PROTON DECAY CATALYZED BY SUPERCONDUCTING COSMIC STRINGS

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Topological defects which form in a grand unification phase transition can catalyze proton decay. For monopoles the catalysis cross section is amplified to a value many orders of magnitude larger than the geometrical cross section. For non-superconducting cosmic strings there is no amplification of the cross section since there are no long range forces. Here we establish that – despite the presence of long range electromagnetic fields – the cross section for uncharged superconducting cosmic strings is the geometrical cross section. Hence, in contrast to the case of monopoles, there are no important new consequences for or constraints from cosmology.

1. Introduction

Several years ago Callan [1] and Rubakov [2] (see also Wilczek [3]) discovered that grand unified monopoles can catalyze proton decay with a strong interaction cross section $\sigma \sim m^{-2}$ rather than with the naive geometrical cross section $\sigma \sim M^{-2}$, where $m \sim 1$ GeV is the proton mass and $M \sim 10^{16}$ GeV is the mass of the monopole. The amplification of the cross section by 32 orders of magnitude has important consequences in cosmology. Monopoles can be trapped by stars. They then catalyze the decay of the protons in these stars. From the lifetime of red giant stars and from limits on X-ray emission it is possible [4] to derive bounds on the flux of monopoles which are more stringent than previous bounds.

Recently it has been shown [5] that the catalysis cross section for ordinary (i.e. non-superconducting) cosmic strings is not amplified. The reason was shown to be the absence of long range electromagnetic fields which can give rise to long range forces which bind a fermion to the topological defect.

In this paper we extend the analysis to uncharged superconducting cosmic strings [6]. We show that despite the presence of long range electromagnetic fields, the catalysis cross section is a geometrical cross section. There is no amplification. Our result has a very simple heuristic physical explanation.

Grand unified monopoles and strings catalyze proton decay since in the cores of these topological defects the gauge and Higgs fields which mediate proton decay are excited. However, in order to participate in the decay process, the fermions must reach the core, and the geometrical cross section for this is very small: it is given by the core radius M^{-1}, where M is the scale of grand unified symmetry breaking.

To see how the geometrical cross section emerges from field theory consider for a moment catalysis by monopoles. The interaction lagrangian for Higgs-mediated proton decay is [#1]

$$\mathscr{L}_1 = g\bar{\psi}\phi\psi\,. \tag{1.1}$$

To first order in perturbation theory (expanding in the coupling constant g) the scattering amplitude is

$$\mathscr{A} = g\int \mathrm{d}t\,\mathrm{d}^3x\,\langle \Psi' \mathrm{M}|\bar{\psi}\phi\psi|\Psi \mathrm{M}\rangle\,, \tag{1.2}$$

where $|\Psi\rangle$ and $|\Psi'\rangle$ are the initial and final fermion states and $|\mathrm{M}\rangle$ denotes the monopole state (the monopole is taken to be very heavy, so that back reaction can be neglected). As discussed in refs. [7–9], the matrix element factorizes

[#1] In eq. (1.1) ϕ stands for a rescaled scalar field with $\phi=0$ everywhere except in the core of the string. Also, as it stands the term is not gauge invariant. As discussed in ref. [7], the term can be made gauge invariant without changing the conclusions. For notational convenience we shall use this simple but imprecise form.

$$\mathscr{A}=g\int \mathrm{d}t\,\mathrm{d}^3x\,\langle\Psi'|\bar\psi\psi|\Psi\rangle\langle \mathrm{M}|\phi|\mathrm{M}\rangle\,. \qquad (1.3)$$

Since the scalar field configuration of the monopole decays exponentially outside the core and since $\phi\sim M$ inside the core, we get

$$\mathscr{A}\sim gM^{-2}\int \mathrm{d}t\,\langle\Psi'|\bar\psi\psi|\Psi\rangle\,. \qquad (1.4)$$

The cross section $\mathrm{d}\sigma/\mathrm{d}\Omega$ is given by taking the square of $\mathscr{A}$ and by integrating over phase space:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\sim\frac{1}{T}V\int \mathrm{d}^3k'\,|\mathscr{A}|^2\,, \qquad (1.5)$$

where k' is the momentum of the final fermion state. T and V are total time and total volume. Using free fermion wave functions for $|\Psi\rangle$ and $|\Psi'\rangle$ we get [5]

$$(\mathrm{d}\sigma/\mathrm{d}\Omega)(\mathrm{geom.})\sim g^2M^{-2}(m/M)^2\,, \qquad (1.6)$$

where the numerator m^2 comes from the spinor sums. We call this the "naive" or "geometrical" cross section.

