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Unification bounds on the possible $N = 2$ supersymmetry-breaking scale \star

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Abstract

In this letter, the possible appearance of $N = 2$ supersymmetry at a low energy scale is investigated in the context of unified theories. Introducing mirror particles for all the gauge and matter multiplets of the minimal supersymmetric extension of the Standard Model (MSSM), the measured values of $\sin^2\theta_w$ and $\alpha_3(M_Z)$ indicate that the $N = 2$ threshold scale M_{S_2} cannot be lower than $\sim 10^{14}$ GeV. If the $U(1)$ normalization coefficient k is treated as a free parameter, M_{S_2} can be as low as 10^9 GeV. On the other hand, if mirror quarks and leptons are absent and a non-standard value for k is used, $N = 2$ supersymmetry breaking could in principle occur at the electroweak scale. © 1997 Published by Elsevier Science B.V.

It has recently been realized [1] that $N = 2$ supersymmetry can be broken spontaneously to $N = 1$ in the context of local quantum field theory, which opens up the possibility that $N = 2$ supersymmetry may become relevant at some intermediate energy scale below the Planck or string scale. Possible $N = 2$ extensions of the Standard Model (SM) have been studied in the past [2] and they are much more restrictive than the $N = 1$ framework. In particular, because of the vanishing of $\text{Str}(M^2)$ after supersymmetry breaking, they guarantee the absence of all field-dependent quadratic divergences in the scalar potential which is a desirable ingredient for solving the hierarchy problem. In this letter, we derive lower bounds on the $N = 2$ breaking scale in the context of unified theories.

It is well known that the $N = 1$ supersymmetric beta-function coefficients b_i allow the three gauge couplings of the electroweak and strong forces to attain a common value at a scale $M_X \sim 10^{16}$ GeV. If $N = 2$ supersymmetry appears at some intermediate threshold scale M_{S_2} , the beta-function coefficients change drastically due to the contributions of the $N = 2$ superpartners of all the SM states. In terms of $N = 1$ superfields, these are one adjoint for each group factor of the gauge symmetry, and one mirror (of opposite chirality) for each matter field. The introduction of mirrors for both Higgs doublets is also necessary for the breaking of the $SU(2)$ gauge symmetry. As a result, gauge coupling unification occurs in general at a different scale M_U , which turns out to be greater than M_X .

In this letter, we study the allowed values of M_U and the corresponding lower bounds for the $N = 2$

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scale M_{S_2} , which are consistent with the low-energy data. We find an interesting correlation between the two scales, namely that higher M_U implies lower M_{S_2} . Fixing the normalization of the $U(1)$ hypercharge to the standard value $k = 5/3$ we find that M_{S_2} cannot be smaller than $\sim 10^{14}$ GeV. However, if a different $U(1)$ hypercharge normalization is allowed, M_{S_2} can be as low as $\sim 10^9$ GeV.

In the energy range between M_{S_2} and the unification scale M_U , the beta-function coefficients read

$$b_1^{N=2} = \frac{66}{5}, \quad b_2^{N=2} = 10, \quad b_3^{N=2} = 6, \quad (1)$$

for the $U(1)$, $SU(2)$ and $SU(3)$ gauge group factors respectively. For simplicity, we assume that $N = 1$ supersymmetry remains exact down to the M_Z scale, so that in the range M_Z to M_{S_2} the beta-function coefficients are those of the $N = 1$ minimal supersymmetric extension of the Standard Model (MSSM), namely

$$b_1^{N=1} = \frac{33}{5}, \quad b_2^{N=1} = 1, \quad b_3^{N=1} = -3. \quad (2)$$

Using the renormalization-group equations for the three gauge couplings, we first eliminate the M_{S_2} scale to obtain the following formula for the unification scale M_U in terms of the experimentally-measured low-energy parameters:

$$\log \frac{M_U}{M_Z} = \frac{\pi}{2\alpha} \left(\sin^2 \theta_W - \frac{\alpha}{\alpha_3} \right), \quad (3)$$

where α , α_3 are the low energy electromagnetic and strong coupling constants, respectively.

