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Seesaw mechanism in the sneutrino sector and its consequences

Athanasios Dedes[a,d], Howard E. Haber[b] and Janusz Rosiek[a,c]

[a]*Institute for Particle Physics Phenomenology, University of Durham, DH1 3LE, UK*

[b]*Santa Cruz Institute for Particle Physics, University of California, Santa Cruz CA 95064*

[c]*Institute of Theoretical Physics, University of Warsaw, Hoża 69, 00-681 Warsaw, Poland*

[d]*Division of Theoretical Physics, University of Ioannina, Ioannina, GR 45110, Greece*

ABSTRACT

The seesaw-extended MSSM provides a framework in which the observed light neutrino masses and mixing angles can be generated in the context of a natural theory for the TeV-scale. Sneutrino-mixing phenomena provide valuable tools for connecting the physics of neutrinos and supersymmetry. We examine the theoretical structure of the seesaw-extended MSSM, retaining the full complexity of three generations of neutrinos and sneutrinos. In this general framework, new flavor-changing and CP-violating sneutrino processes are allowed, and are parameterized in terms of two 3×3 matrices that respectively preserve and violate lepton number. The elements of these matrices can be bounded by analyzing the rate for rare flavor-changing decays of charged leptons and the one-loop contribution to neutrino masses. In the former case, new contributions arise in the seesaw extended model which are not present in the ordinary MSSM. In the latter case, sneutrino–antisneutrino mixing generates the leading correction at one-loop to neutrino masses, and could provide the origin of the observed texture of the light neutrino mass matrix. Finally, we derive general formulae for sneutrino–antisneutrino oscillations and sneutrino flavor-oscillations. Unfortunately, neither oscillation phenomena is likely to be observable at future colliders.

1 Introduction

The Standard Model of particle physics provides a remarkable description of the fundamental interactions of elementary particles at energy scales of order 100 GeV and below. Precision tests at LEP, the Tevatron and other lower energy colliders have detected no significant deviations from the predictions of observed electroweak phenomena [1]. Although the scalar sector responsible for electroweak symmetry breaking has not yet been discovered, the precision electroweak data is consistent with the Standard Model including a scalar Higgs boson of mass 114 GeV $< m_h <$ 182 GeV at 95% CL. Despite its successes, the Standard Model is widely acknowledged to be only a low-energy effective theory, to be superseded (most likely at the TeV energy scale) by a more fundamental theory that can explain the puzzling large hierarchy between the energy scale that governs electroweak symmetry-breaking and the Planck scale [2].

Numerous proposals for a more fundamental theory that supersedes the Standard Model have been advanced over the last thirty years [3]. Low-energy supersymmetric theories (in which supersymmetry breaking effects of order the TeV scale are ultimately responsible for electroweak symmetry breaking) are perhaps the most well-studied framework for TeV-scale physics beyond the Standard Model [4–6]. The simplest supersymmetric extension consists of the particle content of the two-Higgs-doublet extension of the Standard Model and its supersymmetric partners. In addition to the supersymmetric interactions of the particle supermultiplets, one adds the most general set of soft-supersymmetry-breaking terms, which parameterizes the unknown dynamics responsible for supersymmetry breaking [7, 8]. The resulting minimal supersymmetric Standard Model (MSSM) yields a rich phenomenology of new superpartners and interactions, which if present in nature is poised for discovery at the Tevatron and/or Large Hadron Collider (LHC).

Although no significant deviations from Standard Model predictions have been observed at colliders, there is of course one definitive set of observations that are in conflict with (the minimal version of) the Standard Model—the observation of neutrino mixing and its implications for neutrino masses [9]. Since neutrinos are strictly massless in the Standard Model, the latter must be modified in order to incorporate the observed phenomena of neutrino oscillations. The simplest approach is to introduce a gauge invariant dimension-five operator [10][1]

$$\mathscr{L}_5 = -\frac{f_{IK}}{\Lambda}(\epsilon_{ij}L_i^I H_j)(\epsilon_{k\ell}L_k^K H_\ell) + \text{H.c.}\,, \tag{1.1}$$

where H_j is the complex Higgs doublet and $L_i^I \equiv (\nu_L^I\,,\, \ell_L^I)$ is the SU(2)-doublet of two-

[1]Following refs. [7] and [6], we employ a convention where $\epsilon_{12} = -1 = -\epsilon_{21}$.

component lepton fields,[2] where I and K label the three generations.

After electroweak symmetry breaking, the neutral component of the doublet Higgs field acquires a vacuum expectation value, and a Majorana mass matrix for the neutrinos is generated. The dimension-five term [eq. (1.1)] is generated by new physics beyond the Standard Model at the scale Λ. Current bounds on light neutrino masses suggest that $v^2/\Lambda \lesssim 1$ eV [11, 12], or $\Lambda \gtrsim 10^{13}$ GeV. A possible realization of eq. (1.1) is based on the seesaw mechanism, which was independently discovered by a number of different authors [13, 14]. In the seesaw extension of the Standard Model [14], one simply adds SU(2)×U(1) gauge singlet neutrino fields ν_L^{cI} and writes down the most general renormalizable couplings of ν_L^{cI} to the Standard Model fields:

$$\mathscr{L}_{\rm seesaw} = -\epsilon_{ij} Y_\nu^{IJ} H_i L_j^I \nu_L^{c\,J} - \tfrac{1}{2} M^{IJ} \nu_L^{c\,I} \nu_L^{c\,J} + \text{H.c.} \tag{1.2}$$

If $\|M\| \gg v$, then at energy scales below M a dimension-five operator of the form given by eq. (1.1) is generated.

The MSSM is a minimal extension of the Standard Model. Nevertheless, there is a potential source for lepton-number violation and hence neutrino masses. Unlike the Standard Model, it is possible to construct renormalizable operators that violate lepton number and baryon number [15]. In their most generic forms, such operators would lead to extremely fast proton decay in conflict with the observations. The traditional solution is to introduce a discrete symmetry called R parity [16] that distinguishes Standard Model particles and their superpartners. In the R-parity-conserving (RPC) MSSM, neutrinos are massless just as in the Standard Model. Thus, one way to incorporate massive neutrinos in the RPC-MSSM is to formulate a minimal supersymmetric extension of the seesaw-extended Standard Model [17–21]. An alternative approach is to choose a different discrete symmetry that preserves baryon number but violates lepton number [22]. In such an R-parity-violating (RPV) MSSM, a $\mathbb{Z}_3$ baryon triality guarantees that baryon number is conserved by the renormalizable operators of the model (hence preventing fast proton decay). This approach has the advantage that no new fields beyond those of the MSSM need to be introduced. However, certain RPV (lepton-number-violating) couplings must be taken to be quite small in order to explain the scale of neutrino masses [23–25].

In this paper, we shall consider the minimal supersymmetric extension of the seesaw-extended Standard Model [17–21]. In this model, neutrino masses and mixing are governed by the same seesaw mechanism originally introduced into the (non-supersymmetric) Standard Model. In the supersymmetry-extended model, new lepton-violating phenomena enter

[2] To translate the two-component spinor product $L_i^I L_k^K$ into four-component spinor notation, see Appendix A.

due to additional effective lepton-violating operators generated by soft-supersymmetry-breaking. Such effects govern the behavior of the neutrino superpartners—the sneutrinos. Thus, the supersymmetric seesaw model provides new sources for lepton-number-violating phenomena. For example, sneutrinos and antisneutrinos can mix due to effective $\Delta L = 2$ operators [18, 26]. Although such mixing effects are expected to be quite small, there are some scenarios in which sneutrino mixing phenomena could be observed in future collider experiments [18, 27]. Sneutrino mixing also contributes a significant one-loop correction to neutrino masses and could be partially responsible for the observed pattern of neutrino masses and mixing [18,25,28]. The supersymmetric seesaw can also introduce lepton-flavor-violation and CP-violating effects due to the non-trivial flavor structure of the seesaw interactions [19, 20, 29]. Such phenomena are exhibited in the flavor oscillations of the charged sleptons [30] and the sneutrinos, respectively. Moreover, new one-loop processes contribute to $\ell^I \to \ell^J \gamma$ and electric dipole moments, and provide interesting constraints on the model parameters.

In Section 2, we introduce the Lagrangian for the three-generation supersymmetric seesaw model, focusing on the interaction of the lepton and Higgs superfields. Our notation for fermion fields are described in Appendix A. In Section 3, we derive the mass matrices for neutrinos and squared-mass matrices for the sneutrinos. In the limit of $M \gg v$, one can use perturbation theory to obtain accurate analytical expressions for the diagonalization of the effective mass and squared-mass matrices for the light and heavy neutral fermion and scalar states, respectively. The origin of a non-decoupling contribution to sneutrino masses noted in Section 3 is provided in Appendix B. In Section 4, we examine the constraints on the lepton-number conserving parameters of the model due to the observed $g - 2$ of the muon, the (unobserved) electric dipole moment of the electron, and the unobserved radiative decays of charged leptons. In Section 5, constraints on the lepton-number violating parameters of the model are obtained based on observed neutrino mass and mixing data. The general theory and phenomenology of sneutrino oscillations and mixing are addressed in Section 6. Our conclusions are given in Section 7. Although the neutrino are most easily treated as two-component spinor fields, it is convenient to present the Feynman rules of the model using four-component spinor notation. In Appendix A, we demonstrate how to translate between two-component and four-component spinor notation in the interaction Lagrangian. The relevant Feynman rules needed for the computations of this paper are listed in Appendix C. Finally, some order of magnitude estimates for the contributions to one-loop neutrino masses (relevant for the discussion of Section 5.1) are provided in Appendix D.

2 Lagrangian and the scalar potential

In this section, we examine the terms of the Lagrangian that contribute to the masses and the non-gauge interactions of the neutrinos and sneutrinos. That is, we focus on terms that involve the charged leptons, neutrinos, charged sleptons, sneutrinos and the Higgs fields. The relevant superfields (denoted with hats above the corresponding field symbol) are specified in Table 1.

Table 1:

Superfield	hypercharge	Boson Fields	Fermionic Partners
$\widehat{L}^I$	-1	$\widetilde{L}^I_j \equiv (\widetilde{\nu}^I_L\,,\,\widetilde{\ell}^I_L)$	$(\nu^I_L\,,\,\ell^I_L)$
$\widehat{R}^I$	$+2$	$\widetilde{R}^I \equiv (\widetilde{\ell}^I_R)^*$	ℓ^{cI}_L
$\widehat{N}^I$	0	$\widetilde{N}^I \equiv (\widetilde{\nu}^I_R)^*$	ν^{cI}_L
$\widehat{H}^1$	-1	$H^1_j \equiv (H^1_1\,,\,H^1_2)$	$(\widetilde{H}^1_1\,,\,\widetilde{H}^1_2)$
$\widehat{H}^2$	$+1$	$H^2_j \equiv (H^2_1\,,\,H^2_2)$	$(\widetilde{H}^2_1\,,\,\widetilde{H}^2_2)$

The electric charge (in units of e) is given by $Q = T_3 + Y/2$, where Y is the hypercharge specified above. The index j labels components of the SU(2) doublets with $T_3 = \pm 1/2$ for $j = 1, 2$ respectively (and $T_3 = 0$ for the SU(2) singlets). The fermionic partners can be viewed either as two-component fermion fields or the left-handed projections of four-component fermion fields, as explained in Appendix A. The index $I = 1, 2, 3$ labels three possible generations of charged lepton and neutrino superfields. The notation for the scalar field components of the hypercharge-zero superfield is motivated by the fact that in the lepton-number-conserving limit, $\widehat{R}$ and $\widehat{N}$ possess the same lepton number (which is opposite in sign to that of $\widehat{L}$). Consequently, $\widetilde{\nu}_L$ and $\widetilde{\nu}_R$ possess identical lepton numbers [cf. eq. (6.3)].

The most general (renormalizable) form of the superpotential involving the lepton and Higgs superfields in the R-parity-conserving extended MSSM is given by:

$$W = \epsilon_{ij}(\mu \widehat{H}^1_i \widehat{H}^2_j - Y^{IJ}_\ell \widehat{H}^1_i \widehat{L}^I_j \widehat{R}^J + Y^{IJ}_\nu \widehat{H}^2_i \widehat{L}^I_j \widehat{N}^J) + \tfrac{1}{2} M^{IJ} \widehat{N}^I \widehat{N}^J\,, \tag{2.1}$$

where Y_ℓ and Y_ν are complex 3×3 matrices, M is a complex symmetric 3×3 matrix and

μ is a complex parameter.[3] In addition, there are soft-supersymmetry-breaking terms that involve the scalar field components of the above superfields. Before writing these terms explicitly, it is convenient to perform field redefinitions of the (charged and neutral) lepton superfields:

$$\widehat{L}^I \to V_L^{IJ}\widehat{L}^J\,, \qquad \widehat{R}^I \to V_R^{IJ}\widehat{R}^J\,, \qquad \widehat{N}^I \to V_N^{IJ}\widehat{N}^J\,, \tag{2.2}$$

where V_L, V_R and V_N are 3×3 unitary matrices. Note that the kinetic energy terms (and the couplings of the lepton superfields to the gauge fields) are invariant under the above unitary transformations. However, the coefficients of the terms of the superpotential are modified:

$$Y_\ell \to V_L^T Y_\ell V_R\,, \qquad Y_\nu \to V_L^T Y_\nu V_N\,, \qquad M \to V_N^T M V_N\,. \tag{2.3}$$

We shall choose V_L, V_R and V_N such that:

$$V_L^T Y_\ell V_R \;=\; \mathrm{diag}(Y_e\,,\,Y_\mu\,,\,Y_\tau)\,, \tag{2.4}$$

$$V_N^T M V_N \;=\; \mathrm{diag}(M_1\,,\,M_2\,,\,M_3)\,, \tag{2.5}$$

where the elements of the two diagonal matrices above are real and non-negative. It is always possible to find unitary matrices V_L and V_R such that eq. (2.4) is satisfied—this is the singular value decomposition of an arbitrary complex matrix [31]. Likewise, it is always possible to find a unitary matrix V_N such that eq. (2.5) holds—this is the Takagi-diagonalization of an arbitrary complex symmetric matrix [31–33]. Thus, the redefinition of the lepton superfields [eq. (2.2)] implies that one can assume from the beginning without loss of generality that Y_ℓ and M are real non-negative diagonal matrices.[4] Note that the (transformed) Y_ν is in general an arbitrary complex 3×3 matrix.

We next introduce the most general set of R-parity-conserving soft-supersymmetry (SUSY)-breaking terms (following the usual rules of [34]) involving the slepton, sneutrino and Higgs fields:

$$\begin{aligned} V_{\rm SOFT} \;=\;& m^2_{H_1} H_i^{1*}H_i^1 + m^2_{H_2} H_i^{2*}H_i^2 + (m^2_L)^{IJ}\widetilde{L}_i^{I*}\widetilde{L}_i^J + (m^2_R)^{IJ}\widetilde{R}^{I*}\widetilde{R}^J + (m^2_N)^{IJ}\widetilde{N}^{I*}\widetilde{N}^J \\ &- \Big[(m^2_B)^{IJ}\widetilde{N}^I\widetilde{N}^J + \epsilon_{ij}\Big(m^2_{12}H_i^1H_j^2 + A_\ell^{IJ}H_i^1\widetilde{L}_j^I\widetilde{R}^J + A_\nu^{IJ}H_i^2\widetilde{L}_j^I\widetilde{N}^J\Big) + \text{H.c.}\Big]\,, \end{aligned} \tag{2.6}$$

where m^2_L, m^2_R and m^2_N are hermitian matrices, m^2_B is a complex symmetric matrix and A_ℓ and A_ν are complex matrices. In general, these 3×3 matrices do not take a simplified

[3] With the convention for ϵ_{ij} as specified in footnote 1, it is convenient to insert an extra minus sign in front of Y_ℓ in eq. (2.1). This ensures that in a basis where Y_ℓ is a real positive diagonal matrix, the charged lepton masses are also positive. Note that this convention differs from the one adopted in ref. [7].

[4] After electroweak symmetry breaking, eq. (2.4) corresponds to working in a basis in which the charged lepton mass matrices are (real) non-negative and diagonal.

form in the basis defined by eqs. (2.4) and (2.5). The total scalar potential is made up of three contributions: the F-terms, which are derived from eq. (2.1), the D-terms, which arise from the gauge interactions, and and the soft SUSY-breaking terms, which have been specified in eq. (2.6). The total scalar potential is then given by:

$$V = V_F + V_D + V_{\rm SOFT}\,, \qquad \text{where} \qquad V_F \equiv \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 \tag{2.7}$$

and the sum over i is taken over all scalar components of the corresponding superfields.

The Yukawa couplings of the leptons and the Higgs fields and the corresponding fermion mass terms are derived from eq. (2.1) using the well-known formula [6, 7]:

$$-\mathscr{L}_{\rm mass} - \mathscr{L}_{\rm Yuk} = \tfrac{1}{2}\sum_{ij}\left[\frac{\partial^2 W[\phi]}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + \text{H.c.}\right]\,, \tag{2.8}$$

where the ψ_i are the two-component fermion field superpartners of the corresponding ϕ_i, and $W[\phi]$ is the superpotential function with superfields replaced by their scalar components. After electroweak symmetry breaking, the neutral Higgs fields acquire vacuum expectation values,[5]

$$\langle H_1^1\rangle = \frac{v_1}{\sqrt{2}}\,, \qquad\qquad \langle H_2^2\rangle = \frac{v_2}{\sqrt{2}}\,, \tag{2.9}$$

where $v^2 \equiv v_1^2 + v_2^2 = (246~\text{GeV})^2$ and $\tan\beta \equiv v_2/v_1$. Inserting the Higgs field vacuum expectation values into eqs. (2.7) and (2.8), one can isolate the terms of the Lagrangian that are quadratic in the scalar fields and fermion fields, respectively. These terms yield squared-mass matrices for the charged sleptons and sneutrinos and mass matrices for the charged leptons and neutrinos. In the basis defined by eq. (2.4), the charged lepton mass matrix is diagonal, with diagonal elements $m_{\ell^I} = v_1 Y_\ell^I/\sqrt{2}$.

In general, the diagonalization of these mass matrices cannot be performed analytically, and one must resort to numerical techniques. However, the large hierarchy between neutrino masses and charged lepton masses strongly suggests that the parameters $M_I \gg v$, in which case an analytic perturbative diagonalization permits one to isolate the light (s)neutrino sector and integrate out the superheavy (s)neutrino sector, whose particle masses are of order the M_I. This procedure was carried out for the CP-conserving one-generation model in ref. [18]. In Section 3, we shall generalize this analysis to the most general (potentially CP-violating) three-generation model.

First, we clarify the expected magnitudes of the parameters of the model:

[5] We define the overall phases of the neutral Higgs fields, H_1^1 and H_2^2, such that the corresponding vacuum expectation values $v_{1,2}/\sqrt{2}$ are real and positive.

1. We assume that the Yukawa couplings Y_ν^{IJ} satisfy:[6]

$$\|Y_\nu\| \lesssim \mathcal{O}(1)\,. \tag{2.10}$$

2. The Majorana mass M is much heavier than the electroweak scale (seesaw mechanism [13])

$$\|M\| \gg v\,. \tag{2.11}$$

3. Although μ is a supersymmetric parameter, we require it to be of a similar order to the low-energy supersymmetry-breaking scale, $M_{\rm SUSY}$ [35]:

$$\mu \sim M_{\rm SUSY}\,. \tag{2.12}$$

4. The non-singlet soft SUSY-breaking squared-masses are of a similar order to the supersymmetry-breaking scale:

$$\|m_L^2\| \sim \|m_R^2\| \sim M_{\rm SUSY}^2\,. \tag{2.13}$$

5. The parameters m_B^2 and A_ν are unconnected to electroweak symmetry breaking at tree-level. However, these parameters generate a mass-splitting between sneutrinos and antisneutrinos. The latter contributes via loop corrections to neutrino mass splittings, which are experimentally constrained. One expects that [36]:

$$\|A_\nu\| \lesssim M_{\rm SUSY}\,, \qquad\qquad \|m_B^2\| \lesssim M_{\rm SUSY}\|M\|\,, \tag{2.14}$$

although these parameters could conceivably be larger by as much as a factor of 10^3 [18]. Large A_ν also leads also to large corrections to charged slepton masses. Thus, to avoid unnatural fine-tuning in order to prevent charged slepton masses from being larger than about 1 TeV, one again expects that A_ν cannot be much larger than the supersymmetry-breaking scale. The impact of the one-loop effects of m_B^2 on charged lepton radiative decays and the Higgs mass parameters also yield constraints and imply that the bound on m_B^2 given by eq. (2.14) cannot be significantly relaxed.

6. The singlet soft SUSY-breaking parameter m_N^2 is also unconnected to electroweak symmetry breaking at tree-level. However, the one-loop corrections to the Higgs mass parameters depend quadratically on m_N^2, so to avoid unnatural fine-tuning of

[6]The Euclidean matrix norm is defined by $\|A\| \equiv \left[\mathrm{tr}(\mathrm{A}^\dagger \mathrm{A})\right]^{1/2} = \left[\sum_{i,j}|a_{ij}|^2\right]^{1/2}$, for a matrix A whose matrix elements are given by a_{ij}.

the electroweak symmetry breaking scale, one expects that m_N^2 cannot be much larger than $(1 \text{ TeV})^2$. This expectation is confirmed in Appendix B, in which case

$$\|m_N^2\| \lesssim M_{\rm SUSY}^2 \,. \tag{2.15}$$

If significant fine-tuning of the electroweak scale is allowed (as in the split-supersymmetry [37] approach), then the constraints on m_N^2 are significantly relaxed. The one-loop effects of m_N^2 on *physical* observables are rather mild, even as $\|m_N^2\|$ approaches $\|M^2\|$. For example, in ref. [38], the one-loop corrections to Higgs masses in the seesaw-extended MSSM are found to be large and negative if $\|m_L^2\|\,,\, \|m_N^2\| \sim \|M^2\|$. However, these corrections become negligible once these soft-SUSY-breaking masses are taken somewhat below the seesaw scale.

Thus, we shall present results in this paper that allow for the possibility that:

$$\|m_N^2\| \sim \|M^2\| \,. \tag{2.16}$$

If eq. (2.16) holds, then remnants of the heavy neutrino/sneutrino sector can survive in the effective theory of the light sneutrinos. The origin of this non-decoupling effect is explored in Appendix B.

Although naturalness demands that the scale of low-energy supersymmetry-breaking, $M_{\rm SUSY}$, should be (roughly) of $\mathcal{O}(v)$, the absence of observed supersymmetric phenomena (and a light CP-even Higgs boson) suggest that $M_{\rm SUSY}$ may be somewhat larger, of order 1 TeV. Nevertheless, in eqs. (2.12)–(2.15), one could substitute $M_{\rm SUSY}$ with v; the results of this paper are consistent with either choice.

3 The (s)neutrino (squared-)mass matrices

In this section, we examine in detail the neutrino mass matrix and the sneutrino squared-mass matrix. In a three-generation model, the neutrino mass matrix is a 6×6 complex symmetric matrix, which can be written in block (partitioned) form in terms of 3×3 matrix blocks. The sneutrino squared-mass matrix is a 12×12 hermitian matrix, which can be written in block (partitioned) form in terms of 6×6 matrix blocks. Each of these 6×6 matrices can be further partitioned in terms of 3×3 matrix blocks. In order to accommodate the proliferation of matrices of dimension 3, 6 and 12, we adopt a notational device that allows the reader to instantly discern the dimension of a given matrix. Thus, we use a boldface capital letter ($\boldsymbol{M}$) to denote a 12×12 matrix, a calligraphic letter ($\mathcal{M}$)

to denote a 6×6 matrix, and a Latin letter (M or m) to denote a 3×3 matrix. Latin letters will also be used to denote (scalar) mass parameters, with appropriate identifying subscript or superscript labels to distinguish these from the 3×3 matrices introduced in Sections 2 and 3. Following the conventions of Section 2, we shall employ subscript and superscript upper case Latin indices I, J, K as generation labels that run from 1 to 3. Lower case Latin indices i, j, k are employed for other purposes, either as SU(2) gauge indices or as labels representing the six light sneutrino mass eigenstates. Other subscripts appearing in this section will be used to distinguish among different matrix quantities.

3.1 The neutrino mass matrices

Working in a basis where M is a diagonal matrix [cf. eq. (2.5)], we begin by analyzing the neutrino mass matrix. The resulting terms quadratic in the neutrino fields are given in terms of two-component fermion fields[7] by:

$$-\mathscr{L}_{m_\nu} = \tfrac{1}{2}\left(v_2\sqrt{2}\,Y_\nu^{IJ}\nu_L^I\nu_L^{cJ} \;+\; M^{IJ}\nu_L^{cI}\nu_L^{cJ} + \text{H.c.}\right) = \tfrac{1}{2}\,(\nu_L^T \quad \nu_L^{cT})\,\mathcal{M}_\nu \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} + \text{H.c.} \tag{3.1}$$

The neutrino mass matrix $\mathcal{M}_\nu$ is a 6×6 complex symmetric matrix given in block form by:

$$\mathcal{M}_\nu \equiv \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix}, \tag{3.2}$$

where the 3×3 complex matrix

$$m_D \equiv v_2 Y_\nu/\sqrt{2} \tag{3.3}$$

generalizes the neutrino Dirac mass term of the one-generation model [cf. eq. (A.5)].

Provided that $\|M\| \gg \|m_D\|$ [as suggested by eq. (2.11)], $\mathcal{M}_\nu$ is of a seesaw type [13]. The neutrino mass matrix can be Takagi block-diagonalized [21,25,33] as follows. Introduce the 6×6 (approximate) unitary matrix:

$$\mathcal{U} = \begin{pmatrix} \mathbb{1} - \frac{1}{2}m_D^* M^{-2} m_D^T & m_D^* M^{-1} \\ -M^{-1}m_D^T & \mathbb{1} - \frac{1}{2}M^{-1}m_D^T m_D^* M^{-1} \end{pmatrix}, \tag{3.4}$$

where $\mathbb{1}$ is the 3×3 identity matrix.

