Renormalization group analysis and the top quark mass in the SU(4)×O(4) string model

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We perform a two-loop renormalization group analysis for the gauge couplings in the SU(4)×O(4) model. We use the string theory prediction for the unification scale and the experimentally acceptable low energy values for α_3 and $\sin^2\theta_w$, to determine the magnitudes of the various symmetry breaking scales as well as the value of the common gauge coupling at the unification scale. We solve the coupled differential system for the gauge and top and bottom Yukawa couplings, and determine the top mass as a function of two parameters which could be chosen to be the ratio of the Higgs VEV's that give masses to the up and down quarks and the value of the top coupling at the unification scale. We find a relatively heavy top quark mass which lies in the range $130 \leqslant m_t \leqslant 180$ GeV.

Effective low energy models from superstring theories have received much attention the last few years. Various models have been derived from Calabi–Yau compactifications [1], orbifolds [2] and four-dimensional fermionic superstrings [3], but still a lot of work has to be done in order to see which of them – if any – are phenomenologically viable. One class of string derived models is usually based on the standard gauge symmetry with some additional U(1) factors, i.e. $SU(3)\times SU(2)\times U(1)^n$ gauge groups, while a second class incorporates modifications of the old grand unification scenarios. There are certain theoretical difficulties in both classes of models. In the first case, where the standard model is derived directly from the string without invoking any indermediate breaking scale, one of the main difficulties is that of obtaining the right Yukawa couplings which, on the one hand will give the observed fermion mass hierarchy while on the other, will forbid proton decay and flavour changing neutral currents. Since many of these desired properties appear naturally in grand unified theories, an appealing idea would be to proceed through the second class of models mentioned above. However the main obstacle in this case stems from the fact that the old GUT's (like the SU(5) model) need Higgs particles in the adjoint representation in order to break down to the standard model gauge group, while only "small" representations are available in string theories realised at level $k=1$ of the Kac–Moody algebra. Nevertheless, in the case of SU(5) for instance, it has been found that if one extends the gauge group including an additional U(1) factor [4], whose gauge coupling is equal to the SU(5) coupling at the unification scale, then one can break the symmetry without using the adjoint representation [3]. It was found subsequently [5], that the old Pati–Salam $SU(4)\times SU(2)_L\times SU(2)_R$ symmetry [6] without inclusion of any additional U(1) factors, can also break to the standard model with Higgs representations smaller than the adjoint. The model was found to possess may other advantages such as superheavy color triplets and a natural see-saw mechanism for the neutrinos. Moreover it was pointed out that since the symmetry is isomorphic to the product of orthogonal

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groups, namely $SO(6)\times SO(4)$, it could be constructed within the fermionic formulation, using only periodic and antiperiodic boundary conditions for the world sheet fermions. Indeed, recently this model has been derived [7] from the four-dimensional fermionic superstring [8].

In view of these encouraging results and the attractive features of the model, we think that it is time to study thoroughly its phenomenological properties. In this paper we are going to investigate the possible constraints imposed on grand unification and possible intermediate scales, by low energy parameters (e.g. $\sin^2\theta_w(m_W)$ and $\alpha_i(m_W)$), and string unification calculations. To this end, we solve the coupled system of the renormalization group equations (RGE) for the gauge couplings (at the two-loop level) and the Yukawa couplings for the top and the bottom quarks (at the one-loop level), predicting also the range of the top quark mass in this particular model. Obviously, in view of the future experiments at LEP, such a prediction will tell us a lot about the viability of the specific, and perhaps other similar, string constructions.

In order to make the subsequent discussion clear, it is worth reviewing at this point the basic ingredients of the model [5,7]. The quark and lepton fields are accommodated in the following representations of the $SU(4)\times SU(2)_L\times SU(2)_R$ symmetry:

$$F_L(4,2,1)=q(3,2,\tfrac{1}{3})+l(1,2,-1),\quad \bar{F}_R(\bar{4},1,2)=u^c(\bar{3},1,-\tfrac{4}{3})+d^c(\bar{3},1,\tfrac{2}{3})+N^c(1,1,0)\,, \tag{1}$$

where the quantum numbers on the right-hand side are with respect to the $SU(3)_C\times SU(2)_L\times U(1)_Y$ group. Note that $F_L+\bar{F}_R$ makes up the 16-spinorial representation of SO(10). In addition, the model employs the following types of Higgs superfields:

$$\begin{aligned}
&H(4,1,2)=\bar{u}^c_H(3,1,\tfrac{4}{3})+\bar{d}^c_H(3,1,-\tfrac{2}{3})+\bar{e}^c_H(1,1,-2)+\bar{N}^c_H(1,1,0)\,,\\
&\bar{H}(\bar{4},1,2)=u^c_H(\bar{3},1,-\tfrac{4}{3})+d^c_H(\bar{3},1,\tfrac{2}{3})+e^c_H(1,1,2)+N^c_H(1,1,0)\,,\\
&h(1,2,2)=h(1,2,-1)+\bar{h}(1,2,1),\quad D(6,1,1)=D(3,1,-\tfrac{2}{3})+\bar{D}(\bar{3},1,\tfrac{2}{3})\,,\\
&\Phi_i(1,1,0),\quad i=0,1,2,3\,.
\end{aligned} \tag{2}$$

In the string version of the model the above fields carry also non-zero charges under four surplus U(1) factors [7]. These U(1)'s do not participate in the charge operator and they do not play any role in the subsequent discussion. Furthermore, in the string case one also encounters the following kinds of fractionally charged states:

$$H'(4,1,1),\quad \bar{H}'(\bar{4},2,1),\quad h_L(1,2,1),\quad h_R(1,1,2)\,. \tag{3}$$

Then, the most general form of the superpotential includes the following gauge invariant terms:

$$\begin{aligned}
&W_1=F_L\bar{F}_R h+\bar{F}_R H\Phi_i+HHD+\bar{H}\bar{H}D+hh\Phi_i+\Phi_i\Phi_j\Phi_k+F_LF_LD+\bar{F}_R\bar{F}_RD+DD\Phi_i\,,\\
&W_2=H'H'D+\bar{H}'\bar{H}'D+h_Lh_L\Phi_i+h_Rh_R\Phi_i+hh_Lh_R\,.
\end{aligned} \tag{4}$$

It is worth noticing here, that the presence of exotic fractionally charged states is inevitable in all string models realized at the level $k=1$ of the Kac–Moody algebra [9]. Obviously they are going to play an important role in the renormalization group flow, although most of them are expected to receive superheavy masses at an early stage of the symmetry breaking. In principle, one could avoid them in models constructed on higher level Kac–Moody algebras [10]. Although the gauge couplings themselves are level dependent, such an analysis would be interesting and could be done on general grounds for any string derived model [11]. However, in this paper, we are going to concentrate on the level $k=1$ case while a more general analysis including higher level constructions is going to be presented in a future publication.

Before writing down the RGE's for our model, it is worth to describe the symmetry breaking chain from the original gauge group down to the standard one. At the unification scale M_{SU}, the observable SO(10) part of the original string symmetry, breaks down to $SO(6)\times O(4)\sim SU(4)\times SU(2)_L\times SU(2)_R$ gauge group by the GSO projection mechanism. String calculations [12] for the unification point show that, in a wide class of free fermionic, as well as in orbifold constructions, the unification scale is

$$M_{\mathrm{SU}} \sim \frac{1}{4\pi\sqrt{\alpha'}} \hat{M} \sim 1.93 g \times 10^{17}\ \mathrm{GeV} \times \hat{M} ,$$

where the string tension $(a')^{-1} = 8\pi G_{\mathrm{N}}/g$ and $\hat{M}$ is a model dependent parameter. Since a preliminary analysis shows that g lies in the range 0.73–0.75, we take

$$M_{\mathrm{SU}} \sim (1.41\text{–}1.45) \times 10^{17}\ \mathrm{GeV} \times \hat{M} . \tag{5}$$

We will vary M_{SU} in the range $(0.8\text{–}1.2) \times 10^{17}$ GeV. At this point we also have $g_{2\mathrm{L}} = g_{2\mathrm{R}} = g_4 = g$. Next we assume that the symmetry $\mathrm{SO}(6) \times \mathrm{O}(4) \sim \mathrm{SU}(4) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ breaks down to $\mathrm{SU}(3) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)$ at a grand unification scale M_{G}, not far from M_{SU}. In fast SU(4) breaks to $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_{B-L}$ while $\mathrm{SU}(2)_{\mathrm{R}} \to \mathrm{U}(1)_{\mathrm{R}}$. The coupling g_{Y} is given, at this scale, in terms of $g_{2\mathrm{R}}$ and g_{B-L}

$$\frac{1}{g_{\mathrm{Y}}^2(M_{\mathrm{G}})} = \frac{3}{5} \frac{1}{g_{2\mathrm{R}}^2(M_{\mathrm{G}})} + \frac{2}{5} \frac{1}{g_{B-L}^2(M_{\mathrm{G}})} . \tag{6}$$

(At the scale m_{W} we have $g(m_{\mathrm{W}}) = \sqrt{\tfrac{5}{3}} g_{\mathrm{Y}}(m_{\mathrm{W}})$.)

