On the Unitarity Limits of Top-Quark and Higgs-Boson Masses

1990 Europhys. Lett. 12 29

(http://iopscience.iop.org/0295-5075/12/1/006)

EUROPHYSICS LETTERS 1 May 1990

Europhys. Lett., 12 (1), pp. 29-33 (1990)

On the Unitarity Limits of Top-Quark and Higgs-Boson Masses.

C. E. VAYONAKIS

Department of Physics, University of Ioannina, Division of Theoretical Physics Ioannina GR-45110, Ioannina, Greece

(received 3 January 1990; accepted 19 February 1990)

PACS. 14.80D – Quarks and gluons.
PACS. 14.80K – Others (inc. tachyons).

Abstract. – We discuss upper limits on the masses of the top quark and the Higgs boson, as they are derived from unitarity bound and renormalization group considerations. The bounds depend on the scale at which some new physics will appear.

Within the standard model, the most notable parameters which still evade experimental determination are the top-quark mass and the Higgs-boson mass. Recent experimental data from SLC [1] and LEP [2] suggest that the top-quark mass should be in the range $80\,\text{GeV} \leqslant m_t \leqslant 180\,\text{GeV}$, whereas an up-to-date analysis of neutral current data [3] gives m_t around 135 GeV. On the other hand, Higgs boson is as elusive as ever, but most indications are that it is a rather heavy particle, with prospects to be searched at LEP [2, 4] and other future e^+e^- and hadron colliders [5].

On the theoretical side, there are various approaches towards a possible determination of the masses of these particles, before their discovery. These include the use of fixed points in the renormalization group equations [6], the scheme of the reduction of coupling constants [7] and the triviality arguments [8]. Most recently, dynamical symmetry breaking approaches have been also considered [9] (with the Higgs boson appearing, for example, as a $\bar{t}t$ bound state).

In the present letter, we will just consider upper bounds on top-quark and Higgs-boson masses, as inferred from the requirement of perturbative unitarity for the Born amplitudes of relevant elastic-scattering processes. Our improvement on the previous considerations is that we demand the unitarity constraints on the constants involved (top-quark Yukawa coupling and Higgs self-interaction coupling) to be also valid at a scale Λ, where the standard model, considered as an effective, low-energy theory, is replaced by a deeper theory, valid above Λ.

Since, historically, the above unitarity constraint has been first applied to the Higgs-boson mass, we start from that. The unitarity bound on the Higgs boson mass is obtained by considering the S-wave amplitudes for the scattering of longitudinal vector bosons and Higgs bosons, in the limit $s \gg m_W^2$, m_Z^2, m_H^2. Note that the interactions of the Higgs particle H and the longitudinal polarizations of the W and Z are described by the equivalence theorem [10-13] for the weak interactions, which states that S-matrix elements involving longitudinal vector bosons are equal, in the limit $s \gg m_W^2$, m_Z^2, to S-matrix elements with the longitudinal vector bosons replaced by their associated Goldstone bosons (for different physical interpretations of that theorem see ref. [14]; for extension to supersymmetry, see the review article of ref. [15]). The most refined bound on the Higgs mass is obtained [12] by considering the requirements of S wave unitarity on the four-channel system consisting of $W_L^+ W_L^-$, $1/\sqrt{2} Z_L Z_L$, $1/\sqrt{2} HH$ and HZ_L. Then, in the limit $s \gg m_W^2$, m_Z^2, m_H^2, the resulting 4×4 matrix of amplitudes has the biggest eigenvalue

$$a_0 \xrightarrow[s \gg m_W^2, m_Z^2, m_H^2]{} -\frac{G_F m_H^2}{4\pi\sqrt{2}} \cdot \frac{3}{2} = -\frac{3\lambda}{8\pi}, \tag{1}$$

where

$$\lambda = \frac{m_H^2}{2v^2} = \frac{g_2^2 m_H^2}{8 m_W^2} = \frac{G_F m_H^2}{\sqrt{2}}$$

is the Higgs self-coupling. The amplitude a_0 of eq. (1) corresponds to the (unrenormalized) eigenchannel $2W_L^+ W_L^- + Z_L Z_L + HH$ and the simple relation (1) of course reflects the symmetries of the underlying Higgs-Goldstone system [10-13]. Requiring now the unitarity condition [12, 16]

$$|a_0| \leq \frac{1}{2}, \tag{2}$$

we get

$$\frac{3\lambda(\Lambda)}{8\pi} \leq \frac{1}{2}, \tag{3}$$

where now we have explicitly denoted the dependence of λ on the scale Λ, on which we will come back immediately below.

