NEUTRINO MASSES AND OSCILLATIONS IN AN UNCONVENTIONAL MODEL OF LEPTON NUMBER VIOLATION

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Radiatively generated neutrino masses are studied in the framework of a simple model which predicts large mixings for neutrinos independently of the actual value of neutrino masses. The associated phenomenology of neutrino oscillations is analysed in detail. Other lepton violating processes are also discussed.

Neutrino masses can be generated within the gauge symmetries of the standard model [1] provided that the scalar or fermionic sector is suitably expanded [2]. The only fermionic possibility is a gauge singlet Weyl spinor (the "right-handed neutrino") which can give a Dirac mass to the neutrino. Furthermore its quantum numbers allow a Majorana mass term which violates the lepton number L and determines the scale of lepton number breakdown. The Yukawa part of the lagrangian takes the form

$$\mathcal{L}_{\mathbf{Y}} = a_{ij} \, \bar{\boldsymbol{\xi}}_{\mathbf{L}i} \, \mathbf{H} \mathbf{e}_{\mathbf{R}j} + b_{ij} \, \bar{\boldsymbol{\xi}}_{\mathbf{L}i} \, \bar{\mathbf{H}} \mathbf{N}_{\mathbf{R}j} + M_{ij} \, \bar{\mathbf{N}}_{\mathbf{L}i}^{c} \mathbf{N}_{\mathbf{R}j} + \mathbf{h.c.}, \tag{1}$$

where

$$\varrho_{L} = \begin{pmatrix} \nu_{L} \\ e_{L} \end{pmatrix} = (2, -1/2; 1),$$

$$H = \begin{pmatrix} H^{+} \\ H^{0} \end{pmatrix} = (2, +1/2; 0), \quad e_{R} = (1, -1, +1), \quad N_{R} = (1, 0; 1), \quad \bar{H} = \begin{pmatrix} H^{0*} \\ -H^{-} \end{pmatrix} = (2, -1/2; 0).$$
(2)

The numbers in parenthesis indicate the SU(2), Y and L assignments. The neutrino mass M_{ij} can arise either explicitly or as an expectation value of some scalar field that breaks lepton number spontaneously. The indices i, j in (1) are generation indices. The Higgs doublet H' could be the same as H. The neutrino mass matrix resulting from (1) is

$$(\bar{\nu}_{\rm L}, \bar{\rm N}_{\rm L}^{\rm c}) \begin{pmatrix} 0 & b_{ij}v' \\ b_{ij}v' & M_{ij} \end{pmatrix} \begin{pmatrix} \nu_{\rm R}^{\rm c} \\ {\rm N}_{\rm R} \end{pmatrix}.$$
(3)

The parameter v' is the expectation value of the Higgs doublet, $v' = \langle 0|H_0^*|0\rangle$, $b_{ij}v'$ is the Dirac mass matrix and M_{ii} the Majorana mass matrix.

In the framework of SU(3) × SU(2) × U(1) there is no restriction on the Yukawa couplings b_{ij} apart from phenomenology. A constraint, however, emerges if we consider a grand unified scheme [2,3]. If we consider, e.g., Higgs scalars which belong to the 10 and 126 representations [3-4] of SO(10) the neutrino masses $m(v_D)$ and m(N) are related to the up and down quark masses. These predictions can, of course, be affected if we include the 120-dimensional SO(10) representation which necessarily couples antisymmetrically in flavour space.

The above tree level predictions can also be modified by radiative corrections. In fact Witten [5] has shown that it is not even necessary to introduce the 126 representation in order to generate a neutrino mass matrix as



Fig. 1. Right-handed neutrino mass generation via Witten's mechanism.

in (2). Provided that Higgs bosons in the 16 representation of SO(10) are present $^{\pm 1}$, it would be natural to assume the existence of the coupling

$$\delta \mathcal{L} = M(10)_{\rm H}(16)_{\rm H}(16)_{\rm H} \ . \tag{4}$$

The scale M can be left arbitrary and fixed by phenomenology later. Diagrams of the type shown in fig. 1 are nonzero and generate mass terms

$$\overline{\mathbf{N}_{\mathrm{L}i}^{\mathrm{c}}} \mathbf{N}_{\mathrm{R}j} \boldsymbol{A}_{ij} \mathcal{M} \langle 16 \rangle^{2} (\alpha/2\pi)^{2} \mu^{-2}.$$
(5)

