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Non-collapsing membrane instantons in higher dimensions

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Abstract

We introduce a particular embedding of seven-dimensional self-duality membrane equations in $C^3 \times R$ which breaks G_2 invariance down to $SU(3)$. The world-volume membrane instantons define $SU(3)$ special lagrangian submanifolds of C^3. We discuss in detail solutions for spherical and toroidal topologies assuming factorization of time. We show that the extra dimensions manifest themselves in the solutions through the appearance of a non-zero conserved charge which prevents the collapse of the membrane. We find non-collapsing rotating membrane instantons which contract from infinite size to a finite one and then they bounce to infinity in finite time. Their motion is periodic. These generalized complex Nahm equations, in the axially symmetric case, lead to extensions of the continuous Toda equation to complex space.

1. Introduction

Some years ago, we introduced the notion of the self-duality for supermembrane in $4+1$ dimensions and in the light-cone gauge. The corresponding self-duality (s-d) equations proved to be an integrable system with an infinite number of conservation laws and particular solutions were found [1,2] which were collapsing configurations of membrane instantons to point-like or string-like objects. Similar covariant self-duality equations have been introduced before for $2+1$ dimensions [3] and later generalized to $6+1$ dimensions in [4].

These objects, represent world-volume instantons of the supermembrane. In the light cone gauge, the world-volume time and the target time are identical, so these configurations are spacetime membrane instantons and they provide quantum mechanical tunnelling through the membrane self-interaction potential moving with velocities bigger than light. Thus, they can travel infinite distances in finite time.

Their equations of motion, which are Nahm's type equations for the area-preserving diffeomorphism group of the membrane, lead for axially symmetric configurations to continuous Toda equations relating thus the membrane instantons with the self-dual Einstein metrics with isometries [5,6].

The basic ingredients for the study of covariant membrane instantons in higher than $4+1$ dimensions,

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were contained in the pioneering Letter of Ref [4], however, this work was until recently overlooked. The authors in [7] introduced higher-dimensional self-duality equations for the light-cone membranes as well as in the case of the quantized Poisson (i.e., Moyal) bracket. The detailed properties of the octonionic light-cone membrane instantons where studied in [8,9], where the invariance of the seven-dimensional equations under the exceptional group G_2 was exploited. The invariance under this group has as a consequence one remaining supersymmetry consistent with the membrane background. In three dimensions there are eight remaining supersymmetries [10].

In a parallel development, octonionic self-duality for seven- and eight-dimensional gravity was proposed [11,12] and explicit seven and eight gravitational instantons which generalize four-dimensional ones (satisfying first order equations) were found [12,13]. These higher-dimensional gravitational instantons where among the first few explicitly known self-dual metrics with exceptional holonomies G_2 and Spin(7) which were also lifted in 10- and 11-dimensional supergravity. Recently, exceptional holonomy higher-dimensional instantons were studied for their rôle in string and M-theory and an important activity around this subject has been created [14].

In this Letter, we introduce the complexified self-duality equations of the membrane in seven dimensions and represent them as generalized Nahm equations. We show that the extra dimensions manifest themselves in the solutions through the appearance of an non-zero conserved charge which prevents the collapse of the membrane. We integrate completely the three-dimensional complex Nahm's equations for S^2 and T^2 topologies, assuming factorization of time. We find periodic non-collapsing instantons. Starting from infinite size they contract, with increasing angular velocity, to a minimum size and then they bounce back to infinity in finite time.

