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# Q-balls and the proton stability in supersymmetric theories

M. Axenides <sup>a</sup>, E.G. Floratos <sup>a</sup>, G.K. Leontaris <sup>b</sup>, N.D. Tracas <sup>c</sup>

<sup>a</sup> *Institute of Nuclear Physics, N.R.C.S. Democritos, 153 10 Athens, Greece*

<sup>b</sup> *Physics Department, University of Ioannina, GR 45 110 Ioannina, Greece*

<sup>c</sup> *Physics Department, National Technical University, 157 73 Athens, Greece*

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## Abstract

Abelian non-topological solitons with Baryon and/or Lepton quantum numbers naturally appear in the spectrum of the minimal supersymmetric standard model. They arise as a consequence of the existence of flat directions in the potential lifted by non-renormalizable operators and SUSY breaking. We examine the conditions that these operators should satisfy in order to ensure proton stability and present a realistic string model which fulfills these requirements. We further identify a generic  $U(1)$  breaking term in the scalar potential and discuss its effect of rendering Q-balls unstable. © 1999 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Non-topological solitons, abelian or nonabelian, are finite energy configurations which appear in the spectrum of field theories with global symmetries. [1–3]. They arise due to the appearance of appropriate couplings in the scalar potential that effectively cause a  $Q$  number of scalar particles of mass  $m$  to form coherent bound states with binding energy  $E/Q < m$ . Although the general scaling property of their total energy  $E \equiv Q^s (s < 1)$  receives both surface and volume contributions, there is a special class of such configurations in the large  $Q$  limit with  $s = 1$  whose existence persists in the strict “thermodynamic limit” ( $V \rightarrow \infty, E/Q \equiv \text{const}$ ) [4,5].

In the context of the minimal electroweak theory the presence of  $B$  and  $L$  balls associated with the perturbatively conserved Baryon and Lepton quantum numbers is not feasible. Yet for strong interactions, which respect strangeness and isospin, the

possible existence of charged meson balls of strangeness and/or isospin as resonances in the low energy spectrum of QCD has been considered [6,7] as a possibility.

Recently it was pointed out [8] that non-topological solitons generically appear in the Minimal Supersymmetric Standard Model (MSSM). More generally, supergravity (SUGRA) induced logarithmic corrections in the scalar potential, as well as non-renormalizable polynomial interactions that appear naturally in the flat directions of the MSSM [9], give rise to baryon and lepton balls [10]. They are composed of squarks and sleptons and are very efficient “repositories” of baryonic and/or leptonic charge respectively. We will henceforth call them Q-superballs and denote them as Q-sballs. They convert ordinary fermionic matter carrying a net  $B$  and/or  $L$  charge into its bosonic counterpart. In cosmology, large  $B$  and  $L$ -balls can be generated from decaying Affleck-Dine [11] condensates that develop typically

in the aftermath of an inflating SUSY phase. In an expanding universe, a coherently oscillating AD condensate with a net baryon charge is unstable to space dependent perturbations decaying into large baryon sballs [8].  $B$  and  $L$ -sballs, if unstable and rapidly decaying, could have contributed to the net baryon number of the universe [10]. If they are metastable but sufficiently long lived till the present, they can be a component of the much sought after cold dark matter. Non-abelian Q-sballs have been also discussed in Wess-Zumino models [12]. They minimally arise in renormalizable scalar potentials with cubic interactions that respect supersymmetry and constitute domains that break it explicitly.

In the present paper we take a superstring inspired view on the Q-sball bearing flat potentials in supersymmetric extensions of the standard model. We “embed” the  $U(1)$  ball bearing MSSM flat directions in the “effective” low energy superstring picture. We do it by considering low energy string no scale effective lagrangians such as the flipped  $SU(5) \times U(1)$  [15] model.

We establish a precise mapping between the low energy operators of different dimensionality, such as the  $(Q\ell d^c)^2, u^c u^c d^c$  and  $(u^c d^c d^c)^2$ , and their high energy operators they correspond to in these models. Conceivably they are associated with the small distance  $Q$ -sball bearing superstring induced flat directions. We further address the question of proton stability in conjunction with these operators and determine the conditions to avoid fast proton decay.

In the more general context of the effective lagrangians we are considering, we generically identify an explicit  $U(1)$  breaking term. We consider its effect on the stability of the  $B$ -sballs in the low energy regime ( $E < M_s \equiv 1$  TeV, the SUSY breaking scale).

