

Naturally light neutrinos in the flipped $SU(5) \times U(1)$ superstring model

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We analyze the $SU(5) \times U(1)' \times U(1)^4 \times SO(10) \times SU(4)$ superstring model, taking into account non-renormalizable superpotential interactions up to sixth order, and find that all neutrinos stay naturally light within the experimental mass bounds.

In a recent article [1] we analyzed the $SU(5) \times U(1)' \times U(1)^4 \times SO(10) \times SU(4)$ superstring model [2–4] with a spontaneously broken hidden sector down to $SO(7) \times SO(5)$ taking into account non-renormalizable superpotential terms up to eighth order. The result of this analysis was to determine a VEV pattern that satisfies F- and D-flatness and predicts the standard matter spectrum at low energies with an almost realistic hierarchical mass structure for charged matter fermions. The purpose of this letter is to complete the analysis by addressing the problem of neutrino masses.

The chiral massless spectrum of the model [2] contains seventy chiral superfields^{#1}. The solution of the F- and D-flatness equations proposed in ref. [1], assigns non-vanishing VEVs to the singlet fields $\Phi_{31}, \bar{\Phi}_{31}, \Phi_{23}, \bar{\Phi}_{23}, \phi_{45}, \bar{\phi}_{45}, \phi_2, \phi_4, \bar{\phi}_4$, as well as to the $SU(5) \times U(1)'$ breaking VEVs $\bar{F}_5 = F_1 = V$, and the $SO(10) \times SU(4)$ breaking VEVs T_2, T_3, T_4, T_5 (decaplets) and A_2, A_3 (sextets). The VEV pattern for the hidden sector fields can be expressed in terms of four real parameters $\alpha, \beta, \gamma, \delta$ as follows:

$$A_2 = (\alpha, 0, \dots, 0), \quad A_3 = (\beta, 0, \dots, 0), \\ T_4 = (\gamma, i\gamma, 0, \dots, 0), \quad T_5 = \delta(\gamma, i\gamma, 0, \dots, 0), \quad T_3 = (1 + \delta^2)^{1/2}(\gamma, -i\gamma, 0, \dots, 0), \quad T_2 = (0, 0, i\alpha, 0, \dots, 0). \quad (1)$$

According to ref. [1] the flatness constraints determine all the above mentioned VEVs in terms of four independent parameters which can be chosen to be γ, V, ϕ_4 and Φ_{23} . A characteristic of this solution is that the F-flatness constraints are satisfied “approximately” in the sense that they are violated at most by $(10^{10} \text{ GeV})^2$ an amount expected to be tolerated by the supersymmetry breaking. This is achieved by choosing $T_3, T_4, T_5 \sim 10^{14} \text{ GeV}$ which requires $\gamma \equiv T = 10^{-4}$ in units of $M \sim 10^{18} \text{ GeV}$. The solution also requires that $\Phi_{23}, \bar{\Phi}_{23}$ are of $O(1)$ which determines the order of the remaining VEVs:

$$\{\phi_{45}, \bar{\phi}_{45}, \phi_2, \bar{\phi}_4\} \sim \phi_4 \equiv \phi, \quad \{\Phi_{31}, \bar{\Phi}_{31}\} \sim \phi^2, \quad \{\alpha, \beta, \delta\} \sim \phi, \quad (2)$$

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^{#1} For the details and the notation see refs. [2,1,4].

where $\phi \sim 10^{-1}$ as required by the cancelation of the anomalous D-term [2]. Finally for the $SU(5) \times U(1)'$ breaking scale we assume $V \sim 10^{-2} - 10^{-1}$ with preference to the higher value as suggested in ref. [5].

The above flatness solution leads to the following particle assignments:

$$\begin{aligned}
 F_4 &\equiv ((t, b); b^c; \nu_4^c), \quad F_2 \equiv ((c, s); s^c; \nu_2^c), \quad F_3 \equiv ((u, d); d^c; \nu_3^c), \\
 \bar{f}_5 &\equiv ((\mu, \nu_\mu); t^c), \quad \bar{f}_2 \equiv ((e, \nu_e); c^c), \quad \bar{f}_1 \equiv ((\tau, \nu_\tau); u^c), \\
 l_5^c &\equiv \mu^c, \quad l_2^c \equiv e^c, \quad l_1^c \equiv \tau^c.
 \end{aligned} \tag{3}$$

There also remain two pairs of massless Higgs isodoublets, which consist of two linear combinations of h_1, h_2, h_{45} giving masses to down quarks and charged leptons, and two of $\bar{h}_1, \bar{h}_2, \bar{h}_{45}$ giving masses to up quarks and neutrinos [1].