2. The self-duality membrane equations in seven dimensions

Choosing fixed values for the 8th and 9th membrane coordinates, the seven-dimensional self-duality equations [4,7,8] become

$$\dot{X}_i = \frac{1}{2}\Psi_{ijk}\{X_j, X_k\}, \tag{1}$$

where Ψ_{ijk} is the completely antisymmetric tensor that defines the multiplications of octonions [15]. The Gauss law results automatically by making use of the Ψ_{ijk} cyclic symmetry

$$\{\dot{X}_i, X_i\} = 0. \tag{2}$$

The Euclidean equations of motion are obtained as follows

$$\ddot{X}_i = \frac{1}{2}\Psi_{ijk}\big(\{\dot{X}_j, X_k\} + \{X_j, \dot{X}_k\}\big), \tag{3}$$

$$= \big\{X_k, \{X_i, X_k\}\big\}, \tag{4}$$

where use has been made of the identity

$$\Psi_{ijk}\Psi_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} + \phi_{ijlm} \tag{5}$$

and of the cyclic property of the symbol ϕ_{ijlm} [15].

At this point we would like to make a general remark on the nature of the motion described by (1). These equations describe the time evolution of the membrane instanton in flat spacetimes. If the coordinates X_i, $(i = 1, \ldots, 7)$ are periodic functions of the membrane parameters $\sigma_{1,2}$, then integrating both sides of the equations we find that all membrane instantons have their center of mass pinched in a fixed point of space. This implies spontaneous symmetry breaking of translational invariance. If some of the flat space dimensions are compactified, then the center of mass moves with the velocity determined by the cross products of the winding numbers of the membrane in the compactified dimensions. The cross product is defined through the tensor Ψ_{ijk} [9].

In what follows, we proceed to the complexification of the self-duality equations. We embed the seven-dimensional space R^7 into $C^3 \times R$ in a very specific way which depends on the particular definition of the octonionic structure constants used in Ref. [8] which assume the following multiplication table [15]

$$\Psi_{ijk} = \begin{cases} 1 & 2 & 4 & 3 & 6 & 5 & 7 \\ 2 & 4 & 3 & 6 & 5 & 7 & 1 \\ 3 & 6 & 5 & 7 & 1 & 2 & 4 \end{cases}. \tag{6}$$

Thus, if we define

$$z_1 = X_1 + iX_4, \qquad z_2 = X_2 + iX_5,$$
$$z_3 = X_3 + iX_6, \qquad a_0 = X_7$$

then the self-duality equations become

$$D_t z_I = \frac{1}{2}\epsilon_{IJK}\{z_J^*, z_K^*\}, \qquad D_t a_0 = \frac{\iota}{2}\{z_I, z_I^*\}, \quad (7)$$

where I, J, K take the values $1, 2, 3$, whereas D_t is the 'covariant' derivative

$$D_t = \partial_t - i\{a_0, \quad \}. \quad (8)$$

These strikingly simple equations of self-duality, break the G_2 invariance down to $SU(3)$. The $SU(3)$ invariance comes from the unique cross product existing in C^3 which is a remnant of the octonionic cross product in seven dimensions. One consequence is that, the three-dimensional world-volume manifolds described by (7) are $SU(3)$ special lagrangian sub-manifolds of C^3 [16].

In the next section, we will consider the factorization of time and the restriction to three complex dimensions of the above first order equations. Before that, we would like to observe that it is possible to generalize the connection of the three-dimensional self-duality equations with the continuous Toda equations [1,5,6]. This is possible if we consider axially symmetric solutions of the above system. Indeed, the axially symmetric Ansatz,

$$z_1 = R(\sigma_2, t)\cos\sigma_1, \qquad z_2 = R(\sigma_2, t)\sin\sigma_1,$$
$$z_3 = z(\sigma_2, t), \quad (9)$$

where, R, z complex functions, implies $\dot{a}_0 = 0$ for all times and thus a_0 can be fixed to zero by an area preserving transformation. For Eq. (7) we obtain

$$\dot{R} = R^* z^*_{\sigma_2}, \qquad \dot{z} = -R^* R^*_{\sigma_2} \quad (10)$$

and the index σ_2 refers to the derivative with respect to σ_2. This system of equations has as integrability condition the following non-linear equation which extends the continuous Toda equation to three complex dimensions.