[7] In Appendix A, we show how to rewrite eq. (3.1) in terms of four-component neutrino fields. However, the two-component formalism is more economical, so we adopt this notation in what follows.

One can check that:

$$\mathcal{U}^\dagger\mathcal{U} = \begin{pmatrix} \mathbb{1} + \mathcal{O}(m_D^4 M^{-4}) & 0 \\ 0 & \mathbb{1} + \mathcal{O}(m_D^4 M^{-4}) \end{pmatrix} . \tag{3.5}$$

We define transformed (light and heavy) neutrino states ν_ℓ and ν_h^c by:

$$\begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} = \mathcal{U} \begin{pmatrix} \nu_\ell \\ \nu_h^c \end{pmatrix} . \tag{3.6}$$

By straightforward matrix multiplication, one can verify that

$$\mathcal{U}^T\mathcal{M}_\nu\mathcal{U} = \begin{pmatrix} -m_D M^{-1} m_D^T + \mathcal{O}(m_D^4 M^{-3}) & \mathcal{O}(m_D^3 M^{-2}) \\ \mathcal{O}(m_D^3 M^{-2}) & M + \frac{1}{2}(M^{-1} m_D^\dagger m_D + m_D^T m_D^* M^{-1}) + \mathcal{O}(m_D^4 M^{-3}) \end{pmatrix} . \tag{3.7}$$

At this stage, we can identify an effective (complex symmetric) mass matrix M_{ν_ℓ} for the three light (left-handed) neutrinos with respect to the $\{\nu_\ell\}$-basis:

$$M_{\nu_\ell} \simeq -m_D M^{-1} m_D^T \, . \tag{3.8}$$

To identify the physical light neutrino states, we must perform a Takagi-diagonalization of M_{ν_ℓ}. This is accomplished by introducing the unitary MNS matrix [39], $U_{\rm MNS}$, via

$$\nu_\ell^I = U_{\rm MNS}^{IJ} \, (\nu_\ell^J)^{\rm phys} \, , \tag{3.9}$$

where the $(\nu_\ell^J)^{\rm phys}$ $[J = 1, 2, 3]$ denote the physical light neutrino fields. $U_{\rm MNS}$ is determined by the Takagi-diagonalization of M_{ν_ℓ}:

$$U_{\rm MNS}^T M_{\nu_\ell} U_{\rm MNS} = {\rm diag}(m_{\nu_{\ell 1}} \, , \, m_{\nu_{\ell 2}} \, , \, m_{\nu_{\ell 3}}) \, , \tag{3.10}$$

where the $m_{\nu_{\ell J}}$ are the (real non-negative) masses of the light neutrino mass eigenstates.

For completeness, we examine the effective mass matrix of the heavy neutrino states. Although M is diagonal by assumption, the lower right-handed block in eq. (3.7) is no longer diagonal due to the second-order perturbative correction. However, we do not have to perform another Takagi-diagonalization, since the off-diagonal elements are of $\mathcal{O}(m_D^2 M^{-1})$, and would only affect the physical (diagonal) masses at order $\mathcal{O}(m_D^4 M^{-3})$, which we neglect. The corresponding mixing angles would be of $\mathcal{O}(m_D^2 M^{-2})$, which we also neglect here. Thus, we identify the physical heavy neutrino mass eigenstates to leading order by:

$$(\nu_h^{c\,I})^{\rm phys} \simeq \nu_h^{cI} \, , \tag{3.11}$$

with masses

$$m_{\nu_{hI}} = M_I \left(1 + \frac{1}{M_I^2}\sum_J |m_D^{JI}|^2\right) , \tag{3.12}$$

where the M_I are the diagonal elements of M in our chosen basis.

3.2 The sneutrino squared-mass matrices

We now turn to the sneutrino sector. It is convenient to separate out various pieces that comprise the F-term contributions to the scalar potential [eq. (2.7)]:

$$V_F \equiv V_\nu + V_\mu + V_{\rm other} , \tag{3.13}$$

where $V_\nu \equiv \sum_{i=\widetilde{L}_1^I,\,\widetilde{N}^I} |\partial W/\partial\phi_i|^2$ and $V_\mu \equiv |\partial W/\partial H_2^2|^2$ ultimately contribute to the sneutrino squared-mass matrix, whereas $V_{\rm other}$ (which involves derivatives of the superpotential with respect to the other scalar fields) makes no contributions to tree-level sneutrino masses.

As a pedagogical exercise, we first analyze the supersymmetric limit. Although supersymmetry-breaking is required in the MSSM to generate electroweak symmetry breaking, one often finds supersymmetric-like relations between the fermion and sfermion sectors in the limit of $v_1 = v_2$ and $\mu = 0$, *i.e.* for $V_\mu = V_D = 0$. Thus, in the following computation the supersymmetric limit corresponds to taking the total scalar potential [eq. (2.7)] to be $V = V_\nu$. To analyze the contributions of V_ν to sneutrino masses, we can employ the following trick. Focus on the following two terms of the superpotential:

$$W_\nu \equiv Y_\nu^{IJ}\widehat{H}_2^2\widehat{L}_1^I\widehat{N}^J + \tfrac{1}{2}M^{IJ}N^IN^J = \tfrac{1}{2}\begin{pmatrix}\widehat{L}_1^T & \widehat{N}^T\end{pmatrix}\begin{pmatrix} 0 & \widehat{H}_2^2 Y_\nu \\ \widehat{H}_2^2 Y_\nu^T & M\end{pmatrix}\begin{pmatrix}\widehat{L}_1 \\ \widehat{N}\end{pmatrix} . \tag{3.14}$$

Consistent with eq. (3.6), we redefine the neutrino superfields as follows:

$$\begin{pmatrix}\widehat{L}_1 \\ \widehat{N}\end{pmatrix} = \mathcal{U}\begin{pmatrix}\widehat{L}_{1\ell} \\ \widehat{N}_h\end{pmatrix} , \tag{3.15}$$

where the unitary matrix $\mathcal{U}$ is given by eq. (3.4). Defining the matrix $H \equiv \widehat{H}_2^2 Y_\nu$, the effect of eq. (3.15) is to transform W_ν into[8]

$$W_\nu \simeq \tfrac{1}{2}(HM^{-1}H^T)^{IJ}\widehat{L}_{1\ell}^I\widehat{L}_{1\ell}^J + \tfrac{1}{2}\left[M^{IJ} + \tfrac{1}{2}(M^{-1}H^\dagger H + H^TH^*M^{-1})^{IJ}\right]\widehat{N}_h^I\widehat{N}_h^J + \mathcal{O}(H^4M^{-3}) , \tag{3.16}$$

[8]Strictly speaking, this is not a permissible transformation, since W must be holomorphic in the superfields, whereas eq. (3.16) is a function of both $\widehat{H}_2^2$ and $\widehat{H}_2^{2\,*}$. However, since we ultimately set $H_2^2 = v_2/\sqrt{2}$ and only take derivatives of W_ν with respect to $\widetilde{L}_{1\ell}$ and $\widetilde{N}_h$, the procedure outlined here yields correct results.

where there is an implicit sum over I and J. In deriving eq. (3.16), we have used the fact that M^{IJ} is a non-negative diagonal matrix. Setting $H_2^2 = v_2/\sqrt{2}$ and using eq. (2.7), we can directly make use of eq. (3.16) to isolate the contributions to the sneutrino squared-mass matrix that arise from V_ν:

$$-\mathscr{L}_{\rm mass} = \widetilde{L}^\dagger_{1\ell} M^2_{\ell^\dagger\ell} \widetilde{L}_{1\ell} + \widetilde{N}^\dagger_h M^2_{h^\dagger h} \widetilde{N}_h \,, \tag{3.17}$$

where the 3×3 hermitian matrices $M^2_{\ell^\dagger\ell}$ and $M^2_{h^\dagger h}$ are given by:

$$\begin{aligned} M^2_{\ell^\dagger\ell} &= m_D^* M^{-1} m_D^\dagger m_D M^{-1} m_D^T + \mathcal{O}(m_D^6 M^{-4})\,, & (3.18)\\ M^2_{h^\dagger h} &= M^2 + m_D^\dagger m_D + \tfrac{1}{2}(M m_D^T m_D^* M^{-1} + M^{-1} m_D^T m_D^* M) + \mathcal{O}(m_D^4 M^{-2})\,. & (3.19) \end{aligned}$$

Moreover, the effective light and heavy neutrino mass matrices, M_{ν_ℓ} and M_{ν_h}, can also be derived by inserting eq. (3.16) into eq. (2.8). As expected, the resulting neutrino mass matrices are related in a supersymmetric way to the sneutrino squared-mass matrices obtained in eqs. (3.18) and (3.19):

$$M^2_{\ell^\dagger\ell} = M^\dagger_{\nu_\ell} M_{\nu_\ell}\,, \qquad M^2_{h^\dagger h} = M^\dagger_{\nu_h} M_{\nu_h}\,. \tag{3.20}$$

In particular, in the supersymmetric limit,

$$U^T_{\rm MNS}\, M^2_{\ell^\dagger\ell}\, U^*_{\rm MNS} = {\rm diag}\;(m^2_{\nu_{\ell 1}}\,,\, m^2_{\nu_{\ell 2}}\,,\, m^2_{\nu_{\ell 3}})\,, \tag{3.21}$$

which implies that the light neutrino and sneutrino masses coincide.

We now turn to the complete calculation of the sneutrino mass matrix. Although one could perform the computation with respect to the basis of sneutrino states defined by eq. (3.15), this basis is not especially convenient. This is due to the fact that the effective squared-mass matrix of the light sneutrinos is dominated by supersymmetry-breaking effects. In particular, the supersymmetric contribution of $\mathcal{O}(m_D^4 M^{-2})$ [cf. eq. (3.18)] is completely negligible relative to the supersymmetry-breaking contributions. Thus, there is no advantage to performing in the sneutrino sector the same change of basis used to isolate the effective mass matrix of the light neutrinos. Hence we will write the 12×12 hermitian sneutrino squared-mass matrix in block form as:

$$-\mathcal{L}_{\rm mass} = \tfrac{1}{2} \begin{pmatrix} \phi_L^\dagger & \phi_N^\dagger \end{pmatrix} \begin{pmatrix} \mathcal{M}^2_{LL} & \mathcal{M}^2_{LN} \\ (\mathcal{M}^2_{LN})^\dagger & \mathcal{M}^2_{NN} \end{pmatrix} \begin{pmatrix} \phi_L \\ \phi_N \end{pmatrix}, \tag{3.22}$$

where $\phi_L \equiv (\widetilde{L}_1\,,\,\widetilde{L}_1^*)^T$ and $\phi_N \equiv (\widetilde{N}\,,\,\widetilde{N}^*)^T$ are six-dimensional vectors. The 6×6 hermitian matrices $\mathcal{M}^2_{LL}$, $\mathcal{M}^2_{NN}$ and the 6×6 complex matrix $\mathcal{M}^2_{LN}$ can be written in block partitioned form as:

$$\mathcal{M}^2_{AB} \equiv \begin{pmatrix} M^2_{A^\dagger B} & M^{2\,*}_{A^T B} \\ M^2_{A^T B} & M^{2\,*}_{A^\dagger B} \end{pmatrix} , \tag{3.23}$$

where the subscripts A and B can take on possible values L and N [this labeling allows one to keep track of the origin of the various matrix blocks]. The $M^2_{A^\dagger A}$ are 3×3 hermitian matrices and the $M^2_{A^T A}$ are 3×3 complex symmetric matrices, for $A = L\,,\, N$. There are no restrictions on the 3×3 complex matrices $M^2_{A^\dagger B}$ and $M^2_{A^T B}$ for $A \neq B$.

Adding up the contributions of V_ν, V_μ, V_D and $V_{\rm SOFT}$ to the sneutrino masses yields:

$$M^2_{L^\dagger L} = m^2_L + \tfrac{1}{2}M^2_Z\cos 2\beta + m^*_D m^T_D\,, \tag{3.24}$$
$$M^2_{N^\dagger N} = M^2 + m^2_N + m^\dagger_D m_D\,, \tag{3.25}$$
$$M^2_{L^\dagger N} = m^*_D M\,, \tag{3.26}$$
$$M^2_{L^T N} = -X_\nu m_D\,, \tag{3.27}$$
$$M^2_{N^T N} = -2m^2_B\,, \tag{3.28}$$
$$M^2_{L^T L} = 0\,, \tag{3.29}$$

where we have introduced the complex 3×3 matrix parameter X_ν by the following definition:

$$X_\nu m_D \equiv \frac{1}{\sqrt{2}}\left(v_2 A_\nu + \mu^* v_1 Y_\nu\right)\,. \tag{3.30}$$

A quick check of the supersymmetric limit confirms the expected relation between the neutrino mass matrix and the sneutrino squared-mass matrix:

$$\mathcal{M}^\dagger_\nu \mathcal{M}_\nu = \begin{pmatrix} m^*_D m^T_D & m^*_D M \\ M m^\dagger_D & M^2 + m^\dagger_D m_D \end{pmatrix}\,. \tag{3.31}$$

As noted above, because of the dominance of supersymmetry-breaking contributions to the light sneutrino masses, the diagonalization of the light neutrino mass matrix and the light sneutrino squared-mass matrix are completely independent.

Under the assumptions of eqs. (2.10)–(2.15), the 12×12 sneutrino mass matrix, written in terms of 6×6 matrix blocks with estimated magnitudes,

$$\boldsymbol{M}^2_{\tilde\nu} \equiv \begin{pmatrix} \mathcal{M}^2_{LL} & \mathcal{M}^2_{LN} \\ (\mathcal{M}^2_{LN})^\dagger & \mathcal{M}^2_{NN} \end{pmatrix} = \begin{pmatrix} \mathcal{O}(v^2) & \mathcal{O}(vM) \\ \mathcal{O}(vM) & \mathcal{O}(M^2) \end{pmatrix}\,, \tag{3.32}$$

also exhibits a seesaw type behavior, analogous to the seesaw type mass matrix [eq. (3.2)] of the neutrino sector. Following the standard procedure for diagonalizing such matrices (see ref. [25]), we introduce a 12×12 unitary matrix:

$$\boldsymbol{V} = \begin{pmatrix} \mathcal{I} - \frac{1}{2}\mathcal{M}^2_{LN}\mathcal{M}^{-4}_{NN}(\mathcal{M}^2_{LN})^\dagger & \mathcal{M}^2_{LN}\mathcal{M}^{-2}_{NN} \\ -\mathcal{M}^{-2}_{NN}(\mathcal{M}^2_{LN})^\dagger & \mathcal{I} - \frac{1}{2}\mathcal{M}^{-2}_{NN}(\mathcal{M}^2_{LN})^\dagger\mathcal{M}^2_{LN}\mathcal{M}^{-2}_{NN} \end{pmatrix} , \tag{3.33}$$

where $\mathcal{I}$ is the 6×6 identity matrix. One can easily compute:

$$\boldsymbol{V}^\dagger \boldsymbol{M}^2_{\tilde{\nu}}\, \boldsymbol{V} = \begin{pmatrix} \mathcal{M}^2_{LL} - \mathcal{M}^2_{LN}\mathcal{M}^{-2}_{NN}(\mathcal{M}^2_{LN})^\dagger + \mathcal{O}(v^4M^{-2}) & \mathcal{O}(v^3M^{-1}) \\ \mathcal{O}(v^3M^{-1}) & \mathcal{M}^2_{NN} + \mathcal{O}(v^2) \end{pmatrix} . \tag{3.34}$$

Hence, the effective 6×6 hermitian squared-mass matrix for the light sneutrinos reads:

$$\mathcal{M}^2_{\tilde{\nu}_\ell} \equiv \mathcal{M}^2_{LL} - \mathcal{M}^2_{LN}\mathcal{M}^{-2}_{NN}\left(\mathcal{M}^2_{LN}\right)^\dagger + \mathcal{O}(v^4M^{-2})\,, \tag{3.35}$$

analogous to the light effective neutrino mass matrix of eq. (3.8). Likewise, the effective 6×6 hermitian squared-mass matrix for the superheavy sneutrinos reads:

$$\mathcal{M}^2_{\tilde{\nu}_h} \equiv \mathcal{M}^2_{NN} + \tfrac{1}{2}\left[\mathcal{M}^{-2}_{NN}(\mathcal{M}^2_{LN})^\dagger\mathcal{M}^2_{LN} + (\mathcal{M}^2_{LN})^\dagger\mathcal{M}^2_{LN}\mathcal{M}^{-2}_{NN}\right] + \mathcal{O}(v^4M^{-2})\,, \tag{3.36}$$

where for completeness, we have exhibited the $\mathcal{O}(v^2)$ corrections to the leading term. As expected, the masses of half of the sneutrino eigenstates are of order the electroweak symmetry breaking scale, whereas the other half are superheavy, of order M.

Following the notation of Table 1, the (complex) sneutrino interaction eigenstates are denoted by: $\widetilde{\nu}_L \equiv \widetilde{L}_1$ and $\widetilde{\nu}_R \equiv \widetilde{N}^*$. The latter convention reflects the fact that in the lepton-number conserving limit of $M^{IJ} = m^2_B = 0$, the lepton numbers of $\widetilde{\nu}_L$ and $\widetilde{\nu}_R$ are identical, as previously noted. (Of course, the limit of interest in this paper, $\|M\| \gg v$, is very far from the lepton-number conserving limit.) In analogy to ν_ℓ and ν_h, we define transformed (light and heavy) sneutrino states $\tilde{\nu}_\ell$ and $\tilde{\nu}_h$ by:

$$\begin{pmatrix} \phi_L \\ \phi_N \end{pmatrix} = \boldsymbol{V} \begin{pmatrix} \phi_\ell \\ \phi_h \end{pmatrix} , \tag{3.37}$$

where $\phi_\ell \equiv (\widetilde{\nu}_\ell\,, \widetilde{\nu}^*_\ell)^T$ and $\phi_h \equiv (\widetilde{\nu}^*_h\,, \widetilde{\nu}_h)^T$ are six-dimensional vectors. Sneutrino–antisneutrino oscillations are a consequence of the $\Delta L = 2$ elements in the light and heavy sneutrino squared-mass matrices $\mathcal{M}^2_{\widetilde{\nu}_\ell}$ and $\mathcal{M}^2_{\widetilde{\nu}_h}$, and are governed by $M^2_{N^TN}$ and $M^2_{L^\dagger N}$ (note that $M^2_{L^TL}$, which would also violate lepton number by two units, is zero).

Using the form of $\mathcal{M}^2_{AB}$ (A, $B = L$ or N) given by eq. (3.23) with the M^2_{AB} given in eqs. (3.24)–(3.29), the effective 6×6 hermitian squared-mass matrix for the light sneutrinos [eq. (3.35)] is given by:

$$\mathcal{M}^2_{\tilde{\nu}_\ell} \equiv \begin{pmatrix} M^2_{LC} & (M^2_{LV})^* \\ M^2_{LV} & (M^2_{LC})^* \end{pmatrix} , \tag{3.38}$$

where the lepton-number-conserving (LC) and lepton-number-violating (LV) matrix elements are given by:

$$M_{LC}^2 \equiv m_L^2 + \tfrac{1}{2}M_Z^2\cos 2\beta + m_D^* m_D^T - m_D^* M(M^2+m_N^2)^{-1}M m_D^T + \mathcal{O}(v^4M^{-2})\,, \quad (3.39)$$

$$\begin{aligned} M_{LV}^2 \equiv\; & m_D M(M^2+m_N^{2*})^{-1}m_D^T X_\nu^T + X_\nu m_D(M^2+m_N^2)^{-1}M m_D^T \\ & -2m_D M(M^2+m_N^{2*})^{-1}m_B^2(M^2+m_N^2)^{-1}M m_D^T + \mathcal{O}(v^4M^{-2})\,, \end{aligned} \quad (3.40)$$

under the assumption that m_B^2 and m_N^2 can be as large as indicated in eqs. (2.14) and (2.16). Note that M_{LC}^2 is a 3×3 hermitian matrix, and M_{LV}^2 is a 3×3 complex symmetric matrix. Moreover, although M is a diagonal matrix with real positive entries [cf. eq. (2.5)], m_N^2 can be any 3×3 hermitian matrix, not necessarily diagonal nor real. The $M\to\infty$ limit of eqs. (3.39) and (3.40) is noteworthy. In this limit, $M_{LV}^2 = 0$ and the lepton-number-violating effects completely decouple, as expected. If in addition $m_N^2 = 0$, then $M_{LC}^2 = m_L^2 + \frac{1}{2}M_Z^2\cos 2\beta$, which reproduces the well known 3×3 light sneutrino squared-mass matrix of the MSSM. However, according to eq. (2.15), $m_N^2 M^{-2}\sim\mathcal{O}(1)$ is possible, in which case M_{LC}^2 deviates from its MSSM value by a quantity of $\mathcal{O}(v^2)$ *even in the exact decoupling limit of* $M\to\infty$. The origin of this non-decoupling behavior is explained in Appendix B. As a result of this non-decoupling phenomenon, remnants of the heavy sector of the seesaw mechanism may survive in the effective theory of light sneutrinos. These non-decoupling effects can be detected in principle through measurements of the sneutrino and charged slepton properties.

The physical light sneutrino states can be identified by diagonalizing $\mathcal{M}_{\tilde\nu_\ell}^2$. Note that if $M_{LV}^2 = 0$, then the eigenvalues[9] of $\mathcal{M}_{\tilde\nu_\ell}^2$ are doubly degenerate, corresponding to the fact that the conserved lepton number implies that the six light sneutrino states are comprised of three sneutrino antisneutrino pairs. If $M_{LV}^2 \neq 0$, then lepton number is violated and the sneutrinos and antisneutrinos can mix. This mixing splits the degenerate pairs and yields (in general) six non-degenerate light sneutrinos. In particular, the resulting sneutrino mass-eigenstates are self-conjugate real fields, which we denote by $S_1\,, S_2\,,\,\ldots\,,\,S_6$.

To determine the S_k in terms of the interaction sneutrino eigenstates, one must compute the 6×6 unitary matrix $\mathcal{W}$ that diagonalizes $\mathcal{M}_{\tilde\nu_\ell}^2$:

$$\mathcal{W}^\dagger \mathcal{M}_{\tilde\nu_\ell}^2 \mathcal{W} = \mathrm{diag}\,(m_{S_1}^2\,,\, m_{S_2}^2\,,\,\ldots\,,\, m_{S_6}^2)\,. \quad (3.41)$$

Noting that $\Sigma\mathcal{M}_{\tilde\nu_\ell}^2\Sigma = \mathcal{M}_{\tilde\nu_\ell}^{2*}$, where $\Sigma\equiv\left(\begin{smallmatrix}0 & \mathbb{1}\\ \mathbb{1} & 0\end{smallmatrix}\right)$, it follows that if $\mathcal{W}$ satisfies eq. (3.41) then so does $\Sigma\mathcal{W}^*$. However, the unitary matrix that diagonalizes $\mathcal{M}_{\tilde\nu_\ell}^2$ is unique up to

[9]Under the assumption that R-parity is not spontaneously broken, the (real) eigenvalues of the hermitian matrix M_{LC}^2 are non-negative.

a multiplication on the right by a unitary matrix $\mathcal{U}_D$ that is arbitrary within a subspace of degenerate eigenvalues and is otherwise diagonal. Denote the set of all such unitary matrices by $\mathcal{S}$. Hence, one can conclude that $\Sigma\mathcal{W}^* = \mathcal{W}\mathcal{U}_D$ for some $\mathcal{U}_D \in \mathcal{S}$. Since $\mathcal{W}$ is unitary, $\mathcal{U}_D = \mathcal{W}^\dagger\Sigma\mathcal{W}^*$, and it follows that $\mathcal{U}_D\mathcal{U}_D^* = \mathcal{I}$. That is, $\mathcal{U}_D$ must be a symmetric unitary matrix. It then follows that the matrix $\mathcal{W}' \equiv \mathcal{W}U_D^{1/2}$ satisfies $\mathcal{W}' = \Sigma\mathcal{W}'^*$.[10]

Thus, without loss of generality, we may drop the primed superscripts and impose the constraint $\mathcal{W} = \Sigma\mathcal{W}^*$ on the diagonalizing matrix that satisfies eq. (3.41). It then follows that $\mathcal{W}$ has the following form:

$$\mathcal{W} \equiv \begin{pmatrix} X & iY \\ X^* & -iY^* \end{pmatrix} , \tag{3.42}$$

where X and Y are 3×3 complex matrices that satisfy:

$$XX^\dagger + YY^\dagger = \mathbb{1} \,, \qquad\qquad XX^T = YY^T \,, \tag{3.43}$$

$$\mathrm{Re}(X^\dagger X) = \mathrm{Re}(Y^\dagger Y) = \tfrac{1}{2} \,, \qquad \mathrm{Im}(X^\dagger Y) = 0 \,, \tag{3.44}$$

due to the unitarity of $\mathcal{W}$. Consequently, the relation between the sneutrino interaction-eigenstate fields $\widetilde{\nu}_\ell^I$ and the six self-conjugate sneutrino mass-eigenstate fields S_k is given by:

$$\widetilde{\nu}_\ell^I = \sum_{k=1}^{6} \mathcal{W}^{Ik} S_k = \sum_{K=1}^{3} \left(X^{IK} S_K + iY^{IK}\, S_{K+3} \right) , \qquad (I = 1, 2, 3) \,. \tag{3.45}$$

One can then invert eq. (3.45) [using eqs. (3.43) and (3.44)] to obtain:

$$S_K = \sum_{I=1}^{3} \left(X^{IK\,*} \widetilde{\nu}_\ell^I + X^{IK} (\widetilde{\nu}_\ell^I)^* \right) , \quad S_{K+3} = -i \sum_{I=1}^{3} \left(Y^{IK\,*} \widetilde{\nu}_\ell^I - Y^{IK} (\widetilde{\nu}_\ell^I)^* \right) , \quad (K = 1, 2, 3) \,. \tag{3.46}$$

Indeed, the S_k are self-conjugate real fields as noted above.