According to (3) the three left-handed neutrinos ν_e, ν_μ, ν_τ are contained in $\bar{f}_2, \bar{f}_3, \bar{f}_1$ while the three right-handed $\nu_2^c, \nu_3^c, \nu_4^c$ are contained in F_2, F_3, F_4 . There is also an additional right-handed neutrino state corresponding to the surviving combination of F_1 and \bar{F}_5 after the $SU(5) \times U(1)'$ symmetry breaking. This state is $\nu_1^c \equiv (\nu_1^c + \bar{\nu}_5^c) \sqrt{2}$ while $\nu_5^c \equiv (\nu_1^c - \bar{\nu}_5^c) \sqrt{2}$ is absorbed. Both types of neutrinos mix with other neutral fields. These can either be $SU(5) \times U(1)' \times U(1)^4 \times SO(10) \times SU(4)$ singlets or $SU(5) \times U(1)' \times SO(7) \times SO(5)$ singlets that are parts of the broken decaplets or sextets of the hidden sector. We have looked for mixing terms with these fields, up to sixth order of non-renormalizable superpotential couplings. A simplifying fact is that each neutrino mixes with different singlet fields. This allows to consider every neutrino state separately.

For the electron neutrino ν_e the relevant superpotential terms are the following

$$\begin{aligned}
 W_e &\equiv F_2 \bar{f}_2 \phi_4 \bar{h}_{45} + \bar{F}_5 F_2 T_4 T_2 \phi_2 + \Delta_5 \Delta_2 T_4 T_2 \phi_2 + \Delta_5^2 (\bar{\Phi}_{31} + \bar{\phi}_4^2 \Phi_{23}) + \Delta_4^2 \bar{\Phi}_{23} + T_4 T_5 \phi_2 + T_4^2 \Phi_{23} \\
 &+ T_2^2 [\Phi_{31} + \bar{\Phi}_{23} (\phi_2^2 + \phi_4^2)] + T_5^2 [\Phi_{31} + \bar{\Phi}_{23} (\phi_2^2 + \phi_4^2)] + \bar{F}_5 F_3 \Delta_5 \Delta_3 (1 + \Phi_{23} \bar{\Phi}_{23}) + \bar{F}_5 F_2 \Delta_4 \Delta_2 \bar{\Phi}_{23} \phi_4,
 \end{aligned} \tag{4}$$

where we have omitted the calculable coefficients of the Yukawa couplings and we have neglected higher order terms which are corrections to non-vanishing lower order ones. Taking into account the VEV pattern of eq. (1) we conclude that the singlets that mix with ν_e are the singlet components of F_2 (ν_2^c) and F_3 (ν_3^c), a linear combination of the singlet components of T_2 ($T_2^{(1+i2)} \equiv T_2^{(1)} + iT_2^{(2)}$), the third components of T_4 and T_5 ($T_4^{(3)}, T_5^{(3)}$) and the first components of Δ_4 and Δ_5 , ($\Delta_4^{(1)}, \Delta_5^{(1)}$). The corresponding mass matrix takes the form

$$M_1 \equiv \begin{pmatrix} \nu_e & \nu_2^c & T_2^{(1+i2)} & T_4^{(3)} & T_5^{(3)} & \Delta_4^{(1)} & \Delta_5^{(1)} & \nu_3^c \\ \nu_e & 0 & c_1 \phi \nu & 0 & 0 & 0 & 0 & 0 \\ \nu_2^c & c_1 \phi \nu & 0 & c_2 TV \phi & c_3 V \phi^2 & 0 & c_4 V \phi^2 & 0 \\ T_2^{(1+i2)} & 0 & c_2 TV \phi & c_{13} \phi^2 & 0 & 0 & c_5 T \phi^2 & 0 \\ T_4^{(3)} & 0 & c_3 V \phi^2 & 0 & c_6 & c_7 \phi & 0 & c_8 \phi^3 \\ T_5^{(3)} & 0 & 0 & 0 & c_7 \phi & c_9 \phi^2 & 0 & 0 \\ \Delta_4^{(1)} & 0 & c_4 V \phi^2 & 0 & 0 & 0 & c_{10} & 0 \\ \Delta_5^{(1)} & 0 & 0 & c_5 T \phi^2 & c_8 \phi^3 & 0 & 0 & c_{11} \phi^2 \\ \nu_3^c & 0 & 0 & 0 & 0 & 0 & c_{12} V \phi & 0 \end{pmatrix}, \tag{5}$$

where $\nu \sim 10^{-16}$ is the order of the VEV of the Higgs isodoublets. In (5) we have introduced the $O(1)$ coefficients c_i which account for the contribution of both the calculable numerical values of the Yukawa couplings and the numerical part of the various VEVs entering in eq. (2). These coefficients are constrained by the F-flatness equations [1] as follows:

$$c_{13} = 0, \quad c_7^2 = c_6 c_9. \tag{6}$$

Furthermore by definition we have the relation

$$c_3 c_5 = c_2 c_8. \tag{7}$$

The mass matrix in (5), using eqs. (6), (7), is found to have:

(a) One massless eigenstate

$$T_0 \sim \frac{c_5}{c_8} \frac{T}{\phi} \left(\frac{c_6}{c_7} \frac{1}{\phi} T_5^{(3)} - T_4^{(3)} \right) + T_2^{(1+i2)} + O\left(\frac{T}{\phi^2}\right). \quad (8)$$

The state (8) is not a physical state because it corresponds to the linear combination absorbed by the Higgs mechanism which breaks the SO(10) hidden sector group.

