RADIATIVE FERMION MASSES IN SUPERSYMMETRIC THEORIES

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We propose a radiative mass generation mechanism for quarks and leptons in supersymmetric theories, employing intermediate mass particles that mediate intergenerational interactions.

1. Motivation. It has been advocated by many authors that the hierarchical structure of the quark and lepton mass spectrum suggests a radiative mass generation mechanism [1]. In this picture, the heaviest generation obtains mass directly at the tree level by its coupling to the Higgs scalar fields that acquire an expectation value while the lighter generations get a mass only through radiative corrections at the one- and two-loop level correspondingly. This programme has been successfully incorporated [1] in GUTs in various ways, unfortunately at the expense of introducing new scalar particles.

In the currently fashionable globally or locally supersymmetric GUTs it would be natural to try to see whether a radiative mass generation programme can be implemented in a similar way. Recently, Ibañez [2] pointed out that in a supersymmetric theory, any radiative fermion masses, arising after supersymmetry is broken, are typically of the order of

$$(m_{\mathrm{F}})_{\mathrm{rad}} \sim g^n (M_{\mathrm{S}}^2/M^2) M_{\mathrm{W}} , \tag{1}$$

where M_{S} is the supersymmetry breaking mass scale and M the heaviest mass scale, circulating in the internal loops that give rise to eq. (1), while g denotes a generic coupling constant. Identifying this scale with the grand unification scale M_{X} leads to extremely small radiative masses. Remember that the supersymmetry breaking scale is at most $M_{\mathrm{S}}^2 \approx M_{\mathrm{W}} M_{\mathrm{P}}$. Thus, if we employ superheavy particles, as in ordinary GUTs, radiative corrections to fermion masses cannot be more than $g^n (M_{\mathrm{W}}/M_{\mathrm{X}}) M_{\mathrm{W}}$, which is tiny. As usual M_{W} denotes the electroweak scale while M_{P} denotes the Planck scale.

It should not be forgotten however that, due to supersymmetry, radiative corrections to fermion masses are linked to radiative corrections to the masses of their bosonic partners which should not be too large if we do not want any resurgence of the hierarchy problem. Typical radiative corrections to boson masses will be

$$(m_{\mathrm{B}}^2)_{\mathrm{rad}} \sim g^m M_{\mathrm{S}}^4/M^2 . \tag{2}$$

This implies that

$$(m_{\mathrm{F}})_{\mathrm{rad}} \sim g^k (M_{\mathrm{W}}/M)(m_{\mathrm{B}})_{\mathrm{rad}} . \tag{3}$$

The last relation makes the difficulty more striking. If we want light scalars $[(m_{\mathrm{B}})_{\mathrm{rad}} \sim \mathrm{O}(M_{\mathrm{W}})]$ we will probably get stuck with almost massless fermions. The easiest conclusion one can draw from such a situation is that supersymmetry and a radiative mass generation programme seem incompatible. Nevertheless, it is too early to despair. A first attempt to get out of this "impasse" was made in ref. [3] where radiative corrections induced by fermion exchanges were considered. We

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propose here a completely different approach. A quick look at eq. (1) suggests that, if we trust the role of the intergenerational mediator to a set of intermediate mass particles [$M \sim O(M_S)$] instead of superheavy ones, we could obtain radiative masses of the right magnitude $(m_F)_{rad} \sim g^n M_W$. Unfortunately, we must also avoid the consequences of eq. (2) which dictates that for $M \sim O(M_S)$ we would probably get very large corrections to boson masses. There are two ways one could try to avoid the problem. First, one could visualize a situation in which there exists *at least one graph* with fermionic external legs that involves no superheavy particle exchange and at the same time *all graphs* with external bosonic legs involve superheavy particle exchange. Such a distinction between bosons and fermions would probably require the presence of an R symmetry. Secondly, one could ban superheavy particles altogether and construct a model in which only intermediate mass particles communicate between generations but the bosonic radiative corrections appear only in higher orders and are thus suppressed by the high powers of coupling constants.

