Peculiar Velocities and Microwave Background Anisotropies from Cosmic Strings

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ABSTRACT: Using an analytic model we show that the predictions of the cosmic string model for the peculiar velocities and the microwave background (MBR) anisotropy depend on similar combinations of string evolution parameters. Normalizing from the COBE detection of MBR anisotropy, and for certain reasonable values of string network evolution parameters, we find that the magnitude of predicted velocity flows is in good agreement with observations on small scales but is inconsistent with observations on large scales ($> 50h^{-1}$Mpc). The Cosmic Mach Number obtained from the cosmic string scenario is found to depend on a single network evolution parameter and is consistent with observations on scales $5h^{-1}$Mpc to $20h^{-1}$Mpc.

The cosmic string model for large-scale structure formation with hot dark matter promises to be successful in many ways (Vachaspati T. & Vilenkin A. 1991; Vollick D. N. 1992; Albrecht A. & Stebbins A. 1993). The scenario might automatically account for large-scale filaments and sheets (Vachaspati T. 1986; Stebbins A. et al. 1987; Vachaspati T. & Vilenkin A. 1991; Perivolaropoulos L., Brandenberger R. & Stebbins A. 1990; Hara T. & Miyoshi S. 1990), galaxy formation at epochs of $z \sim 2-3$ and galactic magnetic fields (Vachaspati T. & Vilenkin A. 1991; Vachaspati T. 1992b). It can also provide (Vachaspati T. 1992a; Brandenberger R. et al. 1987) large-scale peculiar velocities (Dressler A. et. al. 1987; Burstein D. et. al. 1983; Peebles P. J. E. & Silk J. 1990) and is consistent (Bennett D. P., Stebbins A. & Bouchet F. R. 1992; Perivolaropoulos L. 1993a,b) with the measured anisotropy of the microwave background radiation (Smoot G. et. al. 1992).

Even though the cosmic string model involves a single free parameter (the mass per unit length of the string), its predictions depend also on our understanding of the evolution of the string network. The main features of this evolution can be encoded into a set of parameters which have been constrained by recent detailed numerical simulations (Bennett D. P. & Bouchet F. R. 1988; Allen B. & Shellard E. P. S. 1990) (see also papers by Albrecht & Turok in (Gibbons G. W., Hawking S. W. & Vachaspati T. 1990)). Therefore, in order to test the cosmic string model, it is important to obtain expressions of the model's observational predictions in terms of these parameters and then use observations to fix their values. The consistency of the model can then be tested by comparing the fixed values with numerical simulations of string evolution and with constraints from different observations.

In this letter we show that the predictions of the peculiar velocities and MBR anisotropies resulting from cosmic strings depend on roughly the same combination of undetermined parameters which we call α. The prediction of peculiar velocities also depends on parameters denoted ξ and $f_{5/4}$, denoting the curvature radii of strings and the Newtonian interaction of wiggly strings with matter. On the other hand, the microwave background predictions are independent of $f_{5/4}$ (since there is no Newtonian interaction of strings with photons). By combining the two predictions and comparing with observations, we can impose constraints on these parameters, thus providing an interesting test for the cosmic string model. One outcome is that the string scenario cannot account for the observed peculiar velocities on all the different scales. With the most suitable choice of parameters, there is good agreement with observations on small scales but inconsistency with the observations on large scales. This potential problem of cosmic strings however, is also encountered in several other models (Brandenberger R. et. al. 1987) including the standard CDM model.

It is not difficult to guess that the peculiar velocities and the microwave background anisotropy should depend on roughly the same combination of network parameters. The peculiar velocities arise because with every string sweeping across the horizon, any fixed volume of matter experiences an impulse towards the wake of the string. At the same time, the anisotropy arises because the string gives an impulse to the photons coming towards us from the surface of last scattering. The peculiar velocity that we observe today, would be the sum of all string impulses with some suitable growth factors while the anisotropy would be due to the sum of the string impulses on the photons from the surface of last scattering. Therefore both processes depend on evaluating the magnitude of each impulse, the number of such impulses and, in the case of peculiar velocities, the growth of the velocity following the impulse. Since the calculations are so similar, the dependences on the string network parameters are also very similar.