If there are long range fields and forces due to the topological defect which couple to the fermions, then the amplitude of the fermion wave functions may be very different from that of free Fermi wave functions inside the core of the defect. The amplification of the fermion wave functions near the core of the monopole is the key to understanding the strong interaction catalysis cross section. To determine the amplification factor A we solve the Dirac equation outside the core in the presence of the defect and compare with the free wave function ψ_0. To be more precise, A is the ratio of the amplitudes of the two wave functions evaluated at the core radius $r\sim M^{-1}$:

$$A=\psi(M^{-1})/\psi_0(M^{-1})\,. \qquad (1.7)$$

Once we know the wave function amplification factor A we can easily determine the actual cross section by inserting into eqs. (1.3) and (1.5). We find

$$\mathrm{d}\sigma/\mathrm{d}\Omega\sim A^4(\mathrm{d}\sigma/\mathrm{d}\Omega)(\mathrm{geom.})\,. \qquad (1.8)$$

For monopoles it was shown [10,11] that $A\sim M/m$. Hence the catalysis cross section corresponds to a strong interaction rate, i.e., $\mathrm{d}\sigma/\mathrm{d}\Omega\sim m^{-2}$. For ordinary cosmic strings on the other hand [5] $A\sim 1$ and hence the cross section per unit length

$$\mathrm{d}\sigma/\mathrm{d}\Omega\,\mathrm{d}L\sim g^2M^{-1}(m/M)^2\,. \qquad (1.9)$$

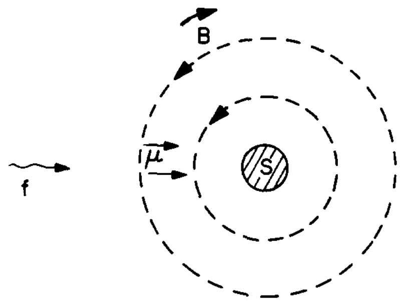

Fig. 1. Sketch of the magnetic field $\boldsymbol{B}$ in the plane perpendicular to a superconducting cosmic string at $x=y=0$. The magnetic moment $\boldsymbol{\mu}$ of fermions in the lowest angular momentum state is orthogonal to $\boldsymbol{B}$.

In this paper we show that eq. (9) is valid also for uncharged superconducting cosmic strings. The heuristic physical argument is simple. For monopoles the amplification of the fermion wave function is due to an attractive force due to magnetic-moment–magnetic-field coupling. Only fermions in the s wave can penetrate to the core (the others are repelled by the centrifugal potential barrier). For s wave fermions the magnetic moment $\boldsymbol{\mu}$ is radial. The monopole magnetic field $\boldsymbol{B}$ is also radial. Hence there is an attractive potential $V(r)\sim\boldsymbol{\mu}\cdot\boldsymbol{B}(r)$. For ordinary cosmic strings there is no such attractive force since there are no long range magnetic fields.

For superconducting cosmic strings the only fermions which can penetrate to the core are those with $L_z=0$ (L_z is the angular momentum in direction of the string which we take to be along the z axis). Without loss of generality we can consider fermions incident in the xy plane. Then the magnetic moment is radial (fig. 1). If it carries a superconducting current then there is a long range magnetic field $B(\rho)$. However, $\boldsymbol{B}(\rho)\sim\boldsymbol{e}_\phi$ has vanishing radial component. Hence there is no long range force and the amplification factor A is of the order 1.

