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Unified models at intermediate energy scales and Kaluza–Klein excitations

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Abstract

We discuss the possibility of intermediate gauge coupling unification in unified models of string origin. Useful relations of the β-function coefficients are derived, which ensure unification of couplings when Kaluza–Klein excitations are included above the compactification scale. We apply this procedure to two models with $SU(3)\times SU(3)_L \times SU(3)_R$ and $SU(4)\times O(4)$ gauge symmetries. © 1999 Published by Elsevier Science B.V. All rights reserved.

Recently, the possibility that the string and the compactification scale are around the energy determined by the geometric mean of the Planck mass and the electroweak scale, has appeared as a viable possibility in Type II string theories [1] with large extra dimensions [2]. On the other hand, as is well known, the minimal supersymmetric standard model (MSSM) spectrum leads to gauge coupling unification at a scale of $M_U \sim 10^{16}$ GeV. To lower down this scale, usually power-law running of the gauge couplings is assumed, due to the appearance of the Kaluza–Klein (KK) tower of states above the compactification scale [3–7].

In a previous paper [8], we studied the possibility of intermediate energy unification of the gauge couplings due solely to the presence of extra matter and Higgs fields under the standard model (SM) group. We have found that unification may happen at the range $\sim 10^{11}$ GeV without the use of power-law running from KK-excitations. In this note we extend our analysis on this issue by considering unified models of string origin which break down to the SM group at some intermediate energy. We further assume the existence of a compactification scale M_C (smaller than the would be unification scale if M_C had not existed) above which KK-excitations are considered. In this context, we find that unification can always be ensured whenever certain conditions of the β-function differences are met.

We apply our results to models with intermediate gauge symmetries which involve no coloured gauge fields and can in principle be safe from proton decay operators. In particular, we study models based on the $SU(3)^3$ and $SU(4)\times O(4)$ gauge symmetries. Such models can be derived from strings and possess various novel properties. Among them, they possess particles with fractional charges while they use small Higgs representations to break the gauge symmetry.

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The superpotential possesses various discrete and other symmetries that may prevent undesired Yukawa couplings, while many unwanted particles are projected out. The original large gauge symmetry breaks down to the intermediate gauge group of the type discussed above owing to the existence of stringy type mechanisms. In the present analysis we assume the existence of the representations that may be obtained in these models, and the corresponding KK-excitations. In our applications, below the intermediate breaking scale, we assume the MSSM particle content, although our analysis can apply to any content respecting the general properties that we will derive in what follows.

We start with the hierarchy of scales as they appear in our present work and which are the following: at the electroweak scale M_W, we use the initial values for the gauge couplings, as they are measured by the experiment. Next we consider $M_S = 1$ TeV, above which the MSSM β functions are operative; M_G, is the scale above which new physics appears and the β functions of the specific grand unified model (GUT) are effective; M_C, is the scale where compactification appears and the KK-states start contributing to the β functions, and M_U denotes the scale where the gauge couplings would unify if there were no compactification scale; M_C is taken to be smaller than M_U. Finally, M_{CU} is the scale where the gauge couplings unify when we include the KK-excitations. We present them in Fig. 1.

We begin our investigation along the lines discussed above, with the presentation of a general property of the β-function coefficients. Let $\beta_{ij} = \beta_i - \beta_j$ denote the β-function differences. We make the following two assumptions:

- There exists an energy scale M_U where the coupling constants α_i's unify, i.e. $\alpha_i(M_U) = \alpha_U$ for all i, assuming conventional logarithmic running (no-compactification scenario). Quantitatively, this is expressed as

$$\frac{\alpha_{ij}^{-1}(M)}{\beta_{ij}} \equiv \frac{\alpha_j^{-1}(M) - \alpha_j^{-1}(M)}{\beta_i - \beta_j} = \frac{\alpha_{ik}^{-1}(M)}{\beta_{ik}} = \frac{1}{2\pi}\log\frac{M_U}{M} > 0, \qquad (1)$$

where M is some initial scale. The positiveness of the ratio ensures the ''convergence'' (and not ''divergence'') of the couplings above M. This point becomes essential when we discuss the cases of GUTs.

