Nonabelian Aharonov–Bohm baryon decay catalysis

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We propose a new mechanism for baryon decay based on the Aharonov–Bohm effect for nonabelian flux lines (e.g. cosmic strings). We discuss in particular the case of cosmic strings produced in a grand unified phase transition in which the gauge group SO(10) breaks down to $SU(5) \times \mathbb{Z}_2$. We comment on the implications for baryogenesis.

1. Introduction

Cosmic strings [1] are one-dimensional topological defects produced during phase transitions in the early Universe. Such phase transitions, predicted by some grand unified models, occur at a temperature of the order $M = 10^{16}$ GeV and produce strings of the right energy density to seed galaxy formation [2,3].

Gauge fields which mediate baryon number violating processes are excited in the core of the string (radius $\sim M^{-1}$) and can catalyze baryon decay [4]. This effect has important cosmological consequences. In particular, a fraction of the primordial baryon to entropy ratio can be erased by cosmic strings [5].

However, interaction of baryons with the string core is not the only way to obtain baryon number violation. According to the well known Aharonov–Bohm effect [6], particles can scatter nontrivially off flux lines even though classically there should be no interaction. This is caused by quantum mechanical phase mixing of the components of the wave function propagating on either side of the flux line.

The situation is even more interesting if the flux carried by the string is nonabelian. In this case, the possibility of mixing among components of a particle multiplet arises. In particular, a beam of protons scattered by a SU(2) isospin flux line can be converted to neutrons [7]. In a similar way, as we show here, a beam of baryons scattered by a GUT string can be converted into leptons if the flux has the appropriate orientation in group space.

In section 2, we review the abelian Aharonov–Bohm (AB) effect and derive some formulas which we use later. In section 3, we consider the nonabelian AB effect. We study the scattering of fermions by a general nonabelian flux line and conclude that mixing of the components of a particle multiplet can occur. A specific example – GUT strings produced in the phase transition in which SO(10) breaks to $SU(5) \times \mathbb{Z}_2$ – is considered in section 4. The cross section is a typical AB cross section $d\sigma/d\Omega\, dl \sim O(1) k^{-1}$, where k is the momentum of the incoming baryon. Section 5 contains a concluding discussion.

2. The abelian Aharonov–Bohm effect

Consider fermions of charge e scattered by an abelian vortex of flux α (in units of $2\pi/e$) in the limit of vanishing thickness of the vortex. The gauge field has the form $A_0 = 0$, $\boldsymbol{A} = \hat{\boldsymbol{e}}_\varphi[(\alpha/e)/\rho]$, where ρ and φ are polar coordinates in the plane perpendicular to the vortex and $\boldsymbol{e}_\varphi$ is the unit vector in the φ direction. We will treat the gauge field as a classical background.

The Dirac equation is

$$i(\partial\!\!\!/ + ieA\!\!\!/)\psi - m\psi = 0. \qquad (2.1)$$

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Solutions with fixed energy ω may be expanded [6,8,9] in eigenstates of angular momentum $n+\frac{1}{2}$

$$\psi(x)=\sum_{n=-\infty}^{\infty} a_n \begin{pmatrix} J_{|\nu|}(x) \\ \lambda B J_{\lambda(\nu+1)}(x)\exp(\mathrm{i}\varphi) \end{pmatrix} \times\exp(\mathrm{i}n\varphi)\exp(-\mathrm{i}\omega t), \tag{2.2}$$

where $x=k\rho$, $k=(\omega^2-m^2)^{1/2}$, $\nu=n+\alpha$, $B=\mathrm{i}k/(\omega+m)$ and $\lambda=\mathrm{sgn}(\nu)$ [#1].