Finally we address the possibility that Q-sballs are present in the “hidden” sectors of supersymmetric theories which are a generic feature of supergravity and more generally superstring theories. Shadow Matter has been a subject of intense scrutiny recently with regard to its possible rich astrophysical and cosmological implications [13,14].

The paper is organized as follows: In Section 2 we review  $Q$ -ball bearing flat directions in the MSSM and consider the most general form of the superstring inspired scalar potential with its one loop

contribution which is presented in Section 3. In Section 4 we identify the leading small distance operators of  $d = 4$  and  $d = 6$  which correspond to the flat directions in the flipped  $SU(5) \times U(1)$  model. Large scalar vevs in those directions with a nonzero baryon number generate the AD type of condensates which can decay into  $B$  and  $L$ -sballs. We present precise results using renormalization group results for the small-large distance evolution of  $U(1)$ -sball configurations.

## 2. Abelian Q-sballs from flat directions

In a scalar field theory with a global continuous symmetry, Q-balls appear if the minimum of the quantity  $[2\mathcal{V}/\phi^2]$  occurs at some point  $\phi_0 \neq 0$ , where  $\mathcal{V}$  is the potential and  $\phi$  is the scalar field.

In supersymmetric theories,  $Q$ -sballs are associated with the  $F$ - and  $D$ -flat directions of the superpotential. In general, the flat directions (usually called the moduli space) are parametrized by expectation values of massless chiral fields (moduli), while along these directions the scalar potential vanishes. In other words, supersymmetric theories have no classical potential along flat directions. In realistic supersymmetric theories, the role of the fields acquiring vacuum expectation values (vevs) is played by particular combinations of the scalar quarks and leptons. The potential along these directions appears as a result of supersymmetry breaking, radiative corrections and non-renormalizable terms. In MSSM, we are interested in forming operators invariant under the gauge group (and therefore acceptable to appear<sup>1</sup> in the superpotential). Such operators can be formed by considering single field flat directions in a gauge invariant way.

As an example, we start with a flat direction in the MSSM, by considering the operator

$$\mathcal{O} = Q_1 \ell_1 d_2^c \quad (1)$$

where indices denote generations. The operator  $\mathcal{O}$  has  $B - L = -1$ . To see why it corresponds to a flat

<sup>1</sup> Additional discrete symmetries and string selection rules may prevent the appearance of otherwise gauge invariant terms in the superpotential.

direction of the superpotential, we consider all relevant Yukawa terms

$$\begin{aligned} \mathcal{W} = & \lambda_u^{11} Q_1 H_u u_1^c + \lambda_d^{11} Q_1 H_d d_1^c + \lambda_d^{22} Q_2 H_d d_2^c \\ & + \lambda_e^{11} \ell_1 H_e e_1^c + \dots \end{aligned} \quad (2)$$

where dots stand for terms not involving the fields  $Q_1, \ell_1, d_2^c$ . The scalar component of such operators could be parametrized by a single scalar field  $\phi$ . For example, in the case of the operator  $\mathcal{Z}$ , by writing

$$Q = \phi \sin \xi, \quad L = \phi \cos \xi \sin \theta, \quad d^c = \phi \cos \xi \cos \theta \quad (3)$$

the operator can be written

$$\mathcal{Z} = (\sin \xi \cos^2 \xi \sin \theta \cos \theta) \phi^3 \equiv c \phi^3 \quad (4)$$

It can be easily checked that  $\mathcal{W}$  is  $F$ -flat with respect to the derivatives of all the fields appearing in the superpotential. This means that for  $\langle \phi \rangle \neq 0$ , all derivatives  $\partial \mathcal{W} / \partial f_a = 0$ , where  $f_a$  stands for all fields.

The connection between Q-sballs and flat directions comes in when addition of new terms alter these flatness and the potential takes the desired form to accommodate Q-sballs. By giving non-zero expectation values to these fields, the gauge group breaks down partially or even totally (in the example of the operator  $\mathcal{Z}$ , the standard model group is broken down to  $SU(2)_{\text{colour}} \times U(1)$ ). Also, since the fields carry baryon  $B$  and lepton  $L$  numbers, the operator may also have a non-zero  $B$ ,  $L$  or  $B - L$  quantum number.