In this paper, we make the self-consistent approximation of ignoring low-energy threshold, two-loop effects in the region below M_{S_2} , and the model-dependent high-energy threshold around M_{S_2} . Above this scale, $N = 2$ supersymmetry is unbroken and there are no higher loop corrections. We should point out that the effects we ignore are potentially important and may alter our results. It is known that these are important for detailed comparisons of $N = 1$ supersymmetric GUTs with the available experimental data (for a review, see [3]). However, since the high-energy threshold effects are currently unknown, we prefer to restrict this analysis to the self-consistent one-loop approximation, and add larger theoretical error bars to the purely experimental errors on the low-energy value of $\sin^2 \theta_W$.

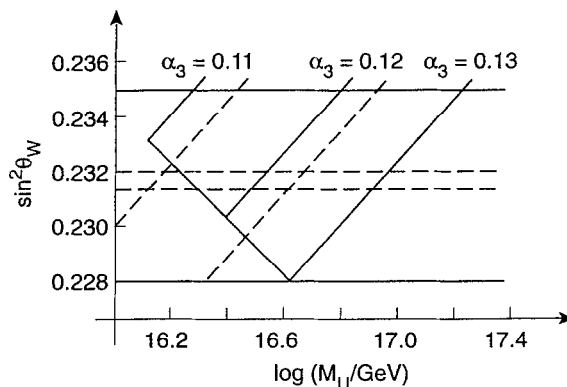


Fig. 1. The range of M_U allowed by the experimental values of α_3 and $\sin^2 \theta_W$ given in Eq. (4). Dotted lines correspond to purely experimental errors, whilst the solid ones include an allowance for theoretical uncertainties.

The experimental values of the low-energy parameters that we use as the basis for our determination of M_U are [4]

$$\begin{aligned} \sin^2 \theta_W &= 0.2316 \pm 0.0004 (\pm 0.003), \\ \alpha_3 &= 0.118 \pm 0.005, \end{aligned} \quad (4)$$

where the second error in $\sin^2 \theta_W$ accounts for the theoretical uncertainties mentioned above, and has been chosen to have the same magnitude as the two-loop effect in the desert in conventional $N = 1$ unification. The resulting M_U region is shown in Fig. 1. The dashed lines represent the first (experimental) error in $\sin^2 \theta_W$ of Eq. (4). We have also indicated the effect of relaxing the experimental constraints on α_3 , allowing it to vary over the range ~ 0.11 – 0.13 . We deduce that, despite the introduction of the new free parameter representing the $N = 2$ threshold scale, the low-energy data give a rather stringent constraint on the unification mass, which has to be less than 2×10^{17} GeV. On the other hand, the assumed hierarchy of scales, $M_{S_2} \leq M_U$, implies the constraint

$$\sin^2 \theta_W \geq \frac{3}{8} - \frac{7\alpha}{4\pi} \ln \frac{M_U}{M_Z}, \quad (5)$$

which requires $M_U \gtrsim 10^{16}$ GeV. The constraint (5) is represented in Fig. 1 by a straight line, corresponding to $M_{S_2} = M_U$, which excludes values of $(\sin^2 \theta_W, M_U)$ below it and to its left.

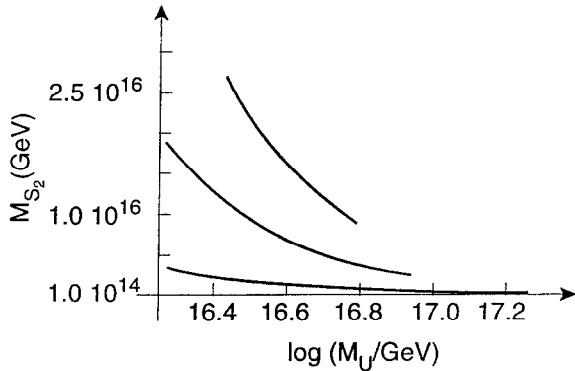


Fig. 2. The $N = 2$ scale M_{S_2} , as a function of M_U , for three values of the minimal $N = 1$ supersymmetric unification scale: $M_X = (0.9, 1.86, 2.69) \times 10^{16}$ GeV.

