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On the two-loop Yukawa corrections to the MSSM Higgs boson masses at large $\tan\beta$

A. Dedes [a], G. Degrassi [b], P. Slavich [c,d]

[a] *Physik Department, Technische Universität München, D-85748 Garching, Germany*
[b] *Dipartimento di Fisica, Università di Roma Tre and INFN, Sezione di Roma III, Via della Vasca Navale 84, I-00146 Rome, Italy*
[c] *Institut für Theoretische Physik, Universität Karlsruhe, Kaiserstrasse 12, Physikhochhaus, D-76128 Karlsruhe, Germany*
[d] *Max Planck Institut für Physik, Föhringer Ring 6, D-80805 München, Germany*

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Abstract

We complete the effective potential calculation of the two-loop, top/bottom Yukawa corrections to the Higgs boson masses in the minimal supersymmetric standard model, by computing the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ contributions for arbitrary values of the bottom Yukawa coupling. We also compute the corrections to the minimization conditions of the effective potential at the same perturbative order. Our results extend the existing $\mathcal{O}(\alpha_t^2)$ calculation, and are relevant in regions of the parameter space corresponding to $\tan\beta \gg 1$. We extend to the Yukawa corrections a convenient renormalization scheme, previously proposed for the $\mathcal{O}(\alpha_b\alpha_s)$ corrections, that avoids unphysically large threshold effects associated with the bottom mass and absorbs the bulk of the corrections into the one-loop expression. For large values of $\tan\beta$, the new contributions can account for a variation of several GeV in the lightest Higgs boson mass.

1. Introduction

One of main features of the minimal supersymmetric standard model (MSSM) [1] is the prediction of the existence of at least one light Higgs boson [2]. After the conclusion of the LEP and Tevatron run I experimental programs that reported no significant evidence for a

E-mail addresses: dedes@ph.tum.de (A. Dedes), degrassi@fis.uniroma3.it (G. Degrassi), slavich@mppmu.mpg.de (P. Slavich).

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Higgs boson, the experimental search for this particle has now become one of the major tasks of the Tevatron run II and of the future LHC. Within the MSSM, the tree-level masses of the neutral Higgs bosons can be parameterized in terms of three input parameters: the mass of the CP-odd Higgs m_A, the Z boson mass m_Z and the ratio of the two Higgs vacuum expectation values, $\tan\beta \equiv v_2/v_1$. At tree level, at least one of the MSSM Higgs bosons is bound to be lighter than the Z boson, thus the failure of detecting it at LEP indicates that the MSSM could be a realistic theory only after the radiative corrections to the Higgs boson masses have been taken into account.

The radiative corrections arise from loop diagrams involving Standard Model particles and their superpartners. Although the earliest computations [3] of radiative corrections to the MSSM Higgs masses date back to the eighties, it was first realized in Ref. [4] that the inclusion of the one-loop top/stop $\mathcal{O}(\alpha_t)$ corrections, where $\alpha_t = h_t^2/(4\pi)$, h_t being the superpotential top coupling, may push the light Higgs mass well above the tree-level bound. In the subsequent years, an impressive theoretical effort has been devoted to the precise determination of the MSSM Higgs masses: full one-loop computations have been provided [5,6], leading logarithmic effects at two loops have been included via appropriate renormalization group equations [7,8], and genuine two-loop corrections of $\mathcal{O}(\alpha_t\alpha_s)$ [9–13], $\mathcal{O}(\alpha_t^2)$ [9,12,14], and $\mathcal{O}(\alpha_b\alpha_s)$ [15] have been evaluated in the limit of zero external momentum. The tadpole corrections needed to minimize the effective potential, $V_{\rm eff}$, have also been calculated [16] at the same perturbative orders. Furthermore, the full two-loop corrections to the MSSM effective potential have been calculated in Ref. [17], together with a first study of the effect of the two-loop corrections to the Higgs masses controlled by the electroweak gauge couplings [18].

The corrections controlled by the top Yukawa coupling are in general the most relevant ones. However, in regions of the MSSM parameter space where $\tan\beta \gg 1$ the superpotential bottom coupling h_b may be large (we recall that, at the classical level, $h_b/h_t = (m_b/m_t)\tan\beta$) and the one-loop bottom/sbottom corrections of $\mathcal{O}(\alpha_b)$, where $\alpha_b = h_b^2/(4\pi)$, can be numerically relevant and compete with those of $\mathcal{O}(\alpha_t)$. At the two-loop level, the evaluation of the corrections controlled by the bottom Yukawa coupling requires the inclusion of one-loop, $\tan\beta$-enhanced threshold corrections to the bottom mass [19]. If the physical bottom mass is used as input parameter in the one-loop part of the computation, potentially large $\tan\beta$-enhanced corrections appear at two loops. To address this problem, a set of renormalization prescriptions for the parameters in the bottom/sbottom sector that avoid the occurrence of unphysically large threshold effects at two loops was proposed in Ref. [15] for the $\mathcal{O}(\alpha_b\alpha_s)$ part of the corrections.

The purpose of this article is to complete the calculation of the two-loop, top/bottom Yukawa corrections to the Higgs boson masses in the effective potential approach. Such corrections were previously computed [9,12,14] in the limit $h_b \to 0$, which is accurate enough only for moderate values of $\tan\beta$. In that limit, the two-loop Yukawa corrections to the MSSM Higgs masses are of $\mathcal{O}(\alpha_t^2 m_t^2)$, which we denote as $\mathcal{O}(\alpha_t^2)$ for brevity. On the other hand, when the bottom Yukawa coupling is left arbitrary, the resulting two-loop corrections are proportional to various combinations of couplings and masses: e.g., we find terms of $\mathcal{O}(\alpha_b^2 m_t^2)$, which might as well be interpreted as $\tan\beta$-enhanced terms of $\mathcal{O}(\alpha_t\alpha_b m_b^2)$. To simplify our notation, we will refer to all such "mixed" terms as to $\mathcal{O}(\alpha_t\alpha_b)$ corrections, and to the terms that depend only on the bottom Yukawa coupling as

to $\mathcal{O}(\alpha_b^2)$ corrections. Our computation will thus provide us with the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ corrections to the MSSM Higgs masses, extending the $\mathcal{O}(\alpha_t^2)$ results obtained in Ref. [14]. As a byproduct, we also calculate the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ corrections to the minimization conditions of the effective potential. We express our results in the $\overline{\text{DR}}$ renormalization scheme, as well as in an "on-shell" scheme which extends the prescription described in Ref. [15] to the case of the Yukawa corrections. The resulting analytical formulae are rather lengthy, thus we make them available, upon request, in the form of a Fortran code (please make your request to P. Slavich, e-mail address: slavich@mppmu.mpg.de).

The structure of this paper is the following. In Section 2 we recall some general issues of the effective potential approach to the calculation of the Higgs masses. Section 3 describes our two-loop computation of the $\overline{\text{DR}}$ tadpoles and CP-odd, CP-even Higgs mass matrices, while Section 4 addresses our on-shell renormalization prescription. Numerical results are given in Section 5 and in Section 6 we present a short discussion of the corrections controlled by the tau Yukawa coupling as well as our conclusions.