(b) Six superheavy eigenstates: Two of O(1), one of O(φ²), one of O(V²) and two of O(Vφ³).

(c) One more massless eigenstate which is essentially ν_e. However, this result is no longer true in higher orders. Already in seventh order, the superpotential coupling $\bar{F}_5^2 F_3^2 A_3^2 \Phi_{31}$, appears, which leads to a term V²φ⁴(ν₃^c)². This term, when included in the matrix M₁, although it has no effect on the superheavy eigenvalues, it gives a tiny Majorana mass to ν_e of the order v²φ⁶/V² ~ 10⁻⁷ eV. For the muon neutrino ν_μ the relevant superpotential terms are

$$W_\mu \equiv F_4 \bar{f}_5 \bar{h}_{45} + F_4 \bar{F}_5 \phi_3 + \Phi_5 \phi_3 \bar{\phi}_4 + \bar{F}_5 F_1 T_4 T_1 \phi_3 + T_1^2 \Phi_{23} + F_1 \bar{f}_5 \bar{h}_{45} T_4 T_1. \quad (9)$$

In this case the singlets that mix with ν_μ are φ₃, $\bar{\phi}_3$, Φ₅, the singlet component of F₄ (ν₄^c) and a linear combination of the singlet components of T₁ (T₁⁽¹⁺ⁱ²⁾ ≡ T₁⁽¹⁾ + iT₁⁽²⁾). The resulting mass matrix takes the form

$$M_2 \equiv \begin{pmatrix} \nu_\mu & \nu_4^c & \phi_3 & \bar{\phi}_3 & \Phi_5 & T_1^{(1+i2)} \\ \nu_\mu & 0 & c'_1 v & 0 & 0 & c'_2 TVv \\ \nu_4^c & c'_1 v & 0 & c'_3 V & 0 & 0 \\ \phi_3 & 0 & c'_3 V & 0 & c'_4 \phi & c'_5 TV^2 \\ \bar{\phi}_3 & 0 & 0 & 0 & c'_6 \phi & 0 \\ \Phi_5 & 0 & 0 & c'_4 \phi & c'_6 \phi & 0 \\ T_1^{(1+i2)} & c'_2 TVv & 0 & c'_5 TV^2 & 0 & c'_7 \end{pmatrix}, \quad (10)$$

The mass matrix M₂ has:

(a) Five superheavy eigenstates: One of O(1), two of O(φ) and two of O(V).

(b) One almost massless eigenstate which is essentially ν_μ; its mass is of the order v²T²V² ~ 10⁻¹⁵ eV. Note that this negligible Majorana mass is much smaller than a possible radiatively generated Dirac mass, after the supersymmetry breaking, which would be of the order v⟨F⟩ ~ 10⁻⁵ eV apart from coupling constant suppressions. Finally for the τ neutrino ν_τ there is only one relevant superpotential term up to sixth order:

$$W_\tau \equiv F_1 \bar{f}_1 \phi_1 \bar{h}_{45}, \quad (11)$$

which mixes ν_τ with the singlet φ₁ through the coupling vV(ν_τφ₁). The state φ₁ does not have a mass neither does it mix with any of the other singlets to the examined order. If a mass is to arise from higher order superpotential terms, then the corresponding see-saw type mass matrix would lead to a Majorana mass for ν_τ in the eV range. In any case, a possible lower bound for the mass of the fermion field in φ₁ is set by the estimate of a radiatively generated mass, after the supersymmetry breaking, which is of the order of ⟨Φ⟩⟨F⟩ ≲ 10² GeV, where ⟨Φ⟩ denotes one of the singlet VEVs. This leads to an estimate of an upper bound for the ν_τ mass of the order v²V²/ (100 GeV) ~ 0.01–0.1 GeV, which is very close to the experimental limit of 0.35 GeV.

As far as the leftover singlet ν_τ^c is concerned, it mixes non-trivially with all remaining singlets of the model. The solution of the general mixing problem is very complicated, especially when the singlets with non-vanishing VEVs are also included. However, one expects that all fields which do not correspond to exact flat directions, will acquire masses. In addition, supersymmetry breaking effects are expected to destroy all flat directions and provide with masses all the remaining fields.

In conclusion, we have studied the coupled neutrino-singlet system in the framework of the flipped SU(5) superstring model with partially broken hidden sector. We have found all neutrino mass terms and mixings up

to the sixth order of non-renormalizable superpotential interactions and concluded that all neutrinos stay naturally light within experimental limits.

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