In this paper we investigate how a radiative mass generation programme can be implemented in supersymmetric theories and demonstrate that intermediate mass particles that mediate interactions between different generations can lead to appreciable radiative fermion masses without any problem from big radiative corrections to boson masses. We illustrate the above ideas on globally supersymmetric models as well as a minimal locally supersymmetric GUT.

2. A model. In order to illustrate our ideas, let us start by introducing a variant of the BFN SU(3) × SU(2) × U(1) × $\widetilde{U}(1)$ model [4] with spontaneously broken global supersymmetry. For simplicity we shall consider only two generations of quarks and leptons. The superfield content of the model is

$$Q, q \to (3, 2, 1/6, 1)\,, \qquad \mathcal{L}, \ell \to (1, 2, -1/2, 1)\,,$$

$$U^c, u^c \to (\bar{3}, 1, -2/3, 1)\,, \quad E^c, e^c \to (1, 1, 1, 1)\,,$$

$$D^c, d^c \to (\bar{3}, 1, 1/3, 1)\,, \qquad P, p \to (1, 1, 0, 1)\,.$$

$$H \to (1, 2, -1/2, -2)\,, \qquad R \to (1, 1, 0, 4)\,,$$

$$H^c \to (1, 2, 1/2, -2)\,, \qquad R^c \to (1, 1, 0, -4)\,,$$

$$S \to (1, 1, 0, 4)\,.$$

Let us begin by writing down the superspace potential

$$f_1 = aQD^cH + bQU^cH^c + c\mathcal{L}E^cH$$
$$+ d\mathcal{L}PH^c + fHH^cS + mSR^c + \mu RR^c\,. \qquad (4)$$

The parameters m, μ are of the order 100 GeV. In f_1 only the "capital" generation appears. As it stands, this model leads to exactly massless "small" quarks and leptons. We can improve this situation, always committed to the dogma of no tree level masses for "small" quarks and leptons, by adding an extra piece

$$f_2 = a'(Qd^c + qD^c)\,\widetilde{H} + b'(Qu^c + qU^c)\,\widetilde{H}^c$$
$$+ c'(\mathcal{L}e^c + \ell E^c)\,\widetilde{H} + d'(\mathcal{L}p + \ell P)\,\widetilde{H}^c$$
$$+ \lambda\widetilde{H}\widetilde{H}^cR + M(\widetilde{H}\overline{\widetilde{H}} + \widetilde{H}^c\overline{\widetilde{H}}^c)\,. \qquad (5)$$

In eq. (5) we have introduced a quartet of isodoublet superfields $\widetilde{H}$, $\widetilde{H}^c$, $\overline{\widetilde{H}}$, $\overline{\widetilde{H}}^c$ (try not to get confused with the notation). We exercise our freedom in neglecting any mixings $H\widetilde{H}$. This is technically natural. These doublets *will not obtain any expectation value* since their mass M [to be of $O(M_S)$] will overpower any negative mass-square contributions coming from the D term. Colour-triplets spoil the minimization leading to a supersymmetric, and therefore global, minimum with broken colour. This is not, however, the reason why we do not introduce them. Ultimately, we must make supersymmetry a local symmetry. $N = 1$ supergravity allows us to pick out the SU(3) × U(1)$_{EM}$ minimum in a unique fashion by arranging it so that it has a vanishing cosmological constant. Colour-triplets could undertake the role of mixing the generations, however, in that case we should be very selective in coupling them to quarks and leptons for fear of proton decay. An intermediate colour-triplet mass of $O(10^{10}$ GeV) is disastrous if we want to keep the Yukawa coupling constants a', b', ... naturally large ($\sim 10^{-1}$–10^{-2}). Having to resort to small couplings would be self-defeating.