Since $M^2_{LC} \sim \mathcal{O}(v^2)$ and $M^2_{LV} \sim \mathcal{O}(v^3 M^{-1})$, the mass-splittings of the would-be sneutrino-antisneutrino pairs are expected to be very small, of order a typical neutrino mass. To compute the magnitude of the corresponding mass-splittings, we can employ perturbative techniques to evaluate the eigenvalues of $\mathcal{M}^2_{\widetilde{\nu}_\ell}$ [eq. (3.38)]. First, we diagonalize the sub-matrix M^2_{LC}:

$$Q_0^\dagger M^2_{LC} Q_0 = D \equiv \mathrm{diag}(d_1 \,,\, d_2 \,,\, d_3) \,, \tag{3.47}$$

[10]We define $\mathcal{U}_D^{1/2} \in \mathcal{S}$ to be the unique square root of $\mathcal{U}_D$ that is symmetric and unitary. This is accomplished by noting that there exists a (unique) real symmetric matrix $\mathcal{H}$ such that $\mathcal{U}_D = \exp(i\mathcal{H})$. Then, $\mathcal{U}_D^{1/2} \equiv \exp(i\mathcal{H}/2)$. Note that there is still some freedom left in the choice of $\mathcal{W}'$, which is unique up to a multiplication on the right by a real orthogonal matrix that is arbitrary within a degenerate subspace and is otherwise diagonal.

where Q_0 is a 3×3 unitary matrix, and the eigenvalues d_I are real. Note that Q_0 is not unique. In Section 4.3, we will argue that the bounds on the radiative flavor-changing charged lepton decay $\ell^J \to \ell^I \gamma$ imply that matrix M^2_{LC} is very close to a diagonal form. In the limit of diagonal M^2_{LC}, we shall take $Q_0 = \mathbb{1}$. We can then determine the off-diagonal elements of Q_0 by writing $M^2_{LC} \simeq \mathrm{diag}(m_1^2\,, m_2^2\,, m_3^2) + m^2_{LC}$, where m^2_{LC} is a matrix made up of the off-diagonal elements of M^2_{LC}, and $Q_0 \simeq \mathbb{1} + q_0$, where $q_0^\dagger = -q_0$. By assumption, the matrix elements of m^2_{LC} are much smaller than the m_I^2, and the matrix elements of q_0 are much smaller than unity. Thus treating eq. (3.47) to first order in the small quantities, we can solve for the off-diagonal elements of q_0 in terms of the elements of m^2_{LC} and the m_I^2. Since at first order $m_I^2 = d_I$, it follows that:

$$(Q_0)_{IJ} \simeq \frac{(M^2_{LC})_{IJ}}{d_J - d_I}\,, \qquad I \neq J\,. \tag{3.48}$$

The diagonal elements of Q_0 can then be determined to the same order by using the unitarity of Q_0. In the remainder of this section, we will not make any assumption regarding the size of the off-diagonal elements of M^2_{LC}, in which case eq. (3.48) does not apply and Q_0 must be obtained numerically from eq. (3.47).

In the following, it will be convenient to define

$$Q = Q_0 T \tag{3.49}$$

where T is a 3×3 diagonal matrix of phases given by:

$$T \equiv \mathrm{diag}\left(e^{-i\phi_1/2}\,, e^{-i\phi_2/2}\,, e^{-i\phi_3/2}\right)\,, \qquad \phi_J \equiv \arg\left(Q_0^T M^2_{LV} Q_0\right)_{JJ}\,. \tag{3.50}$$

Note that the right hand side of eq. (3.47) is unchanged when $Q_0 \to Q_0 T$, so that the unitary matrix Q can also be used to diagonalize M^2_{LC}. It then follows that:

$$\mathscr{D} \equiv \begin{pmatrix} D & B^* \\ B & D \end{pmatrix} = \begin{pmatrix} Q^\dagger & 0 \\ 0 & Q^T \end{pmatrix} \begin{pmatrix} M^2_{LC} & (M^2_{LV})^* \\ M^2_{LV} & (M^2_{LC})^* \end{pmatrix} \begin{pmatrix} Q & 0 \\ 0 & Q^* \end{pmatrix}\,, \tag{3.51}$$

where B is the 3×3 complex symmetric matrix

$$B \equiv Q^T M^2_{LV} Q\,. \tag{3.52}$$

Due to the rephasing of Q_0 as specified by eqs. (3.49) and (3.50), the diagonal elements of B are real and non-negative: $B_{JJ} = |B_{JJ}|$. This is the motivation for our choice of Q in the diagonalization of M^2_{LC}. Note that if M^2_{LC} is approximately diagonal, then $Q_0 \simeq \mathbb{1}$, in which case $\phi_J \simeq \arg[(M^2_{LV})_{JJ}]$. Thus, unless the diagonal elements of M^2_{LV} are non-negative, $Q \simeq T \neq \mathbb{1}$ in this limiting case.

Even though $D \sim \mathcal{O}(v^2)$ and $B \sim \mathcal{O}(v^3 M^{-1})$, the unitary matrix that diagonalizes $\mathscr{D}$ is not close to the identity matrix, due to the double degeneracy of the diagonal elements. In order to perform a perturbative diagonalization of $\mathscr{D}$, we first introduce the following 6×6 unitary matrix $\mathcal{P}$, expressed in block form as:

$$\mathcal{P} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbb{1} & i\mathbb{1} \\ \mathbb{1} & -i\mathbb{1} \end{pmatrix} , \tag{3.53}$$

A straightforward computation yields:

$$\mathcal{P}^\dagger \mathscr{D} \mathcal{P} = \begin{pmatrix} D + \operatorname{Re} B & -\operatorname{Im} B \\ -\operatorname{Im} B & D - \operatorname{Re} B \end{pmatrix} , \tag{3.54}$$

which is a 6×6 real symmetric matrix.

If the elements of the diagonal matrix D are non-degenerate[11] such that $d_I - d_J \sim \mathcal{O}(v^2)$ for all $I \neq J$, then the matrix $\mathcal{P}^\dagger \mathscr{D} \mathcal{P}$ can be diagonalized by a real orthogonal matrix $\mathcal{R}$ that is close to the identity:

$$\mathcal{R} = \begin{pmatrix} \mathbb{1} + \operatorname{Re} R & \operatorname{Im} R \\ \operatorname{Im} R & \mathbb{1} - \operatorname{Re} R \end{pmatrix} + \mathcal{O}(v^2 M^{-2}) , \tag{3.55}$$

where the 3×3 complex antisymmetric matrix R is of order $\mathcal{O}(v M^{-1})$:

$$R_{IJ} = -R_{JI} \equiv \frac{B^*_{IJ}}{d_J - d_I} , \qquad (I \neq J) . \tag{3.56}$$

One can check that:

$$\mathcal{R}^T \mathcal{P}^\dagger \mathscr{D} \mathcal{P} \mathcal{R} = \operatorname{diag}(m^2_{S_1} , m^2_{S_2} , \ldots , , m^2_{S_6}) + \mathcal{O}(v^4 M^{-2}) , \tag{3.57}$$

where the squared-masses of the light sneutrinos are given by:

$$m^2_{S_J , S_{J+3}} = d_J \pm |B_{JJ}| + \mathcal{O}(v^4 M^{-2}) , \qquad (J = 1, 2, 3) , \tag{3.58}$$

and $m^2_{S_J} > m^2_{S_{J+3}}$. Note that the perturbations due to the off-diagonal elements of B contribute only to the $\mathcal{O}(v^4 M^{-2})$ terms of the squared-masses.

[11] In general, we would expect the d_I (which are the eigenvalues of M^2_{LC}) to be non-degenerate. Even if the parameters m^2_L and m^2_N were proportional to the identity matrix at the high energy scale due to some flavor symmetry, this latter symmetry would not be respected by the corresponding low-energy parameters, due to flavor-violating effects that enter the renormalization group running. Moreover, the matrix m_D is likely to reflect some of the flavor-violating effects of the model. Hence, any (near) degeneracy among the d_I would be purely accidental.

Combining the results of eqs. (3.51), (3.53) and (3.55), the light sneutrino mixing matrix [defined in eq. (3.41)] is given by:

$$\mathcal{W} = \frac{1}{\sqrt{2}} \begin{pmatrix} Q(\mathbb{1}+R) & iQ(\mathbb{1}-R) \\ Q^*(\mathbb{1}+R^*) & -iQ^*(\mathbb{1}-R^*) \end{pmatrix} + \mathcal{O}(v^2M^{-2})\,. \tag{3.59}$$

Comparing with eq. (3.42), we identify:

$$X = \frac{1}{\sqrt{2}}Q(\mathbb{1}+R) + \mathcal{O}(v^2M^{-2})\,, \quad \text{and} \quad Y = \frac{1}{\sqrt{2}}Q(\mathbb{1}-R) + \mathcal{O}(v^2M^{-2})\,. \tag{3.60}$$

Inserting these results into eqs. (3.45) and (3.46) yields the desired (approximate) relations between the sneutrino mass eigenstates S_k and the interaction eigenstates $\widetilde{\nu}_\ell^I$.

For completeness, we briefly examine the modifications to eq. (3.58) if some of the d_I are degenerate. In this case, the diagonalizing matrix $\mathcal{R}$ is not close to the identity matrix, and the perturbative analysis above fails. Consider the case of $d_I = d_J \neq d_K$, where $\{I, J, K\}$ is some permutation of $\{1, 2, 3\}$. The first order shift in the eigenvalues of $\mathscr{D}$ will depend on B_{IJ} as well as on the diagonal elements of B. However, the perturbations due to B_{IK} and B_{JK} will only generate second-order shifts to the eigenvalues, which we neglect here. Thus, it is sufficient to solve the characteristic equation of $\mathscr{D}$ in the limit of $d_I = d_J$ and $B_{IK} = B_{JK} = 0$. In this limit, the characteristic polynomial factors into a product of two simpler polynomial factors:[12]

$$\left[(\lambda - d_K)^2 - |B_{KK}|^2\right]\left[(\lambda-d_I)^4 - (\lambda-d_I)^2\left[|B_{II}|^2 + |B_{JJ}|^2 + 2|B_{IJ}|^2\right] + \left|B_{IJ}^2 - B_{II}B_{JJ}\right|^2\right]. \tag{3.61}$$

The resulting sneutrino squared-masses are:

$$m^2_{S_I\,,S_{I+3}} \simeq d_I \pm \left\{\tfrac{1}{2}\left[|B_{II}|^2 + |B_{JJ}|^2 + 2|B_{IJ}|^2 + \sqrt{\Delta}\right]\right\}^{1/2}, \tag{3.62}$$

$$m^2_{S_J\,,S_{J+3}} \simeq d_I \pm \left\{\tfrac{1}{2}\left[|B_{II}|^2 + |B_{JJ}|^2 + 2|B_{IJ}|^2 - \sqrt{\Delta}\right]\right\}^{1/2}, \tag{3.63}$$

$$m^2_{S_K\,,S_{K+3}} \simeq d_k \pm |B_{KK}|\,, \tag{3.64}$$

where

$$\Delta \equiv \left[|B_{II}|^2 + |B_{JJ}|^2 + 2|B_{IJ}|^2\right]^2 - 4\left|B_{IJ}^2 - B_{II}B_{JJ}\right|^2. \tag{3.65}$$

The corresponding mixing matrix can be obtained by performing an exact diagonalization within the two-dimensional degenerate subspace, although we shall omit the details.

[12] In the case of a near degeneracy where $d_I - d_J \lesssim \mathcal{O}(vM^{-1})$, the quartic polynomial factor of the characteristic equation of $\mathscr{D}$ contains a term linear in $\lambda - \frac{1}{2}(d_I + d_J)$. In this case, the resulting expressions for $m^2_{S_I\,,S_{I+3}}$ and $m^2_{S_J\,,S_{J+3}}$ are significantly more complicated than those presented in eqs. (3.62) and (3.63).

Finally, in the very unlikely scenario where $d_1 = d_2 = d_3 \equiv d$, all of the matrix elements of B contribute to the first order shifts of the eigenvalues of $\mathscr{D}$. To determine these shifts, put $\lambda = d + x$ in the characteristic equation of $\mathscr{D}$ to obtain a sixth order polynomial in x. No further perturbative simplification is possible, since all the terms of this polynomial are of the same order of magnitude.

As expected, the mass-splittings of the would-be sneutrino–antisneutrino pairs are nonzero due to the presence of the lepton-number violating matrix M^2_{LV} [cf eq. (3.52)]. If we denote the three sneutrino mass-splittings by $(\Delta m_{\tilde{\nu}_\ell})_J \equiv |m_{S_J} - m_{S_{J+3}}|$ (for $J = 1, 2, 3$), then in the non-degenerate case,

$$(\Delta m_{\tilde{\nu}_\ell})_J \simeq \frac{|B_{JJ}|}{\sqrt{d_J}} \,. \tag{3.66}$$

In the case of degenerate d_I, the mass-splittings $(\Delta m_{\tilde{\nu}_\ell})_J$ also depend on the non-diagonal elements of B.

It is instructive to examine the above results in a simplified one generation model. In this case, $D \equiv M^2_{LC}$ and $B \equiv M^2_{LV}$ are just numbers. In particular, m^2_N is a real parameter and $\mathcal{M}^2_{\tilde{\nu}_\ell}$ is a 2×2 hermitian matrix, with eigenvalues

$$\begin{aligned} m^2_{S_1,S_2} &= M^2_{LC} \pm |M^2_{LV}| \\ &= m^2_L + \tfrac{1}{2} M^2_Z \cos 2\beta + \frac{|m_D|^2 m^2_N}{M^2 + m^2_N} \pm \frac{2|m_D|^2 M}{M^2 + m^2_N} \left| X_\nu - \frac{M m^2_B}{M^2 + m^2_N} \right| \,. \end{aligned} \tag{3.67}$$

The corresponding sneutrino mass-splitting, $\Delta m_{\tilde{\nu}_\ell} \equiv |m_{S_2} - m_{S_1}|$, is given by

$$\frac{\Delta m_{\tilde{\nu}_\ell}}{m_{\nu_\ell}} = \frac{2M^2}{m_{\tilde{\nu}_\ell}(M^2 + m^2_N)} \left| X_\nu - \frac{M m^2_B}{M^2 + m^2_N} \right| \,, \tag{3.68}$$

where $m_{\nu_\ell} \equiv |m_D|^2/M$ is the mass of the light neutrino and $m_{\tilde{\nu}_\ell} \equiv \frac{1}{2}(m_{S_1} + m_{S_2})$ is the average light sneutrino mass. If $m_N \ll M$, then eq. (3.68) coincides with the result given in ref. [18] after taking into account a slight difference in notation.[13]

Assuming that $m^2_B \sim \mathcal{O}(vM)$, it follows that both terms on the right hand side of eq. (3.68) are of the same order, which implies that $\Delta m_{\tilde{\nu}_\ell} \sim \mathcal{O}(m_{\nu_\ell})$. However, as noted below eq. (2.14), it is possible that m^2_B could be as much as a factor of 10^3 larger than its naive estimate [18], in which case the sneutrino-antisneutrino mass splitting could be three orders of magnitude larger than the corresponding light neutrino mass.[14]

[13] If we put $m^2_B \equiv -MB_N$ and change the sign of A_ν (with the corresponding change in X_ν [cf. eq. (3.30)]), we recover the results of ref. [18].

[14] A similarly enhanced sneutrino-antisneutrino mass splitting also arises in the supersymmetric triplet seesaw model of ref. [40].

The same set of manipulations described above can be carried out to obtain the corresponding results for the effective 6×6 hermitian squared-mass matrix for the heavy sneutrinos [eq. (3.36)]:

$$\mathcal{M}^2_{\tilde{\nu}_h} \equiv \begin{pmatrix} M^2_H & -2(m^2_B)^* \\ -2m^2_B & (M^2_H)^* \end{pmatrix} + \mathcal{O}(v^4 M^{-2}) \,, \tag{3.69}$$

where the 3×3 hermitian matrix M^2_H is defined by:

$$M^2_H \equiv M^2 + m^2_N + m^\dagger_D m_D + \tfrac{1}{2}(M^2 + m^2_N)^{-1} M m^T_D m^*_D M + \tfrac{1}{2} M m^T_D m^*_D M (M^2 + m^2_N)^{-1} \,. \tag{3.70}$$

The physical heavy sneutrino mass-eigenstates are determined by diagonalizing $\mathcal{M}^2_{\tilde{\nu}_h}$. At leading order, the mass-eigenstates are mass-degenerate sneutrino/antisneutrino pairs, with masses and mixing angles (with respect to the basis in which M is diagonal) determined by the diagonalization of m^2_N. The lepton-number violating off-block-diagonal matrix m^2_B generates sneutrino-antisneutrino mixing, and yields mass-splittings between nearly degenerate heavy sneutrino pairs of order $\Delta m_{\tilde{\nu}_h} \sim \mathcal{O}(m^2_B M^{-1})$.

The complex elements of the sneutrino squared-mass matrix govern CP-violating sneutrino phenomena, due to the non-degeneracy of masses of the real and imaginary parts of the sneutrino fields. Following the discussion of the CP-properties of the sneutrino fields in Section 6, we find it convenient to define a new basis of sneutrino interaction eigenstates of definite CP. That is, we decompose the complex sneutrino fields into real and imaginary parts:

$$\tilde{\nu}_\ell = \frac{1}{\sqrt{2}} \left[\tilde{\nu}^{(+)}_\ell + i\, \tilde{\nu}^{(-)}_\ell \right] \,, \tag{3.71}$$

$$\tilde{\nu}_h = \frac{1}{\sqrt{2}} \left[\tilde{\nu}^{(+)}_h + i\, \tilde{\nu}^{(-)}_h \right] \,, \tag{3.72}$$

where the $[+,-]$ superscripts indicate that the corresponding sneutrino eigenstates are CP-even and CP-odd. With respect to the CP-basis,

$$-\mathscr{L}_{\rm mass} = \tfrac{1}{2}(\tilde{\nu}^{(+)T}_\ell,\, \tilde{\nu}^{(-)T}_\ell)\mathcal{P}^\dagger \mathcal{M}^2_{\tilde{\nu}_\ell}\mathcal{P} \begin{pmatrix} \tilde{\nu}^{(+)}_\ell \\ \tilde{\nu}^{(-)}_\ell \end{pmatrix} + \tfrac{1}{2}(\tilde{\nu}^{(+)T}_h,\, \tilde{\nu}^{(-)T}_h)\mathcal{P}^T \mathcal{M}^2_{\tilde{\nu}_h}\mathcal{P}^* \begin{pmatrix} \tilde{\nu}^{(+)}_h \\ \tilde{\nu}^{(-)}_h \end{pmatrix} \,, \tag{3.73}$$

where $\mathcal{P}$ is the 6×6 unitary matrix introduced in eq. (3.53).

That is, with respect to the CP-basis, the effective squared-mass matrix for the light sneutrinos is given by:

$$\overline{\mathcal{M}}^2_{\tilde{\nu}_\ell} \equiv \mathcal{P}^\dagger \mathcal{M}^2_{\tilde{\nu}_\ell} \mathcal{P} = \begin{pmatrix} {\rm Re}(M^2_{LC} + M^2_{LV}) & -{\rm Im}(M^2_{LC} + M^2_{LV}) \\ {\rm Im}(M^2_{LC} - M^2_{LV}) & {\rm Re}(M^2_{LC} - M^2_{LV}) \end{pmatrix} \,. \tag{3.74}$$

This is a *real* symmetric matrix (which is easily checked by recalling that M^2_{LC} and M^2_{LV} are, respectively, hermitian and complex symmetric matrices), as the CP-basis consists of real self-conjugate scalar fields.

If $\mathrm{Im}M^2_{LC} = \mathrm{Im}M^2_{LV} = 0$, then the sneutrino mass-eigenstates are also definite eigenstates of CP. If in addition $\mathrm{Re}M^2_{LV} \neq 0$, then the would-be sneutrino-antisneutrino pairs are organized into CP-even/CP-odd pairs of nearly degenerate sneutrinos [18].

Since $\overline{\mathcal{M}}^2_{\tilde{\nu}_\ell}$ is real symmetric, it can be diagonalized by a 6×6 real orthogonal matrix, $\mathcal{Z}_{\tilde{\nu}}$ via:

$$\mathcal{Z}^T_{\tilde{\nu}}\, \overline{\mathcal{M}}^2_{\tilde{\nu}_\ell} \mathcal{Z}_{\tilde{\nu}} = (m^2_{S_1}\,,\, m^2_{S_2}\,,\, \ldots\,,\, m^2_{S_6})\,, \tag{3.75}$$

and the corresponding physical sneutrino mass eigenstates, S_k ($k = 1, \ldots, 6$), can be identified as linear combinations of the CP-even and the CP-odd sneutrino eigenstates:

$$\begin{pmatrix} \widetilde{\nu}^{(+)}_\ell \\ \widetilde{\nu}^{(-)}_\ell \end{pmatrix} = \mathcal{Z}_{\tilde{\nu}} \begin{pmatrix} S_1 \\ \vdots \\ S_6 \end{pmatrix} . \tag{3.76}$$

Matching with the notation employed by our discussion of sneutrino oscillations in Section 6, we note that the sneutrino interaction eigenstates, $\tilde{\nu}_\ell$, can be expressed in terms of the physical (self-conjugate) sneutrino mass eigenstates S_k via:

$$\tilde{\nu}^I_\ell = \frac{1}{\sqrt{2}} \sum_{k=1}^{6} (\mathcal{Z}^{Ik}_{\tilde{\nu}} + i\mathcal{Z}^{I+3,k}_{\tilde{\nu}}) S_k\ . \tag{3.77}$$

Comparing eqs. (3.45) and (3.77), we can identify:

$$X^{IK} = \frac{1}{\sqrt{2}} \left(\mathcal{Z}^{IK}_{\tilde{\nu}} + i\mathcal{Z}^{I+3,K}_{\tilde{\nu}} \right)\,, \qquad Y^{IK} = -\frac{i}{\sqrt{2}} \left(\mathcal{Z}^{I,K+3}_{\tilde{\nu}} + i\mathcal{Z}^{I+3,K+3}_{\tilde{\nu}} \right)\,, \quad (I, K = 1, 2, 3)\,, \tag{3.78}$$

which can be inverted to obtain:

$$\mathcal{Z}_{\tilde{\nu}} = \sqrt{2} \begin{pmatrix} \mathrm{Re}\ X & -\mathrm{Im}\ Y \\ \mathrm{Im}\ X & \mathrm{Re}\ Y \end{pmatrix} . \tag{3.79}$$

One can easily verify that the orthogonality of $\mathcal{Z}_{\tilde{\nu}}$ implies the unitarity of $\mathcal{W}$ defined in eq. (3.42) [and vice versa]. In particular, eqs. (3.41) and (3.75) imply that $\mathcal{Z}_{\tilde{\nu}} = \mathcal{P}^\dagger \mathcal{W}$, in which case

$$\mathcal{Z}^T_{\tilde{\nu}} \mathcal{Z}_{\tilde{\nu}} = \mathcal{W}^T \mathcal{P}^* \mathcal{P}^\dagger \mathcal{W} = \mathcal{W}^T \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \mathcal{W} = \mathcal{W}^\dagger \mathcal{W} = \mathcal{I}\,, \tag{3.80}$$

after using the explicit forms for $\mathcal{W}$ and $\mathcal{P}$.

In summary, we have derived the light effective sneutrino squared-mass matrix by exploiting the seesaw mechanism in the sneutrino as well as in the neutrino sector. Our calculation is quite general under the parameter assumptions specified by eqs. (2.10)–(2.15). We found that $\mathcal{M}^2_{\tilde{\nu}_\ell}$ depends on two 3×3 matrix blocks, M^2_{LC} and M^2_{LV}, given by eqs. (3.39) and (3.40), respectively. In particular, M^2_{LV} is responsible for the splitting of the masses of would-be sneutrino-antisneutrino pairs, or equivalently the mass-splitting of CP-even/CP-odd sneutrino pairs, $\tilde{\nu}^{(\pm)}_\ell$, in the CP-conserving limit. As we shall see in Sections 4 and 5, the matrices M^2_{LC} and M^2_{LV} provide a convenient parameterization for a number of interesting physical observables, such as neutrino masses and radiative lepton decays.

4 Constraints on lepton number conserving parameters

The input parameters that govern sneutrino mixing phenomena and sneutrino decays are encoded in matrices M^2_{LV} and M^2_{LC} given by eqs. (3.40) and (3.39), respectively [or, alternatively, in the physical sneutrino masses and the orthogonal matrix $\mathcal{Z}_{\tilde{\nu}}$ defined in eq. (3.75)]. At present, apart from neutrino oscillations, only lepton number conserving processes are observed in current experiments. These processes constrain the entries of the lepton number conserving matrix M^2_{LC}. In this Section we investigate bounds on the structure of M^2_{LC} imposed by the measurements of the muon magnetic moment anomaly, the $g_\mu - 2$, the electric dipole moment (EDM) of the electron and the radiative flavor changing charged lepton decays, $\ell^J \to \ell^I \gamma$. The latter have also been worked out in detail in ref. [21]. Additional constraints due to $\ell^-_J \to \ell^-_I \ell^-_I \ell^+_I$ decays and μ–e conversion in nuclei are also relevant and have been analyzed in Ref. [21, 41]. These constraints can yield further restrictions on the structure of M^2_{LC}, although we shall not present this analysis here.