Let us first apply the multiple impulse approximation (Vachaspati T. 1992a; Perivolaropoulos L. 1993a,b) to obtain the mean and the standard deviation of cosmic string induced peculiar velocities. According to this approximation, the time between t_{eq} (the time of equal matter and radiation) and the present (t_p) is divided into a set of Hubble time-steps

t_i such that $t_{i+1} = 2t_i$. At every Hubble-step t_i, each long string gives a velocity impulse to matter within a distance ξt_i from it, where ξt_i is the radius of curvature of the string, and, by causality $\xi \leq 1$. It is straightforward to count the total number of string induced impulses for each comoving scale $Lh^{-1}Mpc$. By including the growth factor $(\frac{t_p}{t_n})^{1/3}$ for the nth impulse, the velocity $\vec{v}(t_p, L)$ of the scale L at t_p may be written as:

$$\vec{v}(t_p, L) = \frac{2}{5}\Delta v \sum_{n=0}^{N_L} \sum_{m=1}^{n_s(L)} \hat{k}_{nm} (\frac{t_p}{t_n})^{1/3} \tag{1}$$

where

$$\Delta v = 5\pi G\mu v_s \gamma_s f_{5/4} \tag{2}$$

and the indices n, m count Hubble steps and long strings within a Hubble volume respectively. In (1), (2) and in what follows we use the following notation: N_L is the number of Hubble time steps during which a volume of comoving size L experiences coherent string impulses, $n_s(L)$ is the number of string impulses that the volume experiences per Hubble time, v_s is the string velocity and γ_s the corresponding Lorentz factor, $(G\mu_0)_6$ is the *bare* string tension in units of 10^{-6},

$$f_{5/4} = \frac{4}{5}\left[1 + \frac{1}{2}\frac{1}{(v_s\gamma_s)^2}(1 - \frac{T}{\mu})\right] \tag{3}$$

and T, μ are the renormalized string tension and string mass density. Simulations (Allen B. & Shellard E. P. S. 1990; Bennett D. P. & Bouchet F. R. 1988) indicate $T \sim 0.7\mu_0$ and $\mu \sim 1.4\mu_0$ where μ_0 is the bare string tension. The expression for $f_{5/4}$ in eq. (3) can be thought of in the following way: if we multiply $f_{5/4}$ by Δv, the first term is the induced velocity due to the conical deficit of the string metric while the second term (proportional to $1/v$) is due to the Newtonian potential of the wiggly string. In what follows, we shall not evaluate $f_{5/4}$ using the simulation results but treat it like a free parameter.

If we imagine a volume of comoving size L, the whole volume will get a coherent impulse due to strings moving outside the volume but within a distance equal to the curvature radius of the string. If a string passes through the volume, it will not cause a coherent motion of the volume and so we neglect the effect of such strings. On the other hand, a wiggly string which is distant from the volume would still have some Newtonian gravitational effect for $\xi < 1$ (expressed by second term in $f_{5/4}$) on the volume and, strictly speaking, we should take the distant strings into account with an effectively smaller value of $f_{5/4}$. (We thank Alex Vilenkin for emphasizing this point.) However, since we are keeping the number of strings (n_s) and $f_{5/4}$ as free parameters, the exact values do not concern us and we should view $f_{5/4}$ as an average value that accounts for all the strings. In addition, for $\xi \sim 1$, the curvature radius is comparable to the horizon and our method automatically accounts for all strings. The number of strings that give coherent impulses to the volume can now be found: it is simply the number of strings within the horizon multiplied by the probability that the string passes within a distance ξt_n from the volume of size L. Therefore,

$$n_s(L) = n_s\Big(\frac{\xi t_n - L(t_n)}{t_n}\Big) \tag{4}$$

provided $L(t_n) < \xi t_n$ and $n_s(L) = 0$ otherwise. Here $L(t_n) = L(t_n/t_p)^{2/3}$ is the physical size of the volume at the n^{th} Hubble step and n_s is the number of strings within the horizon.

