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Neutrino masses in flipped SU(5)

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We analyse the fermion masses and mixings in the flipped SU(5) model The fermion mass matrices are evolved from the GUT scale down to m_W by solving the renormalization group equations for the Yukawa couplings The constraints imposed by the charged fermion data are then utilised to make predictions about the neutrino properties It is found that the *generalized* see-saw mechanism which occurs naturally in this model can provide (i) a solution to the solar neutrino problem via the MSW mechanism and (ii) a sufficiently large ν_τ mass to contribute as a hot dark matter component as indicated by the recent COBE data

Nowadays there is a lot of experimental evidence that the neutrinos (or at least some of them) might have non-vanishing – although tiny – masses Recent data from solar neutrino experiments [1] show that the deficiency of solar neutrino flux, i e the discrepancy between theoretical estimates and the experiment, is naturally explained if the ν_e neutrino oscillates to another species during its flight to the earth In addition, new evidence has been reported [2,3] for a significant depletion on the atmospheric ν_μ flux This can also be explained in terms of $\nu_\mu \leftrightarrow \nu_e$ oscillations with mass difference of order $\Delta m^2 \sim 10^{-2} - 10^{-3}\,\mathrm{eV}^2$ and a relatively large mixing angle ($\sin^2 2\theta_{\tau\mu} \geqslant 0\,42$) Furthermore, the COBE measurement [4] of the large scale microwave background anisotropy, might be explained [5] if one assumes an admixture of COLD (~ 75%) plus HOT (~ 25%) dark matter It is hopefully expected that some neutrino (most likely ν_τ) may be the natural candidate of the hot dark matter component

From the theoretical point of view, neutrino masses are zero in the minimal standard model of the electroweak interactions Non-zero neutrino masses arise naturally however, in most of the Grand Unified Theories (GUTs) as well as in supersymmetric ones (SUSY GUTs) In all these models, neutrino masses are related to the quark masses In general one usually obtains a Dirac type neutrino mass matrix very similar (or even identical) to the up-quark mass matrix at the GUT scale Small neutrino masses in these models, compatible with the experimental constraints, are obtained in terms of the see-saw mechanism [6] There, the left-handed ν, and right-handed ν^c components of the neutrinos form the following mass matrix

$$\begin{pmatrix} 0 & m_{\nu_D} \\ m_{\nu_D}^T & M \end{pmatrix}, \tag{1}$$

where m_{ν_D} is the Dirac type 3×3 mass matrix ($\sim m_{\mathrm{up}}$) while M is a 3×3 Majorana mass matrix with entries usually of the order of the GUT scale After diagonalization one obtains small left-handed Majorana masses of the order of $m_\nu \sim m_{\nu_D}^2/M$ and heavy right-handed Majorana states of order $M \sim M_{\mathrm{GUT}}$ Light neutrino eigenmasses may then be evolved down to low energies and be compared with the experimental limits The constraints put by the aforementioned neutrino data and their relation to the quark masses at the GUT scale is a real challenge for most of the proposed GUT models Recently, motivated by the observed merging of the Standard Model gauge coupling constants in SUSY GUTs there has been a revived interest in determining the low energy parameters of the theory in terms of few inputs at the GUT scale [7,8] in the limit of zero neutrino masses Since however most of the GUT models naturally predict the existence of right-handed neutrinos, the proposed framework has now been expanded [9,10] to include non-vanishing neutrino masses as well The general strategy in these

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approaches is to use the minimal number of parameters at the GUT scale so as to have the maximum number of predictions at m_W Ultimately, one hopes that this minimal set of parameters at the GUT scale may be justified in terms of a more fundamental theory, such as the String Theory We should point out, however, that not only m_{ν_D} is related to the up-quark masses but other indirect constraints come also from the rest of fermions It thus appears challenging to utilize all possible such constraints in the mass matrix of eq (1) in order to make definite predictions for the as yet elusive neutrinos which will then be checked by experiment, this way supporting or excluding such GUT scenarios

