# New fermion mass textures from anomalous $U(1)$ symmetries with baryon and lepton number conservation

G.K. Leontaris [a,b], J. Rizos [b]

[a] *Theory Division, CERN, CH 1211 Geneva 23, Switzerland*
[b] *Theoretical Physics Division, Ioannina University, GR-45110 Ioannina, Greece*

## Abstract

In this paper we present solutions to the fermion mass hierarchy problem in the context of the minimal supersymmetric standard theory augmented by an anomalous family dependent $U(1)_X$ symmetry. The latter is spontaneously broken by non-zero vevs of a pair of singlet fields whose magnitude is determined through the *D*- and *F*-flatness conditions of the superpotential. We derive the general solutions to the anomaly cancellation conditions and show that they allow numerous choices for the $U(1)_X$ fermion charges which give several fermion mass textures in agreement with the observed fermion mass hierarchy and mixing. Solutions with $U(1)_X$ fermion charge assignments are found which forbid or substantially suppress the dangerous baryon and lepton number violating operators and the lepton-Higgs mixing coupling while a Higgs mixing mass parameter ($\mu$-term) can be fixed at the electroweak level. We give a general classification of the fermion mass textures with respect to the sum of the doublet-Higgs $U(1)_X$ charges and show that suppression of dimension-five operators naturally occurs for various charge assignments. We work out cases which retain a quartic term providing the left-handed neutrinos with Majorana masses in the absence of right-handed neutrino components and consistent with the experimental bounds. Although there exist solutions which naturally combine all the above features with rather natural $U(1)_X$ charges, the suppression of the $\mu$-term occurs for particular assignments. © 2000 Elsevier Science B.V. All rights reserved.

## 1. Introduction

The minimal supersymmetric extension of the standard model theory (MSSM) has had a remarkable success in explaining the low energy parameters in the context of unification scenario. Among them, the measured values of the strong coupling constant $\alpha_s(m_W)$ and the weak mixing angle $\sin^2\theta_W(m_W)$ are in perfect agreement with those predicted when the unification scale is taken to be of the order $M_U \sim 10^{16}$ GeV and only contribution of the MSSM spectrum is assumed to the beta-function coefficients for the

gauge coupling running. These remarkable properties of the simplest unified model, naturally raise the question whether the fermion mass spectrum observed in low energies is also reproduced from few basic symmetry principles encountered at the unification scale.

The experience from string model building has shown that a natural step towards this simplification is to assume the existence of $U(1)$ symmetries which distinguish the various families. A further indication that additional symmetries beyond the standard gauge group exist, has been the observation that the fermion mixing angles and masses have values consistent with the appearance of ''texture'' zeros in the mass matrices [1]. More precisely, it has been observed that in string model building one usually ends up with the effective field theory model which, in addition to the non-abelian gauge group includes an anomalous abelian gauge symmetry whose anomaly is cancelled by the Green–Schwarz mechanism [2]. In fact, this mechanism allows for the existence of a gauged $U(1)_X$ whose anomaly is cancelled by assigning a non-trivial transformation to an axion which couples universally to all gauge groups. In the spectrum of a string model, there are usually singlet fields $\phi_i, \bar{\phi}_i$ charged under this $U(1)$ symmetry which develop vacuum expectation values (vevs) in order to satisfy the *F*- and *D*-flatness conditions of the superpotential. This results in a spontaneous breaking of the anomalous $U(1)$ symmetry, naturally at some scale one order of magnitude less than the string (unification) scale.

Surprisingly, the existence of an anomalous or non-anomalous $U(1)$ symmetry has remarkable implications in low energy physics: for example, one may try to explain [3–14] the mass hierarchies observed in the quark and charged leptonic sector. In this approach, all quark, lepton and Higgs fields are charged under the extra abelian symmetry. The charges are chosen so that when the $U(1)$ symmetry is unbroken, only the third generation is massive and all mixing angles are zero. However, when the singlet fields obtain a non-zero vev, symmetry breaking terms gradually fill in the fermion mass matrices and generate a hierarchy of mass scales and mixing angles. It turns out that the symmetry breaking terms appearing in the fermion mass matrices may be expressed as powers of a few expansion parameters leading to a rather impressive predictability of the whole scheme. If further the $U(1)$ is anomalous, then the vacuum expectation values of the singlets are also given in terms of the unification (string) scale and definite predictions may arise for the masses and mixing angles. In fact, it will turn out that successful hierarchical mass patterns appear only if the $U(1)_X$ symmetry is anomalous.

It is rather interesting that this scenario may also give the correct prediction for the weak mixing angle without assuming unification. It was shown in Ref. [4] that in the presence of an anomalous $U(1)_X$ symmetry, the value of $\sin^2\theta_W$ could be predicted in terms of the $U(1)_X$ charges of the massless fermions. The anomaly cancellation mechanism may work in a way that the gauge couplings have the correct predictions in low energies. The simplest possibility of symmetric mass matrices was worked out in Ref. [5] and found that the hierarchical pattern of the fermion mass spectrum can be successfully reproduced.

In the present paper, we wish to extend the analysis by considering the most general $U(1)_X$ symmetry with fermion charges respecting the anomaly cancellation conditions. We first find that, even in the simple case of a $U(1)$ factor obtained from linear

combination of the standard model symmetries, the fermion mass matrix structure is richer than that exhibited in [5]. This is a simple consequence of the fact that the vacuum expectation values of the two singlet fields $\phi$ and $\bar{\phi}$ differ from each other since they have to respect the $D$-term anomaly cancellation condition. Going further, we find that a general family dependent $U(1)$ anomalous symmetry generates four approximate texture-zero mass matrices of Table 1. It is found that the Higgs $U(1)_X$ charges play a crucial role, particularly in the determination of the lepton textures as well as baryon and lepton number violation. For a non-zero sum of the $U(1)_X$ Higgs charges it is possible to ban all dangerous dimension-five proton decay operators. We further find that one can choose a consistent set of $U(1)_X$ charges which prevent the appearance of an unacceptably large Majorana mass term for the left-handed neutrino.

The paper is organised as follows: In Section 2, we introduce the notation for fermion charges, we set our assumptions and solve the constraints from mixed anomalies for the fermion and Higgs $U(1)_X$ charges. In Section 3 we derive the general forms of the quark and lepton mass matrices. Using specific values for the charges, consistent with the solutions obtained in Section 2, we classify with respect to the sum of the Higgs-doublet $U(1)_X$ charges all possible fermion mass textures. In Section 4 we analyse the baryon and lepton number violating operators, as well as other dangerous terms which are not prevented from the standard model gauge symmetry. We impose constraints to eliminate these dangerous operators and in Section 5 we present particular examples of fermion matrices which fulfill these requirements. In Section 6 we present our conclusions.

Table 1
The five symmetric texture-zero mass matrices for the up- and down-quarks consistent with the observed hierarchical pattern [1]

| Texture | $m_U$ | $m_D$ |
|---|---|---|
| $T_1$ | $\begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix}$ |
| $T_2$ | $\begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}$ | $\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 2\lambda^3 & 1 \end{pmatrix}$ |
| $T_3$ | $\begin{pmatrix} 0 & 0 & \sqrt{2}\lambda^4 \\ 0 & \lambda^4 & 0 \\ \sqrt{2}\lambda^4 & 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix}$ |
| $T_4$ | $\begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \sqrt{3}\lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}$ | $\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ |
| $T_5$ | $\begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \sqrt{2}\lambda^4 & \dfrac{\lambda^2}{\sqrt{2}} \\ \lambda^4 & \dfrac{\lambda^2}{\sqrt{2}} & 1 \end{pmatrix}$ | $\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ |

## 2. The solutions to anomaly cancellation conditions

The Yukawa terms of the superpotential needed to provide masses to quarks and leptons are $SU(3) \times SU(2)_L \times U(1)_Y$ gauge invariant and are written as follows:

$$\mathcal{W} = \lambda_{ij}^u Q_i U_j^c H_2 + \lambda_{ij}^d Q_i D_j^c H_1 + \lambda_{ij}^e L_i E_j^c H_1. \tag{1}$$

Although all these terms are invariant under the standard model gauge group, there is no explanation why some of the Yukawa couplings are required to be much smaller than others to account for the fermion mass hierarchy. Extending the standard gauge group by one anomalous $U(1)_X$ with the MSSM fields being charged under this abelian factor, only terms which are invariant under this symmetry remain in the superpotential. The observed low energy hierarchy of the fermion mass spectrum and the demand to have natural values of the Yukawa couplings $\lambda_{ij}$ of order one, suggest that only tree-level couplings associated with the third generation should remain invariant. In this case, the mass terms involving some of the lighter fermions are generated through non-renormalizable superpotential couplings at some order. These higher order invariants are formed by adding to the tree-level coupling an appropriate number of singlet fields ($\bar{\phi}$ or $\phi$) which compensate the excess of the $U(1)_X$ charge. Since the anomaly cancellation mechanism [15] requires vevs for the singlet fields that are about an order of magnitude below the unification scale, the above scenario naturally reproduces hierarchical fermion mass spectra. We therefore assume that the anomalous charge of the singlet fields $\phi$ and $\bar{\phi}$ is $+1, -1$ respectively which is equivalent to measuring all charges in $\phi$ charge units.

In the present work we are interested in symmetric fermion mass matrices. To obtain a symmetric structure we need to define proper constraints on the fermion charges under the $U(1)_X$ symmetry. Under the assignments of Table 2 the charges of the mass matrices are $C_{ij}^U = q_i + u_j$, $C_{ij}^D = q_i + d_j$ and $C_{ij}^E = \ell_i + e_j$. The conditions for symmetric mass matrices in the above notation take the form

$$q_i + u_j = q_j + u_i,$$
$$q_i + d_j = q_j + d_i,$$
$$\ell_i + e_j = \ell_j + e_i. \tag{2}$$

The requirement that the third generation has tree-level couplings imposes the constraints

$$q_3 + u_3 + h_2 = 0,$$
$$q_3 + d_3 + h_1 = 0,$$
$$\ell_3 + e_3 + h_1 = 0. \tag{3}$$

By demanding tree-level couplings for the third generation and the minimum number of Higgs doublets (one for the up-quarks and another for the down-quarks and charged leptons) we essentially seek solutions for the case of large $\tan \beta$. In fact, this implies that the top, bottom and $\tau$ Yukawa couplings are equal (up to order one coefficients) at the unification scale. Clearly, the difference between the top mass ($m_t$) and the bottom mass ($m_b$) must arise entirely from a large Higgs vev ratio $\tan \beta = v_2/v_1 \gg 1$. Although we restrict here the analysis only of the large $\tan \beta$ case, yet, there is enough freedom to generate a variety of acceptable mass matrices. The case of small $\tan \beta \sim \mathcal{O}(1)$ can also

Table 2
Charge assignments for MSSM fields under the $U(1)_X$

| Field | Charge |
| --- | --- |
| $Q_i$ | $q_i$ |
| $D_i^c$ | $d_i$ |
| $U_i^c$ | $u_i$ |
| $L_i$ | $\ell_i$ |
| $E_i^c$ | $e_i$ |
| $H_2$ | $h_2$ |
| $H_1$ | $h_1$ |

be worked out easily in a similar way, by modifying conditions (3) so that the $b-\tau$ Yukawa coupling appears at a higher order. In this case, the fermion mass structures obtained are slightly modified [5]. They can be made consistent with the experimentally observed fermion mass spectrum by a suitable choice of the order one coefficients associated with each mass entry.