After this stage of symmetry breaking one is left with the standard model Higgs and matter field content. The colour triplets arising from the decomposition of the sextets and Higgs four-plets receive superheavy masses and decouple from the spectrum. Nevertheless, to make the analysis as general as possible, we will also assume cases where some of them remain massless until some intermediate scale $1.0 \times 10^{13} < M_{\mathrm{I}} < M_{\mathrm{G}}$. The effect of this assumption is to bring M_{G} closer to M_{SU}.

Having described the basic features of the model, we are now ready to present our analysis. The RGE's for the gauge couplings at the two-loop level are given by the formula

$$\frac{\mathrm{d}\alpha_i}{\mathrm{d}t} = \frac{\alpha_i^2}{2\pi} \left(b_i + \frac{1}{4\pi} \sum b_{ij} \alpha_j \right) , \tag{7a}$$

where the β-function coefficients for $\mathrm{SU}(4) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$, can be calculated using the standard formulae [13]. Above M_{G} they are

$$b_i = -\begin{pmatrix} 6 \\ 6 \\ 12 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} n_{\mathrm{G}} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} n_{\mathrm{H}} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} n_{22} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} n_6 + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} n_4 + \begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \end{pmatrix} n_{2\mathrm{L}} + \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} n_{2\mathrm{R}} ,$$

$$b_{ij} = \begin{pmatrix} -24 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -96 \end{pmatrix} + \begin{pmatrix} 14 & 0 & 15 \\ 0 & 14 & 15 \\ 3 & 3 & 31 \end{pmatrix} n_{\mathrm{G}} + \begin{pmatrix} 14 & 0 & 15 \\ 0 & 0 & 0 \\ 3 & 0 & \frac{31}{2} \end{pmatrix} n_{\mathrm{H}} + \begin{pmatrix} 7 & 3 & 0 \\ 3 & 7 & 0 \\ 0 & 0 & 0 \end{pmatrix} n_{22}$$

$$+ \begin{pmatrix} \frac{7}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} n_{2\mathrm{R}} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_{2\mathrm{L}} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{31}{4} \end{pmatrix} n_4 + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 18 \end{pmatrix} n_6 , \tag{7b}$$

where $i, j = (2_{\mathrm{R}}, 2_{\mathrm{L}}, 4)$, while below M_{G}, for $\mathrm{SU}(3) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$, we have

$$b_i = -\begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} n_{\mathrm{G}} + \begin{pmatrix} \frac{1}{10} \\ \frac{3}{2} \\ 1 \end{pmatrix} n_{32} + \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} n_2 + \begin{pmatrix} \frac{1}{5} \\ 0 \\ \frac{1}{2} \end{pmatrix} n_3 ,$$

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} n_{\mathrm{G}} + \begin{pmatrix} \frac{1}{150} & \frac{3}{10} & \frac{8}{15} \\ \frac{1}{10} & \frac{21}{2} & 8 \\ \frac{1}{15} & 3 & \frac{34}{3} \end{pmatrix} n_{32} + \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_2 + \begin{pmatrix} \frac{4}{75} & 0 & \frac{16}{15} \\ 0 & 0 & 0 \\ \frac{2}{15} & 0 & \frac{17}{3} \end{pmatrix} n_3 , \tag{7c}$$

where now $(i,j)=(1,2,3)$ and we have assumed that supersymmetry is effective up to electroweak scale. In eq. (7b), n_6 is the number of sextets fields $(6,1,1)$, n_4 stands for the number of $(4,1,1)$ representations, n_H counts the $(4,1,2)$ Higgses which break SU(4), n_{22} is the number of the $(1,2,2)$ Higgses, $n_{2L(R)}$ are the $(1,2,1)$ and $(1,1,2)$ representations respectively while n_G is the number of generations. Correspondingly in eq. (7c), n_3 is the number of colour triplets, n_2 is the number of W–S doublets while n_{32} are the triplet-doublets.