In the present work we would like to address the question of fermion and in particular the neutrino masses in GUT models which arise [11,12] in the free fermionic construction of four dimensional strings As an application we are going to consider the SU(5) × U(1) model of ref [11], but our results are valid also for the model of ref [12] There has been much fruitful work [13,14] in this kind of models the last few years Recently it was shown [15] that the general see-saw mechanism which occurs naturally in this kind of models, turns out to be consistent with the recent solar neutrino data, but on the other hand suggests that CHOROUS and NOMAD experiments at CERN may have a good chance of observing $\nu_\mu \leftrightarrow \nu_\tau$ oscillations Here we are going to explore the neutrino masses in detail, assuming a specific ansatz for the quark and lepton mass matrices at the GUT scale, which is more or less dictated by some first attempts in deriving the above model from the four dimensional free fermionic Superstrings As stated above, our general strategy is to use the minimum number of inputs so as to have the maximum number of predictions We are going to make use of the GUT relations in order to fix these inputs in terms of well known low energy masses of the charged leptons as well as the m_u and m_c quarks and then to predict the rest of the fermion mass spectrum

The various tree-level superpotential mass terms which contribute to the neutrino mass matrix of the flipped SU(5) model are the following

$$\lambda^u_{ij} F^i \bar{f}^j \bar{h} + \lambda^{\phi\nu^c}_{ij} F^i \overline{H} \phi^j + \lambda^\phi_{ij} \phi^0 \phi^i \phi^j , \tag{2}$$

where in the above terms $F^i, \bar{f}^j$ are the $10, \bar{5}$ matter SU(5) fields while $\overline{H}, \bar{h}, h$ are the $\overline{10}, \bar{5}, 5$ Higgs representations and ϕ^i are neutral SU(5) × U(1) singlets The Higgs field $\overline{H}$ gets a vacuum expectation value (VEV) of the order of the SU(5) breaking scale ($\sim 10^{16}$ GeV), $\bar{h}, h$ contain the standard higgs doublets while ϕ^0 acquires a VEV most preferably at the electroweak scale The neutrino mass matrix may also receive significant contributions from other sources [16,13–15] Of crucial importance are the non-renormalizable contributions [16,14,17] which may give a direct $M_{\nu^c\nu^c} = M^{\rm rad}$ contribution which is absent in the tree-level potential Then, the general 9×9 neutrino mass matrix in the basis (ν_i, ν_i^c, ϕ_i), may be written as follows

$$m_\nu = \begin{pmatrix} 0 & m_U & 0 \\ m_U & M^{\rm rad} & M_{\nu^c,\phi} \\ 0 & M_{\nu^c\phi} & \mu_\phi \end{pmatrix} , \tag{3}$$

where it is understood that all entries in eq (3) represent 3×3 matrices The above neutrino matrix is different from that of eq (1), since now there are three neutral SU(5) × U(1) singlets involved, one for each family

It is clear that the matrix (3) depends on a relatively large number of parameters and a reliable estimate of the light neutrino masses and the mixing angles is a rather complicated task We are going to use however our knowledge of the rest of the fermion spectrum to reduce sufficiently the number of parameters involved Firstly, due to the GUT relation $m_U(M_{\rm GUT}) = m_{\nu_D}(M_{\rm GUT})$, we can deduce the form of $m_{\nu_D}(M_{\rm GUT})$, at the GUT scale in terms of the up-quark masses The heavy Majorana 3×3 matrix $M^{\rm rad}$ depends on the specific kind of generating mechanism For example, if $M^{\rm rad}$ is due to some non-renormalizable interactions, then it is completely model dependent Here, in order to be specific, motivated by the fact that in the non-supersymmetric version of the model this matrix may be generated radiatively [18], we take it to be proportional to the down-quark matrix [14] at the GUT scale

$$M^{\rm rad} = \Lambda^{\rm rad} m_D(M_{\rm GUT}) \tag{4}$$

The $M_{\nu^c,\phi}$ and μ_ϕ 3×3 submatrices are also model dependent In most of the string models however, there is only one entry at the trilinear superpotential in the matrix $M_{\nu^c,\phi}$, which is of the order $M_{\rm GUT}$ Other terms, if any, usually arise from high order non-renormalizable terms We will assume in this work only the existence of the trilinear term, since higher order ones will be comparable to $M^{\rm rad}$ and are not going to change our predictions In particular we will take $M_{\nu^c,\phi} \sim {\rm Diagonal}(M,0,0)$, and $\mu_\phi \sim {\rm Diagonal}(\mu,0,0)$, with $\mu << M \sim M_{\rm GUT}$, thus we will treat (3) as a 7×7 matrix