To find the coefficients a_n and the scattering amplitude $f(\varphi)$, we match (2.2) at infinity to an incoming plane wave plus an outgoing cylindrical wave,

$$\psi(x)=\sum_n \left[(-1)^n \begin{pmatrix} J_n(x) \\ BJ_{n+1}(x)\exp(\mathrm{i}\varphi) \end{pmatrix} + f_n \frac{\exp(\mathrm{i}x)}{\sqrt{x}}\right] \times\exp(\mathrm{i}n\varphi)\exp(-\mathrm{i}\omega t), \tag{2.3}$$

where we have used the standard representation of a plane wave in terms of Bessel functions

$$\exp(\mathrm{i}x\cos\varphi)=\sum_{n=-\infty}^{\infty}(-1)^n J_n(x)\exp(\mathrm{i}n\varphi). \tag{2.4}$$

The scattering amplitude $f(\varphi)$ is given by

$$f(\varphi)=k^{-1/2}\sum_{n=-\infty}^{\infty} f_n\exp(\mathrm{i}n\varphi). \tag{2.5}$$

Matching (2.3) and (2.4) gives

$$a_n=\exp(\mathrm{i}|\nu|\pi/2)$$
$$f_n=(2\pi)^{1/2}\exp(-\mathrm{i}\pi/4)[\exp(-\mathrm{i}|\nu|\pi)-(-1)^n]. \tag{2.6}$$

Therefore (from the large argument scaling of the Bessel functions), the effect of the vortex is to convert the plane wave components $(-1)^n J_n(x)$ to $\exp(-\mathrm{i}|\nu|\pi/2)J_{|\nu|}(x)$, which corresponds to a phase shift $2\delta_n$:

$$\exp(\mathrm{i}x)\to\exp(\mathrm{i}2\delta_n)\exp(\mathrm{i}x),\quad 2\delta_n=-\pi\alpha\,\mathrm{sgn}(\nu). \tag{2.7}$$

We conclude that the pure gauge field of the vortex causes a phase shift which has opposite signs for opposite angular momenta. This leads to phase mixing and a nontrivial scattering amplitude

$$f(\varphi)=\exp[-\mathrm{i}(\varphi-\pi)n_\alpha]\exp(-\mathrm{i}\varphi/2)\exp(-\tfrac{3}{4}\pi\mathrm{i}) \times\frac{\sin\alpha\pi}{\sqrt{2\pi k}\cos(\varphi/2)} \tag{2.8}$$

where n_α is the largest integer smaller than α. Note that this effect is purely quantum mechanical.

[#1] For a solution including a finite core see e.g. ref. [10].

3. The nonabelian Aharonov–Bohm effect

We now extend the previous analysis to nonabelian vortices with zero thickness. Consider a theory based on a symmetry group G with N' generators T_a which admits nonabelian vortex solutions. The nonabelian gauge potential is

$$A_0=0,\quad A_i=\hat{\boldsymbol{e}}_{\varphi,i}\frac{\alpha}{\rho}\sum_{a=1}^{N} T_a C_a\equiv\hat{\boldsymbol{e}}_{\varphi,i}\frac{\alpha}{\rho}M, \tag{3.1}$$

where C_a are constants and the sum runs over the generators of the part of G which is broken by the field configuration.

The Dirac equation is (2.1), where A_μ is given by (3.1) and the generators T_a are evaluated in the representation of G in which the fermions lie. The transformation of fermions and gauge fields under gauge transformations $U(\boldsymbol{\theta})=\exp[-\mathrm{i}e\boldsymbol{T}\cdot\boldsymbol{\theta}(x)]$ is the standard one.

In some cases of interest, M is a hermitean traceless $n\times n$ matrix, where n is the dimension of the representation of ψ. Let us denote its eigenvectors by $\boldsymbol{e}^i$, $i=1,\dots,n$. Then, M is diagonalized by the unitary transformation $U_{ij}=e^i_j$, where e^i_j is the jth component of $\boldsymbol{e}^i$. After diagonalization, the Dirac equation reduces to a set of n decoupled abelian Dirac equations with flux values given by $\alpha_i=\lambda_i\alpha$, where λ_i are the eigenvalues of M. The price to pay for this simplification is a mixing of the physical components of the spinor multiplet ψ. The basis in which the scattering problem is abelian is

$$\psi'_i=U_{ij}\psi_j, \tag{3.2}$$

where ψ_j are the physical eigenstates.

