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On the neutral scalar sector of the general R-parity violating MSSM

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Abstract

Starting out from the most general, gauge invariant and renormalisable scalar potential of the R-parity violating MSSM and performing a calculable rotation to the scalar fields we arrive at a basis where the sneutrino VEVs are zero. The advantage of our rotation is that, in addition, we obtain diagonal soft supersymmetry breaking sneutrino masses and all potential parameters and VEVs real, proving that the MSSM scalar potential does not exhibit spontaneous or explicit CP-violation at tree level. The model has five CP-even and four CP-odd physical neutral scalars, with at least one CP-even scalar lighter than M_Z. We parametrise the neutral scalar sector in a way that resembles the parametrisation of the R-parity conserving MSSM, analyse its mass spectrum, the coupling to the gauge sector and the stability of the potential.

1. Introduction

That none of the terms in the Standard Model (SM) violate lepton number (L) is not due to an imposed symmetry, but merely reflects the fact that all such combinations of SM fields are ruled out by consideration of gauge invariance and renormalisability [1]. For supersymmetric extensions of the SM this is no longer true. In the Minimal Supersymmetric Standard Model (MSSM) [2], lepton number violating terms (and baryon number (B) violating terms) appear naturally, giving rise to tree-level processes, proton decay for example, which are already strongly constrained by experiment. Either, bounds can be set on Lagrangian parameters, or a further discrete symmetry can be imposed on the Lagrangian, such that these processes are absent. The discrete symmetry most commonly imposed is known as R-parity (R_P) [3,4]. Under R-parity the particles of the Standard Model including the scalar Higgs fields are even, while all their superpartners are odd. Imposing this symmetry has a number of effects.

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Firstly, any interaction terms which violate lepton number or baryon number will not appear. Secondly, the decay of the lightest supersymmetric particle (LSP) into SM particles would violate R_P; the LSP is therefore stable. The sneutrino vacuum expectation values (VEVs) are zero; without extending the MSSM field content, spontaneous generation of R_P violating terms is phenomenologically discounted [5].

If R_P conservation is not imposed, fields with different R_P mix [6–12]. In particular, the neutrinos will mix with the neutralinos and the sneutrinos will mix with the neutral scalar Higgs fields; all five complex neutral scalar fields can acquire vacuum expectation values. Minimising this ten parameter potential in general is not straightforward, it is more convenient to simplify the system by choosing an appropriate basis in the neutral scalar sector. As one of the Higgs doublets carries the same quantum numbers as the lepton doublets (apart from the non-conserved lepton number), it is convenient to introduce the notation $\mathcal{L}_\alpha = (H_1, L_i)$ where H_1 and L_i are the chiral superfields containing one Higgs doublet and the leptons, respectively ($\alpha = 0, \dots, 3$ and $i = 1, \dots, 3$). Furthermore, starting from the interaction basis, we are free to rotate the fields and choose the direction corresponding to that of the "Higgs" field. Assuming that the system defining the five complex vacuum expectation values of the fields was solved, four complex VEVs v_α would define a direction in the four-dimensional (H_1, L_i) space. One can then choose the basis vector which defines the Higgs fields to point in the direction defined by the vacuum expectation values. We refer to this basis, in which the "sneutrino" (as we call the fields perpendicular to the "Higgs" field) VEVs are zero, as the vanishing sneutrino VEV basis [13–15]. This basis has the virtue of simplifying the mass matrices and vertices of the theory and thus is better suited for calculations.

Basis independent parameterisations can be chosen which explicitly show the amount of physical lepton number violation [16–18]. Values for physical observables such as sneutrino masses and mass splitting between CP-even and CP-odd sneutrinos have been derived in the literature in terms of these combinations but usually under some approximations (for example the number of generations or CP-conservation). We find this procedure in general complicated for practical applications and we shall not adopt it here.

Instead, we present in the next section a calculable procedure for framing the most general MSSM scalar potential in the vanishing sneutrino VEV basis. An advantage of our procedure is to obtain a diagonal "slepton" mass matrix, two real non-zero vacuum expectation values and real parameters of the neutral scalar potential in the rotated basis. The latter proves that the neutral scalar sector of the most general R-parity violating MSSM exhibits neither spontaneous nor explicit CP-violation in agreement with [19]. In Section 3, the tree-level masses and mixing of the neutral scalar sector is investigated. Using the Courant–Fischer theorem for the interlaced eigenvalues, we prove that there is always at least one neutral scalar which is lighter than the Z-gauge boson. We present approximate formulae which relate the Higgs masses, mixing and Higgs-gauge boson vertices of the R-parity conserving (RPC) case with the R-parity violating (RPV) one. In Section 4, the positiveness of the scalar mass matrices and stability of the vacuum is discussed.

2. Basis choice in the neutral scalar sector

In this section we develop a procedure connecting a general neutral scalar basis with the vanishing sneutrino VEV basis, the latter being more convenient for practical applications. The most general, renormalisable, gauge invariant superpotential that contains the minimal content of fields, is given by

$$\mathcal{W} = \epsilon_{ab}\left[\frac{1}{2}\lambda_{\alpha\beta k}\mathcal{L}^a_\alpha \mathcal{L}^b_\beta \bar{E}_k + \lambda'_{\alpha jk}\mathcal{L}^a_\alpha Q^{bx}_j \bar{D}_{kx} - \mu_\alpha \mathcal{L}^a_\alpha H^b_2 + (Y_U)_{ij} Q^{ax}_i H^b_2 \bar{U}_{jx}\right] + \frac{1}{2}\epsilon_{xyz}\lambda''_{ijk}\bar{U}^x_i \bar{D}^y_j \bar{D}^z_k, \tag{2.1}$$

where $Q^{ax}_i, \bar{D}^x_i, \bar{U}^x_i, \mathcal{L}^a_i, \bar{E}_i, H^a_1, H^a_2$ are the chiral superfield particle content, $i = 1, 2, 3$ is a generation index, $x = 1, 2, 3$ and $a = 1, 2$ are $SU(3)$ and $SU(2)$ gauge indices, respectively. The simple form of (2.1) results when combining the chiral doublet superfields with common hypercharge $Y = -\frac{1}{2}$ into $\mathcal{L}^a_{\alpha=0,\dots,3} = (H^a_1, L^a_{i=1,2,3})$. μ_α is the generalised dimensionful μ-parameter, and $\lambda_{\alpha\beta k}, \lambda'_{\alpha jk}, \lambda''_{ijk}, (Y_U)_{ij}$ are Yukawa matrices with ϵ_{ab} and ϵ_{xyz}

being the totally anti-symmetric tensors, with $\epsilon_{12} = \epsilon_{123} = +1$. Then the five neutral scalar fields, $\tilde{\nu}_{L\alpha}, h_2^0$ from the $SU(2)$ doublets, $\mathcal{L}_\alpha = (\tilde{\nu}_{L\alpha}, \tilde{e}^-_{L\alpha})^T$ and $H_2 = (h_2^+, h_2^0)^T$, form the most general neutral scalar potential of the MSSM,