- The ratios of the differences of the β-functions β_{ij}^{KK} (above the compactification scale M_C) to the corresponding difference β^{ij} (below the compactification scale M_C) have the property:

$$\frac{\beta_{ij}^{\mathrm{KK}}}{\beta_{ij}} = \frac{\beta_{ik}^{\mathrm{KK}}}{\beta_{ik}} > 0. \qquad (2)$$

Again positiveness ensures ''convergence'' of the couplings above M_C.

Then, it can be shown that the gauge couplings do unify, whatever energy scale we choose as a compactification scale M_C, above which the massive KK-states contribute to the running.

Let us sketch the proof of the above statements [9]. Since all couplings unify at M_U we have

$$\alpha_U^{-1} = \alpha_i^{-1}(M) - \frac{\beta_i}{2\pi}\log\frac{M_U}{M}. \qquad (3)$$

Assuming now that there exists a compactification scale $M_C < M_U$, the running of the couplings, for $M' > M_C$, is given by [1]

$$\alpha_i^{-1}(M') = \alpha_i^{-1}(M_C) - \frac{\beta_i^{\mathrm{KK}}}{2\pi}\left(2N\log\frac{M'}{M_C} - 2\log(N!)\right). \qquad (4)$$

where N is an integer such that $(N+1)M_C > M' > NM_C$, which counts the massive KK-states that have

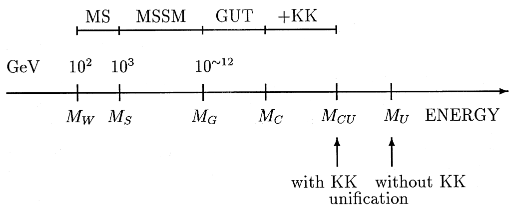

Fig. 1. The energy scales appearing in the paper

[1] We ignore the contribution of the MSSM massless states above M_C since it is negligible compared to that of the KK-excitations. We use the successful approximation of incorporating the massive KK-states with masses less than the running scale [3].

masses below the running scale (we have assumed only one extra dimension and in that case the multiplicity of the states at each mass level is 2). From the running below M_C, we can express $\alpha^{-1}(M_C)$ in the form

$$\begin{aligned}\alpha_i^{-1}(M_C) &= \alpha_i^{-1}(M) - \frac{\beta_i}{2\pi}\log\frac{M_C}{M}\\ &= \alpha_U^{-1} - \frac{\beta_i}{2\pi}\log\frac{M_C}{M_U}\end{aligned}$$

and (4) is written as

$$\begin{aligned}\alpha_i^{-1}(M') = {}& \alpha_U^{-1} - \frac{\beta_i}{2\pi}\log\frac{M_C}{M_U}\\ & - \frac{\beta_i^{\mathrm{KK}}}{2\pi}\left(2N\log\frac{M'}{M_C} - 2\log(N!)\right).\end{aligned} \tag{5}$$

Suppose now that the two couplings α_i and α_j meet at the energy scale M_{CU}. It is easy to check that the following relations hold:

$$2N\log\frac{M_{CU}}{M_C} - 2\log(N!) = -\frac{\beta_{ij}}{\beta_{ij}^{\mathrm{KK}}}\log\frac{M_C}{M_U}$$

$$\begin{aligned}&\alpha_i^{-1}(M_{CU})\\ &\quad = \alpha_j^{-1}(M_{CU})\\ &\quad = \alpha_U^{-1} - \frac{\beta_i}{2\pi}\log\frac{M_C}{M_U} + \frac{\beta_i^{\mathrm{KK}}}{2\pi}\frac{\beta_{ij}}{\beta_{ij}^{\mathrm{KK}}}\log\frac{M_C}{M_U}\\ &\quad = \alpha_U^{-1} - \frac{\beta_j}{2\pi}\log\frac{M_C}{M_U} + \frac{\beta_j^{\mathrm{KK}}}{2\pi}\frac{\beta_{ij}}{\beta_{ij}^{\mathrm{KK}}}\log\frac{M_C}{M_U}.\end{aligned} \tag{6}$$