After imposing the conditions (2), (3) the charges of the possible quark couplings to the appropriate Higgs field take the form

$$C^{QU^cH_2} = C^{QD^cH_1} = \begin{pmatrix} 2(q_1 - q_3) & (q_1 - q_3) + (q_2 - q_3) & q_1 - q_3 \\ (q_1 - q_3) & 2(q_2 - q_3) & q_2 - q_3 \\ (q_1 - q_3) & (q_2 - q_3) & 0 \end{pmatrix}. \quad (4)$$

Similarly, for the charged leptons we have

$$C^{LE^cH_1} = \begin{pmatrix} 2(\ell_1 - \ell_3) & (\ell_1 - \ell_3) + (\ell_2 - \ell_3) & \ell_1 - \ell_3 \\ (\ell_1 - \ell_3) & 2(\ell_2 - \ell_3) & \ell_2 - \ell_3 \\ (\ell_1 - \ell_3) & (\ell_2 - \ell_3) & 0 \end{pmatrix}. \quad (5)$$

We observe that the charges of the up- and down-quark entries are the same. This result is obtained only due to the fact that we require symmetric textures and one-tree level coupling for each one of the quark matrices. Further, the quark charge-entries depend only on two combinations, $q_1 - q_3$ and $q_2 - q_3$. This is also the case for the leptons, with the replacements $q_i \to \ell_i$. Anomaly cancellation conditions will give further relations between $q_i$ and $\ell_i$ charges, so if $U(1)_X$ charges are somehow fixed in the quark sector, then one ends up with predictions in the lepton matrices.

We proceed with the analysis of the quarks. At this stage, one can readily conclude that in order to have acceptable quark masses we must have

$$q_1 - q_3 = \frac{n}{2}, \quad q_2 - q_3 = \frac{m}{2} \quad \text{where } m + n \neq 0, m, n = \pm 1, \pm 2, \dots \quad (6)$$

We do not write down a similar parametrization for the charged lepton entries since, as we will see, due to the various conditions we have imposed we will be able to express them in terms of the quark entries. We will discuss in detail the remaining constraints on the quark and lepton entries in the subsequent section but first we need to deal with the mixed anomalies associated with the MSSM and $U(1)_X$ gauge groups.

It is well known that the MSSM is anomaly free. The introduction of an extra anomalous $U(1)_X$ group factor leads to anomalies which should be absorbed. As already discussed, the Green–Schwarz anomaly cancellation mechanism may cancel the pure $U(1)_X$ anomaly and mixed $U(1)_X$-gravitational anomalies, however there are mixed anomalies of the form $A_i = (G_i G_i U(1)_X)$ where $G_i = (SU(3), SU(2), U(1)_Y)$. In terms of the $U(1)_X$ charges these are written as

$$A_3 : \sum q_i + \frac{1}{2} \sum (u_i + d_i), \tag{7}$$

$$A_2 : \frac{3}{2} \sum q_i + \frac{1}{2} \sum \ell_i + \frac{1}{2}(h_1 + h_2), \tag{8}$$

$$A_1 : \frac{1}{6} \sum q_i + \frac{1}{3} \sum d_i + \frac{4}{3} \sum u_i + \frac{1}{2} \sum \ell_i + \sum e_i + \frac{1}{2}(h_1 + h_2). \tag{9}$$

It was pointed out in Ref. [4] that in a model where the anomalies are canceled through the Green–Schwarz mechanism, the mixed anomalies with the standard model gauge group are proportional to the corresponding Kac–Moody level,

$$\frac{A_3}{A_2} = \frac{k_3}{k_2}, \tag{10}$$

$$\frac{A_2}{A_1} = \frac{k_2}{k_1}. \tag{11}$$

There are also mixed anomalies of the form $A_0 = (U(1)_Y U(1)_X^2)$ which should be zero:

$$A_0 : \sum q_i^2 + \sum d_i^2 - 2 \sum u_i^2 - \sum \ell_i^2 + \sum e_i^2 + (h_2^2 - h_1^2) = 0. \tag{12}$$

We should note here that in the calculation of anomalies we have only considered the fields of the MSSM spectrum and a pair of singlets $\phi, \bar{\phi}$ which are necessary to break the $U(1)_X$ symmetry and create the non-renormalizable terms which fill in the fermion mass matrices. In string constructions, however, there are additional particles (some of them carry fractional charges), which may also be charged under $U(1)_X$. Even in the case that these fields belong to non-trivial representations of $SU(3) \times SU(2) \times U(1)_Y$ group they usually come in pairs with opposite $U(1)_X$ charges so they do not contribute to the mixed anomalies.

The conditions (10), (11) have rather remarkable implications on the determination of the low energy parameters. Indeed, to confront with the standard unification scenario we have to impose the conditions

$$\sin^2 \theta_W(M_U) = \frac{3}{8}, \tag{13}$$

$$k_3 = k_2 \tag{14}$$

which lead to the constraints

$$\frac{A_3}{A_1} = \frac{A_2}{A_1} = \frac{3}{5}. \tag{15}$$

To proceed further, we should combine the equations obtained from the anomaly cancellation with the symmetry constraints (2). These can be solved with respect to the charges $u_1, u_2, d_1, d_2, e_1$ and $e_2$ while the constraints (3) can be solved with respect to

$u_3, d_3$ and $e_3$. Furthermore, we may solve the constraints (15) with respect to the charges $q_3, \ell_3$, while treating as parameters the sums $h_+, q_+, \ell_+$:

$$h_+ = h_1 + h_2, \quad q_+ = q_1 + q_2, \quad \ell_+ = \ell_1 + \ell_2. \tag{16}$$

Then, in terms of (16) we obtain

$$\ell_3 = \tfrac{1}{48}\left[-5h_+ - 18(q_+ - \ell_+)\right], \tag{17}$$

$$q_3 = \tfrac{1}{48}\left[-17h_+ + 6(q_+ - \ell_+)\right]. \tag{18}$$

This parametrization considerably simplifies the analysis of the quadratic constraint (12). Indeed, substituting the above solutions into (12) we arrive at the equation

$$6h_+^2 + 5(11q_+ + \ell_+ - 4h_2)h_+ - 6(\ell_+ + 3q_+)(q_+ - \ell_+ + 4h_2) = 0, \tag{19}$$

which can be solved easily. We find it convenient now to solve the above equation for $h_2$, keeping as parameters the sums $h_+, q_+, \ell_+$ as before. We find two solutions depending on the value of $h_+$:

- For $h_+ = 2(q_+ - 2q_3) \neq 0$ we obtain the simple relations

$$\tfrac{3}{16}\ell_+ = \tfrac{1}{3}q_3 = \tfrac{3}{8}\ell_3 = q_+ \ . \tag{20}$$

- For $h_+ \neq 2(q_+ - 2q_3)$ we obtain the solution

$$h_2 = \frac{-5h_+^2 + h_+(q_+ - 29q_3) + 24q_3(q_+ - 2q_3)}{6(h_+ - 2q_+ + 4q_3)},$$

$$\ell_+ = q_+ - 8q_3 - \tfrac{17}{6}h_+,$$

$$\ell_3 = -\tfrac{7}{6}h_+ - 3q_3. \tag{21}$$

The first solution is characterized by three parameters $q_1, q_2, h_2$ or equivalently $m, n, h_2$ (see Eq. (6)). The second is a four-parameter solution that depends on $m, n, q_3, h_+$ and $\ell_2$. As will become clear later (see Section 3), $\ell_2$ can be exchanged with an integer parameter $k$ and $h_+$ has to be integer too. Thus the first solution depends on two integer

Table 3
$U(1)_X$ charge assignments for the case $h_+ = m + n$ (Eq. (20))

| Field | Generation | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| $Q$ | $\frac{2n - 3m}{10}$ | $\frac{2m - 3n}{10}$ | $-\frac{3(m+n)}{10}$ |
| $U^c$ | $\frac{3m + 8n}{10} - h_2$ | $\frac{8m + 3n}{10} - h_2$ | $\frac{3(m+n)}{10} - h_2$ |
| $D^c$ | $-\frac{7m + 2n}{10} + h_2$ | $-\frac{7n + 2m}{10} + h_2$ | $-\frac{7(n+m)}{10} + h_2$ |
| $L$ | $-\frac{15k + 23m + 8n}{30}$ | $\frac{15k + 7m - 8n}{30}$ | $-\frac{4(m+n)}{15}$ |
| $E^c$ | $-\frac{15k + 37m + 22n}{30} + h_2$ | $\frac{15k - 7m - 22n}{30} - h_2$ | $\frac{11(m+n)}{15} - h_2$ |
| | Higgs | | |
| $H_1$ | $m + n - h_2$ | $H_2$ | $h_2$ |

Table 4

$U(1)_X$ charge assignments for the case $h_+ \neq m + n$. The value of $h_2$ is not an independent parameter but it is related to $q_3, h_+, m, n$ through Eq. (21). The integer $k$ is defined as $k = 2\ell_2 + 6q_3 - m + \frac{7}{3}h_+$ (see Section 3)

| Field | Generation | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| $Q$ | $\frac{n}{2} + q_3$ | $\frac{m}{2} + q_3$ | $q_3$ |
| $U^c$ | $\frac{n}{2} - q_3 - h_2$ | $\frac{m}{2} - q_3 - h_2$ | $-q_3 - h_2$ |
| $D^c$ | $\frac{n}{2} - q_3 - h_+ + h_2$ | $\frac{m}{2} - q_3 - h_+ + h_2$ | $-h_+ + h_2 - q_3$ |
| $L$ | $\frac{n-k}{2} - 3q_3 - 5\frac{h_+}{3}$ | $\frac{m+k}{2} - 3q_3 - 7\frac{h_+}{6}$ | $-3q_3 - 7\frac{h_+}{6}$ |
| $E^c$ | $\frac{n-k}{2} + 3q_3 - \frac{h_+}{3} + h_2$ | $\frac{m+k}{2} + 3q_3 + \frac{h_+}{6} + h_2$ | $\frac{h_+}{6} + h_2 + 3q_3$ |
| | Higgs | | |
| $H_1$ | $h_+ - h_2$ | $H_2$ | $h_2$ |

and one non-integer parameters while the second depends one three integers and one non-integer ones. The detailed charge assignments for each solution are shown in Tables 3 and 4, respectively.

We wish to emphasize here that these are the most general solutions to the anomaly cancellation conditions under the assumptions of symmetric mass matrices and tree-level Yukawa couplings for the third generation with $b - \tau$ unification. Using the parametrization (16), we have succeeded in linearizing the quadratic equation constraint and expressing the solutions in terms of a few parameters. The solutions above are suggestive for a classification with respect to the sum $h_+$ of the Higgs charges.

At this point we find it useful to close this section with a remark on the necessity of the $U(1)$ symmetry of being anomalous. Indeed, one may wonder whether an anomaly free abelian symmetry can comply with the above phenomenological requirements. Actually one can easily derive the most general solution of the constraints (2), (3) together with $A_3 = A_2 = A_1 = A_0 = 0$ that gives also vanishing trace for $U(1)_X$ and vanishing $U(1)_X^3$ anomaly. This solution is $2q_3 = d_3 = -u_3/2 = -\ell_+/3 = e_3/3 = q_+$, $u_1 = -3q_+/2 - q_2$, $u_2 = -5q_+/2 + q_2, d_2 = q_+/2 + q_2$, $d_1 = 3q_+/2 - q_2$, $\ell_1 = -3q_+ - \ell_2$, $e_1 = 3q_+/2 - \ell_2$, $e_2 = 9q_+/2 + \ell_2$, $h_1 = -h_2 = l_3 = -3q_+/2$. Unfortunately this solution predicts two additional tree-level couplings, namely $Q_2 U_1 H_2$ and $Q_1 U_2 H_2$ which are not consistent with phenomenological requirements (see Table 1). Thus one is forced to search for an anomalous $U(1)$ symmetry.

