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# Neutrino masses and abelian lepton flavor symmetry

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Neutrino masses incorporating the controversial 17 keV neutrino can be accounted for in models with only left-handed neutrinos based on the global symmetry  $U(1)_{e-\mu+\tau}$ . Consequences of this being an exact, as well as an approximate symmetry are discussed.

There are some recent crystalline detector experiments [1] which give evidence for the existence of a 17 keV neutrino, which mixes with the electron neutrino at the probability level of 1%. This is by no means new [2] and has been the basis of a continuing experimental controversy over the past seven years, since magnetic spectrometer experiments [3] do not report such an evidence.

A large number of theoretical papers [4–6] have appeared presenting and analyzing models to explain the 17 keV neutrino state. An interesting class of them introduces a sterile neutrino  $v_s$ . Then either [4]  $v_\tau$  and  $v_s$  form a pseudo-Dirac pair with a mass of 17 keV and matter enhanced neutrino  $v_e \leftrightarrow v_\mu$  oscillations explain the solar neutrino deficit, or [5]  $v_\mu$  and  $v_\tau$  form a 17 keV pseudo-Dirac pair and matter enhanced  $v_e \leftrightarrow v_s$  oscillations realize the MSW scenario.

Assuming that such a neutrino really exists, we must have some guiding principles to construct a model – which may very well be some minimal extension of the standard model - incorporating it. Of course, such an extension should respect the various constraints from particle physics, cosmology and astrophysics [7]. In this letter, we will adopt a class of models with only three left-handed neutrinos. Certainly this is the simplest assumption for the neutrino sector. Since a massive neutrino state with a large  $\nu_e$  component is constrained by stringent experimental limits from  $\beta\beta_{0\nu}$ -decay, we prefer to understand the relevant data in terms of a symmetry. In this case we are led uniquely to consider [8] models with a global U(1)<sub>e- $\mu+\tau$ </sub> symmetry <sup>#1</sup>. In order to implement a model with such a symmetry we would also like to avoid the introduction of new symmetry breaking scales below the electroweak one. If we restrict ourselves to only abelian symmetries to start with, we are then led to consider the maximum abelian lepton flavor symmetry  $U(1)_e \times U(1)_u \times U(1)_\tau$  which breaks down to  $U(1)_{e-\mu+\tau}$ . Such models were proposed in ref. [10], and it is in the spirit of this work that we want to direct our discussion. However, such a scheme is so constrained that, for example, it cannot account for the observed deficit of solar neutrinos <sup>#2</sup>. Here we will make one step further and consider an approximate U(1)<sub>e- $\mu$ + $\tau$ </sub> symmetry by an additional breaking. All the breaking procedures will be spontaneous and the associated Goldstone bosons are named flavons. According to our approach, all the corresponding symmetry breaking scales should not be below the electroweak scale.

Thus, we consider models that above some scale V, taken no less than the weak scale  $v \equiv \langle H \rangle$ , have a global abelian lepton flavor symmetry  $G = U(1)_e \times U(1)_{\mu} \times U(1)_{\tau}$ . At scale V the group G is broken to  $H = U(1)_{e-\mu+\tau}$  by VEVs of two SU(2) singlet scalars  $\Phi^{ab}$  (a, b stand for e,  $\mu$ ,  $\tau$ ) giving rise to two Goldstone bosons:

\*1 The same symmetry, but with the addition of heavy singlet neutrino states, was also considered in ref. [9].

<sup>#2</sup> For a recent review, see ref. [11], and references therein.

