

# Phenomenology of a New Minimal Supersymmetric Extension of the Standard Model

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## Abstract

We study the phenomenology of a new Minimally-extended Supersymmetric Standard Model (*nMSSM*) where a gauge singlet superfield is added to the MSSM spectrum. The superpotential of this model contains no dimensionful parameters, thus solving the  $\mu$ -problem of the MSSM. A global discrete  $R$ -symmetry, forbidding the cubic singlet self-interaction, imposed on the complete theory, guarantees its stability with respect to generated higher-order tadpoles of the singlet and solves both the domain wall and Peccei-Quinn axion problems. We give the free parameters of the model and display some general constraints on them. A particular attention is devoted to the neutralino sector where a (quasi-pure) singlino appears to be *always* the LSP of the model, leading to additional cascades, involving the NLSP  $\rightarrow$  LSP transition, compared with the MSSM. We then present the upper bounds on the masses of the lightest and next-to-lightest – when the lightest is an invisible singlet – CP-even Higgs bosons, including the full one-loop and dominant two-loop corrections. These bounds are found to be much higher than the equivalent ones in the MSSM. Finally, we discuss some phenomenological implications for the Higgs sector of the nMSSM in Higgs production at future hadron colliders.

# 1 Introduction

Supersymmetry provides a well defined framework for the study of physics beyond the Standard Model (SM). Its main motivation has been the special properties of supersymmetric (susy) theories with respect to the hierarchy problem. In addition, the low energy data support unification of the gauge couplings in the susy case, in contrast to what happens in the SM scenario. Another interesting feature of susy models is that the breaking of the electroweak (EW) symmetry can be radiatively triggered by the largeness of the top quark mass [1]. The Minimal Supersymmetric Standard Model (MSSM) [2] is defined by promoting each standard field into a superfield, doubling the Higgs fields and imposing  $R$ -parity conservation. Due to the non-observation of superpartners of the standard particles, supersymmetry has to be broken at a scale  $M_{susy}$  not larger than  $O(\text{TeV})$ , so that it still provides a natural solution to the hierarchy problem. Unfortunately, a phenomenologically acceptable realization of EW symmetry breaking in the MSSM requires the presence of the so-called  $\mu$ -term, a direct susy mass term for the Higgs fields, with values of the (theoretically arbitrary) parameter  $\mu$  close to  $M_{susy}$  or  $M_W$ , when its natural value would be either 0 or  $M_P$ . Of course, there exist explanations for an  $O(M_W)$  value of the  $\mu$ -term, alas, all in extended settings [3].

The more or less straightforward solution to the  $\mu$ -problem is to promote the  $\mu$ -parameter into a field whose vacuum expectation value (v.e.v.) is determined, like the other scalar field v.e.v.'s, from the minimization of the scalar potential along the new direction [4, 5, 6, 7]. Naturally, it is expected to fall in the range of the other v.e.v.'s, i.e., of order  $O(M_{susy})$ . Such a superfield has to be a singlet under the SM gauge group. In order to avoid introducing new scales into the model one should stick to dimensionless couplings at the renormalizable level. This can be achieved by imposing a  $\mathbb{Z}_3$  symmetry on the renormalizable part of the superpotential. The resulting model, the Next-to-Minimal Supersymmetric Standard Model (NMSSM), has the following superpotential:

$$W = \lambda S H_1 H_2 + \frac{\kappa}{3} S^3 + \dots \quad (1)$$

where the dots stand for the usual quark and lepton Yukawa couplings (cf eq. (2)). The  $\mathbb{Z}_3$  symmetry is spontaneously broken at the EW scale when the Higgs fields get a non-zero v.e.v. It is well known, however, that the spontaneous breaking of such a discrete symmetry results in disastrous *cosmological domain walls*, unless this symmetry is explicitly broken by the non-renormalizable sector of the theory. Domain walls can be tolerated if there is a discrete-symmetry-violating contribution to the scalar potential larger than the scale  $O(1 \text{ MeV})$  set by nucleosynthesis [8]. Heavy fields interacting with the standard light fields generate in the effective low-energy theory an infinite set of non-renormalizable operators of the light fields scaled by powers of the characteristic mass-scale of the heavy sector ( $M_P, M_{GUT}, \dots$ ). These terms appear either as  $D$ -terms in the Kähler potential or as  $F$ -terms in the superpotential. It is known however that gauge singlet superfields *do not obey decoupling* [3, 9], so that, when supersymmetry is either spontaneously or softly broken, in addition to the suppressed

non-renormalizable terms, they can in general give rise to a large tadpole term in the potential proportional to the heavy scale:  $M_{susy}^2 M_P (S + S^*)$ . Technically, the tadpole is generated through higher-order loop diagrams in which the non-renormalizable interactions participate as vertices together with the renormalizable ones. A discrete global symmetry like the one discussed above would forbid this term but would lead to the appearance of disastrous domain walls upon its unavoidable spontaneous breakdown. The generated large tadpole reintroduces the hierarchy problem, since due to its presence the singlet v.e.v. gets a value  $\langle S \rangle^2 \sim M_{susy} M_P$ . It appears that  $N = 1$  supergravity, spontaneously broken by a set of hidden sector fields, is the natural setting to study the generation of the destabilising tadpoles. A thorough analysis carried out in Ref. [10] shows that the only harmful non-renormalizable interactions are either *even superpotential* terms or *odd Kähler potential* ones. In addition, operators with more than six powers of the cut-off in the denominator are harmless. Finally, a tadpole diagram is divergent only if it contains an *odd* number of ‘dangerous’ vertices.

The solution of the  $\mu$ -problem in the framework of the NMSSM could be rendered a viable one if the destabilization problem were circumvented. What is needed is a suitable symmetry that forbids the dangerous non-renormalizable terms and allows only for tadpoles of order  $M_{susy}^3 (S + S^*)$ . This symmetry should at the same time allow for a large enough  $\mathbb{Z}_3$ -breaking term in the scalar potential in order to destroy the unwanted domain walls [11].