$$\frac{1}{R^*}\partial_t^2 R - \frac{1}{R^{*2}}\partial_t R\partial_t R^* + \frac{1}{2}\partial^2_{\sigma_2} R^2 = 0. \quad (11)$$

This equation maybe relevant for the higher-dimensional self-dual gravity. Setting in (11) $R^2 = e^{\Psi}$ we obtain the form

$$\ddot{\Psi} + \frac{1}{2}(\dot{\Psi} - \dot{\Psi}^*)\dot{\Psi} + e^{-\frac{1}{2}(\dot{\Psi}-\dot{\Psi}^*)}\partial^2_{\sigma_2} e^{\Psi} = 0. \quad (12)$$

In the *real* three-dimensional case [1], we have $R^* = R$ and $z = z^*$, while the continuous Toda equation for $\Psi = \Psi^*$ reads [1,5,6],

$$\partial_t^2\Psi + \partial^2_{\sigma_2} e^{\psi} = 0. \quad (13)$$

In the next section, among other things we find the complete solution of the above (10) system or of the generalized continuous Toda equation (11) restricting the functions R, z so that $R(\sigma_2, t) = \sin\sigma_2\zeta(t)$, $z(\sigma_2, t) = \cos\sigma_2\zeta_3(t)$.

3. Membrane instantons in three complex dimensions

As we show below, it is possible to extend the known three-dimensional instanton solutions into six dimensions where, apart from the radial expansion of the instanton, we observe rotational motion in all of the three planes $(X_1, X_4), (X_2, X_5), (X_3, X_6)$ of the six-dimensional space (which we choose to call them I, II, III complex planes).

We assume factorization of time which will lead to a coherent motion of all the membrane points. These solutions are analogous (but for Euclidean time) to the real time solutions of second order equations of motion for toroidal and spherical membranes recently studied in [17]

$$z_i = \zeta_i(t) f_i(\sigma_1, \sigma_2), \quad (14)$$

where f_i are three complex functions on the surface. First we observe that the Poisson bracket $\{z_i, z_i^*\} = 0$, if the functions f_i, f_i^* are functions of the same combination σ_1, σ_2. From the equation for a_0 we find $\dot{a}_0 = 0$ and therefore by an appropriate area preserving transformation we may fix a_0 to be zero. So we are left with the three complex Nahm's equations for z_i. We shall examine in detail two topologies: spherical (S^2) and toroidal (T^2).

Up to now only three-dimensional solutions of the self-duality equations are known [1]. In order to factorize the time dependence we choose for the case of S^2 the three functions f_i to be

$$f_1 = \cos\phi\sin\theta, \qquad f_2 = \sin\phi\sin\theta,$$
$$f_3 = \cos\theta. \quad (15)$$

The three functions for the algebra $SU(2)$ under Poisson bracket satisfy

$$\{f_i, f_j\} = -\epsilon_{ijk} f_k, \quad i, j, k = 1, 2, 3. \quad (16)$$

For the three complex functions of time, we find the complex Euler equations[1]

$$\dot{\zeta}_i = -\frac{1}{2}\epsilon_{ijk}^2 \zeta_j^* \zeta_k^*. \tag{17}$$

In the case of T^2 we choose the following three functions

$$f_i = e^{\imath \vec{n}_i \cdot \vec{\sigma}}, \quad i = 1, 2, 3, \tag{18}$$

$$\vec{n}_i = (n_{i1}, n_{i2}) \in Z^2. \tag{19}$$

Now we observe that the factorization of time is implemented for any three $\vec{n}_i$'s such that

$$\vec{n}_1 + \vec{n}_2 + \vec{n}_3 = \vec{0}. \tag{20}$$

In this case, we obtain for the corresponding $\zeta_i(t)$

$$\dot{\zeta}_i = -n\frac{1}{2}\epsilon_{ijk}^2 \zeta_j^* \zeta_k^*, \tag{21}$$

where $n = n_{11}n_{22} - n_{12}n_{21} \in Z$.

