NON-PERTURBATIVE EFFECTS IN GUT PHASE TRANSITIONS

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Non-perturbative effects generate a negative squared "mass" term for the Higgs fields, which, in a grand unified theory with Coleman–Weinberg type symmetry breaking, drives the transition from the symmetric high temperature phase to the true vacuum. This mechanism could be of wider interest.

It is generally believed that at the early stages of the expansion of the universe, characterized by very high temperatures in the standard cosmological model, the broken symmetries of elementary particle interactions were intact [1]. The very attractive idea of grand unification [2], in which the SU(3) and SU(2) × U(1) gauge groups of the observed strong and electroweak interactions are unified in a larger gauge group G, requires that this gauge group G suffers successive spontaneous symmetry breakdowns, via some Higgs fields, down to SU(3) × $U(1)_{em}$. In the framework of an expanding universe, these breakdowns correspond to phase transitions, generally of first order [3], which occur as the temperature decreases.

Of particular interest is the case of a Coleman–Weinberg potential, in which the symmetry breaking occurs through radiative corrections [4]. As a consequence of a Coleman–Weinberg symmetry breaking for the GUT phase transition, the universe will supercool below the grand unification scale, before the transition from the symmetric high temperature state to the true vacuum is completed [5]. In this note we point out a mechanism different from the ordinary barrier penetration [6] by which the GUT phase transition will complete itself.

For definiteness, let us consider the SU(5) grand unified theory [7], though most of what will be discussed will not depend on the details of the model under discussion. The one-loop effective potential for the SU(5) model in the Coleman–Weinberg mode is [5]

$$V(\phi) = (5625/1024\pi^2)g^4\phi^4[\ln(\phi^2/\sigma^2) - \tfrac{1}{2}], \quad (1)$$

where $\phi^2 = \frac{2}{15}\mathrm{tr}(\Phi^2)$, with Φ being the adjoint Higgs field and σ the zero temperature minimum of the potential. At finite temperature the potential is altered by the addition of the term [8]

$$V_T(\phi) = (18T^4/\pi^2) \times \int_0^\infty dx\, x^2 \ln\{1 - \exp[-(x^2 + \tfrac{25}{8}g^2\phi^2)^{1/2}]\}. \quad (2)$$

However, for $\phi \ll T \ll \sigma$, this term is well approximated by an effective mass term

$$V_T(\phi) = \tfrac{75}{16} g^2 T^2 \phi^2 . \quad (3)$$

This term has dramatic effects since it stabilizes the false vacuum $\phi = 0$. In fact, $\phi = 0$ ceases to be a local minimum only at zero temperature.

The tunnelling to the true vacuum now proceeds via bubble nucleation. The bubble nucleation rate per unit time per unit volume is approximately

$$A \exp(-S),$$

where S is the euclidean tunnelling action [6] corresponding to an O(3) symmetric solution, and A is a factor roughly proportional to T^4 since T is the relevant scale of the problem. The tunnelling becomes appreciable when it is comparable to the rate of the expansion of the universe per unit time per unit volume, i.e., to ρ^2/M_P^4, where M_P is the Planck mass

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and ρ is the energy density of the universe. In the present case, ρ stands for the energy density of the false vacuum $(5625g^4/2048\pi^2)\sigma^4$. That is, we must have

$$(M_P^4/\rho^2)\, T^4 \exp(-S) \sim O(1)\,, \tag{4}$$

Since the relevant scale of the process is the temperature, the gauge coupling runs with the temperature [9]. It is now known from previous works [6] that eq. (4) is not satisfied and the transition does not complete itself until we reach temperatures ~1 GeV, which is well below the energies at which one can trust perturbation theory.

As we approach lower temperatures, however, gauge field configurations play an important role in the development of the phase transition. One should examine the possibility that the origin of the potential becomes unstable due to gauge boson condensation.

Gauge boson condensation has been considered in QCD [10] where the colour singlet gluon condensate seems to be an important part of the correct vacuum. There the contribution of instantons to the singlet gluon condensate has been considered.

Now, in the SU(5) theory, a non-zero vacuum expectation value to operators made out of gauge boson fields is naturally supplied by SU(5) instanton configurations. In particular, the operator $F^a_{\mu\nu}F^a_{\mu\nu}$ acquires an expectation value. This expectation value is of course temperature dependent. Periodic instantons can be constructed as a periodic superposition of zero temperature instantons.

