

Predictive ansatz for fermion masses in SUSY GUTs

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We reexamine a successful fermion mass ansatz proposed by Giudice for a wide range of the ratio $\tan \beta = \langle \bar{h} \rangle / \langle h \rangle$ (where \bar{h}, h are the two standard Higgs fields), in the context of supersymmetric grand unified theories. We find that the seven predictions of the ansatz, $V_{us}, V_{cb}, V_{ub}, m_u, m_d, m_s$ and m_b are in good agreement with the experiment when either (i) $\tan \beta \approx O(1)$ or (ii) $\tan \beta \geq 30$. A correct prediction for the bottom mass gives a lower limit on $m_t \geq 125$ for case (i), in agreement with the previous analysis, while in case (ii) $m_t \geq 148$.

There is a firm belief among the high energy physicists that in the ultimate theory, all the arbitrary parameters of the standard model will be determined only from a small number of inputs at some unification scale.

Recently, several attempts have been made [1–4] to determine the possible structures of the fermion mass matrices at the grand unification (GUT) scale, which lead to the correct low energy mass spectrum and to the maximal number of predictions. In ref. [2], a simple ansatz for the fermion mass matrices, at the GUT scale, was proposed. The 13 arbitrary parameters of the low energy were determined by six inputs, hence leading to seven predictions.

In a recent paper [5], the original ansatz of ref. [2] for the fermion mass matrices was expanded in order to incorporate non-zero neutrino masses, as these are naturally predicted in most of the GUT models. With only two additional inputs at the GUT scale, it was found that all the previous successful predictions are retained, while one gets seven new low energy predictions: the masses of the three light neutrinos, the leptonic mixing angles and the corresponding CP phase. It was also proposed that the m_{ν_p} mass can give the correct contribution to the hot dark matter, in agree-

ment with the interpretation of the COBE data [6], while the solar neutrino problem is solved via the MSW mechanism [7] and $\nu_\mu \rightarrow \nu_e$ oscillations [8].

All the previous calculations however, have been done for the case of small bottom Yukawa coupling, compared to that of the top quark. This corresponds to a relatively small ratio $\tan \beta$ of the two Higgs VEVs $\langle \bar{h} \rangle$ and $\langle h \rangle$ which give masses to the up and down quarks respectively. In this particular case one can ignore all but the top Yukawa coupling corrections in the renormalization group equations (RGEs) of the Yukawa couplings, and calculate to a good approximation all the low energy parameters from the inputs at the GUT scale.

In many unified models however – and in particular in string derived GUTs – it is quite possible for the top and bottom quark Yukawa couplings to be comparable at the GUT scale. In that case the bottom coupling corrections play also an important role and should not be ignored. Such a case would obviously correspond to a pretty large value of $\tan \beta$, in contrast to the previous case.

In the present letter we are going to explore this latter case. We start with an overview of the basic features of the proposed framework [2]. It is assumed

that there exists some grand unified supersymmetric model (i.e. SO(10), SU(5), SU(4), etc.) with the following form of the mass matrices at the GUT scale:

$$M_u = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \quad M_{\nu\nu c} = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \quad (1)$$

$$M_d = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & d & 2d \\ 0 & 2d & c \end{pmatrix},$$

$$M_e = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & -3d & 2d \\ 0 & 2d & c \end{pmatrix}, \quad (2)$$

$$M_{\nu c\nu c} = M \text{diag}(k^{-2}, k^{-1}, 1). \quad (3)$$

There is a factor of -3 difference in the $\{22\}$ entry of the matrix M_e compared to that of M_d , this arises naturally whenever these entries are coupled to Higgs doublets belonging to specific representations of the GUT group (**45** of SU(5) or **126** of SO(10)). The relation of the $\{22\}$ and $\{23\}$ entries in M_d, M_e matrices is just a phenomenological assumption [2]. The original ansatz for the M_u, M_d and M_e matrices was augmented by a simple assumption for the Dirac, $M_{\nu\nu c}$, and the heavy Majorana, $M_{\nu c\nu c}$, neutrino mass matrices. $M_{\nu\nu c}$ is simply taken to be identical to M_u due to the GUT relations, while $M_{\nu c\nu c}$ is taken for simplicity diagonal, with elements differing by a hierarchy factor $k \approx 10$. The predictions in the neutrino sector have been discussed elsewhere [5], thus we are not going to elaborate them here.