Let us now be more specific on the ways that a flat direction can be lifted. This can be achieved by supersymmetry breaking effects, by non-renormalizable terms that could appear in the superpotential and finally by one loop corrections to the supersymmetry breaking induced soft mass terms. The tree-level part of the scalar potential has the following types of terms:

$$\begin{aligned} \mathcal{V}_{\text{eff}} = & \left| \frac{\partial \mathcal{W}}{\partial f_a} \right|^2 + A(\mathcal{W} + \mathcal{W}^*) + B \left( \frac{\partial \mathcal{W}}{\partial f_a} + c.c. \right) \\ & + \tilde{m}_a^2 |f_a|^2 \end{aligned} \quad (5)$$

The last three terms are related to the supersymmetry breaking effects.  $A$  and  $B$  are soft parameters of the

order of the supersymmetry scale while  $\tilde{m}_a$  are the soft masses of the scalar components of the superfields.

Let us see now how the non-renormalizable terms can lift the flatness of the potential (one loop effects will be discussed in the next section). Suppose that a certain operator, describing a flat direction and parametrized by the scalar  $\phi$ , takes the form  $\phi^m$ ;  $m$  is a power which shows the number of fields entering the operator (in our previous example of the operator  $\mathcal{Z}$ ,  $m = 3$ ). The operator cannot appear by itself in  $\mathcal{W}$ , however, it can show up through NR-terms together with other fields of the theory. Suppose,  $f_x$  is such a field, then a term of the type described above will have the form

$$\mathcal{W} = \lambda_{nr} f_x \frac{\phi^{m \cdot k}}{M^{m \cdot k - 2}} = \lambda_{nr} f_x \frac{\phi^{n-1}}{M^{n-3}} \quad (6)$$

where  $k$  shows the power that the operator appears and  $M$  is some high scale. Obviously, the derivative with respect to the field  $f_x$  leaves a non-zero term

$$\frac{\partial \mathcal{W}}{\partial f_x} \propto |\langle \phi \rangle|^{n-1} \neq 0 \quad (7)$$

and the flat direction is lifted. In addition, there are  $A$ -terms of the form  $A \phi^n / M^{n-3}$ . There are two important points worth mentioning: First, since a flat direction is parametrized by a single (scalar) field, the soft supersymmetry breaking  $A$ -term in the potential violates any possible  $U(1)$  that the superpotential might respect. The absence of a continuous  $U(1)$  forbids the appearance of stable  $Q$ -sball like solutions. Thus, the question arises whether there exist certain conditions such that  $Q$ -sballs can be formed in the MSSM potential. An apparent way out would be to require the initial condition for the  $A$ -term to be zero, however renormalization group running effects will drive its value to magnitudes comparable with other soft parameters. Nevertheless the  $A$ -term could be kept relatively small in the interesting energy region, allowing the possibility for an unstable  $Q$ -sball to develop. Second, in the case that a  $Q$ -sball can be formed, the sign of the  $A$  parameter plays evidently a crucial role in developing the required minimum of the quantity  $\mathcal{V}/|\phi|^2$ .

Returning to the scalar potential, (5) assumes the following general form

$$\mathcal{V}_0 = m_S^2 |\phi|^2 + |\lambda_{nr}|^2 \frac{|\phi|^{2(n-1)}}{M^{2(n-3)}} + \left( A \frac{\phi^n}{M^{(n-3)}} + c.c. \right)$$

where *c.c.* means complex conjugate and we assume for simplicity *A* to be real valued. We observe indeed that the continuous  $U(1)$  symmetry respected by the first two terms, is broken by the *A*-term. Clearly, the usual Q-sball solution  $\phi = e^{i\omega t} \phi_0$  is no longer a solution of the equations of motion, however, it can only be approximate one as long as the *A*-term is relatively small.

In our analysis, we assume for simplicity only one scalar  $\phi$ . In the case of multiple flat directions there will be more scalars, one for each such direction. The soft mass term  $m_S$  is here related to the soft masses of the fields making up the composite operator. Its value is scale dependent  $m(Q)$  and is calculated at any scale using the RGEs once the initial value is known. At tree level, this mass is independent of the scalar vev  $\langle \phi \rangle$  and a minimum of the potential at  $\langle \phi \rangle = 0$  is unavoidable. However, when one-loop corrections are taken into account, there is a  $\phi$ -dependence of  $m_S^2 = m_S^2(\phi)$  which could possibly lead to a minimum away from zero.

