

## Low energy thresholds and the renormalization group in the MSSM

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*Dedicated to the memory of Spyros Vlassopoulos, colleague and friend*

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### Abstract

We derive the 1-loop renormalization group equations for the parameters of the Minimal Supersymmetric Standard Model (MSSM) taking into account the successive decoupling of each sparticle below its threshold. This is realized by a step function at the level of each graph contributing to the renormalization group equations.

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The softly broken version of the Minimal Supersymmetric Standard Model (MSSM) [1] is well known to lead to electroweak symmetry breaking through radiative corrections [2]. The study of radiative corrections is most conveniently done by the use of the renormalization group equations (RGE's) for the running parameters of the model. Ultimately these energy dependent parameters should be related to the physical ones. One popular way to realize this program [3] exclusively in the framework of the (RGE's) is the following:

We derive the renormalization group equations for running masses and couplings in the  $\overline{DR}$  scheme. As we come down in energy and encounter the heaviest particle threshold we switch to another "effective" theory [4] from which the heavy particle is removed and so on. Although the  $\overline{DR}$  scheme is mass independent, we enforce the Decoupling Theorem [5] at the level of the (RGE's) by replacing the full theory by a succession of effective theories. For a particular running mass  $m(Q)$  this treatment of its renormalization group equation will stop when we encounter the corresponding physical mass determined by the condition  $m(m_{r,i}) = m_{ph}$ . Both the step function approximation on the RGE's and this last condition for the physical mass are approximations that keep the leading logarithmic part in 2-point functions and ignore the constant part. The popularity of this scheme is based on the fact that one stays only within the renormalization group equations [6] and does not have to consider the finite parts of the Green functions.

In the present paper we derive in the  $\overline{DR}$  scheme the 1-loop RGE's for both dimensionless (gauge and Yukawa couplings) and dimensionful parameters (soft masses and cubic couplings). At the level of each graph we enforce decoupling by inserting a theta function  $\theta_m \equiv \theta(Q^2 - m^2)$  that counts the contribution of a particle of mass  $m$  at

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energies  $Q > m$ . In order for such a program to be carried out we should calculate the infinities of the two and three point functions of all particles involved. The use of the superfield formalism in a non-supersymmetric theory, although possible through the utilization of spurious superfields, becomes difficult due to the successive decoupling occurring at the component level. One has to resort to the component formulation of the theory where the non-Renormalization Theorem does not apply. The details of the calculation along with the predictions of the MSSM for the particle mass spectrum using the modified RGE's described above will be presented elsewhere [7].

The parameters of the MSSM are defined by the superpotential (suppressing all indices)

$$\mathcal{W} = Y_u Q H_2 U^c + Y_d Q H_1 D^c + Y_e L H_1 E^c + \mu H_1 H_2, \quad (1)$$

and the soft supersymmetry-breaking interaction Lagrangian

$$\begin{aligned} -\mathcal{L}_{\text{soft}} = & \sum_i m_i^2 |\Phi_i|^2 + (Y_u A_u \bar{Q} H_2 \bar{U}^c + Y_d A_d \bar{Q} H_1 \bar{D}^c + Y_e A_e \bar{L} H_1 \bar{E}^c + \text{h.c.}) \\ & + (\mu B H_1 H_2 + \text{h.c.}) + \frac{1}{2} \sum_a M_a \bar{\lambda}_a \lambda_a. \end{aligned} \quad (2)$$