In both cases (S^2 and T^2) the equations for the time evolution are essentially the same. The T^2 case is obtained from the equations of S^2 if we make the replacement $t \to nt$ for n integer. Therefore we only need to investigate Eq. (17) which in component form is written

$$\dot{\zeta}_1 = -\zeta_2^* \zeta_3^*, \qquad \dot{\zeta}_2 = -\zeta_3^* \zeta_1^*, \qquad \dot{\zeta}_3 = -\zeta_1^* \zeta_2^*. \tag{22}$$

There is an obvious symmetry of the above system

$$\zeta_k \to e^{\imath q_k} \zeta_k, \tag{23}$$

where $q_k, k = 1, 2, 3$ are real and $q_1 + q_2 + q_2 = 0$. This invariance leads to the conservation of the three charges

$$Q_i = -\frac{\imath}{2}\big(\dot{\zeta}_i \zeta_i^* - \dot{\zeta}_i^* \zeta_i\big), \quad i = 1, 2, 3. \tag{24}$$

On the other hand, the equations of motion (22) imply that all three charges Q_i are equal to

$$Q_i \equiv Q = -\frac{\imath}{2}\big(\zeta_1\zeta_2\zeta_3 - \zeta_1^*\zeta_2^*\zeta_3^*\big). \tag{25}$$

There are two additional constants of motion in analogy with the Euler equations for the rigid body,

$$c_{ij} = |\zeta_i|^2 - |\zeta_j|^2, \tag{26}$$

where c_{ij} are constants. In polar coordinates

$$\zeta_k = r_k e^{\imath \phi_k} \tag{27}$$

we obtain

$$Q = r_1 r_2 r_3 \sin(\phi_1 + \phi_2 + \phi_3), \tag{28}$$

$$\dot{\phi}_k = \frac{Q}{r_k^2}, \tag{29}$$

$$c_{ij} = r_i^2 - r_j^2. \tag{30}$$

Then, Eq. (22) reduce to

$$\dot{r}_i = r_j r_k \cos\phi, \tag{31}$$

where $\phi = \phi_1 + \phi_2 + \phi_3$. For simplicity we define $s_1 = r_1^2$. We further combine (31) with (28) to obtain the following differential equation

$$\dot{s}_1 = -2\sqrt{s_1 s_2 s_3 - Q^2}. \tag{32}$$

After substitutions, the differential equation obtains a unique form in the right-hand side for all s_i which is

$$\dot{s}^2 = 4\big[s(s-a)(s-b) - Q^2\big], \tag{33}$$

where $s_1 = s$, $s_2 = s - a$, $s_3 = s - b$, where $a = c_{12}, b = c_{12} + c_{23}$.

If we define a new function of time $\mathcal{U}(t) = s(t) - \frac{a+b}{3}$, the differential equation becomes

$$\dot{\mathcal{U}}^2 = 4\mathcal{U}^3 - g_2\mathcal{U} - g_3 \tag{34}$$

which is recognized as the standard form of the Weierstrass equation with solution the (doubly periodic) Weierstrass function $\mathcal{U}(t) = \mathcal{P}(t; g_2, g_3)$, with

$$g_2 = \frac{4}{3}\big(a^2 + b^2 - ab\big), \tag{35}$$

$$g_3 = \frac{4}{27}\big(2b^3 + 2a^3 - 3a^2 b - 3ab^2\big) + 4Q^2. \tag{36}$$

Before we proceed to the analysis of the solution, we would like to point out that there is an isomorphism between the membrane and the matrix model solutions with factorization of time for the spherical and toroidal topologies. This implies that the above solutions have isomorphic matrix model instanton solutions similar to those examined recently in Ref. [18].

[1] For the seven-dimensional system in another context, see also [19].