Those familiar with the specifics of the model can easily recognize that the SU(3) × U(1)$_{EM}$ minimum is controlled by f_1 and corresponds to

$$\langle H \rangle = \begin{pmatrix} \mu/f\sqrt{2} \\ 0 \end{pmatrix}\,, \quad \langle H^c \rangle = \begin{pmatrix} 0 \\ \mu/f\sqrt{2} \end{pmatrix}\,,$$

$$\langle R^c \rangle^2 \simeq \tfrac{1}{4}\,\xi\,[1 - (2\mu^2/\xi)(1/f^2 + 1/8e^2)] + \ldots\,, \qquad (6)$$

while all other fields have zero expectation value. The parameter ξ appears in the Fayet–Iliopoulos D term of the $\tilde{\mathrm{U}}(1)$ gauge group. ξ is naturally of order M_{P}^2. Supersymmetry is spontaneously broken by the auxiliary field expectation values

$$F_{\mathrm{S}} = \tfrac{1}{2}\mu\sqrt{\xi} + \mathrm{O}(\mu^3/\sqrt{\xi}), \quad F_{\mathrm{R}} = \tfrac{1}{2}\mu\sqrt{\xi} + \mathrm{O}(\mu^2),$$

$$\tilde{\mathrm{D}} = \mu^2/4e^2 . \qquad (7)$$

Once supersymmetry is broken, radiative corrections can arise. Only D components of local products of chiral superfields can be generated however, and this restricts the possible corrections one could expect. Denoting by q a general quark or lepton superfield, by h a Higgs superfield and by r a superfield that breaks supersymmetry with its auxiliary F term expectation value, the operators appropriate for fermion masses must be of the form

$$(\mathrm{qqhr}^\dagger)_D . \qquad (8)$$

An expectation value for h is needed in order to have SU(2) × U(1) broken (only then can quarks and leptons obtain a mass) and an expectation value for the F term of r is needed in order to have supersymmetry broken (only then can we have non-vanishing radiative corrections). The operator (8) leads to

$$\int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \theta^2\bar{\theta}^2\, \mathrm{q_F q_F} \langle \mathrm{h}\rangle\langle F_{\mathrm{r}}^\dagger\rangle = \mathrm{q_F q_F}\langle \mathrm{h}\rangle\langle F_{\mathrm{r}}^\dagger\rangle ,$$

which is of dimension six, and thus suppressed by a mass squared that corresponds to the heaviest mass scale that circulates in the loops which gave rise to eq. (8).

For instance, the graph of fig. 1 gives a contribution to the "small" up-quark mass

$$m_{\mathrm{u}} \sim [(a'b'a\lambda)/\pi^2]\,(M_{\mathrm{S}}^2/M^2)\langle \mathrm{H}\rangle . \qquad (9)$$

The expectation value $F_{\mathrm{R}} = M_{\mathrm{S}}^2$ can be read off from eq. (7). It is of order 10^{10} GeV, since $\mu \sim 100$ GeV and $\sqrt{\xi} \sim 10^{18}$–10^{19} GeV.

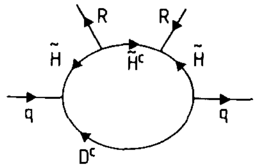

Fig. 2. Possible one-loop boson mass.

Squarks, sleptons and Higgs bosons will obtain radiative masses as well. Operators suitable for boson masses are of the type

$$(\mathrm{qq}^\dagger\mathrm{rr}^\dagger)_D , \quad (\mathrm{hh}^\dagger\mathrm{rr}^\dagger)_D . \qquad (10)$$

Off-diagonal masses are also possible from

$$(\mathrm{qq}^\dagger\mathrm{hrr}^\dagger)_D ,$$

but will be suppressed.

One would naively guess (take the supergraph of fig. 2 for instance) that squarks and sleptons get phenomenologically disastrous contributions of order M_{S}^4/M^2 from one-loop graphs. However, a more careful analysis reveals that no one-loop graph involving the $\tilde{\mathrm{H}}$ fields gives any contribution to boson masses and only higher loop diagrams lead to contributions. For instance, the graph of fig. 3 gives

$$(m_{\tilde{\mathrm{q}}}^2)_{\mathrm{rad}} \sim [(a')^2/(\pi^2)^2]\,\lambda^4\,(M_{\mathrm{S}}^2)^2/M^2 . \qquad (11)$$

Higgs boson radiative masses are even higher order since they do not communicate to $\tilde{\mathrm{H}}$ fields directly but only through the quark-lepton superfields.

