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Planck scale corrections to gauge coupling unification

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An analysis is presented of Planck scale effects through non-renormalizable dimension-five operators on the coupling constant unification in the standard and supersymmetric standard model. These effects turn out to be small.

Ever since the possibility for unification of gauge couplings was first noted [1] by showing that in the standard model radiative corrections drive the strong, electromagnetic and weak couplings together at high energy scales, the unification idea has prevailed in particle physics one way or another [2]. Recently, the interest in this idea has been revived in the light of precision data mainly from LEP, which provide very accurate values for the parameters of the standard model. In fact, using the renormalization group equations for the $SU(3)_C \times SU(2)_L \times U(1)$ gauge couplings, it is straightforward to check whether the low-energy couplings defined by the relations

$$\alpha_1(M_Z) = \frac{5}{3}\frac{g'^2(M_Z)}{4\pi} = \frac{5}{3}\frac{\alpha_{em}(M_Z)}{\cos^2\theta_w(M_Z)},$$
$$\alpha_2(M_Z) = \frac{g^2(M_Z)}{4\pi} = \frac{\alpha_{em}(M_Z)}{\sin^2\theta_w(M_Z)},$$
$$\alpha_3(M_Z) = \frac{g_s^2(M_Z)}{4\pi}, \qquad (1)$$

evolve in energy to meet at the unification value α_U in an energy scale M_X

$$\alpha_1(M_X) = \alpha_2(M_X) = \alpha_3(M_X) = \alpha_U . \qquad (2)$$

In the above relations g_s, g, g' are the coupling constants of the three gauge groups of the standard model and α_{em} the electromagnetic one. Usually, $\sin^2\theta_w(M_Z)$ and all gauge couplings are defined using the modified minimal subtraction $\overline{\text{MS}}$ scheme [3], closely connected with the dimensional reduction $\overline{\text{DR}}$ scheme [4] more appropriate for supersymmetry (SUSY).

The evolution of the coupling constants α_i, $i=1, 2, 3$, is determined by the renormalization group (RG) equations, which up to two-loop order are

$$\mu\frac{d}{d\mu}\alpha_i(\mu) = \frac{1}{2\pi}\left(b_i + \sum_{j=1}^{3}\frac{b_{ij}}{4\pi}\alpha_j(\mu)\right)\alpha_i^2(\mu), \qquad (3)$$

where μ is the energy at which the couplings are evaluated. The coefficients b_i and b_{ij} are well known for the standard model (SM) and the minimal supersymmetric standard model (MSSM) given the number N_f of matter (super)multiplets and the number N_H of Higgs doublets. Neglecting Yukawa couplings, they are summarized, for example, in ref. [5]. It is of course $N_f = 3$ and one usually takes $N_H = 1, 2$ for the SM and the MSSM, respectively. The RG equations can be also written as

$$\frac{d}{d\ln\mu}\alpha_i^{-1}(\mu) = \frac{1}{2\pi}\left(b_i + \sum_{j=1}^{3}\frac{b_{ij}}{4\pi}\alpha_j(\mu)\right), \qquad (4)$$

showing that in one-loop order the equations for the three α_i^{-1} are independent with a linear solution in the α_i^{-1}–$\ln\mu$ plane. When the two-loop order contributions are taken into account, the equations become coupled and the evolution of each α_i^{-1} depends on the other two. One solves eqs. (3), (4) by numerical integration.

Thus, using the RG equations it is straightforward to check whether the low-energy couplings evolve in

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energy to meet at the unification value α_U according to relations (2). It is found [5,6] that for the SM the couplings fail to meet by more than seven standard deviations, whereas for the MSSM they do meet in a point if the mass of the new supersymmetric states (assumed degenerate) is low $M_{\mathrm{SUSY}} \approx 10^{3\pm1}$ GeV. The unification point is at $M_X \approx 10^{16.0\pm0.3}$ GeV. Including threshold effects at the supersymmetry breaking scale [7] does not alter the unification possibility, increasing only the effective SUSY scale (the average scale of the SUSY breaking masses) by a factor of 3–10. The unification assumption has also rather small threshold corrections due to massive states at M_X [8], which may however introduce an inherent uncertainty in the above analysis [9]. Nevertheless, it is still of interest to study the minimal unification scenario, for its success may indicate simplicity in the unification idea either due to the absence of large representations of heavy states or due to the degeneracy of such states.