Next, let us consider the S-wave amplitude for the colour-singlet elastic scattering $\bar{t}t$, in the limit $s \gg m_t^2$, m_H^2, m_Z^2. Although quark scattering amplitudes are not directly observable, if quarks are confined, nevertheless it is correct, within our spirit of reasoning, to insist that they also satisfy partial-wave unitarity. Then, the above amplitude gives [17]

$$a_0 \xrightarrow[s \gg m_W^2, m_Z^2, m_H^2]{} -\frac{3 G_F m_t^2}{4\pi\sqrt{2}} = -\frac{3 g_t^2}{16\pi}, \tag{4}$$

where

$$g_t^2 = \frac{2 m_t^2}{v^2} = 2\sqrt{2}\, G_F m_t^2$$

is the top-quark Yukawa coupling. Requiring again the unitarity condition $|a_0| \leq 1/2$, we get now the bound

$$\frac{3 g_t^2(\Lambda)}{16\pi} \leq \frac{1}{2}, \tag{5}$$

where, as before, we have put explicitly the scale dependence of the Yukawa coupling g_t.

In expressions (3) and (5), the couplings depend on the scale Λ, at which perturbative unitarity will be violated. What actually happens here is the following. The couplings of the standard model are scale dependent for energies up to a scale Λ, which we propose to be the scale where perturbative unitarity is violated and where some new physics will appear, in the sense that the standard model, considered as an effective theory valid below Λ, is replaced by a deeper theory, valid above Λ. In that way, triviality of the scalar field theories may be avoided (see the review article of ref. [8]). In any case, we demand the perturbative unitarity constraints (3) and (5) to be valid at Λ and it is the renormalization group which governs the evolution of the coupling constants up to that scale. At the one-loop level, we recall (see, for example, the first review article of ref. [5], where also a discussion is presented for the Higgs-boson mass upper bounds) that for λ and g_t the relevant full equations are

$$\frac{d\lambda}{dt}=\frac{1}{8\pi^2}\left[12\lambda^2+6\lambda g_t^2-3g_t^4-\frac{9}{2}\lambda g_2^2-\frac{3}{2}\lambda g_1^2+\frac{3}{16}[2g_2^4+(g_2^4+2g_1^2)^2]\right], \tag{6}$$

$$\frac{dg_t^2}{dt}=\frac{1}{8\pi^2}\left(\frac{9}{2}g_t^2-8g_3^2-\frac{9}{4}g_2^2-\frac{17}{12}g_1^2\right)g_t^2. \tag{7}$$

Note that g_t does not depend on λ, at this level. For the gauge couplings, which are independent of both λ and g_t at the one-loop level, we have

$$\frac{dg_3^2}{dt}=\frac{1}{8\pi^2}g_3^4\left(-11+\frac{4}{3}\cdot n_g\right)=-\frac{7}{8\pi^2}g_3^4, \tag{8}$$

$$\frac{dg_2^2}{dt}=\frac{1}{8\pi^2}g_2^4\left(-\frac{22}{3}+\frac{4}{3}n_g+\frac{1}{6}n_H\right)=-\frac{1}{8\pi^2}\cdot\frac{19}{6}g_2^2, \tag{9}$$

$$\frac{dg_1^2}{dt}=\frac{1}{8\pi^2}g_1^4\left(\frac{20}{9}n_g+\frac{1}{6}n_H\right)=\frac{1}{8\pi^2}\cdot\frac{41}{6}g_1^4. \tag{10}$$