In addition to the equations for the gauge couplings we include also the RGE's for the Yukawa couplings for the heaviest fermions namely the top and the bottom quarks. These equations, below M_G read

$$\frac{d\alpha_t}{dt}=\frac{\alpha_t}{\pi}\left(3\alpha_t+\tfrac{1}{2}\alpha_b-\tfrac{8}{3}\alpha_3-\tfrac{3}{2}\alpha_2-\tfrac{13}{30}\alpha_Y\right), \quad \frac{d\alpha_b}{dt}=\frac{\alpha_b}{\pi}\left(3\alpha_b+\tfrac{1}{2}\alpha_t-\tfrac{8}{3}\alpha_3-\tfrac{3}{2}\alpha_2-\tfrac{7}{30}\alpha_Y\right), \tag{8}$$

where $\alpha_t=\lambda_t^2/4\pi$ and $\alpha_b=\lambda_b^2/4\pi$.

One of the main advantages of string unified models is the calculability of the Yukawa couplings [14]. In fact the values of the latter are related to the values of the common gauge coupling at the string unification scale. Thus, although there is an arbitrariness arising from the degeneracy of the string vacua, once one has made a specific choice of a particular vacuum, it is possible to predict the value of the top quark mass and all the other free parameters of the standard model. In particular, in our choice the top Yukawa coupling has been calculated at the string unification scale M_{SU} to be $\lambda_t(M_{SU})=g(M_{SU})\sqrt{2}$ where $g(M_{SU})$ is the value of the gauge coupling at M_{SU}. However to make the analysis more general, we will let the possibility of mixing effects and we will include a multiplicative factor $\cos\theta$ varying from 0.01 to 1, thus $\lambda_t=g\sqrt{2}\cos\theta$. Similarly, for the bottom coupling we define $\lambda_b=g\sqrt{2}\cos\bar{\theta}$.

Let us now set up our calculation. First we run the RGE's for the gauge couplings and determine the M_G and M_I scales as well as the value of the common gauge coupling $a_{SU}=g^2(M_{SU})/4\pi$, successively for the two values of $M_{SU}=0.8\times10^{17}$ GeV and 1.0×10^{17} GeV. The content of the minimal model we are using is

$$n_H=2,\quad n_6=1,\quad n_{22}=1,\quad n_G=3,\quad n_2=2,\quad n_4=n_{2L}=n_{2R}=n_3=0\,.$$

The results are depicted in fig. 1. We have plotted the scale M_G for two values of M_{SU}, namely 0.8×10^{17} GeV and 1.0×10^{17} GeV, in the parameter space $(a_3, \sin^2\theta_w)$. Varying M_I in the range 10^{13} GeV$\leqslant M_I\leqslant M_G$ and α_{SU} in the range 0.042–0.044, for constant values of M_{SU}, we obtain the contours $M_G=$constant. The solid lines correspond to the case of $M_{SU}=0.8\times10^{17}$ GeV while the dashed ones correspond to $M_{SU}=1.0\times10^{17}$ GeV. All lines are drawn up to the point where $M_I=M_G$. The two lines (solid and dashed) on the upper right corner of our figure represent contours of constant M_{SU} ($M_{SU}=0.8\times10^{17}$ GeV and 1.0×10^{17} GeV respectively) for different values of M_G and for the case $M_I=M_G$. We easily notice that for fixed values of α_3 and $\sin^2\theta_W$, M_G

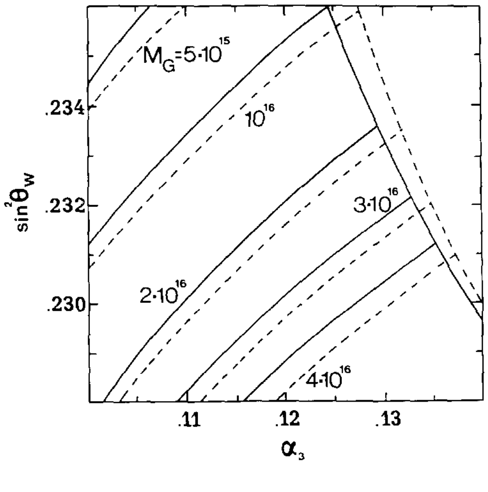

Fig. 1. Contours of M_G=constant for two values of M_{SU} in the parameter space $(\alpha_3, \sin^2\theta_W)$

increases with M_{SU}. The M_G values range from 5×10^{15} GeV up to 5×10^{16} GeV for almost all the phenomenologically acceptable values of α_3 and $\sin^2\theta_w$.