2. Higgs masses in the effective potential approach

We begin our discussion by recalling some general results concerning the computation of the MSSM Higgs masses in the effective potential approach. The effective potential, which we write from the start in terms of $\overline{\text{DR}}$-renormalized fields and parameters, can be decomposed as $V_{\text{eff}} = V_0 + \Delta V$, where V_0 is the tree-level scalar potential and ΔV contains the radiative corrections. Keeping only the dependence on the neutral Higgs fields H_1^0 and H_2^0, the tree-level MSSM potential reads

$$\begin{aligned} V_0 = {} & \left(\mu^2 + m_{H_1}^2\right)\left|H_1^0\right|^2 + \left(\mu^2 + m_{H_2}^2\right)\left|H_2^0\right|^2 + m_3^2\left(H_1^0 H_2^0 + \text{h.c.}\right) \\ & + \frac{g^2 + g'^2}{8}\left(\left|H_1^0\right|^2 - \left|H_2^0\right|^2\right)^2, \end{aligned} \tag{1}$$

where: μ is the Higgs mass term in the superpotential (we assume it to be real, neglecting all possible CP-violating phases); $m_{H_1}^2$, $m_{H_2}^2$ and m_3^2 are soft supersymmetry-breaking masses; g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively. The neutral Higgs fields can be decomposed into their vacuum expectation values (VEVs) plus their CP-even and CP-odd fluctuations as $H_i^0 = (v_i + S_i + i\,P_i)/\sqrt{2}$ $(i = 1, 2)$. The VEVs v_i are determined by solving the minimization conditions of the effective potential, i.e.,

$$\left.\frac{\partial V_{\text{eff}}}{\partial S_i}\right|_{\text{min}} = 0, \qquad \left.\frac{\partial V_{\text{eff}}}{\partial P_i}\right|_{\text{min}} = 0, \tag{2}$$

the second equality being automatically satisfied since we assume that CP is conserved. However, it is also possible to take v_1 and v_2 as input parameters, or equivalently $v^2 \equiv v_1^2 + v_2^2$ and $\tan\beta \equiv v_2/v_1$, where v^2 is related to the squared running mass of the Z boson through $m_Z^2 = (g^2 + g'^2)v^2/4$. In this case, the minimization conditions of V_{eff} can be translated into conditions on μ^2 and m_3^2:

$$\mu^2 = -\frac{m_Z^2}{2} + \frac{m_{H_1}^2 + \Sigma_1 - (m_{H_2}^2 + \Sigma_2)\tan^2\beta}{\tan^2\beta - 1}, \tag{3}$$

$$m_3^2 = \frac{m_Z^2}{2}\sin 2\beta + \frac{1}{2}\tan 2\beta\big(m_{H_1}^2 - m_{H_2}^2 + \Sigma_1 - \Sigma_2\big), \tag{4}$$

where the "tadpoles" Σ_1 and Σ_2 are defined as

$$\Sigma_i \equiv \frac{1}{v_i}\frac{\partial \Delta V}{\partial S_i}\bigg|_{\min}. \tag{5}$$

In the effective potential approach, the mass matrices for the neutral CP-odd and CP-even Higgs bosons can be approximated by

$$\big(\mathcal{M}_P^2\big)_{ij}^{\text{eff}} = \frac{\partial^2 V_{\text{eff}}}{\partial P_i \partial P_j}\bigg|_{\min}, \qquad \big(\mathcal{M}_S^2\big)_{ij}^{\text{eff}} = \frac{\partial^2 V_{\text{eff}}}{\partial S_i \partial S_j}\bigg|_{\min}. \tag{6}$$

Exploiting the minimization conditions of the effective potential, Eq. (2), the CP-odd mass matrix can be written as

$$\big(\mathcal{M}_P^2\big)_{ij}^{\text{eff}} = -m_3^2\frac{v_1 v_2}{v_i v_j} - \delta_{ij}\Sigma_i + \frac{\partial^2 \Delta V}{\partial P_i \partial P_j}\bigg|_{\min}. \tag{7}$$

$(\mathcal{M}_P^2)^{\text{eff}}$ has a single non-vanishing eigenvalue that, in the approximation of zero external momentum, can be identified with the squared physical mass of the A boson. We denote it as $\bar{m}_A^2 = \hat{m}_A^2 + \Delta m_A^2$, where $\hat{m}_A^2 = -2m_3^2/\sin 2\beta$ is the squared running mass of the A boson. The CP-even mass matrix can in turn be decomposed as

$$\big(\mathcal{M}_S^2\big)^{\text{eff}} = \big(\mathcal{M}_S^2\big)^{0,\text{eff}} + \big(\Delta\mathcal{M}_S^2\big)^{\text{eff}}, \tag{8}$$

where the first term in the sum is the tree-level mass matrix expressed in terms of m_Z and $\bar{m}_A$:

$$\big(\mathcal{M}_S^2\big)^{0,\text{eff}} = \begin{pmatrix} m_Z^2 c_\beta^2 + \bar{m}_A^2 s_\beta^2 & -(m_Z^2 + \bar{m}_A^2)s_\beta c_\beta \\ -(m_Z^2 + \bar{m}_A^2)s_\beta c_\beta & m_Z^2 s_\beta^2 + \bar{m}_A^2 c_\beta^2 \end{pmatrix}, \tag{9}$$

($c_\beta \equiv \cos\beta$, $s_\beta \equiv \sin\beta$ and so on), while the second term contains the radiative corrections:

$$\big(\Delta\mathcal{M}_S^2\big)_{ij}^{\text{eff}} = \frac{\partial^2 \Delta V}{\partial S_i \partial S_j}\bigg|_{\min} - (-1)^{i+j}\frac{\partial^2 \Delta V}{\partial P_i \partial P_j}\bigg|_{\min}. \tag{10}$$

It is clear from Eqs. (7)–(10) that, in order to make contact with the physical A mass, the effective potential should be computed as a function of both CP-even and CP-odd fields.

Since V_{eff} generates one-particle-irreducible Green's functions at vanishing external momentum, it is clear that the effective potential approach neglects the momentum-dependent effects in the Higgs self-energies. The complete computation of the physical masses of the CP-even Higgs bosons, m_h and m_H, and of the CP-odd Higgs boson, m_A, requires the full, momentum-dependent two-point functions. A detailed discussion of the correspondence between the effective potential approach and the full computation has been presented in Ref. [14]. Here we just notice that the main conclusions presented in that paper regarding the $\mathcal{O}(\alpha_t\alpha_s)$ and $\mathcal{O}(\alpha_t^2)$ corrections apply also to $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections discussed here. Namely, the two-loop $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections to the lightest Higgs eigenvalue are fully accounted for by the two-loop effective potential evaluation of m_h supplemented by known momentum-dependent one-loop contributions, and the

same is true for m_H when m_A is not too large. Instead, if $m_A > m_t$ a full two-loop $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ computation of m_H requires additional momentum-dependent two-loop contributions, neglected by the effective potential calculation, that have not been computed so far.

3. Computation of the two-loop Yukawa corrections

We shall now describe our two-loop computation of the tadpoles Σ_i, the A-boson mass correction Δm_A^2 and the matrix $(\Delta\mathcal{M}_S^2)^{\text{eff}}$, including terms controlled by the top and/or the bottom Yukawa couplings. The computation is consistently performed in the gaugeless limit, i.e., by setting to zero all the gauge couplings, and by keeping h_t and h_b as the only non-vanishing Yukawa couplings. In this limit, the tree-level (field-dependent) spectrum of the MSSM simplifies considerably: gauginos and Higgsinos do not mix; charged and neutral Higgsinos combine into Dirac spinors with degenerate mass eigenvalues $|\mu|^2$; the only massive SM fermions are the top and bottom quarks, while all other fermions and gauge bosons have vanishing masses; the only sfermions with non-vanishing couplings are the stop and sbottom squarks; the lightest CP-even Higgs boson, h, is massless, and the same is true for the Goldstone bosons; all the remaining Higgs states, $(H, A, H^{\pm})$, have degenerate mass eigenvalues m_A^2. The tree-level mixing angle in the CP-even sector is just $\alpha = \beta - \pi/2$.

The renormalization of the effective potential is performed according to the lines of Ref. [16], i.e., we express V_{eff}, from the beginning, in terms of $\overline{\text{DR}}$-renormalized fields and parameters. In practice, this amounts to dropping all the divergent terms in ΔV and replacing the two-loop integrals $I(m_1^2, m_2^2, m_3^2)$ and $J(m_1^2, m_2^2)$ (see, e.g., Ref. [16] for the definitions) with their "subtracted" counterparts $\hat{I}$ and $\hat{J}$, first introduced in Ref. [20]. Alternatively, we could follow the procedure of Refs. [13,14]: express ΔV in terms of bare parameters and then renormalize the derivatives of ΔV (i.e., the tadpoles and the corrections to the Higgs masses), checking explicitly the cancellation of the divergent terms. The general formulae for the tadpoles and the corrections to the Higgs masses would look slightly more complicated in the latter case. However, we have checked that the two renormalization procedures lead to the same final result, as they should.