The Yukawa coupling matrix is $A_{ij} = u^{-1}m_{ij}(u)$ ($u = \langle 10 \rangle$) while the natural value for the expectation value $\langle 16 \rangle$ and the scale μ should be M. m(u) is the up quark mass matrix. Finally

$$m(N)_{ii} = (\alpha/2\pi)^2 (M/M_W) m_{ii}(u) .$$
(6)

Eq. (6) holds only when the up quark mass comes entirely from a 10 of Higgses. (The relatively induced mass gets modified if the 126 is present as well but from a theoretical point of view it would be difficult to justify the operation of both Witten's mechanism [5] and the tree-level direct generation of a right-handed neutrino mass via the 126.) The off-diagonal tree level Dirac masses are in the simplest case $m_{ij}(v_D) = m_{ij}(u)$. This leads to an "effective" mass matrix for left-handed neutrinos of order $(2\pi/\alpha)^2 (M_W/M)m(u)$. We should bear in mind that there is no SU(3) × SU(2) × U(1) or SU(5) analogue to Witten's mechanism since the propagation of superheavy gauge bosons in the 10 representation of SU(5), contained [2] in the adjoint 45-dimensional representation of SO(10), is crucial (see fig. 1). A general feature of both the tree-level mechanism through the 126 and the radiative mechanism is that the resulting neutrino mass matrix comes out proportional to the quark mass matrices m(d) and m(u). The neutrinos inherit in this way the flavour mixing of quarks but no additional flavour mixing is generated.

It is natural to ask whether there exists a mechanism by which neutral leptons acquire flavour mixing even when there is no quark mixing, i.e. even when m(d) and m(u) are diagonal. To this end we observe that, in the context of $SU(3) \times SU(2) \times U(1)$ symmetry, it is possible to violate lepton number and consequently generate neutrino masses by extending not the fermionic but the bosonic sector of the theory [6,7]. One can include spin zero bosons, in addition to the usual doublets, which can couple in a renormalizable and gauge invariant way to the standard leptons. These are

$$S = (1, 1; -2), T = (3, 1; -2), \Delta = (1, -2; 2).$$
 (7)

The new allowed Yukawa interactions are

$$\Delta \mathcal{L}_{\mathbf{Y}} = (\bar{\nu}_{\mathbf{L}i}, \bar{e}_{\mathbf{L}i}) [c_{ij}\mathbf{S} + d_{ij}\boldsymbol{\tau} \cdot \mathbf{T}/\sqrt{2}] \begin{pmatrix} e_{\mathbf{K}j} \\ -\nu_{\mathbf{K}j}^c \end{pmatrix} + f_{ij}\overline{e_{\mathbf{L}i}^c} e_{\mathbf{R}j} \Delta^* + \text{h.c.}$$
(8)

^{‡1} One way to break SO(10) down to SU(5) is by the use of Higgses in the 16 representation.



Fig. 2. Radiative left-handed neutrino masses in SU(3) \times U(2) \times U(1).

Lepton number violation can arise from boson cubic or quartic couplings like [7]

 $\delta \mathcal{L} = M \text{SHH}' + M \text{HTH} + M \text{SS}\Delta + M \text{TT}\Delta + \lambda \overline{H} \overline{H}' \text{S}\Delta + (\text{other } L \text{-conserving terms}).$ (9)

The masses M need not be the same but they should be of the same order of magnitude. It is not difficult to identify the diagrams originating from (8) and (9) which lead to neutrino masses (see fig. 2).