The beta functions for the gauge and Yukawa couplings are, with  $t \equiv \log(Q^2/M_{\text{GUT}}^2)$  and keeping only the Yukawa couplings  $Y_{i,b,\tau}$  of the third generation fermions,

$$\frac{dg_i}{dt} \equiv \beta(g_i) = \frac{b_i}{2(4\pi)^2} T_i g_i^3, \quad i=1, 2, 3, \quad b_{1,2,3} = \frac{33}{5}, 1, -3, \quad (3)$$

$$\frac{dY_\tau}{dt} \equiv \beta(Y_\tau) = \frac{Y_\tau}{(4\pi)^2} \left( -\frac{3}{2} T_{\tau 2} g_2^2 - \frac{9}{10} T_{\tau 1} g_1^2 + 2T_{\tau\tau} Y_\tau^2 + \frac{3}{2} Y_b^2 \right), \quad (4)$$

$$\frac{dY_b}{dt} \equiv \beta(Y_b) = \frac{Y_b}{(4\pi)^2} \left( -\frac{8}{3} T_{b3} g_3^2 - \frac{3}{2} T_{b2} g_2^2 - \frac{7}{30} T_{b1} g_1^2 + \frac{1}{2} T_{bt} Y_t^2 + 3T_{bb} Y_b^2 + \frac{1}{2} Y_\tau^2 \right), \quad (5)$$

$$\frac{dY_t}{dt} \equiv \beta(Y_t) = \frac{Y_t}{(4\pi)^2} \left( -\frac{8}{3} T_{t3} g_3^2 - \frac{3}{2} T_{t2} g_2^2 - \frac{13}{30} T_{t1} g_1^2 + 3T_{tt} Y_t^2 + \frac{1}{2} T_{tb} Y_b^2 \right). \quad (6)$$

The threshold coefficients  $T_i, T_{ei}$ , etc; appearing in the expressions above are shown in Table 1.

The beta functions for the cubic couplings are<sup>3</sup>

$$\begin{aligned} \frac{dA_\tau}{dt} = & \frac{1}{(4\pi)^2} \left[ -3g_2^2 M_2 \theta_{\bar{W}H_1} - \frac{3}{5} g_1^2 M_1 (2 + \theta_{H_1}) \theta_{\bar{B}} + 3Y_b^2 A_b \theta_{\bar{B}Q} + 4Y_\tau^2 A_\tau \right. \\ & \left. + A_\tau (Z_{\tau 1} g_1^2 + Z_{\tau 2} g_2^2 + Z_{\tau\tau} Y_\tau^2) \right], \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{dA_b}{dt} = & \frac{1}{(4\pi)^2} \left[ -\frac{16}{3} g_3^2 M_3 \theta_G - 3g_2^2 M_2 \theta_{\bar{W}H_1} - \frac{1}{30} g_1^2 M_1 (-4 + 18\theta_{H_1}) \theta_{\bar{B}} + Y_\tau^2 A_\tau \theta_{\bar{E}L} + Y_t^2 A_t \theta_{H_2\bar{U}} \right. \\ & \left. + 6Y_b^2 A_b + A_b (Z_{b3} g_3^2 + Z_{b2} g_2^2 + Z_{b1} g_1^2 + Z_{bt} Y_t^2 + Z_{bb} Y_b^2) \right], \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{dA_t}{dt} = & \frac{1}{(4\pi)^2} \left[ -\frac{16}{3} g_3^2 M_3 \theta_G - 3g_2^2 M_2 \theta_{\bar{W}H_2} - \frac{1}{15} g_1^2 M_1 (4 + 9\theta_{H_2}) \theta_{\bar{B}} + 5Y_t^2 A_t \theta_{H_1\bar{U}} + Y_b^2 A_b \theta_{H_1\bar{D}} \right. \\ & \left. + A_t (Z_{t3} g_3^2 + Z_{t2} g_2^2 + Z_{t1} g_1^2 + Z_{tt} Y_t^2 + Z_{tb} Y_b^2) \right]. \end{aligned} \quad (9)$$

In our notation  $\theta_{ab} \equiv \theta_a \theta_b$ ,  $\theta_{abc} \equiv \theta_a \theta_b \theta_c$ . In all the expressions throughout this paper we assume that the Yukawa couplings are diagonal in family space. The coefficients  $Z_{qi}$  in the expressions for the cubic couplings above are given in Table 2; above all thresholds they vanish and in this case one recovers the well known RGEs.