We now come to the computation of the intermediate $N = 2$ scale. It is useful to express it as a function of the parameters M_U and $\sin^2 \theta_w$, so that we can determine its range in the parameter space of Fig. 1. We obtain

$$M_{S_2} = e^{\frac{4\pi}{3\alpha}(\frac{3}{8} - \sin^2 \theta_w)} \left(\frac{M_Z}{M_U} \right)^{\frac{4}{3}} M_Z. \quad (6)$$

This expression should be compared with the one obtained for the minimal supersymmetric grand unification scenario assuming that the only light particles are those of the MSSM, where the unification scale M_X is given in the one-loop approximation by

$$M_X = e^{\frac{4\pi}{7\alpha}(\frac{3}{8} - \sin^2 \theta_w)} M_Z. \quad (7)$$

Using Eq. (7), we can rewrite M_{S_2} in Eq. (6) as

$$M_{S_2} = \left(\frac{M_X}{M_U} \right)^{\frac{7}{3}} M_U. \quad (8)$$

Thus, for a given M_X , or equivalently $\sin^2 \theta_w$, we can plot M_{S_2} as a function of the unification scale M_U whose range was shown in Fig. 1. This is shown in Fig. 2. We conclude that when M_U is near its lower bound $\sim 10^{16}$ GeV $\sim M_X$, then $M_{S_2} \sim M_X$, while as M_U approaches its higher bound $\sim 2 \times 10^{17}$ GeV then $M_{S_2} \sim 10^{14}$ GeV.

It is important to note that, over the entire allowed M_U range, the value of the gauge coupling at the unification scale remains small: $\alpha_U \ll 1$. In Table 1, we display the value of α_U for three representative cases.

Table 1

Lower bounds on M_{S_2} and the corresponding values of M_U and α_U , for three indicative choices of α_3

α_3	M_U/GeV	M_{S_2}/GeV	$1/\alpha_U$
0.11	$1.91 \cdot 10^{16}$	$2.90 \cdot 10^{15}$	22.14
0.12	$6.30 \cdot 10^{16}$	$5.76 \cdot 10^{14}$	17.92
0.13	$1.70 \cdot 10^{17}$	$1.56 \cdot 10^{14}$	14.46

As we show below, the scale M_{S_2} could decrease if the $U(1)$ normalization coefficient k is larger than its standard value $5/3$ at the unification mass. Conventional $N = 1$ string unification needs small k values to reconcile the high string scale with the low-energy value of the weak mixing angle $\sin^2 \theta_w$. On the other hand, such non-standard $U(1)$ normalizations have been discussed in the context of superstring models [5], which offer the possibility that the k parameter might be larger than $5/3$. This is possible, for example, if the hypercharge generator corresponds to a linear combination of $U(1)$ factors, with an embedding into a higher-rank non-abelian gauge group. Such higher-level constructions have been motivated by two phenomenological considerations: they could guarantee the absence of color-singlet states with fractional electric charges in four dimensional string models [6] (though these could also be confined by hidden-sector interactions, analogously to quarks in QCD [7]).

A key observation is that Eq. (3), which gives the unification scale M_U , is independent of the normalization coefficient k . On the other hand, Eq. (6), which gives the M_{S_2} scale, becomes for arbitrary values of k

$$M_{S_2} = \left[e^{\frac{4\pi}{3\alpha}(\frac{3}{8} - \frac{5}{6} \sin^2 \theta_w)} \left(\frac{M_Z}{M_U} \right)^{\frac{22}{9}} \right]^{-\frac{k-5/3}{k-11/9}} \times M_{S_2} \Big|_{k=5/3}, \quad (9)$$

where we have replaced the expression for M_{S_2} in the $k = 5/3$ case from Eq. (6). It is clear that, as k increases to values larger than $5/3$, the scale M_{S_2} decreases rapidly due to the exponential suppression factor in Eq. (9).