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 $\Phi^{e\mu} = \exp(iF_e/V_e)$  and  $\Phi^{\mu\tau} = \exp(iF_{\mu}/V_{\mu})$ . This corresponds to having an exact symmetry U(1)<sub>e-µ+τ</sub>. In this case the neutrino mass spectrum involves [12] one massless  $m_{\nu_1} = 0$  and two degenerate massive  $m_{\nu_2} = m_{\nu_3}$  mass eigenstates related to weak eigenstates  $\nu_a$  by the relation

$$\nu_a = \sum_{i=1}^{3} U_{ai} \nu_i \,, \tag{1}$$

where U is the unitary lepton mixing matrix. The predictions for neutrino oscillations are now  $P(\nu_{e,\tau} \leftrightarrow \nu_{\mu}) = 0$ ,  $P(\nu_e \leftrightarrow \nu_{\tau}) \neq 0$ . At a lower scale the group  $H = U(1)_{e-\mu+\tau}$  will break by a VEV of a third SU(2) singlet scalar  $\Phi^{ab}$ , giving rise to a third Goldstone boson:  $\Phi^{\tau e} = \exp(iF_{\tau}/V_{\tau})$ . This will correspond to what we call an approximate  $U(1)_{e-\mu+\tau}$  symmetry. Now all neutrino mass eigenvalues  $m_{\nu_i}$  are non-zero and neutrino oscillations between all species will be possible. Constraints on all the relevant parameters will be discussed in the following.

Our model involves the neutral singlets  $\Phi^{ab}$ , the heavy charged singlets  $S^{ab}$  and one extra doublet H' whose VEV is taken to vanish without loss of generality. The new interactions in the lagrangian are

$$g'_a l_a e^{ca} H' + \lambda_{ab} l_a l_b S^{ab} + \tilde{g}_{ab} H H' S^{ab} \Phi^*_{ab}.$$
<sup>(2)</sup>

Neutrino masses are generated by the one-loop diagram shown in fig. 1. The loop calculation for this graph gives

$$m_{ab} = \frac{1}{16\pi^2} \lambda_{ab} \tilde{g}_{ba} \langle \Phi^{ba} \rangle \frac{v(g'_a m_a + g'_b m_b)}{M^2 - m_W^2} \ln \frac{M^2}{m_W^2}.$$
(3)

In the above,  $m_a$ ,  $m_b$  are the masses of the charged leptons, M is the mass of the heavy charged singlet  $S^{ab}$  which appears in the loop, while v is the VEV of the standard Higgs doublet. We have also assumed that the second doublet circulating in the loop has a mass of order  $m_W$ .

In the most general case where all the singlet fields  $\Phi^{ab}$  acquire VEVs the neutrino mass matrix can be parametrized as follows:

$$m_{\nu} = m_0 \begin{pmatrix} 0 & \tan \theta & \cos \phi \\ \tan \theta & 0 & \sin \phi \\ \cos \phi & \sin \phi & 0 \end{pmatrix}.$$
<sup>(4)</sup>

In the above matrix, we have made the approximations  $m_{\tau} + m_{\mu} \approx m_{\tau}$ , and  $m_{\mu} + m_{e} \approx m_{\mu}$ , while  $m_{0}$  sets the mass scale and is given in terms of the various parameters entering eq. (3) from the formula

$$m_0 = \frac{1}{16\pi^2} \frac{v(V_{e\tau}^2 + V_{\mu\tau}^2)^{1/2}}{M^2 - m_W^2} m_\tau \ln \frac{M^2}{m_W^2},$$
(5)

with

 $V_{ab} = \lambda_{ab} \tilde{g}_{ba} g'_a \langle \Phi^{ba} \rangle .$ 



Fig. 1. One-loop diagram for neutrino masses.