## 3. The derivation of the fermion mass matrices

In the subsequent analysis with regard to the derivation of the fermion mass matrices and the investigation of baryon and lepton number violation, a crucial role is played by the value of the parameter $h_+ \equiv h_1 + h_2$. Some first conclusions may be drawn by inspection of the forms of the charge matrices (4), (5), when they are written in terms of the parameters $m, n, h_+$. Thus, the up and down quark mass matrices are independent of the value of $h_+$, while, on the contrary, the structure of the charged lepton mass matrix depends decisively on $h_+$. It can be also easily checked that the baryon and lepton number violating operators are $h_+$-dependent. It is therefore convenient for our subsequent analysis to distinguish three cases for the $h_+$ value: we will first examine the case

$h_+ = 0$ which means that the two Higgs doublets possess opposite charges. Next we consider $h_+$ to be an integer and finally we comment on the non-integer values of $h_+$.

### 3.1. $h_+ = 0$

Starting from this particular value $h_+$, we first find that one of the two solutions (20), (21) does not lead to sensible results. Indeed, by a simple inspection we infer that solution (20) would imply $m + n = 0$ and therefore, a tree-level mass for the 12 and 21 entries. Thus, there is only acceptable one solution for (19), namely (21) which for $h_+ = 0$ takes the form

$$\ell_1 + \ell_2 = (q_1 + q_2) - 8q_3, \quad \ell_3 = -3q_3, \quad h_2 = -h_1 = -2q_3. \tag{22}$$

We can easily calculate the charges which are given in Table 5 in terms of the four free-parameters.

Using these charge assignments one finds that the quark- and lepton-charge matrices take the form

$$C^{QU^cH_2} = C^{QD^cH_1} = \begin{pmatrix} n & \dfrac{m+n}{2} & \dfrac{n}{2} \\ \dfrac{m+n}{2} & m & \dfrac{m}{2} \\ \dfrac{n}{2} & \dfrac{m}{2} & 0 \end{pmatrix}, \tag{23}$$

$$C^{LE^cH_1} = \begin{pmatrix} n-k & \dfrac{m+n}{2} & \dfrac{n-k}{2} \\ \dfrac{m+n}{2} & m+k & \dfrac{m+k}{2} \\ \dfrac{n-k}{2} & \dfrac{m+k}{2} & 0 \end{pmatrix}, \tag{24}$$

where $m,n$ are given in (6) while all the additional dependence of the lepton matrix has been absorbed in the parameter $k$ defined as follows:

$$k = 2\ell_2 + 6q_3 - m. \tag{25}$$

Let us now come to the parameters entering the quark and charged lepton mass matrices. We first note that the only tree-level couplings entering the fermion mass textures are those corresponding to the 33-entries of (23) and (24), i.e. the third generation mass terms for the up, down and lepton fields, i.e. $Q_3 t^c H_2$, $Q_3 b^c H_1$ and $L_3 \tau^c H_1$. The remaining mass matrix entries are expected to be generated from non-re-normalizable terms formed by proper powers of the singlet fields $\phi/M_1$, $\bar{\phi}/M_1$ and $\phi/M_2$, $\bar{\phi}/M_2$. The powers of non-renormalizable terms have to be such that the charges of the entries in the matrices (23), (24) are cancelled out. The singlets are divided by the mass parameters $M_1, M_2$ which refer to some high energy scales. If the dominant source of these terms is from string compactification, then $M_1 = M_2 \equiv M$ and there are only two expansion parameters which enter in the mass matrices, namely $\phi/M$ and $\bar{\phi}/M$. It is also possible that additional vector-like Higgs pairs may acquire their mass via spontaneous breaking after compactification. Then, a strong violation of the $SU(2)_R$ symmetry of the quark sector may occur, and as a result $M_1 \neq M_2$ [5].

Table 5
$U(1)_X$ charge assignments for the case $h_+ = 0$

| Field | Generation | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| $Q$ | $\frac{n}{2} + q_3$ | $\frac{m}{2} + q_3$ | $q_3$ |
| $U^c$ | $\frac{n}{2} + q_3$ | $\frac{m}{2} + q_3$ | $q_3$ |
| $D^c$ | $\frac{n}{2} - 3q_3$ | $\frac{m}{2} - 3q_3$ | $-3q_3$ |
| $L$ | $\frac{n-k}{2} - 3q_3$ | $\frac{m+k}{2} - 3q_3$ | $-3q_3$ |
| $E^c$ | $\frac{n-k}{2} + q_3$ | $\frac{m+k}{2} + q_3$ | $q_3$ |
| | Higgs | | |
| $H_1$ | $2q_3$ | $H_2$ | $-2q_3$ |

We further point out here that in the case of an anomalous $U(1)$ symmetry we should necessarily take $\phi \neq \bar{\phi}$. This is because the cancellation of the $D$-term requires these values to differ from each other. In particular, the Green–Schwarz anomaly cancellation mechanism generates a constant Fayet–Iliopoulos [16] contribution to the $D$-term of the anomalous $U(1)_X$. This is proportional to the trace of the anomalous charge over all fields capable of obtaining non-zero vevs. To preserve supersymmetry the following $D$-flatness condition should be satisfied [15,17]:

$$\sum_{\phi_j} Q_j^X |\phi_j|^2 = -\xi \neq 0, \tag{26}$$

where $Q_j^X$ is the $U(1)_X$ charge of the field $\phi_j$, while the sum extends over all possible singlet fields and the parameter $\xi$ is proportional to the trace of the anomalous $U(1)_X$. Clearly, in the case of two singlets with opposite charges – as in our case – one should have

$$|\langle \phi \rangle|^2 - |\langle \bar{\phi} \rangle|^2 = -\xi. \tag{27}$$

Thus, in the construction of the quark and lepton mass matrices in the general case we may define the following four parameters:

$$\varepsilon = \frac{\phi}{M_1}, \qquad \bar{\varepsilon} = \frac{\bar{\phi}}{M_1}, \tag{28}$$

$$\lambda = \frac{\phi}{M_2}, \qquad \bar{\lambda} = \frac{\bar{\phi}}{M_2}. \tag{29}$$

According to our previous discussion, the parameters $\varepsilon$ and $\bar{\varepsilon}$ should appear in the up-quark mass matrix while the second set, i.e. $\lambda$ and $\bar{\lambda}$, in the down quark and charged lepton mass matrices. Thus the possible mass terms should have one of the following forms:

$$Q_i U_j H_2 \, \varepsilon^{r_{ij}}, \qquad Q_i U_j H_2 \, \bar{\varepsilon}^{r_{ij}}$$

for the up quarks and

$$Q_i D_j^c H_1 \varepsilon^{s_{ij}}, \qquad Q_i D_j^c H_2 \bar{\varepsilon}^{s_{ij}},$$

$$L_i E_j^c H_1 \varepsilon^{p_{ij}}, \qquad L_i E_j^c H_1 \bar{\varepsilon}^{p_{ij}}$$

for the down quarks and charged leptons. Here, $r_{ij}, s_{ij}$ and $p_{ij}$ are numbers which represent the necessary powers of the expansion parameters in order to cancel the charge of the corresponding $ij$-entry in (23) and (24). Clearly these numbers have to be integers.

In the convenient parametrization (25) the charge entries depend only on the three free parameters $m, n, k$. Every charge entry is scaled with the charges of the singlet fields $\phi, \bar{\phi}$. Thus, without loss of generality we may simply take the charge of the latter to be $+1$ and $-1$ respectively. Under a certain choice of the $U(1)_X$ charges of the various fermion and Higgs fields, the entries of the charge matrices (23) and (24) can be either positive or negative. (We exclude the case of charges leading to zeros in these entries since this would lead to an additional tree-level order entry in the mass matrix.) Now, if the charge of an entry is positive we may cancel this only by adding powers of the singlet fields carrying negative $U(1)_X$ charge. On the contrary, entries with negative charge require powers of positively charged singlets. Therefore, in certain choices it is possible that two expansion parameters may enter in the mass matrices, leading thus to new structures. Note the fact that, there is no loss of predictability compared to previous cases [5] where only one expansion parameter was used in the fermion mass textures. Indeed, the two singlet vevs are related through Eq. (26), while the parameter $\xi$ in the right-hand side of this equation is completely determined by the trace of the anomalous symmetry, the value of the common gauge coupling at $M_U$ and the string (unification) scale itself [15,17].

In order for a mass entry to be generated, the specific combination of the free parameters $m, n, k$ entering the charge entry has to be integer. Otherwise the corresponding mass entry is zero since no power of singlet vevs with $\pm 1$ charge could make the relevant Yukawa coupling invariant under this $U(1)$. With the above remarks in mind, we are ready now to proceed in the determination of the viable fermion mass textures.

*Quarks*

We proceed now with the determination of all possible structures of the quark matrix. Although in this section we deal only with the case $h_+ = 0$, we will soon see that even in the most general case where $h_+ \neq 0$ the form of the quark mass matrices does not change as long as we impose the symmetric mass textures conditions on the $U(1)_X$-charges and the requirement of one tree-level coupling for each fermion matrix[1]. Thus, since only the 33-entry is filled up by a renormalizable coupling, clearly as long as the $U(1)_X$ remains unbroken the two lighter generations remain massless. When the singlets $\phi, \bar{\phi}$ develop non-zero vevs along the $D$-flat direction, their magnitudes are of the order $\langle \phi \rangle \sim \langle \bar{\phi} \rangle \sim \xi$, which is approximately one order lower than the unification scale. Then the anomalous symmetry is broken and the remaining fermion mass matrix entries are filled up with mass terms suppressed by powers of the expansion parameters. These powers depend on the certain choice of $U(1)_X$ charges.

Let us now determine the possible viable cases for the parameters $m, n$ which enter the quark mass matrices. In order to have a non-zero value for the second generation

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[1] As a matter of fact, this requirement on the $U(1)_X$-charges only ensures that at least one entry admits a tree-level coupling. It does not exclude the appearance of more than one tree-level couplings. Such solutions *do* appear and are excluded for phenomenological reasons.

quarks it is evident from the structure of the charge matrix (23) that the parameter $m$ has to be an integer. Only in this case at least one of the entries 22, 23/32, may survive. With similar reasoning, while taking also into consideration the necessity of the Cabbibo mixing, we may also conclude that the parameter $n$ has to be an integer too. No constraint from mixing effects can be imposed in the case of leptons, however, due to the fact that $m,n$ are integers we have also to take $k$ to be integer otherwise we would end up with two massless charged lepton states. Therefore there are four possibilities among which we distinguish three viable cases, namely (i) $m,n$ even; (ii) $m$ odd, $n$ even, (iii) $m$ odd, $n$ odd. (The case $m$ even, $n$ odd does not lead to acceptable mixings.) We consider these cases separately and, further, we work out certain choices of $m,n$ pairs which lead to viable mass textures with reasonable values of the expansion parameters. We note that due to our freedom to have two different expansion parameters and to adjust order-one coefficients in the mass matrix entries, additional pairs of $m,n$ values are also possible. They imply different values for the expansion parameters but do not lead to different textures thus they are not elaborated here.