We briefly summarize the constraints from current experiments relevant for the computations presented in this Section. The most recent experimental measurement of the muon anomalous magnetic moment ($a^{\rm exp}_\mu$) exhibits a slight discrepancy [42] relative to the predicted value of the Standard Model ($a^{\rm th}_\mu$). A recent theoretical review of the computation of the Standard Model prediction [43] yielded $\delta a_\mu \equiv a^{\rm exp}_\mu - a^{\rm th}_\mu = (2.94 \pm 0.89) \times 10^{-9}$, where all theoretical and experimental errors are added in quadrature, corresponding to a $3.3\,\sigma$ effect. Thus, we roughly expect that the contribution to the muon anomalous magnetic moment from new physics beyond the Standard Model to be no larger than

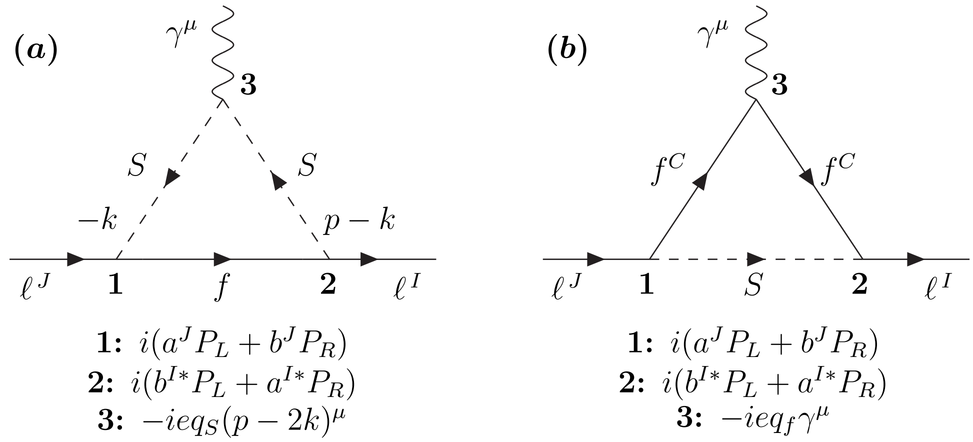

Figure 1: One-loop SUSY diagrams contributing to radiative, $\ell^J \to \ell^I\gamma$, decays. In (a), the scalar S is a charged slepton and the fermion f is a neutralino. In (b), the scalar S is a sneutrino and the fermion f [f^C] is a positively [negatively] charged chargino ($q_f = 1$).

$\delta a_\mu \lesssim 3 \times 10^{-9}$. There is no experimental evidence of an nonzero EDM for the electron (d_e). The most stringent upper bound, obtained in ref. [44], is $d_e \leq 1.6 \times 10^{-27}$ e cm at 90% CL. Likewise, there is no experimental evidence for radiative flavor-changing charged lepton decays. The 90% CL upper limits to the branching ratios for the muon and tau-lepton radiative decays are given by: BR($\mu \to e\gamma$) $\leq 1.2 \times 10^{-11}$, BR($\tau \to e\gamma$) $\leq 1.1 \times 10^{-7}$ and BR($\tau \to \mu\gamma$) $\leq 6.8 \times 10^{-8}$ [11].

4.1 Supersymmetric corrections to the lepton-photon vertex

The amplitudes for the processes of interest are obtained by evaluating triangle diagrams that contribute to the one-loop correction to the lepton-photon $\ell^J\ell^I\gamma$ vertex. Supersymmetric corrections to this vertex arise from the two topologies of diagrams depicted in fig. 1. The corresponding Feynman rules required for the vertices are given in eqs. (C.3) and (C.4) of Appendix C. The anomalous magnetic moment and electric dipole moment (EDM) of the leptons and the lepton flavor violating decays $\ell^J \to \ell^I\gamma$ are derived from the following terms of an effective Hamiltonian:

$$\mathscr{H} \;=\; e\left(C_L^{IJ}\,\bar{\ell}^I\sigma^{\mu\nu}P_L\ell^J \;+\; C_R^{IJ}\,\bar{\ell}^I\sigma^{\mu\nu}P_R\ell^J\right)\,F_{\mu\nu}\,, \tag{4.1}$$

which can be extracted from the computation of the effective one-loop $\ell^I\ell^J\gamma$ vertex.

The computation of the Wilson coefficients C_L, C_R is straightforward. After calculating the contributions of diagrams (a) and (b) of fig. 1 and expanding in momenta of external particles, we find for their total Wilson coefficients

$$
\begin{aligned}
C_L^{iIJ} &= C_{1L}^{iIJ} + m_{\ell^I} C_{4L}^{iIJ} + m_{\ell^J} C_{4R}^{iIJ} \ , \\
C_R^{iIJ} &= C_{1R}^{iIJ} + m_{\ell^I} C_{4R}^{iIJ} + m_{\ell^J} C_{4L}^{iIJ} \ ,
\end{aligned}
\tag{4.2}
$$

where the index i labels the contribution of diagrams $i = a, b$ and the m_{ℓ^I} $(I = 1, 2, 3)$ are the lepton masses. For diagram (a) we obtain,

$$
\begin{aligned}
C_{1L}^{aIJ} &= \frac{1}{2(4\pi)^2} q_S b^{I*} a^J m_f C_{12}(m_S, m_f) \ , \quad & C_{1R}^{aIJ} &= \frac{1}{2(4\pi)^2} q_S a^{I*} b^J m_f C_{12}(m_S, m_f) \ , \\
C_{4L}^{aIJ} &= \frac{1}{2(4\pi)^2} q_S a^{I*} a^J C_{23}(m_S, m_f) \ , \quad & C_{4R}^{aIJ} &= \frac{1}{2(4\pi)^2} q_S b^{I*} b^J C_{23}(m_S, m_f) \ ,
\end{aligned}
\tag{4.3}
$$

and for the diagram (b),

$$
\begin{aligned}
C_{1L}^{bIJ} &= \frac{1}{(4\pi)^2} q_f b^{I*} a^J m_f C_{11}(m_f, m_S) \ , \quad & C_{1R}^{bIJ} &= \frac{1}{(4\pi)^2} q_f a^{I*} b^J m_f C_{11}(m_f, m_S) \ , \\
C_{4L}^{bIJ} &= \frac{1}{2(4\pi)^2} q_f a^{I*} a^J C_{23}(m_f, m_S) \ , \quad & C_{4R}^{bIJ} &= \frac{1}{2(4\pi)^2} q_f b^{I*} b^J C_{23}(m_f, m_S) \ ,
\end{aligned}
\tag{4.4}
$$

where m_f and m_S are the masses of the fermion f and scalar S, respectively, and all other parameters are defined in fig. 1. The loop integrals appearing in eqs. (4.3) and (4.4) are:

$$
\begin{aligned}
C_{11}(x, y) &= -\frac{x^2 - 3y^2}{4(x^2 - y^2)^2} + \frac{y^4}{(x^2 - y^2)^3} \log \frac{y}{x} \ , \\
C_{12}(x, y) &= -\frac{x^2 + y^2}{2(x^2 - y^2)^2} - \frac{2x^2 y^2}{(x^2 - y^2)^3} \log \frac{y}{x} \ , \\
C_{23}(x, y) &= -\frac{x^4 - 5x^2 y^2 - 2y^4}{12(x^2 - y^2)^3} + \frac{x^2 y^4}{(x^2 - y^2)^4} \log \frac{y}{x} \ .
\end{aligned}
\tag{4.5}
$$

The full Wilson coefficients C_L and C_R are obtained by summing over all relevant triangle diagrams in the model. In our case just two of them contribute: diagram (a) with charged slepton and neutralino exchange and diagram (b) with sneutrino and chargino exchange.

4.2 $(g-2)_\mu$ and the electron EDM

The formalism described above leads easily to expressions for the EDM of the electron and for the muon magnetic moment anomaly $(g_\mu - 2)/2$. For both processes $I = J$, so that the flavor-diagonal piece of the effective Hamiltonian is given by

$$
\mathscr{H} = e \, \bar{\ell}^J \sigma_{\mu\nu} \Big[\mathrm{Re} C_{1L}^{JJ} + m_{\ell^J} (C_{4L}^{JJ} + C_{4R}^{JJ}) - i \mathrm{Im} C_{1L}^{JJ} \gamma_5 \Big] \ell^J F^{\mu\nu} \ , \tag{4.6}
$$

where we used the relation $C_{1R}^{JJ} = C_{1L}^{JJ*}$. By matching to the standard form [45, 46]:[15]

$$\mathscr{H} \;=\; -\frac{e}{4m_{l^J}}a_J\bar{\ell}^J\sigma_{\mu\nu}\ell^J F^{\mu\nu} + \frac{id_{\ell^J}}{2}\bar{\ell}^J\sigma_{\mu\nu}\gamma_5\ell^J F^{\mu\nu}\ , \tag{4.7}$$

where $a_J \equiv (g_J-2)/2$ is the magnetic moment anomaly and d_{ℓ^J} is the EDM of the lepton, one can extract the expressions for the electron EDM, d_e, and for $g_\mu - 2$,

$$d_e \;=\; -2e\,\mathrm{Im}C_{1L}^{11}\ , \tag{4.8}$$

$$a_\mu \;=\; -4m_\mu\left[\mathrm{Re}C_{1L}^{22} \;+\; m_\mu(C_{4L}^{22}+C_{4R}^{22})\right]\ . \tag{4.9}$$

In principle, both quantities can be used to set bounds on parameters such as M, m_N^2, m_B^2 and X_ν that govern the heavy sneutrino sector. However, the one-loop contribution to the C_{1L}^{11} from fig. 1(b), which is sensitive to the sneutrino sector, is real if the chargino parameters μ and M_2 are real. Hence, the electron EDM measurement does not yield any constraints on sneutrino parameters at one loop. However, there can be sensitivity due to potentially large two-loop corrections; for further details see Ref. [29]. Similarly, the neutrino magnetic and/or electric dipole moments[16] are also insensitive to the heavy sneutrino sector at one-loop, since there is no possibility of attaching the photon to a one-loop graph that involves the sneutrino-neutrino-neutralino vertex (see Appendix C).

The amplitudes displayed in fig. 1 can give sizable contributions to the anomalous magnetic moment of the muon. These contributions are flavor diagonal and are sensitive mostly to the overall mass scale of the sleptons, gauginos and light sneutrinos—i.e. to the diagonal entries of corresponding mass matrices. Thus, the measurement of a_μ can be used to set lower bound on these SUSY masses. Assuming that the discrepancy between the experimentally observed muon anomalous magnetic moment and the theoretical prediction of the Standard Model, $\delta a_\mu \lesssim 3\times 10^{-9}$, is due to new physics effects arising from the diagrams of fig. 1, one can deduce lower bounds on the magnitude of slepton squared-mass parameter as a function of M_2 and $\tan\beta$. Examples of such bounds are listed in Table 2.

Note that potential contributions to M_{LC}^2 [cf. (3.39)] from the terms containing the Dirac mass m_D are suppressed by a quantity of $\mathcal{O}(m_N^2 M^{-2})$. As we will show in Section 4.3, this ratio can be at most of the order of 10^{-2}, otherwise the Dirac mass term m_D would generate unacceptably large contributions to rare $\ell^J \to \ell^I\gamma$ decays. Thus, the muon anomalous magnetic moment can be effectively used to set a lower bound on the diagonal 22 element

[15] In eq. (4.7), the unit of electric charge e is taken positive, so that the electron charge is $-e$ (which also coincides with the convention adopted by refs. [45] and [46]). Eq. (4.7) is consistent with the corresponding effective Lagrangian of ref. [45], by noting that Commins *et al.* define the anomalous magnetic moment of the electron to be $\kappa = -a_e$ (J.D. Jackson, private communication).

[16] Note that for Majorana particles only transition dipole moments can be nonzero.

	$M_2 = 100$	$M_2 = 200$	$M_2 = 300$
$\tan\beta$	$(m_L)^{min}$	$(m_L)^{min}$	$(m_L)^{min}$
5	170	110	70
10	300	270	210
15	420	420	370
20	530	570	530
25	650	740	700

Table 2: Lower bounds on the square root of $(m_L^2)_{22}$ from the measurement of a_μ. All masses are in GeV.

of the soft slepton squared-mass matrix m_L^2 and on the gaugino mass parameter M_2, as specified in Table 2. The dependence on m_R^2 and μ is significantly weaker.

4.3 Radiative charged lepton decay: $\ell^J \to \ell^I\gamma$

The $\ell^J \to \ell^I\gamma$ decay width is given by

$$\Gamma(\ell^J \to \ell^I\gamma) = \frac{e^2 m_{\ell^J}^3}{4\pi}\left(|C_L^{IJ}|^2 + |C_R^{IJ}|^2\right) . \tag{4.10}$$

The corresponding branching ratio is obtained by dividing the result of eq. (4.10) by the tree level decay width, $\Gamma(\ell^J \to \ell^I \nu^J \bar{\nu}^I) = m_{\ell^J}^5 G_F^2/192\pi^3$ (where we ignore W-propagator effects and a very small correction due to the nonzero mass of the light final state charged lepton). In particular, the branching ratios for the experimentally interesting decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are given by:

$$\mathrm{BR}(\mu \to e\gamma) = \frac{48\pi^2 e^2}{m_\mu^2 G_F^2}\left(|C_L^{12}|^2 + |C_R^{12}|^2\right) , \tag{4.11}$$

and

$$\mathrm{BR}(\tau \to \mu\gamma) = \frac{48\pi^2 e^2}{m_\tau^2 G_F^2}\left(|C_L^{23}|^2 + |C_R^{23}|^2\right) . \tag{4.12}$$

At leading one-loop order, fig. 1(a) yields an amplitude that is proportional to the off-diagonal terms of the slepton soft mass matrix m_L^2, and thus not relevant for setting bounds on heavy sneutrino parameters[17]. The amplitude corresponding to fig. 1(b) depends

[17]Of course this diagram is relevant when Y_ν-dependent corrections to m_L^2 entries are generated by the renormalization group evolution of parameters. This effect has been studied extensively in the literature (see e.g., ref. [47]), and we will not repeat this discussion here.

directly on the lepton flavor conserving part of the light sneutrino mass matrix, M^2_{LC}. This can be verified by using the Feynman rules collected in the Appendix C and employing the mass insertion approximation (MIA) expansion; for more details see e.g. ref. [48]. Assume (at least formally) that sneutrinos are closely degenerate in mass,

$$m^2_{S_k} = m^2_0 + \delta m^2_{S_k}\ , \tag{4.13}$$

and then expand the functions C^{IJ}_L or C^{IJ}_R [denoted generically in eq. (4.14) by f], which depend on the squared-massed $m^2_{S_k}$, up to the first order. This results in

$$f(m^2_{S_k}) \approx f(m^2_0) + (m^2_{S_k} - m^2_0)\,\frac{\partial f}{\partial m^2_{S_k}}\bigg|_{m^2_0} = f(m^2_0) - m^2_0\,\frac{\partial f}{\partial m^2_{S_k}}\bigg|_{m^2_0} + m^2_{S_k}\,\frac{\partial f}{\partial m^2_{S_k}}\bigg|_{m^2_0}\ , \tag{4.14}$$

where there is an implicit sum over k. The advantage of this procedure is that it allows one to perform the sum over the sneutrino flavor index k in evaluating eqs. (4.11) and (4.12). For example, the neutrino squared-masses always appear multiplied by a pair of sneutrino mixing matrices (due to the form of the sneutrino couplings given in Appendix C). Using the inverse of eq. (3.75), one obtains $\mathcal{Z}^{ik}_{\tilde\nu}\mathcal{Z}^{jk}_{\tilde\nu}m^2_{S_k} = (\overline{\mathcal{M}}^2_{\tilde\nu_\ell})^{ij}$.

It is possible to relax the assumption of approximately degenerate sneutrino masses. In particular, it can be shown diagrammatically that it is better to use appropriate ratios in place of the derivatives of eq. (4.14) in the MIA expansion. Thus, for $J > I$ (corresponding to the decay of a heavier lepton ℓ^J into a lighter lepton ℓ^I) and neglecting terms proportional to the lighter lepton mass, one arrives at the simple result:

$$\begin{aligned} C^{IJ}_L &\simeq 0\ ,\\ C^{IJ}_R &\simeq C^{bIJ}_{1R} + m_{\ell^J}\,C^{bIJ}_{4L}\\ &\simeq \frac{m_{\ell^J}}{(4\pi)^2}\frac{e^2}{2s^2_W}\left(M^2_{LC}\right)^{IJ}\left(|Z^{1i}_+|^2\left(\frac{\Delta C_{23}}{\Delta m^2}\right)_{iIJ} - \frac{\sqrt{2}}{\cos\beta}\frac{m_{\chi^+_i}}{M_W}\,Z^{1i*}_+Z^{2i*}_-\left(\frac{\Delta C_{11}}{\Delta m^2}\right)_{iIJ}\right)\ , \end{aligned} \tag{4.15}$$

where the $Z_\pm$ are the chargino mixing matrices defined in ref. [7],

$$\left(\frac{\Delta C_{ij}}{\Delta m^2}\right)_{kIJ} \equiv \begin{cases} \dfrac{C_{ij}(m_{\chi^+_k}, m_{\tilde\nu^I_\ell}) - C_{ij}(m_{\chi^+_k}, m_{\tilde\nu^J_\ell})}{m^2_{\tilde\nu^I_\ell} - m^2_{\tilde\nu^J_\ell}}\ , & \text{for } I \neq J\ ,\\[2ex] \dfrac{\partial C_{ij}(m_{\chi^+_k}, m_{\tilde\nu^I_\ell})}{\partial m^2_{\tilde\nu^I_\ell}}\ , & \text{for } I = J\ . \end{cases} \tag{4.16}$$

and $m_{\tilde\nu^I_\ell}$ are the three "CP-averaged" sneutrino masses, given by the positive square roots of the eigenvalues of M^2_{LC} [cf. eqs. (3.47) and (3.58)].

$\tan\beta$	10	20
M^2_{LC}	$\begin{pmatrix} \gtrsim 270^2 & \lesssim 4^2 & \lesssim 11^2 \\ ... & \gtrsim 270^2 & \lesssim 31^2 \\ ... & ... & \gtrsim 270^2 \end{pmatrix}$	$\begin{pmatrix} \gtrsim 570^2 & \lesssim 8^2 & \lesssim 45^2 \\ ... & \gtrsim 570^2 & \lesssim 150^2 \\ ... & ... & \gtrsim 570^2 \end{pmatrix}$

Table 3: Bounds on the structure of the matrix elements of M^2_{LC} for $M_2 = \mu = 200$ GeV. All masses in the Table are given in GeV.

Clearly, our approximate expression for C_R^{IJ} given by eq. (4.15), which enters the decay rates in eq. (4.10), is proportional to the lepton number conserving squared-mass matrix, M^2_{LC}, defined in eq. (3.39). Even in the case where m^2_L is diagonal, contributions to radiative lepton decays arise from the off-diagonal elements of M^2_{LC} governed by the general form of the matrices m_D and m^2_N [cf. the third term in eq. (3.39)]. Notice that the flavor dependence disappears completely in the limit of diagonal m^2_L and $m^2_N = 0$ in which case M^2_{LC} is diagonal.

The effect of the seesaw contribution to the lepton number conserving part of the sneutrino squared-mass matrix, M^2_{LC}, has not been previously noticed in the literature. This yields an extra contribution to the decay branching ratios BR($\ell^J \to \ell^I\gamma$). Consequently, for a fixed set of chargino sector parameters (μ, M_2 and $\tan\beta$) and soft slepton squared-mass matrix (m^2_L), the experimental bounds on the radiative lepton branching ratios can be used [via eqs. (4.11), (4.12) and (4.15)] to determine upper limits on the off-diagonal matrix elements of M^2_{LC}. Examples of such bounds for $M_2 = \mu = 200$ GeV and two sets of $\tan\beta$ and m_L^{min} (previously exhibited in Table 2) are shown in Table 3. In obtaining these bounds, we assumed that m^2_L is diagonal so that fig. 1(a) does not contribute to the decay amplitude.[18] We then varied the matrix elements of M^2_{LC} until the constraints from measurements were violated. Moreover, we incorporated the full numerical one loop calculation for $\ell^J \to \ell^I\gamma$, presented in Section 4.1 rather than the approximate expressions given, e.g., in eq. (4.15). Notice that there exist lower bounds for the diagonal elements of M^2_{LC} from $(g-2)_\mu$, but upper bounds for the off-diagonal elements of M^2_{LC} from BR($\ell^J \to \ell^I + \gamma$).

The results of Table 3 illustrate that the bounds on the square roots of the off-diagonal elements of M^2_{LC} are at least 10—100 times smaller than the square roots of the diagonal

[18]Non-vanishing off-diagonal elements of m^2_L should in most cases tighten the bounds on M^2_{LC}, barring accidental cancellations between the amplitudes obtained from fig. 1(a) and (b).

elements. It is convenient to rewrite eq. (3.39) in the following form:

$$\begin{aligned} M^2_{LC} &= m^2_L + \tfrac{1}{2}M^2_Z\cos 2\beta + m^*_D M^{-1} m^2_N (\mathbb{1} + M^{-2}m^2_N)^{-1} M^{-1} m^T_D + \mathcal{O}(v^4 M^{-2}) \\ &= m^2_L + \tfrac{1}{2}M^2_Z\cos 2\beta + m^*_D M^{-1} m^2_N M^{-1} m^T_D + \mathcal{O}(v^4 M^{-2}) + \mathcal{O}(v^2 m^4_N M^{-4}) \,, \end{aligned} \quad (4.17)$$

where we have expanded out the quantity $(\mathbb{1} + M^{-2}m^2_N)^{-1}$ under the assumption that $\|M^{-2}m^2_N\| < 1$ (to be justified shortly). Eq. (4.17) implies that the off-diagonal elements of M^2_{LC} are roughly of order $m^2_D m^2_N/M^2$ (barring any accidental cancellations). If we assume that m_D is of order the electroweak scale, then the bounds on the off-diagonal elements given in Table 3 imply that

$$x \equiv \frac{||m^2_N||}{||M^2||} \lesssim \mathcal{O}(10^{-2}) \,, \quad (4.18)$$

with the strongest bound given by $\mu \to e\gamma$ decay. This result suggests that $\|m^2_N\|^{1/2}$ cannot be larger than about 10% of the Majorana mass scale M. Hence, $M^2 + m^2_N \simeq M^2$ and for the estimates of the magnitude of the entries of the lepton number violating mass matrix M^2_{LV} in the next section we henceforth set $m^2_N = 0$.

5 Neutrino masses and the lepton number violating parameters

In this section we examine the constraints on the lepton number violating sneutrino squared-mass matrix M^2_{LV} from our knowledge of the physical (light) neutrino masses and mixing angles.

5.1 One-loop contributions to neutrino masses

The effective operator that describes the light neutrino mass matrix is given by:

$$-\mathscr{L}_{m_{\nu_\ell}} = \tfrac{1}{2} M^{IJ}_{\nu_\ell} \nu^I_\ell \nu^J_\ell + \text{H.c.} \quad (5.1)$$

Note that $\nu^I_\ell \nu^J_\ell$ is a $\Delta L = 2$ operator, since it changes lepton number by two units. In Section 3.1, we evaluated the tree-level contribution to M_{ν_ℓ} [cf. eq. (3.8)]. However, one-loop contributions to the light neutrino mass matrix can be significant, and in some cases these can be as or more important than the tree-level contribution [18, 28]. The dominant one-loop graph involves a loop containing neutralinos and light sneutrinos, as shown in fig. 2(a). Due to the presence of the lepton number-violating sneutrino squared-mass matrix

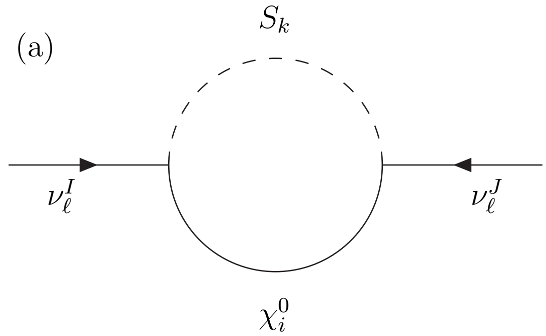

Figure 2: One-loop corrections to light neutrino masses. (a) The loop consisting of light sneutrinos ($S_k,\ k = 1\ldots6$) and neutralinos ($\chi_i^0,\ i = 1\ldots4$) is the dominant contribution. (b) The loop consisting of a neutral Higgs (or Goldstone) boson and a heavy neutrino contributes a relative correction to the light neutrino mass of at most a few percent. The contributions of the corresponding graphs (not shown) in which the light sneutrinos in (a) are replaced by heavy sneutrinos and the heavy neutrinos in (b) are replaced by light neutrinos are suppressed by an additional powers of $\mathcal{O}(vM^{-1})$ as explained in Appendix D.

M^2_{LV}, which violates lepton number by two units, fig. 2(a) can contribute significantly to the light neutrino mass matrix. Other one-loop contributions shown in fig. 2(b), yield corrections to the light neutrino mass matrix of at most a few percent, and thus can be neglected.