Let us next move to fix the mass of the mediators $\tilde{\mathrm{H}}$ in the neighbourhood of M_{S}. For example $M_{\mathrm{S}}/M \sim \mathrm{O}(10)$ whille $M_{\mathrm{S}} \simeq 10^{10}$ GeV. The Yukawa couplings a', b', ... will be assumed to be all of the same order (say $a' \sim 10^{-1}$) while λ could be smaller but

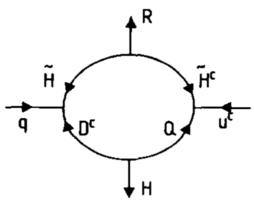

Fig. 1. One-loop fermion mass.

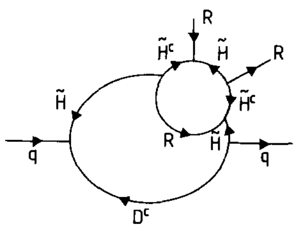

Fig. 3. Two-loop boson mass.

still natural (10^{-2}–10^{-3}). Thus, eq. (9) gives

$$m_u \sim (10^{-2}/\pi^2)\, 10^{-2}\cdot 10^2\, a\langle \mathrm{H}\rangle \sim \mathrm{O}(10\ \mathrm{MeV})\,,$$

while eq. (11) gives

$$m_{\tilde{q}}^2 \sim [10^{-2}\cdot 10^{-8}/(\pi^2)^2]\, 10^2 (10^{10})^2$$

$$\sim (10\text{–}100\ \mathrm{TeV})^2\,.$$

We should not be alarmed by these heavy squarks. Higgs bosons will get masses at the next loop-order and thus, the stability of the tree hierarchy M_W/M will not be upset.

We think that this model has served its purpose as a means of illustrating our basic ideas concerning the feasibility of a radiative mass programme in supersymmetric GUTs.

3. Grand unified examples. An SU(5) × $\tilde{\mathrm{U}}(1)$ model that incorporates the same features as the BFN model [4] can be easily constructed. Unfortunately it is plagued with a supersymmetric minimum. But we know that rigid supersymmetry is not the end of the story and we shall have to look at local supersymmetry at some point. It would be instructive to look at this model even if it is not a realistic theory. Before writing down a superpotential we should bear in mind not to allow for any baryon number violating couplings that would turn out to be dangerous. Again we shall introduce a set of five-plets that contain doublets of an intermediate mass and mediate interactions between generations. The associated colour-triplets must have masses of order M_X in order to avoid rapid proton decay. We still want to avoid unnaturally small Yukawa couplings. Let us start with the superpotential

$$f_1 = a\mathrm{Q}_{10}\mathrm{Q}_{10}\mathrm{H} + b\mathrm{Q}_{10}\mathrm{Q}_{\bar{5}}\,\mathrm{H}^c + c\Theta\Sigma\mathrm{H} + d\Theta^c\Sigma\mathrm{H}^c$$
$$+ M_X\,\mathrm{tr}(\Sigma^2) + h\,\mathrm{tr}(\Sigma^3) + \mu\mathrm{R}\mathrm{R}^c + m\mathrm{S}\mathrm{R}^c + \mathrm{S}\mathrm{H}\mathrm{H}^c$$
$$+ f\mathrm{R}^c\Theta\Theta^c\,. \qquad (12)$$