The coupling to the singlet S is necessarily antisymmetric [6] in flavour space. This is a nice property that gives rise to natural flavour mixing and can serve as a mechanism to generate neutrino mixings independently of quark flavour mixings. The radiatively generated neutrino masses via the diagrams of fig. 2 will be

$$\overline{\nu}_{L}^{i}(\nu_{R}^{c})^{j} \{c_{ij}(m_{i}^{2}-m_{j}^{2})+d_{ij}m_{i}^{2}+c_{ik}f_{kj}c_{ji}m_{j}m_{k}/8\pi^{2}+(d_{ik}f_{kj}/8\pi^{2})d_{ij}m_{j}m_{k}\}M/8\pi^{2}\mu^{2}$$
(10)

 $(m_i \text{ are the charged lepton masses. We have chosen a basis in which the charged lepton mass matrix is diagonal.) The scale <math>\mu^2$ refers to the biggest mass circulating in the loops (that of Δ , T or S) which roughly should be of the same order as the scale M. Concentrating on the dominant one-loop contribution in (10) and taking $d_e = d_\mu = d_\tau = c_{e\mu} = c_{e\tau} = c_{\mu\tau} = c$ we obtain

$$m(\nu_{\rm L})_{ij} \simeq (cm_{\rm T}^2/8\pi^2\mu) \begin{pmatrix} m_{\rm e}^2/m_{\tau}^2 & K & 1\\ K & K & 1\\ 1 & 1 & 1 \end{pmatrix}, \quad K = m_{\mu}^2/m_{\tau}^2.$$
(11)

The neutrino masses arising from (11) are

$$m(v_3) = 2m_0, \quad m(v_2) = m_0, \quad m(v_1) = \frac{1}{2}Km_0, \quad m_0 = cm_\tau^2/8\pi^2\mu.$$
 (12)

The masses have been chosen positive by adjusting the phases so that $(CP)\nu_1 = -\nu_1$, $(CP)\nu_2 = -\nu_2$, $(CP)\nu_3 = \nu_3$. The neutrino mixing matrix takes the form

$$\begin{pmatrix} \nu_{e}^{0} \\ \nu_{\mu}^{0} \\ \nu_{\tau}^{0} \end{pmatrix}_{L} = \begin{bmatrix} \frac{1}{2}\sqrt{2}(1-\frac{1}{4}K) & \frac{1}{3}\sqrt{3}(1+\frac{1}{3}K) & \frac{1}{6}\sqrt{6}(1+\frac{1}{12}K) \\ -\frac{1}{2}\sqrt{2}(1-\frac{1}{4}K) & \frac{1}{3}\sqrt{3}(1-\frac{2}{3}K) & \frac{1}{6}\sqrt{6}(1+\frac{7}{12}K) \\ \frac{1}{2}\sqrt{2}\frac{1}{2}K & -\frac{1}{3}\sqrt{3}(1+\frac{1}{3}K) & \frac{1}{6}\sqrt{6}(1-\frac{1}{6}K) \end{bmatrix} \begin{pmatrix} \nu_{e} \\ \nu_{\mu} \\ \nu_{\tau} \end{pmatrix}_{L} ,$$
(13)

to first order in K. The index zero refers to weak eigenstates. Note that in our model we have large mixings regarless of the actual mass scale. This is a direct consequence of the group theoretic properties of the singlet S which is a component of the antisymmetric 120 representation [4] of SO(10) [which contains the 10 representation of SU(5)]. It goes without saying, of course, that the other members of the same representation should be given a much bigger mass in order to avoid flavour changing neutral currents. The GUT version of the diagrams of fig. 2a are shown in fig. 3a. The presence of the 210 representation [4] of SO(10), which is necessary in order to close



Fig. 3. One-loop radiative left-handed neutrino masses in SO(10) and SU(5).

the lepton number violating vertex, makes SO(10) grand unification of the model rather baroque. On the other hand, the SU(5) scheme which requires only quinteplets and the antisymmetric 10 representation, is a rather minimal extension of the minimal SU(5). The Higgses T and Δ , which can couple symmetrically, belong to the 15 and 50 representation of SU(5) respectively. Both of these belong to the 126 representation of SO(10). The relevant one-loop diagrams are given in fig. 3b. Both one-loop diagrams need the 210 representation of SO(10). In contrast the two-loop diagrams do not need the 210 if the 126 is included. If the model contains only the 126, one encounters the diagram of fig. 4a. If in addition the model contains the 120, i.e. both S and Δ or T, one encounters the diagram of fig. 4b.