2. Dirac equation in the presence of a superconducting cosmic string

To justify the above heuristic arguments, we solve the Dirac equation in the presence of the long range fields produced by a superconducting cosmic string along the z-axis:

$$(\mathrm{i}\partial\!\!\!/-eA\!\!\!/-m)\psi=0\,, \tag{2.1}$$

m is the fermion mass. Two effects make this equation harder to solve than for ordinary cosmic string. First the presence of long range physical fields, second the fact that the z component of the vector potential A prevents us from writing (2.1) as a set of two uncoupled equations for two-component spinors. For a current I on the string

$$A_\mu(\rho)=-(I/2\pi)\ln(\rho M)\delta_{\mu 3},\quad \rho>M^{-1}\,, \tag{2.2}$$

where ρ is the radius in the xy plane.

We shall use the following representation for the γ matrices [12]:

$$\gamma^0=\begin{pmatrix}\sigma_z & 0\\ 0 & -\sigma_z\end{pmatrix},\quad \gamma^1=\begin{pmatrix}\mathrm{i}\sigma_y & 0\\ 0 & -\mathrm{i}\sigma_y\end{pmatrix},$$

$$\gamma^2=\begin{pmatrix}-\mathrm{i}\sigma_x & 0\\ 0 & \mathrm{i}\sigma_x\end{pmatrix},\quad \gamma^3=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}. \tag{2.3}$$

Then, assuming that the spinor ψ is independent of z we get a set of two coupled two-spinor equations

$$(\mathrm{i}\sigma_z\partial_0-\sigma_y\partial_1+\sigma_x\partial_2-m)\psi_1=eA_3\psi_2\,,$$

$$(\mathrm{i}\sigma_z\partial_0+\sigma_y\partial_1-\sigma_x\partial_2-m)\psi_2=-eA_3\psi_1\,. \tag{2.4}$$

In order to make use of the symmetry of the problem, we use cylindrical coordinates ρ, ϕ and z. Eq. (2.4) can then be written as a set of coupled first order differential equations for the four components of ψ

$$\psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\begin{pmatrix}\psi_1^+\\ \psi_1^-\\ \psi_2^+\\ \psi_2^-\end{pmatrix}. \tag{2.5}$$

We look for solutions with definite total angular momentum J about the string axis. In order for the phases to match we must have $\psi_1^-\sim e^{\mathrm{i}\phi}\psi_1^+$ and similarly for ψ_2. The resulting equations simplify if we extract a common factor $\rho^{-1/2}$, i.e., we set

$$\psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\rho^{-1/2}\,e^{\mathrm{i}(J\phi-wt)}\begin{pmatrix}\phi_1^+(\rho)\,e^{-\mathrm{i}\phi/2}\\ \phi_1^-(\rho)\,e^{+\mathrm{i}\phi/2}\\ \phi_2^+(\rho)\,e^{-\mathrm{i}\phi/2}\\ \phi_2^-(\rho)\,e^{+\mathrm{i}\phi/2}\end{pmatrix}. \tag{2.6}$$

Eqs. (2.4) then give the following set of component equations

$$-\mathrm{i}(w-m)\phi_1^+ +(\partial/\partial\rho+J/\rho)\phi_1^- =-\mathrm{i}eA_3\phi_2^+\,,$$

$$-\mathrm{i}(w+m)\phi_1^- +(\partial/\partial\rho-J/\rho)\phi_1^+ =\mathrm{i}eA_3\phi_2^-\,,$$

$$-\mathrm{i}(w+m)\phi_2^+ +(\partial/\partial\rho+J/\rho)\phi_2^- =-\mathrm{i}eA_3\phi_1^+\,,$$

$$-\mathrm{i}(w-m)\phi_2^- +(\partial/\partial\rho-J/\rho)\phi_2^+ =\mathrm{i}eA_3\phi_1^-\,. \tag{2.7}$$