$$V_{\text{neutral}} = \left(m^2_{\tilde{\mathcal{L}}}\right)_{\alpha\beta} \tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\beta} + \mu^*_\alpha\mu_\beta\tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\beta} + \mu^*_\alpha\mu_\alpha h_2^{0*}h_2^0 + m^2_{H_2}h_2^{0*}h_2^0 - b_\alpha\tilde{\nu}_{L\alpha}h_2^0 - b^*_\alpha\tilde{\nu}^*_{L\alpha}h_2^{0*} + \frac{1}{8}\left(g^2+g_2^2\right)\left[h_2^{0*}h_2^0 - \tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\alpha}\right]^2, \tag{2.2}$$

where general complex parameters b_α, an hermitian matrix $(m^2_{\tilde{\mathcal{L}}})_{\alpha\beta}$ and $m^2_{H_2}$ all arise from the supersymmetry breaking sector of the theory. The last term in (2.2) originates from the D-term contributions of the superfields $\mathcal{L}_\alpha, H_2$. Defining

$$\left(\mathcal{M}^2_{\tilde{\mathcal{L}}}\right)_{\alpha\beta} \equiv \left(m^2_{\tilde{\mathcal{L}}}\right)_{\alpha\beta} + \mu^*_\alpha\mu_\beta, \quad \text{and} \quad m_2^2 \equiv m^2_{H_2} + \mu^*_\alpha\mu_\alpha, \tag{2.3}$$

one can rewrite the potential in (2.2) in a compact form as

$$V_{\text{neutral}} = \left(\mathcal{M}^2_{\tilde{\mathcal{L}}}\right)_{\alpha\beta}\tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\beta} + m_2^2 h_2^{0*}h_2^0 - \left(b_\alpha\tilde{\nu}_{L\alpha}h_2^0 + \text{H.c.}\right) + \frac{1}{8}\left(g^2+g_2^2\right)\left[h_2^{0*}h_2^0 - \tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\alpha}\right]^2. \tag{2.4}$$

In order to go to the vanishing sneutrino VEV basis, we redefine the “Higgs-sneutrino” fields

$$\tilde{\nu}_{L\alpha} = U_{\alpha\beta}\tilde{\nu}'_{L\beta}, \tag{2.5}$$

where $\mathbf{U}$ is a 4×4 unitary matrix

$$\mathbf{U} = \mathbf{V}\,\text{diag}\left(e^{i\phi_\alpha}\right)\mathbf{Z}, \tag{2.6}$$

being composed of three other matrices which we define below, $\mathbf{V}$ unitary and $\mathbf{Z}$ real orthogonal. The potential in the primed basis becomes,

$$V_{\text{neutral}} = \left[\mathbf{Z}^T\left(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}}\right)\mathbf{Z}\right]_{\alpha\beta}\tilde{\nu}'^*_{L\alpha}\tilde{\nu}'_{L\beta} + m_2^2 h_2^{0*}h_2^0 - \left[(b'\mathbf{Z})_\alpha\tilde{\nu}'_{L\alpha}h_2^0 + \text{H.c.}\right] + \frac{1}{8}\left(g^2+g_2^2\right)\left(h_2^{0*}h_2^0 - \tilde{\nu}'^*_{L\alpha}\tilde{\nu}'_{L\alpha}\right)^2, \tag{2.7}$$

where

$$\left(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}}\right) = \text{diag}\left(e^{-i\phi_\alpha}\right)\mathbf{V}^\dagger\left(\mathcal{M}^2_{\tilde{\mathcal{L}}}\right)\mathbf{V}\,\text{diag}\left(e^{i\phi_\alpha}\right), \qquad b'^T = b^T\mathbf{V}\,\text{diag}\left(e^{i\phi_\alpha}\right). \tag{2.8}$$

The unitary matrix, $\mathbf{V}$, is chosen such that $(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}})$ is real and diagonal—the hat ($\hat{\ }$) is used to denote a diagonal matrix. The phases ϕ_α are chosen such that b'_α is real and positive [they are equal to the phases of $(b^T\mathbf{V})^*_\alpha$]. The minimisation conditions for the scalar fields are now derived, to obtain conditions for the vacuum expectation values,

$$\begin{aligned}\left.\frac{\partial V}{\partial\tilde{\nu}'^*_{L\alpha}}\right|_{\text{vac}} &= \left[\mathbf{Z}^T\left(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}}\right)\mathbf{Z}\right]_{\alpha\beta}\tilde{\nu}'_{L\beta} - (b'\mathbf{Z})_\alpha h_2^{0*} - \frac{1}{4}\left(g^2+g_2^2\right)\left(h_2^{0*}h_2^0 - \tilde{\nu}'^*_{L\gamma}\tilde{\nu}'_{L\gamma}\right)\tilde{\nu}'_{L\alpha}\Big|_{\text{vac}} = 0,\\ \left.\frac{\partial V}{\partial h_2^{0*}}\right|_{\text{vac}} &= m_2^2 h_2^0 - (b'\mathbf{Z})_\alpha\tilde{\nu}'^*_{L\alpha} + \frac{1}{4}\left(g^2+g_2^2\right)\left(h_2^{0*}h_2^0 - \tilde{\nu}'^*_{L\gamma}\tilde{\nu}'_{L\gamma}\right)h_2^0\Big|_{\text{vac}} = 0,\end{aligned} \tag{2.9}$$

where “vac” indicates that the fields have to be replaced by their VEVs,

$$\langle\tilde{\nu}'_{L\alpha}\rangle = \frac{v_\alpha}{\sqrt{2}}, \qquad \left\langle h_2^0\right\rangle = \frac{v_u}{\sqrt{2}}. \tag{2.10}$$

The $U(1)_Y$ symmetry of the unbroken Lagrangian was used to set the phase of v_u to zero, however, at this stage all other vacuum expectation values will be treated as complex variables. By combining Eqs. (2.9), (2.10) we obtain

$$\left[\mathbf{Z}^T\left(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}}\right)\mathbf{Z}\right]_{\alpha\beta}v_\beta - (b'\mathbf{Z})_\alpha v_u - \frac{1}{8}\left(g^2+g_2^2\right)\left(v_u^2 - v^*_\gamma v_\gamma\right)v_\alpha = 0, \tag{2.11}$$

$$m_2^2 v_u - (b'\mathbf{Z})_\alpha v_\alpha^* + \frac{1}{8}(g^2 + g_2^2)(v_u^2 - v_\gamma^* v_\gamma)v_u = 0. \tag{2.12}$$