Summarizing we stress that the presence of the charged singlet S with antisymmetric coupling results in sizable neutrino mixing. Needless to say that its coloured partners in the SU(5) 10 representation $^{+2}$ can also lead to quark mixing (see fig. 5). But this is a small effect if all neutrinos are light or the coloured bosons are superheavy.

Let us explore the phenomenological implications of the above model on the properties of the neutrinos focusing in the various processes which depend on neutrino masses. These are the following.

(A) Neutrino oscillations [9]. These are the most sensitive means of detecting small neutrino masses especially when the mixing coefficients are of order unity as in our model. Since the neutrinos produced in weak interactions are not eigenstates of the total hamiltonian they can be written in terms of mass eigenstates. Suppose that at t = 0 the neutrino flavour ν_{α} is given by

$$|\nu_{\beta}\rangle = \sum_{j=1}^{3} U_{\beta j} |\nu_{j}\rangle , \qquad (14)$$

 $^{\ddagger 2}$ 10 = (3, 2, 1/6) + (3, 1, -2/3) + (1, 1, 1).





(b)

Fig. 4. Two-loop radiative left-handed neutrino masses in SO(10) and SU(5).

where U is the mixing matrix [see e.g. eq. (13)]. The time-evolved state will be

$$|\nu_{\beta}(t)\rangle = \sum_{j} e^{-iE_{j}t} U_{\beta j} |\nu_{j}\rangle.$$
⁽¹⁵⁾

Then it is straightforward to show that the oscillation probability becomes

$$P(\nu_{\gamma} \rightarrow \nu_{\beta}) = |\nu_{\beta}(L)|\nu_{\gamma}(0)\rangle|^{2} = \delta_{\beta\gamma} - 4\sum_{j < k} U_{\beta k} U_{\gamma k} U_{\gamma j} U_{\beta j} \sin^{2}(\Delta_{kj}) .$$
⁽¹⁶⁾

We assumed that the matrix is real (as is the case in our model) and used



Fig. 5. Quark flavour mixing in the presence of the SO(10) 10-dimensional Higgs representation.

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$$L = ct, \quad \Delta_{kj} = \frac{1}{2}L\left[(p_{\nu}^2 + m_k^2)^{1/2} - (p_{\nu}^2 + m_j^2)^{1/2}\right]. \tag{17}$$

For neutrino masses small compared to the neutrino momentum p_{ν} , we get

$$\Delta_{kj} \approx (\delta m^2) L/4E_{\nu} = 1.269 \, \frac{\delta m^2}{(\text{eV})^2} \left(\frac{L/km}{E_{\nu}/\text{GeV}}\right), \quad \delta m^2 = m_k^2 - m_j^2 \,. \tag{18}$$

To exhibit the periodic nature of $P(\nu_{\gamma} \rightarrow \nu_{\beta})$ as a function of the detector source distance L one defines the oscillation lengths $(L_0)_{kj} = \pi L/\Delta_{kj}$. Thus, in general, the neutrino oscillations are described as three oscillations superimposed on each other with oscillation lengths $(L_0)_{12}, (L_0)_{13}, (L_0)_{23}$.

Most of the analysis of neutrino oscillations has been performed in a two state approximation. Our model is simple enough so that a three-generation discussion is possible. To leading order we have $m_1 = 0$, $m_2 = m_0$ and $m_3 = 2m_0$. Thus, the oscillation lengths are

$$(L_0)_{13} = l_0/4, \quad (L_0)_{23} = l_0/3, \quad l_0 = 2.476 \text{ km} (E_\nu/\text{GeV})/m_0^2(\text{GeV})^2.$$

$$[(L_0)_{12} = l_0 \text{ is a multiple of the above.}] \text{ Then, from eq. (8), with } \Delta_0 = \pi L/l_0 \text{ we obtain}$$

$$|\langle v_e(L) | v_e(0) \rangle|^2 = 1 - [\frac{2}{3} \sin^2 \Delta_0 + \frac{1}{3} \sin^2(4\Delta_0) + \frac{2}{9} \sin^2(3\Delta_0)] = |\langle v_\mu(L) | v_\mu(0) \rangle|^2,$$