The new superfields Θ, Θ^c and Σ appearing in eq. (12) are in the **50**, $\overline{\mathbf{50}}$ and **75** representations and possess $\tilde{\mathrm{U}}(1)$ charges + 2 and 0 correspondingly. We have in the past introduced [5] these representations in order to avoid any fine-tuning in the triplet-doublet problem. It is remarkable that their presence becomes a necessity due to the $\tilde{\mathrm{U}}(1)$ symmetry. Adding to eq. (12) the extra piece

$$f_2 = a'(\mathrm{q}_{10}\mathrm{Q}_{10})\,\tilde{\mathrm{H}} + b'(\mathrm{q}_{\bar{5}}\mathrm{Q}_{10} + \mathrm{q}_{10}\mathrm{Q}_{\bar{5}})\,\tilde{\mathrm{H}}^c$$
$$+ \lambda\tilde{\mathrm{H}}\tilde{\mathrm{H}}^c\mathrm{R} + \tilde{M}(\tilde{\mathrm{H}}\bar{\tilde{\mathrm{H}}} + \tilde{\mathrm{H}}^c\bar{\tilde{\mathrm{H}}}^c) + c'(\bar{\tilde{\mathrm{H}}}\Sigma\mathrm{H} + \bar{\tilde{\mathrm{H}}}^c\Sigma\tilde{\mathrm{H}}^c) \qquad (13)$$

we obtain the grand unified analogue of eq. (4) and (5). The mass of the mediator five-plets comes from the terms

$$\bar{\tilde{\mathrm{H}}}(\tilde{M} + c'\Sigma)\,\tilde{\mathrm{H}}\,,$$

which allow for one fine-tuning. Having no other simple choice, we fine-tune parameters $\tilde{M}[\sim\mathrm{O}(M_X)]$ and $c'\langle\Sigma\rangle$ so that the triplet-mass is of $\mathrm{O}(M_X)$ while the doublets have a mass of $\mathrm{O}(10^{10}\ \mathrm{GeV})$. Finally, at low energies we recover the model of the previous section. Graphs responsible for radiative fermion and boson masses are shown in figs. 4 and 5. They are particular

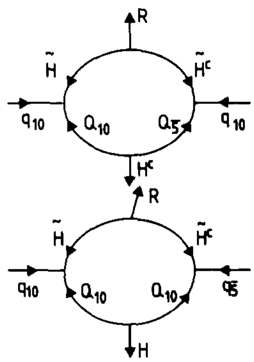

Fig. 4. One-loop fermion masses.

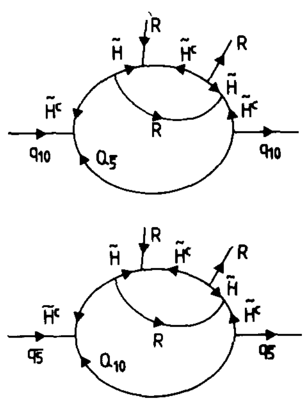

Fig. 5. Two-loop boson masses.

examples of the radiatively generated operators

$$(q_{10}q_{10}H^{c\dagger}R^{\dagger})_D\,,\quad (q_{10}q_{\bar{5}}H^{\dagger}R^{\dagger})_D$$

for fermions,

$$(q_{10}q_{10}^{\dagger}RR^{\dagger})_D\,,\quad (q_{\bar{5}}q_{\bar{5}}^{\dagger}RR^{\dagger})_D$$

for squarks and sleptons, and

$$(HH^{\dagger}RR^{\dagger})_D\,,\quad (H^{c}H^{c\dagger}RR^{\dagger})_D$$

for Higgs bosons.

Again, in this model we obtain qualitatively correct radiative corrections. Despite the fact that we have demonstrated the viability of a radiative mass programme in supersymmetric GUTs, both models are undoubtedly baroque and therefore unsatisfactory.