Applying the results of the abelian example, we may obtain the outgoing spinors $\psi_i'^{\mathrm{out}}$ in the presence of the vortex. Let $\psi'_{i,+}$ (and $\psi'_{i,-}$) denote the sum over modes with positive (and negative) ν. From (2.7) we obtain

$$\psi_i'^{\text{out}}(x) = \exp(-\mathrm{i}\pi\alpha_i)\psi'_{i,+}(x)\exp(\mathrm{i}\pi\alpha_i)\psi'_{i,-}(x), \tag{3.3}$$

and in terms of the physical states

$$\psi_i^{\text{out}}(x) = U_{ij}^{-1}\psi_j'^{\text{out}}(x). \tag{3.4}$$

We can reexpress the above using the definitions (dropping the index i for convenience)

$$\psi = \psi_+ + \psi_-, \quad \tilde{\psi} = \psi_+ - \psi_-. \tag{3.5}$$

Then, in matrix notation, (3.3) becomes

$$\psi^{\text{out}}(x) = D(\cos\pi\boldsymbol{\alpha})\psi(x) + D(-\mathrm{i}\sin\pi\boldsymbol{\alpha})\tilde{\psi}(x), \tag{3.6}$$

where $D(y)$ is the matrix

$$D_{ij}(y) = U_{ki}^{*}y_k U_{kj} = (e_k^i)^* y_k e_k^j. \tag{3.7}$$

Finally, it is easy to verify that for an incoming plane wave

$$\psi(x) = 2\left(\frac{2\pi}{x}\right)^{1/2}\exp(-\mathrm{i}\pi/4)$$
$$\times\,\delta(\varphi-\pi)\begin{pmatrix}1\\ -\mathrm{i}B\exp(\mathrm{i}\varphi)\end{pmatrix}\exp(\mathrm{i}x), \tag{3.8}$$

and

$$\tilde{\psi}(x) = \left(\frac{2}{\pi x}\right)^{1/2}\exp(\mathrm{i}\pi/4)\exp[-\mathrm{i}n_\alpha(\varphi-\pi)]$$
$$\times\frac{\exp(-\mathrm{i}\varphi/2)}{\cos(\varphi/2)}\begin{pmatrix}1\\ -\mathrm{i}B\exp(\mathrm{i}\varphi)\end{pmatrix}\exp(\mathrm{i}x), \tag{3.9}$$

where n_α is defined in section 2.

Since $D(y)$ is in general a nondiagonal matrix, we conclude that a nonabelian vortex can cause mixing among the components of a fermion multiplet. From the form of $\tilde{\psi}$, it follows that the corresponding scattering cross section is a typical AB cross section, i.e.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\,\mathrm{d}l} \sim k^{-1}. \tag{3.10}$$

4. Application to baryon decay catalysis

We now apply the analysis of the previous section to grand unified models with strings. Baryons and leptons then can lie in the same fermion representation. If one of the generators of the gauge group G broken during cosmic string formation connects baryons and leptons, then the AB phase mixing will lead to baryon decay.

Consider for example G = SO(10) breaking down to SU(5) × $\mathbb{Z}_2$. The fermions lie in the 16 representation of SO(10). Symmetry breaking may occur in several stages. The final stage involves a multiplet of Higgs fields ϕ_i which transform according to the 126 representation. This model leads to nonabelian cosmic strings [11–14].

Strings arise in the following way: consider a circle C in physical space with origin O. If ϕ is in a ground state for all angles φ on the circle, then the state at φ is related to the state at O by an SO(10) group element $S(\varphi)$. Strings form if $S(\varphi)$ is an element of SO(10) corresponding to a symmetry which is broken when SO(10) breaks to SU(5) × $\mathbb{Z}_2$ and which, when φ runs from 0 to 2π, connects the two disconnected copies of SU(5) contained in the vacuum manifold of SO(10) [14].