According to Eqs. (5), (7) and (10), the tadpoles and the corrections to the Higgs mass matrices can be computed by taking the derivatives of ΔV with respect to the CP-even and CP-odd fields, evaluated at the minimum of V_{eff}. Following the strategy of Refs. [13,14], we compute ΔV in terms of a set of field-dependent parameters (masses and angles), and use the chain rule to express the corrections in terms of derivatives of ΔV with respect to those parameters. In each sector, the field-dependent parameters can be chosen as

$$m_q,\ m_{\tilde{q}_1}^2,\ m_{\tilde{q}_2}^2,\ \bar{\theta}_{\tilde{q}},\ \varphi_q,\ \tilde{\varphi}_q, \quad (q = t, b), \tag{11}$$

where: m_q and $m_{\tilde{q}_i}^2$ are the quark and squark masses; $\bar{\theta}_{\tilde{q}}$ is the field-dependent squark mixing angle, defined in such a way that $0 \leqslant \bar{\theta}_{\tilde{q}} < \pi/2$ (to be contrasted with the usual field-independent mixing angle $\theta_{\tilde{q}}$, such that $-\pi/2 \leqslant \theta_{\tilde{q}} < \pi/2$); φ_q is the phase in the complex quark mass; $\tilde{\varphi}_q$ is the phase in the off-diagonal element of the squark mass

matrix. For the explicit Higgs field dependence of these parameters, see Refs. [13,14]. In the expression of ΔV relevant to the $\mathcal{O}(\alpha_t^2)$ corrections (i.e., with h_b set to zero), the top and stop phases always combine in the difference $\varphi_t - \tilde{\varphi}_t$, so that a convenient choice for the field-dependent parameter is $c_{\varphi_t-\tilde{\varphi}_t} \equiv \cos(\varphi_t - \tilde{\varphi}_t)$. On the other hand, when both h_t and h_b are non-zero, as it is the case in the present computation of the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ corrections, the situation becomes more complicated: besides the terms involving $\varphi_t - \tilde{\varphi}_t$ and $\varphi_b - \tilde{\varphi}_b$, we find other terms, coming from diagrams with a charged Higgs or Goldstone boson, that involve the combinations $\varphi_t + \tilde{\varphi}_b$, $\varphi_b + \tilde{\varphi}_t$, $\varphi_t + \varphi_b$ and $\tilde{\varphi}_t + \tilde{\varphi}_b$.

Exploiting the field-dependence of the various masses and angles, we get the following general formulae for the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ corrections in the $\overline{\text{DR}}$ renormalization scheme:

$$\begin{aligned}\left(\Delta\mathcal{M}_S^2\right)_{11}^{\text{eff}} &= 2h_b^2 m_b^2 F_1^b + 2h_b^2 A_b m_b s_{2\theta_b} F_2^b + \frac{1}{2}h_b^2 A_b^2 s_{2\theta_b}^2 F_3^b \\ &\quad + \frac{1}{2}h_t^2\mu^2 s_{2\theta_t}^2 F_3^t + 2h_t h_b m_b \mu s_{2\theta_t} F_4^t + h_t h_b \mu A_b s_{2\theta_t} s_{2\theta_b} F_5, \qquad (12)\\ \left(\Delta\mathcal{M}_S^2\right)_{12}^{\text{eff}} &= h_t^2 \mu m_t s_{2\theta_t} F_2^t + \frac{1}{2}h_t^2 A_t \mu s_{2\theta_t}^2 F_3^t + h_t h_b m_b A_t s_{2\theta_t} F_4^t \\ &\quad + h_b^2 \mu m_b s_{2\theta_b} F_2^b + \frac{1}{2}h_b^2 A_b \mu s_{2\theta_b}^2 F_3^b + h_t h_b m_t A_b s_{2\theta_b} F_4^b \\ &\quad + \frac{1}{2}h_t h_b s_{2\theta_t} s_{2\theta_b}\left(A_t A_b + \mu^2\right) F_5 + 2h_t h_b m_t m_b F_6, \qquad (13)\\ \left(\Delta\mathcal{M}_S^2\right)_{22}^{\text{eff}} &= 2h_t^2 m_t^2 F_1^t + 2h_t^2 A_t m_t s_{2\theta_t} F_2^t + \frac{1}{2}h_t^2 A_t^2 s_{2\theta_t}^2 F_3^t \\ &\quad + \frac{1}{2}h_b^2\mu^2 s_{2\theta_b}^2 F_3^b + 2h_t h_b m_t \mu s_{2\theta_b} F_4^b + h_t h_b \mu A_t s_{2\theta_t} s_{2\theta_b} F_5, \qquad (14)\\ v_1^2\Sigma_1 &= m_t\mu\cot\beta s_{2\theta_t} F^t + m_b A_b s_{2\theta_b} F^b + 2m_b^2 G^b, \qquad (15)\\ v_2^2\Sigma_2 &= m_b\mu\tan\beta s_{2\theta_b} F^b + m_t A_t s_{2\theta_t} F^t + 2m_t^2 G^t, \qquad (16)\\ \Delta m_A^2 &= -\frac{1}{c_\beta s_\beta}\left(\frac{h_t^2\mu A_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} F^t + \frac{h_b^2\mu A_b}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} F^b + 2h_t h_b F_A\right). \qquad (17)\end{aligned}$$

In the equations above, A_t and A_b are the soft supersymmetry-breaking trilinear couplings of the Higgs fields to the stop and sbottom squarks, and $s_{2\theta_q} \equiv \sin 2\theta_{\tilde{q}}$ $(q = t, b)$ refer to the usual field-independent squark mixing angles. The functions F_i^q $(i = 1, 2, 3, 4)$, F_5, F_6, F^q, G^q and F_A are combinations of the derivatives of ΔV with respect to the field-dependent parameters, computed at the minimum of the effective potential; their definitions are given in Appendix A. It can be noticed that, as it is predictable from the form of the MSSM Lagrangian, the above results are fully symmetric with respect to the simultaneous replacements $t \leftrightarrow b$ and $H_1 \leftrightarrow H_2$, the latter resulting into $\tan\beta \leftrightarrow \cot\beta$, $v_1 \leftrightarrow v_2$, $(\Delta\mathcal{M}_S^2)_{11}^{\text{eff}} \leftrightarrow (\Delta\mathcal{M}_S^2)_{22}^{\text{eff}}$ and $\Sigma_1 \leftrightarrow \Sigma_2$.

An explicit expression of the two-loop top and bottom Yukawa contribution to ΔV can be found in Ref. [12], while the complete two-loop effective potential for the MSSM was given in the second paper of Ref. [17]. However, those expressions were computed for vanishing CP-odd fields, thus omitting the dependence on the phases φ_q and $\tilde{\varphi}_q$. Since these phases appear in ΔV in many different combinations, it is not possible to infer the

general field-dependent expression of ΔV by means of simple substitutions in Eq. (D.6) of Ref. [12], as it was the case in the computation of the $\mathcal{O}(\alpha_t\alpha_s + \alpha_t^2)$ corrections.[1]

We worked out the general expression of the two-loop top and bottom Yukawa contribution to ΔV in terms of all the field-dependent parameters of Eq. (11). We then computed its derivatives in order to obtain explicit formulae for the various functions appearing in Eqs. (12)–(17). The use of a recursive relation for the derivatives of $I(m_1^2, m_2^2, m_3^2)$, presented in Ref. [16], helped us to keep the number of terms involved under control. However, the resulting analytical formulae are very long and we choose not to display them in print. Instead, we make them available in the form of a Fortran code.

4. On-shell renormalization scheme and input parameters

The results presented in the previous section are valid when the MSSM input parameters are expressed in the $\overline{\rm DR}$ renormalization scheme. This way of presenting the results is convenient for analyzing models that predict, via the MSSM renormalization group equations, the low-energy $\overline{\rm DR}$ values of the parameters in terms of a set of boundary conditions assigned at some scale $M_{\rm GUT}$ much larger than the weak scale (see Refs. [21,22] for a list of public codes that are commonly used in this kind of analyses, and Ref. [23] for a comparison among them). General low-energy analyses of the MSSM, however, do not refer to boundary conditions at high scales, and are usually performed in terms of parameters with a more direct physical interpretation, such as pole masses and appropriately defined mixing angles in the squark sector. Such an approach requires modifications of our two-loop results, induced by the variation of the parameters entering the one-loop corrections when moving from the $\overline{\rm DR}$ scheme to a different scheme (for a generic parameter x, we define the shift from the $\overline{\rm DR}$ value $\hat{x}$ as $\delta x \equiv \hat{x} - x$).