<sup>3</sup>  $\bar{C}, \bar{W}, \bar{B}$  denote the SU(3), SU(2) and U(1) gauge fermions respectively.  $\bar{H}_1, \bar{H}_2$  are Higgs fermions (Higgsinos).

Table 1

Threshold coefficients appearing in the renormalization group equations of the gauge and Yukawa couplings. Above all thresholds these become equal to unity

$$\begin{aligned}
 T_1 &= \frac{1}{33} \left( 20 + \theta_{H_1} + \theta_{H_2} + \frac{1}{2}(\theta_{H_1} + \theta_{H_2}) + \sum_{i=1}^3 (\frac{1}{2}\theta_{L_i} + \theta_{E_i} + \frac{1}{6}\theta_{Q_i} + \frac{1}{3}\theta_{D_i} + \frac{1}{3}\theta_{S_i}) \right) \\
 T_2 &= -\frac{49}{3} + \frac{1}{3}\theta_W + \frac{1}{3}(\theta_{H_1} + \theta_{H_2}) + \frac{1}{6}(\theta_{H_1} + \theta_{H_2}) + \frac{1}{6} \sum_{i=1}^3 (3\theta_{Q_i} + \theta_{L_i}) \\
 T_3 &= \frac{7}{3} - \frac{2}{3}\theta_G - \frac{1}{18} \sum_{i=1}^3 (2\theta_{Q_i} + \theta_{D_i} + \theta_{S_i}) \\
 T_{t_1} &= \frac{1}{4}(-1 + 4\theta_{H_1} - 2\theta_{H_1W} - \theta_{LW} + 4\theta_{H_1LW}) \\
 T_{t_2} &= \frac{1}{12}(11 - 4\theta_{BE} + 8\theta_{BEH_1} - 2\theta_{BH_1} + 4\theta_{H_1} - \theta_{BL} - 4\theta_{BLH_1}) \\
 T_{t_3} &= \frac{1}{4}(2 + \theta_{BE} + 3\theta_{H_1} + 2\theta_{LH_1}) \\
 T_{b_3} &= \frac{1}{4}(6 - \theta_{G\bar{D}} - \theta_{G\bar{Q}}) \\
 T_{b_2} &= \frac{1}{4}(-1 + 4\theta_{H_1} - 2\theta_{H_1\bar{Q}} - \theta_{Q\bar{W}} + 4\theta_{H_1\bar{Q}W}) \\
 T_{b_1} &= \frac{1}{24}(-21 - 4\theta_{B\bar{D}} - 12\theta_{H_1\bar{D}} + 24\theta_{H_1\bar{D}B} + 36\theta_{H_1} - \theta_{B\bar{Q}} + 12\theta_{H_1\bar{Q}B}) \\
 T_{b_2'} &= \frac{1}{2}(\theta_{H_2} + \theta_{H_2\bar{D}}) \\
 T_{b_3'} &= \frac{1}{12}(6 + \theta_{LH_1} + 2\theta_{QH_1}) \\
 T_{\tau_3} &= \frac{1}{4}(6 - \theta_{G\bar{Q}} - \theta_{G\bar{D}}) \\
 T_{\tau_2} &= \frac{1}{4}(-1 + 4\theta_{H_2} - 2\theta_{H_2\bar{W}} - \theta_{Q\bar{W}} + 4\theta_{H_2\bar{Q}W}) \\
 T_{\tau_1} &= \frac{1}{52}(15 - 18\theta_{H_2\bar{B}} + 36\theta_{H_2} - \theta_{B\bar{Q}} - 12\theta_{H_2\bar{Q}B} - 16\theta_{B\bar{D}} + 48\theta_{H_2\bar{D}}) \\
 T_{\nu_1} &= \frac{1}{12}(6 + 3\theta_{H_2} + 2\theta_{H_2\bar{Q}} + \theta_{H_2\bar{D}}) \\
 T_{\nu_2} &= \frac{1}{2}(\theta_{H_1} + \theta_{H_1\bar{D}})
 \end{aligned}$$