An alternative approach is to impose a symmetry which, although it does not forbid the dangerous non-renormalizable terms, only allows for higher-order tadpole graphs that give a  $n$ -loop-suppressed term  $\frac{1}{(16\pi^2)^n} M_{susy}^2 M_P (S + S^*)$ . A case of particular interest is when the cubic self-interaction for the singlet in eq. (1) is forbidden by the symmetry. Actually, it should be noted that if the underlying theory is a Grand Unified one (GUT), although a candidate for the singlet exists, a cubic term does not arise<sup>1</sup>. On the other hand, this case is truly minimal in the sense that, apart from promoting the  $\mu$ -parameter into a field, no new renormalizable terms appear in the superpotential. Of course, a substitute is needed for the twofold role played by the cubic term, namely, its contribution to the mechanism generating the v.e.v. of  $S$  through the soft susy breaking terms and the breaking of the *Peccei-Quinn symmetry* present when  $\kappa = 0$ . This role can be played by the tadpole. (Note that this is not included in the  $\kappa \rightarrow 0$  limit of the existing NMSSM analyses, which up to now have ignored the tadpole term [5, 6, 12].) Recently, a viable solution along these lines was proposed based on *discrete R-symmetries* [13]. The renormalizable superpotential for this *new minimal supersymmetric extension* of the Standard Model (nMSSM) is given by

$$W = \lambda S H_1 H_2 + Y_u Q U^c H_1 + Y_d Q D^c H_2 + Y_e L E^c H_2. \quad (2)$$

Apart from the usual Baryon and Lepton number, it possesses two additional global continuous symmetries, namely, an anomalous Peccei-Quinn symmetry  $U(1)_{PQ}$  with

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<sup>1</sup>In  $E_6$  for example, matter and Higgs fields are contained in the **27** representation together with a singlet. Although the standard trilinear singlet-Higgses term is present in the **27**<sup>3</sup> coupling, no singlet cubic term arises. The same is true for  $E_6$  embeddings of  $SO(10)$  and  $SU(5)$ .

charges

$$Q(-1), U^c(0), D^c(0), L(-1), E^c(0), H_1(1), H_2(1), S(-2) \quad (3)$$

and a non-anomalous  $R$ -symmetry  $U(1)_R$  with charges

$$Q(1), U^c(1), D^c(1), L(1), E^c(1), H_1(0), H_2(0), S(2). \quad (4)$$

One of the solutions worked out consists in imposing the discrete sub-symmetry  $\mathbb{Z}_{5R}$  of the  $U(1)_{R'}$  combination  $R' = 3R + PQ$  on the complete theory, including non-renormalizable operators. The charges under  $\mathbb{Z}_{5R}$  are

$$\begin{aligned} (H_1, H_2) &\rightarrow \alpha(H_1, H_2), \\ (Q, L) &\rightarrow \alpha^2(Q, L), \\ (U^c, D^c, E^c) &\rightarrow \alpha^3(U^c, D^c, E^c), \\ S &\rightarrow \alpha^4 S, \\ \mathcal{W} &\rightarrow \alpha \mathcal{W}, \end{aligned} \quad (5)$$

where  $\alpha = e^{2i\pi/5}$ . An adequately suppressed linear term is generated at six-loop level by combining the non-renormalizable Kähler potential terms  $\lambda_1 S^2 H_1 H_2 / M_P^2 + h.c.$  and  $\lambda_2 S (H_1 H_2)^3 / M_P^5 + h.c.$  with the renormalizable superpotential term  $\lambda S H_1 H_2$ :

$$V_{tadpole} \sim \frac{1}{(16\pi^2)^6} \lambda_1 \lambda_2 \lambda^4 M_{susy}^2 M_P (S + S^*). \quad (6)$$

This tadpole has the desired order of magnitude  $O(M_{susy})$  if  $\lambda_1 \lambda_2 \lambda^4 \sim 10^{-3}$ .

The goal of this paper is the phenomenological study of this nMSSM, where a gauge singlet superfield is added to the MSSM spectrum and a global  $\mathbb{Z}_{5R}$  symmetry is imposed on the complete theory, resulting in the superpotential of eq. (2) and the tadpole term of eq. (6). In section 2 we review the general properties of the parameter space of the model. Phenomenological aspects of the nMSSM are addressed in section 3. The neutralino sector, including the (quasi-pure) singlino, is studied in some detail. Also bounds on CP-even Higgs masses versus their couplings to gauge bosons are displayed along with Higgs production cross sections at future hadron colliders. Section 4 contains our main conclusions.

## 2 Model set-up

The tree-level Higgs scalar potential, namely, the potential which contains the scalar fields  $H_1 = (H_1^0, H_1^-)$ ,  $H_2 = (H_2^+, H_2^0)$  and  $S$ , has the form :

$$\begin{aligned} V^{(0)} &= V_F + V_D + V_{soft} + V_{tadpole}, \\ V_F &= |\lambda|^2 \left[ \left( |H_1|^2 + |H_2|^2 \right) |S|^2 + |H_1|^2 |H_2|^2 \right] \end{aligned} \quad (7)$$

$$- |\lambda|^2 \left( H_1^{0*} H_2^{0*} H_1^- H_2^+ + h.c \right) \quad (8)$$

$$V_D = \frac{g_1^2 + g_2^2}{8} \left[ |H_1|^2 - |H_2|^2 \right]^2 + \frac{g_2^2}{2} |H_1^\dagger H_2|^2, \quad (9)$$

$$V_{soft} = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_S^2 |S|^2 + \left( \lambda A_\lambda S H_1 H_2 + h.c \right). \quad (10)$$

In what follows, we shall assume a phenomenological point of view and write the generated tadpole as

$$V_{tadpole} \equiv \xi^3 (S + S^*), \quad (11)$$

where  $\xi$  is treated as a free parameter.

In order to obtain the correct upper limits on the Higgs boson masses (cf. section 3.2) radiative corrections to the tree-level potential have to be considered. Let us introduce a scale  $Q \sim M_{susy}$  and assume that quantum corrections involving momenta  $p^2 \gtrsim Q^2$  have been evaluated, e.g. by the integration of the Renormalization Group Equations (RGEs) of the parameters from initial values at the GUT scale down to the scale  $Q$ . One is then left with the computation of quantum corrections involving momenta  $p^2 \lesssim Q^2$ . The effective potential  $V_{eff}$  can be developed in powers of  $\hbar$  or loops as

$$V_{eff} = V^{(0)} + V^{(1)} + V^{(2)} + \dots \quad (12)$$

The tree-level potential  $V^{(0)}$  is given by eq. (7). The one-loop corrections to the effective potential read as

$$V^{(1)} = \frac{1}{64\pi^2} \text{STr} M^4 \left[ \ln \left( \frac{M^2}{Q^2} \right) - \frac{3}{2} \right], \quad (13)$$

where  $M^2$  is the field dependent squared mass matrix (in our analysis, we take only top/stop loops into account). Next, we consider the dominant two-loop corrections. These will be numerically important only for large susy breaking terms compared to the Higgs v.e.v.'s  $h_i$ , hence we can expand in powers of  $h_i$ . Since the terms quadratic in  $h_i$  can be absorbed into the tree-level soft terms, we just consider the quartic terms, and here only those which are proportional to large couplings: terms  $\sim \alpha_s h_t^4$  and  $\sim h_t^6$ . Finally, taking only leading logarithms (LLs) into account, the expression for  $V^{(2)}$  reads