Considering instantons of an SU(2) subgroup embedded in SU(5), we write [9]

$$A_\mu = \Pi\bar{\eta}^a_{\mu\nu}(\tfrac{1}{2}\lambda^a)\partial_\nu\Pi^{-1}\,,$$

$$F_{\mu\nu} = \tfrac{1}{2}\Pi\lambda\cdot\partial\bar{\eta}^a_{\mu\nu}(\tfrac{1}{2}\lambda^a)\lambda^+\cdot\partial\Pi^{-1}\,. \tag{5}$$

where λ^a are the appropriate generators of SU(5), $\lambda_\mu = (-\mathrm{i}, \boldsymbol{\lambda})$ and Π is the 't Hooft potential

$$\Pi = 1 + \sum_n \frac{\rho^2}{(x-z_n)^2} = 1 + \frac{\pi\rho^2 T}{r}\,\frac{\sinh(2\pi rT)}{[\cosh(2\pi rT) - \cos(2\pi Tt)]}\,, \tag{6}$$

with t, r being the position of the instanton and ρ its size. Perturbing around the instanton solution, we find that the vacuum expectation value of $(F^a_{\mu\nu})^2$ is

$$\langle (F^a_{\mu\nu})^2\rangle \simeq \int \mathrm{d}\rho\, D(\rho,T) \int \mathrm{d}^4 z (F^a_{\mu\nu}(z,\rho))^2_{\text{inst}}\,, \tag{7}$$

where $D(\rho, T)$ is the instanton density. The above formula reduces to

$$\langle (F^a_{\mu\nu})^2\rangle \simeq 32\pi^2 P_{\text{inst}}(T)\,, \tag{8}$$

with $P_{\text{inst}}(T)$ being the pressure of instantons [9]. This is, for the SU(5) theory,

$$P_{\text{inst}}(T) \simeq C_5 T^4 [4\pi^2/g^2_{\text{SU(5)}}(T)]^{10} \exp[-8\pi^2/g^2_{\text{SU(5)}}(T)] \tag{9}$$

where

$$C_5 \simeq 7.5 \times 10^{-4}\,,$$

and

$$8\pi^2/g^2(T) = \tfrac{20}{3}\ln(T^2/\Lambda^2_{\text{SU(5)}})$$

is the running coupling of the SU(5) theory [$\Lambda_{\text{SU(5)}} \sim 2\times 10^6$ GeV for the standard asymptotically free SU(5) theory]. So eq. (8) gives us an order of magnitude estimate for the contribution of instantons to the vacuum expectation value $\langle F^2\rangle$ [‡1].

The important observation now is that, although not present in the lagrangian [‡2], a gauge invariant effective coupling $(F^a_{\mu\nu})^2 \operatorname{tr}\Phi^2$ of the Higgs fields to the gauge bosons is generated. Of course, at zero temperature no such dimensionful local coupling could be generated due to the absence of a mass parameter. At finite temperatures, however, the longitudinal degrees of freedom develop non-perturbatively a mass $M^2 \propto g^2T^2$ while the transverse gauge bosons remain massless. Diagramslike the one of fig. 1 contain contributions

$$g^4\phi^2 \int_{x,y,z} \langle \epsilon^a(x)\cdot\epsilon^a(y)\rangle \times D_{ii}(x-y)D_{00}(x-z)D_{00}(y-z)\,.$$

These terms appear in the effective potential as

‡1 $\langle F^2\rangle$ has to be properly group-averaged over SU(2) subgroups of SU(5). This is not going to change any order of magnitude estimates.

‡2 A quadratic term for the Higgs field could be also generated non-perturbatively by the operator $A^a_{\mu\nu}A^a_{\mu\nu}$ via the direct coupling $g^2A^2\phi^2$. However, A^2 is a gauge dependent quantity.

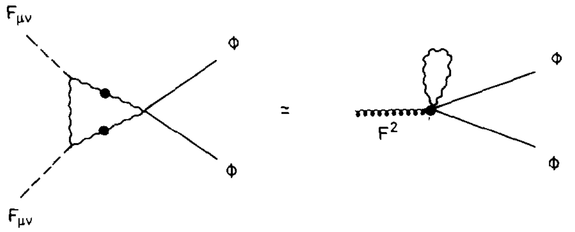

Fig. 1.