In addition to the above expression for M_{S_2} , the formula for the gauge coupling at M_U , α_U , also depends on k :

Table 2
Bounds on k and M_{S_2} for three choices of α_3 , enforcing $\alpha_U = 1$

α_3	M_U/GeV	k	M_{S_2}/GeV	α_U
0.11	$1.35 \cdot 10^{16}$	12.17	$9.25 \cdot 10^8$	1
	$2.00 \cdot 10^{16}$	10.97	$1.21 \cdot 10^9$	
0.12	$2.70 \cdot 10^{16}$	11.95	$2.60 \cdot 10^9$	1
	$5.91 \cdot 10^{16}$	9.30	$3.98 \cdot 10^9$	
0.13	$4.26 \cdot 10^{16}$	11.80	$5.18 \cdot 10^9$	1
	$1.71 \cdot 10^{17}$	7.65	$1.30 \cdot 10^{10}$	

Table 3
The M_{S_2} scale and the value of the unification coupling α_U for two choices of the normalization constant k

α_3	$\sin^2 \theta_W$	M_U/GeV	k	M_{S_2}/GeV	α_U
0.118	0.2316	$2.52 \cdot 10^{16}$	17/3	$5.32 \cdot 10^9$	0.428
0.130	0.2350	$4.26 \cdot 10^{16}$	17/3	$1.35 \cdot 10^{10}$	0.420
0.130	0.2280	$1.63 \cdot 10^{17}$	17/3	$1.17 \cdot 10^{10}$	0.679
0.118	0.2316	$2.52 \cdot 10^{16}$	29/3	$2.46 \cdot 10^9$	0.814
0.130	0.2316	$8.67 \cdot 10^{16}$	29/3	$8.25 \cdot 10^9$	1.0
0.118	0.2348	$4.80 \cdot 10^{16}$	29/3	$3.22 \cdot 10^9$	1.0

$$k = \frac{11}{9} + \frac{\alpha_U}{\alpha} \left(1 + \frac{22}{9} \frac{\alpha}{\alpha_3} - \frac{14}{3} \sin^2 \theta_W \right). \quad (10)$$

Requiring $\alpha_U \leq 1$, we can thus obtain an upper bound on k for any given unification mass. In Table 2, we present these upper bounds for three indicative values of the strong coupling α_3 . We see that, even allowing for a larger value of k , M_{S_2} cannot be lower than about $\sim (10^8\text{--}10^9)$ GeV.

At this point, one may ask whether k can be large enough to be able to impose charge quantization without invoking confinement in the hidden sector [7]. For this, one needs $k \geq 17/3$ [6]. It is obvious from Table 2, that the answer to this question is positive provided that the unification coupling is $\sim \mathcal{O}(1)$. In Table 3 we give the $N = 2$ scale and α_U for the next two allowed k values consistent with the charge quantization condition. For $k = 17/3$, the highest α_U value obtained is ~ 0.68 . We further observe that for $k = 29/3$ the unification coupling can reach the value $\alpha_U = 1$, which corresponds to the self-dual point of the S-duality transformation: $a_U \rightarrow 1/a_U$, for a relatively wide range of the unification mass: $M_U \sim 5 \times 10^{16}\text{--}10^{17}$ GeV.