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Furthermore, since we know nothing about the couplings  $g'_a$ , in the last equation we have used the approximation  $g'_e \approx g'_{\mu} \approx g'_{\tau} \approx g'_{\tau$ 

$$\tan \theta = \frac{m_{\mu} V_{e\mu}}{m_{\tau} (V_{e\tau}^2 + V_{\mu\tau}^2)^{1/2}}, \quad \tan \phi = \frac{V_{\mu\tau}}{V_{e\tau}}.$$
(7,8)

In order to diagonalize the above matrix, guided by phenomenological reasons, we make the natural assumption that  $\sin 2\phi \ll 1$ . In this case the eigenmasses are found to be

$$m_{\nu_1} \approx -\frac{1}{2}m_0 \sin 2\theta \sin 2\phi$$
,  $m_{\nu_2} \approx -\frac{m_0}{\cos \theta} + \frac{1}{4}m_0 \sin 2\theta \sin 2\phi$ ,  $m_{\nu_3} \approx \frac{m_0}{\cos \theta} + \frac{1}{4}m_0 \sin 2\theta \sin 2\phi$ . (9)

Furthermore, we get two different sets of eigenstates, depending upon whether  $\sin \phi \ll 1$  or  $\cos \phi \ll 1$ . However, the case  $\sin \phi \ll 1$  is rejected from the mixing of  $\nu_{17}$  with the  $\nu_e$  as is reported from the experiment [1]. Thus, in the phenomenologically viable case where  $\cos \phi \ll 1$ , the eigenstates are related to the weak states as follows:

$$\nu_e \approx N_1 \cos \theta \, (\sin \phi - \cos^2 \theta \cos \phi \sin 2\phi) \nu_1 + N_2 (\sin \phi \sin \theta - \cos \phi) \nu_2 + N_3 (\sin \phi \sin \theta + \cos \phi) \nu_3 ,$$
  

$$\nu_\mu \approx N_1 \cos \theta \, (\cos \phi - \cos^2 \theta \sin \phi \sin 2\phi) \nu_1 + N_2 (\cos \phi \sin \theta - \sin \phi) \nu_2 + N_3 (\cos \phi \sin \theta + \sin \phi) \nu_3 ,$$
  

$$\nu_\tau \approx -N_1 \sin \theta \, \nu_1 + N_2 \cos \theta (1 - \sin 2\phi \sin \theta) \nu_2 + N_3 \cos \theta (1 + \sin \theta \sin 2\phi) \nu_3 ,$$
 (10)

where the normalization constants  $N_1$ ,  $N_2$  and  $N_3$  are given by

$$N_{1} = \frac{1}{\sqrt{1 - \sin^{2}2\phi\cos^{4}\theta (1 + \sin^{2}\theta)}},$$

$$N_{2,3} = \frac{1}{\sqrt{1 + \sin^{2}\theta + \cos^{2}\theta (1 \mp \sin\theta\sin 2\phi)^{2} \mp 2\sin\theta\sin 2\phi}}.$$
(11)

A simplified version of the above model arises when one considers the particular value  $\cos \phi = 0$ . This case corresponds to the particular choice  $\langle \Phi_{e\tau} \rangle = 0$ , which exhibits an exact  $L_{e-\mu+\tau}$  symmetry. In this latter case one finds a zero mass for the first neutrino and two completely degenerate states for the other two, i.e.,

$$m_{\nu_1} = 0, \quad m_{\nu_2} = -\frac{m_0}{\cos\theta}, \quad m_{\nu_3} = \frac{m_0}{\cos\theta},$$
 (12)

while the diagonalizing matrix is given by

$$U = \begin{pmatrix} \cos\theta & \sin\theta/\sqrt{2} & \sin\theta/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ -\sin\theta & \cos\theta/\sqrt{2} & \cos\theta/\sqrt{2} \end{pmatrix}.$$
 (13)