$$V_{LL}^{(2)} = 3 \left( \frac{h_t^2}{16\pi^2} \right)^2 h_t^4 \left( 32\pi\alpha_s - \frac{3}{2} h_t^2 \right) t^2, \quad (14)$$

where  $t \equiv \ln \left( \frac{Q^2}{m_t^2} \right)$ ,  $m_t$  being the top quark mass. One-loop corrections to the tree-level relations between bare parameters and physical observables, once reinserted in the one-loop effective potential, also appear as two-loop effects. These are: corrections to the kinetic terms of the Higgs bosons, which lead to a wave function renormalization factor  $Z_{H_2}$  in front of the  $D_\mu H_2 D^\mu H_2$  term with, to order  $h_t^2$

$$Z_{H_2} = 1 + 3 \frac{h_t^2}{16\pi^2} t; \quad (15)$$

and corrections to the top quark Yukawa coupling with, to orders  $h_t^2$ ,  $\alpha_s$

$$h_t(m_t) = h_t(Q) \left( 1 + \frac{1}{32\pi^2} \left( 32\pi\alpha_s - \frac{9}{2} h_t^2 \right) t \right). \quad (16)$$

In general, the parameters  $\lambda$ ,  $A_\lambda$  and  $\xi$  could be complex. However, by redefining the fields  $H_2$  (or  $H_1$ ) and  $S$ , one can always get – without loss of generality – that  $\lambda A_\lambda, \xi^3 \in R$ . Note that in the NMSSM with the cubic singlet superpotential term  $\frac{1}{3}\kappa S^3$  one has to further *assume* that the combination  $\lambda\kappa^*$  (or, equivalently,  $A_\lambda/A_\kappa$ ) is real [5, 6]. By  $SU(2)_L \times U(1)_Y$  gauge invariance one can get rid of the phase of  $H_1$ , by taking  $\langle H_1^- \rangle = 0$  and  $h_1 \equiv \langle H_1^0 \rangle \in R^+$ . One can then show that the condition for a local minimum with  $\langle H_2^+ \rangle = 0$  is equivalent to a positive mass squared for the charged Higgs. It has been proven that a sufficient condition is  $\lambda < g_2$  [14] which, as we shall see below, is always verified in the universal case. By taking  $h_2 \equiv \langle H_2^0 \rangle = \rho_2 e^{i\phi_2}$ ,  $s \equiv \langle S \rangle = \rho_0 e^{i\phi_0}$  and minimizing the complete (two-loop) effective potential with respect to  $\phi_0$  and  $\phi_2$ , we find that there is one and only one global vacuum for which the two phases relax to zero, i.e.,  $\phi_0 = \phi_2 = 0$ . This implies that there is *no spontaneous CP-violation*. Therefore one can choose  $h_1 \in R^+$  and  $h_2, s \in R$ . This result distinguishes the nMSSM from the usual NMSSM where loop corrections can generate spontaneous CP-violation [15].

The soft terms of the model can be constrained by requiring universality at the GUT scale. The independent parameters of the model are then a universal gaugino mass  $M_{1/2}$  (always positive in our convention), a universal mass for the scalars  $m_0^2$ , a universal trilinear coupling  $A_0$  (either positive or negative), the (positive) Yukawa coupling  $\lambda_0$  at the scale  $M_{GUT}$  and the tadpole coefficient  $\xi$ . The (well-known) value of the  $Z$ -boson mass fixes one of these parameters with respect to the others, so that we end up with *four free parameters at the GUT scale*, i.e., as many as in the MSSM with universal soft terms. In principle, one could choose the same set of free parameters as in the MSSM, i.e.,  $M_{1/2}$ ,  $m_0^2$ ,  $A_0$  and  $\tan\beta$  ( $\equiv \frac{h_2}{h_1}$ ), with  $\lambda$ ,  $s$  and  $\xi$  being determined by the three minimization equations, demanding also radiative electroweak symmetry breaking [1]. However, this appears to be a non-trivial issue, as  $\lambda$  also influences the running of the RGEs of the soft parameters between the GUT and the EW scale. In other terms, one would need a lot of fine tuning of the dimensionful  $A_0$  in order to get a dimensionless parameter,  $\lambda$ , of the desired value at the EW scale. Therefore, in the case of universality, we conveniently adopt in our numerical analysis the following input parameters:  $m_0^2/M_{1/2}$ ,  $A_0/M_{1/2}$ ,  $\xi/M_{1/2}$  and  $\lambda_0$  ( $\tan\beta$  and  $s$  being calculated from the minimization of the potential and the overall scale  $M_{1/2}$  fixed by  $M_Z$ ).

If one requires the absence of a Landau singularity for  $\lambda$  below the GUT scale, one obtains an upper bound on  $\lambda$  at the EW scale. This upper bound depends on the value of the top quark Yukawa coupling  $h_t$ , i.e., on  $\tan\beta$  (cf. fig. 1). Requiring furthermore, universality at the GUT scale, one ends up with the more restrictive constraint  $\lambda \lesssim .3$ , higher values leading to unphysical global minima of the effective potential.

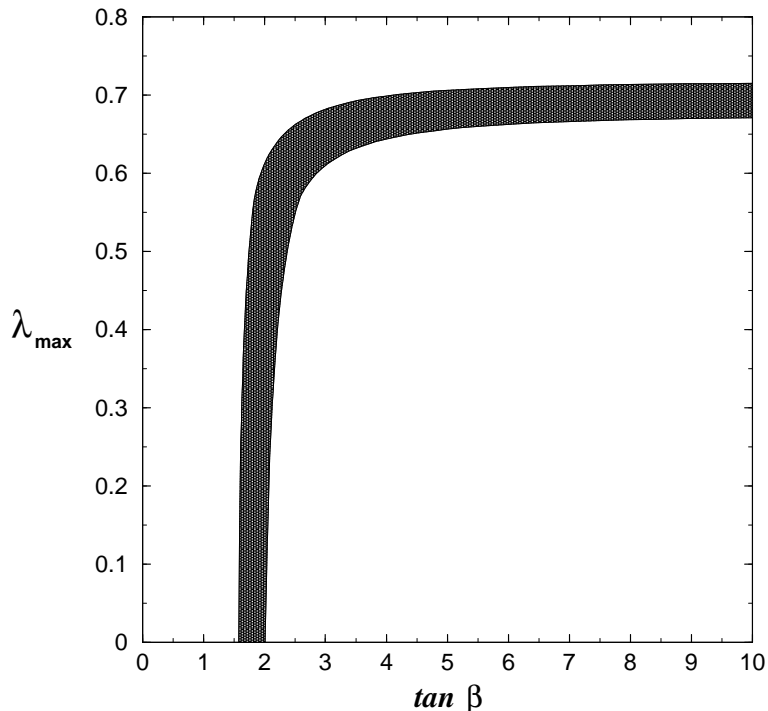


Figure 1: Upper bound on  $\lambda$  as a function of  $\tan \beta$  for  $m_t^{pole} = 173.8 \pm 5.2$  GeV [18]. The width of the curve is due to the uncertainty on  $m_t^{pole}$ .