The reason that α_U becomes strong before the M_{S_2} scale can be lowered considerably is essentially the large positive contribution to the beta functions from

all the extra $N = 2$ superpartners, which include in particular the mirrors of the conventional quarks and leptons. The existence of the latter is of course problematic, since it is difficult to invent a mechanism which gives them masses and at the same time generates chirality together with partial supersymmetry breaking. Some examples overcoming this difficulty have been discussed in the context of string theory and/or using compactifications involving constant magnetic fields [8]. These examples suggest that it might be possible for the mirror fermions to form massive pairs with the Kaluza-Klein excitations, whose spectrum is shifted by the symmetry breaking. In these cases, the $N = 2$ scale is linked to the compactification radius of an extra dimension, and one needs special models with no large thresholds in order to be able to continue the renormalization-group equations above M_{S_2} [9].

In order to cover this possibility, we now repeat our analysis assuming no mirrors for the known chiral fermions (quarks and leptons). The beta-function coefficients then read

$$\tilde{b}_1^{N=2} = \frac{36}{5}, \quad \tilde{b}_2^{N=2} = 4, \quad \tilde{b}_3^{N=2} = 0. \quad (11)$$

We note that the differences $(\tilde{b}_i^{N=2} - \tilde{b}_j^{N=2})$ remain the same as the $(b_i^{N=2} - b_j^{N=2})$ of Eq. (1). Consequently, the k -independent expression (3) for the M_U scale still holds. However, relation (9) for M_{S_2} is modified to become

$$\tilde{M}_{S_2} = \left[e^{\frac{4\pi}{3\alpha} \left(\frac{3}{8} - \frac{1}{2} \sin^2 \theta_W \right)} \left(\frac{M_Z}{M_U} \right)^{\frac{8}{3}} \right]^{-\frac{k-5/3}{k-1/3}} \times M_{S_2} \Big|_{k=\frac{5}{3}}, \quad (12)$$

where again we used the expression for M_{S_2} in the $k = 5/3$ case from Eq. (6). In addition, the relation (10) is modified as follows:

$$k = \frac{1}{3} + \frac{\alpha_U}{\alpha} \left(1 + \frac{8}{3} \frac{\alpha}{\alpha_3} - 4 \sin^2 \theta_W \right) \quad (13)$$

It is easy to see now that k is allowed in principle to attain values much larger than previously, whilst keeping α_U in the perturbative region. Moreover, the scale \tilde{M}_{S_2} can be arbitrarily small even for moderate values of k . Now, Eq. (12) provides an upper bound

for k , based on the phenomenological requirement that M_{S_2} cannot be lower than the weak scale M_Z :

$$k \leq 3 + \frac{\alpha_3}{\alpha} (1 - 4 \sin^2 \theta_W). \quad (14)$$

Thus, from (3), we find the upper bound $k \lesssim 4.24$, attained when $\alpha_U \lesssim 0.11$. Note that on the boundary $\tilde{M}_{S_2} = M_Z$ one has $\alpha_U = \alpha_3$, since the beta function (11) of $SU(3)$ vanishes. Moreover, uncertainties from two-loop corrections in this latter case are eliminated, as the $N = 2$ scale remains down to M_Z . Note also that, unlike the previous case, the present bound on k is smaller than the minimum value $k = 17/3$ required from the charge quantization condition.

If in addition to omitting mirrors of the quarks and leptons, we also assume there are no mirrors for the Higgses, we are left to consider only the effect of adjoint matter ($SU(3)$ octets and $SU(2)$ triplets) at some intermediate scale. This possibility has been considered previously with the aim of increasing the unification mass close to the string scale [10].

In conclusion: in the context of unified models having as effective low-energy theory the minimal supersymmetric extension of the Standard Model, we have derived bounds on a possible $N = 2$ supersymmetry-breaking scale. Assuming the presence of mirror partners for all the chiral matter and Higgs fields and assuming the canonical normalization of the $U(1)$ of hypercharge, we have found that the $N = 2$ scale cannot be lower than 10^{14} GeV. On the other hand, if one allows a non-standard $U(1)$ normalization, the $N = 2$ scale could be as low as 10^9 GeV. If there are no mirrors for quarks and leptons, the $N = 2$ breaking scale could be as low as the electroweak scale, but there are still significant restrictions on the normalization of the $U(1)$ of hypercharge.

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