*Case (i): n,m even*

$i_A$: We first start with positive $n,m$ values. In this case all quark charge entries in (23) are positive, so the lowest power of singlets needed to cancel this charge involves only the singlet $\bar{\phi}$ to a proper power. (Additional contributions involving pairs $(\phi\bar{\phi})^v$ are always possible but relatively suppressed.) Taking $n = 2m > 0$ we obtain a quark mass structure similar to the texture $T_5$ of Table 1. For example, for $m = 4, n = 8$ we get[2]

$$m_U = \begin{pmatrix} \bar{\varepsilon}^8 & \bar{\varepsilon}^6 & \bar{\varepsilon}^4 \\ \bar{\varepsilon}^6 & \bar{\varepsilon}^4 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^4 & \bar{\varepsilon}^2 & 1 \end{pmatrix}. \tag{30}$$

This matrix is not actually an exact texture-zero as the $T_5$ case, however, one can observe that the entries replacing the zeros of texture $T_5$ are highly suppressed here. Remarkably, this texture is also an outcome of the string derived flipped $SU(5)$ model [18,19].

The corresponding down quark matrix has the same form but in general involves a different expansion parameter, namely $\bar{\lambda}$. This gives the freedom to adjust the two parameters so that the correct hierarchy and Cabbibo mixing arise[3]. Thus, the matrix takes the form

$$m_D = \begin{pmatrix} \bar{\lambda}^8 & \bar{\lambda}^6 & \bar{\lambda}^4 \\ \bar{\lambda}^6 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \bar{\lambda}^4 & \bar{\lambda}^2 & 1 \end{pmatrix}. \tag{31}$$

We note here that the mass entries in the above textures are accurate up to order one coefficients which are not calculable in this approach. As far as we know the calculation

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[2] The mass matrices are scaled by the appropriate Higgs vev.

[3] Note, however, that the same expansion parameter $\bar{\lambda}$ enters in the lepton sector, so additional constraints will also come from the charged lepton mass eigenvalues.

of the coefficients is only possible in string models. The remarkable fact, however, in the present simple approach is that one does not need to introduce unnaturally small Yukawa couplings to explain the huge ratios of mass eigenstates. The present procedure tell us that the hierarchical pattern is just a simple consequence of the $U(1)_X$ symmetry.

$i_B$: Next, we give an example where both parameters enter in the structure of the quark mass matrices. Thus, taking one of the integers to be negative, we may obtain textures with $\varepsilon$ and $\bar{\varepsilon}$ powers in the matrices. For example, an appropriate choice is $n = -4m$, where we obtain a structure which is very close to the texture $T_4$ of Table 1. In particular, choosing $m = 4, n = -16$, the lower $2 \times 2$ charge entries in (23) are positive, while the rest are negative so we have the following structure of the up- and down-quark mass matrices:

$$
m_U = \begin{pmatrix} \varepsilon^{16} & \varepsilon^6 & \varepsilon^8 \\ \varepsilon^6 & \bar{\varepsilon}^4 & \bar{\varepsilon}^2 \\ \varepsilon^8 & \bar{\varepsilon}^2 & 1 \end{pmatrix}, \qquad m_D = \begin{pmatrix} \lambda^{16} & \lambda^6 & \lambda^8 \\ \lambda^6 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \lambda^8 & \bar{\lambda}^2 & 1 \end{pmatrix}. \tag{32}
$$

$i_C$: We finally give the mass matrices for one more choice, which, as we will see coincide with the one presented in Ref. [5]. With our definitions of charges, this case arises if we put $m = 2$ and $n = -8$,

$$
m_U = \begin{pmatrix} \varepsilon^8 & \varepsilon^3 & \varepsilon^4 \\ \varepsilon^3 & \bar{\varepsilon}^2 & \bar{\varepsilon} \\ \varepsilon^4 & \bar{\varepsilon} & 1 \end{pmatrix}, \qquad m_D = \begin{pmatrix} \lambda^8 & \lambda^3 & \lambda^4 \\ \lambda^3 & \bar{\lambda}^2 & \bar{\lambda} \\ \lambda^4 & \bar{\lambda} & 1 \end{pmatrix}. \tag{33}
$$

Notice, however, the appearance of two expansion parameters in (33) compared to only one used to appear in Ref. [5].

Above, we have provided examples based on a different charge assignment ($m, n$ values) which naturally give hierarchical patterns for the quark sector. A natural question now arises as to which of these cases fits better the observed hierarchy and mixing effects. There are mainly three sources of further constraints that would definitely guide us to pick up one definite case. First, one needs an exact value of the parameter $\xi$ which determines the singlet Higgs vevs. Second, the order one coefficients which are not calculable, may point to a certain choice. Finally, the structure of the lepton mass matrix will provide further information on the parameters $\lambda, \bar{\lambda}$. We proceed now to the other two possibilities for $m, n$.

*Case (ii): m odd, n even*

This case assumes odd values for $m$ and even for $n$ which lead to an exact texture-zero as in the case of Table 1. To obtain viable matrices, we may take either $n = 2m$ or $n = -2m$. These choices lead to the same texture $T_3$ but with different expansion parameters. Taking $m = 3, n = \pm 6$ we get

$$
m_U = \begin{pmatrix} \bar{\varepsilon}^6 & 0 & \bar{\varepsilon}^3 \\ 0 & \bar{\varepsilon}^3 & 0 \\ \bar{\varepsilon}^3 & 0 & 1 \end{pmatrix} \quad \text{and} \quad m_U = \begin{pmatrix} \varepsilon^6 & 0 & \varepsilon^3 \\ 0 & \bar{\varepsilon}^3 & 0 \\ \varepsilon^3 & 0 & 1 \end{pmatrix}, \tag{34}
$$

respectively. How do these zero entries arise? Bearing in mind that $m$ was taken to be odd and $n$ even, the charge entries 11, 12, 21 and 23, 32 in the charge matrix of (23) are

half-integers. Since the singlet charges are $\pm 1$, it is not possible to generate contributions in these entries from non-renormalizable terms. These are exact texture-zero mass matrices and their form was proposed purely from phenomenological analysis in Ref. [20]. As in the first case discussed above, the down-quark mass matrices of these two cases are obtained with the replacements $\varepsilon \to \lambda$ and $\bar{\varepsilon} \to \bar{\lambda}$, thus these are

$$m_D = \begin{pmatrix} \bar{\lambda}^6 & 0 & \bar{\lambda}^3 \\ 0 & \bar{\lambda}^3 & 0 \\ \bar{\lambda}^3 & 0 & 1 \end{pmatrix} \quad \text{and} \quad m_D = \begin{pmatrix} \lambda^6 & 0 & \lambda^3 \\ 0 & \bar{\lambda}^3 & 0 \\ \lambda^3 & 0 & 1 \end{pmatrix}. \tag{35}$$

We will work out this case further when we will discuss the corresponding lepton matrix for total Higgs charge $h_+ \neq 0$.

*Case (iii): n,m odd*

We finally examine the case where both $n,m$ are odd. Here we obtain mass matrices similar to the up-quark texture $T_1$. We have the freedom to use several sets of $m,n$ pairs. A suitable choice is $m = -3, n = 11$ which gives

$$m_U = \begin{pmatrix} \bar{\varepsilon}^{11} & \bar{\varepsilon}^4 & 0 \\ \bar{\varepsilon}^4 & \varepsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{36}$$

A slightly different matrix involving only one parameter arises for $n = 5$ and $m = 3$. It leads to the same texture zero, however different powers of the expansion parameters appear. One gets

$$m_U = \begin{pmatrix} \bar{\varepsilon}^5 & \bar{\varepsilon}^4 & 0 \\ \bar{\varepsilon}^4 & \bar{\varepsilon}^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{37}$$

A general comment for the case (iii) is necessary here: due to the same structure of the up- and down-quark mass matrices, the exact texture-zero mass matrices in this case have small chance to reproduce the correct Kobayashi–Maskawa (KM) mixing. Indeed, since the down-quark mass matrix has the same form with the up, the KM mixing of the third generation with the other two – although experimentally is measured to be small – cannot be generated due to the complete decoupling of the third generation. However, in a realistic case, as in string model building, more than one pair of singlet fields acquire non-zero vevs. Usually, some other singlets with different charge assignment form some higher order Yukawa couplings with the fermions and generate small but nevertheless important contributions to the zero entries of the fermion matrices. Another source of induced small mixing arises from renormalization group effects. If charged lepton and Dirac–neutrino Yukawa couplings are not flavour diagonal (and as we will see, this is exactly what happens in the present case), then small calculable non-zero entries will replace the zeros in the above $m_U$ texture.

Having completed the analysis of the quark textures, we now need to consider the implications on the lepton mass matrix structure. Closing this subsection we simply note the remarkable fact that, even with one $U(1)_X$ anomalous symmetry and only one pair of singlet fields one is able to reproduce four out of the five phenomenological textures of Table 1.

*Leptons*

The analysis of the quark mass matrices in the previous section, has put several constraints on the values of $m, n$ parameters. Note also that already the phenomenological constraint which implies the successful relation $m_\tau = m_b$ at the unification scale has also been imposed on the $U(1)_X$ charges. Thus, the only remaining freedom to construct the charged lepton mass matrices in the case $h_+ = 0$ is the value of the parameter $k$. Note further that there is no freedom to adjust the 12, 21 elements of the charge–lepton matrix since they are fixed completely by the quark matrix. Bearing in mind that the parameters $m, n$ are integers, we can easily see that only the case of integer values $k = 0, \pm 1, \pm 2, \dots$ can lead to acceptable lepton mass matrices. In the following, we examine viable lepton textures with respect to the value of $k$ for each of the three cases in the quark sector discussed above.

(i): As in the corresponding case for quarks, we derive here the lepton mass matrices for three $(m, n)$ sets and viable choices for $k$.

$i_A$: For $n = 2m > 0$, we may take for example $m = 4$, $n = 8$ and $k = 0$ or $k = -1$, so we obtain

$$m_L = \begin{pmatrix} \bar\lambda^8 & \bar\lambda^6 & \bar\lambda^4 \\ \bar\lambda^6 & \bar\lambda^4 & \bar\lambda^2 \\ \bar\lambda^4 & \bar\lambda^2 & 1 \end{pmatrix}, \quad \text{or} \quad m_L = \begin{pmatrix} \bar\lambda^7 & \bar\lambda^6 & 0 \\ \bar\lambda^6 & \bar\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{38}$$

respectively. These correspond to the approximate hierarchies $m_e : m_\mu : m_\tau \approx \bar\lambda^8 : \bar\lambda^4 : 1$ or $m_e : m_\mu : m_\tau \approx \bar\lambda^7 : \bar\lambda^3 : 1$. The first matrix predicts exactly the correct relation $\det m_D = \det m_L$ [21] while the second gives also a quite satisfactory result up to order one coefficients.