The value of the third coupling $\alpha_k^{-1}(M_{CU})$ at the scale M_{CU} is given by

$$\begin{aligned}&\alpha_k^{-1}(M_{CU})\\ &\quad = \alpha_U^{-1} - \frac{\beta_k}{2\pi}\log\frac{M_C}{M_U}\\ &\qquad - \frac{\beta_k^{\mathrm{KK}}}{2\pi}\left(2N\log\frac{M_C}{M'} - 2\log(N!)\right)\\ &\quad = \alpha_U^{-1} - \frac{\beta_k}{2\pi}\log\frac{M_C}{M_U} + \frac{\beta_k^{\mathrm{KK}}}{2\pi}\frac{\beta_{ij}}{\beta_{ij}^{\mathrm{KK}}}\log\frac{M_C}{M_U}.\end{aligned} \tag{7}$$

It is now straightforward to check, using the second condition (2), that $\alpha_k^{-1}(M_{CU})$ equals the values of α_i^{-1} and α_j^{-1} at the same scale. Therefore, the couplings unify, no matter what compactification scale M_C we choose. The positiveness condition of (2) comes from the ''convergence'' requirements of the couplings above M_C. From (4) we get

$$\frac{\alpha_{ij}^{-1}(M_C)}{\beta_{ij}^{\mathrm{KK}}} = \frac{1}{2\pi}\left(2N\log\frac{M_{CU}}{M_C} - 2\log(N!)\right),$$

which should be positive, since the unification scale $M_{CU} > NM_C$. But from the running below M_C we get

$$\alpha_{ij}^{-1}(M_C) = \frac{\beta_{ij}}{2\pi}\log\frac{M_C}{M}$$

and the positivity condition can be put in the form

$$\frac{\beta_{ij}}{\beta_{ij}^{\mathrm{KK}}} > 0.$$

Let us note also that the initial scale M in (1) could be either an intermediate one where a group larger than the SM one appears, or could be just M_W if no GUT is assumed.

We now come to the β-function, both below and above M_C. Below the compactification scale, the (one-loop) β-function is given by

$$\frac{1}{16\pi^2}\left(-3C_2(G) + \sum_i T(R_i)\right), \tag{8}$$

where the first term corresponds to the vector supermultiplet (gauge bosons and gauginos) contribution while, the second corresponds to the chiral (quarks, leptons, higgs and superpartners) supermultiplets. $C_2(G)$ is the quadratic Casimir operator for the adjoint representation, R_i is the representations of the matter multiplets and $T(R)$ is defined by the relation $\mathrm{Tr}[R^aR^b] = T(R)\delta^{ab}$. Above M_C, the massive KK-states give the following β-function

$$\frac{1}{16\pi^2}\left(-2C_2(G) + \sum_i T(R_i)\right). \tag{9}$$

The difference from (8) comes from the fact that the massive vector supermultiplet is actually a $N=2$ hypermultiplet with a vector plus a chiral supermultiplet.

As a first example we discuss the MSSM where we know that the three couplings α_1, α_2 and α_3 unify at the scale $\sim 10^{16}$ GeV. Now assuming that only the gauge bosons and the higgs acquire KK-states (the matter fields are placed on the fixed points of the heterotic string and therefore no KK-states appear for them), the above formulae give

$$16\pi^2\beta_{31} = -9.6, \quad 16\pi^2\beta_{32} = -4,$$
$$16\pi^2\beta_{21} = -6.4,$$
$$16\pi^2\beta_{31}^{\mathrm{KK}} = -6.6, \quad 16\pi^2\beta_{32}^{\mathrm{KK}} = -3,$$
$$16\pi^2\beta_{21}^{\mathrm{KK}} = -4.4. \tag{10}$$

Therefore, with an error of less than 10%, the ratio β_{ij}/β_{ik} is the same below and above M_C. Note here that, since the matter multiplets are complete $SU(5)$ ones (the equal contribution of matter in the three β-functions is due to that), even in the case where they had KK-excitations, the relations between the β-function ratio would still hold. Therefore, whatever energy scale we choose as our compactification scale, the three couplings will unify. We now apply this idea to the two models mentioned above. Some details on the β-functions and the string spectra of the models may be found in [10].