Thus, having defined a certain flat direction, the next task is the determination of the expectation value of the corresponding scalar parametrizing this direction. As stressed above, the tree-level classical potential leaves the scalar vev undetermined. Radiative corrections to the classical potential will determine this vev. Therefore, one has to add also at least one-loop corrections to  $\mathcal{V}_0$ . The directions used to form condensates, should be chosen with great care. The reason is that there are *R*-parity breaking terms (like  $d^c d^c u^c$ ) which create proton decay at low energies. We will work out cases where fast proton decay is forbidden.

### 3. A superstring inspired Q-sball bearing flat potential

A necessary presupposition to obtain a global minimum of  $\mathcal{V}_0/\phi_0^2$  away from  $\langle \phi_0 \rangle = 0$ , as can

be seen from the form of the effective scalar potential, is to have a  $\phi$ -dependent scalar mass parameter  $m_S^2$ . At the tree level, this is a sum of soft mass parameters related to the fields forming up the condensate, independent of the value of  $\phi$ . Thus, at tree level,  $\mathcal{V}_0/\phi_0^2$  has a minimum at  $\langle \phi_0 \rangle = 0$ . At the one loop level the soft mass parameter  $m_S^2$  is modified by one-loop corrections proportional to the logarithm  $\log \phi_0^2/Q^2$  where  $Q$  is the running scale. Thus,  $m_S^2$  depends on  $\phi$  and a minimum away from the origin is possible. For a scalar field  $\phi$ , the one loop corrected potential is

$$\mathcal{V}_1(\phi) = \mathcal{V}_0(\phi) + \frac{1}{64\pi^2} m_S^2 \left( \ln \frac{\phi^2}{Q^2} - \frac{3}{2} \right) \quad (8)$$

$V_0(Q)$  is the (R.G.E. improved) tree-level potential while the appearance of the last term is due to the radiative corrections (at one-loop level). Thus, in the case of a scalar field  $\phi$ , as that described above the one loop correction results to a shift of the soft mass parameter of the condensate. This makes the mass parameter of the last term in (6) a function of  $\phi$ , which is essential in the determination of the minimum of (1) at values  $\langle \phi \rangle \neq 0$  as required. The soft mass parameter will be in general a linear combination of the scalar mass terms forming the condensate,  $m_S^2 = \sum_i \alpha_i m_i^2$ . The general form of the mass coefficient of the logarithmic term is then [16]

$$m_S^2 = \sum_i \alpha_i \frac{d\tilde{m}_i^2}{dt} = \sum_i \alpha_i \left( - \sum_A c_A^i g_A^2 m_A^2 + c_i^i \lambda_i^2 \sum_{n_3} \tilde{m}_{n_3}^2 \right) \quad (9)$$

where  $t = \log Q$  while  $\alpha_i$  are coefficients. Furthermore,  $m_A$  are the gaugini masses and  $g_A$  are the three gauge couplings; further,

$$\sum m_{n_3} = \tilde{m}_{Q_3}^2 + \tilde{m}_{u_3^c}^2 + m_{\tilde{h}}^2$$

is the sum of the scalar mass parameters of the third generation and the higgs while only  $\lambda_t$  Yukawa contributions have been included. Let's assume the particular case of  $n = 3$  which will be useful in our subsequent analysis. In this case, the general form of the quantity  $\mathcal{V}/\phi^2$  becomes

$$\frac{\mathcal{V}}{\phi^2} = \kappa + \nu \log \phi + \alpha \phi + \lambda \phi^2 \quad (10)$$

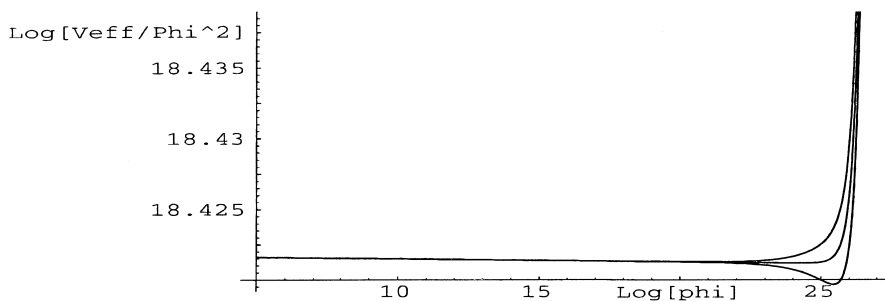


Fig. 1. Plot of the quantity  $\log[\mathcal{V}_{\text{eff}}/\phi^2]$  versus  $\log[\phi_0]$  in the effective supergravity model described in the text and for the operator  $u^c d^c d^c$ , for three values of the parameter  $A_{\text{eff}}$ . The minimum is formed in a very high scale, when the conditions discussed in the text are met. The corresponding superheavy  $B$ -ball is unstable.