$i_B$: Let us now take $n = -4m$, with the additional restriction that $m + k > 0$. Certainly, by inspection of the charge matrix (24) we conclude that the entries 22, 23, 32 have positive charges whilst all entries connected to the first generation obtain negative ones. This means that to lowest order we can cancel the charge of the first with $\bar\lambda$-powers and the charge of the second with powers of the expansion parameter $\lambda$. Thus, the following texture arises:

$$m_L = \begin{pmatrix} \lambda^{(4m+k)} & \lambda^{3m/2} & \lambda^{(4m+k)/2} \\ \lambda^{3m/2} & \bar\lambda^{m+k} & \bar\lambda^{(m+k)/2} \\ \lambda^{(4m+k)/2} & \bar\lambda^{(m+k)/2} & 1 \end{pmatrix}. \tag{39}$$

Choosing now $m = 4$ (as in the corresponding quark case) and $k = -1$ we arrive at a texture-zero matrix of the form

$$m_L = \begin{pmatrix} \lambda^{15} & \lambda^6 & 0 \\ \lambda^6 & \bar\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{40}$$

Comparing with the previous case (39) we see that we now have the possibility of adjusting the value of the 22-entry independently from the other matrix elements. Indeed, recall that the down-quark mass hierarchy in this case is $\lambda^{12} : \bar\lambda^4 : 1$, which implies the hierarchical relation $\bar\lambda \approx \lambda^{3/2}$ to $\lambda^2$ (depending on the order one coefficients). This relation fits also well the charged-lepton mass hierarchy.

$i_C$: Finally, we derive the lepton matrix which corresponds to the case $i_C$ of the quark sector. For $m = 2$, $n = -8$ and $k = 3$ we obtain

$$m_L = \begin{pmatrix} \lambda^5 & \lambda^3 & 0 \\ \lambda^3 & \lambda^1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{41}$$

in accordance with the texture derived in [5]. We note also that we may have more possibilities by choosing $m + k < 0$, obtaining a different lepton structure but such cases will not be elaborated here.

(ii): Here, as in the case of quark mass matrices, we take the cases $m = 3, n = 6$ (the case $m = 3, n = -6$ can be worked out similarly). Now we are free to choose the value of $k$ in order to obtain a natural charged-lepton mass hierarchy. Assuming $k$-values in the range $-3 < k < 6$ we can write the lepton mass matrix in form

$$m_L = \begin{pmatrix} \bar{\lambda}^{6-k} & 0 & \lambda^{\frac{6-k}{2}} \\ 0 & \lambda^{3+k} & \lambda^{\frac{1+k}{2}} \\ \bar{\lambda}^{\frac{6-k}{2}} & \lambda^{\frac{1+k}{2}} & 1 \end{pmatrix}. \tag{42}$$

An interesting texture arises for the $k = -2$. This gives a lepton matrix which has the same structure with the quarks:

$$m_L = \begin{pmatrix} \bar{\lambda}^8 & 0 & \bar{\lambda}^4 \\ 0 & \bar{\lambda} & 0 \\ \bar{\lambda}^4 & 0 & 1 \end{pmatrix}. \tag{43}$$

This matrix gives eigenvalues in the ratios $-\bar{\lambda}^8 : \bar{\lambda}$ to be compared with the mass eigenstates $m_e/m_\tau : m_\mu/m_\tau$ at the unification scale. We note however that this relation is satisfied for a rather large value of the expansion parameter $\bar{\lambda}$. Further, for $k = -1$ we obtain

$$m_L = \begin{pmatrix} \bar{\lambda}^7 & 0 & 0 \\ 0 & \bar{\lambda}^2 & \bar{\lambda} \\ 0 & \bar{\lambda} & 1 \end{pmatrix}, \tag{44}$$

which gives the ratios $-\bar{\lambda}^7 : \bar{\lambda}^2$ for $m_e/m_\tau : m_\mu/m_\tau$. We will see soon that the matrices obtained for the case (ii) are phenomenologically more promising when we assume $h_+ \neq 0$. We note here that this texture implies large mixing in the $\mu$–$\tau$ sector and it could be distinguished from the first one, due to the different flavour violating processes it implies. In particular, we should expect an enhancement of the $\tau \to \mu\gamma$ branching ratio compared to the first case.

(iii): In this last case we choose $n = 11, m = -3$. We have observed that in the quark sector there is no mixing between the two heavier generations. In contrast, in the case of

charged leptons this mixing may arise from a suitable choice of the additional parameter $k$. Thus, for the quark matrix (37) choosing $k = 1$ we obtain

$$
m_L = \begin{pmatrix} \bar{\lambda}^{10} & \bar{\lambda}^4 & \bar{\lambda}^5 \\ \bar{\lambda}^4 & \lambda^2 & \lambda^2 \\ \bar{\lambda}^5 & \lambda^2 & 1 \end{pmatrix}.
\tag{45}
$$

### 3.2. $h_+ \neq 0$

We come now to the most general case where the sum of the Higgs doublet charges is different than zero. As explained in Section 2, there are two solutions of the anomaly equations under the symmetry requirements and the tree-level constraints. The full $U(1)_X$-charge assignment of the two solutions for the matter and Higgs fields are now shown in Tables 3 and 4.

It is a welcome fact that the quark mass matrices (as can be easily checked), do not change at all under this generalization thus, our analysis concerning the up and down quark textures remains intact. We therefore turn our attention to the case of the charged lepton mass matrices. In this case we can easily see that only the entries connected with the first generation in (24) receive additional contribution. The charge-lepton matrix in the general case $h_+ \neq 0$ becomes

$$
C^{LE^c H_1} = \begin{pmatrix} n - k - h_+ & \dfrac{m + n - h_+}{2} & \dfrac{n - k - h_+}{2} \\ \dfrac{m + n - h_+}{2} & m + k & \dfrac{m + k}{2} \\ \dfrac{n - k - h_+}{2} & \dfrac{m + k}{2} & 0 \end{pmatrix}
\tag{46}
$$

with the replacement now of a new value for $k = 2\ell_2 + 6q_3 - m + \frac{7}{3}h_+$.

There is an additional contribution which equals the minus sum of the Higgs charge $(-h_+)$ in the entries 11,12/21 and 13/31 thus, in the general case the elements 12 and 21 are no-longer equal to the corresponding ones of the quark matrix. Our notation here might be confusing in the sense that there appear four different parameters in the lepton case, namely $m, n, k$ and $h_+$. In fact, (as is clear from (5)), there are only two parameters $\ell_1 - \ell_3$ and $\ell_2 - \ell_3$ which enter in this structure; here they can be taken to be the combinations $n + m - h_+$ and $m + k$. This is the price we have to pay in order to keep the parametrization already used, and transfer the constraints from the quark sector.

In the above parametrization we can easily see now that the case $h_+ \neq 0$ has some important implications on the lepton mass matrix structure. First, from our analysis in the quark sector we observe that we are forced to take integer values for the parameters $m, n$. We can easily see that a non-integer value of the total Higgs charge $h_+$ would lead to a massless state. As a result we are forced to assume only integer values for both, $k$ and $h_+$ parameters.

Let us consider now the first solution (20). As seen from Table 3 the sum of the Higgs doublet charges is fixed $h_+ = m + n$ and thus the elements 12 and 21 of the lepton matrix (46) vanish. This means that the associated couplings become of the order of the

$\tau$ mass and this texture leads to two heavy eigenstates. Thus we will not consider this solution further and from this point we will refer only to Solution (21) when discussing $h_+ \neq 0$.

Another important constraint arises from the relation $\det m_D = \det m_L$ [21]. Assuming[4] $n > 0$ we have

$$\det m_D = \begin{cases} \lambda^{-m} \, \bar{\lambda}^n & m < 0, \, m + n = \text{odd} \\ \bar{\lambda}^{m+n} \text{ or } \lambda^{-m-n} & \text{otherwise} \end{cases} \tag{47}$$

for the quarks. Similarly for $k + n - h_+ > 0$ we have

$$\det m_L = \begin{cases} \lambda^{-k-m} \, \bar{\lambda}^{n-k-h_+} & k + m < 0, \, m + n - h_+ = \text{odd} \\ \bar{\lambda}^{m+n-h_+} \text{ or } \lambda^{-m-n+h_+} & \text{otherwise} \end{cases} \tag{48}$$

The eigenvalues of the lepton mass matrix can also be worked out. They have the form

$$(\lambda \text{ or } \bar{\lambda})^{|h_+ + k - n|}, (\lambda \text{ or } \bar{\lambda})^{|k + m|}, 1. \tag{49}$$

Notice that the presence of $h_+$ affects only the lightest eigenvalue.

We wish now to give one more example where we can obtain a realistic texture-zero matrix. We choose the values $m = 1, n = 2$ which correspond to quark matrices of case type $T_3$ as in *ii*. Taking $k = -2$ and $h_+ = 9$ we obtain

$$m_D = \begin{pmatrix} \bar{\lambda}^2 & 0 & \bar{\lambda} \\ 0 & \bar{\lambda} & 0 \\ \bar{\lambda} & 0 & 1 \end{pmatrix}, \qquad m_L = \begin{pmatrix} \lambda^5 & \lambda^3 & 0 \\ \lambda^3 & \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{50}$$

while $m_U$ has the same texture-zero as $m_D$ provided that we replace $\lambda \to \varepsilon$. The above texture-zero charged lepton matrix is different from the $m_D$-matrix. It implies no mixing for the $\tau$ lepton while it predicts the correct hierarchy, provided we impose the relation $\bar{\lambda} \approx \lambda^2$.

We give a final example by taking $m = 3, n = 6, \, h_+ = 12, \, h_1 = 1, \, k = -2$ and $q_3 = -4$. Then, we obtain the same texture-zero for both, down quark and lepton matrices:

$$m_D = \begin{pmatrix} \bar{\lambda}^6 & 0 & \bar{\lambda}^3 \\ 0 & \bar{\lambda}^3 & 0 \\ \bar{\lambda}^3 & 0 & 1 \end{pmatrix}, \qquad m_L = \begin{pmatrix} \lambda^4 & 0 & \lambda^2 \\ 0 & \bar{\lambda} & 0 \\ \lambda^2 & 0 & 1 \end{pmatrix}. \tag{51}$$

## 4. Baryon and lepton number violating operators

In addition to the standard Yukawa couplings which provide with masses quarks and leptons, the gauge symmetry of the MSSM allows also terms which violate baryon and

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[4] Similar relations are obtained for $n < 0$ by interchanging $\varepsilon \leftrightarrow \bar{\varepsilon}$.

lepton number already at the tree-level. Suppressing generation indices, the terms relevant to proton decay are written

$$\lambda LLE^c + \lambda' LQD^c + \lambda'' U^c D^c D^c. \tag{52}$$

There are also gauge invariant Higgs, and lepton-Higgs mixing terms of the form

$$\mu H_1 H_2 + \mu' L H_2. \tag{53}$$

If all terms (52) are allowed in the superpotential they lead to fast proton decay. In particular, the combination of the terms $LQD^c$ and $U^c D^c D^c$ generates an effective dimension-four operator via the diagram generated by exchanging the scalar component of the $D^c$ superfield. Imposing the $R$-parity [22,23] multiplicative symmetry $R = (-1)^{3B+2S+L}$ under which matter fields (quarks and leptons) change sign while the Higgs doublets transform to themselves, all dangerous terms change sign and are eliminated from the superpotential.

$R$-parity prevents also the appearance of the second Higgs mixing term in (53). However, the usual $\mu$-term, i.e. the direct mixing between the two electroweak Higgs fields is invariant under the $R$-symmetry. In the model under consideration this may lead to a disaster, as this mixing can be generated by a term of the form $\phi^{\bar{s}} \bar{\phi}^r H_1 H_2$ where $r,s$ are suitable powers matching the sum of the charge of the two Higgs doublets. With vevs $\langle \phi \rangle, \langle \bar{\phi} \rangle \sim 10^{-1} M_U$ – as required by the $D$-term cancellation condition and the fermion mass textures – a large power (at least $r + s > 15$) is needed to sufficiently suppress the $\mu$-mass parameter and bring it down to the electroweak scale.