$$-(g^2)^2\phi^2T^2 \int_0^{1/T} dt \int d^3x\, \langle -\epsilon^a(x,t)\cdot\epsilon^a(0,0)\rangle$$

$$\times \sum_{m=-\infty}^{+\infty} \int d^3k\, \frac{\exp(ik\cdot x + 2im\pi Tt)}{[k^2+(2m\pi T)^2+M^2]^2}$$

$$\times \sum_{m=-\infty}^{+\infty} \int d^3p\, \frac{\exp(ip\cdot x + i2m\pi Tt)}{[p^2+(2m\pi T)^2]}$$

$$= -(g^2)^2\phi^2T^2 \int_0^{\infty} dr\, r \int d\Omega \int_0^{1/T} dt$$

$$\times \langle -\epsilon^a(r,t)\cdot\epsilon^a(0,0)\rangle \frac{\sinh(2\pi rT)}{\cosh(2\pi rT)-\cos(2\pi tT)}$$

$$\times \sum_{m=-\infty}^{+\infty} \frac{\exp(2im\pi Tt)\exp\{-r[M^2+(2m\pi T)^2]^{1/2}\}}{[M^2+(2m\pi T)^2]^{1/2}}.$$

The last expression, being infra-red singular when the mass M goes to zero or equivalently when the temperature goes to zero, can be replaced by its most singular term

$$\frac{-(g^2)^2\phi^2T^2}{M} \int_0^{\infty} dr\, r \int_0^{1/T} dt \langle -\epsilon\cdot\epsilon\rangle$$

$$\times \frac{\sinh(2\pi rT)}{\cosh(2\pi rT)-\cos(2\pi Tt)} \exp(-rM)$$

$$\simeq -g^3\phi^2T \int_0^{1/gT} dr\, r \int_0^{1/T} dt\, \langle -\epsilon\cdot\epsilon\rangle$$

$$\times \frac{\sinh(2\pi rT)}{\cosh(2\pi rT)-\cos(2\pi tT)}.$$

Assuming now that the non-perturbative expectation value will not vary much over a range $1/T$, we obtain

$$-(g^3\phi^2/T_2)\langle -\epsilon^2\rangle \int_0^{1/g} dr\, r \int_0^{2\pi} dt\, \frac{\sinh r}{\cosh r - \cos t}$$

$$\simeq -(g^3\phi^2/T^2)\langle F^2\rangle\, O(1). \tag{10}$$

Of course, the zero temperature limit is infra-red singular. A term like (10) has the right sign [‡3] in order to destabilize the false vacuum $\phi = 0$ [see eqs. (1) and (3)].

The consequences of this term are easy to analyze. At high temperatures ($\Lambda \ll T \ll M_X$) this term is negligible and the transition proceeds through tunnelling very slowly. As we approach lower temperatures there will be some temperature at which this term will have enough strength to destabilize the origin. When this happens there is a very rapid transition to the broken phase. The adjustment of parameters at zero temperature is of course such that the transition to SU(3) × SU(2) × U(1) will be favoured over SU(4) × U(1).

Since instantons are only one of different factors contributing to the gauge boson condensate, we cannot rely on the value of $\langle F^2\rangle$ we have obtained or its temperature dependence. Hence although we can be pretty sure that the condensate will destabilize the potential, we can only get a rough estimate of the transition temperature from instantons. Thus, our naive estimate gives a critical temperature close to Λ, $T \sim O(1)\Lambda$.

The emergence of instantons at low temperatures naturally destabilizes the wrong vacuum and causes a rapid transition to the broken phase. After that, instantons are suppressed by a factor $\exp(-\langle\phi^2\rangle\rho^2)$.

In considering gauge boson condensates, amongst the attractive channels the singlet is the most attractive one (MAC). However, the formation of non-singlet channels is also a possibility. In this respect there

‡3 $\langle F^2\rangle$ is positive definite quantity in Minkowski space.

seems to be a difference between QCD and other SU(N) ($N \geqslant 5$) theories. QCD remains unbroken and confining the MAC criterion, although approximate, seems to be correct. On the other hand, for pure SU(N) ($N \geqslant 5$) theories, Monte Carlo calculations in lattice gauge theories [11] indicate a different behaviour.

Fermion condensation could also take place. This is known [12] to break the symmetry down to SU(4). There is no singlet in this case and the MAC corresponds to the condensate of the **10** of fermions ($\psi_{ij}^{T} \times \epsilon_{mijkl}\psi_{kl}$). However, it seems to us energetically more favourable for the symmetry breaking to occur via a singlet than a non-singlet condensate.

It is not a coincidence that in the Weinberg–Salam phase transition with a Coleman–Weinberg potential the supercooling is halted by the fermion condensate that breaks chiral symmetry at temperatures close to Λ_{QCD} [13]. The SU(2) instantons are of course there but their effect would be non-negligible only at temperatures much lower than Λ_{QCD}.