The general form of an SO(10) Lie algebra element in the spinor representation, the representation the fermions of the standard model lie in, is given in ref. [15]. We pick a generator which corresponds to a broken symmetry and which connects baryons and leptons, in particular the u quark and the right-handed neutrino: $S(\varphi) = \exp(\mathrm{i}M\varphi)$, with

$$M = \tfrac{1}{2}\begin{pmatrix}A & 0 & 0 & 0\\ 0 & A & 0 & 0\\ 0 & 0 & -A & 0\\ 0 & 0 & 0 & -A\end{pmatrix}, \quad A = \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}. \tag{4.1}$$

Indeed, $S(\varphi)$ connects the two copies of SU(5) contained in the vacuum manifold of SO(10) as φ varies from 0 to 2π. Note, that in the above we have used the following ordering of the 16 fermions of the spinor representation:

$$F = (\mathrm{u}_1, \mathrm{u}_2, \mathrm{u}_3, \nu_\mathrm{e}, \mathrm{d}_1, \mathrm{d}_2, \mathrm{d}_3, \mathrm{e}^-,$$
$$\mathrm{d}_1^\mathrm{c}, \mathrm{d}_2^\mathrm{c}, \mathrm{d}_3^\mathrm{c}, \mathrm{e}^+, -\mathrm{u}_1^\mathrm{c}, -\mathrm{u}_2^\mathrm{c}, -\mathrm{u}_3^\mathrm{c}, -\nu_\mathrm{e}^\mathrm{c}), \tag{4.2}$$

where the subscripts 1–3 denote the three colors.

The important fact about (4.1) is that in all two-dimensional subspaces which experience mixing, baryons and leptons are mixed. Thus, $S(\varphi)$ leads to AB catalyzed baryon decay.

Let us focus on a two-dimensional subspace. Restricted to this subspace, the matrix M (see section 3) is

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{4.3}$$

It is then easy to determine the eigenvalues and eigenvectors and calculate the matrix D. If the incoming state is $[\psi_l(x), 0]^{\mathrm{T}}$, then from (3.6)

$$\psi(x)^{\mathrm{out}} = \cos \pi\alpha \begin{pmatrix} \psi_l(x) \\ 0 \end{pmatrix} + \frac{1}{\mathrm{i}} \sin \pi\alpha \begin{pmatrix} 0 \\ \tilde{\psi}_l(x) \end{pmatrix}. \tag{4.4}$$

For a vortex appropriately oriented in internal space, $\psi_l(x)$ and $\psi_{l+1}(x)$ can be a baryon and lepton respectively. Hence, AB phase mixing can convert directly baryons to leptons. The cross section for this process is a typical AB cross section and hence much larger than the geometrical one. Note that the physics is completely independent of the vortex core structure, in contrast to the process discussed in refs. [4,10].

5. Discussion

The large AB cross section for baryon decay catalysis discussed in this paper may lead to interesting cosmological effects. First, there are possible constraints on the number density of string loops, similar to the bounds on the flux of magnetic monopoles [15] from the Callan–Rubakov effect. However, in the case of cosmic strings such bounds must compete with the bound on the number density of loops coming from the scaling solution [1].

Second, the enhanced catalysis cross section in this paper may increase the rate at which strings can destroy a primordial baryon to entropy ratio. However, most of the destruction will occur at very early times [5], shortly after the strings are produced. At early times (high temperature), the AB scattering cross section, like all other cross sections, is modified by finite temperature effects [16]. The cross section becomes [17]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\,\mathrm{d}l} = \mathrm{O}(1)\frac{1}{k(T)} \sim \frac{1}{T} \tag{5.1}$$

which for T close to the unification scale M approaches the catalysis cross section of ref. [5]. Hence, the fraction of the primordial baryon to entropy ratio which can be erased by AB induced catalysis is of the same order of magnitude as that erased by "ordinary" catalysis [5]. The conclusion from ref. [5] is that only if coupling constants are large, this fraction is significant.

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