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A NOTE ON R-PARITY VIOLATION AND FERMION MASSES

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Abstract

We consider a class of supersymmetric $SU(3)\times SU(2)\times U(1)$ multihiggs models in which R-parity is violated through bilinear Higgs-lepton interactions. The required, due to R-parity violation, higgs-lepton rotations introduce an alternative way to generate the phenomenologically desirable fermion mass matrix structures independently of the equality of Yukawas, possibly imposed by superstring or other unification.

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One of the many consequences of modern unified theories are the relations they imply among Yukawa couplings. Such relations exist in GUT models[1] as well as in models derived in the context of Superstring Theory[2]. These relations reduce the number of free parameters required to fit the fermion masses and mixing angles in the framework of the Standard Model. The theoretical and phenomenological success of unification ideas in explaining certain parameters of the Standard Model has made the *supersymmetric unification*[3][4] framework very attractive despite the fact that some of the predicted mass relations are not automatically successful in a minimal context. In SUSY models certain Ansätze incorporating "texture" zeroes at superlarge energies have been proven to be an effective method to explain fermion masses while reducing the number of free parameters [5].

Simple $SU(5)$ requires at the unification scale $\lambda_b = \lambda_\tau$. This leads to the experimentally observed m_b/m_τ mass ratio [6]. Nevertheless, the identification of the down and lepton Yukawa couplings at high energies does not lead to the correct prediction for the lightest generation mass ratios. A solution to this problem was proposed initially by Georgi and Jarlskog [7]. Ramond, Roberts and Ross [8] performed a systematic search for symmetric mass matrices with a maximum number of texture zeroes and were led to several satisfactory Ansätze. In these proposals the fermion mass ratios are brought in agreement with the experimental data by taking the lepton Yukawa matrix to be almost identical to the down Yukawa matrix apart from a factor of three in (2,2) entry. Several attempts have been made to incorporate such Ansätze and explain the down/lepton relative factor of three in $SO(10)$ and $SU(5)$ models introducing extra Higgses in various representations [9]. Here we present an alternative approach in which the relative factor of three in the lepton matrix is generated as a consequence of field redefinitions required by the presence of interactions that violate R-parity.

In the five solutions presented in ref. [8] the down quark Yukawa matrix takes the following form

$$Y_d = \begin{pmatrix} 0 & F & 0 \\ F^* & E & E' \\ 0 & E' & D \end{pmatrix} \tag{1}$$

Where E' can be taken either as 0 or of the same order of magnitude as E , depending on the choice of up-quark Yukawa matrix. The lepton Yukawa matrix is taken to be

$$Y_e = \begin{pmatrix} 0 & F & 0 \\ F^* & 3E & E' \\ 0 & E' & D \end{pmatrix} \tag{2}$$

The elements of the above matrix obey the approximate relations

$$\frac{F}{E} = \lambda \qquad \frac{E}{D} = 2\lambda^3 \qquad E \approx E' \tag{3}$$

$\lambda \approx .22$ stands for the *Cabbibo angle.*

Since $D >> E, E'$, the diagonalization of the matrix corresponding to the first two generations gives a ratio of masses in good agreement with the experimental data

$$\frac{m_d}{m_s} = 9\frac{m_e}{m_\mu} \tag{4}$$

Note that for this kind of textures proposed in ref. [8] the masses of the first and second generations are independent of the third generation Yukawa. In what follows we shall consider a two generation model in which down quark and lepton masses are described by matrices

$$\mathcal{D} = \begin{pmatrix} 0 & F \\ F & E \end{pmatrix} \qquad \mathcal{E} = \begin{pmatrix} 0 & F \\ F & -3E \end{pmatrix} \tag{5}$$

All the above parameters are taken to be real.

Let us now consider a two generation model with two pairs of Higgs isodoublets. The down quark and lepton Yukawas are equal as is the case in a large class of unified models. R-parity is broken in this model by bilinear terms in the superpotential

$$W = \mu_i h_i h_i^c + \xi_{ij} h_i^c l_j + Y_{ijk}\left(q_i d_j^c + l_i e_j^c\right) h_k \tag{6}$$