In the above equations $t=\ln\mu$ as usual and we take $n_g=3$ fermion generations and $n_H=1$ Higgs boson for the standard model. In the complete theory, one should take care of all equations (6)-(10). However, for our purposes, since we are dealing with heavy-top-quark and Higgs-boson masses and since we are looking for upper limits, it suffices to take into account only λ, g_t and g_3 and drop g_2 and g_1. Then, in a first approximation, it is enough to consider the system of coupled equations

$$\frac{d\lambda}{dt}=\frac{1}{8\pi^2}(12\lambda^2+6\lambda g_t^2-3g_t^4), \tag{11}$$

$$\frac{dg_t^2}{dt}=\frac{1}{8\pi^2}\left(\frac{9}{2}g_t^4-8g_3^2g_t^2\right), \tag{12}$$

$$\frac{dg_3^2}{dt}=-\frac{1}{8\pi^2}\cdot 7g_3^4. \tag{13}$$

The system of coupled differential equations (11)-(13) has been studied in the past, using different approaches [6-8]. In our case, we solve it numerically, using as «initial» conditions

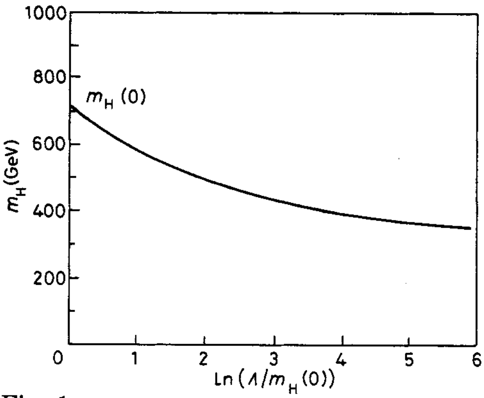

Fig. 1.

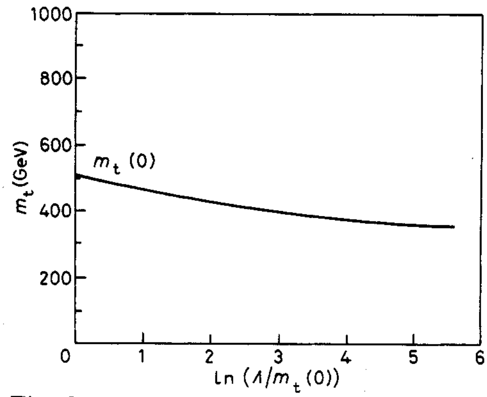

Fig. 2.

Fig. 1. – The unitarity bound on Higgs-boson mass as a function of the scale Λ.

Fig. 2. – The unitarity bound on top-quark mass as a function of the scale Λ.

the relations

$$\lambda(m_{\rm H}) = \frac{G_{\rm F}\, m_{\rm H}^2}{\sqrt{2}}, \tag{14}$$

$$g_{\rm t}^2(m_{\rm t}) = 2\sqrt{2}G_{\rm F}\, m_{\rm t}^2 \tag{15}$$

and the value

$$\frac{g_3^2(m_{\rm W})}{4\pi} = 0.107. \tag{16}$$

Taking all these, we can translate the bounds given by eqs. (3) and (5) into bounds on Higgs-boson and top-quark masses as functions of Λ. These are depicted grafically in fig. 1 and 2, respectively.

Figure 1 shows the dependence of the unitarity bound of $m_{\rm H}$ on the scale $\ln(\Lambda/m_{\rm H}(0))$, where $m_{\rm H}(0) = (4\pi\sqrt{2}/3G_{\rm F})^{1/2} = 713$ GeV is the old value of the unitarity bound. As we see, the unitarity bound becomes more stringent with increasing value of the scale, in which, as we said, some new physics can be expected. Note that only for an extremely large scale Λ (close to the Landau pole) the unitarity bounds get transformed into triviality bounds, whereas for smaller values of Λ the unitarity bounds are stronger than triviality bounds. Similarly, fig. 2 shows the unitarity bound on $m_{\rm t}$ as a function of the scale $\ln(\Lambda/m_{\rm t}(0))$, where $m_{\rm t}(0) = (2\pi\sqrt{2}/3G_{\rm F})^{1/2} \approx 500$ GeV is the old unitarity bound. These figures show that for any Higgs-boson and top-quark mass there is an energy, at which some new physics will appear. As an example, in the scheme of dynamical symmetry breaking of Bardeen *et al.* [9], we see that their favoured results $m_{\rm t} \approx 230$ GeV and $m_{\rm H} \approx 260$ GeV with $\Lambda \approx 10^{13} \div 10^{15}$ GeV are what is expected from our bounds. On the other hand, in our approach, the presently favoured experimental range for $m_{\rm t}$ around 135 GeV signifies that the scale of the supposedly new physics lies somewhere beyond the Planck mass region. Here, once again, experiment will have the final word on what could exist ahead.