In addition to the tree-level couplings there are also higher order gauge invariant terms leading to dangerous dimension-five operators which induce proton decay. The ones surviving $R$-parity are [24]

$$\frac{\lambda_4^{ijkl}}{M_U} Q_i Q_j Q_k L_l, \qquad \frac{\lambda_5^{ijkl}}{M_U} U_i^c U_j^c D_k^c E_l^c, \tag{54}$$

where the indices $i,j,k,l = 1,2,3$ refer to the three generations. Although the induced amplitudes of dimension-five operators are relatively suppressed compared to those arising from the terms (52), due to the fact that they arise as non-renormalizable interactions, the baryon decay bounds on their Yukawa coupling constants are very restrictive. In the general case one has to impose $\lambda_4 < 10^{-7}$ for operators involving light quarks while the constraints are less important for $\lambda_5$ [24]. If an expansion parameter $\epsilon \sim 0.23$ is involved in the coupling, we should require a power $\epsilon^9$ for a coupling involving only first and second generation fermions to comply with the experimental bound. Couplings involving third generation fields suffer additional suppression from mixing angles and the bounds are less restrictive. Therefore it is crucial to examine whether the charge assignment of the fermion fields under the anomalous $U(1)_X$ symmetry is also capable of eliminating these baryon and lepton number violating operators.

We have classified all possible non-zero couplings involving the various generations together with their $U(1)_X$ charges and exhibit them in Table 10. The total $U(1)_X$ charge of each operator is now expressed only in terms of the free parameters $m,n,k$ and the sum of the Higgs charges $h_+ = h_1 + h_2$.

In the first column of this table we write the particular operator in terms of its family indices while in the second column we present its charge. Since everything here is

parametrized in terms of the charge of the singlet, we should simply check whether the charge of a particular operator is integer or non-integer. We now distinguish two cases:

- A: $h_1 + h_2 \equiv h_+ \equiv 0$

  To analyse the effects of the anomalous abelian symmetry on these operators, let us start with the case $h_+ = h_1 + h_2 = 0$. As seen in Table 10, the charges of these operators depend only on the integer parameters $m, n, k$ and they involve $1/2$ fractions of these parameters. Therefore, we consider which of these operators survive for various choices of $m, n, k$. We assume that the value of the parameter $k$ is odd. This choice of $k$ fits perfectly with the findings in the lepton mass matrices. (Indeed, in Section 3, most of the acceptable lepton mass textures where constructed choosing odd values for $k$.) Clearly, the most favorable case is when both $m$ and $n$ are even as it eliminates most of the operators involving the light generations. The rest of the operators are needed to be suppressed with appropriate selection of $m$ and $n$ and $k$. As it will become clear in the next section one can easily find charge assignments (see e.g. solution A of Table 11) that give acceptable fermion mass textures and adequately suppress all these operators.

- B: $h_1 + h_2 \equiv h_+ \neq 0$

  Now let us come to the most general case. It is interesting that the dimension-five proton decay operators can also be expressed in terms of integer parameters, namely $m, n, k, h_+$ and they do not involve $h_2$. Actually the dimension-five operators of Table 10 are receiving additional charge, the first seven (of the form $QQQL$) obtain an $-\frac{5}{3}h_+$ additional charge while the remaining receive a contribution of $-\frac{7}{6}h_+$. The charges of the operators of the type $U^c U^c D^c E^c$ are obtained by adding $\frac{h_+}{3}$ to the charge of the $QQQL$ operator in the same line.

  We can choose the Higgs charges so that the contributions $\frac{5}{3}h_+$ and $\frac{7}{6}h_+$ are neither integers nor half-integers. Then, all operators are eliminated simultaneously.

Another non-renormalizable operator allowed by $R$-parity is the following [26]:

$$\frac{\lambda_8}{M_U}\left(L_i \bar{H}\right)\left(L_j \bar{H}\right), \tag{55}$$

where $i, j$ refer to generations. This operator, which violates lepton number by two units, may have interesting phenomenological consequences as it is capable of generating a Majorana mass for the left-handed neutrino. A coupling $\lambda_8 \approx 1\text{–}10^{-2}$ would be of the right order for such a mass term. In Table 6 we present all relative operators and their

Table 6
$U(1)_X$ charges of the operators $L_i L_j H_2 H_2$ for the case $h_+ = 0$. Indices refer to generations

| Operator | $U(1)_X$ charge |
|---|---|
| $L_1 H_2 L_1 H_2$ | $n - k - 10 q_3$ |
| $L_1 H_2 L_2 H_2$ | $\frac{m+n}{2} - 10 q_3$ |
| $L_1 H_2 L_3 H_2$ | $\frac{n-k}{2} - 10 q_3$ |
| $L_2 H_2 L_2 H_2$ | $m + k - 10 q_3$ |
| $L_1 H_2 L_3 H_2$ | $\frac{k+m}{2} - 10 q_3$ |
| $L_3 H_2 L_3 H_2$ | $- 10 q_3$ |

charges for the case $h_+ = 0$. When $h_+ \neq 0$ the relative charges can be calculated using Table 4. The role of this term in specific examples will be presented in the next section.

A more difficult problem however is related to the $\mu$-term. As is well known, there must be a Higgs mixing via a term of the form $\mu h \bar{h}$ with $\mu \sim m_W$ in order to prevent the appearance of an unwanted axion. In the simple scenario of one $U(1)$ symmetry and the two singlet fields we discuss here, this is not easy. In general, if the charge $h_+ = h_1 + h_2$ is an integer, then the singlet fields $\phi, \bar{\phi}$ may couple to the combination $h\bar{h}$, giving rise to a $\mu$ 'mass' parameter of the order $(\bar{\phi}/M_U)^{h_+ - 1}\phi$!! Since the vev of $\langle \phi \rangle \sim 10^{-1} M_U$ one has to impose the condition $h_+ \geqslant 15$, otherwise the Higgs doublets receive unacceptably large masses [25].

There are also other possible ways of avoiding such a large mass term for the Higgs doublets. For example, one may introduce a Peccei–Quinn symmetry [27] to ban [28] simultaneously the Higgs mixing as well as the proton decay operators discussed above. In our case, since the Higgs charges are basically unconstrained, it is possible to work out cases where their sum is not an integer. Therefore, the Higgs term does not appear. We note however, that solutions which eliminate completely the $\mu$-term are not favourable; if a term is completely forbidden for symmetry reasons in the superpotential, it is not obvious how it can appear in the Kähler potential. We think that the suppression of the Higgs mixing coupling by an appropriate choice of the Higgs charges is a rather natural solution. In Table 11 we give cases with field charges which lead to a large $\mu$-term suppression and a viable set of Yukawa mass matrices.

We note that, even if we ignore the above problem of the Higgs mixing – assuming the existence of another type of solution – and impose a half-integer value of $h_+$, we

Table 7
$U(1)_X$ charges of the $R$-parity violating couplings $L_i Q_j D_k^c$ for the case $h_+ = 0$. The indices refer to the generations

| Operator | $U(1)_X$ charge |
|---|---|
| $L_1 Q_1 D_1^c$ | $\frac{3n-k}{2} - 5q_3$ |
| $L_1 Q_1 D_2^c,\ L_1 Q_2 D_1^c$ | $n + \frac{m-k}{2} - 5q_3$ |
| $L_1 Q_1 D_3^c,\ L_1 Q_3 D_1^c$ | $n - \frac{k}{2} - 5q_3$ |
| $L_1 Q_2 D_2^c$ | $m + \frac{n-k}{2} - 5q_3$ |
| $L_1 Q_2 D_3^c,\ L_1 Q_3 D_2^c$ | $\frac{m+n-k}{2} - 5q_3$ |
| $L_1 Q_3 D_3^c$ | $\frac{n-k}{2} - 5q_3$ |
| $L_2 Q_1 D_1^c$ | $n + \frac{k+m}{2} - 5q_3$ |
| $L_2 Q_1 D_2^c, L_2 Q_2 D_1^c$ | $m + \frac{k+n}{2} - 5q_3$ |
| $L_2 Q_1 D_3^c,\ L_2 Q_3 D_1^c$ | $\frac{k+m+n}{2} - 5q_3$ |
| $L_2 Q_2 D_2^c$ | $\frac{k+3m}{2} - 5q_3$ |
| $L_2 Q_2 D_3^c,\ L_2 Q_3 D_2^c$ | $m + \frac{k}{2} - 5q_3$ |
| $L_2 Q_3 D_3^c$ | $\frac{k+m}{2} - 5q_3$ |
| $L_3 Q_1 D_1^c$ | $n - 5q_3$ |
| $L_3 Q_1 D_2^c,\ L_3 Q_2 D_1^c$ | $\frac{m+n}{2} - 5q_3$ |
| $L_3 Q_1 D_3^c,\ L_3 Q_3 D_1^c$ | $\frac{n}{2} - 5q_3$ |
| $L_3 Q_2 D_2^c$ | $m - 5q_3$ |
| $L_3 Q_2 D_3^c,\ L_3 Q_3 D_2^c$ | $\frac{m}{2} - 5q_3$ |
| $L_3 Q_3 D_3^c$ | $-5q_3$ |

Table 8
$U(1)_X$ charges of the $R$-parity violating couplings $L_i L_j E_k^c$ for the case $h_+ = 0$. The indices refer to the generations

| Operator | $U(1)_X$ charge |
|----------|-----------------|
| $L_1 L_2 E_1^c$ | $n + \frac{m-k}{2} - 5\,q_3$ |
| $L_1 L_2 E_2^c$ | $m + \frac{n+k}{2} - 5\,q_3$ |
| $L_1 L_2 E_3^c$ | $\frac{m+n}{2} - 5\,q_3$ |
| $L_1 L_3 E_1^c$ | $n - k - 5\,q_3$ |
| $L_1 L_3 E_2^c$ | $\frac{m+n}{2} - 5\,q_3$ |
| $L_1 L_3 E_3^c$ | $\frac{n-k}{2} - 5\,q_3$ |
| $L_2 L_3 E_1^c$ | $\frac{m+n}{2} - 5\,q_3$ |
| $L_2 L_3 E_2^c$ | $k + m - 5\,q_3$ |
| $L_2 L_3 E_3^c$ | $\frac{k+m}{2} - 5\,q_3$ |

encounter another difficulty; we know from the analysis of the quark mass matrices that $m, n$ are integers while from the lower $2 \times 2$ charged-lepton mass matrix, we find that $k$ also has to be integer. Then, we infer that the non-integer values of $h_+$ lead unavoidably to a massless electron state. In a more complicated theory we may hope that radiative effects or other weakly coupled singlets could generate a small entry adequate to provide the electron with a mass.

We would like now to abandon the $R$-parity symmetry and investigate the possibility of constructing a set of charges which give viable fermion mass textures with baryon and lepton violation within the existing limits. In Tables 7, 8, 9 we present all dangerous trilinear operators capable of inducing proton decay. In the second column we exhibit their total charge under the $U(1)_X$ anomalous symmetry. We have expressed the total charge in terms of the parameters $m, n, k$ (which parameterize all quark and charged lepton mass matrices) and the charge of the third generation quark doublet $q_3$. Thus, in order to generate a gauge invariant baryon violating term we should be able to add a singlet $\phi$ or antisinglet $\overline{\phi}$ to the proper power $r$, $\phi^r$ ($\overline{\phi}^r$) to cancel the charge. For example, if $q_i + q_j + \ell_k = \pm r$, then the operator $Q_i Q_j L_k \overline{\phi}^r$ ($\phi^r$) cannot be avoided.