Our ansatz for the other fermion mass matrices is

$$m_U = q\begin{pmatrix} 0 & 0 & x \\ 0 & y & z \\ x & z & 1 \end{pmatrix} \equiv m_{\nu_D}, \tag{5a}$$

$$m_D = s\begin{pmatrix} 0 & \alpha & 0 \\ \alpha^* & b & 0 \\ 0 & 0 & f \end{pmatrix}, \quad m_E = s\begin{pmatrix} 0 & \alpha & 0 \\ \alpha^* & -3b & 0 \\ 0 & 0 & f \end{pmatrix}, \tag{5b}$$

where

$$s = \frac{v}{\sqrt{2}}\sin\beta, \quad q = \lambda_t(t_0)\frac{v}{\sqrt{2}}\cos\beta \tag{6}$$

and $\tan\beta = \bar{v}/v \equiv \bar{h}/h$ The form of the above mass matrices is considered at the GUT scale Note that the m_D and m_E matrix elements are not necessarily related in the flipped model However, our choice minimizes the number of arbitrary parameters and moreover, it leads to definite predictions in the neutrino sector In order to find the structure of the mass matrices at the low energy scale and calculate the mass eigenstates as well as the mixing matrices and compare them with the experimental data, we need to evolve them down to m_W, using the renormalization group equations Using the results of ref [19] we obtain the renormalization group equations for the Yukawa couplings at one-loop level

$$16\pi^2\frac{\rm d}{{\rm d}t}\lambda_U = (I\,{\rm Tr}[3\lambda_U\lambda_U^\dagger] + 3\lambda_U\lambda_U^\dagger + \lambda_D\lambda_D^\dagger - IG_U)\lambda_U, \tag{7}$$

$$16\pi^2\frac{\rm d}{{\rm d}t}\lambda_N = (I\,{\rm Tr}[\lambda_U\lambda_U^\dagger] + \lambda_E\lambda_E^\dagger - IG_N)\lambda_N, \tag{8}$$

$$16\pi^2\frac{\rm d}{{\rm d}t}\lambda_D = (I(3\,{\rm Tr}[\lambda_D\lambda_D^\dagger] + {\rm Tr}[\lambda_E\lambda_E^\dagger]) + 3\lambda_D\lambda_D^\dagger + \lambda_U\lambda_U^\dagger - IG_D)\lambda_D, \tag{9}$$

$$16\pi^2\frac{\rm d}{{\rm d}t}\lambda_E = (I({\rm Tr}[\lambda_E\lambda_E^\dagger] + {\rm Tr}[\lambda_D\lambda_D^\dagger]) + 3\lambda_E\lambda_E^\dagger - IG_E)\lambda_E, \tag{10}$$

where λ_α, $\alpha = U,N,D,E$, represent the 3×3 Yukawa matrices which are defined in terms of the mass matrices given in eqs (4)–(6), and I is the 3×3 identity matrix We have neglected one-loop corrections proportional to λ_N^2 $t \equiv \ln(\mu/\mu_0)$, μ is the scale at which the couplings are to be determined and μ_0 is the reference scale, in our case the GUT scale The gauge contributions are given by

$$G_\alpha = \sum_{i=1}^{3} c_\alpha^i g_i^2(t), \tag{11}$$

$$g_i^2(t) = \frac{g_i^2(t_0)}{1 - (b_i 8\pi^2)g_i^2(t_0)(t - t_0)} \tag{12}$$

The g_i are the three gauge coupling constants of the Standard Model and b_i are the corresponding beta functions in minimal supersymmetry The coefficients c_i^α are given by

$$\{c_U^i\}_{i=1,2,3} = \{\tfrac{13}{15}, 3, \tfrac{16}{3}\}, \quad \{c_D^i\}_{i=1,2,3} = \{\tfrac{7}{15}, 3, \tfrac{16}{3}\}, \tag{13}$$

$$\{c_E^i\}_{i=1,2,3} = \{\tfrac{9}{5}, 3, 0\}, \quad \{c_N^i\}_{i=1,2,3} = \{\tfrac{3}{5}, 3, 0\}, \tag{14}$$

In the following, we find it convenient to redefine the quark and lepton fields such that λ_U and λ_N are diagonal,

$$\lambda_U \to \lambda_U = K^\dagger \lambda_U K, \quad \lambda_N \to \lambda_N = K^\dagger \lambda_N K, \tag{15}$$