The RGEs for the Yukawa couplings at the one loop level are

$$16\pi^2 \frac{d}{dt} \lambda_U = (I \text{Tr}[3\lambda_U \lambda_U^\dagger] + 3\lambda_U \lambda_U^\dagger + \lambda_D \lambda_D^\dagger - IG_U) \lambda_U, \quad (4)$$

$$16\pi^2 \frac{d}{dt} \lambda_N = (I \text{Tr}[\lambda_U \lambda_U^\dagger] + \lambda_E \lambda_E^\dagger - IG_N) \lambda_N, \quad (5)$$

$$16\pi^2 \frac{d}{dt} \lambda_D = (I \text{Tr}[3\lambda_D \lambda_D^\dagger + \lambda_E \lambda_E^\dagger] + 3\lambda_D \lambda_D^\dagger + \lambda_U \lambda_U^\dagger - IG_D) \lambda_D, \quad (6)$$

$$16\pi^2 \frac{d}{dt} \lambda_E = (I \text{Tr}[\lambda_E \lambda_E^\dagger + 3\lambda_D \lambda_D^\dagger] + 3\lambda_E \lambda_E^\dagger - IG_E) \lambda_E, \quad (7)$$

where λ_α , $\alpha = U, N, D, E$, represent the 3×3 Yukawa matrices which are defined in terms of the mass matrices given in eqs. (1)–(3), and I is the 3×3 identity matrix and

$$G_\alpha = \sum_{i=1}^3 c_\alpha^i g_i^2(t), \quad (8)$$

$$g_i^2(t) = \frac{g_i^2(t_0)}{1 - (b_i/8\pi^2) g_i^2(t_0)(t - t_0)}, \quad (9)$$

$$\{c_U^i\}_{i=1,2,3} = \left\{ \frac{13}{15}, 3, \frac{16}{3} \right\},$$

$$\{c_D^i\}_{i=1,2,3} = \left\{ \frac{7}{15}, 3, \frac{16}{3} \right\}, \quad (10)$$

$$\{c_E^i\}_{i=1,2,3} = \left\{ \frac{9}{5}, 3, 0 \right\},$$

$$\{c_N^i\}_{i=1,2,3} = \left\{ \frac{3}{5}, 3, 0 \right\}. \quad (11)$$

Following ref. [2] we diagonalize the up quark Yukawa matrix at the GUT scale and redefine properly the lepton and quark fields

$$\lambda_U \rightarrow \tilde{\lambda}_U = K^\dagger \lambda_U K, \quad \lambda_N \rightarrow \tilde{\lambda}_N = K^\dagger \lambda_N K,$$

$$\lambda_D \rightarrow \tilde{\lambda}_D = K^\dagger \lambda_D K, \quad \lambda_E \rightarrow \tilde{\lambda}_E = K^\dagger \lambda_E K, \quad (12)$$

where now $\tilde{\lambda}_U$ is diagonal and K is

$$K = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}, \quad \tan 2\theta = \frac{2b}{a}. \quad (13)$$

Now assuming that the only significant Yukawa terms are $\tilde{\lambda}_{U33}$, $\tilde{\lambda}_{D33}$ and $\tilde{\lambda}_{E33}$, to a good approximation we may drop all other terms in the parentheses of the right-handed side of the RGEs above, and write them formally as follows:

$$\tilde{\lambda}_U(t) = \gamma_U \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \zeta \zeta^3 \end{pmatrix} \zeta^3 \tilde{\lambda}_U(t_0), \quad (14)$$

$$\tilde{\lambda}_D(t) = \gamma_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \zeta \zeta^3 \end{pmatrix} \zeta^3 \zeta' \tilde{\lambda}_D(t_0), \quad (15)$$

$$\tilde{\lambda}_E(t) = \gamma_E \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \zeta^3 \end{pmatrix} \zeta^3 \zeta' \tilde{\lambda}_E(t_0), \quad (16)$$

$$\tilde{\lambda}_N(t) = \gamma_N \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \zeta' \end{pmatrix} \xi \tilde{\lambda}_N(t_0), \quad (17)$$

where

$$\gamma_\alpha(t) = \exp\left(-\frac{1}{16\pi^2} \int G_\alpha(t) dt\right), \quad (18)$$

$$\xi = \exp\left(\frac{1}{16\pi^2} \int_{t_0}^t \tilde{\lambda}_t dt\right), \quad (19)$$

$$\zeta = \exp\left(\frac{1}{16\pi^2} \int_{t_0}^t \tilde{\lambda}_b dt\right), \quad (20)$$