In a general basis, it is difficult to solve the above system with respect to the VEVs without making some approximations, for example assuming small "sneutrino" VEVs [11]. In order to simplify calculations we would like to find a basis where the "sneutrino" VEVs vanish, $v_1 = v_2 = v_3 = 0$. In other words, we are seeking an orthogonal matrix $\mathbf{Z}$, such that the following equation,

$$\left[\mathbf{Z}^T (\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}})\mathbf{Z}\right]_{\alpha 0} v_0 - (b'\mathbf{Z})_\alpha v_u - \frac{1}{2} M_Z^2 \frac{v_u^2 - v_0^2}{v_u^2 + v_0^2} v_0 \delta_{0\alpha} = 0, \tag{2.13}$$

holds. If the above system is satisfied, then a solution with zero "sneutrino" VEVs exists. The other solutions, with non-vanishing "sneutrino" VEVs will be discussed later. In Eq. (2.13),

$$M_Z^2 = \frac{1}{4}(g^2 + g_2^2)(v_u^2 + v_0^2), \tag{2.14}$$

is the Z-gauge boson mass squared. It is obvious that when $v_i = 0$, v_0 is real. It is now useful to define

$$\tan\beta \equiv \frac{v_u}{v_0}. \tag{2.15}$$

To determine $\mathbf{Z}$, multiplying (2.13) by $Z_{\gamma\alpha}$, summing over α and solving for $Z_{\alpha 0}$, yields,

$$Z_{\alpha 0} = \frac{b'_\alpha \tan\beta}{(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}})_{\alpha\alpha} - \frac{1}{2} M_Z^2 \frac{\tan^2\beta - 1}{\tan^2\beta + 1}}. \tag{2.16}$$

For given set of model parameters, $Z_{\alpha 0}$ depends only on $\tan\beta$ which we can now fix by solving the orthonormality condition,

$$\sum_{\alpha=0}^{3} Z_{\alpha 0} Z_{\alpha 0} = \sum_{\alpha=0}^{3} \frac{b'^2_\alpha \tan^2\beta}{[(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}})_{\alpha\alpha} - \frac{1}{2} M_Z^2 \frac{\tan^2\beta - 1}{\tan^2\beta + 1}]^2} = 1. \tag{2.17}$$

This equation can be easily be solved numerically for any given set of model parameters.

It is worth noting that when $b_i = 0$ and using notation more typical for this case, $b'_0 \equiv m_{12}^2$, $(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}})_{00} \equiv m_1^2$, Eq. (2.17) reduces to one of the standard RPC MSSM equations for the Higgs VEVs:

$$m_{12}^2 v_d = v_u \left[m_1^2 - \frac{1}{8}(g^2 + g_2^2)(v_u^2 - v_d^2) \right]. \tag{2.18}$$

For some parameter choices Eq. (2.17) may admit multiple solutions for $\tan\beta$. Each of the possible $\tan\beta$ specify a different basis, and each of these bases has one solution of the minimisation conditions with vanishing "sneutrino" VEVs. The subtlety highlighted earlier is the following: all possible solutions of the minimisation conditions can be found in each basis, so, in general, each basis contains a number of extrema equal to the number of possible solutions for $\tan\beta$. Hence, a solution with $v_i = 0$ in one basis, is a solution with $v_i \neq 0$ in another basis. The important point to note is that by considering all possible values of $\tan\beta$, and selecting the value which corresponds to the deepest minima for the solution with vanishing sneutrino VEVs, all the solutions will have been accounted for, and the vanishing sneutrino VEV basis will have been determined correctly. The value of the potential at the vacuum, in terms of $\tan\beta$ is given by

$$V(\tan\beta) = -\frac{M_Z^4}{2(g^2 + g_2^2)} \left(\frac{\tan^2\beta - 1}{\tan^2\beta + 1} \right)^2. \tag{2.19}$$

The obvious conclusion from the equation above is that the deepest minimum of the potential is given by the solution for $\tan\beta$ or $\cot\beta$ which is greatest.

Knowing $\tan\beta$, one should fix m_2^2 using Eqs. (2.12), (2.14)–(2.16) (again in the analogy with RPC MSSM where m_2^2 is usually given in terms of $M_A, \tan\beta$). Namely

$$m_2^2 = Z_{\alpha 0} b'_\alpha \cot\beta - \frac{1}{2} M_Z^2 \frac{\tan^2\beta - 1}{\tan^2\beta + 1}. \tag{2.20}$$

In this way m_2^2 is chosen to give the correct value of the Z-boson mass.

Only the first column of the $\mathbf{Z}$ matrix, $Z_{\alpha 0}$, is defined by Eq. (2.16). The remaining elements of $\mathbf{Z}$ must still be determined. Having fixed the first column of the matrix, the other three columns can be chosen to be orthogonal to the first column and to each other. This leaves us with an $O(3)$ invariant subspace, such that the matrix $\mathbf{Z}$ is given by

$$\mathbf{Z} = \mathbf{O} \begin{pmatrix} 1 & 0 \\ 0 & \mathbf{X}_{3\times 3} \end{pmatrix}, \tag{2.21}$$

where

$$\mathbf{O} = \begin{pmatrix} Z_{00} & -\sqrt{Z_{10}^2 + Z_{20}^2 + Z_{30}^3} & 0 & 0 \\ Z_{10} & \frac{Z_{00} Z_{10}}{\sqrt{Z_{10}^2+Z_{20}^2+Z_{30}^3}} & -\frac{\sqrt{Z_{20}^2+Z_{30}^3}}{\sqrt{Z_{10}^2+Z_{20}^2+Z_{30}^3}} & 0 \\ Z_{20} & \frac{Z_{00} Z_{20}}{\sqrt{Z_{10}^2+Z_{20}^2+Z_{30}^3}} & \frac{Z_{10} Z_{20}}{\sqrt{Z_{20}^2+Z_{30}^3}\sqrt{Z_{10}^2+Z_{20}^2+Z_{30}^3}} & -\frac{Z_{30}}{\sqrt{Z_{20}^2+Z_{30}^3}} \\ Z_{30} & \frac{Z_{00} Z_{30}}{\sqrt{Z_{10}^2+Z_{20}^2+Z_{30}^3}} & \frac{Z_{10} Z_{30}}{\sqrt{Z_{20}^2+Z_{30}^3}\sqrt{Z_{10}^2+Z_{20}^2+Z_{30}^3}} & \frac{Z_{20}}{\sqrt{Z_{20}^2+Z_{30}^3}} \end{pmatrix}, \tag{2.22}$$

and $\mathbf{X}$ is an, as yet, undetermined 3×3 orthogonal matrix determined by three angles. This remaining freedom can be used to diagonalise $[\mathbf{Z}^T(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}})\mathbf{Z}]_{ij}$, i.e. the (real symmetric) "sneutrino" part of the $\mathbf{Z}^T(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}})\mathbf{Z}$ matrix, with entries $(\hat{M}^2_{\tilde{L}})_i$. We have now accomplished our aim of finding the matrices $\mathbf{V}$ and $\mathbf{Z}$ which, after inserting into potential of Eq. (2.7) and dropping the primes, reduce the scalar potential to the form