In order to establish the results just quoted, we begin by reviewing the relevant interactions that govern the one-loop contributions to the light neutrino masses. The light neutrino couplings arise from eq. (2.8) and the supersymmetric sneutrino-neutrino-neutral gaugino interactions. After isolating the interaction terms containing one neutrino field, one arrives at

$$\mathscr{L}_\nu = -Y_\nu^{IJ}\left(\nu_L^I\nu_L^{cJ}H_2^2 + \widetilde{H_2^2}\nu_L^I\tilde{\nu}_R^{J*} + \widetilde{H_2^2}\nu_L^{cJ}\tilde{\nu}_L^{I*}\right) + \frac{i}{\sqrt{2}}(g_2\widetilde{W}^3 - g_1\widetilde{B})\nu_L^I\tilde{\nu}_L^{I*} + \text{H.c.}\,, \quad (5.2)$$

where $\widetilde{W}^3$ and $\widetilde{B}$ are the SU(2) and U(1) neutral (two-component) gaugino fields, and g_2 and g_1 are the corresponding gauge couplings. Using eqs. (3.4) and (3.6), it follows that $\nu_L \simeq \nu_\ell + m_D^* M^{-1}\nu_h^c$ and $\nu_L^c \simeq \nu_h^c - M^{-1}m_D^T\nu_\ell$. Likewise, it follows from eqs. (3.33) and (3.37) that

$$\tilde{\nu}_L \simeq \tilde{\nu}_\ell + m_D^* M(M^2 + m_N^2)^{-1}\tilde{\nu}_h^*\,, \quad (5.3)$$

$$\tilde{\nu}_R^* \simeq \tilde{\nu}_h^* - (M^2 + m_N^2)^{-1}Mm_D^T\tilde{\nu}_\ell\,. \quad (5.4)$$

Thus, the effective interaction involving (at least) one light neutrino field is given by:

$$\begin{aligned}\mathscr{L}_{\nu_\ell} \simeq -Y_\nu^{IJ}\Big\{&\widetilde{H_2^2}\nu_\ell^I\tilde{\nu}_h^{J*} + \nu_\ell^I\nu_h^{cJ}H_2^2 - (m_D M^{-1})^{KJ}\left(\widetilde{H_2^2}\nu_\ell^K\tilde{\nu}_\ell^{I*} + \nu_\ell^I\nu_\ell^K H_2^2\right) \\ &-[(M^2+m_N^2)^{-1}Mm_D^T]^{JK}\widetilde{H_2^2}\nu_\ell^I\tilde{\nu}_\ell^K\Big\} \\ &+\frac{i}{\sqrt{2}}(g_2\widetilde{W}^3 - g_1\widetilde{B})\left[\nu_\ell^I\tilde{\nu}_\ell^{I*} + m_D M(M^2+m_N^{2*})^{-1}\nu_\ell^I\tilde{\nu}_h^I\right] + \text{H.c.}\end{aligned} \tag{5.5}$$

In order to perform the explicit loop computations, it is convenient to rewrite eq. (5.5) in terms of mass eigenstate fields. The Higgs field H_2^2 is expressed as [49]:

$$H_2^2 = \frac{1}{\sqrt{2}}\left[v_2 + h^0\cos\alpha + H^0\sin\alpha + i(\cos\beta A^0 + \sin\beta G^0)\right], \tag{5.6}$$

in terms of the CP-even Higgs fields h^0 and H^0 (where $m_{h^0} \le m_{H^0}$), the CP-odd Higgs field A^0 and the Goldstone field G^0, where $\tan\beta \equiv v_2/v_1$ and α is the CP-even Higgs mixing angle. We also define two-component mass-eigenstate neutralino fields κ_j^0 ($j = 1, \ldots, 4$) following ref. [7] by

$$\psi_i \equiv Z_N^{ij}\kappa_j^0, \qquad \text{where} \qquad \psi_i \equiv (-i\widetilde{B}, -i\widetilde{W}^3, \widetilde{H}_1^1, \widetilde{H}_2^2), \tag{5.7}$$

and Z_N is a unitary matrix that governs the Takagi-diagonalization of the complex symmetric 4×4 neutralino mass matrix, M_{χ^0} via $Z_N^T M_{\chi^0} Z_N = \text{diag}(M_{\chi_1^0}, \ldots, M_{\chi_4^0})$.

Before presenting the explicit computations, let us first estimate the order of magnitude of the loop-contributions to the neutrino mass due to the loop graphs of fig. 2(a) and (b), and the corresponding graphs (not shown) in which the light sneutrinos [heavy neutrinos] in graph (a) [(b)] are replaced by heavy sneutrinos [light neutrinos]. This analysis is presented in Appendix D—the results obtained there imply that the graphs of fig. 2(a) and (b) both yield contributions to the one-loop light neutrino mass matrix of order the tree-level light neutrino masses, multiplied by the appropriate vertex couplings and a typical loop factor. Other one-loop contributions not shown in fig. 2 are suppressed by additional powers of $\mathcal{O}(vM^{-1})$ and are utterly negligible.

We begin with an examination of the loop amplitude of fig. 2(b), which is governed by the light neutrino-heavy neutrino-Higgs interaction term of eq. (5.5). The internal heavy neutrino line is marked with an $\times$ to indicate the lepton-number violating propagator proportional to its (diagonal) mass $M\delta^{KL}$. Summing over all the internal neutral Higgs and Goldstone states, the leading $\mathcal{O}(M)$ term vanishes, leaving a subleading term of $\mathcal{O}(v^2M^{-1})$, which is the magnitude of the *light* neutrino mass. We find that fig. 2(b) yields a leading contribution to the light neutrino mass that is proportional to the tree-level light neutrino mass matrix [cf. eq. (3.8)]:

$$\delta M_{\nu_\ell} \approx -\frac{M_{\nu_\ell}}{32\pi^2}\frac{g_2^2}{c_W^2}\log\frac{\overline{M}}{\overline{m}_H}\,, \tag{5.8}$$

where $\overline{M}$ and $\overline{m}_H$ denote average heavy neutrino and Higgs boson masses. This correction turns out to be of the order of at most few percent. Additional corrections can also arise that modify the flavor structure of M_{ν_ℓ}, but these are not logarithmically enhanced and are thus even smaller.

Hence, the possibility of a *significant* one-loop contribution to the light neutrino mass matrix can only arise from fig. 2(a), which is governed by the light sneutrino-neutrino-gaugino interaction term of eq. (5.5). In the following, we examine the corresponding loop graph in which the external light neutrino fields are mass eigenstates $(\nu_\ell^J)^{\rm phys}$ [cf. eq. (3.9)]. Using four-component spinor methods, the amplitude for this graph (with incoming four-momentum p) will be denoted by

$$-\,i[(\not p\Sigma_V^{IJ} + \Sigma_S^{IJ})P_L + (\not p\Sigma_V^{IJ*} + \Sigma_S^{JI*})P_R]\,, \tag{5.9}$$

where the generic self energies $\Sigma_{V,S}^{IJ}(p^2)$ of the Majorana neutrino must be symmetric in its indices I, J. To evaluate this graph, we express the neutrino-sneutrino-gaugino interaction Lagrangian in terms of the four-component self-conjugate Majorana neutrino fields ν_M^I and the Majorana neutralino fields χ_i^0 [cf. Appendix A]:[19]

$$\mathscr{L}_{\chi\nu\tilde\nu} = -\tfrac{1}{2}\,(g_2 Z_N^{2i} - g_1 Z_N^{1i})(\mathcal{Z}_{\tilde\nu}^{Ik} - i\mathcal{Z}_{\tilde\nu}^{(I+3)k})U_{MNS}^{IJ}\,\bar\chi_i^0 P_L \nu_M^J S_k + \text{H.c.}\,, \tag{5.10}$$

where the neutralino mixing matrix Z_N is defined in eq. (5.7). The resulting $\overline{\rm DR}$-renormalized neutrino mass matrix at one-loop order is given by:

$$(M_{\nu_\ell}^{(1-\text{loop})})^{IJ} \;=\; m_{\nu_\ell I}(\mu_R)\,\delta^{IJ} + \text{Re}\left[\Sigma_S^{IJ}(\overline{m}^2_{\nu_\ell}) + \tfrac{1}{2}m_{\nu_\ell I}\Sigma_V^{IJ}(\overline{m}^2_{\nu_\ell}) + \tfrac{1}{2}m_{\nu_\ell J}\Sigma_V^{JI}(\overline{m}^2_{\nu_\ell})\right], \tag{5.11}$$

where the loop diagrams are regularized by dimensional reduction and the tree level diagonal mass, $m_{\nu_\ell I}$, is defined at the renormalization scale μ_R. In addition, $\overline{m}^2_{\nu_\ell}$, is some average neutrino mass scale, which to a very good approximation can be taken to be zero in the explicit loop calculations presented below.

In order to determine the masses of the light neutrinos at one-loop accuracy, it is usually sufficient to calculate the diagonal matrix elements of the self energies (i.e., by setting $I = J$ in eq. (5.11)), assuming that the tree-level neutrino masses are non-degenerate. However,

[19] More explicitly, the non-zero components of $P_L\nu_M^I$ are the two-component neutrino fields $(\nu_\ell^I)^{\rm phys}$, and the non-zero components of $P_L\chi^0$ are the two-component neutralino fields κ_i^0 introduced in eq. (5.7).

in some cases $\Sigma_{S,V}^{IJ}$ can be numerically large for $I \neq J$. If the latter holds, then one must re-diagonalize the neutrino mass matrix, $(M_{\nu_\ell}^{\rm one-loop})^{IJ}$, in order to obtain the loop-corrected physical neutrino masses and corresponding mixing matrix $U_{\rm MNS}$ (more details of a similar procedure in the context of R-parity violating models can be found, e.g., in refs. [50] and [25]).

An explicit calculation of the diagram shown in fig. 2(a), in the limit $\overline{m}_{\nu_\ell}^2 \to 0$, yields

$$\Sigma_S^{IJ} = \frac{-m_{\chi_i^0}}{4(4\pi)^2}\,(g_2 Z_N^{2i} - g_1 Z_N^{1i})^2\,(\mathcal{Z}_{\tilde\nu}^{Lk} - i\mathcal{Z}_{\tilde\nu}^{(L+3)k})\,(\mathcal{Z}_{\tilde\nu}^{Mk} - i\mathcal{Z}_{\tilde\nu}^{(M+3)k})U_{\rm MNS}^{LI}U_{\rm MNS}^{MJ}\,B_0(m_{\chi_i^0}, m_{S_k}), \tag{5.12}$$

$$\Sigma_V^{IJ} = \frac{-1}{4(4\pi)^2}\,|g_2 Z_N^{2i} - g_1 Z_N^{1i}|^2\,(\mathcal{Z}_{\tilde\nu}^{Lk} - i\mathcal{Z}_{\tilde\nu}^{(L+3)k})\,(\mathcal{Z}_{\tilde\nu}^{Mk} + i\mathcal{Z}_{\tilde\nu}^{(M+3)k})U_{\rm MNS}^{LI}U_{\rm MNS}^{MJ*}\,B_1(m_{\chi_i^0}, m_{S_k}), \tag{5.13}$$

with an implicit sum over repeated indices, where $m_{\chi_i^0}$ and m_{S_k} are the neutralino and sneutrino masses, respectively, and B_0, B_1 are the standard 2-point loop-integrals [51] evaluated at $p^2 = 0$,

$$B_0(x,y) \;=\; \Delta - \log\frac{xy}{\mu_R^2} + 1 - \frac{x^2+y^2}{x^2-y^2}\log\frac{x}{y}\;, \tag{5.14}$$

$$B_1(x,y) \;=\; -\frac{1}{2}\Delta + \frac{1}{2}\log\frac{xy}{\mu_R^2} - \frac{3}{4} - \frac{y^2}{2(x^2-y^2)} + \left(\frac{x^4}{(x^2-y^2)^2} - \frac{1}{2}\right)\log\frac{x}{y}\;, \tag{5.15}$$

with $\Delta \equiv 2/(4-d) - \gamma + \ln 4\pi$ set to $\Delta = 0$ in the minimal subtraction renormalization scheme. Note that Σ_S is finite, i.e. in the sum over k the dependence on Δ and μ_R cancels exactly due to the orthogonality of $\mathcal{Z}$. Likewise, Σ_V^{IJ} is finite for $I \neq J$, which is easily verified after using the orthogonality of $\mathcal{Z}$ and the unitarity of $U_{\rm MNS}$. This is to be expected since in the mass basis there are (by definition) no tree-level off-diagonal neutrino mass matrix elements. In contrast, Σ_V^{JJ} is divergent, and after minimal subtraction it is here that the μ_R dependence resides.

We now examine the relative magnitudes of the various contributions in eq. (5.11) to the loop-corrected neutrino mass. First, we observe that Σ_V [given by eq. (5.13)] is dimensionless and has a magnitude of the order of a typical electroweak correction (this has been numerically confirmed). Thus, the one loop contribution of the terms proportional to the minimally subtracted Σ_V in eq. (5.11) is at most a few percent of the tree-level neutrino mass. Given the current experimental accuracy of neutrino data, this latter correction can be neglected, as it does not provide any constraints on sneutrino parameters. Thus, we focus on Σ_S [given by eq. (5.12)], which can be simplified by employing the MIA expansion described in Section 4.3. The end result is:

$$\begin{aligned}\delta M^{IJ}_{\nu_\ell} &\equiv (M^{1-\text{loop}}_{\nu_\ell})^{IJ} - m_{\nu_{\ell I}}\delta^{IJ} \\ &\simeq \frac{-1}{32\pi^2}\sum_{i,K,M} m_{\chi^0_i}\text{Re}\left[(g_2 Z_N^{2i} - g_1 Z_N^{1i})^2\, U^{KI}_{\text{MNS}} U^{MJ}_{\text{MNS}} \left(M^2_{LV}\right)_{KM}\right] \left(\frac{\Delta B_0}{\Delta m^2}\right)_{iKM}, \end{aligned} \tag{5.16}$$

where in analogy to (4.16) we define

$$\left(\frac{\Delta B_0}{\Delta m^2}\right)_{kIJ} \equiv \begin{cases} \dfrac{B_0(m_{\chi^0_k}, m_{\tilde{\nu}^I_\ell}) - B_0(m_{\chi^0_k}, m_{\tilde{\nu}^J_\ell})}{m^2_{\tilde{\nu}^I_\ell} - m^2_{\tilde{\nu}^J_\ell}}, & \text{for } I \neq J\,, \\ \dfrac{\partial B_0(m_{\chi^0_k}, m_{\tilde{\nu}^I_\ell})}{\partial m^2_{\tilde{\nu}^I_\ell}}, & \text{for } I = J\,. \end{cases} \tag{5.17}$$

and the CP-averaged sneutrino masses, $m_{\tilde{\nu}^I_\ell}$, are defined below eq. (4.16). As expected, this contribution is finite and is explicitly lepton number violating, as it is proportional to the matrix M^2_{LV}. Eq. (5.16) is a generalization of eq. (7) of ref. [18] to the 3-flavor seesaw model.[20]

The results given in Section 5.1 can be used to estimate the bounds on the heavy sneutrino soft parameters m^2_N, m^2_B, X_ν imposed by the current experimental measurements of neutrino masses and mixing. These bounds allow for a significant one-loop correction to the light neutrino mass matrix, $\delta M^{IJ}_{\nu_\ell}$, which could even compete with the corresponding tree-level masses. Further details will be given in Sections 5.3 and 5.4.

5.2 Radiative generation of neutrino masses and mixing

It is very tempting to explain the characteristics of the neutrino mass spectrum as a consequence of radiative corrections. The most economical possibility is one in which the pattern of neutrino masses is entirely radiatively generated by the loop corrections. However, in the supersymmetric seesaw model this is not possible. If one sets $m_{\nu_{\ell I}} = 0$ (for all I) in eq. (5.16), then $m_D = 0$ (or equivalently, $Y_\nu = 0$), in which case only the light sneutrino-neutrino-gaugino interaction of eq. (5.5) survives. However, this interaction generates a one-loop neutrino mass that is proportional to M^2_{LV} [cf. eq. (5.16)], which vanishes in the limit of $m_D = 0$.

Here, we shall be less ambitious and investigate whether the hierarchy and/or the flavor mixing of neutrinos can be generated entirely by loop effects. As we shown below, such a scenario seems to be possible. However, in order to obtain the correct values of the light neutrino mixing matrix elements, a fine-tuning of sneutrino parameters may be required.

[20] We correct here a typographical in eq. (7) of ref. [18] where $(g_2 Z_N^{2i} - g_1 Z_N^{1i})^2$ is incorrectly written as $|g_2 Z_N^{2i} - g_1 Z_N^{1i}|^2$.

To be more specific, consider the following scenario. At tree level we assume the Yukawa coupling matrix Y_ν to be real, non-negative and flavor diagonal, i.e. $Y_\nu^{IJ} = Y_\nu^I \delta^{IJ}$ (with $Y_\nu^I \geq 0$). Consequently, the tree level neutrino mass matrix [eq. (3.8)] is also real, non-negative and diagonal so that $U_{\rm MNS}^{\rm tree} = i\mathbb{1}$. Then, the one-loop correction to the neutrino mass matrix [eq. (5.16)] is proportional to:

$$\alpha_{IJ} \equiv \frac{1}{32\pi^2} \sum_{i=1}^{4} m_{\chi_i^0} (g_1 Z_N^{1i} - g_2 Z_N^{2i})^2 \left(\frac{\Delta B_0}{\Delta m^2} \right)_{iIJ} . \tag{5.18}$$

If one assumes that the flavor splitting of the light sneutrino masses is small, then the ratio $(\Delta B_0/\Delta m^2)_{iIJ}$ is approximately constant with the respect to the indices I, J, so that $\alpha_{IJ} \approx \alpha$ is roughly constant. Therefore, the one-loop corrected neutrino mass matrix [eq. (5.11)] can be written as

$$m_{\nu_\ell}^{(1-\rm loop)} \simeq -m_D M^{-1} m_D + {\rm Re}\left(\alpha M_{LV}^2\right) . \tag{5.19}$$

Since we have assumed above that Y_ν is diagonal, it follows that $m_D \equiv v_2 Y_\nu/\sqrt{2}$ is also diagonal, in which case there is no need to distinguish between m_D and its transpose. For simplicity, we shall further assume that $m_N^2 \ll M^2$. Then, using eq. (3.40) for M_{LV}^2, in which only the leading $\mathcal{O}(vM^{-1})$ terms are kept [under the assumption that $m_B^2 \sim \mathcal{O}(vM)$ as suggested by eq. (2.14)], we may express eq. (5.19) in the following form:

$$\begin{aligned} m_{\nu_\ell}^{(1-\rm loop)} \simeq\ & -[\mathbb{1} - {\rm Re}(\alpha X_\nu)]\, m_D M^{-1} m_D\, [\mathbb{1} - {\rm Re}(\alpha X_\nu^T)] \\ & -\ 2\, m_D \frac{1}{M} {\rm Re}(\alpha m_B^2) \frac{1}{M} m_D\ + {\rm Re}(\alpha X_\nu)\, m_D M^{-1} m_D\, {\rm Re}(\alpha X_\nu^T) . \end{aligned} \tag{5.20}$$

To achieve the correct hierarchy of neutrino masses and mixings, one possible strategy is to demand that the sum of the last two terms on the right hand side of eq. (5.20) is negligible, in which case the first term yields the correct physical neutrino masses and the mixing matrix. Then, using eq. (2.5), we perform a Takagi-diagonalization to identify the physical (loop-corrected) neutrino masses and mixing matrix elements:

$$-\,[\mathbb{1} - {\rm Re}(\alpha X_\nu)]\, m_D M^{-1} m_D\, [\mathbb{1} - {\rm Re}(\alpha X_\nu^T)] \ = \ (U_{\rm MNS}^{\rm phys})^*\, m_{\nu_\ell}^{phys}\, (U_{\rm MNS}^{\rm phys})^\dagger , \tag{5.21}$$

where $m_{\nu_\ell}^{phys}$ is the (non-negative) diagonal physical neutrino mass matrix. One can solve eq. (5.21) analytically for ${\rm Re}(\alpha X_\nu)$, which yields:

$${\rm Re}(\alpha X_\nu) = \mathbb{1} - i (U_{\rm MNS}^{\rm phys})^* (m_{\nu_\ell}^{phys})^{1/2} R M^{1/2} m_D^{-1} , \tag{5.22}$$

where R is a complex orthogonal matrix, subject to the restriction that the right hand side of eq. (5.22) is real. Thus, starting from *any* hierarchy of the tree-level diagonal, non-vanishing Yukawa couplings Y_ν^I, the special choice of X_ν given in eq. (5.22) allows us to reproduce the correct neutrino mass hierarchy and the mixing matrix.

Clearly, the scenario just presented is not very realistic from the phenomenological point of view. To achieve the desired result, a specific form of the X_ν parameter, very close to perturbativity limit of Y_ν and the charged slepton masses is required, as well as a rather precise cancellation between the last two terms of eq. (5.20). Nevertheless, our example above provides an analytical existence proof for a radiative mixing scenario. In general, for given Y_ν and M, many choices of sneutrino parameters leading to the correct pattern of neutrino masses and mixing at the one-loop level exist, but they need to be determined numerically. Presumably, all successful scenarios require a certain degree of fine-tuning, but perhaps some solutions would be deemed acceptable.

5.3 Universal parameters at the scale M

The magnitudes of the parameters A_ν, m_B^2 and m_N^2 that govern the behavior of the heavy sneutrino sector are connected with the mechanism of supersymmetry breaking [cf. eq. (2.6)]. These parameters decouple at the scale $M \gg M_Z$ where the sneutrino superfield $\widehat{N}$ decouples. If the scale M is close to the GUT scale then soft SUSY breaking parameters are restricted by GUT symmetry considerations. Further assumptions on the minimality of the Kähler potential in supergravity simplify our input parameters considerably, at the scale $M \sim M_{GUT}$,

$$A_\nu \;=\; A_0\, Y_\nu \quad , \quad m_B^2 \;=\; m_0\, M \quad , \quad m_N^2 \;=\; xM^2 \,, \tag{5.23}$$

where A_0 is a complex number, m_0 and x are real numbers, M is a diagonal 3×3 Majorana neutrino matrix [cf. eq. (2.5)] and Y_ν is the neutrino Yukawa coupling [cf. eq. (2.1)].

Under the universality assumptions of eq. (5.23), the matrices M_{LC}^2 and M_{LV}^2 assume the following simple forms at the GUT scale:

$$M_{LC}^2 \;=\; m_L^2 \;+\; \frac{1}{2}M_Z^2\cos 2\beta + \frac{x}{1+x} m_D^* m_D^T \,, \tag{5.24}$$

$$M_{LV}^2 \;=\; \frac{2\, M_{\nu_\ell}}{1+x}\left(A_0 + \mu^* \cot\beta - \frac{m_0}{1+x}\right) \,, \tag{5.25}$$

where the light tree-level neutrino mass matrix M_{ν_ℓ} is given in eq. (3.8). As parameters "run" from the GUT scale to low energies, m_L^2 receives renormalization from other Yukawa and gauge interactions. In contrast, all the parameters associated with the superfield $\widehat{N}$ are hardly affected since $M \sim M_{GUT}$. Moreover, the neutrino mass matrix M_{ν_ℓ} and the superpotential parameter μ are both multiplicatively renormalized. Hence, just above the scale of low-energy supersymmetry breaking, the low-energy value of M_{LV}^2 is still given by eq. (5.25), with the parameters on the right-hand side defined at the low scale. At the low-energy supersymmetry-breaking scale the $\overline{\rm DR}$ running neutrino mass matrix $M_{\nu_\ell}(\mu_R)$ [or its diagonal form $m_{\nu_\ell^I}(\mu_R)$] receives finite threshold corrections from the neutralino–sneutrino loop in fig. 2(a). The one-loop correction to the neutrino mass matrix given in eq. (5.16) is proportional to the diagonal tree-level neutrino mass matrix.[21] Hence, the one-loop corrected neutrino masses assume the very simple and suggestive form

$$m_{\nu_\ell I}^{(1-{\rm loop})} \;=\; m_{\nu_\ell I}\left[1+2{\rm Re}\frac{\alpha}{(1+x)}\left(A_0+\mu^*\cot\beta-\frac{m_0}{1+x}\right)\right]\,, \tag{5.26}$$

where α is defined in eq. (5.18) and all parameters are now defined at the scale $\mu_R = M_Z$.

We next examine the light sneutrino mass difference. Since the results of Table 3 imply that M^2_{LC} is very close to diagonal form, it follows that $Q_0 \simeq 1$ (cf. discussion above eq. (3.48)]. Combining the results of eqs. (3.49), (3.52) and (3.66), we derive

$$\left(\frac{\Delta m_{\tilde\nu_\ell}}{m_{\nu_\ell}}\right)_I \;=\; \frac{2}{m_{\tilde\nu_\ell I}m_{\nu_\ell I}}\left|\frac{(M_{\nu_\ell})_{II}}{1+x}\left(A_0+\mu^*\cot\beta-\frac{m_0}{1+x}\right)\right|\,, \tag{5.27}$$

which is identical to the one flavor case found in eq. (3.68) and in Ref. [18] if the neutrino mass matrix M_{ν_ℓ} is diagonal. In the more general case of non-diagonal M_{ν_ℓ}, the diagonal elements of the neutrino mass matrix do not coincide with the neutrino masses $m_{\nu_\ell I}$. Consequently, the quantity $(\Delta m_{\tilde\nu_\ell}/m_{\nu_\ell})_I$ exhibits non-trivial dependence on the flavor index I.

To produce quantitative results, we need to initialize the neutrino Yukawa couplings in such a way that we always reproduce the "observed" MNS mixing matrix. Using eqs. (3.8) and (3.10), it follows that

$$m_{\rm D} = iU^*_{\rm MNS}\,(m^{\rm phys}_{\nu_\ell})^{1/2}\,R^T\,M^{1/2}\,, \tag{5.28}$$

where R is an arbitrary complex orthogonal matrix [47], with three (complex) angles, $\theta_{1,2,3}$. (As the sign of R is undetermined, one may choose det $R = 1$ without loss of generality.) In the plots that follow, we assume a hierarchical spectrum for the neutrinos, and all relevant input parameters are displayed in Table 4. The value for m_L adopted in Table 4 is consistent with a supersymmetric interpretation of the observed experimental excess for δa_μ.