Since in our case we take always  $\lambda > 0$ , it can be checked that the necessary minima with respect to  $\phi$  exist for the cases  $\nu < 0, \alpha > 0$ , and  $\nu > 0, \alpha < 0$ . Clearly,  $\kappa, \nu, \alpha$  and  $\lambda$  are scale dependent. Their relation with the MSSM mass parameters  $m_S^2$  etc are easily found. To find the minima therefore, one has to examine the values of the above quantities at any scale while varying the coefficients  $\alpha_i$  in such a way so as the above conditions are met.

In Fig. 1 we plot the logarithm  $\mathcal{V}_{\text{eff}}/\phi_0^2$  against the logarithm of  $\phi_0$ , at the energy scale  $Q = 10^{13}$  GeV. We have considered in the  $\mathcal{V}_{\text{eff}}$  the zero and one loop order potential (8) plus NR- and trilinear  $A$ -terms of the form

$$\lambda_{\text{eff}} \phi^6 / M^2 + A_{\text{eff}} \phi^3$$

We give to  $\lambda_{\text{eff}}$  the value  $\mathcal{O}(0.1)$  while the three curves correspond to negative, zero and positive  $A_{\text{eff}}$ -values starting from the one that produces the deeper minimum. Yukawa effects have been included only due to top quark. (For presentation purposes, in the vertical axis an arbitrary positive constant has been added to  $\mathcal{V}_{\text{eff}}/\phi_0^2$ .) The minimum exists only when  $A_{\text{eff}}$  obtains negative values ( $\sim -0.1 m_{3/2}$ ) in a narrow range, while it shifts to unacceptably high values as  $A_{\text{eff}}$  changes due to renormalization group running.

#### 4. An SU(5) superstring model

A natural ground for the above ideas is offered by models which possess extra  $U(1)$  symmetries. This is because such symmetries (if properly chosen) may

prevent disastrous combinations of  $R$ -parity breaking terms which lead to fast proton decay. Models with these properties are found in string constructions. As an example, we will work out the relevant non-renormalizable operators which are obtained in the case of the string derived flipped  $SU(5)$  model. The details can be found elsewhere [15,17]. Here we recall only the necessary parts. The generations and higgses are accommodated in

$$F = (10, -1/2), \quad \bar{f} = (\bar{5}, 3/2), \quad \ell^c = (1, -5/2) \quad (11)$$

$$H = (10, -1/2), \quad \bar{H} = (10, 1/2);$$

$$h = (5, 1), \quad \bar{h} = (\bar{5}, -1) \quad (12)$$

The quark and lepton fields are found in the following representations

$$F = (Q, d^c, \nu^c), \quad \bar{f} = (u^c, \ell), \quad \ell^c = e^c \quad (13)$$

and the tree level superpotential relevant to the above terms is

$$FFh + F\bar{f}\bar{h} + H\bar{H}h + \bar{H}\bar{H}\bar{h} \quad (14)$$

Additional  $U(1)$  symmetries can be chosen to distinguish between the various generations that appear at this level. However, operators of the form described above, allowed by these symmetries, can be generated in the non-renormalizable part of the superpotential. Then, terms of the form (6) appear in the effective potential of the MSSM after the breaking of the GUT symmetry. We will concentrate in the case of lowest dimension operators. Let us make the above by describing an example. Suppose we are

interested in the operator  $\mathcal{X} = u^c d^c d^c$  describing a flat direction of the MSSM and we would like to check whether it could appear through the flipped  $SU(5)$  non-renormalizable term

$$FFF\bar{f}\Phi \quad (15)$$

Here,  $\Phi$  is a possible singlet (or a power of singlet fields) which may appear in such terms. The fields  $F, \bar{f}$  are of the form (11) which may accommodate the ordinary quarks and leptons, or other heavy fields of the same quantum numbers if the model is non-minimal. The above NR-term gives the following two low energy operators:

$$\begin{array}{ccccc} F & F & F & \bar{f} & \Phi \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ d^c(\bar{3},1) & d^c(\bar{3},1) & (1,1) & u^c(\bar{3},1) & (1,1) \end{array} \quad (16)$$

and

$$\begin{array}{ccccc} Q & (1,1) & d^c & \ell & (1,1) \end{array}$$

where the numbers in parentheses are with respect to the  $SU(3) \times SU(2)$ . When we take the derivative of this term with respect to the singlet (1,1) belonging to  $F$ , a term of the form

$$|u^c d^c d^c \Phi|^2 \quad (17)$$

will appear in the effective potential. Likewise, a similar term corresponding to the  $QLd^c$  flat direction can appear through the same fifth order non renormalizable term, again taking the derivative with respect to the (1,1) component of a  $F$  field, the only one which can be used to form a MSSM singlet term in the superpotential.

The  $LLe^c$  MSSM flat direction can appear through the term  $F\bar{f}\bar{f}^c\Phi$  where again the differentiation is taken with respect to the singlet of the  $F$  field.

$$F\bar{f}\bar{f}^c\Phi \rightarrow LLe^c \quad (18)$$

These three directions, namely  $u^c d^c d^c$ ,  $QLd^c$  and  $LLe^c$ , exhaust the 3-field composite operators describing flat directions in the MSSM.

Going now to the 4-field operators of the MSSM describing flat directions, namely  $QQu^c d^c$ ,  $QQQL$ ,  $QLu^c e^c$  and  $u^c u^c d^c e^c$ , it is easily checked that:

- $QQu^c d^c$  and  $QQQL$  can appear either from the  $FFF\bar{f}\Phi$  operators (differentiating with respect to  $\Phi$ ) and from the  $FFF\bar{f}\phi_{\pm}\Phi$  ones (differentiating with respect to  $\phi_{\pm}$ ).
- $QLu^c e^c$  and  $u^c u^c d^c e^c$  can appear through the  $F\bar{f}\bar{f}^c\Phi$  operators (differentiating with respect to  $\Phi$ ).

Higher order terms can lead to the same type of operators with some additional suppression factors that make such contributions less important. It is therefore, adequate to find the minimum dimension NR-terms which contribute to a certain type of operator.

A basic problem encountered with this type of operators, however, is the undesirable fast proton decay. In particular, the simultaneous existence of terms as  $u^c d^c d^c$  and  $QLd^c$  in the low energy effective theory will induce a fast decay of the proton. Thus, at first sight, it seems that terms forming condensates for Q-balls should be banned due to their possible catastrophic consequences and contradiction with the low energy data. There are certain conditions, however, under which these terms can exist without causing the aforementioned problems. In particular:

- If the field  $\Phi$  has a vanishing vev,  $\langle \Phi \rangle = 0$ , this operator cannot contribute to proton decay. However, the corresponding condensate survives in the scalar potential when differentiating with respect to  $\Phi$ .
- If, as in the case of non-minimal models (which is often the case in string constructions), one of the fields  $F, \bar{f}$  entering the operator is a heavy state, not related to the ordinary quarks and leptons, proton decay is avoided.

Although the above requirements look rather unlikely to be fulfilled, it is interesting that they do occur in certain string models. In the following, we will investigate this possibility in the case of the flipped  $SU(5)$  string model. We will not exhaust all possible cases here, but we will concentrate in a particular operator.

To avoid confusion, we remark here that in the following, the indices in the representations  $F_i, \bar{f}_i, \ell_i^c$  indicate the sector of the string basis they belong to rather than the generation. In fact, the accommodation of the three generations takes place as follows:

$$\begin{aligned} \bar{f}_1: u^c, \tau, \quad \bar{f}_2: c^c, e/\mu, \quad \bar{f}_3: t^c \mu/e, \\ F_2: Q_2, s^c, \quad F_3: Q_1, d^c, \quad F_4: Q_3, b^c, \\ \ell_1^c: \tau^c, \quad \ell_2^c: e^c, \quad \ell_5^c: \mu^c \end{aligned} \quad (19)$$

where  $F_i = (10, -1/2)$ ,  $\bar{f}_i = (\bar{5}, -1)$  and  $\ell_i^c = (1, -5/2)$ . They also carry charges under the four surplus

$U(1)$  factors which will play a crucial role in determining the NR-terms.