$$|\langle v_\tau(L) | v_\tau(0) \rangle|^2 = 1 - \frac{8}{9} \sin^2(3\Delta_0),$$

$$|\langle v_\tau(L) | v_\tau(0) \rangle|^2 = \frac{2}{3} \sin^2 \Delta_0 + \frac{1}{3} \sin^2(4\Delta_0) - \frac{2}{3} \sin^2(3\Delta_0) \rangle$$
(19)

$$\begin{aligned} |\langle \nu_{e}(L) | \nu_{\mu}(0) \rangle|^{2} &= \frac{1}{3} \sin^{2} \Delta_{0} + \frac{1}{3} \sin^{2} (4\Delta_{0}) - \frac{1}{9} \sin^{2} (3\Delta_{0}) , \\ |\langle \nu_{\mu}(L) | \nu_{\tau}(0) \rangle|^{2} &= |\langle \nu_{e}(L) | \nu_{\tau}(0) \rangle|^{2} = \frac{4}{9} \sin^{2} (3\Delta_{0}) . \end{aligned}$$
⁽²⁰⁾

From the neutrino oscillation data [10]⁺³ at Fermilab the following limits have been obtained:

$$|\langle \tilde{\nu}_{\rm e}(L)|\tilde{\nu}_{\mu}(0)\rangle|^2 < 6.5 \times 10^{-3} \,, \quad |\langle \tilde{\nu}_{\tau}(L)|\tilde{\nu}_{\mu}(0)\rangle|^2 < 4.4 \times 10^{-2} \,, \tag{21}$$

for $L/E_{\nu} = 0.03 \text{ km/GeV}$. In our model such limits can only emerge if the quantity Δ_0 is very small or near π . For $\Delta_0 \leq 1$ we get ^{*4} $|\langle \tilde{\nu}_{\mu}(L) | \tilde{\nu}_{\mu}(0) \rangle|^2 \approx 4\Delta_0^2$. Using the above experimental limit we get $\Delta_0 \leq 4 \times 10^{-2}$ which leads to the constraint $m_0 \lesssim 1$ eV. Thus the limit in the mass scale of lepton violation becomes $\mu \gtrsim 10^6$ GeV. We also get

$$m_{\nu_1} \approx 1.8 \times 10^{-3} \,\mathrm{eV} \,, \quad m_{\nu_2} \approx 1 \,\mathrm{eV} \,, \quad m_{\nu_3} \approx 2 \,\mathrm{eV} \,,$$
 (22)

$$\begin{aligned} |\langle \nu_{e}(L)|\nu_{e}(0)\rangle|^{2} &= |\langle \nu_{\mu}(L)|\nu_{\mu}(0)\rangle|^{2} \ge 0.987 , \quad |\langle \nu_{\tau}(L)|\nu_{\tau}(0)\rangle|^{2} \ge 0.990 , \\ |\langle \nu_{e}(L)|\nu_{\mu}(0)\rangle|^{2} &< 6.4 \times 10^{-3} , \quad |\langle \nu_{\mu}(L)|\nu_{\tau}(0)\rangle|^{2} &= |\langle \nu_{e}(L)|\nu_{e}(0)\rangle|^{2} < 6.4 \times 10^{-3} . \end{aligned}$$

$$(23)$$

The resulting oscillation lengths are

$$(L_0)_{12} = l_0/4 = 0.619 E_{\nu}/\text{GeV}, \quad (L_0)_{23} = l_0/3 = 0.825 E_{\nu}/\text{GeV}.$$
 (24)

(B) Decay experiments. Neutrino oscillations only enable us to extract the neutrino mass differences $|m_j^2 - m_k^2|$ once the mixing parameters are determined. The masses themselves can be extracted directly from the kinematics of decay experiments [12] if they can be resolved. If they are not resolved one measures

$$\langle m_{\alpha} \rangle \sum_{j} |U_{\alpha j}|^2 m_j ,$$
 (25)

if the flavour α is weakly produced. Note that there is always constructive interference in such experiments. In recent years there has been considerable excitement due to the decay experiment of Lubimov et al. which

⁺³ For a review of the experimental data, see e.g. ref. [11].

