RENORMALIZATION EFFECTS ON FLAVOUR MIXING IN SUPERGRAVITY THEORIES

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In this letter the renormalization effects on the charged slepton mass matrix are discussed in the context of supergravity models. It is found that such effects enhance by many orders of magnitude the branching ratios, previously obtained at the tree level, for the flavour violating processes ($\mu\to e\gamma$, $\mu\to \bar{e}ee$, etc.). They are still, however, two or three orders of magnitude smaller than the present experimental limits.

Even though the standard model is phenomenologically very successful, it is viewed not as the ultimate theory of nature but as a low energy approximation. Among its many possible extensions grand unified theories based on supergravity have recently become very promising [1]. Such theories also arise naturally as low energy approximations of the currently fashionable superstring models [2]. All such theories, however, predict a plethora of new particles beyond those of the standard model. The discovery of the possible indirect effects of such particles is indeed a challenge for these creative models [3,4].

In this paper we shall examine some new effects of minimal supersymmetric extensions of the standard model on the subject of lepton flavour non-conservation. In the traditional models such effects are very small since the amplitudes are proportional to the square of the mass of the light neutrinos or inversely proportional to the square of the mass of the heavy neutrinos. Higgs mediated processes are also not favoured [5]. In supersymmetric theories, however, one has entirely new possibilities through diagrams involving intermediate supersymmetric particles [3,4,6].

The question of flavour violating processes, $\mu\to e\gamma$, $\mu\to eee$, etc., induced by intermediate SUSY particles, has been considered previously. It was found, however, that at tree level the relevant supersymmetric partners of leptons remain almost degenerate. As a result the GIM mechanism remains effective and the flavour violating processes are suppressed. It was subsequently shown that this degeneracy can be removed if one goes beyond the tree level and includes radiative corrections to the charged slepton mass matrix [4]. Such corrections turned out to be equal to $cm_D m_D^\dagger$, where m_D is the Dirac mass matrix for the neutrinos and c a constant. It may thus become significant in case the neutrinos are Majorana particles, since then the Dirac mass need not be small.

The above conclusion, if true, is very important because one may indirectly infer from flavour violating processes whether the neutrinos are Majorana particles. This privilege has hitherto been exclusively reserved for lepton violating processes (neutrinoless double β-decay and μ^-, e^+ conversion). Crucial, of course, to the above questions is the numerical value of the parameter c and its stability against variations of the model parameters. We shall attempt to make a careful estimate of the parameter c including the renormalization effects. Furthermore we will employ a more realistic form of the Dirac mass matrix which takes into account constraints resulting from the up-quark mass spectrum.

In the supersymmetric extension of the standard model, the Yukawa couplings can be written

$$V = Q\bar{H}U^c + LHE^c + QHD^c , \qquad (1)$$

where

$$Q=(u_L,\tilde{u};d_L,\tilde{d})\ ,\quad U^c=(u_L^c,\tilde{u}^c)\ ,$$
$$L=(e_L,\tilde{e};-\nu_L,\tilde{\nu})\ ,\quad D^c=(d_L^c,\tilde{d}^c)\ ,\quad E^c=(e_L^c,\tilde{e}^c)\ ,$$
$$H=(\tilde{H}_L,H)\ ,\quad \bar{H}=(\tilde{\bar{H}}_L,\bar{H})\ . \tag{2}$$

Including now the right-handed neutrino N^c in the theory, we are able to introduce the additional terms $LN^c\bar{H}$ and MN^cN^c in the superpotential. Thus the Yukawa lagrangians and the non-gauge part of the scalar potential can be derived from the superpotential

$$W=\lambda' LE^cH+\lambda LN^c\bar{H}+\tfrac{1}{2}MN^cN^c+\ldots\ , \tag{3}$$

where $N=(\tilde{N}_L^c, N^c)$. From the above, we can easily derive, after supersymmetry breaking, the charged and the neutral boson mass matrices. For example the charged boson mass matrix squared, in the $\tilde{e}$, $\tilde{e}^c$, $\tilde{e}^*$, $\tilde{e}^{c*}$ basis, is the 6×6 mass matrix

$$\begin{pmatrix} m^\dagger m+m_{3/2}^2 & A^*m^\dagger m_{3/2} \\ Amm_{3/2} & m^\dagger m+m_{3/2}^2 \end{pmatrix}, \tag{4}$$

where m is the usual charged lepton mass matrix and $m_{3/2}$ is the gravitino mass. This means that the charged lepton and slepton mixing mass matrices are essentially the same.

