry $\phi \to e^{ia}\phi$. Therefore, the most general $F(\phi)$ depends on the coefficients p_6 and q_6, and these are in general complex numbers. If we expand $\delta\mathcal{L}_6$, however, we will see that it is only affected by $Re\, p_6$, $Re\, q_6$ and $Im(p_6 - \lambda q_6)$. Therefore, we are left with three free parameters instead of four for our redefinition. Thus, the D6 Warsaw basis Lagrangian has exactly two degrees of freedom, as can be seen by our results. In general, the operator B_6 (the one with Wilson coefficient b_6) isn't modified by our field redefinitions and therefore is always present in the Warsaw basis. The second degree of freedom can be chosen to be one of the operators A_6 or C_6, or the linear combination $(D_6 + D_6^\dagger)$.

To derive the D8 Warsaw basis we start with the most general[5] D8 local function for the field redefinition, i.e.

$$\begin{aligned} G(\phi) = {} & p_8|\phi|^4\phi^* + q_8\phi^*|\partial\phi|^2 + r_8\phi(\partial\phi^*)^2 \\ & + s_8|\phi|^2\partial^2\phi^* + t_8(\phi^*)^2\partial^2\phi + u_8\partial^4\phi^*\,. \end{aligned} \tag{G.44}$$

If every parameter was complex then we would have twelve free parameters to choose, and therefore we could restrict our Lagrangian beyond cancelling the ten operators containing contracted higher derivatives in eq. (G.37). Let us ignore for a moment the D6^2 effects, since these will only redefine our Wilsons, and concentrate on how our field redefinitions will

[5] Here we consider the most general function *without* taking into account the massive parameter m^2.

affect the Lagrangian to first order, i.e. through the EoMs. We will see that, without loss of generality, we can take the parameters s_8 and u_8 to be real, which brings us down to the ten degrees of freedom that are dictated by the nature of the problem. Denoting with X_n the operator multiplied by the Wilson x_n, the genuine D8 contribution reads:

$$\begin{aligned}
-\Lambda^4 \delta\mathcal{L}_8 \supset\, & p_8\big[F_8 + m^2 A_6 + \lambda A_8\big] \\
& + q_8\big[G_8 + m^2 B_6 + \lambda B_8\big] \\
& + r_8\Big[H_8 - m^2(2B_6 + D_6^\dagger) - \tfrac{1}{2}\lambda(B_8 + F_8^\dagger)\Big] \\
& + s_8\Big[E_8 + m^2 D_6^\dagger + \lambda F_8^\dagger\Big] \\
& + t_8\big[I_8 + m^2 D_6 + \lambda F_8\big] \\
& + u_8\Big[J_8 + m^2 C_6 + \lambda(2E_8 + 4G_8^\dagger + 2H_8^\dagger + I_8^\dagger)\Big]\,.
\end{aligned} \tag{G.45}$$

We see a major problem here. From the outset we decided to use perturbative field redefinitions so that we can fix our Lagrangian order by order, *without* affecting the lower order terms. But the mass-square term in the Lagrangian reduces the dimensionality of the operators by a degree of two. Therefore our D8 redefinitions, which should affect only D8 and higher terms change our D6 terms and actually introduce again "redundant" D6 operators, suppressed by an extra factor of m^2/Λ^2. To fix this, we must include suitable terms in the field redefinitions by taking into account this m^2 term. Therefore, our full redefinition function would be:

$$\begin{aligned}
G(\phi) = \,& p_8|\phi|^4\phi^* + q_8\phi^*|\partial\phi|^2 + r_8\phi(\partial\phi^*)^2 \\
& + s_8|\phi|^2\partial^2\phi^* + t_8(\phi^*)^2\partial^2\phi + u_8\partial^4\phi^* \\
& + v_8 m^2|\phi|^2\phi^* + w_8 m^2\partial^2\phi^*\,.
\end{aligned} \tag{G.46}$$

These new Wilsons, the complex v_8 and the real w_8, will be utilised to cancel these unwanted contributions to D6. Of course now the redefinition at D8 will introduce D4 terms proportional to those in eq. (G.43), but this will be taken care off when we do the canonicalisation of the kinetic term with a (this time *constant*) field redefinition. Following the procedure we used for the D6 case and taking into account the extra minus sign this time, we choose $w_8 = -u_8$ and $v_8 = -t_8 - s_8^* + v_8^* + \lambda u_8$.

This phenomenon happens iteratively for every subsequent higher order redefinition. For example a purely D10 (with only genuine D10 operators included) redefinition would introduce D8 Green operators. Therefore, we should include redefinitions with m^2/Λ^2 suppression (therefore D8 operators), which in turn would introduce D6 Green operators, and these should be cancelled by introducing m^4/Λ^4 suppressed D6 operators in our redefinition.

Ultimately, for any EFT order, this complicated procedure results in (i) contributions in the Warsaw basis to lower orders, and (ii) changes in the D4 terms of our renormalisable Lagrangian, which will be dealt with later when we canonically normalise our fields to define the mass basis.

The methodology explained in this appendix may prove useful in the future to augment the top-down applications of the SMEFT beyond the leading EFT order, in which case simply ignoring the EoM redundant operators would lead to erroneous results. This project is left for future work.

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