⁺⁴ For $\Delta_0 \approx \pi$ we obtain $m_0 \approx 9.1$ eV. This cannot be excluded but it seems very coincidental.

(10)

predicts a non-zero neutrino mass [13], $14 \text{ eV} < m_{\nu} < 46 \text{ eV}$. This experimental result is, of course, not universally accepted due to the uncertainties regarding the analysis of the experimental data. In our model the mass eigenstates cannot be resolved. We thus predict

$$\langle m_{\widetilde{\nu}_{\rm e}} \rangle = \frac{2}{3} m_0 \approx 0.67 \, {\rm eV}, \quad \langle m_{\nu_{\mu}} \rangle = \frac{2}{3} m_0 \approx 0.67 \, {\rm eV}, \quad \langle m_{\nu_{\tau}} \rangle = \frac{5}{3} m_0 \approx 1.67 \, {\rm eV},$$

for triton decay, $\pi \to \mu \nu_{\mu}$ and $\tau \to \pi \nu_{\tau}$ decay respectively. We note that our predictions are inconsistent with the experiment of Lubimov et al., but they are, of course, consistent with the other experimental limits,

$$\langle m_{\nu_e} \rangle < 33 \text{ eV}, \quad \langle m_{\nu_{\mu}} \rangle < 0.57 \text{ MeV}, \quad \langle m_{\nu_{\tau}} \rangle < 250 \text{ MeV},$$

of Tretyakov et al. [14], Daum et al. [15] and Blocker et al. [16], respectively.

(C) Neutrinoless double beta decay ($0\nu \beta\beta$ -decay). The relevant mass for this process [17-19] is

$$\langle m_{\nu} \rangle = \sum_{j} U_{ej}^2 CP(j) m_j$$

Note that due to the phases CP(j) there can be destructive interference. In our model we indeed have such destructive interference and find $\langle m_{\nu} \rangle \approx (m_e/m)^2 m_0 \approx 10^{-7} m_0$. This is much smaller than the present experimental limit $\langle m_{\nu} \rangle \lesssim 4 \text{ eV}$. This suppression is, of course, expected since in our model m_{ee} is very small.

(D) The (μ^-, e^+) reaction [21]. In this case the relevant parameter is

$$\eta'_{\nu} = \sum_{j} U_{ej} U_{\mu j} CP(j) m_j / m_e ,$$

which in our model is $\langle \eta_{\nu} \rangle = Km_0/m_e \approx 7 \times 10^{-9}$. This suppression is easily understood due to the smallness of $m_{e\mu}$. If $\langle \eta'_{\nu} \rangle$ is indeed so small this process is not observable.

(E) Lepton flavour changing interactions ($\mu \rightarrow e\gamma$, $\mu \rightarrow ee^+e^-$, etc.). The neutrino mass contribution ⁺⁵ to such a process in our model is given by the lepton violating parameter

$$\widetilde{\eta}_{\nu} = \sum_{j} U_{ej} U_{\mu j}^{*} (m_{j}/m_{e})^{2} = (m_{0}/m_{e})^{2} = 4 \times 10^{-12}$$

Even though the effective neutrino mass in this case is m_0 , the predicted rate is many orders of magnitude below the present experimental limits. Even if the light neutrinos are in the range of eq. (24), $\tilde{\eta}_{\nu}$ yields a branching ratio which is much below the present experimental limits [21].

To conclude our discussion we stress that in our model we have large mixing angles regardless of the neutrino mass scales. Hence our predictions can be tested in neutrino oscillations where the smallness of the neutrino mass can be compensated by a suitable choice of the source detector distance. Our mass scale ($\sim 1 \text{ eV}$) was determined from such limits on such oscillations. Thus all lepton and family number violating processes are suppressed due to the smallness of the neutrino masses. Such neutrino masses can be seen in decay experiments provided that they can be done at the eV level. The mass scale, however, can be larger if the above neutrino oscillation limits are discarded.

⁺⁵ In our model there exist additional contributions to such processes. These can be sizable and they will be discussed elsewhere [7].

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