Thus the amplitude for flavour violating processes becomes diagonal in a fashion analogous to neutral currents [6]. The neutral slepton mass matrix is a 12×12 matrix. The amplitude for flavour violating processes, like $\mu\to e\gamma$, etc., now contains different mixing matrices at each vertex as in the case of charged currents. The mediating sleptons become quite massive, but they still remain almost degenerate. Thus the GIM mechanism remains very effective and the resulting amplitudes become very small.

The question which now arises is how the above picture is modified if one takes into account renormalization corrections due to the existence of the new trilinear term $\lambda LN^c\bar{H}$. In fact one should mainly worry about radiative corrections in the charged slepton mass matrix [4]. In general, as we already mentioned, one expects corrections of the type

$$\Delta m_L^2=cm_D^\dagger m_D \tag{5}$$

where c is a parameter to be specified by the renormalization group equations (RGE's).

The RGE's for the parameters of our superpotential, eq. (3), are given by the equations [7,8]

$$d\lambda/dt=(6\lambda^3-3\lambda g_2^2-\tfrac{3}{5}\lambda g_1^2)/16\pi^2\ , \tag{6}$$
$$d\lambda'/dt=(6\lambda^3-3\lambda' g_2^2-\tfrac{9}{5}\lambda' g_1^2)/16\pi^2\ , \tag{7}$$
$$dA_\lambda/dt=3(2\lambda^2A_\lambda+g_2^2M_2+\tfrac{1}{5}g_1^2M_1)/8\pi^2\ , \tag{8}$$
$$dA_{\lambda'}/dt=3(2\lambda'^2A_{\lambda'}+g_2^2M_2+\tfrac{3}{5}g_1^2M_1)/8\pi^2\ , \tag{9}$$
$$dm_L^2/dt=[(m_L^2+m_D^2+m_{\bar{H}}^2+|A_\lambda|^2)\lambda^2 + (m_L^2+m_H^2+m_{E^c}^2+|A_{\lambda'}|^2)\lambda'^2 - 8\cdot\tfrac{3}{4}g_2^2M_2^2-8\cdot\tfrac{3}{20}g_1^2M_1^2]/16\pi^2\ , \tag{10}$$
$$dm_D^2/dt=(m_D^2+m_{\bar{H}}^2+m_L^2+|A_\lambda|^2)\lambda^2/16\pi^2\ , \tag{11}$$
$$dm_{E^c}^2/dt=[(m_L^2+m_H^2+m_{E^c}^2+|A_{\lambda'}|^2)\lambda'^2 - 8\cdot\tfrac{3}{5}g_1^2M_1^2]/16\pi^2\ , \tag{12}$$

where $t=\ln E$, g_1 and g_2 are the gauge couplings while M_1 and M_2 are the gaugino masses. Finally A_λ and $A_{\lambda'}$ are the factors relating the coefficient η of the trilinear coupling in the soft breaking potential with the coefficient λ of the trilinear term in the superpotential: $\eta=A_\lambda\lambda$ [8].

We would like to estimate from the above equations the generation mixing which is induced in the slepton mass matrix through the term $\lambda LN^c\bar{H}$. We firstly observe that we can estimate the Yukawa couplings λ, λ' by solving the system of the eqs. (6)–(9). Thus, assuming natural values of λ and λ', A_λ and $A_{\lambda'}$ at the grand unification scale $M_X\approx O(10^{15}\text{ GeV})$

$$\lambda^2/4\pi=\lambda'^2/4\pi=10^{-2}\ ,\quad \text{and}\quad A_\lambda=A_{\lambda'}=O(m_{3/2}) \tag{13}$$

[the gravitino mass $m_{3/2}\approx O(100\text{ GeV})$], we get the following values at the low energy, M_W, scale:

$$\lambda^2(M_W)/4\pi\approx1.14\times10^{-2}\quad \text{and}\quad \lambda'^2(M_W)/4\pi\approx1.20\times10^{-2}\ . \tag{14}$$