Let us now briefly address the problem of Charge and Color Breaking (CCB) minima. The most dangerous CCB direction involves the trilinear coupling  $h_e A_e E_{R,1} L_1 H_1$  where  $h_e$  denotes the electron Yukawa coupling ( $\sim 10^{-5}$ ),  $E_{R,1}$  is the right-handed selectron,  $L_1$  the left-handed slepton doublet of the first generation, and  $H_1$  the corresponding Higgs doublet. From the absence of a non-trivial minimum of the scalar potential in the  $D$ -flat direction  $|E_{R,1}| = |L_1| = |H_1|$ , the following inequality among the soft susy breaking terms can be derived [4]:

$$A_e^2 < 3(m_E^2 + m_L^2 + m_1^2), \quad (17)$$

where  $m_E^2$ ,  $m_L^2$  and  $m_1^2$  are the soft susy breaking mass terms associated with the three fields above. If the inequality (17) is violated, the fields develop v.e.v.'s of  $O(A_e/h_e)$  and the depth of the minimum is of  $O(A_e^4/h_e^2)$ . Accordingly, (17) has to be imposed at a scale  $Q \sim A_e/h_e \sim 10^7$  GeV. Assuming universal soft terms at the GUT scale, (17) then becomes [6]:

$$(A_0 - 0.5M_{1/2})^2 < 9m_0^2 + 2.67M_{1/2}^2. \quad (18)$$

So-called UFB directions (for Unbounded From Below, which actually never occurs in the universal case) can also be considered. Assuming universality for the soft terms, the absence of a global minimum in these directions typically implies [16, 17]

$$\frac{m_0}{M_{1/2}} \gtrsim 1. \quad (19)$$

However, the tunnelling rate from the standard EW minimum to a UFB one is in general quite small [16], so that this constraint can be avoided if one is ready to assume that the standard EW vacuum is metastable.

## 3 Phenomenological aspects of the nMSSM

### 3.1 Singlino LSP and additional cascades

The nMSSM contains additional gauge singlet states in the Higgs sector (one neutral CP-even and one CP-odd state) and in the neutralino sector (a two component Weyl fermion). These states are mixed with the corresponding ones of the MSSM, and the physical states have to be obtained from the diagonalization of the mass matrices in each sector. In the basis  $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S})$  the (symmetric) neutralino mass matrix reads as

$$\mathcal{M}^0 = \begin{pmatrix} M_1 & 0 & -g_1 h_1 / \sqrt{2} & g_1 h_2 / \sqrt{2} & 0 \\ & M_2 & g_2 h_1 / \sqrt{2} & -g_2 h_2 / \sqrt{2} & 0 \\ & & 0 & \lambda_s & \lambda h_2 \\ & & & 0 & \lambda h_1 \\ & & & & 0 \end{pmatrix}. \quad (20)$$

Note that the diagonal singlino ( $\tilde{S}$ ) mass term is zero. Furthermore, the singlino mixings (with Higgsinos) are proportional to  $\lambda$ , which, as remarked earlier, turns out to be quite small, especially in the universality scenario. Consequently, the singlino state appears to be an almost pure singlet state with a very small mass, so that it is always the lightest supersymmetric particle (LSP) of the model. Actually, in the universal case, we find:  $\text{few MeV} \lesssim m_{\tilde{S}} \lesssim 3 \text{ GeV}$ , with a singlet component  $\gtrsim 99\%$ . This state has only small couplings to the gauge bosons and to the other sparticles. (We have explicitly checked that its contribution to the invisible  $Z$ -boson width is  $< 4.2 \text{ MeV}$  [18].) Thus, the production cross sections of the singlino are small and it seems to be nearly impossible to observe this particle in any experiment, this rendering the nMSSM apparently similar to the ordinary MSSM. However, as the singlino is the LSP of the model, it will appear at the end of all sparticle decay chains, giving rise to additional cascades compared with the MSSM signals. Such additional cascades are common to many supersymmetric extensions involving singlets [12, 19]. It should be noticed, however, that unlike in the NMSSM, where the singlino LSP scenario requires strong constraints on the parameter space (i.e.,  $M_{1/2} \gg m_0, A_0$ ) [12], the singlino is *always* the LSP in the nMSSM.

Besides, by assuming universality at the GUT scale, we find that the next-to-lightest supersymmetric particle (NLSP) is always the second lightest neutralino, which turns out to be a quasi-pure bino ( $\tilde{B}$ ). Depending on the region of the param-



eter space under scrutiny, the following channels can play a role in the NLSP  $\rightarrow$  LSP cascade:

- $\tilde{B} \rightarrow \tilde{S}\nu\bar{\nu}$  (sneutrino/ $Z$  exchange) giving an invisible cascade;
- $\tilde{B} \rightarrow \tilde{S}l^+l^-$  (slepton/ $Z$  exchange) where the leptons could be mainly  $\tau$ 's, the stau being lighter than the other sleptons;
- $\tilde{B} \rightarrow \tilde{S}q\bar{q}$  (squark/ $Z$  exchange) the branching ratio being quite small ( $\lesssim 10\%$ ), as the squarks are usually heavy;
- $\tilde{B} \rightarrow \tilde{S}Z$  if the  $\tilde{B}$  is heavy enough;
- $\tilde{B} \rightarrow \tilde{S}S$  where  $S$  is a light quasi-pure singlet Higgs boson, decaying to  $b\bar{b}$  or  $\tau\bar{\tau}$  depending on its mass;
- $\tilde{B} \rightarrow \tilde{S}\gamma$  through loops.

The properties of these cascades have been analyzed in detail in Ref. [12] for the case of the NMSSM and most of the results can equally apply to the case of the nMSSM. As for experimental searches, high multiplicity events have been under investigation already in the context of models with gauge mediated supersymmetry breaking [20] or with  $R$ -parity violation [21]. In principle, small  $\lambda$ 's ( $\lesssim 10^{-4}$ ) could give rise to a delayed NLSP  $\rightarrow$  LSP transition, i.e. a displaced neutral vertex [12, 19]. However, such values of  $\lambda$  are disfavoured if one wants the tadpole term of eq. (6) to be large enough, so that displaced neutral vertices are not expected as typical signatures of the nMSSM.