4. Locally supersymmetric GUTs. Let us now address ourselves to the question of a radiative mass programme in a locally supersymmetric GUT. We begin with minimal SU(5) coupled to $N = 1$ supergravity and write down the superpotential [6,7]

$$f_1 = Az + B + M_X\,\mathrm{tr}(\Sigma^2) + h\,\mathrm{tr}(\Sigma^3) + c\bar{H}\Sigma H + aQ_{10}Q_{10}H + bQ_{10}Q_{\bar{5}}\bar{H} + m\bar{H}H\,, \tag{14}$$

featuring the "capital" generation only. The superfield z is the Polonyi field coupled in the simplest possible way. The mass parameter A is of the order $M_W M_P$ rougly. An extra piece can be again introduced as

$$f_2 = a'Q_{10}q_{10}\tilde{H} + b'(Q_{10}q_{\bar{5}} + q_{10}Q_{\bar{5}})\tilde{H}^c + \tilde{M}\tilde{H}\tilde{H}^c + c'\tilde{H}^c\Sigma\tilde{H}\,. \tag{15}$$

The combined superpotential $f = f_1 + f_2$ leads to an SU(3) × SU(2) × U(1) theory with zero cosmological constant, massless [‡1] H doublets and $\tilde{H}$ doublets of an intermediate mass M to be fixed. The last two points are achieved with the technically natural fine-tunings [‡2]

$$-36c + m = 0\,,\quad -36c' + \tilde{M} = M\,. \tag{16}$$

Supersymmetry is softly broken by boson (squark, slepton and Higgs) masses of the order of the gravitino mass as well as cubic couplings of the same order. The gravitino mass is

$$m_{3/2}^2 = (A^2/M_P^2)\,e^{(\sqrt{3}-1)^2}\,,$$

and sets the scale of the supersymmetry breaking. In most models it is taken to be of the order of M_W. Of course, this is strongly related to the particular mode of SU(2) × U(1) symmetry breaking that Nature follows. The only viable known way for the SU(3) × SU(2) × U(1) breakdown to SU(3) × $U(1)_{EM}$ is through radiative corrections [7,8], and this scenario allows different possibilities; in one of them [8], the gravitino mass, and therefore the supersymmetry breaking are decoupled from M_W, the latter arising dynamically à la Coleman–Weinberg. For the time being, it would be better not to commit ourselves to any specifics of the radiative symmetry breaking, and concentrate on the question of mass generation for the "small" quarks and leptons.

The scale M should not be too low for two reasons. First, we should, of course, avoid any conflict with the phenomenological status of transgenerational interactions. Secondly, we should be careful to keep the family mixing fields $\tilde{H}$ at zero expectation value. Since in the analysis of the SU(2) × U(1) radiative symmetry breaking it became clear that the value of certain Yukawa couplings (t-quark) were crucial in deciding that the Higgses will obtain an expectation value, it is clear that the $\tilde{H}$ fields will stay at zero expectation value as long as they do not participate in any such couplings. The soft breaking of supersymmetry resulting from eqs. (14) and (15) will contain terms

$$m_{3/2}(f + f^*)\,,$$

and in particular

$$m_{3/2}M(\tilde{H}_2\tilde{H}_2^c + \mathrm{h.c.})\,.$$

Diagrams like those of fig. 6 lead to contributions

$$(a'b'a/\pi^2)[(m_{3/2}M)/M^2]M_W\,. \tag{17}$$

We cannot take M big any more as in the case of global supersymmetry. This would result in tiny radiative masses. However, $M \sim O(m_{3/2})$ (for $m_{3/2}$ in the TeV region) is also a possibility leaving us with very slightly suppressed fermion masses.

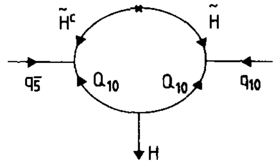

Fig. 6. One-loop fermion mass in SU(5) coupled to $N = 1$ supergravity.

‡1 That is, of $O(M_W)$.
‡2 $\langle\Sigma\rangle = \sigma\,\mathrm{diag}(2, 2, 2, -3, -3)$.

We believe that our analysis has convinced the reader of the viability of a radiative mass mechanism for the fermions in supersymmetric GUTs. Of course, we have left our many important questions such as flavour mixing, non-minimal supergravity effects [7,9] (non-renormalizable terms, ... etc) which deserve a thorough study.

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