In the last two equations there are only two parameters, namely the mass scale  $m_0$  and the angle  $\theta$ . If we adopt the result from the  $\beta$  spectrum of the <sup>3</sup>H decay experiment, which predicts a  $m_{17} = 17$  keV "heavy" neutrino that mixes with  $\nu_e$  at 10% of the time, both parameters can be fixed uniquely. From this mixing one concludes that sin  $\theta = 0.1$ . Moreover, bearing in mind that tan  $\theta = V_{e\mu}m_{\mu}/V_{\tau\mu}m_{\tau}$ , we can extract the following relation for the singlet VEVs:

$$V_{\mu\tau} = 10 \, \frac{m_{\mu}}{m_{\tau}} \, V_{e\mu} \,. \tag{14}$$

On the other hand, from the one-loop formula (3) and the 17 keV neutrino mass, one can fix the ratio

$$\frac{M^2}{V_{\mu\tau}} = \frac{v}{16\pi^2} \frac{m_{\tau}}{m_{17}\cos\theta} \ln\frac{M^2}{m_W^2} \approx 0.32 \times 10^6 \ln\frac{M}{m_W} \,(\text{GeV}) \,.$$
(15)

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The precise value for the singlet mass M circulating in the loop cannot be determined, but an upper value can be extracted from cosmological bounds on the lifetime  $\tau_{17}$  of the 17 keV neutrino. A stringent limit for  $\tau_{17} \leq 3 \times 10^8$  s arises, since bigger decay times are incompatible with microwave background anisotropy limits and galaxy clustering observations [13]. This requires

$$\tau_{17} \approx 2 \times 10^{-3} \,\mathrm{s} \cdot \left(\frac{V_{\mu\tau}}{300 \,\,\mathrm{GeV}}\right)^2 \leq 3 \times 10^8 \,\mathrm{s} \,,$$
 (16)

which gives the bound  $V_{\mu\tau} \le 10^8$  GeV. It is straightforward to note that a similar bound is applied also on the mass M of the charged singlet, i.e.,  $M \le 2 \times 10^7$  GeV.

The above model, in the case of the exact  $L_{e-\mu+\tau}$  symmetry, gives also definite predictions for neutrino oscillations and flavor changing processes which we now describe in brief.

(i) Neutrino oscillations. There is only one oscillation length characterized by the difference  $\Delta m_{31}^2 \approx m_{17}^2$ which mixes  $\nu_{17}$  with  $\nu_e$ . For distances far from the source, the oscillation probability is found within the experimental bound  $P(\nu_e \leftrightarrow \nu_\tau) = \frac{1}{2} \sin^2 2\theta \approx 0.02$ . (See eqs. (22)–(25) below.) On the contrary  $P(\nu_{e,\tau} \leftrightarrow \nu_{\mu}) = 0$ .

(ii)  $\beta\beta_{0\nu}$ -decay,  $(\mu^-, e^+)$  reaction. In the case of an exact  $L_{e-\mu+\tau}$  symmetry, neutrino-less double beta  $\beta\beta_{0\nu}$ -decay is strictly forbidden, since the effective neutrino mass parameter is zero:

$$\langle m_{\nu} \rangle = \sum_{i=1}^{3} U_{ei} U_{ei}^* \xi_i |m_i| = 0.$$
 (17)

This happens because the electron neutrino mass is zero, while there is a complete cancellation of the other two terms due to the opposite CP-phases  $-\xi_2 = \xi_3 = 1$ .

On the contrary, the reaction  $(\mu^-, e^+)$  is enhanced in this case. This reaction can proceed through the well known box diagrams [14]. Thus in the case of the box diagram with the gauge boson exchange, the corresponding effective neutrino mass parameter is written

$$\langle m'_{\nu} \rangle = \sum_{i=1}^{3} U_{ei} U_{ei} \xi_i m_i = m_{17} \sin \theta$$
 (18)

This value is significantly larger than the one obtained in previous models [14]. Nevertheless, despite this enhancement factor, the branching ratio BR ( $\mu^- \rightarrow e^+$ ) is still very small ( $\leq 10^{-20}$ ) and beyond the goals of the present experiments. Note that, even if all nuclear final states are included, the 17 keV neutrino is not large enough to make this reaction detectable.