In this case, the Yukawa couplings of the terms (52) should be highly suppressed, in particular those involving first generation quark and lepton states. According to our natural assumption that the non-calculable coefficients should be of order one, we infer

Table 9
$U(1)_X$ charges of the $R$-parity violating couplings $U_i^c D_j^c D_k^c$ for the case $h_+ = 0$. The indices refer to the generations

| Operator | $U(1)_X$ charge |
|----------|-----------------|
| $U_1^c D_1^c D_2^c$ | $n + \frac{m}{2} - 5\,q_3$ |
| $U_1^c D_1^c D_3^c$ | $n - 5\,q_3$ |
| $U_1^c D_2^c D_3^c$ | $\frac{m+n}{2} - 5\,q_3$ |
| $U_2^c D_1^c D_2^c$ | $m + \frac{n}{2} - 5\,q_3$ |
| $U_2^c D_1^c D_3^c$ | $\frac{m+n}{2} - 5\,q_3$ |
| $U_2^c D_2^c D_3^c$ | $m - 5\,q_3$ |
| $U_3^c D_1^c D_2^c$ | $\frac{m+n}{2} - 5\,q_3$ |
| $U_3^c D_1^c D_3^c$ | $\frac{n}{2} - 5\,q_3$ |
| $U_3^c D_2^c D_3^c$ | $\frac{m}{2} - 5\,q_3$ |

Table 10
Dimension-five operators leading to proton decay are presented in the first and second column. The associated charges for the case $h_+ = 0$ are presented in the third column. The symbol $\surd$ marks the surviving operators for the allowed values of $m$ and $n$ assuming $k$ is odd. When $h_+ \neq 0$, all $QQQL$ operators receive an additional charge ($\frac{5}{3}h_+$ or $\frac{7}{6}h_+$), thus for appropriate $h_+$ values are forbidden. The $D^cU^cU^cE^c$ receive similar contributions. (For details see Section 6)

| Operator | | $U(1)_X$ charge | $m,n$ even | $m$ odd, $n$ even | $n,m$ odd |
|---|---|---|---|---|---|
| $Q_1Q_1Q_2L_1$ | $D_1^cU_1^cU_2^cE_1^c$ | $\frac{3n+m-k}{2}$ | $\surd$ | | |
| $Q_1Q_1Q_3L_1$ | $D_1^cU_1^cU_3^cE_1^c$ | $\frac{3n-k}{2}$ | | | $\surd$ |
| | $D_1^cU_2^cU_3^cE_1^c$ | | | | |
| $Q_1Q_2Q_3L_1$ | $D_2^cU_1^cU_3^cE_1^c$ | $n+\frac{m-k}{2}$ | $\surd$ | | $\surd$ |
| | $D_3^cU_1^cU_2^cE_1^c$ | | | | |
| $Q_2Q_1Q_2L_1$ | $D_2^cU_1^cU_2^cE_1^c$ | $m+n-\frac{k}{2}$ | | | |
| $Q_2Q_2Q_3L_1$ | $D_2^cU_2^cU_3^cE_1^c$ | $m+\frac{n-k}{2}$ | | | $\surd$ |
| $Q_3Q_1Q_3L_1$ | $D_3^cU_1^cU_3^cE_1^c$ | $n-\frac{k}{2}$ | | | |
| $Q_3Q_2Q_3L_1$ | $D_3^cU_2^cU_3^cE_1^c$ | $\frac{m+n-k}{2}$ | $\surd$ | | |
| $Q_1Q_1Q_2L_2$ | $D_1^cU_1^cU_2^cE_2^c$ | $n+m+\frac{k}{2}$ | | | |
| $Q_1Q_1Q_3L_2$ | $D_1^cU_1^cU_3^cE_2^c$ | $n+\frac{k+m}{2}$ | $\surd$ | | $\surd$ |
| | $D_1^cU_2^cU_3^cE_2^c$ | | | | |
| $Q_1Q_2Q_3L_2$ | $D_2^cU_1^cU_3^cE_2^c$ | $m+\frac{k+n}{2}$ | | | $\surd$ |
| | $D_3^cU_1^cU_2^cE_2^c$ | | | | |
| $Q_2Q_1Q_2L_2$ | $D_2^cU_1^cU_2^cE_2^c$ | $\frac{n+3m+k}{2}$ | $\surd$ | | |
| $Q_2Q_2Q_3L_2$ | $D_2^cU_2^cU_3^cE_2^c$ | $\frac{3m+k}{2}$ | $\surd$ | $\surd$ | |
| $Q_3Q_1Q_3L_2$ | $D_3^cU_1^cU_3^cE_2^c$ | $\frac{m+k+n}{2}$ | $\surd$ | | |
| $Q_3Q_2Q_3L_2$ | $D_3^cU_2^cU_3^cE_2^c$ | $m+\frac{k}{2}$ | | | |
| $Q_1Q_1Q_2L_3$ | $D_1^cU_1^cU_2^cE_3^c$ | $n+\frac{m}{2}$ | $\surd$ | | |
| $Q_1Q_1Q_3L_3$ | $D_1^cU_1^cU_3^cE_3^c$ | $n$ | $\surd$ | $\surd$ | $\surd$ |
| | $D_1^cU_2^cU_3^cE_3^c$ | | | | |
| $Q_1Q_2Q_3L_3$ | $D_2^cU_1^cU_3^cE_3^c$ | $\frac{m+n}{2}$ | $\surd$ | | $\surd$ |
| | $D_3^cU_1^cU_2^cE_3^c$ | | | | |
| $Q_2Q_1Q_2L_3$ | $D_2^cU_1^cU_2^cE_3^c$ | $m+\frac{n}{2}$ | $\surd$ | $\surd$ | |
| $Q_2Q_2Q_3L_3$ | $D_2^cU_2^cU_3^cE_3^c$ | $m$ | $\surd$ | $\surd$ | $\surd$ |
| $Q_3Q_1Q_3L_3$ | $D_3^cU_1^cU_3^cE_3^c$ | $\frac{n}{2}$ | $\surd$ | $\surd$ | |
| $Q_3Q_2Q_3L_3$ | $D_3^cU_2^cU_3^cE_3^c$ | $\frac{m}{2}$ | $\surd$ | | |

that the $U(1)_X$ symmetry should prevent the appearance of such terms at the renormalizable superpotential. These operators should appear at high orders so that their couplings are suppressed by proper powers of the expansion parameter. In order to put appropriate constraints on the $U(1)_X$-charges, we first need the experimental bounds on the relevant Yukawa couplings. The most severe bounds are imposed on the Yukawa couplings $\lambda'_{111}$ and $\lambda'_{133}$ of this operator. In particular, from the absence of the exotic reaction of $\beta\beta$-decay we have $\lambda'_{111} < 10^{-3}$ and from the bounds on the left-handed neutrino Majorana mass, $\lambda'_{133} < 2 \times 10^{-3}$. Other exotic decays imply bounds to various combinations of couplings, while more restrictive bounds arise for products of the form $\lambda\lambda'$; a recent analysis on the various Yukawa couplings predicted in $U(1)$ models and a relevant discussion of the above operators can be found in [29][5].

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[5] See also Ref. [30].

Note also that, when *R*-parity is absent, additional dimension-five operators involving Higgs multiplets are also dangerous when they are combined with the couplings (52) leading to proton decay via loop-graphs (For a complete list of these operators see Ref. [24]). In particular, operators of the form $[QQQH_1]_F$ are dangerous in the presence of $LQD^c$ couplings while the operators $[QU^cE^cH_1]_F$ are also dangerous in the presence of $U^cD^cD^c$ terms. The former, leads to a tree-level proton decay diagram via the Higgs vev $H_1$ and its coupling to the down quark $QD^cH_1$, and similarly the second leads to an effective $U^cU^cD^cE^c$ operator. Finally, one should avoid the simultaneous existence of the term $U^cD^cD^c$ with the lepton number violating operators $[QU^cL^*]_D$ and $[QU^cL^*]_D$. It is now straightforward to turn the above bounds to constraints on the $U(1)_X$ charges. Since in our subsequent analysis we will present cases where all the tree-level operators are either suppressed, or banned by the symmetry, we will not pursue this issue further.

## 5. A few typical solutions

We now pass to an investigation of possible solutions which are in accordance with the phenomenological requirements discussed in the previous section. There are numerous case of $U(1)_X$ charge assignments which give textures consistent with the hierarchical fermion mass pattern. Here, we present only few characteristic examples which mainly fall into two categories: Those, which allow baryon and lepton number violating operators and need additional underlying symmetries to evade them and, those which strictly forbid any lepton and baryon violating operator.

- *Solution A*. It is a remarkable fact that one of the most promising texture-zero mass matrices found in Section 3 arises from a simple generation independent charge assignment. The first generation fermions are assigned with charge 4, the second with 2 and third with 0 (see Table 11). This yields (30) for the up quarks

$$m_U = \begin{pmatrix} \bar{\varepsilon}^8 & \bar{\varepsilon}^6 & \bar{\varepsilon}^4 \\ \bar{\varepsilon}^6 & \bar{\varepsilon}^4 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^4 & \bar{\varepsilon}^2 & 1 \end{pmatrix}, \tag{56}$$

and similarly for down quarks and leptons

$$m_L \sim m_D = \begin{pmatrix} \bar{\lambda}^8 & \bar{\lambda}^6 & \bar{\lambda}^4 \\ \bar{\lambda}^6 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \bar{\lambda}^4 & \bar{\lambda}^2 & 1 \end{pmatrix}. \tag{57}$$

The above charge assignment, although it allows dimension-five operators, sufficiently suppresses the dangerous ones. The suppression factors are

$$\lambda_4^{3233} \sim \bar{\lambda}^2,$$
$$\lambda_4^{3133}, \lambda_4^{3223}, \lambda_4^{3232} \sim \bar{\lambda}^4,$$
$$\lambda_4^{3231}, \lambda_4^{3222}, \lambda_4^{3132}, \lambda_4^{1233} \sim \bar{\lambda}^6,$$
$$\lambda_4^{2231}, \lambda_4^{3131}, \lambda_4^{1232}, \lambda_4^{1133}, \lambda_4^{2123} \sim \bar{\lambda}^8,$$
$$\lambda_4^{1231}, \lambda_4^{1132}, \lambda_4^{2122}, \lambda_4^{1123} \sim \bar{\lambda}^{10},$$
$$\lambda_4^{1131}, \lambda_4^{2121}, \lambda_4^{1122} \sim \bar{\lambda}^{12},$$
$$\lambda_4^{1121} \sim \bar{\lambda}^{14}, \tag{58}$$