### 3.2 Higgs couplings and mass bounds

The Higgs sector of the nMSSM consists of three CP-even neutral states, denoted by  $S_i$  with masses  $m_1 < m_2 < m_3$ , plus two CP-odd neutral states, labelled as  $P_i$  with masses  $m'_1 < m'_2$ . The tree-level mass matrix for the CP-even states in the basis  $(ReH_1^0, ReH_2^0, ReS)$  reads

$$\mathcal{M}_S^2 = \begin{pmatrix} g^2 h_1^2 - \lambda s A_\lambda \tan \beta & (2\lambda^2 - g^2) h_1 h_2 + \lambda s A_\lambda & \lambda(2\lambda s h_1 + A_\lambda h_2) \\ & g^2 h_2^2 - \lambda s A_\lambda \cot \beta & \lambda(2\lambda s h_2 + A_\lambda h_1) \\ & & -\lambda^2 A_\lambda \frac{h_1 h_2}{\lambda s} - \lambda \frac{\xi^3}{\lambda s} \end{pmatrix} \quad (21)$$

where  $g^2 = (g_1^2 + g_2^2)/2$ . In the reminder of this section, we study the upper bounds on the lightest CP-even states with general soft susy breaking terms, in a non-universal scenario. By taking into account the full one-loop and the dominant two-loop top/stop corrections displayed in section 2, and assuming  $h_i \ll M_{susy}$ , one obtains the following upper limit on the lightest CP-even Higgs mass:

$$m_1^2 \leq M_Z^2 \left( \cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right) \left( 1 - \frac{3h_t^2}{8\pi^2} t \right) + \frac{3h_t^2(m_t)}{4\pi^2} m_t^2(m_t) \sin^2 \beta \left( \frac{1}{2} \tilde{X}_t + t + \frac{1}{16\pi^2} \left( \frac{3}{2} h_t^2 - 32\pi\alpha_s \right) (\tilde{X}_t + t)t \right) \quad (22)$$

where

$$\tilde{X}_t \equiv 2 \frac{\tilde{A}_t^2}{M_{susy}^2} \left( 1 - \frac{\tilde{A}_t^2}{12M_{susy}^2} \right), \quad (23)$$

$$\tilde{A}_t \equiv A_t - \lambda s \cot \beta, \quad (24)$$

$A_t$  being the top trilinear soft term.

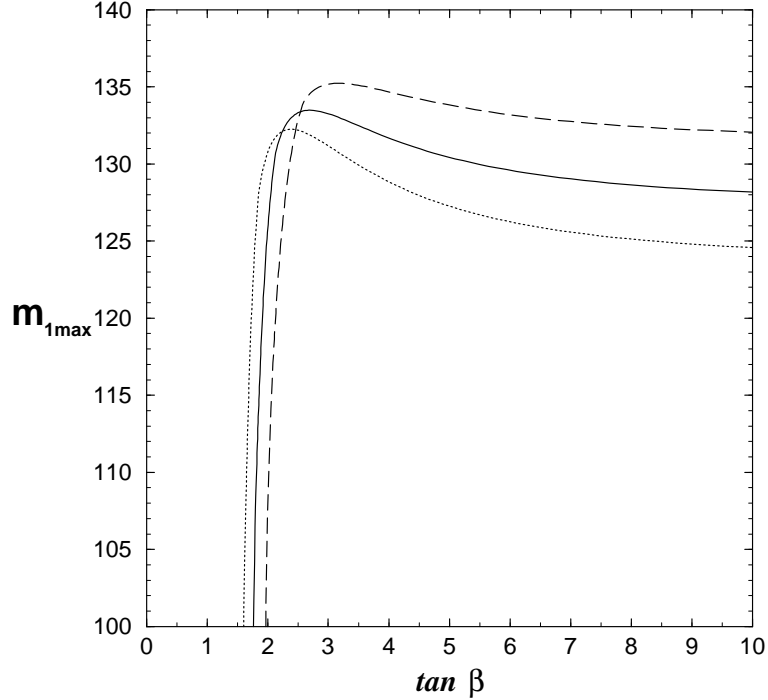


Figure 2: Upper bound on  $m_1$  [GeV] versus  $\tan \beta$  for  $m_t^{pole} = 173.8 \pm 5.2$  GeV (straight, dashed, dotted line respectively), and  $M_{susy} \leq 1$  TeV.

The only difference between the MSSM bound [22] and eq. (22) is the ‘tree-level’ contribution  $\sim \lambda^2 \sin^2 2\beta$ . This term is important for moderate values of  $\tan \beta$ . Hence, the maximum of the lightest Higgs mass in the nMSSM is not obtained for large  $\tan \beta$  values, like in the MSSM, rather for moderate ones (cf. fig. 2). In contrast, the radiative corrections are identical in the nMSSM and in the MSSM. In particular, the linear dependence in  $\tilde{X}_t$  is the same in both models. Hence, from eq. (23), the upper bound on  $m_1^2$  is maximised for  $\tilde{X}_t = 6$  (corresponding to  $\tilde{A}_t = \sqrt{6}M_{susy}$ , the

'maximal mixing' case) and minimized for  $\widetilde{X}_t = 0$  (corresponding to  $\widetilde{A}_t = 0$ , the 'no mixing' case).

However, the upper limit on  $m_1$  is not necessarily physically relevant, since the coupling of the lightest CP-even Higgs boson to the  $Z$ -boson can be very small. Actually, this phenomenon can also appear in the MSSM, if  $\sin^2(\beta - \alpha)$  is small. In this case though, the CP-odd Higgs boson  $A$  is necessarily light ( $m_A \sim m_h < M_Z$  at tree-level) and the process  $e^+e^- \rightarrow Z \rightarrow hA$  can be used to cover this region of the MSSM parameter space. In the nMSSM, a small gauge boson coupling of the lightest Higgs  $S_1$  is usually related to a large singlet component, in which case no (strongly coupled) light CP-odd Higgs boson is available. Hence, Higgs searches in the nMSSM have to possibly rely on the search for the second lightest Higgs scalar  $S_2$ .