In the second part of our analysis we determine the top quark mass. Once we have found the scales M_G and M_I, as well as α_{SU}, we run the coupled differential system of RGE's for the gauge and the Yukawa couplings in eqs. (7) and (8) for given M_{SU}, $\alpha_3(M_W)$ and $\sin^2\theta_w(M_W)$ values, treating λ_t and λ_b at the unification scale as free parameters. Defining

$$k=\langle \bar{h}_0\rangle/\langle h_0\rangle\ ,$$

the ratio of the Higgs VEV's $\langle \bar{h}_0\rangle$ and $\langle h_0\rangle$ that give masses to the up and down quarks correspondingly, the top and bottom quark masses are determined by the equations

$$m_t^2=\frac{\sqrt{2}\pi\alpha_t(2m_t)}{G_F}\frac{k^2}{k^2+1}\ ,\quad m_b^2=\frac{\sqrt{2}\pi\alpha_b(2m_t)}{G_F}\frac{1}{k^2+1}\ , \tag{9a,b}$$

where $\alpha_{t,b}=\lambda_{t,b}^2/4\pi$.

Using $m_b(m_W)\sim5.2$ GeV, for any $(\lambda_t(M_{SU}),\lambda_b(M_{SU}))$ pair we can determine k and m_t uniquely for specific values of the parameters α_{SU}, M_{SU}, $\sin^2\theta_w$ and α_3. Our results are presented in figs. 2, 3. Since we have found that the results do not alter significantly for various $(a_3, \sin^2\theta_w)$ pairs, we have chosen certain values for them, namely $a_3(m_W)=0.122$ and $\sin^2\theta_w=0.234$ to present our results.

In fig. 2 we plot the contours $m_t(\chi_t,\chi_b)=$ constant in the parameter space (χ_t,χ_b) where $\chi_t=\cos\theta=\lambda_t/g\sqrt{2}$ and $\chi_b=\cos\bar{\theta}=\lambda_p/g\sqrt{2}$.

The corresponding values of the ratio $k=\langle\bar{h}_0\rangle/\langle h_0\rangle$, can be read off from the diagram of fig. 3, where we have plotted the contours k=constant in the same parameter space as for the top quark mass. The following general observations are in order. For most of the parameters space, $0.1<\chi_{t,b}<0.9$ and for the chosen values of $(\alpha_3(m_W),\sin^2\theta_w)$, the top quark mass lies between 130 GeV and 180 GeV. This range of m_t do not alter significantly for different $(\alpha_3(m_W),\sin^2\theta_w)$ pairs, provided that the latter are taken within the experimental bounds. From fig. 3, we notice also that a large ratio k is always needed in order to satisfy eq. (9b). Thus, even for small χ_t (which means that λ_t is also small) a large value of k is needed to give the correct m_b mass. Furthermore, for small χ_b's ($\chi_b<0.2$) the ratio k is not sensitive to χ_t changes. On the contrary, for small χ_t's ($\chi_t<0.2$), the top