Now we square (1) and take averages with the assumption that the $\hat{k}$ vectors are random. That is,

$$< \hat{k}_{nm} \cdot \hat{k}_{n'm'} >= \delta_{nn'}\delta_{mm'} \tag{5}$$

This gives (Vachaspati T. 1992a) the present average (rms) peculiar velocity magnitude on a comoving scale $Lh^{-1}Mpc$:

$$\bar{v}(t_p, L \geq \xi t_{eq}) = 44\alpha\xi f_{5/4} L_{100}^{-1} \ km/s \tag{6}$$

where

$$\alpha = \sqrt{n_s \xi}(G\mu_0)_6(v_s\gamma_s) \ , \tag{7}$$

$L_{100} \equiv L/(100h^{-1}Mpc)$ and we have taken $N_L >> 1$. The result (6) is valid on scales Lh^{-1} larger than the curvature radius of strings at $t_{eq} = 10h^{-2}Mpc$ because in deriving it we assumed $L(t_0) > \xi t_{eq}$. For scales $L < \xi t_{eq}$ we get

$$\bar{v}(t_p, L \leq \xi t_{eq}) = 437 \left[1 + 2.9 \left(1 - \frac{L}{\xi t_{eq}}\right)\right]^{1/2} \alpha f_{5/4} h \ km/s \ . \tag{8}$$

The dispersion $\sigma(L)$ of $\bar{v}(L)$ may also be obtained (Vachaspati T. 1992a) as:

$$\sigma(t_p, L) = 0.36 \, \bar{v}(t_p, L) \ . \tag{9}$$

As discussed above, the free parameter α involved in the peculiar velocity calculation can be pinned down by demanding that the cosmic string model be consistent with the recent detection of MBR anisotropy by COBE. The multiple impulse approximation can be used (Perivolaropoulos L. 1993a) to express the string induced MBR anisotropies in terms of α. In this case the approximation is realized as follows: A given photon experiences the impulses due to the various strings along its way from the last scattering surface to us. Each impulse can either red shift or blue shift the photon, depending on the various possible directions of the string velocity, photon momentum and string orientation. The temperature variations at some fixed angular separation will be correlated only when the photon beams at that angular separation will experience the same impulse. The correlation function is thus obtained by counting the number of common impulses on each angular scale θ. In this case, due to the lack of a growth factor, the dependence on ξ (the string curvature radius) comes only as a square root dependence in counting the number of strings that come within a radius of curvature distance from the photon. From the correlation function it is straightforward to obtain the rms temperature fluctuation for angular beam separation equal to $60°$ and smoothing on a scale of $10°$ as in the COBE experiment. The result is (Perivolaropoulos L. 1993a):

$$\left(\frac{\Delta T}{T}\right)_{rms} = 1.6 \, \alpha \times 10^{-5} \tag{10}$$

where α is the same product of parameters as in (7).

Now the COBE observations fix the rms temperature fluctuations to be $(\frac{\Delta T}{T})_{rms} = (1.1 \pm 0.2) \times 10^{-5}$ which when inserted in (6) gives

$$\alpha \simeq 0.7 \pm 0.2 \tag{11}$$

It is of interest to use the values of evolution parameters n_s, $v_s \gamma_s$ and ξ obtained in numerical simulations to extract from the value of α in (11), a rough value of the single free parameter of the string model $G\mu_0$. From (Allen B. & Shellard E. P. S. 1990) we have $n_s \simeq 10$, $v_s \simeq 0.15$ and $\xi \simeq 0.7$. Using these values in (7) we obtain (Perivolaropoulos L. 1993a):