The SU(4) × O(4) case. We first take as an example the $SU(4)\times SU(2)_L\times SU(2)_R$ model, which is assumed to break to the SM-symmetry at some scale M_G. Above M_G, apart from the MSSM matter content, we have the following extra states

$$n_6=(6,1,1), \quad n_4=(4,1,1), \quad n_L=(1,2,1), \quad n_R=(1,1,2),$$
$$n_{22}=(1,2,2), \quad n_H=(4,1,2)/(\bar{4},1,2).$$

where we show the quantum numbers under the GUT group. The subscript H refers to the Higgs fields that break the $SU(4)$ and the $SU(2)_R$ groups, while the 22 gives the Standard Model Higgs. The one loop β-functions are

$$\beta_R = -6+2n_G+2n_H+2n_{22}+n_R/2,$$
$$\beta_L = -6+2n_G+2n_{22}+n_L/2,$$
$$\beta_4 = -12+2n_G+n_H+n_6+n_4/2. \tag{11}$$

where n_G is the number of generations. The relations between the MSSM and the GUT model couplings, at M_G, are

$$\alpha_4=\alpha_3, \quad \alpha_L=\alpha_2, \quad \alpha_R^{-1}=\tfrac{5}{3}\alpha_1^{-1}-\tfrac{2}{3}\alpha_4^{-1}.$$

Assuming now that the ''turning'' point from MSSM to the GUT content is 10^{11-14} GeV, the ratios of the coupling constant differences are in the ranges

$$\frac{\alpha_{4R}^{-1}}{\alpha_{4L}^{-1}} = 3.54-3.79,$$
$$\frac{\alpha_{L4}^{-1}}{\alpha_{LR}^{-1}} = (-0.39)-(-0.36).$$

Above the compactification scale we assume that all extra (beyond that of the MSSM) matter could have KK-states. Allowing a difference at most 3% between the ratio of the coupling constants and the ratio of the β-functions, and for $M_G=10^{12}$ GeV and $M_G=10^{13}$ GeV, the only values that the β-function can give (all $n's$ take even integer values) are

$$\frac{\beta_{4R}}{\beta_{4L}} = \frac{\beta_{4R}^{\mathrm{KK}}}{\beta_{4L}^{\mathrm{KK}}} = \tfrac{33}{9}, \qquad \frac{\beta_{L4}}{\beta_{LR}} = \frac{\beta_{L4}^{\mathrm{KK}}}{\beta_{LR}^{\mathrm{KK}}} = -\tfrac{3}{8}.$$

If we require M_G to be either 10^{11} GeV or 10^{14} GeV, then we should raise the acceptable error between the ratios to 5% and the only values that the ratios, below M_C, can have are

$$\frac{\beta_{4R}}{\beta_{4L}} = \tfrac{33}{9} \text{ or } \tfrac{7}{2}, \qquad \frac{\beta_{L4}}{\beta_{LR}} = -\tfrac{3}{8} \text{ or } -\tfrac{2}{5},$$

while the ratios above M_C remain the same.

Of course, several particle contents below and above the compactification scale, render the above values for the ratios. In the following table we give one example, where the content below M_C can, in

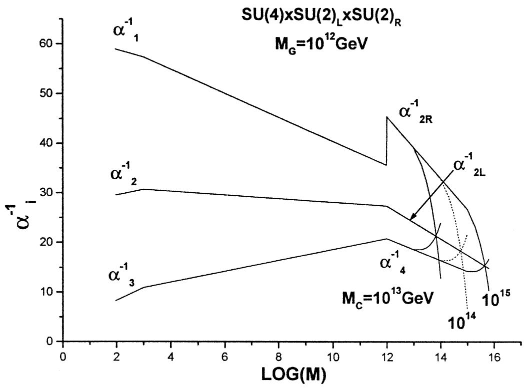

Fig. 2. The inverse of the three gauge couplings as a function of energy, for the $SU(4)\times SU(2)_L\times SU(2)_R$ GUT with the specific content appearing in (12). We have chosen $M_G=10^{12}$ GeV and three values of the compactification scale $M_C=10^{13}$,10^{14}, 10^{15} GeV.

principle, be reproduced by the string $SU(4)\times SU(2)_L\times SU(2)_R$ model, while we have chosen $M_G = 10^{12}$ GeV

$$\begin{array}{lcccccc} & n_6 & n_4 & n_L & n_R & n_H & n_{22} \\ \text{below } M_C & 4 & 8 & 10 & 10 & 4 & 4 \\ \text{above } M_C & 0 & 2 & 0 & 0 & 4 & 4. \end{array} \qquad (12)$$

In Fig. 2 we show the running of the coupling constants for the above content and for several values of M_C. In Fig. 3 a scatter plot is presented showing the (inverse) of the unified coupling for several contents of the model.