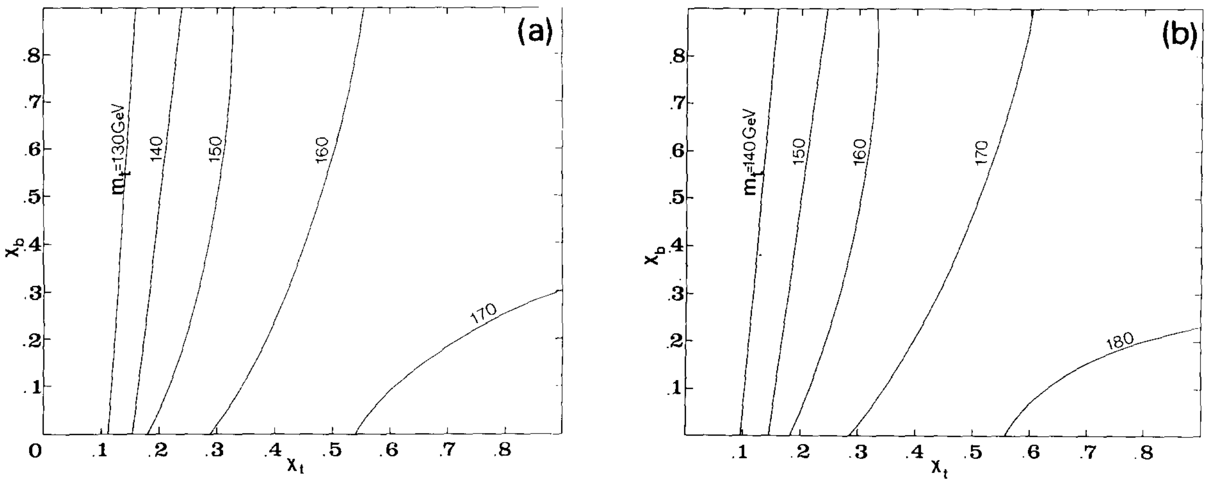

Fig. 2. Contours of constant m_t in the parameter space (χ_t,χ_b) for two values of M_{SU}, (a) $M_{SU}=0.8\times10^{17}$ GeV and (b) $M_{SU}=1.0\times10^{17}$ GeV.

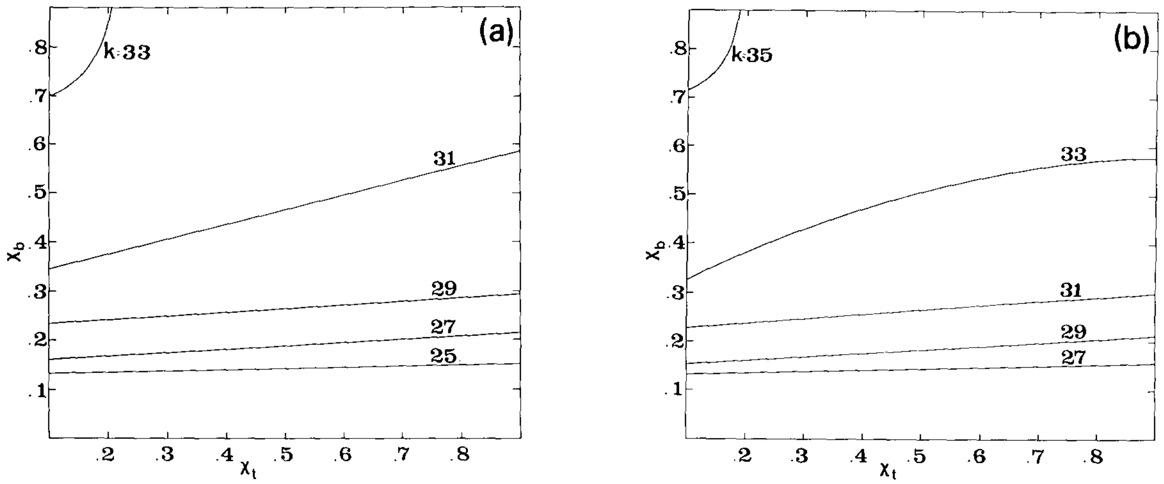

Fig. 3. Contours of constant $k=\langle \bar{h}_0 \rangle / \langle h_0 \rangle$ in the parameter space (χ_t, χ_b) for two values of M_{SU}, (a) $M_{SU}=0.8\times 10^{17}$ GeV and (b) $M_{SU}=1.0\times 10^{17}$ GeV.

mass appears to be insensitive to the variation of the parameter χ_b, while for higher χ_t-values, keeping χ_t constant m_t decreases as χ_b increases. Finally m_t and k both increase as the value of M_{SU} increases.

In conclusion we have performed the renormalization group analysis for the gauge couplings at the two-loop level and we have calculated the top quark mass as a function of the Yukawa couplings $\lambda_t(M_{SU})$ and $\lambda_b(M_{SU})$ for the SU(4)×O(4) model. Adopting the value $M_{SU}\sim 1.93g\times 10^{17}$ GeV for the superunification scale, we find that the SU(4)×O(4) symmetry breaks down to SU(3)×SU(2)×U(1) at the "grand unification" scale $M_G\sim 1.0\times 10^{16}$ GeV. The top quark mass is found to lie in the range 130–180 GeV, for natural values of the λ_b and λ_t Yukawa couplings at the unification scale.

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