(iii) Decays of charged leptons. Another consequence of the above model is the decay of the  $\tau$  lepton to an electron and a Goldstone boson (flavon F). This decay proceeds through the one-loop diagram of fig. 2, which is realized via the quartic coupling

$$\hat{g}\boldsymbol{\Phi}_{e\mu}\boldsymbol{\Phi}_{\mu\tau}S^{e\mu}S^{\mu\tau}.$$





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The branching ratio of the above decay can be written as follows:

$$BR(\tau \to eF) \approx \frac{12\pi^2}{5} \frac{1}{G_F^2 m_\tau^2} \frac{\hat{g}_{e\tau}^2}{g'^2} \left[ \left( \frac{\lambda_{e\mu}}{\tilde{g}_{\mu\tau}} \right)^2 + \left( \frac{m_\tau}{10m_\mu} \right)^2 \left( \frac{\lambda_{\mu\tau}}{\tilde{g}_{e\mu}} \right)^2 \right] \left( \frac{m_{17} \cos \theta}{v m_\tau} \right)^2.$$
(20)

In the last expression, we have substituted the mass ratio  $V_{\mu\tau}/M^2$  from eq. (15). Thus there is no direct dependence in this formula on the unknown mass scales  $V_{\mu\tau}$  and  $M^2$ . Thus, for a branching ratio of the order of  $10^{-4}$  and making some natural assumptions for the various couplings which are involved in (20), we would have the following constraint on the quartic coupling  $\hat{g}$ :

$$\hat{g} \leq g' \tilde{g}_{e\mu} / 10 \lambda_{e\mu}$$
.

(21)

Obviously, this inequality can be satisfied with natural values of the involved Yukawa couplings.

In the subsequent we would like to consider the general case where the third singlet field is allowed to develop a non-zero VEV, i.e.,  $\langle \Phi^{e\tau} \rangle \neq 0$ . This VEV will break the  $L_{e-\mu+\tau}$  symmetry and allow oscillations to take place between all neutrino species. The flavor violating processes  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow e$  conversion and  $\mu \rightarrow 3e$ , which were previously forbidden, are now possible. Evidently, experimental bounds on the above processes are expected to put a bound on the magnitude of the singlet VEV  $\langle \Phi^{e\tau} \rangle$ .

Let us now discuss the constraints on the entries of the general mass matrix (5) that emerge from the neutrino oscillations. As we already argued, there exist now oscillations between all neutrino species. In the limit in which  $\cos \phi = 0$ , the predictions of the model are not very interesting since it yields  $P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_\mu \leftrightarrow \nu_\tau) = 0$  while

$$P(\nu_e \leftrightarrow \nu_\tau) = [\sin 2\phi \sin(\pi L/l)]^2, \qquad (22)$$

where  $l_{23} = 0$  and

$$l = l_{31} = l_{21} = \frac{4\pi E_{\nu}}{m_0^2} = 2.467 \text{ km} \cdot \frac{E_{\nu}/1 \text{ GeV}}{(m_0/1 \text{ eV})^2}$$
(23)

is the oscillation length. In our case for  $E_{\nu} \approx 1$  GeV and  $m_0 \approx 17$  keV, one obtains

$$l \approx 8.6 \times 10^{-9} \,\mathrm{km} = 8.67 \times 10^{-6} \,\mathrm{m} \,. \tag{24}$$

In other words one has very small oscillations even within the detector itself. Thus one can set  $\sin^2(\pi L/l) \rightarrow \langle \sin^2(\pi L/l) \rangle = \frac{1}{2}$ , so

$$P(\nu_e \leftrightarrow \nu_\tau) = \frac{1}{2} \sin^2 2\theta \,. \tag{25}$$

Now, going beyond the zeroth order, the above results are modified primarily due to the fact that the two heavy neutrino components are no longer degenerate. Thus

$$l_{23} = 2.467 \,\mathrm{km} \cdot \frac{E_{\nu}/1 \,\mathrm{GeV}}{2(m_{\nu_1}/1 \,\mathrm{eV})(m_0/1 \,\mathrm{eV})}.$$
(26)