Tree level couplings of the above model, do not lift flat directions of the ones discussed above. There are fifth and sixth order terms of this type which may lift the above flatness of quark and lepton fields which might lead to fast proton decay. These are [17]

$$F_4 F_4 F_3 \bar{f}_3 \Phi_{31}, \quad F_2 F_2 F_3 \bar{f}_3 \bar{\Phi}_{23}, \quad F_1 F_1 F_3 \bar{f}_3 \Phi_{31} \quad (20)$$

$$F_3 \bar{f}_3 \bar{f}_1 \ell_1^c \Phi_{31}, \quad F_3 \bar{f}_3 \bar{f}_5 \ell_5^c \bar{\Phi}_{23}, \quad F_3 \bar{f}_3 \bar{f}_2 \ell_2^c \bar{\Phi}_{23} \quad (21)$$

$$F_3 \bar{f}_2 \bar{f}_2 \ell_3^c \Phi_{31}, \quad F_3 \bar{f}_1 \bar{f}_1 \ell_3^c \Phi_{31}, \quad F_3 \bar{f}_5 \bar{f}_5 \ell_3^c \bar{\Phi}_{23} \quad (22)$$

However, with a proper choice of vacuum expectation values ([17]) which also respects  $F$  and  $D$  flat directions, none of these terms are dangerous since they do not involve particles in the light Standard Model part of the spectrum. At sixth order, the following potentially-dangerous operators appear:

$$F_4 F_3 F_3 \phi_+ \bar{f}_5 \bar{\Phi}_{23}, \quad F_4 F_3 F_3 \bar{f}_5 \bar{\phi}_- \Phi_{31} \quad (23)$$

The singlet fields  $\phi_+$  and  $\bar{\phi}_-$  do not acquire vevs and proton decay is avoided. Being safe from proton decay problems, we turn now to the possible role of these terms on the  $Q$ -ball formation. This will be manifest in the way described for the term of the form (6). Thus in a number of cases, for example, the role of the field  $f_x$  is played here by the field  $\bar{f}_3$  which appears in a number of terms of fifth order. Differentiating with respect to this field, we may create the quantity analogous to that in (7). (In fact, here, the operator is multiplied by one additional singlet vev, namely  $\langle \Phi_3 \rangle$ , which will offer an additional suppression factor to the NR-coupling:  $\lambda_{nr} \sim g_U \langle \Phi_3 \rangle / M_U$ .) The preceding discussion does not intend to systematically exhaust all possible sources associated with  $Q$ -ball formation. Rather, it is indicative in the way these finite energy configurations may occur in realistic theories. The question on the stability of these objects and a thorough study of the related equations should be a first priority before a complete analysis in the context of such theories is done.

## 5. Q-sballs in the shadow world – concluding remarks

A Hidden Sector appears to be a generic element in supergravity theories and the low energy limit of any superstring theory. In addition to providing a gravity mediated mechanism to break supersymmetry its possible role as being the origin of a purely gravitation matter component in our universe such as dark matter has been previously discussed [14]. As so far our discussion of Q-sballs appearing in the spectrum of susy gauge theories with unbroken global symmetries concerned the observable sector from the point of view of the superstring we would now like to deal with Q-sballs appearing in the hidden sector of any such theory. In the context of superstring inspired models a hidden sector typically contains scalar particles with only gravitational interactions which are described to a very good approximation by sigma models [18]. These are parametrized in general by a coset  $G/H$  space. The group  $G$  acts nonlinearly whereas  $H$  acts linearly and can be viewed as a global symmetry of the  $\sigma$ -model. Hence we would expect the presence of abelian or nonabelian global symmetries and hence of Q-sballs in the hidden sectors of such theories [19]. In our present model such a sector has an  $SU(4) \times SO(10)$  gauge symmetry. As such the  $SU(4)$  coupling may become strong at around  $\sim 10^{10-12} Gev$  mimicking pretty much QCD. The confinement of the nonabelian gauge charge greatly restricts the meaningfulness of extended Q-sball configurations with a net nonabelian charge. In a more general setting, however, in the presence of unbroken global symmetries the hidden sector should be expected to possess nontopological solitons which are solutions to the field equations of motion including gravity.