However, it is not always possible to find a sensible definition with a direct physical interpretation for all the relevant parameters. For example, while there is a well-known physical ($\equiv$ pole) definition for the masses, the so-called "on-shell" (OS) definition, and an OS definition for the squark mixing angles can be also conceived [24], it is not clear what meaning should be assigned to an OS definition of parameters like $(A_t, A_b, \mu, \tan\beta)$. For instance, they could be related to specific physical amplitudes. However, given our present ignorance of any supersymmetric effect, such a choice does not seem particularly useful. In these cases it seems sometimes simpler to stick to a $\overline{\rm DR}$ definition.

It is rather easy to devise an OS renormalization scheme for the parameters in the top/stop sector, based on the OS prescription for the top and stop masses and the stop mixing angle and treating A_t as a derived quantity, while retaining a $\overline{\rm DR}$ definition for μ and $\tan\beta$ (see, e.g., Refs. [13,14]). Instead, some additional care is required in the choice of an OS scheme for the parameters in the bottom/sbottom sector, due to the potentially large one-loop threshold corrections [19], proportional to $\tan\beta$, that contribute to the pole bottom mass. For example, a definition of A_b in terms of the OS bottom and sbottom masses and sbottom mixing angle, similar to the definition of A_t, would produce a shift δA_b

[1] Also, we do not agree with Ref. [12] on the sign of the penultimate line of Eq. (D.6).

proportional to $\tan^2\beta$ [25]. When $\tan\beta$ is large, this would induce very large corrections to the Higgs masses at two loops, questioning the validity of the perturbative expansion.

To overcome this problem, we adopt a set of renormalization prescriptions for the parameters in the bottom/sbottom sector, first introduced in Ref. [15] for the case of the strong corrections, that avoid the occurrence of unphysically large threshold effects and at the same time enforce other desirable properties such as the decoupling of heavy particles, the infrared finiteness and gauge-independence. Generalizing these prescriptions to the case of the Yukawa corrections, and combining them with the usual prescriptions for the top/stop parameters [14], we obtain a convenient OS renormalization scheme for the $\mathcal{O}(\alpha_t\alpha_b+\alpha_b^2)$ part of the corrections to the Higgs masses. Since the corrections controlled by the bottom Yukawa coupling can be sizeable only for large values of $\tan\beta$, we work directly in the physically relevant limit of $\tan\beta\to\infty$, i.e., $v_1\to 0$, $v_2\to v$.

For the OS squark masses and mixing angles, top quark mass and electroweak parameter $v\equiv(\sqrt{2}\,G_\mu)^{-1/2}$ we adopt the definitions

$$\delta m^2_{\tilde{q}_i}=\Pi^{\tilde{q}}_{ii}\big(m^2_{\tilde{q}_i}\big),\qquad \delta\theta_{\tilde{q}}=\frac{1}{2}\frac{\Pi^{\tilde{q}}_{12}(m^2_{\tilde{q}_1})+\Pi^{\tilde{q}}_{12}(m^2_{\tilde{q}_2})}{m^2_{\tilde{q}_1}-m^2_{\tilde{q}_2}},$$

$$\delta m_t=\Sigma_t(m_t),\qquad \delta v=\frac{v}{2}\frac{\Pi^T_{WW}(0)}{m^2_W},\tag{18}$$

where $\tilde{q}=(\tilde{t},\tilde{b})$, while $\Pi^{\tilde{q}}_{ij}(p^2)$, $\Sigma_t(p)$ and $\Pi^T_{WW}(p^2)$ denote the real and finite parts of the self-energies of squarks, top quark and W boson, respectively. Following Ref. [14], we further treat μ as a $\overline{\rm DR}$ parameter computed at a reference scale $Q_0=175$ GeV, and h_t and A_t as derived quantities that can be computed by means of the tree-level formulae for m_t and $s_{2\theta_t}$, respectively. In principle, we still have to define m_b, h_b and A_b. However, in the large $\tan\beta$ limit, the bottom mass is just zero, and the sbottom mixing angle becomes

$$s_{2\theta_b}=\frac{\sqrt{2}\,h_b\mu v}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}},\tag{19}$$

which is independent of m_b and A_b. We can thus treat h_b as a quantity derived from the sbottom mixing, and use Eqs. (18) and (19) to obtain a prescription for δh_b:

$$\delta h_b=h_b\left(\frac{\delta m^2_{\tilde{b}_1}-\delta m^2_{\tilde{b}_2}}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}+\frac{\delta s_{2\theta_b}}{s_{2\theta_b}}-\frac{\delta v}{v}\right).\tag{20}$$

In Ref. [15], an OS definition for the quantity $\tilde{A}_b\equiv h_bA_b$, or equivalently for $\delta A_b=(\delta\tilde{A}_b-\delta h_bA_b)/h_b$, was proposed in terms of the $(\tilde{b}_1\tilde{b}_2^*A)$ proper vertex $i\Lambda_{12A}(p_1^2,p_2^2,p_A^2)$ for the case of the strong corrections. A generalization of that definition that can also en-

compass the Yukawa corrections is given by

$$\begin{aligned}\delta\tilde{A}_b = &-\frac{i}{\sqrt{2}}\big[\Lambda_{12A}\big(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_1}, 0\big) + \Lambda_{12A}\big(m^2_{\tilde{b}_2}, m^2_{\tilde{b}_2}, 0\big)\big] \\ &+ \frac{\tilde{A}_b}{2}\Bigg[\frac{\Pi^{\tilde{b}}_{11}(m^2_{\tilde{b}_1}) - \Pi^{\tilde{b}}_{11}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}} + \frac{\Pi^{\tilde{b}}_{22}(m^2_{\tilde{b}_1}) - \Pi^{\tilde{b}}_{22}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}} \\ &\qquad + \frac{\Pi_{AA}(m^2_{\tilde{b}_1}) - \Pi_{AA}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}\Bigg]. \end{aligned} \tag{21}$$

Having fully specified our OS renormalization prescriptions in the limit $\tan\beta \to \infty$, physically relevant for the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections, we can proceed to obtain formulae for the CP-even Higgs masses in our OS scheme and merge them with the known $\mathcal{O}(\alpha_t^2)$ OS results [14] that contain an explicit dependence on $\tan\beta$. This can be done in three steps: first, we take the limit of $\tan\beta \to \infty, m_b \to 0$ in the general $\overline{\text{DR}}$ results for the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ part of $(\Delta\mathcal{M}^2_S)^{\text{eff}}$; then we add the contributions due to the shifts of the parameters entering the one-loop corrections (this requires the computation of the $\mathcal{O}(\alpha_t + \alpha_b)$ part of the counterterms in the large $\tan\beta$ limit); finally, we subtract from this results the pure $\mathcal{O}(\alpha_t^2)$ part which, being relevant for all values of $\tan\beta$, must instead be computed separately with the formulae of Ref. [14]. Notice that we do not encounter any terms that blow up when we take the limit of large $\tan\beta$ in the $\overline{\text{DR}}$ results: unphysically large contributions could only be introduced by hand, as the result of a poor choice of the renormalization conditions for the parameters in the bottom/sbottom sector.

We discuss now the parameters that we will actually use as inputs of our calculation. In particular, although we have used Eqs. (19), (20) to define an OS bottom Yukawa coupling h_b through the sbottom mixing, we still need to exploit the experimental information on the bottom mass in order to obtain the $\overline{\text{DR}}$ running coupling $\hat{h}_b$. The OS coupling will then be computed through the relation $h_b = \hat{h}_b - \delta h_b$. Following Ref. [15], we define the running coupling $\hat{h}_b$ at the reference scale $Q_0 = 175$ GeV to be

$$\hat{h}_b \equiv h_b(Q_0)^{\overline{\text{DR}}}_{\text{MSSM}} = \frac{\bar{m}_b\sqrt{2}}{v_1}\frac{1+\delta_b}{|1+\epsilon_b|}, \tag{22}$$

where: $\bar{m}_b \equiv m_b(Q_0)^{\overline{\text{DR}}}_{\text{SM}} = 2.74 \pm 0.05$ GeV is the standard model bottom mass, evolved up to the scale Q_0 to take into account the resummation of the universal large QCD logarithms; ϵ_b contains the $\tan\beta$-enhanced threshold corrections from both the gluino-sbottom and the higgsino-stop loops (denoted as ϵ_b and ϵ'_b, respectively, in Eqs. (25) and (26) of Ref. [15]); δ_b contains the residual threshold corrections that are not enhanced by $\tan\beta$. Notice that, as shown in Ref. [26], keeping ϵ_b in the denominator of Eq. (22) allows to resum the $\tan\beta$-enhanced threshold corrections to all orders in the perturbative expansion. On the other hand, there is no preferred way of including the threshold corrections parametrized by δ_b, whose effect on the value of $\hat{h}_b$ is anyway very small.