$$\zeta' = \exp\left(\frac{1}{16\pi^2} \int_{t_0}^t \tilde{\lambda}_\tau dt\right), \quad (21)$$

where $\tilde{\lambda}_t$, $\tilde{\lambda}_b$ and $\tilde{\lambda}_\tau$ stand for $\tilde{\lambda}_{U_{33}}$, $\tilde{\lambda}_{D_{33}}$ and $\tilde{\lambda}_{E_{33}}$ respectively. In the above equations $\tilde{\lambda}_\tau$ is also included since it satisfies the same initial condition with $\tilde{\lambda}_b$. Of course the evolution down to M_Z is different and ζ' stays very close to 1 as long as the initial value of $\tilde{\lambda}_\tau$ does not get too large. Note that in the limit where $\tilde{\lambda}_t \gg \tilde{\lambda}_b$, $\tilde{\lambda}_\tau$ we get $\zeta \approx \zeta' \approx 1$ and the above equations reduce to the simple uncoupled form

$$\tilde{\lambda}_\alpha(t) = \xi^k \gamma_\alpha \lambda_\alpha(t_0), \quad (23)$$

where now

$$\xi = \left(1 - \frac{k}{8\pi^2} \tilde{\lambda}_t(t_0) \int_{t_0}^t \gamma_\alpha^2(t) dt\right)^{-1/2k} \quad (24)$$

and we recover the previous results [2,5]. In the general case, however, the differential equations remain coupled and only a numerical solution is possible.

We obtain the following relations among the masses:

$$m_t = \zeta \xi^3 \frac{\eta_u m_c^2}{\eta_c^2 m_u}, \quad (25)$$

and for the down quarks and leptons

$$m_b \approx \frac{\gamma_D}{\gamma_E} \frac{\zeta^3}{\zeta'^3} \xi m_\tau \eta_b, \quad (26)$$

$$m_s \approx \eta_s \frac{\gamma_D}{\gamma_E} \frac{m_\mu}{3} \left(1 - \frac{4}{9} (1 + 3\zeta'^3) \frac{m_\mu}{m_\tau}\right), \quad (27)$$

$$m_d \approx \eta_d \frac{\gamma_D}{\gamma_E} 3m_e \left(1 + \frac{4}{9} (1 + 3\zeta'^3) \frac{m_\mu}{m_\tau}\right), \quad (28)$$

where in the above relations η_α ($\alpha = b, c$) are taking into account the QCD renormalization effects of the corresponding quark masses from the energy scale m_t down to their masses $\eta_\alpha = m_\alpha(m_\alpha)/m_\alpha(m_t)$, while for ($\alpha = u, d, s$) we use $\eta_\alpha = m_\alpha(1 \text{ GeV})/m_\alpha(m_t)$. In what follows, we use the values $\eta_b = 1.4$, $\eta_c = 1.8$ and $\eta_u = \eta_d = \eta_s = 2.0$. The bottom quark mass is taken to lie in the range $m_b(m_b) = 4.25 \pm 0.1 \text{ GeV}$. Thus, from the relation (26), the correct prediction for m_b fixes the combination $\xi(\zeta/\zeta')^3 \approx 0.81 \pm 0.02$ (the rest of the renormalization group parameters involved in (26) vary slowly in terms of the input parameters). In order to express the predictions for the Kobayashi-Maskawa (KM) mixing angles, we use the following parameterization for the KM matrix:

$$V_{\text{KM}} = \begin{pmatrix} c_1 c_3 e^{i\phi} - s_1 s_2 s_3 & s_1 c_3 e^{i\phi} + c_1 s_2 s_3 & -c_2 c_3 \\ -s_1 c_2 & c_1 c_2 & s_2 \\ c_1 s_3 e^{i\phi} + s_1 s_2 c_3 & s_1 s_3 e^{i\phi} - c_1 s_2 c_3 & c_2 c_3 \end{pmatrix}, \quad (29)$$

where $c_1 = \cos \theta_1$, $s_1 = \sin \theta_1$, etc. Then the predictions are

$$V_{us} \approx 3s_1 \approx \sqrt{\frac{m_e}{m_\mu}} \left(1 - \frac{25}{2} \frac{m_e}{m_\mu} + \frac{16}{9} \frac{m_\mu}{m_\tau}\right), \quad (30)$$

$$V_{cb} \approx s_2 \approx \frac{2}{3} (\xi \zeta'^3)^{-1} \frac{m_\mu}{m_\tau} \left(1 - \frac{m_e}{m_\mu} - \frac{1}{9} \frac{m_\mu}{m_\tau}\right), \quad (31)$$

$$v_{ub} \approx s_3 \approx \left(\frac{\xi}{\zeta}\right)^2 \frac{m_c}{\eta_c m_t}. \quad (32)$$