The matrix which diagonalizes the up-quark mass matrix at the GUT scale is given by $(x < y < z)$

$$K = \begin{pmatrix} \frac{y - z^2}{D_1} & -\frac{xz}{D_2} & \frac{xy(1-y+z^2)}{(1+z^2)D_3} \\ \frac{xz}{D_1} & \frac{y-z^2}{D_2} & \frac{zy}{D_3} \\ -\frac{xy}{D_1} & -\frac{z(y-z^2)}{D_2} & \frac{y(1-y+z^2)}{D_3} \end{pmatrix}, \tag{16}$$

with

$$D_1 \approx [(y-z^2)^2(1+x^2) + x^2z^2]^{1/2},$$
$$D_2 \approx [(y-z^2)^2(1+z^2) + x^2z^2]^{1/2},$$
$$D_3 \approx [y^2(1-y+z^2) + y^2z^2]^{1/2},$$

The mass eigenvalues at the GUT scale read

$$m_1 \approx q\frac{-x^2y}{y-(x^2+z^2)}, \quad m_2 - m_1 \approx q\left(y - \frac{x^2+z^2}{1-y}\right), \quad m_3 \approx q\left(1 + \frac{x^2+z^2}{1-y}\right) \tag{17}$$

We apply the field redefinitions (15) to the differential equations (7)–(10) and within the parentheses on the right hand side we retain only the dominant Yukawa coupling $\lambda_t^2(t)$,

$$16\pi^2 \frac{d}{dt}\lambda_U = [\lambda_t^2(t)\begin{pmatrix} 3 & & \\ & 3 & \\ & & 6 \end{pmatrix} - G_U(t)I]\lambda_U, \tag{18}$$

$$16\pi^2 \frac{d}{dt}\lambda_N = [\lambda_t^2(t)\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} - G_N(t)I]\lambda_N, \tag{19}$$

$$16\pi^2 \frac{d}{dt}\lambda_D = [\lambda_t^2(t)\begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} - G_D(t)I]\lambda_D, \tag{20}$$

$$16\pi^2 \frac{d}{dt}\lambda_E = -G_E(t)I\lambda_E \tag{21}$$

Solving eqs (18)–(21), we obtain

$$\lambda_t(t) = \lambda_t(t_0)\zeta^6\gamma_U(t), \tag{22}$$

where

$$\gamma_\alpha(t) = \exp\left(-\int G_\alpha(t)\,\mathrm{d}t/(16\pi^2)\right) \tag{23}$$

$$= \prod_{j=1}^{3} \left(\frac{\alpha_{j,0}}{\alpha_j}\right)^{c_\alpha^j/2b_j} \tag{24}$$

$$= \prod_{j=1}^{3} \left(1 - \frac{b_{j,0}\alpha_{j,0}(t-t_0)}{2\pi}\right)^{c_\alpha^j/2b_j}, \tag{25}$$

$$\zeta = \exp\left(\frac{1}{16\pi^2}\int_{t_0}^{t} \lambda_t^2(t)\,\mathrm{d}t\right) \tag{26}$$

$$\times \left(1 - \frac{3}{4\pi^2}\lambda_\alpha(t_0)\int_{t_0}^{t} \gamma_t^2(t)\,\mathrm{d}t\right)^{-1/12} \tag{27}$$

Then, the up-quark masses are predicted to be

$$m_u \approx \gamma_U \zeta^3 q \frac{x^2 y}{y - x^2 - z^2} n_u, \tag{28}$$

$$m_c \approx \gamma_U \zeta^3 q \left(y - \frac{z^2 + x^2}{1-y}\right) n_c + \frac{\eta_c}{\eta_u} m_u, \tag{29}$$

$$m_t \approx \gamma_U \zeta^6 q \left(1 + \frac{x^2 + z^2}{1-y}\right) \tag{30}$$

In the above formulae, η_u and η_c are taking into account the effects of QCD renormalization from the scale m_t down to 1 GeV for m_u and to m_c for m_c

Similarly, renormalizing λ_N down to m_t and expressing the eigenvalues in terms of the up-quark masses, we find that the Dirac-neutrino masses are

$$m_{\nu_{D_1}} \approx \frac{\gamma_N}{\gamma_U} \frac{1}{\eta_u \zeta^2} m_u, \quad m_{\nu_{D_2}} \approx \frac{\gamma_N}{\gamma_U} \frac{1}{\eta_t \zeta^2} m_c, \quad m_{\nu_{D_3}} \approx \frac{\gamma_N}{\gamma_U} \frac{1}{\zeta^5} m_t \tag{31}$$