Neglecting all the terms controlled by the electroweak gauge couplings, ϵ_b reads

$$\epsilon_b = -\frac{2\alpha_s}{3\pi}\frac{m_{\tilde{g}}\mu\tan\beta}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}\left[\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{b}_1}-m^2_{\tilde{g}}}\ln\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{g}}}-\frac{m^2_{\tilde{b}_2}}{m^2_{\tilde{b}_2}-m^2_{\tilde{g}}}\ln\frac{m^2_{\tilde{b}_2}}{m^2_{\tilde{g}}}\right]$$
$$-\frac{\alpha_t}{4\pi}\frac{A_t\mu\tan\beta}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}\left[\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_1}-\mu^2}\ln\frac{m^2_{\tilde{t}_1}}{\mu^2}-\frac{m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_2}-\mu^2}\ln\frac{m^2_{\tilde{t}_2}}{\mu^2}\right]. \tag{23}$$

It appears from Eq. (22) that $\hat{h}_b$ blows up when ϵ_b approaches -1, in which case the correct value of the bottom mass cannot be reproduced with $\hat{h}_b$ in the perturbative regime, and the corresponding set of MSSM parameters must be discarded. It can also be noticed from Eq. (23) that, since we take $m_{\tilde{g}} > 0$, for $A_t > 0$ ($A_t < 0$) the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_t)$ contributions enter ϵ_b with the same (the opposite) sign. Moreover, if we take only the $\mathcal{O}(\alpha_s)$ part of ϵ_b into account, $\hat{h}_b$ can be enhanced by the threshold correction only for large values of $m_{\tilde{g}}$ and large and positive[2] values of μ, whereas, when we include the $\mathcal{O}(\alpha_t)$ part, $\hat{h}_b$ can be enhanced also for small values of $m_{\tilde{g}}$ and large and negative values of both μ and A_t.

For the top/stop sector, we take as input the current central value of the top pole mass, $m_t = 174.3$ GeV [27], and the parameters $(m_{Q,\tilde{t}}, m_U, A_t)$ that can be derived by rotating the diagonal matrix of the OS stop masses by the angle $\theta_{\tilde{t}}$, defined in Eq. (18). Concerning the sbottom sector, additional care is required, because of our non-trivial definition of h_b and of the fact that, at one loop, the parameter $m_{Q,\tilde{b}}$ entering the sbottom mass matrix differs from the corresponding stop parameter $m_{Q,\tilde{t}}$ by a finite shift [25]. We start by computing the renormalized coupling h_b as given by Eqs. (20) and (22). Then we compute $m_{Q,\tilde{b}}$ following the prescription of Ref. [25]. Finally, we use the parameters h_b and $m_{Q,\tilde{b}}$ to compute the actual values of the OS sbottom masses and mixing angle. Concerning the A-boson mass, which enters the tree-level mass matrix for the CP-even Higgses, we take as input the physical mass m_A, dropping the distinction between m_A and the effective potential mass $\bar{m}_A$ (this amounts to neglecting the effect of the uncomputed momentum-dependent two-loop corrections). The renormalization of the Z-boson mass, whose numerical value we take equal to the physical mass $m_Z = 91.187$ GeV, does not affect the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections. The remaining numerical inputs are the OS electroweak parameter $v = 246.218$ GeV and the strong coupling constant, which we fix as $\alpha_s(Q_0) = 0.108$.

5. Numerical results

We are now ready to discuss the numerical effect of our two-loop corrections. In the previous sections we have discussed how to express our results in either the $\overline{\text{DR}}$ renormalization scheme or an OS scheme suitably chosen to separate the genuine two-loop corrections from the threshold corrections in the relation between m_b and h_b. Our $\overline{\text{DR}}$ results for the two-loop corrections to the Higgs masses and to the electroweak symmetry

[2] Our convention for the sign of μ is such that, e.g., the sbottom mixing parameter reads $X_b = A_b + \mu\tan\beta$.

breaking conditions can be easily implemented in the existing codes [21,22] that compute the MSSM mass spectrum from a set of unified parameters at the scale $M_{\rm GUT}$. A study of the implications of our results in the framework of gravity (mSUGRA), gauge (GMSB) or anomaly (AMSB) mediated supersymmetry breaking models goes beyond the scope of this paper, and will appear elsewhere [28]. In the following discussion, we will adopt a low-energy point of view and assume that the various input parameters are related, when possible, to physical quantities. To this aim, we will make use of the OS renormalization scheme presented in Section 4. We recall that, although our OS prescription is defined in the limit $\tan\beta \to \infty$, the corrections have an indirect dependence on $\tan\beta$ coming from the input value for $\hat{h}_b$, see Eq. (22).

In Fig. 1(a) and (b) we show the light Higgs mass m_h as a function of $\tan\beta$, for $m_A = 120$ GeV and $m_A = 250$ GeV, respectively. The other input parameters are chosen as $A_t = 1$ TeV, $A_b = 2$ TeV, $\mu = m_{Q,\tilde{t}} = m_U = m_D = m_{\tilde{g}} = 1$ TeV. In each plot, the long-dashed curve corresponds to the value of m_h obtained at $\mathcal{O}(\alpha_t + \alpha_t\alpha_s + \alpha_t^2)$, i.e., by including only the one- and two-loop corrections controlled by the top Yukawa coupling; the dot-dashed curve includes in addition the one-loop $\mathcal{O}(\alpha_b)$ corrections, controlled by the bottom Yukawa coupling;[3] the short-dashed curve includes the two-loop $\mathcal{O}(\alpha_b\alpha_s)$ corrections computed in Ref. [15]; finally, the solid curve corresponds to the full two-loop Yukawa computation of m_h, i.e., it includes also the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections discussed in the previous sections. We can see from Fig. 1(a) and 1(b) that the corrections controlled by the top Yukawa coupling depend very weakly on $\tan\beta$ when the latter is large. On the other hand, the $\mathcal{O}(\alpha_b)$ corrections lower considerably m_h when $\tan\beta$ increases. Concerning the two-loop corrections controlled by the bottom Yukawa coupling, the comparison between the dot-dashed and short-dashed curves shows that the $\mathcal{O}(\alpha_b\alpha_s)$ corrections amount to a small fraction of the $\mathcal{O}(\alpha_b)$ ones, but they can still lower m_h by several GeV when $\tan\beta$ is large. The comparison between the short-dashed and solid curves shows that the effect of the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections can also amount to several GeV when $\tan\beta$ is large. From Fig. 1(a) we see that, when m_A is small and the correction to m_h is mainly driven by $(\Delta\mathcal{M}_S^2)_{11}$, the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections enter with the same sign as the $\mathcal{O}(\alpha_b\alpha_s)$ corrections, reducing further the value of m_h. On the other hand, Fig. 1(b) shows that for larger values of m_A, when the correction to m_h is sensitive to $(\Delta\mathcal{M}_S^2)_{22}$, the new corrections account for an increase in m_h of a few GeV at moderately large values of $\tan\beta$ (i.e., $\tan\beta \approx 30$–40). This is basically due to a positive contribution to $(\Delta\mathcal{M}_S^2)_{22}$ coming from the $\mathcal{O}(\alpha_t\alpha_b)$ part of corrections. When $\tan\beta$ takes on larger values, however, the overall effect of the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections to m_h turns again to negative.