The $SU(3)_C\times U(3)_L\times U(3)_R$ model. Another interesting string derived model, which admits a low (intermediate) unification scale (no dangerous dimension-six operators), is based on the $SU(3)\times SU(3)_L\times SU(3)_R$ symmetry. The MSSM content is found in the 27 representation of the E_6 group

$$27\to(3,\bar{3},1)+(\bar{3},1,3)+(1,3,\bar{3}). \qquad (13)$$

where

$$(3,\bar{3},1)=\begin{pmatrix} u\\ d\\ D\end{pmatrix},\quad (\bar{3},1,3)=\begin{pmatrix} u^c\\ d^c\\ D^c\end{pmatrix},$$
$$(1,3,\bar{3})=\begin{pmatrix} h^0 & h^+ & e^c\\ h^- & \bar{h}^0 & \nu^c\\ e & \nu & N\end{pmatrix}. \qquad (14)$$

The breaking chain we adopt here is the following: the first group is the colour $SU(3)$. The second

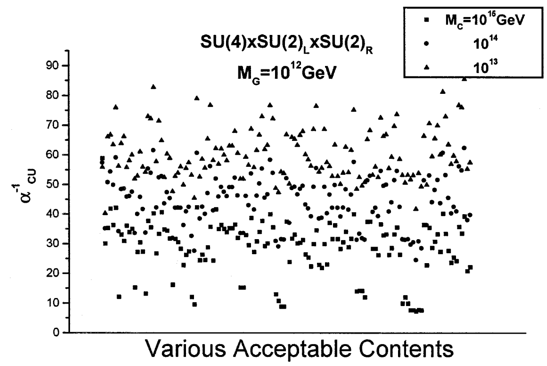

Fig. 3. Scatter plot of the inverse of the unified gauge coupling, for the $SU(4)\times SU(2)_L\times SU(2)_R$ GUT, choosing $M_G=10^{12}$ GeV for three values of the compactification scale $M_C = 10^{13}, 10^{14}, 10^{15}$ GeV. The horizontal axes enumerates the various acceptable contents of the model (the order of appearance along the x-axis is irrelevant). The highest the compactification scale the greater the value of the α_{CU}.

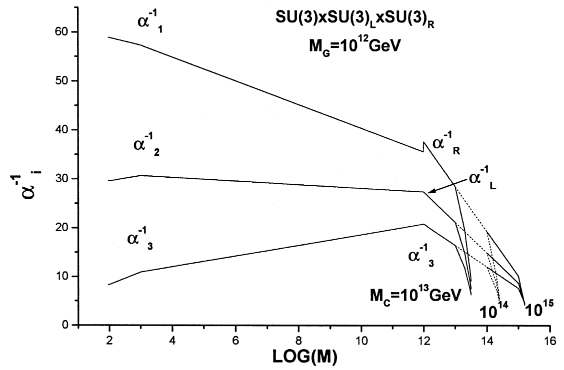

Fig. 4. Same as in Fig. 2 for the $SU(3)\times SU(3)_L\times SU(3)_R$ model and the specific content of (19).

breaks to $SU(2)_L\times U(1)_L$, while the third breaks to a $U(1)_R$. The SM $U(1)_Y$ emerges as a linear combination of the two $U(1)_{L,R}$. The conventional hypercharge Y is related to the X and Z charges of $U(1)_L$ and $U(1)_R$ correspondingly, by the relation

$$Y=\frac{1}{\sqrt{5}}X+\frac{2}{\sqrt{5}}Z,$$

while the corresponding relations of the couplings at the breaking scale is

$$\alpha_L=\alpha_2,\quad \alpha_R^{-1}=\tfrac{5}{4}\alpha_Y^{-1}-\tfrac{1}{4}\alpha_L^{-1}.$$