In the above basis where the up-quark and neutrino matrices are diagonal, the renormalized down quark mass matrix is found to be

$$m_D^{\mathrm{ren}} \approx \gamma_D \begin{pmatrix} 1 & & \\ & 1 & \\ & & \zeta \end{pmatrix} s K^\dagger \begin{pmatrix} 0 & \alpha & 0 \\ \alpha^* & b & 0 \\ 0 & 0 & f \end{pmatrix} K, \tag{32}$$

while for the leptons one gets the matrix

$$m_E^{\mathrm{ren}} \approx \gamma_E s K^\dagger \begin{pmatrix} 0 & \alpha & 0 \\ \alpha^* & -3b & 0 \\ 0 & 0 & f \end{pmatrix} K \tag{33}$$

We consider the lepton masses as inputs, and we find the approximate expressions for the down quarks in terms of the leptons to be

$$m_b \approx \frac{\gamma_D}{\gamma_E} \zeta m_\tau n_b, \tag{34}$$

$$m_d \approx -n_d \frac{\gamma_D}{6\gamma_E} \left(m_\mu - m_e - \sqrt{(m_\mu - m_e)^2 + 36 m_\mu m_e}\right) , \tag{35}$$

$$m_s \approx n_s \frac{\gamma_D}{6\gamma_E} \left(m_\mu - m_e + \sqrt{(m_\mu - m_e)^2 + 36 m_\mu m_e}\right) , \tag{36}$$

where now η_d, η_s and η_b are taking into account the QCD renormalization effects for the corresponding down quarks and ζ, γ_α are given in terms of (23)–(27) We will take $\eta_{d,s,u} \approx 2, \eta_c \approx 1\ 8$ and $\eta_b \approx 1\ 4$ Now, since the range of the charged lepton masses are well known, one can use the above equations to determine the corresponding range of the down quarks and compare it with the running masses of d,s and b The range of the latter, is determined via SU(4) mass relations or QCD sum rules [20] Thus, for example, from SU(4) mass relations one gets

$$m_d = 7\ 9 \pm 2\ 4\,\text{MeV} , \quad m_s = 155 \pm 50\,\text{MeV} , \tag{37}$$

and from QCD sum rules

$$m_d = 8\ 9 \pm 2\ 6\,\text{MeV} , \quad m_s = 175 \pm 55\,\text{MeV} , \tag{38}$$

while $m_b = 4\ 25 \pm 0\ 10$ GeV An interesting fact is that the renormalization parameter ζ is constrained in a narrow region in order to give the correct prediction for the bottom mass For all the acceptable m_t range $\gamma_D/\gamma_E \approx 2\ 1$ and $\eta_b \approx 1\ 4$, thus $\zeta \approx 0\ 81 \pm 0\ 2$ The predictions of the other two down quark masses are $m_s \approx 153$ MeV and $m_d \approx 6\ 3$ MeV The m_s value is within the acceptable ranges given in (37), (38) The m_d value is somewhat low but still in the range of (37)

The Kobayashi–Maskawa (KM) matrix can be determined by diagonalizing the down-quark matrix in (32) In order to determine the KM mixing angles, we first determine the values of the parameters of x, y, z which give the correct masses $m_u = 5\ 1 \pm 1\ 5$ MeV, $m_c = 1\ 27 \pm 0\ 05$ GeV, while always we adjust properly $\tan\beta$ and $\lambda_t(t_0)$, so as to obtain the correct value for m_b It is worth noting here that the restricted region of ζ has a significant impact on the m_t value Indeed, as m_t gets smaller, the range $(M_{\text{GUT}} - m_t)$ becomes bigger, thus the value of ζ increases For $M_{\text{GUT}} \approx 10^{16}$ GeV, (using $\eta_b \approx 1\ 4$), we find that m_b is pushed to its upper limit, when m_t is around 125 GeV m_b goes to its lower limit as m_t approaches 175 GeV, while $m_u(m_c)$ gets its lower (higher) acceptable value We also keep track of the ratios $15 \leqslant m_s/m_d \leqslant 25, 0\ 2 \leqslant m_u/m_d \leqslant 0\ 7$, which are constrained by chiral Lagrangian analyses [21] Here, they are found $\approx 24\ 5$ and $\approx 0\ 65$ respectively Proceeding further, we determine numerically the KM matrix for each case separately Then, we return to the neutrino mass matrix and find the mass eigenstates as well as the diagonalizing matrix Then, if S^ν is the matrix which diagonalizes the effective 3×3 light neutrino sector and S_e^L the charged lepton mixing matrix, the leptonic mixing matrix is defined as follows