It is interesting to realize that the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections can be sizeable also for parameter choices that make the $\mathcal{O}(\alpha_b\alpha_s)$ corrections irrelevant. In Fig. 2 we show m_h as a function of $\tan\beta$, for $m_A = 120$ GeV, $\mu = A_t = -2$ TeV, $A_b = -3$ TeV, $m_{Q,\tilde{t}} = m_U = m_D = 1$ TeV and $m_{\tilde{g}} = 200$ GeV. The meaning of the various curves is the same as in Fig. 1. Due to the small value of the gluino mass with respect to the sbottom masses, the $\mathcal{O}(\alpha_b\alpha_s)$ corrections to m_h are negligible (in fact, the dot-dashed and short-dashed curves

[3] In the calculation of the one-loop $\mathcal{O}(\alpha_t)$ and $\mathcal{O}(\alpha_b)$ corrections we include the effects proportional to m_Z^2 and the momentum corrections as in [5].

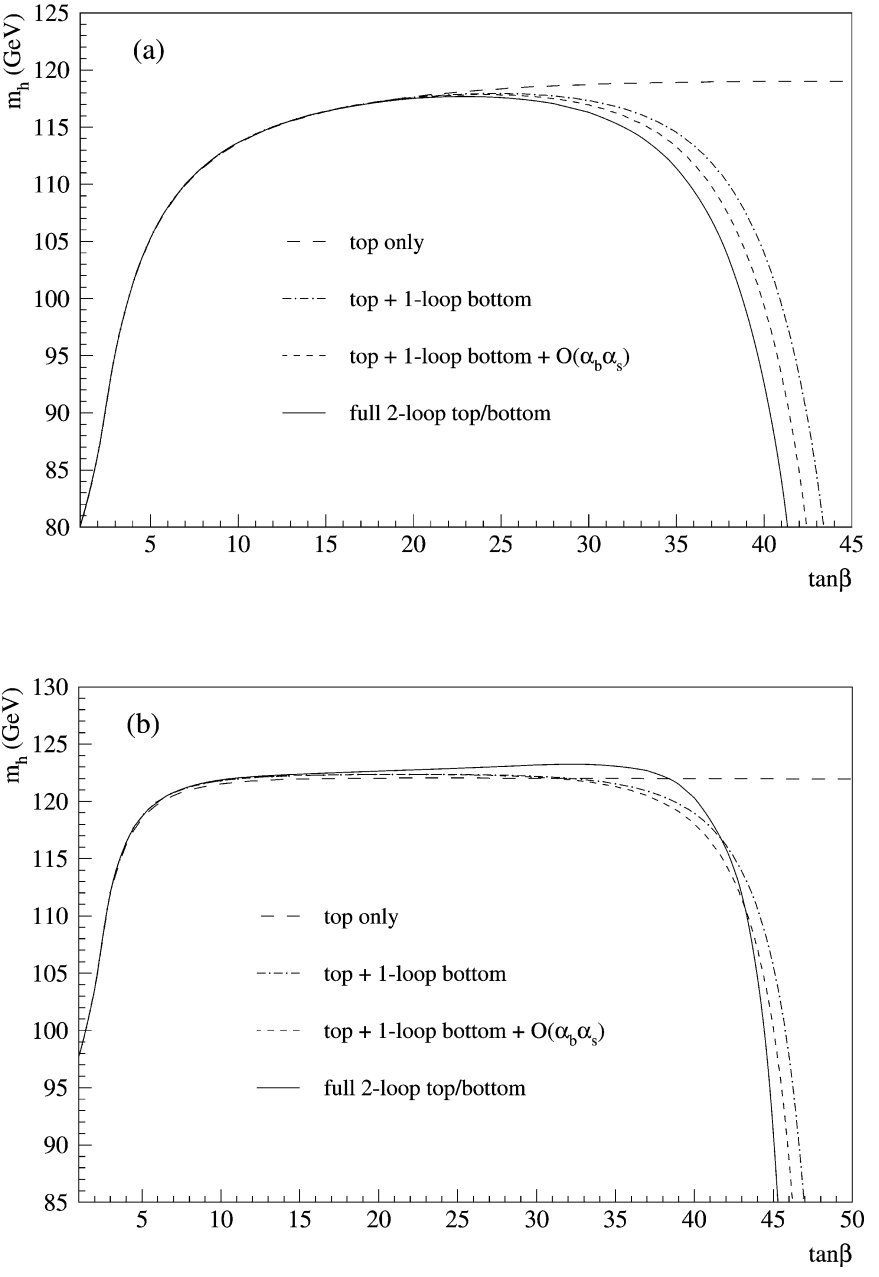

Fig. 1. The mass m_h as a function of $\tan\beta$, for $m_A = 120$ GeV (upper panel) or 250 GeV (lower panel). The other input parameters are $A_t = 1$ TeV, $A_b = 2$ TeV, $\mu = m_{Q,\tilde{t}} = m_U = m_D = m_{\tilde{g}} = 1$ TeV. The meaning of the different curves is explained in the text.

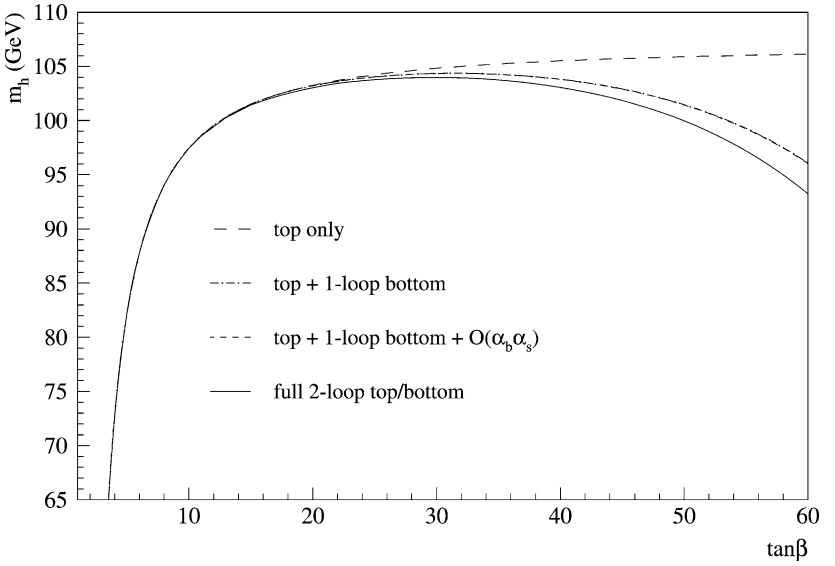

Fig. 2. The mass m_h as a function of $\tan\beta$, for $m_A = 120$ GeV, $\mu = A_t = -2$ TeV, $A_b = -3$ TeV, $m_{Q,\tilde{t}} = m_U = m_D = 1$ TeV and $m_{\tilde{g}} = 200$ GeV. The meaning of the different curves is explained in the text.

overlap). On the other hand, comparing the short-dashed and solid curves we see that the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections can still amount to a few GeV when $\tan\beta$ is large enough.

Finally, Fig. 3 shows both CP-even Higgs masses, m_h and m_H, as functions of the CP-odd Higgs mass, in the region of relatively small m_A (100 GeV $< m_A <$ 180 GeV), for $\tan\beta = 40$. The other input parameters are chosen as $A_t = 1$ TeV, $A_b = 2$ TeV, $\mu = m_{Q,\tilde{t}} = m_U = m_D = m_{\tilde{g}} = 1$ TeV. The meaning of the various curves is the same as in Fig. 1. Comparing the short-dashed and solid curves we see that, for this choice of parameters, the effect of the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections is particularly evident in the region where m_A is small, and can account for variations of several GeV (around 5 in this example) in both m_h and m_H.

In general, it appears from Figs. 1–3 that the two-loop $\mathcal{O}(\alpha_b\alpha_s)$ and $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections are usually a small fraction of the one-loop $\mathcal{O}(\alpha_b)$ ones. We stress that this is a desirable consequence of our renormalization prescription, which allows to set apart the $\tan\beta$-enhanced threshold corrections, resummed to all orders in the renormalized coupling h_b. If we were to adopt for the bottom/sbottom sector the same renormalization prescription that we use for the top/stop sector, the dependence on $\tan\beta$ of the one-loop corrections would be smoother, but very large corrections would appear at two loops, questioning the validity of the perturbative expansion.