4. Analysis of the non-collapsing instanton solutions

The equation of motion (33) for the membrane radii ($r_1^2 \equiv s, r_2^2 = s - a$ and $r_3^2 = s - b$), is analogous to the motion of a particle in a potential V. This is indeed the motion of any point of the Euclidean membrane:

$$\dot{s}^2 \equiv V(s, Q) = 4\big[s(s-a)(s-b) - Q^2\big], \tag{37}$$

where a, b are positive constants. Without loss of generality we may choose $a > b > 0$. The left-hand side of Eq. (37) is always positive, thus the permitted regions for the variable s are such that $V(s, Q)$ is also positive. These regions depend on the values of Q. There is a limiting position of the qubic curve when $Q^2 = 0$ ($V(s, 0) \equiv V_0(s)$). This is the upper curve shown in Fig. 1 and the three real roots are $s_1 = a$, $s_2 = b$ and $s_3 = 0$. For all other values of Q^2 the curve is below the limiting one and whenever there are three real roots, they are $b > s_3 > 0$ $s_2 < b, s_1 > a$, respectively. $V(s, Q)$ possesses extrema at the values

$$s_{\max/\min} = \frac{1}{3}\big(a + b \mp \sqrt{a^2 + b^2 - ab}\big). \tag{38}$$

There is a critical value $Q^2 = Q_c^2$ for which the $V(s, Q)$ has a double root which is the $s_{\max}$. Q_c is determined as follows

$$Q_c^2 = V_0(s_{\max}). \tag{39}$$

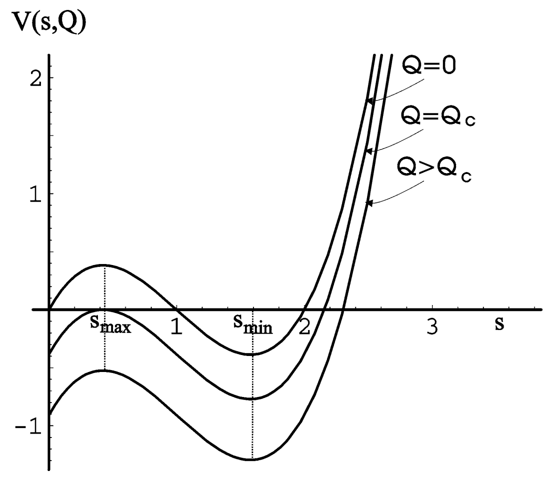

Fig. 1. The ‘Potential’ for three characteristic Q-values: in all cases, the allowed region is beyond the highest root to the infinity.

In this case we calculate the maximum root to be

$$s_1^c = \frac{1}{2}\big(a + b + 2\sqrt{a^2 + b^2 - ab}\big). \tag{40}$$

The physical region is this case is beyond s_1^c.

When $Q^2 > Q_c^2$ there are two complex conjugate roots (the maximum of V is below the real axis) the physical region is $s > s_1$ where s_1 is the real root. The three cases described above are presented in Fig. 1. In what follows, we proceed in the detailed description of the dynamics of the membrane instanton in the three cases discussed above ($Q = 0$, $Q \neq 0$, $Q = Q_c$).

• When $Q = 0$ it is possible to redefine the time-independent phases (29) to zero values and the self-duality equation reduces to the ones of the three-dimensional case [1]. Because of the conservation laws we only need the equation for $s = r_1^2$ which reads

$$\dot{s} = -2\sqrt{s(s-a)(s-b)}. \tag{41}$$

We distinguish the following cases [1]:

• $a = b = 0$. The solution is a spherical membrane with the radius varied with time as

$$r = \frac{r_0}{1 + r_0(t - t_0)}. \tag{42}$$

There is a critical value $t_c = t_0 - 1/r_0$ where $r \to \infty$, whilst for $t \to \infty$ the radius shrinks to zero.