Let us define the reduced coupling  $R_i$  as the square of the  $ZZS_i$  coupling divided by the corresponding standard model Higgs -  $Z$  boson coupling

$$R_i = (S_{i1} \cos\beta + S_{i2} \sin\beta)^2, \quad (25)$$

where  $S_{i1}, S_{i2}$  are the  $H_1, H_2$  components of the CP-even Higgs boson  $S_i$ , respectively. Evidently, we have  $0 \leq R_i \leq 1$  and unitarity implies

$$\sum_{i=1}^3 R_i = 1. \quad (26)$$

We are interested in upper bounds on the two lightest CP-even Higgs bosons  $S_{1,2}$ . These are obtained in the limit where the third Higgs,  $S_3$ , is heavy and decoupled, i.e.,  $R_3 \sim 0$  (this scenario is similar the so-called decoupling limit in the MSSM: the upper bound on the lightest Higgs  $h$  is saturated when the second Higgs  $H$  is heavy and decouples from the gauge bosons). In this limit, we have  $R_1 + R_2 \simeq 1$ .

In the regime  $R_1 \geq 1/2$ , experiments will evidently first discover the lightest Higgs (with  $m_1 \leq 133.5$  GeV for  $m_t^{pole} = 173.8$  GeV and  $M_{susy} = 1$  TeV). The 'worst case scenario' in this regime corresponds to  $m_1 \simeq 133.5$  GeV and  $R_1 \simeq 1/2$ : the presence of a Higgs boson with these properties has to be excluded in order to test this part of the parameter space of the nMSSM.

In the regime  $R_1 < 1/2$  (i.e.  $1/2 < R_2 \leq 1$ ) the lightest Higgs may escape detection because of its small coupling to gauge bosons, and it may be easier to look for the second lightest Higgs  $S_2$ . In fig. 3 we show the upper limit on  $m_2$  as a function of  $R_2$  as a thin straight line. For  $R_2 \rightarrow 1$  (i.e.  $R_1 \rightarrow 0$ ), the upper limit on  $m_2$  is actually given by the previous upper limit on  $m_1$ , even if the corresponding Higgs boson is the second lightest one. For  $R_2 \rightarrow 1/2$ , on the other hand,  $m_2$  can be as large as 190 GeV. However, one finds that the upper limit on  $m_2$  is saturated when the mass  $m_1$  of the lightest Higgs boson tends to 0. Clearly, one has to take into account the constraints from Higgs boson searches which apply to reduced couplings  $R < 1/2$ , i.e., lower limits on  $m_1$  as a function of  $R_1 \simeq 1 - R_2$ , in order to obtain realistic upper limits on  $m_2$  versus  $R_2$ . The dotted curves in fig. 3 show the upper limit on  $m_2$  as a function of  $R_2$  for different fixed values of  $m_1$  (as indicated on each curve). They can be used to obtain upper limits on the mass  $m_2$ , in the regime  $R_1 < 1/2$ , for arbitrary experimental lower limits on the mass  $m_1$  versus  $R_1$ . For each value of the

coupling  $R_1$ , which would correspond to a vertical line in fig. 3, one has to find the point where this vertical line crosses the dotted curve associated to the corresponding experimental lower limit on  $m_1$ . Joining these points by a curve leads to the upper limit on  $m_2$  as a function of  $R_2$ . We have indicated as a thick straight line in fig. 3 the present LEP-II bounds [23], which give, in the ‘worst case’ scenario, an upper limit on  $m_2$  of  $\simeq 160$  GeV for  $R_2 \simeq 1/2$ .

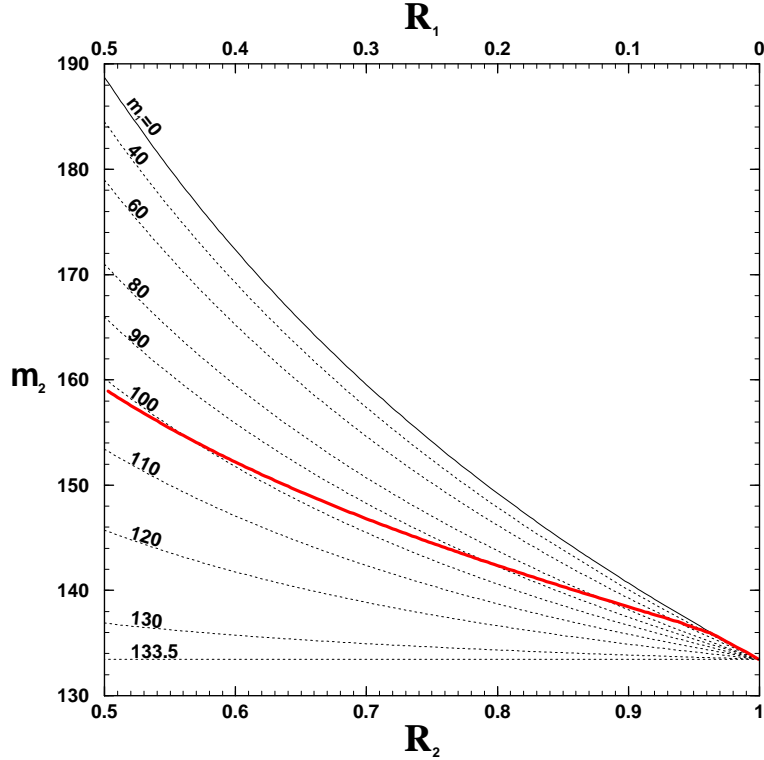


Figure 3: Upper limits on the mass  $m_2$  (in GeV) against  $R_2$ , for different values of  $m_1$  (as indicated on each line in GeV), assuming  $m_t^{pole} = 173.8$  GeV and  $M_{susy} \leq 1$  TeV.  $R_1 = 1 - R_2$  is shown on the the top axis. The thick straight line corresponds to LEP-II lower limits on  $m_1$  vs.  $R_1$ .

Lower experimental limits on a Higgs boson with  $R > 1/2$  restrict the allowed regime for  $m_2$  (for  $R_2 > 1/2$ ) in fig. 3 from below. The present lower limits on  $m_2$  from LEP-II are not visible in fig. 3, since we have only shown the range  $m_2 > 130$  GeV. Possibly Higgs searches at Tevatron Run II will push the lower limits on  $m_2$  upwards into this range. This would be necessary if one aims at an exclusion of this regime of the nMSSM. Then, lower limits on the mass  $m_2$  – for any value of  $R_2$  between  $1/2$  and  $1$  – of at least  $133.5$  GeV are required. The precise experimental lower limits on  $m_2$  as a function of  $R_2$ , which would be needed to this end, will depend on the achieved lower limits on  $m_1$  as a function of  $R_1$  in the regime  $R_1 < 1/2$ .

In principle, from eq. (26), one could have  $R_2 > R_1$  with  $R_2$  as small as  $1/3$ . However, in the regime  $1/3 < R_2 < 1/2$ , the upper bound on  $m_2$  as a function of

$R_2$  for different fixed values of  $m_1$  can only be saturated if  $R_1 = R_2$ . It is then sufficient to look for the lightest Higgs  $S_1$  (i.e. for a Higgs boson with a coupling  $1/3 < R < 1/2$  and a mass  $m \lesssim 133.5$  GeV) to cover this region of the parameter space of the nMSSM.