Table 2

Threshold coefficients appearing in the renormalization group equations of the trilinear scalar couplings. Above all thresholds these are vanishing

$$\begin{aligned}
 Z_{t_1} &= \frac{3}{40}(11 + 10\theta_B - 8\theta_E - 4\theta_{BE} + 8\theta_{BEH_1} + 2\theta_{H_1} - 8\theta_{E_1L} - 2\theta_L - \theta_{BL} - 8\theta_{EL} - 4\theta_{BH_1L} + 4\theta_{H_1L}) \\
 Z_{t_2} &= \frac{1}{8}(-3 + 6\theta_{H_1} - 6\theta_L - 12\theta_{H_1L} + 6\theta_W - 3\theta_{LW} + 12\theta_{WH_1L}) \\
 Z_{t_3} &= \frac{1}{4}(-16 + 6\theta_{H_1} - \theta_{H_1L} - 3\theta_{H_1} + 4\theta_{H_1L} + 4\theta_{EL} - 2\theta_{H_1L} + 8\theta_{H_1L}) \\
 Z_{b_3} &= \frac{3}{4}(6 - 2\theta_D + 4\theta_G - \theta_{G\bar{D}} - 2\theta_{\bar{Q}} - 4\theta_{D\bar{Q}} - \theta_{G\bar{Q}}) \\
 Z_{b_2} &= \frac{1}{4}(-3 + 6\theta_{H_1} - 6\theta_{\bar{Q}} - 12\theta_{H_1\bar{Q}} + 6\theta_W - 3\theta_{Q\bar{W}} + 12\theta_{WH_1\bar{Q}}) \\
 Z_{b_1} &= \frac{1}{120}(-21 + 10\theta_B - 8\theta_D - 4\theta_{B\bar{D}} + 24\theta_{B\bar{D}H_1} + 18\theta_{H_1} - 24\theta_{H_1\bar{D}} - 2\theta_{\bar{Q}} - \theta_{Q\bar{B}} + 8\theta_{Q\bar{D}} + 12\theta_{B\bar{Q}H_1} - 12\theta_{H_1\bar{Q}}) \\
 Z_{b_2'} &= \frac{1}{4}(-24 + 6\theta_{H_1} - \theta_{H_1\bar{D}} - 3\theta_{H_1} + 4\theta_{H_1\bar{D}} + 12\theta_{Q\bar{D}} - 2\theta_{Q\bar{H}_1} + 8\theta_{H_1\bar{Q}}) \\
 Z_{b_3'} &= \frac{1}{4}(2\theta_{H_1} - 6\theta_{H_2} - \theta_{H_2\bar{D}}) \\
 Z_{\tau_3} &= \frac{3}{4}(6 - 2\theta_{\bar{Q}} + 4\theta_G - \theta_{G\bar{Q}} - 2\theta_{\bar{D}} - 4\theta_{D\bar{Q}} - \theta_{G\bar{D}}) \\
 Z_{\tau_2} &= \frac{1}{8}(-3 + 6\theta_{H_2} - 6\theta_{\bar{Q}} - 12\theta_{H_2\bar{Q}} + 6\theta_W - 3\theta_{Q\bar{W}} + 12\theta_{WH_2\bar{Q}}) \\
 Z_{\tau_1} &= \frac{1}{120}(15 + 34\theta_B + 18\theta_{H_2} - 2\theta_{\bar{Q}} - \theta_{B\bar{Q}} - 12\theta_{H_2\bar{Q}} + 12\theta_{H_2\bar{Q}} - 32\theta_{\bar{D}} - 16\theta_{B\bar{D}} + 48\theta_{B\bar{D}H_2} - 48\theta_{H_2\bar{D}} - 16\theta_{D\bar{Q}}) \\
 Z_{\nu_1} &= \frac{1}{4}(2\theta_{H_1} - \theta_{H_1} - \theta_{H_1\bar{D}}) \\
 Z_{\nu_2} &= \frac{1}{4}(-24\theta_{H_1\bar{D}} + 6\theta_{H_2} - \theta_{H_2\bar{D}} - 3\theta_{H_2} + 4\theta_{H_2\bar{D}} + 12\theta_{Q\bar{D}} - 2\theta_{Q\bar{H}_2} + 8\theta_{H_2\bar{Q}})
 \end{aligned}$$