$$V_{\text{neutral}} = \left(M^2_{\tilde{\mathrm{L}}}\right)_{\alpha\beta} \tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\beta} + m_2^2 h_2^{0*} h_2^0 - \left[B_\alpha \tilde{\nu}_{L\alpha} h_2^0 + \text{H.c.}\right] + \frac{1}{8}\left(g^2 + g_2^2\right)\left(h_2^{0*}h_2^0 - \tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\alpha}\right)^2, \tag{2.23}$$

where

$$\left(M^2_{\tilde{\mathrm{L}}}\right)_{\alpha\beta} \equiv \left[\mathbf{Z}^T\left(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}}\right)\mathbf{Z}\right]_{\alpha\beta} \quad \text{and} \quad B_\alpha \equiv (b'\mathbf{Z})_\alpha, \tag{2.24}$$

with $(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}})$ and b' given by Eq. (2.8). In this basis the matrix $\mathbf{M}^2_{\tilde{\mathrm{L}}}$ adopts a particularly simple form

$$\left(M^2_{\tilde{\mathrm{L}}}\right)_{\alpha\beta} = \begin{pmatrix} B_0 \tan\beta - \frac{1}{2} M_Z^2 \cos 2\beta & B_j \tan\beta \\ B_i \tan\beta & (\hat{M}^2_{\tilde{\mathrm{L}}})_i \delta_{ij} \end{pmatrix}, \tag{2.25}$$

where there is no sum over i in the down-right part of the matrix. Notice that we did not only succeed to self consistently go to a basis where the sneutrino VEVs are zero, but also we managed to have the sneutrino masses $(\hat{M}^2_{\tilde{\mathrm{L}}})_i$ diagonal and all the parameters of the scalar potential in Eq. (2.23) real.

As a byproduct of our procedure, we denote here that the potential of Eq. (2.23) exhibits neither spontaneous nor explicit CP-violation at the tree level. The latter is in agreement with the results of Ref. [19] following a different method. Of course, the parameters μ_α of the superpotential and the soft supersymmetry breaking couplings stay in general complex. The result that the neutral scalar potential is CP invariant can also be seen directly from Eq. (2.4). By forming the complex basis $(\tilde{\nu}_{L\alpha}, h_2^{0*})$ the first line of the potential can be rewritten as a matrix; a rotation can then be performed such that the matrix is real and diagonal. After the rotation, the second line, being the contribution from D-terms, contains complex parameters in general, but the rotation matrix can be chosen such that these phases are set to zero.

A question arises when we include high order corrections to the potential. Then the vanishing "sneutrino" VEVs will be shifted to non-zero values by tadpoles originating, for example, from the $\mathcal{L}QD$ contribution in the superpotential (2.1). The "sneutrino" VEVs maybe set back to zero by a renormalisation condition such that a counterterm for these VEVs set their one particle irreducible (1PI) tadpole corrections to zero.

To conclude, it is worth making a remark about the sign of B_0. As is clear from the form of Eqs. (2.24), (2.8), (2.16), if $(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}})_{\alpha\alpha} - \frac{1}{2}M_Z^2\frac{\tan^2\beta-1}{\tan^2\beta+1} > 0$ for all α, B_0 is always positive in the vanishing sneutrino VEV basis.

3. Parametrising the neutral scalar mass matrices

The neutral scalar sector of the R-parity violating MSSM is in general very complicated. This is due to the fact that the scalars mix through the lepton number violating terms proportional to B_i, M^2_{L} and unless all of these parameters and VEVs are real one has a 10×10 matrix to consider. However, for any given set of model parameters, one can always perform the basis change described in the previous section and arrive to the potential defined by Eq. (2.23), with only real parameters. Consequently, the physical neutral scalars are, at the tree level, exact CP-eigenstates. This implies that the neutral scalar mass matrix decouples into two 5×5 matrices, one for the CP-odd particles and one for CP-even. In the same manner as in the R-parity conserving MSSM, once quantum corrections are considered, the CP invariance will generically be broken [20].

Ultimately, one would like to parametrise the scalar sector resulting from the potential in (2.23) with as few parameters as possible in order to make contact with phenomenology. These parameters in the case of the R-parity conserving MSSM are: the physical mass of the CP-odd Higgs boson

$$M_A^2 = \frac{2B_0}{\sin 2\beta}, \tag{3.1}$$

and $\tan\beta$. An advantage of the form of potential in Eqs. (2.23)–(2.25) is that, M_A and $\tan\beta$ can still be used for parameterising the general Higgs sector in the R-parity violating MSSM. M_A^2 is the mass of the lightest CP-odd Higgs boson in the R-parity conserving MSSM; as such, it is used here as a parameter. m_A^2 is used to denote the physical tree-level mass of the lightest CP-odd Higgs in the R-parity violating MSSM (the convention adopted is that masses in the RPC case, parameters in this model, are denoted by M, and the masses in the RPV model are denoted by m).

3.1. CP-even neutral scalar masses and couplings

The Lagrangian after spontaneous gauge symmetry breaking contains the terms

$$\mathcal{L} \supset -\big(\mathrm{Re}\, h_0^2 \quad \mathrm{Re}\,\tilde{\nu}_{L0} \quad \mathrm{Re}\,\tilde{\nu}_{Li}\big)\mathcal{M}^2_{\mathrm{EVEN}}\begin{pmatrix}\mathrm{Re}\, h_0^2\\ \mathrm{Re}\,\tilde{\nu}_{L0}\\ \mathrm{Re}\,\tilde{\nu}_{Lj}\end{pmatrix}. \tag{3.2}$$

As such, the scalar CP-even Higgs squared mass matrix becomes

$$\mathcal{M}^2_{\mathrm{EVEN}} = \begin{pmatrix} \cos^2\beta M_A^2 + \sin^2\beta M_Z^2 & -\frac{1}{2}\sin 2\beta(M_A^2+M_Z^2) & -B_j \\ -\frac{1}{2}\sin 2\beta(M_A^2+M_Z^2) & \sin^2\beta M_A^2 + \cos^2\beta M_Z^2 & B_j\tan\beta \\ -B_i & B_i\tan\beta & M_i^2\delta_{ij} \end{pmatrix}, \tag{3.3}$$

where

$$M_i^2 \equiv \big(\hat{M}^2_{\tilde{\mathrm{L}}}\big)_i + \frac{1}{2}\cos 2\beta M_Z^2, \tag{3.4}$$

are in fact the sneutrino physical masses of the RPC case. It is important here to notice that the top-left 2×2 sub-matrix is identical to the RPC case, for which the Higgs masses are given by

$$M^2_{h,H} = \frac{1}{2}\Big(M^2_Z + M^2_A \pm \sqrt{\big(M^2_Z + M^2_A\big)^2 - 4M^2_A M^2_Z \cos^2 2\beta}\Big), \tag{3.5}$$

and will be used as parameters in the RPV model.