Table 11

Some typical $U(1)_X$ charge assignments consistent with anomaly cancellation and acceptable fermion mass matrices

| Solution A | | | | Solution B | | | | Solution C | | | | Solution D | | | | Solution E | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Field | Generation | | | Field | Generation | | | Field | Generation | | | Field | Generation | | | Field | Generation | | |
| | 1 | 2 | 3 | | 1 | 2 | 3 | | 1 | 2 | 3 | | 1 | 2 | 3 | | 1 | 2 | 3 |
| $Q$ | 4 | 2 | 0 | $Q$ | 6 | 4 | 2 | $Q$ | $\frac{9}{2}$ | $\frac{5}{2}$ | $\frac{1}{2}$ | $Q$ | $\frac{7}{3}$ | $\frac{5}{6}$ | $-\frac{2}{3}$ | $Q$ | $-\frac{1}{2}$ | $-\frac{5}{2}$ | $-\frac{9}{2}$ |
| $D^c$ | 4 | 2 | 0 | $D^c$ | $-2$ | $-4$ | $-6$ | $D^c$ | $\frac{5}{2}$ | $\frac{1}{2}$ | $-\frac{3}{2}$ | $D^c$ | 5 | $\frac{7}{2}$ | 2 | $D^c$ | $-\frac{17}{6}$ | $-\frac{29}{6}$ | $-\frac{41}{6}$ |
| $U^c$ | 4 | 2 | 0 | $U^c$ | 6 | 4 | 2 | $U^c$ | $\frac{9}{2}$ | $\frac{5}{2}$ | $\frac{1}{2}$ | $U^c$ | $\frac{7}{3}$ | $\frac{5}{6}$ | $-\frac{2}{3}$ | $U^c$ | $\frac{23}{6}$ | $\frac{11}{6}$ | $-\frac{1}{6}$ |
| $L$ | 4 | 2 | 0 | $L$ | $-2$ | $-4$ | $-6$ | $L$ | $\frac{5}{2}$ | $\frac{1}{2}$ | $-\frac{3}{2}$ | $L$ | 5 | $\frac{7}{2}$ | 2 | $L$ | $-\frac{55}{6}$ | $-\frac{19}{6}$ | $-\frac{31}{6}$ |
| $E^c$ | 4 | 2 | 0 | $E^c$ | 6 | 4 | 2 | $E^c$ | $\frac{9}{2}$ | $\frac{5}{2}$ | $\frac{1}{2}$ | $E^c$ | $\frac{7}{3}$ | $\frac{5}{6}$ | $-\frac{2}{3}$ | $E^c$ | $-\frac{61}{6}$ | $-\frac{25}{6}$ | $-\frac{37}{6}$ |
| Higgs | | | | Higgs | | | | Higgs | | | | Higgs | | | | Higgs | | | |
| $H_1$ | 0 | $H_2$ | 0 | $H_1$ | 4 | $H_2$ | $-4$ | $H_1$ | 1 | $H_2$ | $-1$ | $H_1$ | $-\frac{4}{3}$ | $H_2$ | $\frac{4}{3}$ | $H_1$ | $\frac{34}{3}$ | $H_2$ | $\frac{14}{3}$ |
| Singlets | | | | Singlets | | | | Singlets | | | | Singlets | | | | Singlets | | | |
| $\phi$ | 1 | $\bar{\phi}$ | $-1$ | $\phi$ | 1 | $\bar{\phi}$ | $-1$ | $\phi$ | 1 | $\bar{\phi}$ | $-1$ | $\phi$ | 1 | $\bar{\phi}$ | $-1$ | $\phi$ | 1 | $\bar{\phi}$ | $-1$ |

and similarly for $\lambda_5^{ijkl}$, where the couplings refer to Eq. (54).

This solution has also the advantage of *not* suppressing the quartic couplings $L_i L_j H_2 H_2$. Actually they have the form

$$L_i L_j H_2 H_2 \sim \frac{m_W^2}{M_U} \begin{pmatrix} \bar{\lambda}^8 & \bar{\lambda}^6 & \bar{\lambda}^4 \\ \bar{\lambda}^6 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \bar{\lambda}^4 & \bar{\lambda}^2 & 1 \end{pmatrix}. \tag{59}$$

Therefore, this simple charge assignment predicts also a hierarchical texture for the left-handed neutrino Majorana mass. The mass scale is determined by the suppression mass factor $m_W^2/M_U$ so there is a sufficient suppression without the use right-handed neutrino fields and the see-saw mechanism. As in all other matrices, only the third-generation diagonal coupling $L_3 L_3 H_2^2$ appears at the tree level.

This solution does not suppress *R*-parity violating couplings so one has to assume that *R*-parity is a good symmetry. It does not also suppress the $\mu$-term so one has to assume the existence of another mechanism that deals with this problem.

· *Solution B.* Here we present another example which results in the same mass matrices as in solution A, and with similar suppression of dimension-five operators. The difference here is that $L_i L_j H_2 H_2$ operators are also suppressed (the stronger coupling is of the order $\bar{\lambda}^{12}$). Similar comments with solution A hold for the *R*-parity violating couplings and the $\mu$ term.

In the above two cases, we have used integer $U(1)_X$ charges for fermions and Higgs fields. Going further, we present few more examples where now we introduce fractional $U(1)_X$ charge assignments.

· *Solution C.* This solution gives mass matrices similar to solution A. The dimension-five operators are suppressed according to (58). The difference here is that all

$R$-parity violating couplings in (52), (53) vanish explicitly. However, the Majorana neutrino mass operator survives and takes the form

$$L_i L_j H_2 H_2 \sim \frac{m_W^2}{M_U} \begin{pmatrix} \bar{\lambda}^3 & \bar{\lambda}^1 & \lambda^1 \\ \bar{\lambda}^1 & \lambda^1 & \lambda^3 \\ \lambda^1 & \lambda^3 & \lambda^5 \end{pmatrix}. \tag{60}$$

- *Solution D.* The charges in this case appear also in Table 11. They yield up-quark fermion mass textures of the form

$$m_U = \begin{pmatrix} \bar{\varepsilon}^6 & 0 & \bar{\varepsilon}^3 \\ 0 & \bar{\varepsilon}^3 & 0 \\ \bar{\varepsilon}^3 & 0 & 1 \end{pmatrix} \tag{61}$$

and similarly for down quarks and leptons

$$m_D = m_L = \begin{pmatrix} \bar{\lambda}^6 & 0 & \bar{\lambda}^3 \\ 0 & \bar{\lambda}^3 & 0 \\ \bar{\lambda}^3 & 0 & 1 \end{pmatrix}. \tag{62}$$

This solution completely eliminates all dimension-five proton decay as well as all $R$-parity violating couplings (52). $L_i L_j H_2 H_2$ operators are also suppressed. The only additional mechanism one needs is for the suppression of the $\mu$ term as the solution belongs to the category $h_+ = 0$.

- *Solution E.* The quark and lepton matrices are

$$m_U = \begin{pmatrix} \bar{\varepsilon}^8 & \bar{\varepsilon}^6 & \bar{\varepsilon}^4 \\ \bar{\varepsilon}^6 & \bar{\varepsilon}^4 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^4 & \bar{\varepsilon}^2 & 1 \end{pmatrix}, \tag{63}$$

and similarly for $m_D$ with $\bar{\varepsilon} \to \bar{\lambda}$, while the charged leptons are given by

$$m_L = \begin{pmatrix} \lambda^8 & \lambda^2 & \lambda^4 \\ \lambda^2 & \bar{\lambda}^4 & \bar{\lambda}^2 \\ \lambda^4 & \bar{\lambda}^2 & 1 \end{pmatrix}. \tag{64}$$

This solution forbids all dimension-five operators as well as baryon and lepton number violating couplings (52). One concludes that all additional dimension-five operators are also suppressed whatever the charges of these operators. At the same time it suppresses the Higgs mixing as the $\mu$-term appears now through the non-renormalizable term

$$W_{NR} \to \left( \frac{\bar{\phi}}{M_U} \right)^{16} M_U H_1 H_2, \tag{65}$$

therefore the Higgs doublets are protected from receiving an unacceptable large mass. Surprisingly the operator $L_i L_j H_2 H_2$ is not suppressed; it gives a left-handed

Majorana neutrino texture,

$$L_i L_j H_2 H_2 \sim \frac{m_W^2}{M_U} \begin{pmatrix} \lambda^9 & \lambda^3 & \lambda^5 \\ \lambda^3 & \bar{\lambda}^3 & \bar{\lambda}^1 \\ \lambda^5 & \bar{\lambda}^1 & \lambda^1 \end{pmatrix}, \tag{66}$$

which exhibits the phenomenologically interesting feature of a rather large mixing in the $\nu_\mu - \nu_\tau$ sector. The price one has to pay for all these welcome features is the rather exotic charges. This is a feature also pointed out in Refs. [3,7,8].

## 6. Conclusions

In this work we have attempted to generate the hierarchical standard model fermion mass spectrum by means of an anomalous abelian family symmetry $U(1)_X$ and in the context of the minimal unification scenario. We have extended previous analyses by considering the $U(1)_X$ to be family dependent a possibility that naturally arises in superstring model building. A minimum number of fields – one singlet and its conjugate – were used to break the anomalous $U(1)_X$ symmetry at a high scale. We have assumed that the $U(1)_X$ anomaly is cancelled by the string Green–Schwarz anomaly cancellation mechanism. We have imposed conditions on the $U(1)_X$-matter and Higgs charges by requiring symmetric mass matrices and tree-level couplings for the third generation. We have demanded the mixed $SU(3)^2 U(1)_X$, $SU(2)^2 U(1)_X$ and $U(1)_Y^2 U(1)_X$ anomalies to be proportional to the Kac–Moody constants $k_3 = k_2 = 3k_1/5 = 1$ as well as cancellation of the $U(1)_Y^2 U(1)_X$ mixed and $U(1)_X^3$ anomalies. The general solution of the resulting equations has been determined and all possible textures of the fermion mass matrices were classified in terms of the admissible values of the sum of the two $U(1)_X$-Higgs charges. The cases of zero and integer values of the Higgs sum charge where considered while non-integer values, although possible, were not discussed since they lead to a massless charged-lepton eigenstate and prevent the appearance of a $\mu$-term to all orders. Using the freedom left by the anomaly conditions on the $U(1)_X$ charges, four distinct phenomenologically acceptable texture-zero solutions for the fermion mass hierarchy problem have been predicted. The mass hierarchy is determined from powers of parameters defined as the dimensionless ratio of the singlet vevs over some high (string) scale. The magnitude of the expansion parameters is constrained due to the *D*-term cancellation mechanism which determines the singlet vevs in terms of the unification scale and the common (string) coupling. We note that up- and down-quark mass matrices are predicted to have the same form due to the initial assumptions that the matrices are symmetric and the requirement that both top and bottom Yukawa couplings appear at the tree-level. However, the predicted quark masses can be reconciled with the low energy measured values due to the possible appearance of different expansion parameters in the matrices and renormalization running effects. The success of the above scenario might look more impressive if some of the simplifying assumptions were relaxed. Nevertheless, we find it remarkable that even in this simple extension of the minimal supersymmetric standard model one may predict to a good approximation the

big mass gaps observed in the particle spectrum. It is tempting to extend the analysis by relaxing some of the unnecessary assumptions and re-examine the above model.

A rather remarkable fact is that this simple $U(1)_X$ anomalous symmetry with the constraints implied by the anomaly cancellation conditions allows fermion charge assignments which can suppress, or in certain cases eliminate, all dangerous baryon and lepton number violating operators. In the case that $R$-parity is a good symmetry we have found solutions that can suppress the dangerous dimension-five proton decay operators (allowed by $R$-parity). We have also found solutions that do not need the introduction of $R$-parity since there, all $R$-parity violating couplings are naturally suppressed.

We have further shown that these solutions may also suppress sufficiently the Higgs doublets mixing parameter ($\mu$-term) and keep them massless down to the electroweak scale. This latter possibility requires the introduction of a rather large charge for the sum of the Higgs doublets which demand rather peculiar $U(1)_X$ assignments for MSSM fields at least in the case that the singlet charge is unity. In the context of these solutions we have also succeeded to find cases which provide the left-handed neutrino with acceptable Majorana masses.

It is remarkable that most or all of the good features mentioned above can occur simultaneously in a few simple solutions which we presented in this work.

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