One can notice that these results are the same as those obtained in the NMSSM [24]. This comes from the fact that the non-singlet part of the CP-even mass matrix (21) as well as the singlet/non-singlet mixing terms are the same in both models, giving the same upper limit on  $m_1$ , eq. (22) and fig. 2. Even though they are not similar, the CP-even singlet mass term (the  $3 \times 3$  element in eq. (21)) is ‘free’ in both models, i.e., it can take any value between 0 and  $\sim 1$  TeV. This explains why the curves displayed in fig. 3 are the same in both models, the upper bounds on  $m_2$  stemming from degenerate singlet/non-singlet mass terms, degeneration lifted by the mixings (off-diagonal  $1 \times 3$  and  $2 \times 3$  terms in eq. (22)).

Thus, the phenomenological potential of the nMSSM is not less exciting in the Higgs sector than it is in the sparticle sector. On the one hand, the necessary (but not sufficient) condition for testing the complete parameter space of the nMSSM is to rule out a CP-even Higgs boson with a coupling  $1/3 < R < 1$  and a mass below 135 GeV. On the other hand, the sufficient condition (i.e., the precise upper bound on  $m_2$  versus  $R_2$ ) depends on the achieved lower bound on the mass of a ‘weakly’ coupled Higgs (with  $0 < R < 1/2$ ) and can be obtained from fig. 3. At the Tevatron Run II this would probably require an integrated luminosity of up to  $30 \text{ fb}^{-1}$  [25]. If this cannot be achieved, one has to wait for the advent of the LHC, in order to know whether the nMSSM is actually realized in nature.

Imposing universality of the soft terms at the GUT scale, CCB constraint (eq. (17)) and LEP II experimental constraints on Higgs masses [23] one finds the more restrictive bound  $m_1 < 122$  GeV under the same hypotheses ( $m_t^{pole} = 173.8 \pm 5.2$  GeV and  $M_{susy} < 1$  TeV) and  $m_2 < 135$  GeV in the case where  $S_1$  is mainly singlet ( $R_1 < R_2$ ).

An intriguing example of possible evidence of the nMSSM at future hadron-hadron colliders is the following. Recall that a crucial signature for an intermediate Higgs boson at both the Tevatron and the LHC is the one produced via  $q\bar{q}' \rightarrow W^\pm$  Higgs (with a smaller contribution from  $q\bar{q} \rightarrow Z$  Higgs as well), with the gauge vector yielding high transverse-momentum and isolated leptons and the Higgs scalar decaying into  $b\bar{b}$  pairs. Now, let us imagine that Higgs searches in this mode have finally revealed the evidence of a light scalar Higgs resonance, but no further Higgs states are detected up to well above the EW scale. (In this respect, the LHC is a better example to illustrate, as compared to the Tevatron, because of its much extended scope in mass.) This scenario could be realized in the MSSM, if the latter is in the above mentioned decoupling regime, where one has  $m_{H^\pm} \approx m_H \approx m_A \gg M_Z$  and the strength of the lightest Higgs boson couplings to the gauge vector bosons  $W^\pm$  and  $Z$  approaches unity,  $R_h \equiv \sin^2(\beta - \alpha) \simeq 1$ . Indeed, this phenomenology can also be realized in the nMSSM when, e.g.,  $R_1 = 1$  and  $R_2 = 0^2$ . Under these circumstances, the mass of the

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<sup>2</sup>Further notice that, no matter their actual mass value, pseudoscalar neutral Higgs states of either model cannot be produced via the above two processes, as their coupling to  $W^\pm$  and  $Z$  vectors is prohibited at tree-level.

$h$  scalar would only depend on  $\tan\beta$  and the minimum of the Higgs production cross section is obtained, in both susy models, in correspondence of the maximum Higgs mass. Fig. 4 shows this dependence for both the MSSM and the nMSSM at the LHC, with  $\sqrt{s} = 14$  TeV. (The trend is very similar at the Tevatron, where  $\sqrt{s} = 2$  TeV.) Over a broad range in  $\tan\beta$ , the production rates of the latter are significantly below those of the former, with a minimum at  $\tan\beta \simeq 2.7$  (for  $m_t = 173.8$  GeV), owing to the peculiar  $\tan\beta$  dependence of the maximum value of  $m_1$ , as seen in fig. 2 (recall instead that the maximum mass of the lightest Higgs boson of the MSSM increases monotonically with  $\tan\beta$  [22], hence the cross section here decreases correspondingly, for  $\tan\beta \lesssim 40$ ).

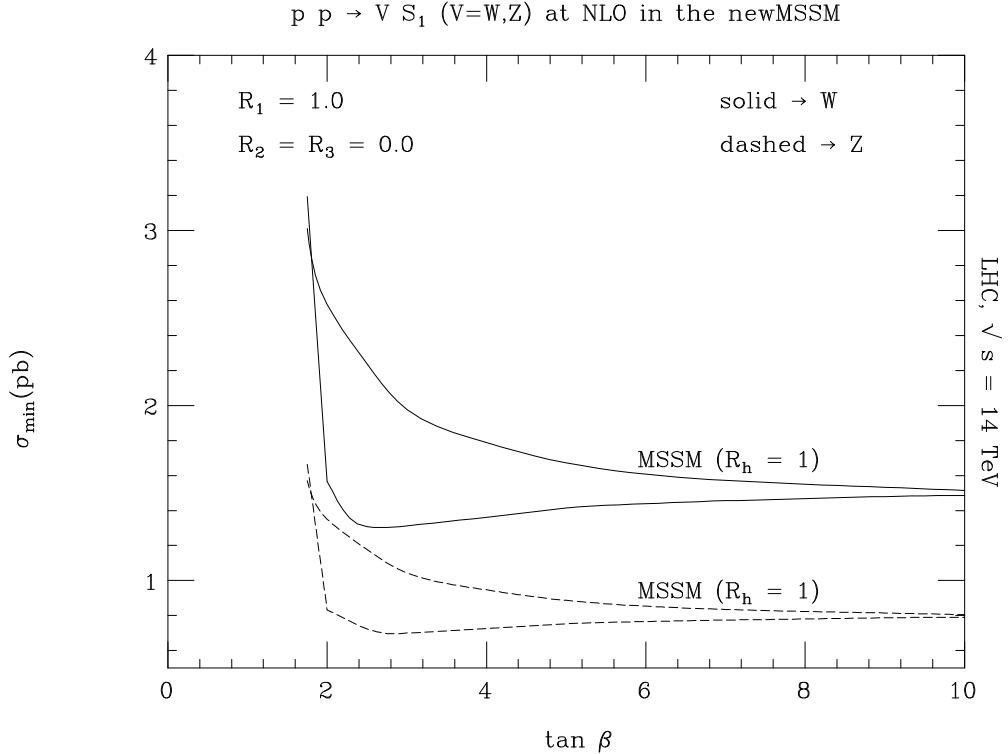


Figure 4: Minimum of the production cross section of a neutral scalar Higgs with SM-like couplings to  $W^\pm$  and  $Z$  gauge bosons at the LHC, as a function of  $\tan\beta$ , in correspondence of the maximum values of its mass, in both the MSSM and the nMSSM.