• $a = 0, b \neq 0$. The equation becomes $\dot{s} = -2s\sqrt{s - b}$ and the solution obtained is

$$r = r_0 \frac{1 + \frac{\sqrt{b}}{r_0} \tan\{\sqrt{b}\,(t - t_0)\}}{1 - \frac{r_0}{\sqrt{b}} \tan\{\sqrt{b}\,(t - t_0)\}}. \tag{43}$$

At $t_c = \frac{1}{\sqrt{b}} \tan^{-1} \frac{\sqrt{b}}{r_0}$ the membrane has an infinite radius. On the contrary, when $t_{\text{in}} = -\frac{1}{\sqrt{b}} \tan^{-1} \frac{\sqrt{b}}{r_0}$ the configuration collapses to a string.

• $a > b > 0$. This is the most general four-dimensional case. The s-d equation reads

$$\dot{s} = -2\sqrt{s(s-a)(s-b)}.$$

In order to write it in a more familiar form, we make the transformations $x = \frac{\sqrt{a}}{r}$, $k^2 = \frac{b}{a} < 1$, with $r > a$ and, therefore, $x < \frac{1}{\sqrt{a}}$. Then, separating variables we have

$$t = \frac{1}{\sqrt{a}} \int_0^{\sin\phi} \frac{dx}{\sqrt{(1 - x^2)(1 - k^2 x^2)}},$$

$$-\frac{\pi}{2} < \phi < \frac{\pi}{2}. \tag{44}$$

The right-hand side is the elliptic integral [20] $\mathcal{F}(\phi, k)$, and the radius r is given

$$r = \frac{\sqrt{a}}{sn\sqrt{a}\,t}. \tag{45}$$

Here, we assumed as initial condition $t = 0$ and $r_0 = r(t = 0) = \infty$. The positivity of r restricts the t-range in a half period of the elliptic sn. The real period is the complete elliptic integral, i.e., when the upper limit of (44) is equal to unity, $\sin\phi = 1$,

$$\frac{T}{2} = \frac{1}{\sqrt{a}}\int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} = \frac{1}{\sqrt{a}} K\left(k^2 = \frac{b}{a}\right). \tag{46}$$

With the above initial conditions, at $t = 0$ the volume of the ellipsoidal membrane is infinite, whereas at time $t = \frac{T}{4}$ reaches a minimum value with $r_{\min} = \frac{\sqrt{a}}{sn\sqrt{a}\frac{T}{4}}$ and the membrane collapses to an elliptic disc.

It is worth mentioning that in three dimensions, assuming simple factorization of time, we do not find non-collapsing membranes. On the other hand we find all possible collapsed configurations for the membrane, that is, points, strings and discs.

• Now we consider the case $Q \neq 0$. This case differentiates from the $Q = 0$ case because it exists only in dimensions higher than three (see Eq. (29)).

As we shall see, a remarkable fact is that the dynamics of the membrane in higher dimensions is encoded in the higher-dimensional angular momentum Q which, from the point of view of three dimensions, it behaves like a charge.

In the case of spherical topology, there are three different geometries, spherical, ellipsoidal with axial symmetry, and anisotropic ellipsoidal ones. These three cases correspond to the degeneracy of the roots of the polynomial in the right-hand side of Eq. (33).

If the degeneracy g is $g = 3$, we have the spherically symmetric membrane which from any initial condition it approaches the radius equal to the largest real root of the Eq. (33) in finite time and it goes back to infinity.

If $g = 2$, we have the axially symmetric ellipsoid which from an initial configuration it decreases its volume until a limiting one which is determined also by the largest real root. The same also happens to the anisotropic ellipsoidal membrane. The general solution in terms of elliptic Jacobi functions or Weierstrass function of Eq. (34), can be found in a similar way with the case $Q^2 = 0$ in Eq. (41)

$$\frac{ds}{\sqrt{s(s-a)(s-b) - Q^2}} = \frac{dx}{\sqrt{x(x-e_{13})(x-e_{23})}}, \tag{47}$$

where $x = s - e_3$ and $e_{ij} = e_i - e_j$.

If the topology is toroidal, the radii $r_{1,2,3}$ of the torus T^3 inside which the T^2 toroidal membrane is embedded, at any moment of time they are equal to $\sqrt{s}$, $\sqrt{s-b}$, $\sqrt{s-a}$, respectively. We note that in this case there are no non-degenerate solutions below four dimensions.