On the other hand, in the context of superstring theories all interactions are unified at the string unification scale, which is calculable in terms of the Planck mass M_{Pl} and threshold corrections due to massive string states [10–12]. At lowest order the string unification scale can be taken to be equal to the square root of the inverse of the Regge slope:

$$M_{\mathrm{SU}} = \frac{2}{\sqrt{\alpha'}} = \frac{g_{\mathrm{string}} M_{\mathrm{Pl}}}{\sqrt{8\pi}} = 1.7\times10^{18}\ \mathrm{GeV}\,, \qquad (5)$$

and string threshold corrections turn out to give

$$M_{\mathrm{SU}}^{\mathrm{thr}} = \frac{\exp[\frac{1}{2}(1-\gamma)]3^{-3/4}}{\sqrt{2\pi}} M_{\mathrm{SU}}$$
$$= 0.216\, M_{\mathrm{SU}} = 3.6\times10^{17}\ \mathrm{GeV} \qquad (6)$$

[13]. The question whether one can make consistent the unification scale M_X of the minimal unification scenario, i.e. of the MSSM, with the relevant string unification scale $M_{\mathrm{SU}}^{\mathrm{thr}}$ is a challenge. Some suggestions have been offered in the literature along this direction [14–16], either by considering additional matter representations at relatively low energies beyond those of the content of the MSSM pushing thus M_X towards $M_{\mathrm{SU}}^{\mathrm{thr}}$, or by having a grand unified theory (GUT) broken to the standard model gauge groups in a scale M_X in fact different from $M_{\mathrm{SU}}^{\mathrm{thr}}$ with additional evolution between these two scales.

In this respect, all possible sources capable of influencing the scale M_X should be examined. Some time ago [17], there has been considered the modification of the unification scale M_X due to the presence in the full lagrangian of non-renormalizable dimension-five terms [#1] of the form

$$L_{\mathrm{dim.-5}} = \frac{c\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}H)}{M_{\mathrm{nr}}}\,, \qquad (7)$$

where $G_{\mu\nu}$ is the field strength of a unified gauge group G and H is an appropriate Higgs field, which breaks G to the standard model gauge groups. These terms present for energies $M_X < \mu < M_{\mathrm{Pl}}(M_{\mathrm{SU}}^{\mathrm{thr}})$ could naturally result from quantum gravitational (string) effects. The impact of these terms for arbitrary c on the unification scale M_X has been found in ref. [17] to be non-negligible. Now, however, that we have more accurate data it is interesting to reconsider this problem within the present-day context of the unification idea.

Let us first remind the reader about the effect such terms have on the gauge couplings. Consider the breaking of the theory at a scale M_X including these terms. Then, for energies $\mu \ll M_X$ we will have the following modified gauge kinetic terms for the SU(3), SU(2) and U(1) gauge fields

$$-\tfrac{1}{4}(1+\varepsilon_3)\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})_{\mathrm{SU(3)}}$$
$$-\tfrac{1}{4}(1+\varepsilon_2)\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})_{\mathrm{SU(2)}}$$
$$-\tfrac{1}{4}(1+\varepsilon_1)\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})_{\mathrm{U(1)}}\,, \qquad (8)$$

where for the case of (7) the ε_i are given by $C_i(cV/M_{\mathrm{nr}})$, the C_i being appropriate Clebsch–Gordan coefficients associated with the breaking of the unified theory by the vacuum expectation value V of H. Thus, the effect of the terms under discussion is to induce effective "dielectric constants", which are slightly different for the three gauge couplings below M_X, but which vanish as a power of energy above M_X. As a result, below M_X there is a redefinition of the physically observed coupling constants relative to the values that actually meet at the unification point. In

[#1] Non-renormalizable dimension-five operators have been first discussed in ref. [18]. They have been considered in relation to neutrino masses in ref. [19] and have been recently reanalysed in ref. [20] for neutrino physics in and in ref. [21] for majoron physics.

fact, by absorbing the ε_i terms in a finite coupling constant and field renormalization, the physical coupling constants and fields are given by

$$g_i = (1+\varepsilon_i)^{-1/2} g_{i0}, \quad A_{\mu i} = (1+\varepsilon_i)^{1/2} A_{\mu i0}, \tag{9}$$

where g_{i0} and $A_{\mu i0}$ are the bare coupling constants and fields of the underlying theory. The g_i are the physical coupling constants we observe experimentally at a low energy scale. They obey the usual RG equations, while the g_{i0} are the bare coupling constants that join to define the unification scale. Thus, the overall effect is to modify the unification boundary conditions at M_X, which instead of (2) are now

$$(1+\varepsilon_1)\alpha_1(M_X) = (1+\varepsilon_2)\alpha_2(M_X) = (1+\varepsilon_3)\alpha_3(M_X)$$
$$= \alpha_U \tag{10}$$