$$V^{\text{lep}} = S^\nu S_e^{L\dagger} \tag{39}$$

In the following we present numerical results for some characteristic values of the m_t mass We start running the RGEs from the scale $M_{\text{GUT}} \approx 10^{16}$ GeV (which is known to be the scale where the standard model gauge couplings meet [22]), while the value for the common gauge coupling at M_{GUT} is taken $g_{\text{GUT}} = 1/25\ 1$ We will assume that supersymmetry is valid down to the scale m_t while we run the system with the non-supersymmetric beta function coefficients below m_t First we determine the quark and charged lepton masses, mixings etc which are described in terms of 13 free parameters in the context of the standard model, only with the eight input parameters $(x, y, z, q, \phi, a, b, f)$ at the GUT scale Using only two additional inputs which are the scales of the $\nu^c\nu^c$ and $\nu^c\phi$ entries in the neutrino mass matrix, we give predictions for the light neutrino masses and leptonic mixing angles which can be tested in recent neutrino experiments Taking into account all the constraints and mass relations mentioned above, we present in the following our results for $m_t = 130, 150$ and 160 GeV We always choose to fix a, b and f parameters in terms of the charged lepton masses, hence we give our results only in terms of the set (x, y, z, ϕ) and $\tan\beta$ Then, $\lambda_t(t_0)$ coupling is also fixed once $\tan\beta$ and m_t are chosen

For $m_t = 130\,\mathrm{GeV}, \tan\beta = 1\ 1$ and $\phi = \pi/6\ 5$, we obtain the following results

$$m_d \approx 6\ 3\,\mathrm{MeV}, \quad m_s \approx 154\,\mathrm{MeV}, \quad m_b \approx 4\ 33\,\mathrm{GeV}, \quad m_u \approx 4\ 0\,\mathrm{MeV}, \quad m_c \approx 1\ 27\,\mathrm{GeV}, \tag{40a}$$

in agreement with the values obtained by the approximation formulae (28)–(30) and (34)–(36) The Kobayashi–Maskawa matrix elements $|(V_{\mathrm{KM}})_{ij}|$, are

$$V_{\mathrm{KM}} = \begin{pmatrix} 0\ 9754 & 0\ 2205 & 0\ 0032 \\ 0\ 2202 & 0\ 9748 & 0\ 0356 \\ 0\ 0108 & 0\ 0340 & 0\ 9994 \end{pmatrix} \tag{40b}$$

For $m_t = 150\,\mathrm{GeV}$, we use $\tan\beta = 2\ 2$ and $\phi = \pi/4\ 5$ We get

$$m_d \approx 6\ 2\,\mathrm{MeV}, \quad m_s \approx 153\,\mathrm{MeV}, \quad m_b \approx 4\ 25\,\mathrm{GeV}, \quad m_u \approx 4\ 05\,\mathrm{MeV}, \quad m_c \approx 1\ 26\,\mathrm{GeV}, \tag{41a}$$

$$V_{\mathrm{KM}} = \begin{pmatrix} 0\ 9752 & 0\ 2212 & 0\ 0028 \\ 0\ 2109 & 0\ 9744 & 0\ 0429 \\ 0\ 0117 & 0\ 0413 & 0\ 9991 \end{pmatrix} \tag{41b}$$

Finally, for $m_t = 160\,\mathrm{GeV}$, we take $\tan\beta \approx 3\ 3$ and $\phi = \pi/4\ 5$ Then,

$$m_d \approx 6\ 2\,\mathrm{MeV}, \quad m_s \approx 152\,\mathrm{MeV}, \quad m_b \approx 4\ 26\,\mathrm{GeV}, \quad m_u \approx 3\ 9\,\mathrm{MeV}, \quad m_c \approx 1\ 26\,\mathrm{GeV}, \tag{42a}$$

$$V_{\mathrm{KM}} = \begin{pmatrix} 0\ 9751 & 0\ 2219 & 0\ 0025 \\ 0\ 2216 & 0\ 9741 & 0\ 0445 \\ 0\ 0120 & 0\ 0430 & 0\ 9990 \end{pmatrix} \tag{42b}$$

It is worth noting here, that as the top mass gets higher the phase ϕ should also become larger in order for the KM entries to lie within the experimental limits A larger $\tan\beta$ is also required