[21]Indeed, assuming universal parameters at the GUT scale, and noting that $x \lesssim \mathcal{O}(10^{-2})$ [cf. eq. (4.18)], it follows that $M^2_{LC} \simeq m^2_{LC}\mathbb{1}$ at the GUT scale, where m^2_{LC} is one of the approximately degenerate eigenvalues of M^2_{LC}. The positive square roots of the eigenvalues of M^2_{LC}, evaluated at the low-energy scale, are identified as the three CP-averaged light sneutrino masses. Although m^2_L is no longer proportional to the identity matrix at low-energies, this latter effect is formally of higher order in the loop expansion of $\delta M^{IJ}_{\nu_\ell}$ [cf. eq. (5.16)]. Consequently, we can neglect the flavor splitting of the CP-averaged light sneutrino masses in the evaluation of the ratio $(\Delta B_0/\Delta m^2)_{iKM}$, in which case this ratio is roughly constant with respect to the indices K and M as discussed below eq. (5.18).

Input Parameters			
Neutrino Sector		SUSY Sector	
$m^{\rm phys}_{\nu_{\ell 1}}$	10^{-14}	A_0	0
$m^{\rm phys}_{\nu_{\ell 2}}$	$\sqrt{\Delta m^2_{\rm sol}}$	m_0	0
$m^{\rm phys}_{\nu_{\ell 3}}$	$\sqrt{\Delta m^2_{\rm atm}}$	μ	350
θ_1	0.2+0.1 i	$\tan\beta$	10
θ_2	0.3	$M_{\tilde{B}}$	95
θ_3	0.1 + 0.5i	$M_{\tilde{W}}$	189
M_1	10^{14}	x	0.0
M_2	2×10^{14}	m_L	197
M_3	5×10^{14}	m_R	135

Table 4: If not otherwise indicated, the input parameters that govern the neutrino and SUSY sectors listed above have been employed in our numerical analysis. We take $\Delta m^2_{\rm sol} = (8.0^{+0.4}_{-0.3})\times 10^{-5}$ eV^2 and $\Delta m^2_{\rm atm} = (2.45\pm 0.55)\times 10^{-3}$ eV^2 from Ref. [11]. The values for $\theta_{1,2,3}$ above are representative choices (as these angles are not fixed by the light neutrino data). All mass parameters in the above table are in GeV units.

In fig. 3 we plot the ratios $(\Delta m_{\tilde{\nu}_\ell}/m_{\nu_\ell})_I$ [upper panels] and $(m^{(1-{\rm loop})}_{\nu_\ell}/m_{\nu_\ell})_I$ [lower panels] as functions of the SUSY-breaking parameters m_0 [left panels] and A_0 [right panels]. When varying m_0 we set $A_0 = 0$ and when varying A_0 we set $m_0 = 0$. Otherwise, our input parameters are as specified in Table 4. In obtaining these results, we have incorporated the full one-loop contribution to the neutrino masses. In the two lower panel plots, the ratios $(m^{(1-{\rm loop})}_{\nu_\ell}/m_{\nu_\ell})_I$ are nearly independent of the flavor I, and thus only one curve is shown. Our numerical results confirm our analytical approximate formulae of eqs. (5.26) and (5.27) and demonstrate that one must have $m_0 \lesssim 10^5$ GeV ($|A_0| \lesssim 10^5$ GeV) to guarantee that the radiative corrections to neutrino masses are less than 80% of the tree level neutrino mass. In this case, the sneutrino mass difference is at most $\Delta m_{\tilde{\nu}_\ell} \lesssim 300\,\Delta m_{atm} \simeq 15$ eV.

For completeness, we plot in fig. 4 the results for $g_\mu - 2$ anomaly and the branching ratios for the decays $\ell^J \to \ell^I\gamma$ in the case of universal parameters at the SUGRA scale. The results shown in fig. 4 confirm our choices of a lower bound for m_L [cf. Table 2] obtained in Section 4.1 and an upper bound for x [cf. eq. (4.18)] obtained in Section 4.3.

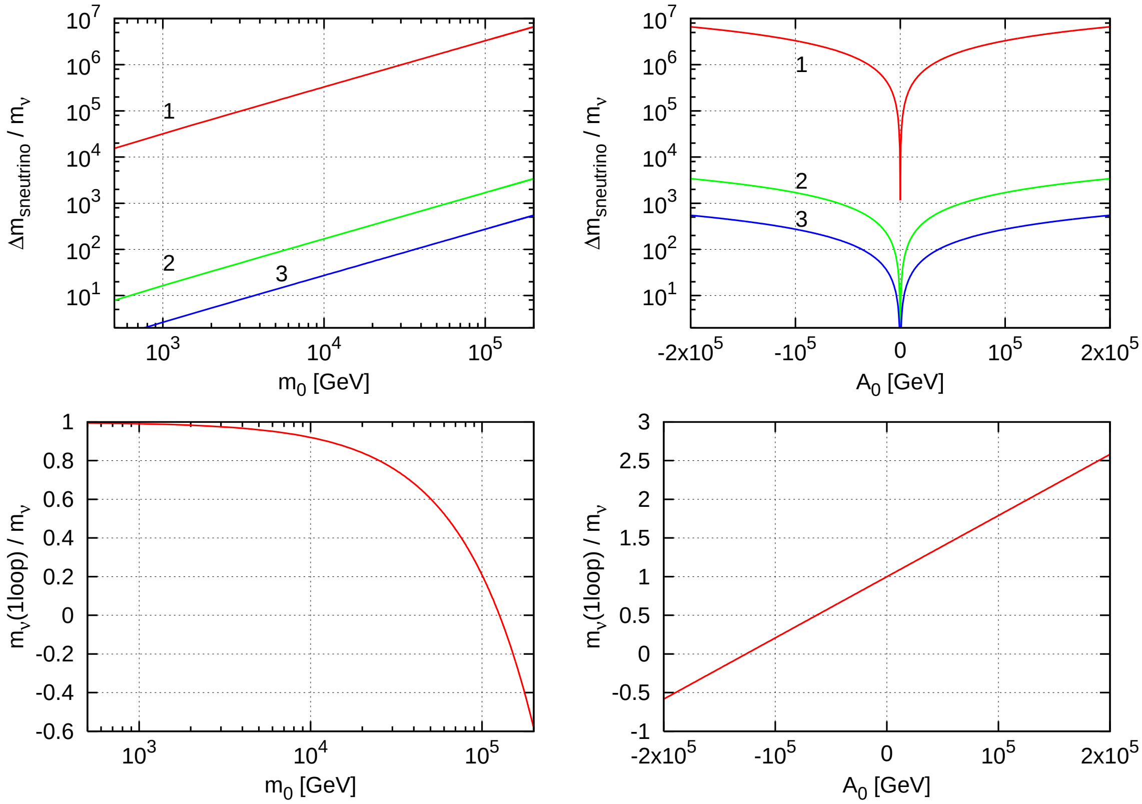

Figure 3: Predictions for the ratios $(\Delta m_{\tilde{\nu}_\ell}/m_{\nu_\ell})_I$ and $(m_{\nu_\ell}^{(1-\text{loop})}/m_{\nu_\ell})_I$ for the three neutrino states $(I = 1, 2, 3)$ as functions of the soft SUSY-breaking parameters m_0 and A_0. When varying m_0 [left panels] we set $A_0 = 0$ and when varying A_0 [right panels] we set $m_0 = 0$.

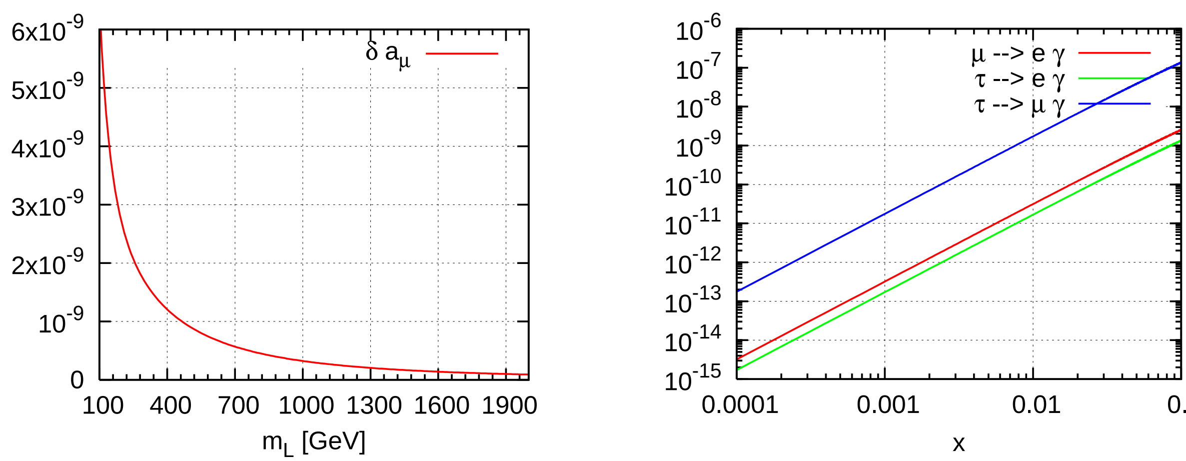

Figure 4: (a) In the left panel, the contribution to the muon anomalous magnetic moment from the diagrams in fig. 1 as a function of $m_L = m_R$ is exhibited. (b) In the right panel, the prediction for $\mathrm{BR}(\ell^J \to \ell^I \gamma)$ is shown as a function of the parameter $x = m_N^2/M^2$. The upper [lower] curves correspond to $\tau \to \mu\gamma$ [$\tau \to e\gamma$], and the middle curve to $\mu \to e\gamma$.

5.4 General case

So far we have dealt with universal boundary conditions for the supersymmetric parameters. One can set general bounds for the lepton number violating matrix elements of M_{LV}^2 from eq. (5.16) and the "naturalness" assumption of $\delta m_{\nu_\ell} \lesssim m_{\nu_\ell}$. In the general case, appropriate bounds can be derived only numerically and depend on the particular form of the MNS matrix. Analytical estimates can be obtained using the following approach. Let us require that the one-loop corrections to the neutrino mass matrix do not significantly affect the physical neutrino masses and their mixing. Combining Eqs. (3.10) and (5.16), one gets for any I, J:

$$|U_{\rm MNS}^{MI} \left(M_{\nu_\ell}\right)_{MN} U_{\rm MNS}^{NJ}| \geq \left| \frac{m_{\chi_i^0}}{32\pi^2} \,{\rm Re}\left[(g_2 Z_N^{2i} - g_1 Z_N^{1i})^2 \, U_{\rm MNS}^{MI} U_{\rm MNS}^{NJ} \left(M_{LV}^2\right)_{MN}\right] \left(\frac{\Delta B_0}{\Delta m^2}\right)_{iMN} \right| . \tag{5.29}$$

The structure of the $U_{\rm MNS}$ factors on both sides of eq. (5.29) is identical, so roughly [barring possible cancellations between terms and the effects of truncating a potential imaginary part[22] of $U_{\rm MNS}^{MI} \left(M_{\nu_\ell}\right)_{MN} U_{\rm MNS}^{NJ}$], the condition above can be rewritten as:

$$\begin{aligned} |\left(M_{\nu_\ell}\right)_{MN}| = |\left(m_D M^{-1} m_D^T\right)_{MN}| \; &\geq \left| \frac{m_{\chi_i^0}}{32\pi^2} \,{\rm Re}\left[(g_2 Z_N^{2i} - g_1 Z_N^{1i})^2 \; \left(M_{LV}^2\right)_{MN}\right] \left(\frac{\Delta B_0}{\Delta m^2}\right)_{iMN} \right| \\ &\approx \; |\alpha_{MN} \left(M_{LV}^2\right)_{MN}| \, , \end{aligned} \tag{5.30}$$

with α_{MN} defined in eq. (5.18).

Further estimates depend on the particular choice of the m_D (or Y_ν) and M and on the neutralino sector parameters. For example, using the parameters specified in Table 4, one has $\alpha_{MN} \approx \alpha \sim 4 \times 10^{-6}$ GeV^{-1}, so that

$$|\left(M_{LV}^2\right)_{MN}| \leq 2.5 \times 10^5 \text{ GeV} \, |\left(M_{\nu_\ell}\right)_{MN}| \, . \tag{5.31}$$

Eq. (5.31) implies that in the general case one should expect the entries of the matrix M_{LV}^2 to be no more than 5 or 6 orders of magnitude larger then the typical scales in the effective neutrino mass matrix; i.e. of the order of a few MeV^2. Bounds on M_{LV}^2 can be also translated into bounds on X_ν and m_B^2. From eq. (3.40) one can see that, barring fine tuning, we have approximate relations $M_{LV}^2 \sim M_{\nu_\ell} X_\nu$ or $M_{LV}^2 \sim M_{\nu_\ell} m_B^2/M$. Thus the rough estimates we made above suggest that both X_ν and m_B^2/M should be smaller than approximately 100 TeV.

[22]If the Higgsino mixing parameter μ and the lepton trilinear coupling A_ℓ are real (the case of complex μ and A_ℓ has been extensively discussed in the literature, see e.g. [52]) then there is no bound on the imaginary parts of the matrices M_{LC}^2 and M_{LV}^2.

Stronger bounds on the matrix elements of M^2_{LV} can be obtained numerically after assuming some particular form of the MNS matrix. As an example, under the assumption of tri-bimaximal mixing of ref. [53] and the parameters given in Table 4,

$$M^2_{LV} \lesssim \begin{pmatrix} 2\times 10^{-9} & ... & ... \\ ... & 2\times 10^{-6} & ... \\ ... & ... & 10^{-5} \end{pmatrix} \text{GeV}^2 \quad , \tag{5.32}$$

where the dots indicate elements with similar bounds as the diagonal ones. The significant suppression of the lepton number violating matrix elements of M^2_{LV} relative to the lepton number conserving matrix elements $M^2_{LC} \sim \mathcal{O}(v^2)$ is particularly noteworthy.

6 Sneutrino Oscillations

The theory behind sneutrino oscillations follows closely the very well known theory of oscillations in the neutral Kaon-meson system. The light sneutrino state [cf. eq. (5.3)], $\widetilde{\nu}_\ell \simeq \widetilde{\nu}_L - m^*_D M(M^2+m^2_N)^{-1}\widetilde{\nu}^*_R$ is to leading order in vM^{-1} the supersymmetric partner of left-handed neutrino ν_L, and therefore couples to the $W^\pm$ and Z gauge bosons. For the present discussion, it suffices to approximate: $\widetilde{\nu}^I_\ell \simeq \widetilde{\nu}^I_L$, which we shall denote simply by $\widetilde{\nu}_I$ in this Section. The $\widetilde{\nu}_I$ can be produced, for example, in e^+e^- annihilation via s-channel Z exchange:

$$e^+ + e^- \to \widetilde{\nu}_I + \widetilde{\nu}^*_I \ . \tag{6.1}$$

When lepton number is conserved, the $\tilde{\nu}_I$ ($\tilde{\nu}^*_I$) possess a definite lepton number equal to -1 ($+1$) and they are produced in definite flavor eigenstates $I = 1, 2, 3$.

It is convenient to introduce a two-dimensional complex vector space spanned by a basis of vectors consisting of the sneutrinos states of a given flavor I, $|\tilde{\nu}_I\rangle$ and $|\tilde{\nu}^*_I\rangle$. Two important operators that act on this state are:

$$\hat{L} \equiv \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \ , \qquad \text{and} \qquad CP \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \tag{6.2}$$

where $\hat{L}$ is the lepton number operator and CP is the CP-operator in the $\{|\tilde{\nu}_I\rangle, |\tilde{\nu}^*_I\rangle\}$ basis. That is, $|\tilde{\nu}_I\rangle$ and $|\tilde{\nu}^*_I\rangle$ are eigenstates of $\hat{L}$:

$$\hat{L}|\tilde{\nu}_I\rangle = -|\tilde{\nu}_I\rangle \quad , \quad \hat{L}|\tilde{\nu}^*_I\rangle = +|\tilde{\nu}^*_I\rangle \ , \tag{6.3}$$

and the charge-conjugate parity operator CP transforms particle states into antiparticle states:

$$CP|\tilde{\nu}_I\rangle = |\tilde{\nu}_I^*\rangle \quad , \quad CP|\tilde{\nu}_I^*\rangle = |\tilde{\nu}_I\rangle \, . \tag{6.4}$$

The eigenstates of CP are given by

$$|\widetilde{\nu}_I^{(+)}\rangle \equiv \frac{1}{\sqrt{2}}\left(|\tilde{\nu}_I\rangle + |\tilde{\nu}_I^*\rangle\right) \quad , \quad |\widetilde{\nu}_I^{(-)}\rangle \equiv \frac{1}{i\sqrt{2}}(|\tilde{\nu}_I\rangle - |\tilde{\nu}_I^*\rangle) \, , \tag{6.5}$$

with definite eigenvalues

$$CP|\widetilde{\nu}_I^{(+)}\rangle = +|\widetilde{\nu}_I^{(+)}\rangle \quad , \quad CP|\widetilde{\nu}_I^{(-)}\rangle = -|\widetilde{\nu}_I^{(-)}\rangle \, . \tag{6.6}$$

The CP-even sneutrino state of flavor I, $|\widetilde{\nu}_I^{(+)}\rangle$, and the CP-odd sneutrino state of flavor I, $|\widetilde{\nu}_I^{(-)}\rangle$, are states of indefinite lepton number. Of course, these states are the real and imaginary parts of the sneutrino field of definite lepton number,

$$\tilde{\nu}_I = \frac{1}{\sqrt{2}}(\widetilde{\nu}_I^{(+)} + i\widetilde{\nu}_I^{(-)}) \, . \tag{6.7}$$

Inevitably, in a supersymmetric model with a mechanism that yields neutrino flavor oscillations, the sneutrino flavor states should oscillate as well. The sneutrino mass eigenstates, S_k, $(k = 1, 2...6)$ are linear combinations of the CP eigenstates $|\tilde{\nu}_I^{(\pm)}\rangle$, and for a three flavor system $(I = 1, 2, 3)$ they are related by:

$$|\widetilde{\nu}_I^{(+)}\rangle = \mathcal{Z}_{\tilde{\nu}}^{Ik}|S_k\rangle \quad , \quad |\widetilde{\nu}_I^{(-)}\rangle = \mathcal{Z}_{\tilde{\nu}}^{(I+3)k}|S_k\rangle \, , \tag{6.8}$$

where the real orthogonal 6×6 matrix with $\mathcal{Z}_{\tilde{\nu}}^{ij}$ has been introduced in eq. (3.75). The $|S_k\rangle$ are states of definite CP unless the following CP-violating conditions hold:

$$\mathcal{Z}_{\tilde{\nu}}^{I(J+3)} \neq 0 \quad , \quad \mathcal{Z}_{\tilde{\nu}}^{(I+3)J} \neq 0 \, , \qquad I, J = 1, 2, 3 \, . \tag{6.9}$$

In the presence of complex parameters in the Lagrangian (whose phases cannot be absorbed by field redefinition), one expects the conditions specified in eq. (6.9) to be satisfied (even in the case of a one-generation model).

Let us initially focus our analysis on the CP-conserving one-generation model. Consider the time evolution of the sneutrino states. The time dependence of a sneutrino in the state $|\widetilde{\nu}^{(\pm)}\rangle$ is governed by a definite frequency $\omega_\pm = E_\pm/\hbar$ where $E_\pm = (p^2c^2 + m_\pm^2 c^4)^{1/2}$. where m_+ and m_- are the masses of $|\widetilde{\nu}^{(+)}\rangle$ and $|\widetilde{\nu}^{(-)}\rangle$ respectively. If these masses are large compared to momentum p then the corresponding energies are $E_\pm \simeq m_\pm c^2$ (in which case, $\omega_\pm \simeq m_\pm$ in units where $\hbar = c = 1$). In addition to the time-dependent phase, we must also account for the fact that the sneutrinos decay exponentially (e.g. into a chargino and a lepton) with a lifetime of $\tau_\pm$ (for $\widetilde{\nu}^\pm$ respectively). We exhibit this time dependence explicitly by writing

$$\Psi_+(t) = e^{-i\omega_+ t - \frac{t}{2\tau_+}} |\widetilde{\nu}^{(+)}\rangle \qquad , \qquad \Psi_-(t) = e^{-i\omega_- t - \frac{t}{2\tau_-}} |\widetilde{\nu}^{(-)}\rangle \ , \tag{6.10}$$

where the $\widetilde{\nu}^{(\pm}$ are time-independent state vectors, That is, starting at $t = 0$, the probability for finding particle in the sneutrino state $\widetilde{\nu}^{(+)}$ is given by $|\langle\widetilde{\nu}^{(+)}|\Psi(t)\rangle|^2 = e^{-t/\tau_+}$, as expected.

The well known striking effects of the K-system (e.g., K–$\overline{K}$ mixing and regeneration) can also occur in the sneutrino system. For example, we demonstrate how sneutrinos states $|\tilde{\nu}\rangle$ can turn to states $|\tilde{\nu}^*\rangle$. If we start off with a sneutrino state that is $\Psi(0) = |\tilde{\nu}\rangle = \frac{1}{\sqrt{2}}(|\widetilde{\nu}^{(+)}\rangle + i|\widetilde{\nu}^{(-)}\rangle)$ at $t = 0$, then it follows that at time t,

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left[e^{-i\omega_+ t - \frac{t}{2\tau_+}} |\widetilde{\nu}^{(+)}\rangle + i e^{-i\omega_- t - \frac{t}{2\tau_-}} |\widetilde{\nu}^{(-)}\rangle \right] \ . \tag{6.11}$$

Then, the probability amplitude that the sneutrino $|\tilde{\nu}\rangle$ is in state $|\tilde{\nu}^*\rangle$ is

$$P_{\tilde{\nu}\to\tilde{\nu}^*}(t) = |\langle\tilde{\nu}^*|\Psi(t)\rangle|^2 = \frac{1}{4}\left[e^{-t/\tau_+} + e^{-t/\tau_-} - 2\, e^{-\frac{1}{2}\left(\frac{t}{\tau_+} + \frac{t}{\tau_-}\right)} \cos[(\omega_+ - \omega_-)t] \right] \ . \tag{6.12}$$

The quantum interference effects can only be seen if $t \simeq \tau_+ \simeq \tau_-$ and $(m_+ - m_-)\, t \equiv (\Delta m)t = \mathcal{O}(1)$. That is,

$$\frac{\Delta m}{\Gamma_{\tilde{\nu}}} \simeq \mathcal{O}(1) \ , \tag{6.13}$$

where $\Gamma_{\tilde{\nu}}$ is an average decay rate for the sneutrino, and Δm is the mass difference of the CP-even and CP-odd sneutrino states. Eq. (6.12) describes the oscillations of sneutrinos into antisneutrinos, or equivalently the oscillation between states of definite CP quantum number. We shall call this phenomena CP-driven oscillations.

Similarly, one may compute the probability that the initial state $|\tilde{\nu}\rangle$ is in the state $|\tilde{\nu}\rangle$ at time t. We find

$$P_{\tilde{\nu}\to\tilde{\nu}}(t) = |\langle\tilde{\nu}|\Psi(t)\rangle|^2 = \frac{1}{4}\left[e^{-t/\tau_+} + e^{-t/\tau_-} + 2\, e^{-\frac{1}{2}\left(\frac{t}{\tau_+} + \frac{t}{\tau_-}\right)} \cos[(\omega_+ - \omega_-)t] \right] \ . \tag{6.14}$$

One can also easily verity that $P_{\tilde{\nu}^*\to\tilde{\nu}^*} = P_{\tilde{\nu}\to\tilde{\nu}}$ and $P_{\tilde{\nu}^*\to\tilde{\nu}} = P_{\tilde{\nu}\to\tilde{\nu}^*}$. However, the probability $P_{\tilde{\nu}\to\tilde{\nu}}$ is proportional to the number of negatively charged leptons (N_{l^-}) due to the decay $\tilde{\nu} \to l^- + \chi^+$ while $P_{\tilde{\nu}\to\tilde{\nu}^*}$ is proportional to the number of positively charged leptons (N_{l^+}) due to the decay $\tilde{\nu}^* \to l^+ + \chi^-$. Then the asymmetry,

$$A_l = \frac{N_{l^-} - N_{l^+}}{N_{l^-} + N_{l^+}} \ , \tag{6.15}$$

is proportional to the quantum interference term $\cos(\Delta m\, t)$ in eqs. (6.12) and (6.14). That is, the lepton charge asymmetry A_l oscillates in time and provides a possible method for experimentally determining the value of Δm.

The signal for sneutrino–antisneutrino oscillations can be interpreted as the observation of a sneutrino that decays into a final state with a "wrong-sign" charged lepton. The phenomenological implications of such wrong-sign charged lepton final states at future colliders have been explored recently in Ref. [54].