Thus, if  $\tan\beta$  has already been measured in another context (e.g., in susy sparticle processes), to detect an isolated scalar resonance in the  $b\bar{b}$  channel (the dominant decay model in either model for  $M_{\text{Higgs}} \lesssim 133.5$  GeV), at a rate well below the minimum one predicted by the MSSM, could imply that physics beyond the latter would be realized in nature. In fact, the reader should recall that the  $\text{BR}(\text{Higgs} \rightarrow b\bar{b})$  is basically the same in both models [26] and the combined uncertainties on the rates of the latter and of the production modes (due to higher-order effects, parton distri-

bution functions, hard scale dependence, etc.) are of the order of just a few percent [27]. In contrast, the differences between the MSSM and nMSSM rates can be as large as a factor of two, in the vicinity of  $\tan\beta \simeq 2.7$ , and well above the mentioned uncertainties for  $\tan\beta$  up to 10 or so. Besides, kinematical analysis of the  $b\bar{b}$  system might provide further evidence in this respect, if the mass resolution is larger than the difference between the Higgs mass values, as predicted by the two models for a given  $\tan\beta$ . Similar arguments can be made for the case of the  $gg \rightarrow \text{Higgs} \rightarrow \gamma\gamma$  signature too, however, the production and decay phenomenology is here much more involved (because loop processes take place at either stage), so that we leave it aside for future consideration [28].

## 4 Summary and conclusions

We have studied the phenomenology of the nMSSM, which promotes the  $\mu$ -parameter into a singlet superfield, hence solving the well-known  $\mu$ -problem of the MSSM. Besides, cubic self-couplings and possible dimensionful couplings are avoided in the nMSSM thanks to a global discrete R-symmetry, in turn broken by supergravity-induced tadpole corrections, which solves both the so-called ‘domain wall’ and ‘axion’ problems. The new model is truly minimal, in the sense that – despite incorporating new fields – it can be parametrised by the same number of inputs as in the MSSM in the universal case. Its phenomenology stands out quite different from that of both the ordinary MSSM and the so-called Next-to-Minimal Supersymmetric Standard Model (NMSSM) – where the cubic self-interacting coupling is instead present. In particular, the following aspects emerged as crucial from our analysis.

- Assuming an accuracy up to the dominant top-stop contributions at two-loop level and depending on the (reduced) couplings of the two lighter CP-even Higgs bosons to the  $Z$ -boson, we have found that the upper limit on the mass of the lightest Higgs state of the nMSSM can be 133.5 GeV, in correspondence of the central value of the top mass (i.e.,  $m_t = 173.8$  GeV), that is, about 10 GeV higher than the MSSM value and within the Tevatron Run-II reach. In addition, the upper limit on the mass of the lightest ‘observable’ Higgs boson (i.e., the next-to-lightest one, when the lightest one couples invisibly to the  $Z$ -boson) could be as high as 160 GeV but still within the LHC scope.
- To remain with the Higgs sector, we also have described a benchmark example that could allow one to phenomenologically distinguish the nMSSM from the MSSM in the search for the lightest Higgs state at future hadron colliders, such as the Tevatron Run-II and the LHC. If only one neutral Higgs state is accessible through associated production with an EW gauge vector  $W^\pm$  or  $Z$ , via its  $b\bar{b}$  decays, the knowledge of  $\tan\beta$  and of the production rate of such Higgs process could be enough to assign such a Higgs state to one or the other of the two models, even prior to the investigation of the mass resonance that can eventually be reconstructed from the  $b\bar{b}$  system. Production and decay studies of all other Higgs states of the nMSSM are now in progress [28].

- Despite the remarkable dissimilarities seen so far between the nMSSM and the MSSM, one might quite rightly question that the phenomenology of the nMSSM is very similar to that of the NMSSM, as far as the Higgs sector is concerned. However, a dramatic difference is revealed between these two models, if one investigates CP-violation effects. In fact, no matter the actual value of  $\lambda$ , a peculiar feature of the nMSSM is the following: Contrary to the NMSSM case, spontaneous CP-violation cannot occur in the Higgs sector of the nMSSM, neither at tree-level nor at one-loop.
- Finally, further differences between the nMSSM and any other model can be appreciated in the sparticle sector. In fact, here, two concurrent aspects render the phenomenology of the former both more ‘spectacular’ and ‘natural’, in comparison to the MSSM and the NMSSM, respectively. Firstly, the lightest neutralino appears to be an almost pure singlet state. Secondly, such a state (the ‘singlino’) is *always* the LSP of the theory, with a very small mass (varying from a few MeV to a few GeV). The consequence is twofold. On the one hand, in comparison to the NMSSM, the singlino LSP scenario requires no strong constraints to be imposed on the parameter space. On the other hand, in comparison to the MSSM, any sparticle decay chain involves a further step, the NLSP  $\rightarrow$  LSP transition, giving rise to additional cascades.
- The stimulating issue that such an LSP could be a good dark matter candidate or, alternatively, could be excluded from cosmological arguments, also deserves attention. However, a quantitative analysis in this respect was far beyond the intention of this note.

Concluding, the nMSSM is, at the same time, the *best theoretically motivated* and the *most economical* susy extension of the SM. Here, we have pointed out differences or similarities between the new model, the NMSSM and the MSSM, in both the Higgs and neutralino sectors. Future collider experiments, at Tevatron Run-II and LHC, will be able to prove whether or not the nMSSM is the realization of Supersymmetry that nature has chosen.

**Note** While finalising this paper, whose main results were made public already in Ref. [29], we became aware of Ref. [30], by C. Panagiotakopoulos and A. Pilaftsis, which deals with a similar subject. In this respect, although we agree with the more general statements given in this other paper, we would like to remark that there only the top-stop one-loop corrections were used, whereas here we have calculated also the corresponding two-loop contributions. This explains the significant discrepancy between their and our upper limit on the lightest Higgs boson mass:  $m_1 < 150$  GeV versus  $m_1 < 133.5$  GeV.

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