An interesting realization of this possibility was recently put forward for the minimal non-susy electroweak theory [20]. Q-Balls are shown to be present and induced by the coupling of the unobservable so far Higgs to a gauge singlet complex scalar field in a theory with an additional unbroken global abelian symmetry. As these stable solitons presumably interact gravitationally and only through a Higgs exchange to the observable sector they can be a dark matter component.

In summary we investigated the possible existence of Abelian non-topological solitons associated with global  $B$ ,  $L$ , or  $B - L$  quantum numbers in low energy effective supergravity models arising from superstring theories. We described the conditions in the effective potential for a  $B$ -ball to appear and discussed the role of radiative corrections,  $A$ -terms and non-renormalizable contributions. In particular, we found that  $Q$ -balls are likely to appear at high scales, however, we showed that  $A$ -terms lead to a potential instability of the associated  $Q$ -ball. We further discussed the ways to ensure proton stability triggered by the above non-renormalizable operators and presented a string example where all baryon and lepton violating terms associated with these finite energy configurations are suppressed.

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### References

- [1] J. Rosen, *J. Math. Phys.* 9 (1998) 996; 9 (1968) 999.
- [2] R. Friedberg, T.D. Lee, R. Sirlin, *Phys. Rev. D* 13 (1976) 2739.
- [3] T.D. Lee, Y. Pang, *Phys. Rept.* 221 (1992) 251, and refs. therein.
- [4] S. Coleman, *Nucl. Phys. B* 262 (1985) 263.
- [5] A. Safian, S. Coleman, M. Axenides, *Nucl. Phys. B* 297 (1988) 498.
- [6] J. Distler, B. Hill, D. Spector, *Phys. Lett. B* 182 (1986) 71; A.P. Balachandran, B. Rai, A.M. Srivastava, *Phys. Rev. Lett.* 59 (1987) 853; *Int. J. Mod. Phys. A* 3 (1988) 2621.
- [7] M. Axenides, Harvard U., Ph.D. thesis, 1986, unpublished; *Int. J. Mod. Phys. A* 7 (1992) 7169.
- [8] A. Kusenko, *Phys. Lett. B* 405 (1997) 108; A. Kusenko, M. Shaposhnikov, *Phys. Lett. B* 418 (1998) 46; G. Dvali, A. Kusenko, M. Shaposhnikov, *Phys. Lett. B* 417 (1998) 99.
- [9] M. Dine, L. Randall, S. Thomas, *Nucl. Phys. B* 458 (1996) 291.
- [10] K. Enqvist, J. McDonald, *Phys. Lett. B* 425 (1998) 309.
- [11] I. Affleck, M. Dine, *Nucl. Phys. B* 249 (1985) 361.
- [12] M. Axenides, E. Floratos, A. Kehagias, hep-ph/9810230, Non-Abelian Q-Balls in Supersymmetric Theories.
- [13] A. Kusenko, V. Kuzmin, M. Shaposhnikov, P.G. Tinyakov, *Phys. Rev. Lett.* 80 (1998) 3185.
- [14] E. Kolb, D. Seckel, M. Turner, *Nature* 314 (1985) 415; L. Krauss, A. Guth, D. Spergel, G. Field, *W. Press, Nature* 319 (1986) 748.
- [15] I. Antoniadis, J. Ellis, J. Hagelin, D.V. Nanopoulos, *Phys. Lett. B* 231 (1989) 69.
- [16] C. Kounnas, A. Lahanas, D.V. Nanopoulos, M. Quiros, *Phys. Lett. B* 132 (1982) 95.
- [17] G.K. Leontaris, K. Tamvakis, *Phys. Lett. B* 260 (1991) 333; I. Antoniadis, J. Rizos, K. Tamvakis, *Phys. Lett. B* 278 (1992) 257; G.K. Leontaris, *Zeit. Phys. C* 53 (1992) 87; J. Ellis, G.K. Leontaris, S. Lola, D.V. Nanopoulos, *Phys. Lett. B* 425 (1998) 86.
- [18] J. Wess, J. Bagger in: *Supersymmetry and Supergravity*, Princeton Univ. Press, 1983.
- [19] A. Kehagias, private communication.
- [20] D.A. Demir, hep-ph/9810453, Weak Scale Hidden Sector and Electroweak Q-Balls.