We now turn to the three-generation model (allowing for the possibility of CP-violation) and consider the additional possibility of flavor metamorphosis. We pose the following question: Given the state $|\tilde{\nu}_I\rangle$ at time $t = 0$, what is the probability that the sneutrino at time t is in the state $|\tilde{\nu}^*_J\rangle$ or $|\tilde{\nu}_J\rangle$? Following the arguments given above eq. (6.11), we find that a sneutrino wave function involves with time according to

$$|\Psi_I(t)\rangle = \frac{1}{\sqrt{2}}\left(\mathcal{Z}_{\tilde{\nu}}^{Ik} + i\mathcal{Z}_{\tilde{\nu}}^{(I+3)k}\right) e^{-i\omega_k t - \frac{t}{2\tau_k}}\, |S_k\rangle\ . \tag{6.16}$$

Hence, the probabilities to be in the state $|\tilde{\nu}^*_J\rangle$ or $|\tilde{\nu}_J\rangle$ at time t are given by:

$$\begin{aligned} P_{\tilde{\nu}_I\to\tilde{\nu}^*_J}(t) &= P_{\tilde{\nu}^*_I\to\tilde{\nu}_J}(t) = \frac{1}{4}\sum_{k,s=1}^{6} e^{-t\left[\frac{1}{2\tau_k}+\frac{1}{2\tau_s}\right]}\cos\left[(\omega_k-\omega_s)t\right]\ \times \\ &\quad \Big(\mathcal{Z}_{\tilde{\nu}}^{Jk}\mathcal{Z}_{\tilde{\nu}}^{Ik}\mathcal{Z}_{\tilde{\nu}}^{Js}\mathcal{Z}_{\tilde{\nu}}^{Is} + \mathcal{Z}_{\tilde{\nu}}^{(J+3)k}\mathcal{Z}_{\tilde{\nu}}^{(I+3)k}\mathcal{Z}_{\tilde{\nu}}^{(J+3)s}\mathcal{Z}_{\tilde{\nu}}^{(I+3)s} - 2\mathcal{Z}_{\tilde{\nu}}^{Jk}\mathcal{Z}_{\tilde{\nu}}^{Ik}\mathcal{Z}_{\tilde{\nu}}^{(J+3)s}\mathcal{Z}_{\tilde{\nu}}^{(I+3)s} \\ &+\ \mathcal{Z}_{\tilde{\nu}}^{Jk}\mathcal{Z}_{\tilde{\nu}}^{(I+3)k}\mathcal{Z}_{\tilde{\nu}}^{Js}\mathcal{Z}_{\tilde{\nu}}^{(I+3)s} + \mathcal{Z}_{\tilde{\nu}}^{(J+3)k}\mathcal{Z}_{\tilde{\nu}}^{Ik}\mathcal{Z}_{\tilde{\nu}}^{(J+3)s}\mathcal{Z}_{\tilde{\nu}}^{Is} + 2\mathcal{Z}_{\tilde{\nu}}^{Jk}\mathcal{Z}_{\tilde{\nu}}^{(I+3)k}\mathcal{Z}_{\tilde{\nu}}^{(J+3)s}\mathcal{Z}_{\tilde{\nu}}^{Is}\Big)\ , \end{aligned} \tag{6.17}$$

$$\begin{aligned} P_{\tilde{\nu}_I\to\tilde{\nu}_J}(t) &= P_{\tilde{\nu}^*_I\to\tilde{\nu}^*_J}(t) = \frac{1}{4}\sum_{k,s=1}^{6} e^{-t\left[\frac{1}{2\tau_k}+\frac{1}{2\tau_s}\right]}\cos\left[(\omega_k-\omega_s)t\right]\ \times \\ &\quad \Big(\mathcal{Z}_{\tilde{\nu}}^{Jk}\mathcal{Z}_{\tilde{\nu}}^{Ik}\mathcal{Z}_{\tilde{\nu}}^{Js}\mathcal{Z}_{\tilde{\nu}}^{Is} + \mathcal{Z}_{\tilde{\nu}}^{(J+3)k}\mathcal{Z}_{\tilde{\nu}}^{(I+3)k}\mathcal{Z}_{\tilde{\nu}}^{(J+3)s}\mathcal{Z}_{\tilde{\nu}}^{(I+3)s} + 2\mathcal{Z}_{\tilde{\nu}}^{Jk}\mathcal{Z}_{\tilde{\nu}}^{Ik}\mathcal{Z}_{\tilde{\nu}}^{(J+3)s}\mathcal{Z}_{\tilde{\nu}}^{(I+3)s} \\ &+\ \mathcal{Z}_{\tilde{\nu}}^{Jk}\mathcal{Z}_{\tilde{\nu}}^{(I+3)k}\mathcal{Z}_{\tilde{\nu}}^{Js}\mathcal{Z}_{\tilde{\nu}}^{(I+3)s} + \mathcal{Z}_{\tilde{\nu}}^{(J+3)k}\mathcal{Z}_{\tilde{\nu}}^{Ik}\mathcal{Z}_{\tilde{\nu}}^{(J+3)s}\mathcal{Z}_{\tilde{\nu}}^{Is} - 2\mathcal{Z}_{\tilde{\nu}}^{Jk}\mathcal{Z}_{\tilde{\nu}}^{(I+3)k}\mathcal{Z}_{\tilde{\nu}}^{(J+3)s}\mathcal{Z}_{\tilde{\nu}}^{Is}\Big)\ . \end{aligned} \tag{6.18}$$

Note that the probabilities in eqs. (6.17) and (6.18) are unchanged under the interchange of flavor indices I and J, respectively. The three-generation model possesses both flavor and CP-driven oscillations.

In the supersymmetric seesaw model, neutrino mixing and masses are governed by a variety of parameters that contribute to the tree-level and one-loop neutrino mass matrix (cf. Section 5.2). Some of these parameters also are relevant for determining the structure of the real orthogonal sneutrino mixing matrix $\mathcal{Z}_{\tilde{\nu}}^{ij}$, which controls the properties of the sneutrino mixing as shown above. Consequently, the bounds on the model parameters discussed in Sections 4 and 5 can be used to significantly constrain the general form of eqs. (6.17) and (6.18).

The mass splittings among sneutrinos of different flavors is typically much larger than the sneutrino–antisneutrino mass splitting between sneutrino states of a given flavor. In particular, due to the renormalization group evolution of parameters, Δm^2_{IJ} is generally larger than few GeV^2, even in the case of universality assumptions at the high scale, whereas sneutrino–antisneutrino mass splittings are typically of order the light neutrino masses. The observability of oscillations depends on the ratio $\Delta m/\Gamma$ [cf. eq. (6.13)]. Because the total decay width, Γ, is universal for a given sneutrino, whereas the scales of the corresponding mass splittings are so different, it follows that $\Delta m/\Gamma \sim \mathcal{O}(1)$ can be satisfied only for one of the two oscillation phenomena. That is, at most one oscillation phenomenon, either flavor oscillations *or* CP-driven oscillations, can be observed.

Consider first the CP-driven oscillations. These oscillations can be observed if the lifetime of the sneutrinos is sufficiently long (the appropriate numerical requirements are given later in this section). In this case, flavor-driven oscillations are much faster and have a very short "baseline", so these oscillations are unobservable in collider experiments. Therefore, one can take a time average over flavor-changing terms in the sums in Eqs. (6.17) and (6.18), setting them effectively to zero, and retain only those terms where the mass splitting is CP-driven and not flavor-driven (i.e. keep only those terms with $s = k$ or $s = k + 3$). Now, the sum over s can be performed, and eqs. (6.17) and (6.18) simplify to:

$$\begin{aligned} P_{\tilde{\nu}_I\to\tilde{\nu}^*_J} &= \sum_{K=1}^{3} \left(e^{-t/\tau_{K_+}} \left|X^{IK}X^{JK}\right|^2 + e^{-t/\tau_{K_-}} \left|Y^{IK}Y^{JK}\right|^2\right) \\ &- 2\sum_{K=1}^{3} e^{-t\left[\frac{1}{2\tau_{K_+}}+\frac{1}{2\tau_{K_-}}\right]} \cos\left[\Delta_K t\right]\, \mathrm{Re}\left(X^{IK}X^{JK}Y^{IK}Y^{JK}\right)\,, \end{aligned} \tag{6.19}$$

$$\begin{aligned} P_{\tilde{\nu}_I\to\tilde{\nu}_J} &= \sum_{K=1}^{3} \left(e^{-t/\tau_{K_+}} \left|X^{IK}X^{JK}\right|^2 + e^{-t/\tau_{K_-}} \left|Y^{IK}Y^{JK}\right|^2\right) \\ &+ 2\sum_{K=1}^{3} e^{-t\left[\frac{1}{2\tau_{K_+}}+\frac{1}{2\tau_{K_-}}\right]} \cos\left[\Delta_K t\right]\, \mathrm{Re}\left(X^{IK}X^{JK}Y^{IK}Y^{JK}\right)\,, \end{aligned} \tag{6.20}$$

where $\Delta_K \equiv \omega_K - \omega_{K+3}$ and we have used eq. (3.79) to express the 6×6 matrices $\mathcal{Z}_{\tilde{\nu}}$ in terms of the 3×3 matrices X and Y.

Eqs. (6.19) and (6.20) are easily interpreted. For "long baseline" oscillations, one needs first to project flavor I onto some K (via the X^{IK}, Y^{IK} factors), then the CP-driven oscillation takes place between the would-be sneutrino–antisneutrino states S_K and S_{K+3}, and finally the result is projected back onto flavor J.

Further simplification is possible if we exploit the bounds on the parameters due to the $\ell^J \to \ell^I\gamma$ decays obtained in Section 4.3 to conclude that the matrix M^2_{LC} is very close to diagonal form. In this case, the matrix Q_0 that diagonalizes M^2_{LC} [cf. eq. (3.47)] is close to the identity matrix. Moreover, the matrix elements of R [cf. eq. (3.59)] are suppressed by the ratio of $\Delta m_{\tilde{\nu}}/m_{\tilde{\nu}}$, and are therefore negligible. It then follows that $X \simeq Y \simeq T/\sqrt{2}$, where $T \equiv \mathrm{diag}(e^{-i\phi_1/2}\,,\, e^{-i\phi_2/2}\,,\, e^{-i\phi_3/2})$ and $\phi_J \simeq \arg(M^2_{LV})_{JJ}$ [cf. eq. (3.50)]. If we consider flavor conserving (i.e. $I = J$) sneutrino–antisneutrino oscillations, then there is one large contribution in eq. (6.19) in the sum over K for $I = K$, whereas the contributions of $I \neq K$ are strongly suppressed by the squares of mixing angles. Therefore, the dominant contribution to the probability for sneutrino–antisneutrino oscillations is given by:

$$P_{\tilde{\nu}_I \to \tilde{\nu}^*_I} \approx \frac{1}{4}\left[e^{-t/\tau_{I_+}} + e^{-t/\tau_{I_-}} - 2\, e^{-t\left[\frac{1}{2\tau_{I_+}}+\frac{1}{2\tau_{I_-}}\right]} \cos(\Delta_I t)\, \cos(2\phi_I)\right] , \qquad (6.21)$$

which coincides exactly with the formula obtained previously for the one generation case [cf. eq. (6.12)] in the CP-conserving limit (where M^2_{LV} is a real matrix so that $\cos 2\phi_I = 1$). Similarly, for $P_{\tilde{\nu}_I \to \tilde{\nu}_I}$, one reproduces eq. (6.14) in the same limiting case.

To complete the analysis of the sneutrino oscillation formulae, we must compute the total sneutrino decay width, $\Gamma_k \equiv \Gamma(S_k \to \text{anything}) = 1/\tau_{S_k}$. Supposing that the neutralino is the lightest supersymmetric particle (LSP), the sneutrino decay width is the sum of the partial widths of the following two kinematically available decay chains,[23]

$$\Gamma(S_k \to \ell^{\mp I} + \chi^{\pm}_i) = g_2^2\,\frac{m_{S_k}}{32\pi}\left(1 - \frac{m^2_{\chi_i}}{m^2_{S_k}}\right)^{3/2} |Z^{1i}_+|^2 \left(|\mathcal{Z}^{Ik}_{\tilde{\nu}}|^2 + |\mathcal{Z}^{(I+3)k}_{\tilde{\nu}}|^2\right) , \qquad (6.22)$$

$$\Gamma(S_k \to \nu^I + \chi^0_i) = \frac{g_2^2}{c_W^2}\,\frac{m_{S_k}}{64\pi}\left(1 - \frac{m^2_{\chi^0_i}}{m^2_{S_k}}\right)^{3/2} |Z^{1i}_N s_W - Z^{2i}_N c_W|^2 \sum_{J=1}^{3}\left|(\mathcal{Z}^{Jk}_{\tilde{\nu}} - i\mathcal{Z}^{(J+3)k}_{\tilde{\nu}})U^{JI}_{\rm MNS}\right|^2 . \qquad (6.23)$$

In deriving the formulae above, we have used the Feynman Rules eqs. (C.1) and (C.4) from Appendix C and have taken the lepton masses to zero. Eqs. (6.22) and (6.23) agree with Ref. [18] in the limit $U_{\rm MNS} = \mathcal{Z}_{\tilde{\nu}} = 1$. Writing $\mathcal{Z}_{\tilde{\nu}}$ in terms of X and Y [cf. eq. (3.79)], it easily follows that the decay rates of the sneutrinos S_k with $k = 1, 2, 3$ [$k = 4, 5, 6$] depend on X [Y] alone. Since X and Y differ only by the "small" R matrix [cf. eq. (3.60)], it follows that $\tau_{I_+} \simeq \tau_{I_-}$, which can be used to further simplify the expression given by eq. (6.21).

[23] $\Gamma(S_k \to \ell^{\mp I} + \chi^{\pm}_i)$ indicates the sum of the sneutrino partial widths to the lepton–chargino and its charge-conjugated final states.

The total sneutrino decay width is given by:

$$\begin{aligned}\Gamma_k &= \sum_{I=1}^{3}\sum_{i=1}^{2}\Gamma(S_k \to \ell^{\mp I} + \chi_i^{\pm}) + \sum_{I=1}^{3}\sum_{i=1}^{4}\Gamma(S_k \to \nu^I + \chi_i^0) \\ &= g_2^2\,\frac{m_{S_k}}{32\pi}\left[\sum_{i=1}^{2}\left(1-\frac{m_{\chi_i}^2}{m_{S_k}^2}\right)^{3/2}|Z_+^{1i}|^2 + \frac{1}{2c_W^2}\sum_{i=1}^{4}\left(1-\frac{m_{\chi_i^0}^2}{m_{S_k}^2}\right)^{3/2}|Z_N^{1i}s_W - Z_N^{2i}c_W|^2\right] ,\end{aligned} \tag{6.24}$$

where the summation over the lepton indices can be performed in the limit of vanishing lepton masses, with the use of the orthogonality [unitarity] relations for the matrices $\mathcal{Z}_{\tilde{\nu}}$ [$U_{\rm MNS}$].

How can one observe sneutrino CP-oscillations? Consider the following scenario: suppose that the LHC finds sneutrinos with masses that are accessible at a future International Linear Collider (ILC). Then, at the ILC, the sneutrinos are produced through the annihilation process of eq. (6.1), and subsequently decay into [leptons + charginos] and [neutrinos + neutralinos] following the decay widths given by eqs. (6.22) and (6.23), respectively. Sneutrino CP-oscillations will then be observed only if the asymmetry A_l defined in eq. (6.15), is appreciable, *i.e.*, $A_l \sim \mathcal{O}(1)$, which can be realized if both Δm_k is small (providing a long enough oscillation base) and the sneutrino decay rate is sufficiently slow such that $\Delta m_k/\Gamma_k \sim \mathcal{O}(1)$. This scenario is impossible if the sneutrinos are sufficiently heavy compared to the neutralinos and/or charginos, in which case (neglecting the phase space suppression in eq. (6.24) and performing the summation over the chargino and neutralino indices) the sneutrino decay rate is approximately given by:

$$\Gamma_k \approx g_2^2\,\frac{m_{S_k}}{32\pi}\left[\sum_{i=1}^{2}|Z_+^{1i}|^2 + \frac{1}{2c_W^2}\sum_{i=1}^{4}|Z_N^{1i}s_W - Z_N^{2i}c_W|^2\right] = g_2^2\,\frac{m_{S_k}}{32\pi}\left(1+\frac{1}{2c_W^2}\right) . \tag{6.25}$$

The expression above depends only on the sneutrino mass and cannot be suppressed by a particular choice of mixing angles of the $\mathcal{Z}_{\tilde{\nu}}, Z_+$ or Z_N matrices. Thus, using the results of Section 5, one can check that the ratio $\Delta m_k/\Gamma_k$ is always much too small for the sneutrino oscillations to be observed. As an example, in the case of universal parameters discussed in Section 5.3, for the lightest sneutrino and m_0, $|A_0| \lesssim 10^5$ GeV we obtain

$$\frac{\Delta m_S}{\Gamma_S} \lesssim 2.7\times 10^{-6} , \tag{6.26}$$

which is very far from the value $\mathcal{O}(1)$ required for the observability of sneutrino oscillations.

In the case of 2-body decays, the decay width Γ_k can be only suppressed by choosing an appropriate hierarchy of particle masses. Most of the decay channels in Eqs. (6.22) and (6.23) would have to be closed kinematically, with the open channels strongly suppressed either by the very small phase space factors (which requires rather unnatural degeneracy between sneutrino and neutralino or chargino masses), or by sufficiently small mixing angles for the relevant channel. An alternative possibility is one where the sneutrinos are lighter then all charginos and neutralinos, so that all 2-body decay channels are closed, but heavier than some charged slepton. In this case, $\widetilde{\nu} \to \widetilde{\ell}^{\pm} W^{\mp}$, and assuming that the W is produced off-shell the end result is a 3-body decays that can produce an observable charged lepton. Three-body phase space significantly suppresses the sneutrino decay rate (relative to the two-body decay rates discussed above), and can yield observable sneutrino–antisneutrino oscillations, as shown in ref. [18]. However in such a scenario, either the charged slepton is the LSP, which is strongly disfavored by astrophysical data, or the charged slepton decays to some new lighter supersymmetric particle, which requires extending the model beyond the seesaw-extended MSSM considered in this paper [55]. As we have shown, the oscillations in the three-generation case does not differ much from the one-generation case, where the flavor indices are summed over [cf. eqs. (6.21) and (6.24)]. Thus, the results of ref. [18] can also be used without significant changes in the three-generation case discussed in this paper.

Finally, we discuss the case of sneutrino *flavor* oscillations. These oscillations are described by eqs. (6.17) and (6.18) with indices $I \neq J$. For any choice of $I \neq J$, both equations can be significantly simplified using the bounds on the structure of sneutrino mixing matrices derived in Sections 4 and 5. These bounds imply that the off-diagonal elements of matrices Q and R [defined in eqs. (3.49) and (3.56)] are small, which then imply [via eqs. (3.60) and (3.79)] that the off-diagonal elements of the matrices X, Y and $\mathcal{Z}_{\tilde{\nu}}$ are likewise small. Thus, to a good approximation one can keep in eqs. (6.17) and (6.18) only terms at most quadratic in the non-diagonal elements of $\mathcal{Z}_{\tilde{\nu}}$. For example, in the sum of the first term of the product of four $\mathcal{Z}_{\tilde{\nu}}$'s in eq. (6.17), it is sufficient to keep only terms with $s, k = I, I+3, J, J+3$. Assuming that the lifetimes of all eigenstates are very similar (i.e., $\tau \simeq \tau_k$), all the dominant terms can be summed to give a simple final expression valid for $I \neq J$,

$$P_{\tilde{\nu}_I \to \tilde{\nu}_J} \approx e^{-\frac{t}{\tau}} \left\{ |Q^{IJ} Q^{JJ*}|^2 + |Q^{JI} Q^{II*}|^2 + 2\mathrm{Re}\left(Q^{IJ} Q^{JJ*} Q^{JI*} Q^{II}\right) \cos \Delta m_{IJ} t \right\}, \quad (6.27)$$

where $\Delta m_{IJ} \equiv m_{\tilde{\nu}_I} - m_{\tilde{\nu}_J}$.

The analogous expression for the sneutrino-antisneutrino oscillation probability $P_{\tilde{\nu}_I \to \tilde{\nu}^*_J}$ is bilinear in the matrix elements of R [cf. eq. (3.56)]. The latter are at most of

$\mathcal{O}(10^5 m_\nu/v) \lesssim 10^{-6}$ and thus lead to completely negligible sneutrino–antisneutrino transition rates.[24]

The form of eq. (6.27) is explicitly invariant with respect to rephasing, $Q^{IJ} \to Q^{IJ} e^{i\phi_J}$. Thus, without loss of generality, we may replace Q by Q_0 [cf. eq. (3.49)] in eq. (6.27), where the off-diagonal matrix elements of the unitary matrix Q_0 are given approximately by eq. (3.48) and the diagonal elements of Q_0 are fixed by unitarity. As Q_0 is close to the identity matrix, the following approximations are valid: $Q_0^{JJ} \simeq 1$ and $Q_0^{JI*} \simeq -Q_0^{IJ}$ for $I \neq J$. In this approximation, eq. (6.27) simplifies for $I \neq J$ to:

$$P_{\tilde{\nu}_I \to \tilde{\nu}_J} \approx 2e^{-t/\tau} \left[|Q_0^{IJ}|^2 - \mathrm{Re}(Q_0^{IJ})^2 \cos \Delta m_{IJ} t \right] . \tag{6.28}$$

If one uses the approximate expression given in eq. (3.48), $Q_0^{IJ} \simeq (M^2_{LC})^{IJ}/(m^2_{\tilde{\nu}_J} - m^2_{\tilde{\nu}_I})$, then eq. (6.28) yields the oscillation probabilities directly in terms of the sneutrino squared-mass matrix elements. As expected, the sneutrino flavor-transition depends on the flavor-conserving matrix M^2_{LC}.

Defining the oscillation length by $L = ct$ we can write

$$\Delta m_{IJ} t = 5.06 \times \Delta m_{IJ}\,(\mathrm{GeV})\, L(\mathrm{fm}) . \tag{6.29}$$

As in neutrino oscillations, it is useful to define $\Delta m_{IJ}\, L = 2\pi L/L_0$ where L_0 is the characteristic length of the oscillation :

$$L_0 = 1.24\,\mathrm{fm} \times \frac{1}{\Delta m_{IJ}\ (\mathrm{GeV})} . \tag{6.30}$$

If the sneutrino mass difference is of $\mathcal{O}(1\ \mathrm{GeV})$, the characteristic oscillation length is of order 1 fm. Of course, the characteristic length of oscillation must be smaller than or at most comparable to the decay length of the particle for oscillations to be observable. In the case of the sneutrino, the decay length is [using eq. (6.25)]:

$$L_{\tilde{\nu}} = c\tau \simeq \frac{28\ (\mathrm{fm})}{m_{\tilde{\nu}}\ (\mathrm{GeV})} . \tag{6.31}$$

Hence, the condition $L_{\tilde{\nu}} \gtrsim L_0$ requires that

$$\frac{\Delta m_{IJ}}{m_{\tilde{\nu}}} \gtrsim \frac{1}{25} . \tag{6.32}$$

[24] An accurate estimate of $P_{\tilde{\nu}_I \to \tilde{\nu}_J^*}$ should also take into account similarly small effects produced by the admixture of the heavy sneutrino states in the definition of the $\tilde{\nu}_I$, which were neglected in derivation of eqs. (6.17) and (6.18). However, given the extremely small transition probabilities, we do not present the full analysis here.

Such a mass splitting between the sneutrino states of different flavors is sensible. Thus, the likelihood of observing flavor sneutrino oscillations at colliders depends primarily on the degree of suppression caused by the mixing angles in the matrix Q. It is instructive to input some representative numbers in eq. (6.27). Thus, for $\Delta m_{12} = 10$ GeV, $m_{\tilde{\nu}} = 270$ GeV, $\tan\beta = 10$ and taking into account the bounds of Table 3, we obtain for $\tilde{\nu}_\mu \to \tilde{\nu}_e$ oscillations at time $t = \tau = \Gamma^{-1}$ [cf. eq. (6.24)]:

$$P_{\tilde{\nu}_\mu \to \tilde{\nu}_e} \;\approx\; 1.25 \times 10^{-5}\,[1 \;-\; \cos(\Delta m_{12}\tau)]\;, \tag{6.33}$$

Thus, as a consequence of the bounds from neutrino masses and radiative flavor changing decays obtained in Sections 4 and 5, we conclude that *in the see-saw extended MSSM, sneutrino flavor oscillations are difficult to observe at colliders.*

If the bounds of Sections 4 and 5 could be avoided, say with some cancellation mechanism (which in the absence of such a mechanism would appear unnatural), then it may be possible to find regions of the supersymmetric parameter space where flavor oscillations are observable. Then, at the ILC, one can define a flavor asymmetry for the number of muons vs. electrons in the final state, analogous to eq. (6.15). A time-variation of this flavor asymmetry would indicate the presence of flavor oscillations.

7 Conclusions

In this paper, we have studied sneutrino mixing phenomena in the seesaw-extended MSSM, allowing for the full complexity of the three-generation model (which includes both flavor-changing and CP-violating effects). We have focused primarily on the soft-SUSY-breaking matrix parameters m^2_N, m^2_B and A_ν, which govern the structure of the sneutrino squared-mass matrices. We have found a convenient parameterization of the sneutrino sector, where all relevant physical observables depend analytically on a pair of 3×3 mass matrices M^2_{LV} and M^2_{LC} given in eqs. (3.40) and (3.39), respectively. The elements of M^2_{LV} violate lepton number by two units, whereas elements of M^2_{LC} are lepton-number conserving parameters.

Within this framework, we have analyzed the constraints arising from one-loop neutrino masses and mixings, from radiative flavor-changing charged lepton decays, and from the electron electric dipole moment (EDM). We discovered new and potentially significant contributions to radiative lepton decays $\ell^J \to \ell^I + \gamma$ due to the dependence of m^2_N which modifies the MSSM value of M^2_{LC}. We also observed that although the $(g-2)_\mu$ measurement places non-trivial constraints on the SUSY-breaking parameters, the electron EDMs do not yield any additional constraints (at one loop) on the seesaw-extended MSSM parameters.

All conclusions presented here are based on a complete numerical analysis of the processes described above.[25] In all cases, we have also provided useful analytic approximations, which have served as a check of our numerical work.