To obtain the neutrino spectrum and lepton mixing, we must introduce values for the two additional parameters $M, \Lambda^{\mathrm{rad}}$ of the neutrino mass matrix (3) We assume naturally $M = \langle\overline{H}\rangle \approx 10^{16}\,\mathrm{GeV}$ In order to study the properties of the neutrino matrix, we let Λ^{rad} vary in a reasonable range between 10^{11} and 10^{13} and fix its value later with the available neutrino data

Next, we parametrize the lepton mixing matrix in a convenient way, i e

$$V^{\mathrm{lep}} = \begin{pmatrix} c_1c_3 - s_1s_2s_3\,\mathrm{e}^{i\phi} & s_1c_3 + c_1s_2s_3\,\mathrm{e}^{i\phi} & -c_2s_3 \\ -s_1c_2\,\mathrm{e}^{i\phi} & c_1c_2\,\mathrm{e}^{i\phi} & s_2 \\ c_1s_3 + s_1s_2c_3\,\mathrm{e}^{i\phi} & s_1s_3 - c_1s_2c_3\,\mathrm{e}^{i\phi} & c_2c_3 \end{pmatrix} \tag{43}$$

The predictions of the relevant mixing for the neutrino oscillations can now be presented in terms of the angles defined in the parametrization of V^{lep}

In our model described above V^{lep} is fixed by the quark and charged lepton data In fact, due to the assumed form of the matrix $M_{\nu^c,\phi}$, it only depends on the ratio $M_{33}^{\mathrm{rad}}/M_{11}^{\mathrm{rad}}$ which in our model is equal to b/f (see eqs (4) and (5a)) The neutrino eigenvalues, however, cannot be accurately predicted due to the scale quantity Λ^{rad} which is not specified in our model Thus they can be written as

$$m_{\nu_e} \approx 0, \quad m_{\nu_\mu} = \frac{\Lambda^\mu}{\Lambda^{\mathrm{rad}}} \times 10^{-2}\,\mathrm{eV}, \quad m_{\nu_\tau} = \frac{\Lambda^\tau}{\Lambda^{\mathrm{rad}}} \times 10\,\mathrm{eV} \tag{44a}$$

For $m_t \approx 130\,\mathrm{GeV}$ we get $\Lambda^\mu \approx 0\ 80 \times 10^{12}$ and $\Lambda^\tau \approx 1\ 85 \times 10^{12}$

Next, we present the light neutrino mixing matrices for two choices of m_t, i e $m_t = 130\,\mathrm{GeV}$ and $m_t = 150\,\mathrm{GeV}$

For $m_t = 130\,\mathrm{GeV}$, we obtain

$$V^{\rm lep} = \begin{pmatrix} -0\ 9958 + 7\ 7i \times 10^{-4} & (-8\ 5 + 3\ 2i) \times 10^{-2} & 0\ 00347 \\ (-8\ 5 - 3\ 2i) \times 10^{-2} & 0\ 9954 + 7\ 7i \times 10^{-4} & -0\ 0307 \\ (-8\ 4 + 9\ 9i) \times 10^{-4} & -0\ 031 + 0\ 09i \times 10^{-3} & -0\ 9995 \end{pmatrix} \tag{44b}$$

For $m_t = 150\,\mathrm{GeV}$ we get

$$V^{\rm lep} = \begin{pmatrix} -0\ 9955 + 1\ 4i \times 10^{-3} & (8\ 4 - 4\ 5i) \times 10^{-2} & 0\ 0034 \\ (-8\ 4 - 4\ 5i) \times 10^{-2} & -0\ 9947 - 1\ 37i \times 10^{-3} & -0\ 0388 \\ (-0\ 13 + 1\ 7i) \times 10^{-3} & 0\ 039 - 0\ 098i \times 10^{-3} & -0\ 9993 \end{pmatrix} \tag{44c}$$