Thus for the various types of neutrino oscillations that now take place we get

$$P(\nu_e \leftrightarrow \nu_\tau) \approx \frac{1}{2} \sin^2 2\theta \left[ 1 - \frac{1}{2} \sin^2 (\pi L/l_{23}) \right],$$
(27)

$$P(\nu_{\tau} \rightarrow \nu_{\mu}) \approx \cos^2\theta \sin^2(\pi L/l_{23}) , \qquad (28)$$

$$P(\nu_e \to \nu_\mu) \approx \sin^2\theta \sin^2(\pi L/l_{23}) . \tag{29}$$

Again, the short oscillations have been averaged out. We notice that the above oscillation probabilities are expressed solely in terms of two parameters,  $\theta$  and  $l_{23}$ , but they do not coincide with the familiar two generation expressions [15]. We also notice that  $\nu_{\mu}$  oscillates primarily into  $\nu_{\tau}$ , since the oscillation probability in this case is proportional to  $\cos^2\theta$  which according to the experiment is close to unity. Thus from the experimental limit associated with the maximum mixing [16] one has the bound

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 $2m_0m_{\nu_1} < 7.9 \ (eV)^2$ ,

which converts to a bound for the light neutrino mass

$$m_{\nu_1} \leq 4.6 \times 10^{-4} \, \mathrm{eV}$$
.

The tiny electron neutrino mass puts a stringent bound on the angle  $\phi$  through formula (9). Indeed

$$\sin 2\phi \leq 3 \times 10^{-7}$$

which, according to our previous approximation, is consistent with  $\phi \approx \frac{1}{2}\pi$ . This of course results in a stringent bound on the mass scale  $V_{e\tau}$ , compared to the bounds put upon  $V_{\mu\tau}$  derived from eq. (16). In fact one finds that  $V_{e\tau} \leq 10^{-7} V_{\mu\tau}$ . For natural values of the Yukawa couplings (i.e.,  $\lambda, g', \tilde{g} \simeq 10^{-1}$ ), this tells us that the VEV  $\langle \Phi^{e\tau} \rangle$ may very well be above the electroweak scale as desired. We should note however that other non-accelerator experimental limits [5] put more severe bounds.

Let us finally mention that another bound on  $\langle \Phi^{e\tau} \rangle$  could be derived from the decay  $\mu \rightarrow eF$ , but exactly as in the previous case the branching ratio depends on the quartic coupling  $\hat{g}_{e\mu}$  which is unrelated to the neutrino masses. Therefore, it can be chosen to have the desired value. If, however, we accept that all couplings  $\hat{g}_{ij}$  are of the same order and that BR $(\tau \rightarrow eF) \approx 10^{-4}$ , then we obtain the bound  $V_{e\tau} \leq 10^{-3} V_{\mu\tau}$ . This is of course weaker than the bound derived from neutrino oscillations.

In conclusion, in order to implement the 17 keV neutrino scenario, we have assumed a simple extension of the standard model of the electroweak interactions, with the introduction of three singly-charged states and three neutral singlets  $\Phi_{\mu\tau}$ ,  $\Phi_{e\mu}$  and  $\Phi_{e\tau}$ . When the latter are allowed to acquire VEVs, neutrino masses are generated at the one-loop level, while, as a result of the spontaneous lepton flavor symmetry, there appear three Goldstone bosons (flavons). When  $\langle \Phi^{e\tau} \rangle = 0$ , the model exhibits an exact  $L_{e-\mu+\tau}$  symmetry, while  $\nu_{\mu} \leftrightarrow \nu_{\tau}$  neutrino oscillations put severe bounds on the  $L_{e-\mu+\tau}$  symmetry breaking:  $\langle \Phi^{e\tau} \rangle \leq 10^{-7} \langle \Phi^{\mu\tau} \rangle$ .

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