The matrix (3.3) *always* has one eigenvalue which is smaller than M^2_Z. This may be proved as follows: one first observes that the upper left 2×2 submatrix of (3.3), call it A, has at least one eigenvalue smaller than or equal to M^2_Z. Then using the Courant–Fischer theorem [21] of the linear matrix algebra one proves that, for one flavour, the eigenvalues of the 3×3 matrix $\mathcal{M}^2_{\mathrm{EVEN}}$, are interlaced with those of A. This means that the matrix $\mathcal{M}^2_{\mathrm{EVEN}}$ with $i = 1$ has at least one eigenvalue smaller or equal than M^2_Z. Repeating this procedure twice, proves our statement. Furthermore, it is interesting to notice that in the region where $\tan\beta \gg 1$, the eigenvector $(\sin\beta, \cos\beta, 0, 0, 0)^T$ corresponds to the eigenvalue with mass approximately M^2_Z. Notice that this is the same eigenvector as in the RPC case which corresponds to the Higgs boson which couples almost maximally to the Z-gauge boson.

Lepton flavour violating processes have not been observed as yet and therefore, bearing in mind cancellations, the parameters $B_i \tan\beta$ have to be much smaller than $\min(M^2_A, M^2_i)$. To get a rough estimate, consider the dominant contribution from neutral scalars and neutralinos in the loop [22],

$$m_\nu \sim \frac{a_{ew}}{16\pi}\frac{B^2\tan^2\beta}{\tilde{m}^3} \lesssim 1\ \mathrm{eV}, \tag{3.6}$$

with $\tilde{m} = \max(M_A, M_i)$ and $B \sim \mathcal{O}(B_i)$. This shows that

$$\frac{B_i \tan\beta}{\tilde{m}^2} \sim \frac{1.2 \times 10^{-3}}{\sqrt{\tilde{m}}} \sim 0.1\%. \tag{3.7}$$

With this approximation, it is not hard to find a matrix $\mathbf{Z}_R$ which rotates the fields into the mass basis, such that

$$\mathbf{Z}^T_R \mathcal{M}^2_{\mathrm{EVEN}} \mathbf{Z}_R = \mathrm{diag}\big[m^2_{h^0}, m^2_{H^0}, \big(m^2_{\tilde{\nu}_+}\big)_i\big], \tag{3.8}$$

with $m^2_{h^0}$ being the lightest neutral scalar mass and

$$\mathbf{Z}_R \approxeq \begin{pmatrix} \cos\alpha & \sin\alpha & -\frac{\cos(\beta-\alpha)\cos\alpha B_j}{\cos\beta(M_j^2-M_h^2)} + \frac{\sin(\beta-\alpha)\sin\alpha B_j}{\cos\beta(M_j^2-M_H^2)} \\ -\sin\alpha & \cos\alpha & \frac{\cos(\beta-\alpha)\sin\alpha B_j}{\cos\beta(M_j^2-M_h^2)} + \frac{\sin(\beta-\alpha)\cos\alpha B_j}{\cos\beta(M_j^2-M_H^2)} \\ \frac{\cos\beta P_i^h B_i}{\cos(\beta-\alpha)} & \frac{\cos\beta P_i^H B_i}{\sin(\beta-\alpha)} & \delta_{ij} \end{pmatrix}, \tag{3.9}$$

where there is no sum over i and (M^2_j, M^2_h, M^2_H) are defined in (3.4), (3.5). In addition,

$$\tan 2\alpha = \tan 2\beta \frac{M^2_A + M^2_Z}{M^2_A - M^2_Z} \quad \text{and} \quad P^{h,H}_i = \frac{M^2_Z\cos^2 2\beta - M^2_{h,H}}{\cos^2\beta(M^2_H - M^2_h)(M^2_i - M^2_{h,H})} \tag{3.10}$$

(the common convention is to choose $0 \leqslant \beta \leqslant \pi/2$ and $-\pi/2 \leqslant \alpha \leqslant 0$). The mass eigenstates of the RPV model are therefore given by

$$h^0 \simeq \cos\alpha\,\mathrm{Re}\,h^2_0 - \sin\alpha\,\mathrm{Re}\,\tilde{\nu}_{L0} + \left(\frac{\cos\beta P^h_i B_i}{\cos(\beta-\alpha)}\right)\mathrm{Re}\,\tilde{\nu}_{Li},$$

$$H^0 \simeq \sin\alpha\,\mathrm{Re}\,h^2_0 + \cos\alpha\,\mathrm{Re}\,\tilde{\nu}_{L0} + \left(\frac{\cos\beta P^H_i B_i}{\sin(\beta-\alpha)}\right)\mathrm{Re}\,\tilde{\nu}_{Li},$$

$$(\tilde{\nu}_+)_i \simeq \left(-\frac{\cos(\beta-\alpha)\cos\alpha B_j}{\cos\beta(M_j^2-M_h^2)} + \frac{\sin(\beta-\alpha)B_j\sin\alpha B_j}{\cos\beta(M_j^2-M_H^2)}\right)\mathrm{Re}\, h_0^2$$
$$+\left(\frac{\cos(\beta-\alpha)\sin\alpha B_j}{\cos\beta(M_j^2-M_h^2)} + \frac{\sin(\beta-\alpha)\cos\alpha B_j}{\cos\beta(M_j^2-M_H^2)}\right)\mathrm{Re}\,\tilde{\nu}_{L0} + \mathrm{Re}\,\tilde{\nu}_{Li}, \tag{3.11}$$

with corresponding masses,

$$m_{h^0}^2 \simeq M_h^2 - \frac{M_Z^2\cos^2 2\beta - M_h^2}{(M_H^2-M_h^2)\cos^2\beta}\sum_{i=1}^{3}\frac{B_i^2}{M_i^2-M_h^2} + \mathcal{O}\left(\frac{B^4}{M^6\cos^4\beta}\right), \tag{3.12}$$

$$m_{H^0}^2 \simeq M_H^2 + \frac{M_Z^2\cos^2 2\beta - M_H^2}{(M_H^2-M_h^2)\cos^2\beta}\sum_{i=1}^{3}\frac{B_i^2}{M_i^2-M_H^2} + \mathcal{O}\left(\frac{B^4}{M^6\cos^4\beta}\right), \tag{3.13}$$

$$\left(m_{\tilde{\nu}_+}^2\right)_i \simeq \left(\hat{M}_{\tilde{\nu}}^2\right)_i + \frac{B_i^2}{\cos^2\beta}\frac{M_i^2 - M_Z^2\cos^2 2\beta}{[M_i^4 - M_i^2(M_A^2+M_Z^2) + M_A^2 M_Z^2\cos^2 2\beta]} + \mathcal{O}\left(\frac{B^4}{M^6\cos^4\beta}\right). \tag{3.14}$$

The above expressions, are useful in relating the masses of the neutral scalars in the RPC and RPV case in the valid approximation $B\tan\beta \ll \min(M_A^2, M_i^2)$. They are presented here for the first time except the mass in (3.14) which agrees with Ref. [15]. We note here that these formulae are not valid if some of the diagonal entries in the mass matrix are closely degenerated—in such case even small B_i terms lead to the strong mixing of respective fields. However in many types of calculations (e.g. various loop calculations) one can still formally use such expansion—in the final result one often gets expressions of the type $\frac{f(m_1)-f(m_2)}{m_1-m_2}$ which have a well defined and correct limit also for degenerate masses, even if the expansion used in the intermediate steps was, in principle, wrong.