The non-perturbative generation of a Higgs mass with the wrong sign, by instantons, naturally leads to the speculation that the negative mass squared, put in by hand in the perturbative treatment of spontaneous symmetry breaking, might be dynamically generated in just this way.

It is by now rather evident that both in the case of the GUT and the SU(2) × U(1) phase transition with a Coleman–Weinberg potential it is nonperturbative effects that stop the supercooling. Even in cases where the symmetry breaking occurs at tree level [14], extensive supercooling to low temperatures would make the gauge loop corrections and eventually non-perturbative effects important. Thus, in all cases the rich structure of gauge theories seems to prevent long drawn out phase transitions. In the light of all this, it seems rather questionable that the supercooling required by Guth's scenario [15] of the inflationary universe can be realized in gauge models. The tangle between the solution to the the horizon and flatness cosmological problems in the scenario seems accidental and it is likely that they can be solved independently without the need for supercooling.

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Note added. We want to point out that, with the degree of supercooling suggested by the present work, a very ambitious picture of the universe can emerge. As it has been first observed by Linde [16] in the meantime, after the formation of a bubble, the field ϕ inside the bubble gradually grows from the value ϕ' determined by $V(\phi', T_c) = V(0, T_c)$ to the value $\phi(T_{reh}) \sim \sigma$, where T_{reh} is the reheating temperature $T_{reh} \sim O(10^{14}$ GeV). However, it approaches the value only after a period of time $\tau \sim T_c^{-1}$. During this period of time the universe expands exponentially $e^{H\tau}$ times, where $H \approx [\frac{8}{3}\pi(\rho/M_p^2)]^{1/2} \sim 10^{10}$ GeV is the Hubble constant. The bubble at the moment of its creation has a typical size $O(T_c^{-1}) \sim 10^{-20}$ cm, if $T_c \sim 10^6$ GeV, as it is suggested by our work. Then after the above discussed time period, it will have a size $O(10^{-20}\, e^{H\tau}$ cm$) \sim 10^{10^4}$ cm, which is many orders of magnitude greater than the observable part of the universe $\sim 10^{28}$ cm. Therefore, all the observable part of the universe is contained inside one bubble. With this picture, it is easily seen that the isotropy, the homogeneity-horizon and the flatness cosmological problems Barrow and Turner [17] can be resolved, as well as the primordial magnetic monopoles problem, simply because in this scenario no monopoles are created in the observable part of the universe.

References

[1] D.A. Kirzhnits and A.D. Linde, Phys. Lett. 42B (1972) 471;
for a review, see: A.D. Linde, Rep. Prog. Phys. 42 (1979) 389.

[2] For a review and references, see: J. Ellis, CERN preprint TH 2942 (1980);
P. Langacker, Phys. Rep. 72 (1981) 185.

[3] A.D. Linde, Phys. Lett. 99B (1981) 391, and references therein.

[4] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.

[5] L.F. Abbott, CERN preprint TH. 3018 (1981), to be published;
A. Billoire and K. Tamvakis, CERN preprint TH-3019 (1981), to be published;
G.P. Cook and D.T. Mahanthappa, Phys. Rev. D23 (1981) 1321;
M.A. Sher, UCSC preprint 81/139 (1981), revised version, to be published.

[6] S. Coleman, Phys. Rev. D15 (1977) 2929;
S. Callan and S. Coleman, Phys. Rev. D16 (1977) 1762;
A. Guth and E. Weinberg, Phys. Rev. D23 (1981) 876.

[7] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

[8] L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320; S. Weinberg, Phys. Rev. D9 (1974) 3357.
[9] D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. 53 (1981) 43, and references therein.
[10] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (!979) 385, 448, 519.
[11] M. Creutz, Phys. Rev. Lett. 46 (1981) 1441; A. Bohr and K.J.M. Moriarty, ICTP/80/81-19 preprint (1981).
[12] S. Raby, S. Dimopoulos and L. Susskind, Nucl. Phys. B169 (1980) 373.
[13] E. Witten, Nucl. Phys. B177 (1981) 477.
[14] A. Guth and E. Weinberg, Phys. Rev. D23 (1981) 876, and references therein.
[15] A. Guth, Phys. Rev. D23 (1981) 347.
[16] A.D. Linde, Lebedev Physical Institute preprint no. 229 (Moscow, 1981)
[17] J.D. Barrow and M.S. Turner, Nature 292 (1981) 35.