From the above, it can be seen that the mixing between all neutrino species is small For values of $\Lambda^{\rm rad} \approx 10^{12}$ the obtained neutrino masses are much too small to be detected directly in present experiments like neutrinoless double beta decay, muon number violating processes etc At present, the only place to detect such small neutrino masses are neutrino oscillation experiments or astrophysics We can approximate the oscillation probabilities relevant to this latter case with a high accuracy in terms of the 2×2 familiar case, as follows

$$P(\nu_e \leftrightarrow \nu_\mu) \approx 3\ 1 \times 10^{-2} \sin^2\left(\pi \frac{L}{l_{12}}\right), \tag{45a}$$

$$P(\nu_\tau \to \nu_\mu) \approx 4\ 0 \times 10^{-3} \sin^2\left(\pi \frac{L}{l_{13}}\right), \tag{45b}$$

$$P(\nu_e \to \nu_\tau) \approx 4\ 0 \times 10^{-5} \sin^2\left(\pi \frac{L}{l_{13}}\right), \tag{45c}$$

where L is the source–detector distance and

$$l_{ij} = \frac{4\pi E_\nu}{|m_i^2 - m_j^2|} \tag{46}$$

Notice that the oscillation length l_{23} does not appear in the above formulae Since however, $m_{\nu_e} \ll m_{\nu_\mu}$ and $m_{\nu_\mu} \ll m_{\nu_\tau}$, one can in principle constrain both m_{ν_μ} and m_{ν_τ} from such data It is clear from the relations (45a)–(45c) that our results are not compatible with large mixing angle experimental limits Neutrino oscillations in the medium [23] via the MSW effect [24] provide a solution to the solar neutrino problem The GALLEX solar neutrino data [25],

$$5\ 0 \times 10^{-3} \leqslant \sin^2 2\theta_{ij} \leqslant 1\ 6 \times 10^{-2}, \tag{47}$$

$$0\ 32 \times 10^{-5} \leqslant \delta m_{ij}^2 \leqslant 1\ 2 \times 10^{-5} (\mathrm{eV})^2, \tag{48}$$

can be accomodated in our model Our result (45a) is a bit outside the above range but the mass constraint can be easily satisfied by choosing $\Lambda^{\rm rad}$ in the range

$$0\ 7 \times 10^{12} \leqslant \Lambda^{\rm rad} \leqslant 7 \times 10^{12}$$

Our neutrino masses can also easily be made to fall into the range of the Fréjus atmospheric neutrinos [26]

$$10^{-3} \leqslant \delta m_{ij}^2 \leqslant 10^{-2} (\mathrm{eV})^2, \tag{49}$$

but our mixing is much too small Our results are also consistent with the data on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations [26],

$$\sin^2 2\theta_{\mu\tau} \leqslant 4\,0 \times 10^{-3}, \quad \delta m^2_{\nu_\mu \nu_\tau} \geqslant 50(\mathrm{eV})^2 \tag{50}$$

Our results however cannot be made to fall on the $\sin^2 2\theta$ versus δm^2 of the BNL $\nu_\mu \leftrightarrow \nu_e$ oscillation results [27]

Moreover, it is always possible to obtain $m_{\nu_\tau} \approx$ (few $\sim$ 20) eV, hence one can obtain simultaneously the cosmological HOT-dark matter component, in agreement with the interpretation of the COBE data [28] Indeed an upper limit on the ν_τ mass can be obtained from the formula

$$7\,5 \times 10^{-2} \leqslant \Omega_\nu h^2 \leqslant 0\,3 \tag{51}$$

Translating this into a constraint on m_{ν_τ}, arising from the relation $m_{\nu_\tau} \approx \Omega_\nu h^2\, 91\,5\,\mathrm{eV}$ where $h = 0\,5 \sim 1\,0$ is the Hubble parameter, one gets the range

$$6\,8 \leqslant m_{\nu_\tau} \leqslant 27\,\mathrm{eV}, \tag{52}$$

which can be easily achieved with the above range of Λ^{rad}

In conclusion, we have proposed a structure of the fermion mass matrices in the flipped SU(5) model By allowing the Yukawa couplings to evolve from the GUT scale down to m_W, using only 8 input parameters at the GUT scale, we can fix all the 13 measurable parameters (masses and mixings angles) at m_W Furthermore, with the above information our model allows us to make definite predictions for the neutrino masses and the leptonic "Kobayashi–Maskawa" matrix In particular, we have found that the generalized see-saw mechanism which occurs naturally in this model can provide a solution to the solar neutrino problem via the MSW mechanism Moreover, a sufficiently large ν_τ mass is always possible in this model in order to contribute as a hot dark matter component, as indicated by the recent COBE data

Note added After the completion of this work we received a copy of ref [29], where it is shown that the generalized see-saw mechanism in this model can also account for the baryon asymmetry of the Universe Indeed, in our model we also have at least one singlet neutrino state with mass of order 10^{11} GeV

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