It is interesting to note that the rotation matrix $\mathbf{U}$ defined in (2.6), although explicitly calculated in this Letter, does not appear to all the neutral scalar vertices. For example, the vertices of the CP-even neutral scalars with the gauge bosons read as,[1]

$$\mathcal{L}_{VVH} = \frac{1}{2}\frac{g_2 M_Z}{\cos\theta_w}(\cos\beta Z_{R2s} + \sin\beta Z_{R1s})Z^\mu Z_\mu H_s^0 + \frac{1}{2}g_2 M_W(\cos\beta Z_{R2s} + \sin\beta Z_{R1s})W^{+\mu}W_\mu^- H_s^0, \tag{3.15}$$

where $H^0_{s=1,\dots,5}$ are the Higgs boson fields, h^0, H^0, $(\tilde{\nu}_+)_1$, $(\tilde{\nu}_+)_2$, $(\tilde{\nu}_+)_3$, respectively. From (3.9) and $\mathcal{L}_{VVH}$ above, it is easy to see that the light Higgs boson coupling to the vector bosons ($V = Z, W$), is proportional to $\sin(\beta-\alpha)$ as in the RPC case.[2] In fact, the coupling sum rule,

$$\sum_{s=1}^{5} g_{VVH_s^0}^2 = g_{VV\phi}^2, \tag{3.16}$$

valid in the RPC case for $s = 1, 2$, persists also here, where $g_{VVH_s^0}$ are the couplings appearing in (3.15) and $g_{VV\phi}$ the corresponding coupling appearing in the Standard Model.

3.2. CP-odd neutral scalar masses and couplings

For the CP-odd case one finds,

$$\mathcal{L} \supset -\left(\mathrm{Im}\, h_0^2 \quad \mathrm{Im}\,\tilde{\nu}_{L0} \quad \mathrm{Im}\,\tilde{\nu}_{Li}\right)\mathcal{M}^2_{\mathrm{ODD}}\begin{pmatrix}\mathrm{Im}\, h_0^2\\ \mathrm{Im}\,\tilde{\nu}_{L0}\\ \mathrm{Im}\,\tilde{\nu}_{Lj}\end{pmatrix}, \tag{3.17}$$

[1] Note that the matrix $\mathbf{Z}$ defined in (2.21) has nothing to do with neither $\mathbf{Z}_R$ nor $\mathbf{Z}_A$ defined in this section.

[2] We follow the conventions of Ref. [23].

where the CP-odd mass matrix reads,

$$\mathcal{M}^2_{\mathrm{ODD}} = \begin{pmatrix} \cos^2\beta M_A^2 + \xi \sin^2\beta M_Z^2 & \frac{1}{2}\sin 2\beta (M_A^2 - \xi M_Z^2) & B_j \\ \frac{1}{2}\sin 2\beta (M_A^2 - \xi M_Z^2) & \sin^2\beta M_A^2 + \xi \cos^2\beta M_Z^2 & B_j \tan\beta \\ B_i & B_i \tan\beta & M_i^2 \delta_{ij} \end{pmatrix}, \tag{3.18}$$

and ξ is the gauge fixing parameter in R_ξ gauge. In fact, by using an orthogonal rotation

$$\mathcal{V} = \begin{pmatrix} \sin\beta & -\cos\beta & 0 \\ \cos & \sin\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.19}$$

we can always project out the would-be Goldstone mode, of the CP-odd scalar matrix and thus

$$\mathcal{V}^T \mathcal{M}^2_{\mathrm{ODD}} \mathcal{V} = \begin{pmatrix} \xi M_Z^2 & 0 & 0 \\ 0 & M_A^2 & \frac{B_j}{\cos\beta} \\ 0 & \frac{B_i}{\cos\beta} & M_i^2 \delta_{ij} \end{pmatrix}. \tag{3.20}$$

Under the approximation of small bilinear RPV couplings [see Eq. (3.7)], a solution is determined for the matrix $\mathbf{Z}_A$ which rotates the fields into the mass basis, such that

$$\mathbf{Z}_A^T \mathcal{M}^2_{\mathrm{ODD}} \mathbf{Z}_A = \mathrm{diag}\big[m^2_{G^0}, m^2_{A^0}, \big(m^2_{\tilde{\nu}_-}\big)_i\big], \tag{3.21}$$

$$\mathbf{Z}_A \approxeq \begin{pmatrix} \sin\beta & \cos\beta & \frac{B_j}{M_j^2 - M_A^2} \\ -\cos\beta & \sin\beta & \frac{B_j \tan\beta}{M_j^2 - M_A^2} \\ 0 & \frac{B_i}{\cos\beta (M_i^2 - M_A^2)} & \delta_{ij} \end{pmatrix}, \tag{3.22}$$

with the mass eigenstates given by

$$\begin{aligned} G^0 &\simeq \sin\beta \, \mathrm{Im}\, h_0^2 - \cos\beta \, \mathrm{Im}\, \tilde{\nu}_{L0}, \\ A^0 &\simeq \cos\beta \, \mathrm{Im}\, h_0^2 + \sin\beta \, \mathrm{Im}\, \tilde{\nu}_{L0} + \frac{B_i}{\cos\beta (M_i^2 - M_A^2)} \mathrm{Im}\, \tilde{\nu}_{Li}, \\ (\tilde{\nu}_-)_i &\simeq \frac{B_i}{M_j^2 - M_A^2} \mathrm{Im}\, h_0^2 + \frac{B_j \tan\beta}{M_i^2 - M_A^2} \mathrm{Im}\, \tilde{\nu}_{L0} + \mathrm{Im}\, \tilde{\nu}_{Li}, \end{aligned} \tag{3.23}$$

with corresponding masses,

$$m_A^2 \simeq M_A^2 - \frac{1}{\cos^2\beta} \sum_{i=1}^{3} \frac{B_i^2}{M_i^2 - M_A^2} + \mathcal{O}\left(\frac{B^4}{M^6 \cos^4\beta}\right), \tag{3.24}$$

$$\big(m^2_{\tilde{\nu}_-}\big)_i \simeq M_i^2 - \frac{B_i^2}{(M_A^2 - M_i^2)\cos^2\beta} + \mathcal{O}\left(\frac{B^4}{M^6 \cos^4\beta}\right). \tag{3.25}$$