Apart from the above states, in the string model, fractionally charged and other exotic states usually appear, belonging to the representations

$$\begin{array}{ccc} \left(\overset{(-)}{3},1,1\right) & \left(1,\overset{(-)}{3},1\right) & \left(1,1,\overset{(-)}{3}\right) \\ 0 & \pm 1/3 \text{ and} \pm 2/3 & \pm 1/3 \text{ and} \pm 2/3 \end{array} \qquad (15)$$

where the second line shows the corresponding (electric) charges. One should not be misled by the values of these charges: the neutral states are coloured, while the others are singlet under the colour group. Therefore, after the symmetry breaking, these states will result in exotic lepton doublets and singlets carrying charges like those of the down and up quarks. Note that such states are not common in GUTs, however, they are generic in string models.

The one-loop β-functions are given by

$$\beta_3=-9+\tfrac{1}{2}\left(3n_Q+3n_{Q^c}+n_C\right), \qquad (16)$$
$$\beta_L=-9+\tfrac{1}{2}\left(3n_Q+3n_L+n_{L'}\right), \qquad (17)$$
$$\beta_R=-9+\tfrac{1}{2}\left(3n_{Q^c}+3n_L+n_{L''}\right), \qquad (18)$$

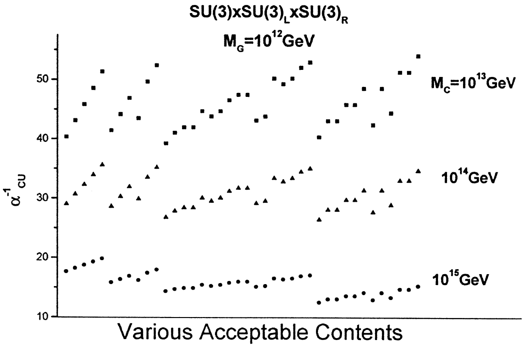

Fig. 5. Same as in Fig. 3 for the $SU(3)\times SU(3)_L \times SU(3)_R$ model.

where n_Q, n_{Q^c} and n_L are the number of the representations appearing in the complete 27, Eq. (13), while n_C, $n_{L'}$ and $n_{L''}$ are the number of the exotic representations of (15).

As in the case of the previous model, several massless spectra pass the two conditions and provide unification of the three couplings. Although it seems that the $SU(3)^3$ is probably less constrained (giving a lot of possible contents, presumably because of the symmetric form of the β-functions), one should be careful, since the unification coupling could be high enough in some cases and get out of the perturbative region. This of course happens for high matter content, when the β-functions become large and positive. We should note at this point (and it is a general remark not applicable only to the specific GUT) that the value of M_C starts playing a significant role in the case where the unification coupling constant is getting large: if the β-functions between M_G and M_C are already large, M_C cannot be much larger than M_G if we want to avoid a non-perturbative value of the unification coupling.

In the following table, we give, as an example, the content below and above M_C, for the $SU(3)^3$ model, where we have chosen $M_G = 10^{12}$ GeV and a 3% error in the equality of the ratios

	n_Q	n_L	n_{Q^c}	n_C	$n_{L'}$	$n_{L''}$	
below M_C	10	4	6	10	0	2	(19)
above M_C	6	0	2	10	0	2.	

In Fig. 4 we show the running of the couplings for the above content and for several values of M_C while Fig. 5 is a scatter plot of the (inverse of the) inified coupling foe several contents of the model.

We conclude with a few remarks: the possibility of lowering the unification scale is a fascinating one, both from the theoretical and from the experimental point of view. Experimentally, it would be exciting to have a low enough unification scale for the possibility of testing its implications in the near-future machines. Theoretically, it would give a solution to the desert-puzzle invoked in previous Planck-mass unification scenarios. However, when lowering the unification scale in most of the GUTs, one faces the notorious problem of proton decay. A possible solution, which combines the idea of a relatively low unification and a reasonable solution to the proton decay problem, is the one presented in this note. We have considered GUTs that do not lead to proton decay via dimension-six operators and implemented the idea that the unification occurs at an intermediate scale so that, for appropriate Yukawa couplings, other dangerous operators may be sufficiently suppressed. We have shown that there exist numerous cases of massless spectra (which can be derived from the superstring), implying naturally intermediate scale unification.

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