$$G\mu_0 = (1.8 \pm 0.5) \times 10^{-6} \tag{12}$$

which is the value required from galaxy and large scale structure calculations (Perivolaropoulos L., Brandenberger R. & Stebbins A. 1990; Turok N. & Brandenberger R. 1986; Sato H. 1986; Stebbins A. 1986). (The value of $G\mu$ given in (Perivolaropoulos L. 1993a) refers to the *effective* $G\mu$ which includes the string small scale structure and is therefore slightly different from (12)). The same value for $G\mu$ has been obtained in (Bennett D. P., Stebbins A. & Bouchet F. R. 1992) by using the numerical simulations of (Bennett D. P. & Bouchet F. R.1988; Bouchet F. R., Bennett D. P. & Stebbins A. 1988) to simulate the MBR sky and compare with the COBE results. This is an additional test of consistency for our results.

Now, using (11) in (6) and taking into account a 1σ spread (eq. (9)) we get

$$\bar{v}(t_p, L \geq \xi t_{eq}) = (31 \pm 11)\, \xi f_{5/4} L_{100}^{-1} \;\; km/s \tag{13}$$

$$\bar{v}(t_p, L \leq \xi t_{eq}) = (306 \pm 110) \left[1 + 2.9 \left(1 - \frac{L}{\xi t_{eq}}\right)\right]^{1/2} f_{5/4} h \;\; km/s \; . \tag{14}$$

The dependence of $\bar{v}$ on the length scale is plotted in Fig. 1 for $f_{5/4} = 1.8$, $\xi = 1$ and $h = 0.5$. The observations (Burstein D. et. al. 1983; Collins C., Joseph R. & Robertson

N. 1986; Dressler A. et. al. 1987; Groth J.,Juszkiewicz R. & Ostriker J. 1989) are also indicated with their error bars.

The figure shows that the peculiar velocities from cosmic strings are in good agreement with observations on small scales but not on large scales. The inconsistency is not an artifact of our choice of parameters. For example, we could take $\xi = 1$ (the largest permissible value), $h = 0.5$ (the smallest value of h) and $f_{5/4} = 7$ so that the observation on $60h^{-1}$Mpc is barely accommodated within the 1σ prediction of the string model. But then, the predicted 1σ velocity on scales of $8h^{-1}$Mpc range from 1135km/s to 2409km/s. These velocities are clearly inconsistent with observations. In other words, no choice of parameters will allow us to explain the velocity observations on all scales.

We now consider the Cosmic Mach Number (CMN) induced due to strings. The CMN has been shown to be independent of various uncertainties such as biasing and hence may be a valuable statistic in comparing theory with observation (Ostriker J. & Suto Y. 1990). As we shall see, the string induced CMN only depends on the parameter ξ and for reasonable values of ξ the agreement between the string predictions and observations on small scales is quite good. Hence we believe that cosmic strings can successfully explain the observations on small scales but fail in explaining the large scale data.

The Cosmic Mach Number $M(L,a)$ is defined (Ostriker J. & Suto Y. 1990) as the ratio of the rms velocity $\bar{v}(t_p, L)$ of a volume of size Lh^{-1} over the dispersion of the velocity field smoothed on a scale a, in the rest frame and within the same volume:

$$M(L,a) \equiv \frac{\bar{v}(t_p, L)}{\sqrt{< (\vec{v}(t_p, a) - \vec{v}(t_p, L))^2 >}} \tag{15}$$

Inserting (8) for the peculiar velocities gives us (for $a, L < \xi t_{eq}$),

$$M(L,a) = \sqrt{\frac{1.35\xi t_{eq} - L}{L - a}}\ . \tag{16}$$

If $a < \xi t_{eq} < L$, we get,

$$M(L,a) = \sqrt{\frac{0.7\xi^2 t_{eq}^2}{2.7L^2 - 2aL^2/\xi t_{eq} - 0.7\xi^2 t_{eq}^2}} \ . \tag{17}$$