We finally investigate the renormalization effects on the slepton mass matrices through the eqs. (10)–(12). To first approximation the mixing in m_L^2 is estimated proportional to m_D^2 through the relation

$$\Delta m_L^2\approx(\lambda^2/16\pi^2)\,m_D^2\Delta t\ . \tag{15}$$

Thus the coefficient c in eq. (5) is given by

$$c=(\lambda^2/16\pi^2)\,\Delta t=(\lambda^2/16\pi^2)\,\ln(M_X/M_W) \qquad (16)$$

and using eq. (14) we obtain

$$c=2.4\times10^{-2} \qquad (16a)$$

which is two orders of magnitude smaller than the estimated value in ref. [4] (we have solved numerically the coupled differential equations (8)–(12), ignoring the m_H and $m_{\bar{H}}$ terms, and checked that the naive solution of eq. (15) is actually valid for a high enough value of m_D at the M_X scale).

Whether this can lead to a sizeable contribution to the flavour violating processes depends on the matrix m_D. Obviously if the neutrinos are Dirac particles the matrix m_D is constrained to have small elements and the above contribution is negligible. On the other hand if the neutrinos are Majorana particles the present experimental limits on the neutrino masses are consistent with the matrix m_D being analogous to that of the up-quark mass matrix. In this case the above contribution can be sizeable. Let us in fact incorporate our knowledge on the mass matrix m_D in the context of grand unified theories. It is well known that the light Majorana neutrino mass matrix $m_\nu^{\rm eff}$, which is constrained from ββ-decay and other similar processes [3], is obtained in the most general case from the following approximate form:

$$m_\nu^{\rm eff}=m_\nu-m_D m_N^{-1} m_D\ , \qquad (17)$$

where m_N is the Majorana mass of the right-handed neutrino. Thus even if $m_\nu\equiv0$, by assigning appropriate values to m_D and m_N, we can get light Majorana neutrino masses of the order of 10 eV. Therefore the light neutrino spectrum cannot impose any restriction on m_D. Even if $m_\nu\equiv0$, one needs information about the matrix m_N. Such a restriction, however, can arise from the up-quark mass spectrum.

Let us take a specific model [9,10] based in SO(10) grand unified theory. In this model the matrix m_D depends on the same parameters as those of the matrix m_α of the up-quarks [10]

$$m_\alpha=\begin{pmatrix}0 & P & 0\\ P & 0 & Q\\ 0 & Q & V\end{pmatrix}, \qquad (18)$$

$$m_D=\begin{pmatrix}0 & P & 0\\ P & 0 & -3Q\\ 0 & -3Q & V\end{pmatrix}. \qquad (18\text{ cont'd})$$

Thus in order to get sensible results from the quark masses, one is also obliged to constrain the entries in the neutrino Dirac mass matrix. By assuming a 45 GeV value for the t quark, the values if P, Q and V in the above matrices are calculated to be 0.125, 8.130 and 43.510 (taking $m_u=10$ MeV and $m_c=1.4$ GeV). By further assuming that the heavy Majorana mass is of the same type

$$M_N=\begin{pmatrix}0 & A & 0\\ A & 0 & B\\ 0 & B & C\end{pmatrix}, \qquad (19)$$

we immediately observe that it is indeed possible to get light Majorana masses of the order of 10 eV, if we push the parameters A, B and C in the range of 10^8 to 10^{14} GeV [11,12].

From the above discussion it is clear that the slepton mass matrix, ignoring $\tilde{e}$, $\tilde{e}^*$ mixing, is

$$mm^{\rm T}=m_{3/2}^2\mathbb{1}+m_e m_e^{\rm T}+cm_D m_D^{\rm T}$$
$$\approx m_{3/2}^2\mathbb{1}+cm_D m_D^{\rm T}\ . \qquad (20)$$

Thus the mixing matrix for the sleptons takes the approximate form

$$S_{\tilde{e}}=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\vartheta & \sin\vartheta\\ 0 & -\sin\vartheta & \cos\vartheta\end{pmatrix}, \qquad (21)$$

where $\frac{1}{2}\tan\vartheta=3Q/V$ or $\cos\vartheta=0.9125$ and $\sin\vartheta=0.4089$, and with eigenvalues

$$m_{\tilde{L}}^2=m_{3/2}^2+\mu_L^2c^2\ , \qquad (22)$$

where $\mu_1^2\approx0$, $\mu_2^2=120$ GeV^2 and $\mu_3^2=2968$ GeV^2.