Where $i, j, k = 1, 2$. We shall take the ξ -matrix to be

$$\xi_{ij} = \begin{pmatrix} 0 & \xi_2 \\ \xi_1 & 0 \end{pmatrix} \tag{7}$$

At this point let us introduce the angles

$$\sin\theta_1 = \frac{\xi_2}{\sqrt{\mu_1^2+\xi_2^2}} \quad \sin\theta_2 = \frac{\xi_1}{\sqrt{\mu_2^2+\xi_1^2}} \tag{8}$$

and the *Higgs mass-eigenstate fields*

$$H_1 = \cos\theta_1 h_1 + \sin\theta_1 l_2 \tag{9}$$

$$H_2 = \cos\theta_2 h_2 + \sin\theta_2 l_1 \tag{10}$$

$$L_1 = -\sin\theta_1 h_1 + \cos\theta_1 l_2 \tag{11}$$

$$L_2 = -\sin\theta_2 h_2 + \cos\theta_2 l_1 \tag{12}$$

Since our model serves only a demonstrative purpose it is not restrictive to assume that the parameters μ_i and ξ_i are of the same order of magnitude M_W and chose angles $\theta_1 \approx \theta_2 = \theta$. Note that for $\xi_1 = \xi_2 = \mu_1 = \mu_2$, $\theta = \pi/4$. The superpotential becomes

$$\begin{aligned} W &= \sqrt{\mu_1^2+\xi_2^2} H_1 h_1^c + \sqrt{\mu_2^2+\xi_1^2} H_2 h_2^c + Y_{ijk}\cos\theta H_k q_i d_j^c \\ &+ \left[Y_{2j1}\cos 2\theta H_1 + \left(Y_{2j2}\cos^2\theta - Y_{1j1}\sin^2\theta\right) H_2\right] L_1 e_j^c \\ &+ \left[\left(Y_{1j1}\cos^2\theta - Y_{2j2}\sin^2\theta\right) H_1 + Y_{1j2}\cos 2\theta H_2\right] L_2 e_j^c + W_{\Delta R} \end{aligned} \tag{13}$$

$W_{\Delta R}$ includes R-parity violating terms:

$$W_{\Delta R} = -sin\theta Y_{ijk} q_i d_j^c L_k + sin\theta cos\theta (Y_{1j1} + Y_{2j2})(H_1 H_2 e_j^c - L_1 L_2 e_j^c) \tag{14}$$

R-parity violating terms of the type qd^cL , although phenomenologically chalenging, are not dangerous in themselves for proton decay, provided no bare terms of the type $u^cd^cd^c$ exist. In an $SU(5)$ version of the Lagrangian (6) terms like $\xi_{ij} D_{Hi} d_j^c + Y_{ijk}(u_i^c d_j^c \overline{D}_{Hk} + q_i l_j \overline{D}_{Hk})$ should be present arising from $\overline{\mathbf{5}}\ \mathbf{5}_H + \mathbf{10}\ \overline{\mathbf{5}}\ \overline{\mathbf{5}}_H$, D_H and $\overline{D}_H$ denote the color triplets contained in $\mathbf{5}_H$ and $\overline{\mathbf{5}}_H$ respectively . Models of this kind have been analyzed in [10]. In contrast, $H_1 H_2 e_j^c$ terms are dangerous and should not be present. Note however that due to $SU(2)$ antisymmetry these terms are not present in a one-Higgs model. Considering the vev's

$$\langle H_1 \rangle = v_1 \quad \langle H_2 \rangle = v_2 \tag{15}$$

and defining

$$v_1\ Y_{ij1} = Y_{ij} \quad v_2\ Y_{ij2} = I_{ij} \quad \alpha = \frac{v_2}{v_1} \tag{16}$$

we see that these terms are eliminated if the following conditions are met

$$\begin{aligned} \alpha Y_{11} + I_{21} &= 0 \\ \alpha Y_{12} + I_{22} &= 0 \end{aligned} \tag{17}$$

In the case $\xi = 0$, or equivalently for $\sin\theta = 0$ and $L_1 = l_2, L_2 = l_1$ the down quark and lepton masses are given by

$$M^d = M^e = Y_{ij} + I_{ij} \tag{18}$$

For non-zero ξ these matrices are modified to

$$M^d_{ij} = \cos\theta\,(Y_{ij} + I_{ij}) \tag{19}$$

and

$$M^e = \begin{pmatrix} \cos 2\theta Y_{21} - \sin^2\theta\alpha Y_{11} + \cos^2\theta I_{21} & \cos 2\theta Y_{11} - \sin^2\theta\alpha Y_{12} + \cos^2\theta I_{22} \\ \cos^2\theta Y_{11} - \frac{\sin^2\theta}{\alpha} I_{21} + \cos 2\theta I_{11} & \cos^2\theta Y_{12} - \frac{\sin^2\theta}{\alpha} I_{22} + \cos 2\theta I_{12} \end{pmatrix} \tag{20}$$