In order to compute the various renormalization group parameters which enter the various relations given above, we solve numerically eqs. (14)–(16) assuming the initial condition at $M_{\text{GUT}} \approx 10^{16} \text{ GeV}$, with $g_{\text{GUT}} \approx 1/25.1$. We are taking supersymmetric beta function coefficients from M_{GUT} down to m_b , while below m_t we run the system with non-supersymmetric ones. We ensure that the gauge couplings lie in the experimentally accepted region at m_W and we compute the quark masses for a wide range of $\tan \beta$, each time using the proper initial values for the couplings $\lambda_{0,t}, \lambda_{0,b}$. Obviously, when $\tan \beta \approx 1$, a small ratio $r = \lambda_{0,b}/\lambda_{0,t}$ is needed, while when $r \approx 1$ then

$\tan \beta \gg 1$. Note however, that $\lambda_{0,b} = \lambda_{0,\tau}$, while $\lambda_{0,\tau}$ and $\tan \beta$ should also be chosen so as to give the correct τ mass. Thus, a consistency check is done for each chosen pair of values $(\lambda_{0,\tau}, \tan \beta)$ separately, where the τ mass is taken to be $m_\tau = 1784.1$ MeV. Our numerical analysis reproduces the previous results when $\tan \beta \leq 5$ and extends the analysis to the case where $\tan \beta \gg 1$.

In order to see clearly the effect of a large $\tan \beta$ (or equivalently a large $\lambda_{0,b}$ coupling), in fig. 1a we plot the bottom mass versus the bottom coupling $\lambda_{0,b}$, at the GUT scale for constant top mass $m_t = 145$ GeV, for three successive approximations: Contour (I) represents the case where $\lambda_{0,b}, \lambda_{0,\tau}$ -corrections are neglected. Contour (II) represents the solution where only the $\lambda_{0,\tau}$ correction is neglected, while case (III) is the contour which corresponds to the complete differential system (14)–(16) where the corrections from all three couplings are taken into account in the running. All curves are almost identical for small λ_b and $\tan \beta < 5$. Curve (I) is no longer valid for $\tan \beta > 5$ whole curve (II) is not a good approximation for $\tan \beta > 10$. Case (III), but in terms of the ratio $r = \lambda_{0,b}/\lambda_{0,\tau}$ is shown in fig. 1b. However, for this particular value of m_b , reasonably large values of $\tan \beta$ (~ 10), are excluded from the bottom mass which is shown as the shaded region of these figures. Acceptable m_b values are possible only for $\tan \beta \geq 40$ (!), but this corresponds to the unlikely case of $r > 1$. Moreover, additional constraints arise from the KM-mixing angles which are given in (29)–(32). In particular, for the $m_t = 145$ GeV case, the element V_{bc} is in its upper

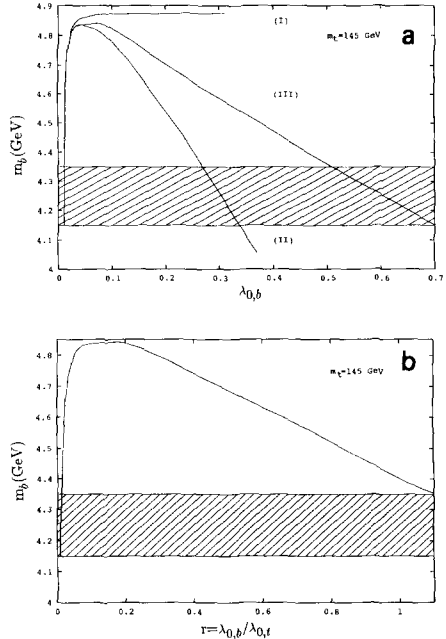


Fig. 1. Bottom mass for $m_t = 145$ GeV, (a) as a function of $\lambda_{0,b}(M_{GUT})$, and (b) as a function of $r = \lambda_{0,b}/\lambda_{0,\tau}$ (see text for details).

experimentally allowed limit ($\approx 5.4 \times 10^{-2}$, only when $r = \lambda_{0,b}/\lambda_{0,\tau} \approx 1.1$, while – unless $r \ll 1$ – all the rest of the region of r is excluded either from m_b or from V_{bc} constraints. In fact, the value $m_t \approx 148$ GeV, is the lower top mass which can be obtained for a large λ_b coupling, while smaller top mass values can be obtained only for a negligible λ_b coupling. In table 1 we collect various results for the experimentally mea-