To conclude this section, we notice that our knowledge of the general formulae for the corrections to the CP-even Higgs mass matrix in the $\overline{\text{DR}}$ scheme allows us to estimate the uncertainty connected with the fact that we take the limit of $\tan\beta \to \infty, m_b \to 0$ in the corresponding OS results. In the numerical examples considered above we find that, in the regions where the corrections are sizeable, the $\overline{\text{DR}}$ results for the $\mathcal{O}(\alpha_b\alpha_s)$ part of $(\Delta\mathcal{M}_S^2)^{\text{eff}}$ vary by less than 20% when the limit $\tan\beta \to \infty, m_b \to 0$ is taken. The $\overline{\text{DR}}$

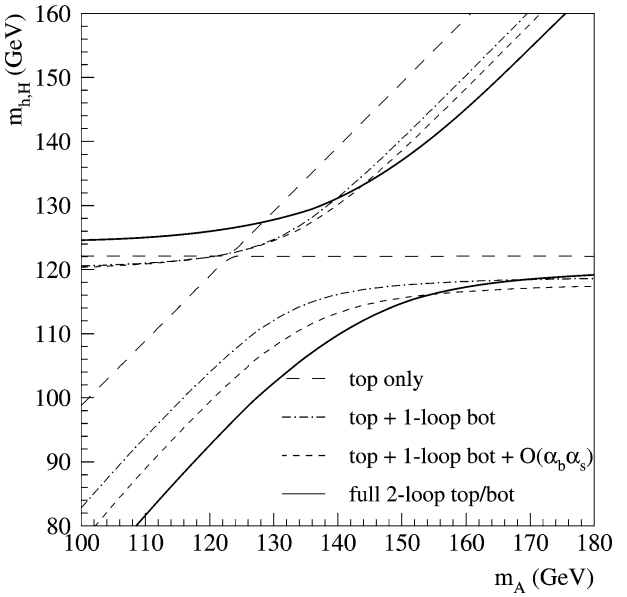

Fig. 3. The masses m_h and m_H as a function of m_A, for $\tan\beta = 40$, $A_t = 1$ TeV, $A_b = 2$ TeV, $\mu = m_{Q,\tilde{t}} = m_U = m_D = m_{\tilde{g}} = 1$ TeV. The meaning of the different curves is explained in the text.

results for the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ part of the corrections vary instead by less than 10%. We can assume that similar variations occurr also in the corresponding OS results, which leads to shifts in m_h typically smaller than 1 GeV.

6. Conclusions and discussion

In this paper we computed the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections to the MSSM neutral Higgs boson masses and to the minimization conditions of the MSSM effective potential. Such corrections are relevant when the ratio of the Higgs VEVs, $\tan\beta$, is large. Combined with the previously computed $\mathcal{O}(\alpha_t\alpha_s)$ [13], $\mathcal{O}(\alpha_t^2)$ [14] and $\mathcal{O}(\alpha_b\alpha_s)$ [15] corrections to the neutral Higgs masses, and with the corresponding corrections to the minimization conditions of the effective potential [16], these results provide us with a complete computation of the leading two-loop corrections controlled by the top and bottom Yukawa couplings.

Using the formalism of the effective potential, we obtained complete analytic expressions for the momentum-independent part of the $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ corrections, valid for arbitrary values of the MSSM input parameters when the latter are expressed in the $\overline{\text{DR}}$ renormalization scheme. We also discussed an OS renormalization prescription for the parameters of the bottom/sbottom sector in the physically relevant limit of large $\tan\beta$. Such prescription, first introduced in Ref. [15] for the case of the $\mathcal{O}(\alpha_b\alpha_s)$ corrections, allows to separate the large threshold corrections appearing in the relation between h_b and the pole bottom mass from the genuine two-loop effects. Finally, we discussed the numerical

impact of our results in a few representative examples, showing that, for large values of $\tan\beta$, the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ corrections can induce variations in the Higgs masses of the order of a few GeV. Since our analytic expressions, both in the $\overline{\rm DR}$ and the OS schemes, are too long to be useful if explicitly written on paper, we choose to make them available in the form of a Fortran code.

Although the terms controlled by the top and bottom Yukawa couplings undoubtedly account for the bulk of the two-loop corrections, several pieces are still missing for a complete two-loop computation of the MSSM Higgs masses. When $\tan\beta$ is large, the corrections controlled by the tau Yukawa coupling h_τ might in principle be non-negligible. In the approximation of neglecting the electroweak gauge couplings, the only two-loop corrections involving the tau Yukawa coupling are those of $\mathcal{O}(\alpha_b\alpha_\tau)$ and those of $\mathcal{O}(\alpha_\tau^2)$, where $\alpha_\tau = h_\tau^2/(4\pi)$. While the mixed $\mathcal{O}(\alpha_b\alpha_\tau)$ corrections would require a dedicated computation,[4] explicit formulae for the $\mathcal{O}(\alpha_\tau^2)$ corrections can be obtained from the formulae of Refs. [14,16] for the purely $\mathcal{O}(\alpha_t^2)$ corrections, with the replacements $t \to \tau$, $\tilde{b}_L \to \tilde{\nu}_\tau$, $N_c \to 1$ and $H_1 \leftrightarrow H_2$ (the latter resulting into $\tan\beta \leftrightarrow \cot\beta$, $v_1 \leftrightarrow v_2$, $(\Delta\mathcal{M}_S^2)_{11}^{\rm eff} \leftrightarrow (\Delta\mathcal{M}_S^2)_{22}^{\rm eff}$ and $\Sigma_1 \leftrightarrow \Sigma_2$). If the input parameters are given in the OS scheme, a suitable definition of δA_τ is required in order to avoid introducing $\tan\beta$-enhanced terms in the two-loop part of the result. Anyway, we find that the $\mathcal{O}(\alpha_\tau^2)$ corrections to the Higgs masses are in general very small compared with those controlled by the bottom Yukawa coupling. Besides the hierarchy between m_b and m_τ, the suppression of the tau corrections is motivated by the absence of color enhancements, and by the fact that the only $\tan\beta$-enhanced threshold corrections to the relation between h_τ and m_τ are those controlled by the electroweak gauge couplings.

A full two-loop determination of the MSSM Higgs masses will require going beyond the gaugeless limit and the effective potential approximation, i.e., including both the corrections controlled by the electroweak gauge couplings and the effect of the momentum-dependent part of the Higgs self-energies. It can also be expected that, among the three-loop corrections, at least those involving the top Yukawa coupling affect the Higgs masses in a non-negligible way. In Ref. [18] the two-loop, zero-momentum electroweak corrections have been computed numerically in a representative scenario, and found to yield a shift in the lightest Higgs boson mass m_h of about 1 GeV with respect to the result obtained in the gaugeless approximation. In Ref. [29], the theoretical uncertainty in the prediction for m_h arising from the combined effect of the missing two-loop corrections and the leading three-loop corrections has been estimated to be around 3 GeV.

If the MSSM is a viable theory for physics at the weak scale, a light Higgs boson will be discovered either at the Tevatron or at the LHC. Subsequently, its properties will be determined with high precision at a future linear collider: for example, the predicted experimental accuracy in the determination of m_h at TESLA is about 50 MeV [30]. It is thus clear that further effort will be required in the coming years, in order to improve the accuracy of the theoretical predictions up to the level required to compare with the experimental results expected at the next generation of colliders.

[4] We thank A. Brignole for drawing our attention on this point.