Our task now is to solve the RG equations (3) (or 4)) by numerically integrating them using relations (10) as the unification condition. The principal uncertainty is the unknown value of c. We would like to take an agnostic approach regarding the origin of the ε_i. Nevertheless, to be specific we will take the simple ratio

$$\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = -\tfrac{1}{4} : \tfrac{3}{2} : 1 \tag{11}$$

valid for an SU(5) theory broken to the standard model by an adjoint Higgs field H and parametrize the corrections in terms of $\varepsilon \equiv \varepsilon_2$. Our conclusions do not essentially depend on this specific ratio. We further use as initial conditions at $M_Z = 91.173 \pm 0.020$ GeV the world average values [22]

$$\alpha_{em}(M_Z) = \frac{1}{127.9 \pm 0.02},$$
$$\sin^2\theta_w(M_Z) = 0.2325 \pm 0.0008,$$
$$\alpha_3(M_Z) = 0.113 \pm 0.004, \tag{12}$$

or, equivalently,

$$\alpha_1(M_Z) = 0.01698 \pm 0.00003,$$
$$\alpha_2(M_Z) = 0.0336 \pm 0.0001,$$
$$\alpha_3(M_Z) = 0.113 \pm 0.004. \tag{13}$$

Our unification criterion that all three couplings $\alpha_i(\mu$ meet at a single unification point M_X with unified coupling α_U is the coincidence of α_i at M_X up to three significant figures. That is what we call a unification point specified by M_X and α_U. There is a further constraint. Neglecting non-gauge contributions, the proton lifetime is estimated as

$$\tau_p \approx \frac{1}{\alpha_U^2} \frac{M_X^4}{m_p^5}, \tag{14}$$

where m_p is the proton mass. Such an estimate of the proton lifetime must not contradict the present experimental limit $\tau_p \geqslant 5.5 \times 10^{32}$ yr, a non-trivial constraint.

(1) *Standard model* (*SM*). Consider first the case $\varepsilon = 0$, i.e. without corrections. We verify the known situation that a single unification point cannot be obtained within the present errors and this result cannot be modified by threshold effects near the unification scale or by higher order corrections. To test the sensitivity to the number of Higgses, we have checked that unification is possible with $N_H = 6$ or 7, but no unification is possible for less than six or more than seven Higgs doublets. However, even in these cases the unification point is too low $M_X \approx (3\text{–}4) \times 10^{13}$ GeV, giving an unacceptable proton lifetime $\tau_p \approx (0.4-0.7) \times 10^{24}$ yr.

Let us now examine the effect of non-zero corrections $\varepsilon \neq 0$, which is our subject. Consider first the case $\varepsilon > 0$. We find that for ε in the range 10^{-3}–10^{-2} it is possible to have unification for all values of Higgs doublets up to six. All these, however, correspond to a low unification point $M_X \approx (3\text{–}5) \times 10^{13}$ GeV leading to an unacceptable proton lifetime $\tau_p \approx (0.2\text{–}3) \times 10^{24}$ yr. No unification is found for more than six Higgs doublets. In the case of negative values of ε, we have unification only for seven or more Higgs doublets, but still at an unacceptable low scale. The conclusion is that, although some improvement is seen as far as the unification possibility is concerned, nevertheless this happens at a too low scale giving unacceptably low proton lifetimes.

(2) *Minimal supersymmetric standard model* (*MSSM*). Let us now come to the more interesting case of the MSSM, where we already know that unification really takes place, and see then possible modifications arise from the corrections under discussion. In fact, assuming that either both Higgs doublets are effective already from M_Z or one doublet is effective from M_Z and the other one from M_{SUSY} (both cases give practically the same results), we verify [4,5] that unification is indeed possible for M_{SUSY}

Table 1
Values of the correction parameter ε in the MSSM with only one Higgs doublet effective between M_Z and M_{SUSY}, for which unification is possible with acceptable proton decay rates.