Table 1

The predictions for the ansatz of ref. [2] for large values of $\lambda_{0,b}$ and $\tan \beta$. The corresponding experimental ranges are $V_{ub} = (4 \pm 3) \times 10^{-3}$, $V_{cb} = (4.3 \pm 1.1) \times 10^{-2}$ [9]. From (27)–(30) we also get $m_d \approx 6.8$ MeV, $m_s \approx 138$ MeV and $V_{ub} \approx 0.218$, in agreement with the experiment. Only the cases denoted with \checkmark are acceptable.

m_t (GeV)	r	$V_{ub} \times 10^3$	$V_{cb} \times 10^2$	m_b (GeV)	$\tan \beta$	
148	~ 0.97	4.90 ± 0.20	5.40	4.35	45	\checkmark
	~ 1.3	5.12 ± 0.20	5.91	4.26	50	
155	~ 0.83	4.30 ± 0.17	5.40	4.31	45	\checkmark
	~ 1.0	4.53 ± 0.18	5.98	4.20	50	
165	~ 0.35	3.58 ± 0.14	5.02	4.34	35	\checkmark
	~ 0.63	3.75 ± 0.15	5.56	4.19	45	
170	~ 0.33	3.25 ± 0.15	4.91	4.32	30	\checkmark
	~ 0.38	4.39 ± 0.17	5.57	4.26	35	

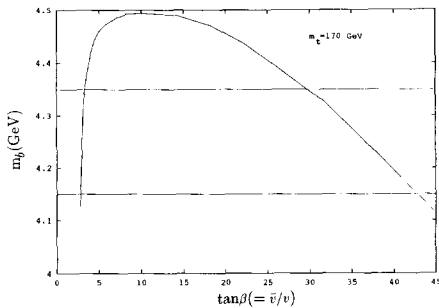


Fig. 2. Bottom mass for $m_t = 170$ GeV as a function of $\tan \beta$. Values of m_b corresponding to the dashed part of the curve are not accepted due to experimental limits on V_{cb} (see table 1).

sured parameters for various values of m_t . For all these cases we find that $m_d \approx 6.8$ MeV and $m_s \approx 138$ MeV.

As m_t gets higher however, smaller $\tan \beta$ values are possible. In fig. 2 we show such a case for $m_t = 170$ GeV where we plot m_b versus $\tan \beta$. A comparison of the two curves in terms of the $\lambda_{0,b}$ values is presented in fig. 3. The shaded area (whose upper bound corresponds to $m_t \approx (170 \pm 3)$ GeV), is prevented due to the bad ratio m_u/m_c . Thus, it is remarkable that this ratio which is derived only in terms of the running m_u, m_c masses determined by well known methods [10], can put an upper bound on the top-quark mass. If, on the other hand, from the electroweak breaking

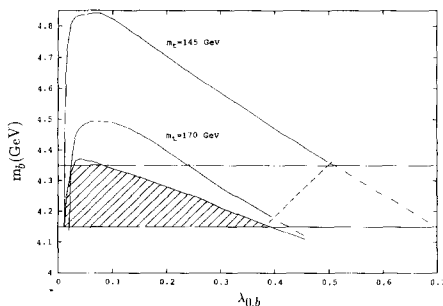


Fig. 3. Bottom mass for $m_t = 145$ and 170 GeV. The shaded area is prevented due to an unacceptable m_u/m_c ratio and the dashed curve due to V_{cb} bounds.

mechanism in supersymmetric models we demand the condition $\tan \beta > 1$, then we obtain a lower bound $m_t \geq 125$ GeV, although, this second bound is referred to a very small region as can be seen from fig. 3.

In conclusion, we have reconsidered a proposed [2] ansatz for the fermion mass matrices at the GUT scale, and studied in detail the effects of a large bottom Yukawa coupling on the various experimentally measured parameters of the low energy theory. We have shown that the renormalization corrections of the λ_b Yukawa coupling have a significant impact whenever $r = \lambda_{0,b}/\lambda_{0,t} \geq 0.1$, while when $r \ll 1$, they can safely be neglected. Furthermore, for $r \geq 1$, m_b and V_{cb} low energy bounds put a lower limit on the top mass $m_t \geq 148$ GeV, while in the case of $r \ll 1$, one gets a less restrictive top mass $m_t \geq 125$ GeV.

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