The beta functions for the scalar masses are given by the following RGEs<sup>4</sup>. The masses  $m_{1,2}^2$  appearing in the RGEs below refer to the Higgs masses squared which are related to the their soft masses  $m_{H_1,H_2}^2$  and the mixing parameter  $\mu$  by  $m_{1,2}^2 = m_{H_1,H_2}^2 + \mu^2$ .

<sup>4</sup> The RGEs for the soft masses of the squarks and sleptons presented here refer to the third generation of fermions. For the first two generations the Yukawa couplings should be set to zero.

$$\begin{aligned} \frac{dm_{\tilde{Q}}^2}{dt} = & \frac{1}{(4\pi)^2} \{ -[\frac{8}{3}g_3^2(\theta_{\tilde{Q}} - \theta_G) + \frac{2}{3}g_2^2(\theta_{\tilde{Q}} - \theta_W) + \frac{1}{30}g_1^2(\theta_{\tilde{Q}} - \theta_B)]m_{\tilde{Q}}^2 \\ & - \frac{16}{3}g_3^2M_3^2\theta_G - 3g_2^2M_2^2\theta_W - \frac{1}{15}g_1^2M_1^2\theta_B + \frac{1}{10}g_1^2S \\ & + Y_t^2[m_{\tilde{Q}}^2\theta_t + m_{\tilde{Q}}^2\theta_U + m_{\tilde{Q}}^2\theta_{H_2} + A_t^2\theta_{H_2\tilde{Q}} + \mu^2(\theta_{H_1\tilde{Q}} - 2\theta_{H_2})] \\ & + Y_b^2[m_{\tilde{Q}}^2\theta_b + m_{\tilde{Q}}^2\theta_D + m_{\tilde{Q}}^2\theta_{H_1} + A_b^2\theta_{H_1\tilde{Q}} + \mu^2(\theta_{H_2\tilde{Q}} - 2\theta_{H_1})] \}. \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{dm_{\tilde{U}}^2}{dt} = & \frac{1}{(4\pi)^2} \{ -[\frac{8}{3}g_3^2(\theta_{\tilde{U}} - \theta_G) + \frac{2}{3}g_2^2(\theta_{\tilde{U}} - \theta_B)]m_{\tilde{U}}^2 - \frac{16}{3}g_3^2M_3^2\theta_G - \frac{16}{15}g_1^2M_1^2\theta_B - \frac{2}{3}g_1^2S \\ & + 2Y_t^2[m_{\tilde{U}}^2\theta_t + m_{\tilde{U}}^2\theta_Q + m_{\tilde{U}}^2\theta_{H_2} + A_t^2\theta_{H_2\tilde{Q}} + \mu^2(\theta_{H_1\tilde{Q}} - 2\theta_{H_2})] \}. \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{dm_{\tilde{D}}^2}{dt} = & \frac{1}{(4\pi)^2} \{ -[\frac{8}{3}g_3^2(\theta_{\tilde{D}} - \theta_G) + \frac{2}{3}g_2^2(\theta_{\tilde{D}} - \theta_B)]m_{\tilde{D}}^2 - \frac{16}{3}g_3^2M_3^2\theta_G - \frac{4}{15}g_1^2M_1^2\theta_B + \frac{1}{3}g_1^2S \\ & + 2Y_b^2[m_{\tilde{D}}^2\theta_b + m_{\tilde{D}}^2\theta_Q + m_{\tilde{D}}^2\theta_{H_1} + A_b^2\theta_{H_1\tilde{Q}} + \mu^2(\theta_{H_2\tilde{Q}} - 2\theta_{H_1})] \}. \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{dm_{\tilde{L}}^2}{dt} = & \frac{1}{(4\pi)^2} \{ -[\frac{2}{3}g_2^2(\theta_{\tilde{L}} - \theta_W) + \frac{1}{10}g_1^2(\theta_{\tilde{L}} - \theta_B)]m_{\tilde{L}}^2 - 3g_2^2M_2^2\theta_W - \frac{2}{3}g_1^2M_1^2\theta_B - \frac{2}{10}g_1^2S \\ & + Y_e^2[m_{\tilde{L}}^2\theta_e + m_{\tilde{L}}^2\theta_E + m_{\tilde{L}}^2\theta_{H_1} + A_e^2\theta_{H_1\tilde{L}} + \mu^2(\theta_{H_2\tilde{L}} - 2\theta_{H_1})] \}. \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{dm_{\tilde{E}}^2}{dt} = & \frac{1}{(4\pi)^2} \{ -[\frac{2}{3}g_1^2(\theta_{\tilde{E}} - \theta_B)]m_{\tilde{E}}^2 - \frac{12}{5}g_1^2M_1^2\theta_B + \frac{1}{5}g_1^2S \\ & + 2Y_e^2[m_{\tilde{E}}^2\theta_e + m_{\tilde{E}}^2\theta_L + m_{\tilde{E}}^2\theta_{H_1} + A_e^2\theta_{H_1\tilde{L}} + \mu^2(\theta_{H_2\tilde{L}} - 2\theta_{H_1})] \}. \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{dm_{\tilde{H}}^2}{dt} = & \frac{1}{(4\pi)^2} \{ -[\frac{2}{3}g_2^2(\theta_{H_1} - \theta_{H_1,W}) + \frac{1}{10}g_1^2(\theta_{H_1} - \theta_{H_1,B})]m_{\tilde{H}}^2 - 3g_2^2(M_2^2 + \mu^2)\theta_{H_1,W} - \frac{2}{5}g_1^2(M_1^2 + \mu^2)\theta_{H_1,B} \\ & - \frac{1}{10}g_1^2S + Y_t^2(m_{\tilde{Q}}^2 + m_{\tilde{U}}^2\theta_U + m_{\tilde{D}}^2\theta_D + A_t^2\theta_{Q\tilde{U}}) + 3Y_b^2(m_{\tilde{Q}}^2 + m_{\tilde{D}}^2\theta_D + A_b^2\theta_{Q\tilde{D}}) + 3Y_e^2\mu^2\theta_{Q\tilde{E}} \}. \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{dm_{\tilde{H}'}^2}{dt} = & \frac{1}{(4\pi)^2} \{ -[\frac{2}{3}g_2^2(\theta_{H_2} - \theta_{H_2,W}) + \frac{1}{10}g_1^2(\theta_{H_2} - \theta_{H_2,B})]m_{\tilde{H}'}^2 - 3g_2^2(M_2^2 + \mu^2)\theta_{H_2,W} - \frac{2}{5}g_1^2(M_1^2 + \mu^2)\theta_{H_2,B} \\ & - \frac{1}{10}g_1^2S + 3Y_t^2(m_{\tilde{Q}}^2 + m_{\tilde{U}}^2\theta_U + m_{\tilde{D}}^2\theta_D + A_t^2\theta_{Q\tilde{U}}) + 3Y_b^2\mu^2\theta_{Q\tilde{D}} + Y_e^2\mu^2\theta_{E\tilde{E}} \}. \end{aligned} \quad (16)$$

The quantity  $S$  appearing in the equations above is defined as

$$S \equiv \text{Tr}[\frac{1}{2}Y\theta_m m^2]. \quad (17)$$

This vanishes if universal boundary conditions are assumed for all soft scalar masses involved at the unification scale, as long as we are above all particle thresholds. This is due to the fact that  $S$  is multiplicatively renormalized.