ε ($\times 10^{-2}$)	M_{SUSY} ($\times 10^3$ GeV)	M_X ($\xi 10^{16}$ GeV)	α_U	τ_p ($\times 10^{33}$ yr)
1.1	3.8–5.1	0.77–0.78	0.0379–0.0382	0.57–0.59
0.9	1.7–6.7	0.77–0.89	0.0377–0.0390	0.55–0.97
0.7	0.80–7.8	0.77–1.0	0.0376–0.0398	0.55–1.5
0.5	0.54–8.5	0.77–1.1	0.0376–0.0402	0.55–2.1
0.3	0.29–7.9	0.77–1.3	0.0376–0.0409	0.55–3.8
0.1	0.22–6.0	0.77–1.4	0.0378–0.0413	0.55–4.9
0	0.20–5.7	0.77–1.4	0.0378–0.0414	0.55–6.3
−0.1	0.17–5.0	0.77–1.5	0.0379–0.0416	0.55–7.8
−0.3	0.10–4.1	0.77–1.7	0.0380–0.0422	0.55–12
−0.5	0.10–3.8	0.77–1.9	0.0380–0.0423	0.55–17
−0.7	0.10–3.2	0.77–1.9	0.0381–0.0423	0.55–18
−0.9	0.10–2.7	0.77–2.1	0.0383–0.0424	0.55–25
−1.1	0.10–1.6	0.82–2.1	0.0388–0.0424	0.70–25
−1.3	0.10–0.98	0.90–2.1	0.0393–0.0423	1.0–25
−1.5	0.10–0.66	1.0–2.0	0.0397–0.0422	1.5–21
−1.7	0.10–0.44	1.2–1.9	0.0401–0.0421	2.8–18
−1.9	0.10–0.26	1.3–1.8	0.0408–0.0421	3.7–15
−2.1	0.10–0.16	1.4–1.6	0.0412–0.0419	6.2–9.9

Table 2.
The same as in table 1, but with both Higgs doublets effective from M_Z.

ε ($\times 10^{-2}$)	M_{SUSY} ($\times 10^3$ GeV)	M_X ($\times 10^{16}$ GeV)	α_U	τ_p ($\times 10^{33}$ yr)
0.7	1.5–5.6	0.77–0.87	0.0379–0.0392	0.55–0.89
0.5	0.77–7.0	0.77–1.0	0.0378–0.0399	0.55–1.5
0.3	0.39–8.2	0.77–1.2	0.0377–0.0406	0.55–2.8
0.1	0.29–7.1	0.77–1.4	0.0378–0.0410	0.55–4.7
0	0.20–6.9	0.77–1.4	0.0378–0.0414	0.55–6.2
−0.1	0.17–6.5	0.77–1.5	0.0378–0.0416	0.55–7.8
−0.3	0.10–5.4	0.77–1.7	0.0379–0.0422	0.55–12
−0.5	0.10–4.1	0.77–1.9	0.0381–0.0423	0.55–17
−0.7	0.10–3.8	0.77–1.9	0.0382–0.0423	0.55–17
−0.9	0.10–3.2	0.77–2.1	0.0382–0.0424	0.55–25
−1.1	0.10–2.6	0.77–2.1	0.0384–0.0424	0.55–25
−1.3	0.10–1.7	0.83–2.1	0.0388–0.0423	0.73–25
−1.5	0.10–1.1	0.94–2.0	0.0392–0.0422	1.2–21
−1.7	0.10–0.61	1.1–1.9	0.0398–0.0421	2.0–18
−1.9	0.10–0.31	1.3–1.8	0.0405–0.0421	3.7–15
−2.1	0.10–0.20	1.4–1.6	0.0411–0.0419	4.8–9.9

ranging from 200 GeV up to 7×10^3 GeV with unification point at $M_X \approx (0.8-1.4)\times10^{16}$ GeV. with $\alpha_U \approx 0.0378-0.0414$ and acceptable proton lifetime $\tau_p \approx (0.55\text{–}6.3)\times10^{33}$ yr.

Let us next include non-zero corrections $\varepsilon \neq 0$. We find that unification persists with an acceptable proton decay rate if the correction parameter ε is in the range $|\varepsilon| \leqslant 10^{-2}$. Table 1 summarizes relevant numerical results for the case where only one Higgs doublet is effective between M_Z and M_{SUSY}, whereas table 2 for the case where both Higgs doublets are already effective from M_Z. As we see, there are rather small non-significant changes when non-renormalizable term corrections are present. Only for negative values of ε we have a slight increase in the unification scale M_X. More significant in that case is the tendency to have a lower supersymmetry scale M_{SUSY}. For positive values of ε we have somehow a reverse situation with a slight decrease in the unification scale M_X and a larger supersymmetry scale M_{SUSY}. There is not much difference between the two cases, namely when only one or both Higgs doublets are effective from M_Z. We conclude that Planck-scale effects through non-renormalizable terms are essentially incapable of changing the unification picture quantities of the minimal supersymmetric standard model.

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