We can combine eqs. (2.7) into the following set of second order differential equations for the components

$$[\partial^2/\partial\rho^2-J(J-1)/\rho^2+w^2-m^2-e^2A_3^2]\phi_1^+$$
$$=\mathrm{i}eB\phi_2^-\,,$$
$$[\partial^2/\partial\rho^2-J(J+1)/\rho^2+w^2-m^2-e^2A_3^2]\phi_2^-$$
$$=\mathrm{i}eB\phi_1^+\,, \tag{2.8}$$

and an identical set of equations for the pair of components ϕ_2^+ and ϕ_1^-. $B(\rho)$ is the absolute value of the magnetic field

$$B(\rho)=I/2\pi\rho\,,\quad \rho>M^{-1}\,. \tag{2.9}$$

Note that the symmetry of the problem has led to a significant simplification of the problem. However, we have not obtained uncoupled second order differential equations for each component as we did for ordinary cosmic strings. The magnetic field leads to a coupling of ϕ_1^+ with ϕ_2^- and of ϕ_2^+ with ϕ_1^-.

We shall now investigate the system (2.8) of second order differential equations and show that despite the presence of the magnetic field there is no amplification of the fermion wave function. Note that if ϕ_1^+ is real then ϕ_2^- and ϕ_1^- must be imaginary. Eq. (2.8) has the form of a system of coupled Schrödinger equations ($k^2=w^2-m^2$)

$$(\partial^2/\partial\rho^2)\phi_1^+ +k^2\phi_1^+ =+V_J(\rho)\phi_1^+ +\mathrm{i}eB(\rho)\phi_2^-\,, \tag{2.10}$$

with potential

$$V_J(\rho)=J(J-1)/\rho^2+(eI/2\pi)^2\ln^2(\rho M)\,. \tag{2.11}$$

The current I defines a length scale R_I,

$$R_I=2\pi/eI\,. \tag{2.12}$$

For $I=I_0=10^{20}$ A (the critical current for bosonic superconducting cosmic strings) we get $R_I\sim 10^{-12}$ GeV^{-1}. Thus for currents smaller than the critical current R_I is larger than the string width $w\sim M^{-1}$. The potential $V_J(\rho)$ is sketched in fig. 2 for $J=1/2$. The

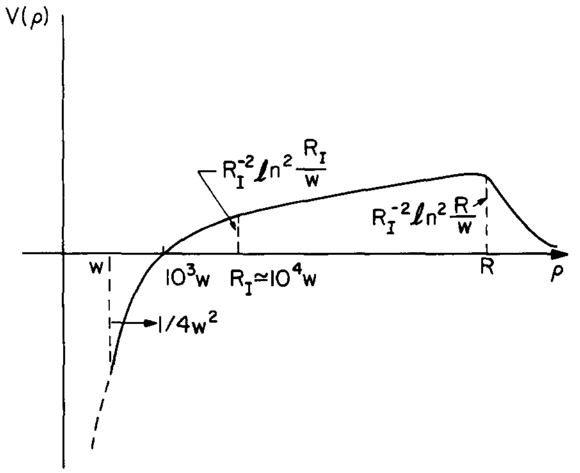

Fig. 2. Sketch of the potential $V_J(\rho)$ for the radial Schrödinger equation (2.10).

cutoff at large ρ corresponds to the length of the string. For cosmic strings of interest in cosmology $\rho\sim 1$ pc $\sim 10^{32}\,\mathrm{GeV}^{-1}$ and $\ln^2(\rho M)\sim 10^4$.

$V_J(\rho)$ is repulsive for large ρ. Hence only high energy fermions have a chance to penetrate to the core. The requirement on k is

$$k>R_I^{-1}\cdot 10^2\,. \tag{2.13}$$

The contribution from the magnetic field to $V_J(\rho)$ is strictly positive and does not lead to an amplification of the wave function. Such an amplification can only come from the coupling terms on the RHS of eq. (2.8).