The coupling of the Z-gauge boson to the CP-even and CP-odd neutral scalar fields is given by

$$\mathcal{L}_{ZHA} = \frac{-i g_2}{2 c_W} \left[(p_{H_s^0} - p_{A_p^0})_\mu \left(\sum_{\alpha=0}^{3} Z_{R(2+\alpha)s} Z_{A(2+\alpha)p} - Z_{R1s} Z_{A1p} \right) \right] Z^\mu H_s^0 A_p^0, \tag{3.26}$$

where the four momenta $p^\mu_{H_s^0}$, $p^\mu_{A_p^0}$ are incoming and the fields $A^0_{p=1,\dots,5}$ correspond to G^0, A^0, $(\tilde{\nu}_-)_1$, $(\tilde{\nu}_-)_2$, $(\tilde{\nu}_-)_3$, respectively. One may check that the coupling $Z - G^0 - h^0$ derived from (3.26) is proportional to $\sin(\alpha - \beta)$ as it should be.

4. Positiveness and stability of the scalar potential

4.1. Positiveness

In general, one should inspect whether all physical masses in the CP-odd and CP-even sector are positive. For that, all diagonal square subdeterminants of mass matrices should be positive. One can easily check that both CP-odd and CP-even mass matrices in (3.3), (3.18) respectively, lead, in the rotated basis, to the same set of conditions,

$$M_i^2 > 0 \quad \text{with } i = 1, 2, 3 \quad \text{and} \quad M_A^2 > \frac{1}{\cos^2\beta} \sum_{i=1}^{3} \frac{B_i^2}{M_i^2}. \tag{4.1}$$

Using the form of M_A^2 in (3.1), the last equation can be rewritten in the form

$$B_0 > \tan\beta \sum_{i=1}^{3} \frac{B_i^2}{M_i^2}. \tag{4.2}$$

Excluding some very singular mass configurations, the above conditions are rather trivially fulfilled if one takes into account the bound of Eq. (3.7).

4.2. Stability

The question of whether the potential is stable, i.e. bounded from below, is far more complicated. In most cases the quartic (D-)term dominates and there is no problem. The only exception being when the fields follow the direction $|h_2^0|^2 = \sum_{i=0}^{4} |\tilde{\nu}_{Li}|^2$. In such a case, one should check whether the remaining part of the potential is positive along this direction.

Denoting $R \equiv \sqrt{\sum_{i=0}^{3} |\tilde{\nu}_i|^2}$ and $h_2^0 = Re^{-i\phi}$, where ϕ is a free phase, and using Eqs. (2.20), (2.25), (3.4), one can write down the scalar potential along this direction in the vanishing snueutrino VEV basis as

$$\begin{aligned} V_{\text{neutral}} &= \frac{B_0}{\sin\beta\cos\beta} \tilde{\nu}_{L0}^* \tilde{\nu}_{L0} + \left[M_i^2 + B_0 \cot\beta\right] \tilde{\nu}_{Li}^* \tilde{\nu}_{Li} + B_i \tan\beta \left(\tilde{\nu}_{L0}^* \tilde{\nu}_{Li} + \tilde{\nu}_{L0} \tilde{\nu}_{Li}^*\right) - B_\alpha \left(\tilde{\nu}_{L\alpha} h_0^2 + \text{H.c.}\right) \\ &\equiv \tilde{\nu}_L^\dagger \mathbf{Q} \tilde{\nu}_L - \left(\mathbf{B}^T \tilde{\nu}_L R e^{-i\phi} + \text{H.c.}\right), \end{aligned} \tag{4.3}$$

where the real symmetric matrix $\mathbf{Q}$ is

$$\mathbf{Q} = \begin{pmatrix} M_A^2 & B_i \tan\beta \\ B_j \tan\beta & [M_i^2 + B_0 \cot\beta]\delta_{ij} \end{pmatrix}. \tag{4.4}$$

Finding the stability conditions for the potential (4.3) is difficult, it depends on nine real variables (4 moduli and five phases of the fields). To simplify the problem, we perform one more field rotation to the basis in which the matrix $\mathbf{Q}$ is diagonal. This can be done, in general, by numerical routines (routines where already used in calculating the vanishing sneutrino VEV basis, and therefore, finding the stability conditions for the general scalar potential always has to involve some numerical analysis). We thus define the matrix $\mathbf{P}$, $\tilde{\nu}_L \to \mathbf{P}\tilde{\nu}_L$, as

$$\mathbf{P}^\dagger \mathbf{Q} \mathbf{P} = \text{diag}(X_0, X_1, X_2, X_3). \tag{4.5}$$

In fact, $\mathbf{Q}$ is real, so we can choose $\mathbf{P}$ to be real orthogonal. Also, we denote $D_\beta \equiv B_\alpha P_{\alpha\beta}$. Obviously, the rotation $\mathbf{P}$ preserves the value of $R = |h_2^0|$.

The potential becomes:

$$V_{\text{neutral}} = \sum_{\alpha=0}^{3} \left[X_\alpha |\tilde{\nu}_{L\alpha}|^2 - D_\alpha R\left(\tilde{\nu}_{L\alpha} e^{-i\phi} + \text{H.c.}\right)\right], \tag{4.6}$$

where X_0 has to be positive, otherwise for $\phi = 0$ along the direction $\tilde{\nu}_{Li} = \operatorname{Im}\tilde{\nu}_{L0} = 0$ the potential $V_{\text{neutral}} = |\operatorname{Re}\tilde{\nu}_{L0}|^2[X_0 - D_0 \operatorname{sign}(\operatorname{Re}\tilde{\nu}_{L0})]$ falls to $-\infty$ at least for one direction along the $\operatorname{Re}\tilde{\nu}_{L0}$ axis. In fact the condition on X_α is $X_\alpha \geqslant 2|D_\alpha|$. Thus our first conclusion is that the matrix $\mathbf{Q}$ has to be positively defined. One can write down appropriate conditions in the same manner as for the scalar mass matrices; comparing with Eq. (4.1), it can be observed that this condition is automatically fulfilled if relation (4.1) holds.