* * *

We wish to thank the CERN Theory Division (where we acknowledge critical comments by S. POKORSKI) for partial support and hospitality and, also, the Greek Ministry of Research and Technology for partial support.

REFERENCES

[1] SLAC Lepton-Photon Symposium, Summary given at CERN by J. KIRKBY, August 29, 1989.
[2] LEP Physics Workshop, CERN September 4-5, 1989.
[3] ELLIS J. and FOGLI G. F., CERN preprint CERN-TH. 5457/89 and to be published.
[4] DREES M. *et al.*, CERN preprint CERN-TH. 5487/89 (1989), to appear in the *Proceedings of the Workshop on* Z *Physics at* LEP, edited by G. ALTARELLI, R. KLEISS and C. VERZEGNASSI.
[5] For reviews, see DAWSON S., GUNION J. F., HABER H. E. and KANE G. K., *The Higgs Hunter's Guide*, to be published; CHANOWITZ M. S., *Ann. Rev. Nucl. Part. Sci.*, 38 (1988) 323; CAHN R. N., *Rep. Prog. Phys.*, 52 (1989) 389.
[6] PENDLETON B. and ROSS G. G., *Phys. Lett. B*, **98** (1981) 291; HILL C. T., *Phys. Rev. D*, **24** (1981) 691; HILL C. T., LEUNG C. N. and RAO S., *Nucl. Phys. B*, **262** (1985) 517.
[7] KUBO J., SIBOLD K. and ZIMMERMAN W., *Nucl. Phys. B*, **259** (1985) 331; *Phys. Lett. B*, **220** (1989) 185, 191.
[8] MAIANI L., PARISI G. and PETRONZIO R., *Nucl. Phys. B*, **136** (1978) 115; CABBIBO N., MAIANI L., PARISI G. and PETRONZIO R., *Nucl. Phys. B*, **158** (1979) 295; DASHEN R. and NEUBERGER H., *Phys. Rev. Lett.*, **50** (1983) 1847; BEG M. A., PANAGIOTAKOPOULOS C. and SIRLIN A., *Phys. Rev. Lett.*, 52 (1984) 883; HASENFRATZ A., NEUHAUS T., JANSEN K., HONEYAMA H. and LANG C. B., *Phys. Lett. B*, **199** (1987) 531; HASENFRATZ P. and NAGER J., *Z. Phys. C*, **37** (1988) 477; LÜSHER M. and WEISZ P., *Phys. Lett. B*, **212** (1988) 472; for a review, see CALLAWAY D. J. E., *Phys. Rep.*, **167** (1988) 241.
[9] NAMBU Y., Enrico Fermi Institute preprint EFI-89-08 (1989); MARCIANO W., *Phys. Rev. Lett.*, **62** (1989) 2793; MIRANSKY V. A., TANABASHI M. and YAMAWAKI K., *Mod. Phys. Lett. A*, **4** (1989) 1043; *Phys. Lett. B*, **221** (1989) 177; BARDEEN W. A., HILL C. T. and LINDNER M., Fermilab, preprint FERMI-PUB-89/127-T (1989).
[10] CORNWALL J., LEVIN D. and TIKTOPOULOS G., *Phys. Rev. D*, **10** (1974) 1145.
[11] VAYONAKIS C. E., *Lett. Nuovo Cimento*, **17** (1976) 383.
[12] LEE B. W., QUIGG C. and THACKER H., *Phys. Rev. D*, **16** (1977) 1519.
[13] CHANOWITZ M. S. and GAILLARD M. K., *Nucl. Phys. B*, **261** (1985) 379.
[14] BAGGER J. and SCHMIDT C., Harvard preprint HUTP-89/A030 (1989).
[15] GATTO R., UGVA-DPT 1989/06-619 (1989).
[16] LÜSHER M. and WEISZ P., ref. [8].
[17] CHANOWITZ M. S., FURMAN M. A. and HINCHLIFFE I., *Nucl. Phys. B*, **153** (1979) 402.