To estimate the effect of the coupling terms we consider eq. (2.8) in the small ρ limit which [because of (2.8) and (2.13)] is valid for $w<\rho<R_I$. We make the power law ansatz [13]

$$\phi_1^+(\rho)\sim(k\rho)^p\,,\quad \phi_2^-(\rho)\sim(k\rho)^q\,. \tag{2.14}$$

For $B=0$ the normalized wave functions for $J=1/2$ scale with the powers

$$p=1/2\quad\text{and}\quad q=3/2\,. \tag{2.15}$$

Now we turn on the current in the string. Since $B(\rho)\sim\rho^{-1}$ the RHS of eq. (2.8) will not influence the power p of the "most singular" component $\phi_1^+(\rho)$, i.e., we still have

$$p=1/2\,. \tag{2.16}$$

Given $\phi_1^+(\rho)\sim(k\rho)^{1/2}$ we can consider the equation for $\phi_2^-(\rho)$. For any power $q<3/2$ the LHS of the equation would dominate, leading to an inconsistency. Hence, while the magnetic field does change the solution of $\phi_2^-(\rho)$ it does not produce a larger amplitude as $k\rho\to 0$.

For $J=-1/2$ the discussion is identical with the substitutions $\phi_1^+\to\phi_2^+$ and $\phi_2^-\to\phi_1^-$. We conclude that the wave function amplification factor is of the order 1:

$$A=\psi(w)/\psi_0(w)\sim 1 \tag{2.17}$$

(ψ_0 is the free fermion wave function).

3. Discussion

Let us now compare the above analysis of the small distance behavior of the fermion wave function in the presence of a superconducting cosmic string with the results for monopoles [11]. For superconducting cosmic strings the lowest angular momentum fermion wave function scales as $\rho^{+1/2}$ as $\rho\to 0$ both with and without magnetic field. For monopoles the small r behavior is very different. In the absence of the magnetic field the lowest angular momentum wave function scales as r for $r\to 0$, with non-vanishing magnetic field there is an angular momentum eigenfunction which approaches a non-zero constant as $r\to 0$. We would like to connect the above observations with the heuristic reasons for the wave function amplification discussed in the introduction.

As stressed in ref. [14], the key to understanding the wave function amplification in the presence of a monopole is the fact that the interaction of the fermion charge with the monopole magnetic field induces an additional contribution to the orbital angular momentum $\boldsymbol{L}$ proportional to $\boldsymbol{B}\cdot\boldsymbol{r}$. If g is the magnetic monopole charge, then $q=eg$ is half integer and [15]

$$\boldsymbol{L}=\boldsymbol{r}\times(\boldsymbol{p}-e\boldsymbol{A})-q\boldsymbol{r}/r\,. \tag{3.1}$$

The total angular momentum is

$$\boldsymbol{J}=\boldsymbol{L}+\tfrac{1}{2}\boldsymbol{\sigma}\,. \tag{3.2}$$

For $q=0$ the eigenvalues of $\boldsymbol{J}^2$ are $j=n+1/2$ with $n\in\mathbb{Z}^+$. For $q\neq 0$ the extra term in $\boldsymbol{L}$ leads to a change in the eigenvalues. For $q=1/2$ there is an eigenvector of J^2 and J_z with $j=0$ (see ref. [11]). The corresponding solution of the radial Dirac equation approaches a non-vanishing constant as $r\to 0$ as opposed to the solutions with $j>0$ which all vanish at $r=0$. Thus, the key to understanding the strong interaction cross section for fermion scattering by monopoles is the observation that the extra term in the angular momentum operator induced by the monopole allows a solution of the Dirac equation with $j=0$ for which the angular momentum potential barrier is completely absent.

If $\boldsymbol{B}\cdot\boldsymbol{r}=0$ as in the case of superconducting cosmic strings the orbital angular momentum operator is unchanged in the presence of a non-vanishing current. Thus the orbital eigenfunctions are the same and the only change occurs in the radial Dirac equation. However, the small distance scaling of the wave function is given by j. The magnetic field contributions are subdominant as $\rho\to 0$. Hence since j is unchanged with and without the magnetic field, the $\rho\to 0$ scalings of the wave functions will be identical and there will be no wave function amplification.

In our discussion we have implicitly assumed that the vacuum about the string is unperturbed. Recently, several preprints have appeared claiming that this may not be true [16]. It would be interesting to investigate the consequences of this effect for proton decay catalysis.

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