With X_α positive, one can write down the potential as:

$$V_{\text{neutral}} = \sum_{\alpha=0}^{3} \left| \sqrt{X_\alpha}\tilde{\nu}_{L\alpha} - \frac{D_\alpha}{\sqrt{X_\alpha}} R e^{i\phi} \right|^2 - R^2 \sum_{\alpha=0}^{3} \frac{D_\alpha^2}{X_\alpha}. \tag{4.7}$$

To further simplify the problem, denote $\tilde{\nu}_{L\alpha} = u_\alpha e^{i(\phi-\phi_\alpha)}$, where $u_\alpha \geqslant 0$ are field moduli and ϕ_α are free phases. Then

$$V_{\text{neutral}} = R^2 \left(\sum_{\alpha=0}^{3} \left| \sqrt{X_\alpha}\frac{u_\alpha}{R} - \frac{D_\alpha}{\sqrt{X_\alpha}} e^{i\phi_\alpha} \right|^2 - \sum_{\alpha=0}^{3} \frac{D_\alpha^2}{X_\alpha} \right), \tag{4.8}$$

where $R = \sqrt{\sum_{i=0}^{3} |\tilde{\nu}_i|^2} = \sqrt{\sum_{i=0}^{3} u_i^2}$. Phases ϕ_α can be adjusted independently of u_α. The worst case from the point of view of potential stability, the smallest first term inside the parenthesis, occurs for $D_\alpha e^{i\phi_\alpha} = |D_\alpha|$. Denoting further $\epsilon_\alpha = u_\alpha/R$, $0 \leqslant \epsilon_\alpha \leqslant 1$, one can reduce our initial problem to the question whether the function

$$g(\epsilon_\alpha) = \sum_{\alpha=0}^{3} \left| \sqrt{X_\alpha}\epsilon_\alpha - \frac{|D_\alpha|}{\sqrt{X_\alpha}} \right|^2 - \sum_{\alpha=0}^{3} \frac{D_\alpha^2}{X_\alpha} = \sum_{\alpha=0}^{3} \left(X_\alpha \epsilon_\alpha^2 - 2|D_\alpha|\epsilon_\alpha \right), \tag{4.9}$$

depending now on four real positive parameters, is non-negative on the unit sphere $\sum_{\alpha=0}^{3} \epsilon_\alpha^2 = 1$. In general such problem can be solved numerically using the method of Lagrange multipliers. For $X_i > X_0 - D_0$, the minimum occurs for

$$\epsilon_\alpha = \frac{|D_\alpha|}{X_\alpha + \lambda}, \tag{4.10}$$

where λ can be found numerically as a root of the following equation:

$$\sum_{\alpha=0}^{3} \frac{D_\alpha^2}{(X_\alpha + \lambda)^2} = 1. \tag{4.11}$$

For smaller X_i, the minimum is realised for $\epsilon_i = 0$ for one or more values of i and requires analysis of various special cases. Having found the correct minimum, to prove the stability of the potential one needs to show that the function g at the minimum is non-negative.

As shown in Eq. (3.7), B_i terms and thus also D_i terms are usually very small. In this case one can set approximate, sufficient conditions for the stability of the potential, without resorting to solving Eq. (4.11), numerically. Denote $D = \sum_{i=1}^{3} D_i^2$ and $X_{\min} = \min(X_1, X_2, X_3)$. Then, using the inequality $D_i\epsilon_i \leqslant \sqrt{\sum_{i=1}^{3} D_i^2}\sqrt{\sum_{i=1}^{3} \epsilon_i^2} = D\sqrt{1-\epsilon_0^2}$, one has

$$g(\epsilon_\alpha) \geqslant X_0\epsilon_0^2 + X_{\min}\left(1-\epsilon_0^2\right) + (X_i - X_{\min})\epsilon_i^2 - 2|D_0|\epsilon_0 - 2D\sqrt{1-\epsilon_0^2}. \tag{4.12}$$

Terms $(X_i - X_{\min})\epsilon_i^2$ are always non-negative. The worst case being when the vector $(\epsilon_1, \epsilon_2, \epsilon_3)$ is along the minimal X_i axis, where these terms vanish. Other terms are rotation invariant in the 3-dimensional space $(\epsilon_1, \epsilon_2, \epsilon_3)$, so Eq. (4.12) is equivalent to finding parameters $X_0, X_{\min}, D_0, D$ for which the expression (4.13), depending on just one real variable, is positive:

$$g'(\epsilon_0) = X_0\epsilon_0^2 + X_{\min}\left(1-\epsilon_0^2\right) - 2|D_0|\epsilon_0 - 2D\sqrt{1-\epsilon_0^2} \geqslant 0. \tag{4.13}$$

Analysis of (4.13) is further simplified by one more approximation, justified for small D:

$$g'(\epsilon_0) \geqslant X_0\epsilon_0^2 + X_{\min}\left(1-\epsilon_0^2\right) - 2|D_0|\epsilon_0 - 2D. \tag{4.14}$$

The rhs of Eq. (4.14) is now trivial. Following approximate conditions for the stability of the potential can be summarised as follows:

$X_{\min}$ range	Stability requires
$X_{\min} \geqslant X_0 - D_0$	$X_0 \geqslant 2\vert D_0\vert + 2D$
$0 < X_{\min} < X_0 - D_0$	$(X_0 - X_{\min})(X_{\min} - 2D) \geqslant D_0^2$

Both conditions are sufficient, but not minimal—we have made some approximations and there may be parameters which do not fall into either of the categories above, and yet still give a stable potential. For example, if $X_0 = X_1 = X_2 = X_3 \equiv X$, one can easily derive the exact necessary and sufficient condition for potential stability as $X \geqslant 2\sqrt{D_0^2 + D^2}$, less strict than $X \geqslant 2(|D_0| + |D|)$ which would be given by the table above.

For complementary work the reader is referred to Ref. [24].

5. Conclusions

In this Letter we present a procedure for calculating the rotation matrix which brings the neutral scalar fields of the general R-parity violating MSSM onto the vanishing sneutrino VEV basis where they develop n zero VEVs, with n being the number of flavour generations. In doing so, we have made no assumption about the complexity of the parameters. We consider the case of $n = 3$ generations, but our approach immediately applies to other cases, apart from obvious modifications of the form of $\mathbf{Z}$ matrix defined in (2.21), (2.22). As a byproduct of basis change, we prove that the tree level MSSM potential does not exhibit any form of CP-violation, neither explicit nor spontaneous. Consequently, the neutral scalar fields can be divided into CP-even and CP-odd sectors with the 5×5 neutral scalar squared mass matrices, taking a very simple form with only small RPV masses sitting on their off diagonal elements. We can thus expand along small RPV masses and find analytic approximate formulae which relate the RPC and the RPV neutral scalar masses. Furthermore we also find, that in general there is always at least one neutral scalar field with mass lighter than M_Z which couples maximally to the Z-gauge boson in the case of large $\tan\beta$ and large M_A. Our procedure for finding the rotation matrix $\mathbf{U}$ has been coded[3] and is numerically stable.

In the end, we are aiming to construct the most general MSSM quantum field theory structure resorting neither to R-parity violation nor to other approximations. This will be useful for examining the phenomenology of the MSSM as a whole. The convenient choice of the basis for the neutral sector found in this Letter is a first step towards this direction.

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[3] The code will be available from http://www.fuw.edu.pl/~rosiek/physics/rpv/scalar.html.

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