In the following, we discuss the $Q \neq 0$ spherical case ($a = b = 0$). Integrating the equation we find the solution in terms of the incomplete beta function [20]

$$t = \frac{1}{6Q^{1/3}} \mathrm{Beta}\left(\frac{Q^2}{r^6}; \frac{1}{6}, \frac{1}{2}\right). \tag{48}$$

We assume here the following initial conditions: at $t = 0$ the spherical membrane has infinite volume and in finite time $T = \frac{1}{6Q^{1/3}} \mathrm{Beta}(\frac{1}{6}, \frac{1}{2})$ contracts at the minimum permitted radius $r_0 = Q^{1/3}$ and goes back to infinity. From the angular velocity Eq. (29), at $t = 0$ or infinite radius, the angular velocity is zero and contracting it develops at the limiting time T angular velocity $\omega_T = \dot{\phi} = Q^{1/3}$.

The ellipsoidal cases follow similar pattern and we parametrize the solution in terms of the Weierstrass function [20]:

$$s(t) = \mathcal{P}(t; g_2, g_3) + \frac{a+b}{3}, \tag{49}$$

where $g_{2,3}$ are functions of a, b given by (35), (36) above.

Due to the non-zero angular momentum the brane obtains a minimum size given by the radii squared, s_1, $s_1 - a$, $s_1 - b$ where as discussed in the beginning of this section, s_1 stands for the largest root of $V(s, Q)$ (see Fig. 2). At this minimum size, there are limiting

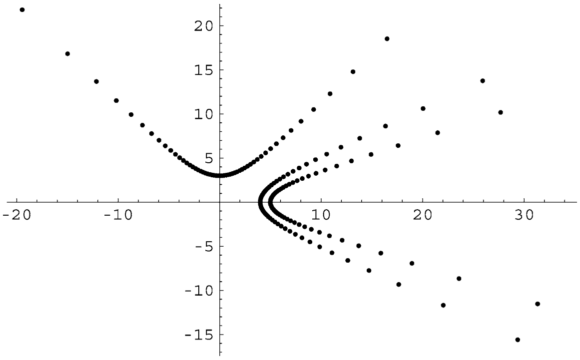

Fig. 2. The points show the equidistant time-evolution of the coordinates of the ellipsoidal membrane (45) in the three planes $I = (X_1, X_4)$, $II = (X_2, X_5)$ and $III = (X_3, X_6)$. All of them reach a minimum size and go back to infinity.

angular velocities given by

$$\omega_1 = \frac{Q}{s_1}, \qquad \omega_2 = \frac{Q}{s_1 - a}, \qquad \omega_3 = \frac{Q}{s_1 - b}. \qquad (50)$$

In the solution (34) we assume initial conditions $s(0) = \infty$ and s_1 is given by $s_1 = \mathcal{P}(\frac{T}{2}; g_2, g_3) + \frac{a+b}{3}$, where T is the real period of the Weierstrass function. In the special case of $Q = Q^c$, we have a simple algebraic solution (similar to the $Q = 0$ case), and $s_1 = s_1^c$ with s_1^c given by (40).

5. Conclusions

Breaking the G_2 invariance of the octonionic self-duality equations for the membrane in seven dimensions down to $SU(3)$, we found explicit solutions of non-collapsing rotating membrane instantons which they have periodic motion starting at some initial moment from infinite size, shrinking down to a finite one in a half period and then bouncing back to infinity. The rôle of these instantons for the quantum mechanical vacuum of the membrane depends on the period which is the inverse temperature in membrane plasma of finite temperature. In the case of infinite period (zero temperature) the membrane instantons collapse to point-, string- and disc-like objects which represent the vacua of the quantum mechanical membrane. Since up to now it is not known how to quantize the supermembrane, we hope that the information we provided in this work is a step towards this direction.

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