The RGEs given above refer to the third generation. For the first two generations we have just to make the appropriate replacements for the Yukawa couplings which, due to their smallness, we have assumed zero.

For the Higgs and Higgsino mixing parameters  $m_{\tilde{H}}^2 \equiv B\mu$  and  $\mu$  respectively we have

$$\begin{aligned} \frac{dm_{\tilde{g}}^2}{dt} = & \frac{1}{(4\pi)^2} \left[ \left( -\frac{3}{2}g_3^2(\theta_{H_1} + \theta_{H_2} + 2\theta_{H_1H_2} - \theta_{H_1\bar{W}} - \theta_{H_2\bar{W}}) - \frac{3}{20}g_2^2(\theta_{H_1} + \theta_{H_2} + 2\theta_{H_1H_2} - \theta_{H_1B} - \theta_{H_2B}) \right. \right. \\ & \left. \left. + \frac{3}{2}Y_1^2 + \frac{3}{2}Y_2^2 + \frac{1}{2}Y_3^2 \right) m_{\tilde{g}}^2 \right. \\ & \left. + \mu \left( -3g_3^2 M_2 \theta_{H_1H_2\bar{W}} - \frac{3}{2}g_2^2 M_1 \theta_{H_1H_2B} + 3A_1 Y_1^2 \theta_{Q\bar{Q}} + 3A_2 Y_2^2 \theta_{Q\bar{Q}} + A_3 Y_3^2 \theta_{L\bar{L}} \right) \right], \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{d\mu}{dt} = & \frac{1}{(4\pi)^2} \left[ \frac{3}{2}g_3^2(\theta_{H_1} + \theta_{H_2} - 8\theta_{H_1H_2} + \theta_{H_1\bar{W}} + \theta_{H_2\bar{W}}) + \frac{3}{20}g_2^2(\theta_{H_1} + \theta_{H_2} - 8\theta_{H_1H_2} + \theta_{H_1B} + \theta_{H_2B}) \right. \\ & \left. + \frac{3}{2}Y_2^2(\theta_Q + \theta_{\bar{Q}}) + \frac{3}{2}Y_1^2(\theta_Q - \theta_{\bar{Q}}) + \frac{1}{2}Y_3^2(\theta_L + \theta_{\bar{L}}) \right] \mu. \end{aligned} \quad (19)$$

Finally the beta functions for the three gaugino masses are

$$\frac{dM_i}{dt} = S_i \frac{b_i}{(4\pi)^2} M_i, \quad i=1, 2, 3 \quad (20)$$

where  $b_i$  are the beta function coefficients of the gauge couplings above all particle thresholds, i.e.

$$b_{1,2,3} = \frac{11}{3}, 1, -3, \quad (21)$$

and  $S_i$  are threshold function coefficients given by

$$S_1 = -3\theta_3 - \frac{1}{2} \sum_{i=1}^{N_c} (2\theta_{Q_i} + \theta_{L_i} + \theta_{\bar{Q}_i}), \quad (22)$$

$$S_2 = -6\theta_2 - \frac{1}{2} \sum_{i=1}^{N_c} (3\theta_{Q_i} + \theta_{L_i}) - \frac{1}{2}(\theta_{H_1H_2} + \theta_{H_1H_2\bar{W}}), \quad (23)$$

$$S_3 = \frac{1}{11} \left( \sum_{i=1}^{N_c} (\frac{1}{2}\theta_{Q_i} + \frac{1}{2}\theta_{L_i} + \frac{1}{2}\theta_{\bar{Q}_i} + \frac{1}{2}\theta_{L_i} + \theta_{H_i}) + \frac{1}{2}(\theta_{H_1H_2} + \theta_{H_1H_2\bar{W}}) \right). \quad (24)$$

Electroweak symmetry breaking effects, among which especially those expressed directly through  $m_t$ , might play a role, have not been included above. Note that the threshold effects computed above at the one-loop level are expected to be of the same order of magnitude as the standard 2-loop contributions to the RGE's [2]. Work on these subjects is in progress.

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