On the other hand the lepton mass matrix is

$$mm^{\rm T}=\begin{pmatrix}R^2 & -3RS & 0\\ -3RS & R^2+9S^2 & 0\\ 0 & 0 & T^2\end{pmatrix}, \qquad (23)$$

where

$$T=m_\tau=1.784\ {\rm GeV}\ ,$$

$$S=\tfrac{1}{3}(m_\mu-m_e)\approx0.035\ {\rm GeV}\ ,$$

$$R\approx\sqrt{m_e m_\mu}=7.32\times10^{-3}\ {\rm GeV}\ .$$

We note that in this case there are no corrections à la Dirac. The lepton mixing mass matrix takes the form

$$S_e = \begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\sin\beta \approx \tan\beta = \sqrt{m_e/m_\mu} .$$

We thus get

$$S_e^\dagger S_{\tilde e} = \begin{pmatrix} \cos\beta & -\sin\beta\cos\vartheta & -\sin\beta\sin\vartheta \\ \sin\beta & \cos\beta\cos\vartheta & \cos\beta\sin\vartheta \\ 0 & -\sin\vartheta & \cos\vartheta \end{pmatrix}, \tag{24}$$

$$S_{\tilde e}^\dagger S_e = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \cos\vartheta\sin\beta & \cos\vartheta\cos\beta & -\sin\vartheta \\ \sin\vartheta\sin\beta & \sin\vartheta\sin\beta & \cos\vartheta \end{pmatrix}. \tag{25}$$

The amplitude for flavour violating processes takes now the form

$$A = (S_e^\dagger S_{\tilde e})_{2j} A(j) (S_{\tilde e}^\dagger S_e)_{j1} , \tag{26}$$

where $A(j)$ is the corresponding amplitude associated with the slepton mass eigenstate j. To leading order in $\mu_j^2/m_{3/2}^2$ we get for the branching ratio of $\mu \to e\gamma$

$$R = R_0 c^2 (\mu_1^2/m_{3/2}^2 + \cos^2\vartheta\, \mu_2^2/m_{3/2}^2 + \sin\vartheta\, \mu_3^2/m_{3/2}^2)^2 \sin^2\beta \cos^2\beta ,$$

where

$$R_0 = \alpha^3 12\pi [f(x)]^2 / G_F m_{3/2}^2 , \quad x = m_{\tilde\gamma}^2/m_{3/2}^2 ,$$

and $f(x) = \frac{1}{12}[17x^3 - 9x^2 - 9x + 1 - 6x^2(x+3)\ln x] / (1-x)^5$ [4].

Using the values $m_{3/2} \approx 150$ GeV, $Q = 8$ GeV, $f(x) \approx \frac{1}{20}$ and $\sin^2\beta \approx m_e/m_\mu$ we get (fig. 1)

$$B(\mu \to e\gamma) \approx 2.5 \times 10^{-13} c^2 .$$

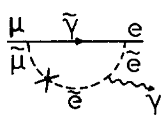

Fig. 1. Contribution to $\mu \to e\gamma$ decay through the exchange of a photino.

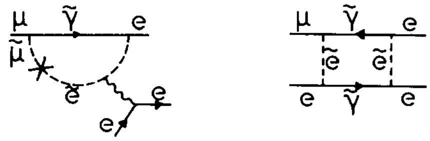

Fig. 2. Dominant diagrams in $\mu \to eee$ decay.

Thus for the above value of $c \approx 2.4 \times 10^{-2}$ we finally get

$$B(\mu \to e\gamma) \approx 1.44 \times 10^{-16} .$$

Similarly, for $\mu \to \bar{e}ee$, taking into account the contribution both from the photonic and the box diagrams, we find (fig. 2)

$$B(\mu \to \bar{e}ee) \approx 2.3 \times 10^{-18} .$$

Thus we observe that the above rates are impressively improved although still remaining far from being detectable. Furthermore, investigation of other rare processes [4,13] is not expected to affect the above estimates.

In conclusion we have examined the effects of the one loop renormalization group equations on the flavour violating processes. Using reasonable simplified assumptions, we have applied them in a realistic grand unified model [9,10] and we have seen that the branching ratios of the rate processes $\mu \to e\gamma$ and $\mu \to \bar{e}ee$ are within the present experimental bounds [14,15]. Although our results depend on the specific model we have chosen, we stress that the above features will persist in all models in which the neutrinos are Majorana particles, and the Dirac neutrino mass matrix is related to that of the up-quark.

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