Acknowledgements

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Appendix A

We present here the expressions for the functions F_i^t $(i = 1, 2, 3, 4)$, F_5, F_6, F^t, G^t and F_A, appearing in Eqs. (12)–(17), in terms of derivatives of the $\overline{\text{DR}}$-renormalized ΔV, computed at the minimum of V_{eff}:

$$
\begin{aligned}
F_1^t = {} & \frac{\partial^2 \Delta V}{(\partial m_t^2)^2} + \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_1}^2)^2} + \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_2}^2)^2} + 2\frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde{t}_1}^2} + 2\frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde{t}_2}^2} + 2\frac{\partial^2 \Delta V}{\partial m_{\tilde{t}_1}^2 \partial m_{\tilde{t}_2}^2} \\
& + \frac{1}{4m_t^4}\left(\frac{\partial \Delta V}{\partial c_{\varphi_t+\varphi_b}} + z_t \frac{\partial \Delta V}{\partial c_{\varphi_t-\tilde{\varphi}_t}} + z_b \frac{\partial \Delta V}{\partial c_{\varphi_t+\tilde{\varphi}_b}} \right), \qquad \text{(A.1)} \\
F_2^t = {} & \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_1}^2)^2} - \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_2}^2)^2} + \frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde{t}_1}^2} - \frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde{t}_2}^2} \\
& - \frac{4c_{2\theta_t}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left(\frac{\partial^2 \Delta V}{\partial c_{2\tilde{\theta}_t}^2 \partial m_t^2} + \frac{\partial^2 \Delta V}{\partial c_{2\tilde{\theta}_t}^2 \partial m_{\tilde{t}_1}^2} + \frac{\partial^2 \Delta V}{\partial c_{2\tilde{\theta}_t}^2 \partial m_{\tilde{t}_2}^2} \right) \\
& - \frac{z_t}{s_{2\theta_t}^2 m_t^2 (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} \frac{\partial \Delta V}{\partial c_{\varphi_t-\tilde{\varphi}_t}}, \qquad \text{(A.2)} \\
F_3^t = {} & \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_1}^2)^2} + \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_2}^2)^2} - 2\frac{\partial^2 \Delta V}{\partial m_{\tilde{t}_1}^2 \partial m_{\tilde{t}_2}^2} - \frac{2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left(\frac{\partial \Delta V}{\partial m_{\tilde{t}_1}^2} - \frac{\partial \Delta V}{\partial m_{\tilde{t}_2}^2} \right) \\
& + \frac{16c_{2\theta_t}^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2}\left(c_{2\theta_t}^2 \frac{\partial^2 \Delta V}{(\partial c_{2\tilde{\theta}_t}^2)^2} + 2\frac{\partial \Delta V}{\partial c_{2\tilde{\theta}_t}^2} \right) \\
& - \frac{8c_{2\theta_t}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left(\frac{\partial^2 \Delta V}{\partial c_{2\tilde{\theta}_t}^2 \partial m_{\tilde{t}_1}^2} - \frac{\partial^2 \Delta V}{\partial c_{2\tilde{\theta}_t}^2 \partial m_{\tilde{t}_2}^2} \right) \\
& + \frac{4z_t}{s_{2\theta_t}^4 (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2}\left(\frac{\partial \Delta V}{\partial c_{\varphi_t-\tilde{\varphi}_t}} + \frac{\partial \Delta V}{\partial c_{\varphi_b+\tilde{\varphi}_t}} + z_b \frac{\partial \Delta V}{\partial c_{\tilde{\varphi}_t+\tilde{\varphi}_b}} \right), \qquad \text{(A.3)}
\end{aligned}
$$

$$
\begin{aligned}
F_4^t = {} & \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_1}^2\,\partial m_b^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_1}^2\,\partial m_{\tilde{b}_1}^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_1}^2\,\partial m_{\tilde{b}_2}^2} - \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_2}^2\,\partial m_b^2} - \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_2}^2\,\partial m_{\tilde{b}_1}^2} - \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_2}^2\,\partial m_{\tilde{b}_2}^2} \\
& - \frac{4c_{2\theta_t}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left(\frac{\partial^2\Delta V}{\partial m_{\tilde{b}_1}^2\,\partial c_{2\bar{\theta}_t}^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{b}_2}^2\,\partial c_{2\bar{\theta}_t}^2} + \frac{\partial^2\Delta V}{\partial m_b^2 \partial c_{2\bar{\theta}_t}^2}\right) \\
& - \frac{z_t}{s_{2\theta_t}^2 m_b^2 (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}\frac{\partial\Delta V}{\partial c_{\varphi_b+\tilde{\varphi}_t}},
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
F_5 = {} & \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_1}^2\,\partial m_{\tilde{b}_1}^2} - \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_1}^2\,\partial m_{\tilde{b}_2}^2} - \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_2}^2\,\partial m_{\tilde{b}_1}^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_2}^2\,\partial m_{\tilde{b}_2}^2} \\
& - \frac{4c_{2\theta_t}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left(\frac{\partial^2\Delta V}{\partial m_{\tilde{b}_1}^2\,\partial c_{2\bar{\theta}_t}^2} - \frac{\partial^2\Delta V}{\partial m_{\tilde{b}_2}^2\,\partial c_{2\bar{\theta}_t}^2}\right) \\
& - \frac{4c_{2\theta_b}^2}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2}\left(\frac{\partial^2\Delta V}{\partial m_{\tilde{t}_1}^2\,\partial c_{2\bar{\theta}_b}^2} - \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_2}^2\,\partial c_{2\bar{\theta}_b}^2}\right) \\
& + \frac{16c_{2\theta_t}^2 c_{2\theta_b}^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)}\frac{\partial^2\Delta V}{\partial c_{2\bar{\theta}_t}^2\,\partial c_{2\bar{\theta}_b}^2} \\
& - \frac{4z_t z_b}{s_{2\theta_t}^2 s_{2\theta_b}^2 (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)}\frac{\partial\Delta V}{\partial c_{\tilde{\varphi}_t+\tilde{\varphi}_b}},
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
F_6 = {} & \frac{\partial^2\Delta V}{\partial m_t^2 \partial m_b^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_1}^2\,\partial m_b^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_2}^2\,\partial m_b^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{b}_1}^2\,\partial m_t^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{b}_2}^2\,\partial m_t^2} \\
& + \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_1}^2\,\partial m_{\tilde{b}_1}^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_1}^2\,\partial m_{\tilde{b}_2}^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_2}^2\,\partial m_{\tilde{b}_1}^2} + \frac{\partial^2\Delta V}{\partial m_{\tilde{t}_2}^2\,\partial m_{\tilde{b}_2}^2} - \frac{1}{4m_t^2 m_b^2}\frac{\partial\Delta V}{\partial c_{\varphi_t+\varphi_b}},
\end{aligned}
\tag{A.6}
$$

$$
F^t = \frac{\partial\Delta V}{\partial m_{\tilde{t}_1}^2} - \frac{\partial\Delta V}{\partial m_{\tilde{t}_2}^2} - \frac{4c_{2\theta_t}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\frac{\partial\Delta V}{\partial c_{2\theta_t}^2},
\tag{A.7}
$$

$$
G^t = \frac{\partial\Delta V}{\partial m_t^2} + \frac{\partial\Delta V}{\partial m_{\tilde{t}_1}^2} + \frac{\partial\Delta V}{\partial m_{\tilde{t}_2}^2},
\tag{A.8}
$$

$$
\begin{aligned}
F_A = {} & \frac{1}{4m_t m_b}\frac{\partial\Delta V}{\partial c_{\varphi_t+\varphi_b}} + \frac{4(A_t A_b - \mu^2)^2 m_t m_b z_t z_b}{s_{2\theta_t}^2 s_{2\theta_b}^2 (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2 (m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)^2}\frac{\partial\Delta V}{\partial c_{\tilde{\varphi}_t+\tilde{\varphi}_b}} \\
& + \frac{m_t z_t}{s_{2\theta_t}^2 m_b (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2}\left(A_t^2\frac{\partial\Delta V}{\partial c_{\varphi_b+\tilde{\varphi}_t}} + \mu^2\cot^2\beta\frac{\partial\Delta V}{\partial c_{\varphi_t-\tilde{\varphi}_t}}\right) \\
& + \frac{m_b z_b}{s_{2\theta_b}^2 m_t (m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)^2}\left(A_b^2\frac{\partial\Delta V}{\partial c_{\varphi_t+\tilde{\varphi}_b}} + \mu^2\tan^2\beta\frac{\partial\Delta V}{\partial c_{\varphi_b-\tilde{\varphi}_b}}\right).
\end{aligned}
\tag{A.9}
$$

In the above formulae, $z_q \equiv \text{sign}(X_q)$,[5] where X_q $(q = t, b)$ is the squark mixing parameter. The functions F_i^b, F^b and G^b can be obtained from their top counterparts through the replacement $t \leftrightarrow b$.

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