Remember that the desired structure of the fermion mass matrices is

$$\mathcal{D} = \begin{pmatrix} 0 & F \\ F & E \end{pmatrix} \quad \mathcal{E} = \begin{pmatrix} 0 & F \\ F & -3E \end{pmatrix} \tag{21}$$

In order to match that by the derived mass matrices we must have four equations from the down quark mass

$$M^d_{ij} = \mathcal{D}_{ij} = \cos\theta(Y_{ij} + I_{ij}) \tag{22}$$

plus four additional equations derived from the lepton mass matrix

$$\cos 2\theta Y_{21} - \alpha\sin^2\theta Y_{11} + \cos^2\theta I_{21} = 0 \tag{23}$$

$$\cos 2\theta Y_{22} - \alpha\sin^2\theta Y_{12} + \cos^2\theta I_{22} = F \tag{24}$$

$$\cos^2\theta Y_{11} - \frac{\sin^2\theta}{\alpha} I_{21} + \cos 2\theta I_{11} = F \tag{25}$$

$$\cos^2\theta Y_{12} - \frac{\sin^2\theta}{\alpha} I_{22} + \cos 2\theta I_{12} = -3E. \tag{26}$$

Note that

$$I_{ij} = \frac{\mathcal{D}_{ij}}{\cos\theta} - Y_{ij} \tag{27}$$

After inserting this result in the previous equations, we find *two compatibility conditions*

$$\cos 2\theta = \alpha \cos\theta \tag{28}$$

$$F = \frac{4\alpha}{1-\alpha^2} E \tag{29}$$

The deduced matrix elements of Y, after imposing conditions (17) are:

$$\begin{aligned} Y_{11} &= \frac{\cos 2\theta}{\alpha \sin\theta \sin 2\theta} F && (30) \\ Y_{12} &= \frac{1}{\alpha \sin\theta} \left(\frac{\cos 2\theta}{\sin 2\theta} E - \frac{F}{2\sin\theta} \right) && (31) \\ Y_{21} &= \frac{F}{\sin\theta \sin 2\theta} && (32) \\ Y_{22} &= \frac{E - \cos\theta F}{\sin\theta \sin 2\theta} && (33) \\ & && (34) \end{aligned}$$

Note that from experiment the ratio F/E has to be

$$F/E = \lambda \approx .22 \tag{35}$$

The compatibility condition (29) led to a *Cabbibo hierarchy* for the vev ratio

$$\alpha = \frac{v_2}{v_1} \approx \frac{\lambda}{4} \approx .055 \tag{36}$$

Hence the condition (28) is satisfied for an angle

$$\theta = \frac{\pi}{4} - \epsilon; \tag{37}$$

with

$$\epsilon = \frac{\sqrt{2}}{4}\alpha + \frac{\alpha^2}{8} + \mathcal{O}(\alpha^3) \tag{38}$$

Therefore, the matrices Y and I turn out to be, to order α^2

$$Y = \begin{pmatrix} F & -3E - \frac{3\sqrt{2}}{8}F \\ \sqrt{2}F & \sqrt{2}E - \frac{7}{8}F \end{pmatrix} \qquad I = \begin{pmatrix} -F & 3E + \frac{11\sqrt{2}}{8}F \\ 0 & \frac{3}{4}F \end{pmatrix} \tag{39}$$

Finally, we obtain, up to a multiplicative factor

$$Y = \begin{pmatrix} \lambda & -3 - \frac{3\sqrt{2}}{8}\lambda \\ \sqrt{2}\lambda & \sqrt{2} - \frac{7}{8}\lambda \end{pmatrix} \qquad I = \begin{pmatrix} -\lambda & 3 + \frac{11\sqrt{2}}{8}\lambda \\ 0 & \frac{3}{4}\lambda \end{pmatrix} \tag{40}$$

The model analyzed serves as an existence proof of an alternative possible mechanism responsible for the structure of fermion mass matrices. Instead of inducing a disparity between Yukawa couplings due to the presence of Higgs multiplets with different couplings due to their different representations, the possibility explored in this note indicates that the phenomenologically required differentiation is achieved due to the necessary rotation in the *(Higgs isodoublet)- (left handed lepton doublet)* space induced by R-parity violation. Independently of the features of the particular example employed here it should be stressed that the circumstances suitable for this mechanism to operate could most naturally arise in an effective $SU(3) \times SU(2) \times U(1)$ theory resulting from Superstrings. Since higher Higgs representations are notoriously difficult to arise in Superstring embedable models and since the equality of quark and lepton Yukawas is required by string unification at the string scale without the presence of any extra gauge symmetry, R-parity violation through the bilinear Higgs-lepton couplings seems like an interesting possibility.

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