Observations on scales larger than $5h^{-1}Mpc$ indicate (Ostriker J. & Suto Y. 1990) that

$$\begin{aligned} M(L = 8h^{-1}Mpc, a = 5h^{-1}Mpc) &= 2.2 \pm 0.5 \\ M(L = 18h^{-1}Mpc, a = 5h^{-1}Mpc) &= 1.3 \pm 0.4 \quad . \end{aligned} \tag{18}$$

while the corresponding predictions of the cosmic string scenario obtained from (16) for $\xi = 1$, are

$$\begin{aligned} M_s(L = 8h^{-1}Mpc, a = 5h^{-1}Mpc) &= 2.5 \\ M_s(L = 18h^{-1}Mpc, a = 5h^{-1}Mpc) &= 0.8 \quad . \end{aligned} \tag{19}$$

The CMN predictions are smaller for smaller values of ξ. For example, if $\xi = 0.7$, one gets 1.9 and 0.5 in eq. (19). Simulations indicate $\xi \simeq 0.7$, and so we can say that the cosmic string predictions are in rough agreement with the CMN observations. If, however, the simulations can pin down ξ better, the CMN observations would be a powerful test for the cosmic string scenario.

The success of the string scenario with the observations on small scales suggests that we should choose our parameters so that both the peculiar velocity magnitudes and the CMN are in good agreement on these scales. That is, we choose $\xi = 1$, $f_{5/4} = 1.8$ and $h = 0.5$. With this choice of parameters, the peculiar velocity on $60h^{-1}$Mpc is 89 ± 30km/s and is off from the observations of roughly 600km/s by a factor of about 7. This gives us an idea of the magnitude of the problem.

We are unable to give a spread on the predicted values of the CMN because our analytic model does not take the correlations of velocity impulses on the large and small scales into account. These correlations are important for the CMN because the dispersion occurring in the denominator of eq. (15) is the dispersion of the smaller scale volumes (size a) within a bigger volume (size L). For example, it is quite possible that if the bigger volume has a larger than average velocity, then the smaller volumes will also have a larger than average velocity. Such correlations cannot be taken into account in our model since we can only find the average velocities of the small and big volumes. It would also be wrong to include the spread in peculiar velocities (eq. (9)) and simply propagate these into the CMN because the calculated spread includes all volumes of size a and not just those contained within some larger volume of size L.

The prediction of the cosmic string model is in better agreement with observations than adiabatic CDM which predicts (Ostriker J. & Suto Y. 1990) $M(L, a = 5h^{-1}Mpc)$ to be well below 1 on both $8h^{-1}Mpc$ and $18h^{-1}Mpc$.

In conclusion, we have used the multiple impulse approximation to identify two new features of the cosmic string model valid with both hot and cold dark matter:

a) The Cosmic Mach Number predicted by the model is consistent with observations on scales $5 - 20h^{-1}$Mpc.

b) The magnitude of the predicted large scale velocity flows normalized by the COBE detection of MBR anisotropy is consistent with observations on small scales for reasonable values of the string evolution parameters but the large scale peculiar velocities cannot be explained with these parameters.

The potential problem pointed out in b) is also encountered in other theories like standard CDM where a similar L^{-1} scaling of the magnitude of large scale velocity flows has been shown (Kaiser N. 1983; Vittorio N. & Silk J. 1985; Vittorio N. & Turner M. S. 1987; Kolb E. W. & Turner M. S. 1990).

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FIGURE CAPTIONS

1. The predicted peculiar velocity versus the scale lies in the shaded area between the two shown curves within a 1σ error including the standard deviation of $\bar{v}$. The data points with error bars are the observations. The parameter values $f_{5/4} = 1.8$, $\xi = 1$, $h = 0.5$ were used.