Sneutrino mixing phenomena takes on two different forms. The mixing of sneutrinos and antisneutrinos violates lepton number by two units, whereas sneutrino flavor mixing is a lepton-number conserving process. Both forms of mixing are in present in principle in the three-generation seesaw-extended MSSM. In this paper, we have generalized the sneutrino-antisneutrino mixing formalism, originally presented in a one-generation model [18], to the three-generation model. This sneutrino-antisneutrino mixing then acts back on the neutrino sector, and provides an important loop correction to the neutrino mass matrix. In this paper, we examined the possibility that starting from a diagonal neutrino mass matrix at tree-level, the nontrivial flavor structure of the neutrino mass matrix is generated entirely by the one-loop diagram that directly involves the sneutrino–antisneutrino transition. Our analysis shows that this is indeed possible, although in practice certain fine-tunings among SUSY breaking parameters in the leptonic sector seem to be unavoidable.

Returning to the sneutrino sector, we have derived analytical expressions for both sneutrino-flavor oscillations and sneutrino-antisneutrino oscillations in eqs. (6.17) and (6.18). We determined that if the constraints analyzed above are combined with the assumption that sneutrinos can decay into two-body final states, then sneutrino-antisneutrino oscillations are not observable at colliders. This is consistent with a similar result of the one-generation model obtained in Ref. [18]. This conclusion is easily understood, by noting that the sneutrino-antisneutrino mass difference, $\Delta m_{\tilde{\nu}}$, is proportional to the neutrino mass and is at most of the order of 1 keV. This is much smaller than the corresponding width of the sneutrino, $\Gamma_{\tilde{\nu}}$, of order 1 GeV or larger. The observability of sneutrino-antisneutrino oscillations at colliders requires that $\Delta m_{\tilde{\nu}} \sim \Gamma_{\tilde{\nu}}$. A sneutrino width of order 1 keV or less is possible only if there are no kinematically allowed two-body final states in sneutrino decay. In the seesaw-extended MSSM, this scenario is possible only if a charged slepton is the lightest supersymmetric particle, a possibility strongly disfavored by astrophysical data. Other possibilities exist if one introduces new degrees of freedom beyond the seesaw-extended MSSM, but this lies beyond the scope of this paper.

Sneutrino flavor oscillations are more likely to be observable at colliders, since the mass splitting between sneutrinos of difference flavors can be of order 1 GeV or larger. We have derived simple approximate formulae for such oscillations and have estimated their magnitudes. Unfortunately, in the seesaw-extended MSSM, after imposing bounds on

[25] Fortran-77 and Maple-10 numerical codes are available from the authors.

bounding sneutrino mixing angles determined from the analysis of radiative charged lepton decays, the resulting probabilities for sneutrino flavor oscillations are likely to be too small to be observed directly at colliders.

At present, within the seesaw framework for neutrino masses, few handles exist for probing the physics at the seesaw scale. At most, one can hope to measure the MNS mixing angles, and determine neutrino mass differences (and with a little luck, the absolute scale of neutrino masses). In the seesaw-extended MSSM, some of the physics of the seesaw scale is imprinted on parameters that govern the properties of the light sneutrinos. With a precision program at future colliders for measuring sneutrino observables, there are new opportunities to explore the fundamental physics that is responsible for the origin of neutrino masses.

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Appendix A Notation for fermion fields

Fermion fields in quantum field theory can be described by employing either two-component or four-component fermion notation [56]. In models where lepton number is not conserved, two-component fermion notation is generally simpler and more efficient. In this appendix, we briefly discuss the relation between the two treatments.

In Table 1, the fermionic fields associated with the lepton and Higgs sectors of the seesaw-extended MSSM are listed. These fermion fields can be viewed either as two-component fermion fields or the left-handed projections of four-component fermion fields, with $\Psi_L \equiv \frac{1}{2}(1-\gamma_5)\Psi$ and

$$\Psi^c \equiv C\overline{\Psi}^T, \qquad \overline{\Psi^c} = -\Psi^T C^{-1}, \tag{A.1}$$

where $\overline{\Psi} \equiv \Psi^\dagger \gamma^0$ and $C = -C^T$ is the charge conjugation matrix.

For example, in four-component notation, given a four-component (anticommuting) Dirac spinor ν_D, we define the following four-component spinors:

$$\nu_L \equiv P_L \nu_D\,, \qquad \nu_L^c \equiv P_L \nu_D^c\,, \qquad \nu_R \equiv P_R \nu_D\,, \quad \text{and} \quad \nu_R^c \equiv P_R \nu_D^c\,, \tag{A.2}$$

where $P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$, respectively. The corresponding two-component (anticommuting) fields are given by the non-zero components of $\nu_L \equiv P_L \nu_D$ and $\nu_L^c \equiv P_L \nu_D^c$. Consequently, we shall use the same symbols ν_L and ν_L^c for the corresponding two-component neutrino fields. However, one must be careful to note that in our notation

$$\nu_L^c = C\overline{\nu_R}^T, \qquad \overline{\nu_R^c} = -\nu_L^T C^{-1}, \tag{A.3}$$

since, e.g., $\nu_L^c \equiv P_L C \overline{\nu}_D^T = C(\overline{P_R \nu_D})^T$. The same notation also applies to charged fermion fields. Our conventions for left and right-handed charged conjugated fields follow those of ref. [57]. Note that eq. (A.3) implies that anticommuting fermion fields satisfy:

$$\overline{\nu_R^c}\nu_L^c = \overline{\nu_R}\nu_L\,, \qquad \overline{\nu_L^c}\nu_R^c = \overline{\nu_L}\nu_R\,. \tag{A.4}$$

In the text, the effective Lagrangians for fermion mass and interaction terms are given in terms of two-component fermion fields. These terms can be easily translated into the four-component spinor notation . As a first example, the dimension-five operator that governs the standard seesaw mechanism [eq. (1.1)] contains a product of two-component fermion fields, $L_i^I L_k^K$. In terms of four-component spinors, this product is given by $-(L^T)_i^I C^{-1} L_k^K = (\overline{R^c})_i^I L_k^K$, where $L_k^K \equiv (\nu_L^K\,,\,\ell_L^K)$ is now interpreted as a doublet of four-component fermion fields as described above and $(R^c)_i^I \equiv (\nu_R^{cI}\,,\,\ell_R^{cI})$.

As a second example, we derive the four component version of eq. (3.1) in the one-generation model. One can redefine the phases of the neutrino fields such that m_D and M are real and non-negative. The two-component spinor product $\nu_L \nu_L^c$ + H.c. translates to the product of four-component spinors: $-\nu_L^T C^{-1} \nu_L^c + \text{H.c.} = \overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R$, which is the usual Dirac mass term. Similarly, the two-component spinor product $\nu_L^c \nu_L^c$ translates to the four-component spinor product $-\nu_L^{cT} C^{-1} \nu_L^c = \overline{\nu_R}\nu_L^c$. Hence, if the Majorana mass term $M \neq 0$ in eq. (3.1), one cannot identify the physical mass eigenstates as Dirac fermions. For example, the mass terms of the one-generation neutrino Lagrangian, which in terms of two-component fermion fields is given by $-\mathscr{L}_{\rm mass} = m_D \nu_L \nu_L^c + \frac{1}{2} M \nu_L^c \nu_L^c + \text{H.c.}$, translates in four-component notation to

$$
\begin{aligned}
-\mathscr{L}_{\rm mass} &= \tfrac{1}{2} m_D (\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L + \overline{\nu_L^c}\nu_R^c + \overline{\nu_R^c}\nu_L^c) + \tfrac{1}{2} M (\overline{\nu_R}\nu_L^c + \overline{\nu_L^c}\nu_R) \\
&= \tfrac{1}{2} \begin{pmatrix} \overline{\nu_R^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} + \tfrac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\nu_L^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} \\
&= -\tfrac{1}{2} \begin{pmatrix} \nu_L^T & \nu_L^{cT} \end{pmatrix} C^{-1} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} + \text{H.c.}\,, \qquad \text{(A.5)}
\end{aligned}
$$

where we have used eq. (A.4) to write the first line of eq. (A.5) in a symmetrical fashion and eq. (A.3) to obtain the final form above.

The Takagi-diagonalization of the neutrino mass matrix yields two (self-conjugate) Majorana fermion mass-eigenstates. This is accomplished by introducing a unitary matrix $\mathcal{U}$,

$$
\begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} = \mathcal{U} \begin{pmatrix} P_L \nu_\ell \\ P_L \nu_h^c \end{pmatrix}, \qquad \text{(A.6)}
$$

such that

$$
\mathcal{U}^T \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \mathcal{U} = \begin{pmatrix} m_{\nu_\ell} & 0 \\ 0 & m_{\nu_h} \end{pmatrix}, \qquad \text{(A.7)}
$$

where $m_{\nu_\ell} \simeq m_D^2/M$ and $m_{\nu_h} \simeq M + m_D^2/M$. The resulting neutrino mass Lagrangian is:

$$
-\mathscr{L}_{\rm mass} = -\tfrac{1}{2}\left[m_{\nu_\ell} \nu_\ell^T C^{-1} P_L \nu_\ell + m_{\nu_h} \nu_h^{cT} C^{-1} P_L \nu_h^c \right] + \text{H.c.} \qquad \text{(A.8)}
$$

We can define four-component self-conjugate Majorana fields by:

$$
\psi_M \equiv P_L \nu_\ell + P_R C \overline{\nu}_\ell^T\,, \qquad \overline{\psi}_M \equiv \overline{\nu}_\ell P_R - \nu_\ell^T C^{-1} P_L\,, \qquad \text{(A.9)}
$$

$$
\Psi_M \equiv P_L \nu_h^c + P_R C \overline{\nu}_h^{cT}\,, \qquad \overline{\Psi}_M \equiv \overline{\nu}_h^c P_R - \nu_h^{cT} C^{-1} P_L\,. \qquad \text{(A.10)}
$$

Thus, eq. (A.8) reduces to the expected form:

$$-\mathscr{L}_{\rm mass} = \tfrac{1}{2}\left[m_{\nu_\ell}\overline{\psi}_M\psi_M + m_{\nu_h}\overline{\Psi}_M\Psi_M\right] . \tag{A.11}$$

Appendix B A non-decoupling contribution to sneutrino masses when $m_N^2 \sim \mathcal{O}(M^2)$

B.1 Non-decoupling effects when $m_N^2 \gg v^2$

In Section 3.2, we noted below eq. (3.40) non-decoupling in the limit of $\|M\| \to \infty$ with $\|m_N^2 M^{-2}\|$ fixed. The lepton-number conserving 3×3 squared-mass matrix of the light sneutrinos [eq. (3.39)] can be written as:

$$M_{LC}^2 = m_L^2 + \tfrac{1}{2}M_Z^2\cos 2\beta + m_D^* M^{-1} m_N^2 M^{-1} m_D^T + \mathcal{O}(v^4M^{-2}) + \mathcal{O}(v^2 m_N^4 M^{-4})\,, \tag{B.1}$$

after expanding the quantity $(\mathbb{1} + M^{-2}m_N^2)^{-1}$ under the assumption that $\|M^{-2}m_N^2\| < 1$. Thus, we have a non-decoupling correction to the usual MSSM result of $\mathcal{O}(m_N^2M^{-2})$ as previously noted.

To understand the origin of this non-decoupling phenomenon, we use eq. (5.4) which relates the original right-handed sneutrino with the light and heavy sneutrino states after block diagonalization of the sneutrino mass matrix. To formally integrate out the heavy sector and obtain the effective theory of the light sneutrinos, we must write:

$$\widetilde{N}^I = \tilde{\nu}_h^I - \epsilon_{kn}[(M^2+m_N^2)^{-1}MY_\nu^T]^{IJ}\widetilde{L}_n^J H_k^2\,, \tag{B.2}$$

before electroweak symmetry breaking, where we have used $\widetilde{N}^I \equiv \tilde{\nu}_R^{I*}$. Note that when H_2^2 is replaced by its vacuum expectation value $v_2/\sqrt{2}$, we recover eq. (5.4) after using $m_D \equiv v_2 Y_\nu/\sqrt{2}$. In addition, we have used $\widetilde{L}_1^J \simeq \tilde{\nu}_\ell^J + \mathcal{O}(vM^{-1})$ and have worked consistently to leading order in vM^{-1}.

Consider the contribution of $|dW/dN^J|^2$ to the scalar potential, where W is given by eq. (2.1). Then,

$$\frac{dW}{dN^J} = M^{JK}N^K + \epsilon_{ij}Y_\nu^{KJ}H_i^2 L_j^K\,. \tag{B.3}$$

After squaring, and including the soft-SUSY-breaking term $\widetilde{N}^* m_N^2 \widetilde{N}$ (where m_N^2 is hermitian), we find:

$$\widetilde{N}^* m_N^2 \widetilde{N} \;+\; \left(\frac{dW}{dN^J}\right)\left(\frac{dW}{dN^J}\right)^* = \epsilon_{ij}\epsilon_{kn}Y_\nu^{KJ}Y_\nu^{IJ*}H_i^2H_k^{2*}\widetilde{L}_j^K\widetilde{L}_n^{I*}$$
$$+\left[\epsilon_{ij}(Y_\nu M)^{KI}\widetilde{N}^{I*}H_i^2\widetilde{L}_j^K + \text{H.c.}\right] + (M^2+m_N^2)^{KJ}\widetilde{N}^{K*}\widetilde{N}^J\,. \tag{B.4}$$

To obtain the relevant operator that survives in the low-energy effective theory, we insert eq. (B.2) for $\widetilde{N}^I$ in eq. (B.4), and then take the limit as $\|M\| \to \infty$, In addition, we set $\tilde{\nu}_h = 0$. The end result is:

$$\epsilon_{kn}\epsilon_{ij}\left[Y_\nu^* Y_\nu^T - Y_\nu^* M(M^2 + m_N^2)^{-1} M Y_\nu^T\right]^{JK} \widetilde{L}_n^{J\,*} \widetilde{L}_j^K H_k^{2\,*} H_i^2 \,. \tag{B.5}$$

Note that this is a dimension-4 (hard) SUSY-violating operator [58] which vanishes if $m_N^2 = 0$ [as m_N^2 is the only SUSY-breaking source in eq. (B.5)]. If $m_N^2 < M^2$, one can expand $(M^2 + m_N^2)^{-1}$ in eq. (B.5), which yields:

$$\epsilon_{kn}\epsilon_{ij}[Y_\nu^* M^{-1} m_N^2 M^{-1} Y_\nu^T + \mathcal{O}(m_N^4 M^{-4})]^{JK} \widetilde{L}_n^{J\,*} \widetilde{L}_j^K H_k^{2\,*} H_i^2 \,. \tag{B.6}$$

We now replace $H_2^2 \to v_2/\sqrt{2}$. If $m_N^2 \sim \mathcal{O}(v^2)$, then the hard SUSY-breaking operator is of $\mathcal{O}(v^2 M^{-2})$, which is the expected result. Such corrections are extremely small, assuming that $v \ll \|M\|$, and can be be dropped from the low-energy effective field theory of the light $\mathcal{O}(v)$ degrees of freedom. On the other hand, if $x \equiv \|m_N^2\|/\|M^2\|$ is held fixed to a finite positive value as $M \to \infty$, then the hard SUSY-breaking operator is of $\mathcal{O}(x)$, which must be kept in the low-energy effective theory if x is not too small.

In the latter case, we see the presence of a non-decoupling effect in the low-energy effective field theory of the $\mathcal{O}(v)$ degrees of freedom as $M \to \infty$. We identify this as a hard SUSY-breaking effect described by the dimension-4 operator given by eq. (B.6). Ultimately, this non-decoupling effect can be traced to the fact that although ν_L [ν_L^c] and $\tilde{\nu}_L$ [$\tilde{\nu}_R^*$] are superpartners, it is not quite true that ν_ℓ [ν_h] and $\tilde{\nu}_\ell$ [$\tilde{\nu}_h$] are superpartners. Explicitly [cf. eqs. (5.3) and (5.4)], whereas

$$\nu_h^c \simeq \nu_L^c + M^{-1} m_D^T \nu_L \,, \tag{B.7}$$

to leading order in vM^{-1}, we have:

$$\tilde{\nu}_h^* \simeq \tilde{\nu}_R^* + (M^2 + m_N^2)^{-1} M m_D^T \tilde{\nu}_L \,. \tag{B.8}$$

Clearly, with $m_N^2 \neq 0$, there is a slight discrepancy between $\tilde{\nu}_h$ and the superpartner of ν_h.

If we replace H_2^2 with its vacuum expectation value $v_2/\sqrt{2}$ in eq. (B.5) and again make use of $\widetilde{L}_1^J \simeq \tilde{\nu}_\ell^J + \mathcal{O}(vM^{-1})$, we obtain a contribution to M_{LC}^2: Then eq. (B.5) becomes:

$$[m_D^* m_D^T - m_D^* M(M^2 + m_N^2)^{-1} M m_D^T]^{JK} \tilde{\nu}_\ell^{J\,*} \tilde{\nu}_\ell^K \,, \tag{B.9}$$

which correctly reproduces the last two terms of M_{LC}^2 given in eq. (3.39). Of course, the non-seesaw MSSM result of M_{LC}^2 derives from the soft-SUSY-breaking term, $\widetilde{L}_i^* m_L^2 \widetilde{L}_i$, and the D-term contribution, $\frac{1}{2} M_Z^2 \cos 2\beta$. As expected, in the $M \to \infty$ limit (with $x \to 0$), the low-energy effective theory reproduces the non-seesaw MSSM result. In this appendix, we have explained the origin of the non-decoupling correction to the non-seesaw MSSM result in the $M \to \infty$ limit with x held fixed to a finite positive value.

Finally, we address the question of the allowed size of the matrix parameter m_N^2. Does it make sense to have x close to $\mathcal{O}(1)$? In ref. [38], it is shown that for values of $x \sim 1$, there is a very large negative shift in the mass of the lightest CP-even Higgs boson due to radiative corrections from the heavy neutrino/sneutrino sector of the seesaw-extended MSSM. If we demand that there should be no unusually large radiative correction to a *physical observable* generated as a result of $m_N \neq 0$, we can apply the results of ref. [38] for the radiatively-corrected physical Higgs masses to conclude that $x \lesssim 0.1$. Note that this upper bound is less severe than the bound of $x \lesssim 0.01$ given in eq. (4.18). The latter was obtained in Section 4.3 from the bounds on rare charged lepton radiative decay rates, which imply that the matrix M_{LC}^2 should be close in form to a diagonal matrix.

B.2 Naturalness constraints on the magnitude of m_N^2

It seems that phenomenological constraints allow for the possibility that $\|m_N^2\|$ is significantly larger than $\mathcal{O}(v^2)$, in which case the non-decoupling contribution to M_{LC}^2 may be significant (perhaps as large as a few percent of the non-seesaw MSSM result). However, if one imposes the usual fine-tuning (or naturalness) requirements for the stability of the electroweak scale, one can show that $\|m_N^2\|$ cannot be significantly larger than $\mathcal{O}(v^2)$. This can be verified by computing the one-loop correction to the H_2^2 self-energy. The computation in the supersymmetric limit is performed explicitly in Appendix E, section 7 of ref. [6] for the Wess-Zumino model. This computation is easily adapted to the present case of interest (in which the Higgs boson couples the the neutrino/sneutrino system). We then modify the supersymmetric computation in the case of the one-generation seesaw model by setting the boson (heavy sneutrino) squared-mass to $M^2 + m_N^2$ and the fermion (heavy neutrino) mass to M. [Here, we are dropping terms of $\mathcal{O}(v^2)$.] If $m_N^2 \neq 0$ (which softly breaks the supersymmetry), the quadratic divergence does not cancel exactly. The surviving contribution to the sqaured-mass term of H_2^2 is of the form

$$m_N^2 |Y_\nu|^2 \mathcal{I}(M^2, m_N^2) |H_2^2|^2 \,, \tag{B.10}$$

where $\mathcal{I}$ is a logarithmically divergent integral (that can be regularized by dimensional reduction [59]).

We now add this one-loop result to the corresponding tree-level contribution to the scalar potential:

$$(m_{H_2}^2 + |\mu|^2) |H_2^2|^2 \,. \tag{B.11}$$

In order to achieve successful electroweak symmetry breaking with $v = 246$ GeV, the complete coefficient multiplying $|H_2^2|^2$ must be of $\mathcal{O}(v^2)$. By assumption, we take $\mu \sim \mathcal{O}(v)$ [cf. eq. (2.12)]. If $m_N^2 \gg v^2$, the correct scale of electroweak symmetry breaking can be achieved only by an unnatural fine-tuning of the parameter $m_{H_2}^2$. Thus, naturalness requires that $m_N^2 \sim v^2$. We have not distinguished between $\mathcal{O}(v^2)$ and $\mathcal{O}(M^2_{\rm SUSY})$ in the above discussion. It is likely that there is a slight separation of scales with $M_{\rm SUSY} \lesssim 1$ TeV. By imposing the naturalness condition on the dynamics of electroweak symmetry breaking (which ultimately is the motivation for TeV-scale supersymmetry in the first place), we conclude that the expected natural order of magnitude for $\|m_N^2\|$ is:

$$\|m_N^2\| \sim \mathcal{O}(M_{\rm SUSY})\,, \tag{B.12}$$

as indicated by eq. (2.15).

For completeness, we note that the same conclusion can be drawn by considering the one-loop effective scalar potential, $V^{(1)}(\phi)$. In particular, if we introduce a hard momentum cutoff Λ, one obtains a one-loop contribution of [60]

$$V^{(1)}(\phi) = \frac{\Lambda^2}{32\pi^2}\sum_i {\rm Str}\, M_i^2(\phi) + \frac{1}{64\pi^2}{\rm Str}\left\{M_i^4(\phi)\left[\ln\frac{M_i^2(\phi)}{\Lambda^2} - \frac{1}{2}\right]\right\}\,, \tag{B.13}$$

where $M_i^2(\phi)$ are the contributing squared-mass matrices of particles whose masses originate from their couplings to the Higgs boson, with the vacuum expectation values replaced by the corresponding Higgs fields, ϕ, and

$${\rm Str}\,\{\cdots\} = \sum_i (-1)^{2J_i}(2J_i+1)C_i\,\{\cdots\}\,. \tag{B.14}$$

In eq. (B.14), C_i counts the electric charge and color degrees of freedom of particle i (e.g., $C = 2$ for the $W^\pm$ gauge boson and $C = 6$ for a colored quark, since we count both particle and antiparticle). It is convenient to absorb the factor of $1/2$ in the last term on the right hand side of eq. (B.13), by defining μ such that:

$$\ln\frac{M_i^2(\phi)}{\Lambda^2} - \frac{1}{2} \equiv \ln\frac{M_i^2(\phi)}{\mu^2}\,. \tag{B.15}$$

Using the results of eqs. (3.12), (3.69) and (3.70), we focus on the contributions to the supertraces from the heavy neutrinos and sneutrinos. Indeed,

$$\sum_i {\rm Str}\, M_i^2(\phi) = 2\,{\rm Tr}\; m_N^2 + \mathcal{O}(v^2)\,, \tag{B.16}$$

although m_N^2 is field independent and thus contributes only to the vacuum energy. Here, we are interested in the implications of naturalness associated with electroweak symmetry breaking (and not the cosmological constant). Thus we focus on the field-dependent part of the scalar potential that is quadratic in the Higgs fields. To do this, we simply replace m_D with $H_2^2 Y_\nu$. For simplicity, we shall examine the one generation seesaw model. In this case, we obtain the following scalar field-dependent squared-masses:

$$m^2_{\nu_h} \simeq M^2 + 2|Y_\nu|^2|H_2^2|^2\,, \tag{B.17}$$

$$m^2_{\tilde{\nu}_h} \simeq M^2 + m_N^2 + |Y_\nu|^2|H_2^2|^2\left[1+\frac{M^2}{M^2+m_N^2}\right]\,. \tag{B.18}$$

Inserting these results into the last term on the right hand side of eq. (B.13), and using eq. (B.15) to replace Λ with μ, we end up with the following terms in $V^{(1)}(\phi)$ that contribute to the coefficient of $|H_2^2|^2$

$$\begin{aligned}
&2\left\{(M^2+m_N^2)^2 + 2(2M^2+m_N^2)|Y_\nu|^2|H_2^2|^2\right\}\ln\left[\frac{M^2+m_N^2+|Y_\nu|^2|H_2^2|^2\left(\frac{2M^2+m_N^2}{M^2+m_N^2}\right)}{\mu^2}\right] \\
&\qquad -2\left\{(M^4+4M^2|Y_\nu|^2|H_2^2|^2\right\}\ln\left[\frac{M^2+2|Y_\nu|^2|H_2^2|^2}{\mu^2}\right]\,,
\end{aligned} \tag{B.19}$$

where we have dropped terms of $\mathcal{O}(v^2|H_2^2|^2)$. Expanding out the logarithms, the above expression reduces to

$$\begin{aligned}
&2\left\{(M^2+m_N^2)^2 + 2(2M^2+m_N^2)|Y_\nu|^2|H_2^2|^2\right\}\left\{\ln\left[\frac{M^2+m_N^2}{\mu^2}\right] + |Y_\nu|^2|H_2^2|^2\frac{2M^2+m_N^2}{(M^2+m_N^2)^2}\right\} \\
&\qquad -2\left\{(M^4+4M^2|Y_\nu|^2|H_2^2|^2\right\}\left\{\ln\frac{M^2}{\mu^2}+\frac{2|Y_\nu|^2|H_2^2|^2}{M^2}\right\}\,.
\end{aligned} \tag{B.20}$$

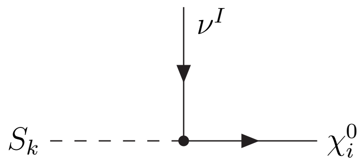
If we keep only terms proportional to $|H_2^2|^2$, we end up with:

$$4|Y_\nu|^2|H_2^2|^2\left\{2M^2\ln\left(1+\frac{m_N^2}{M^2}\right)+m_N^2\left[\ln\left(\frac{M^2+m_N^2}{\mu^2}\right)+\frac{1}{2}\right]+\mathcal{O}(v^2)\right\}\,. \tag{B.21}$$

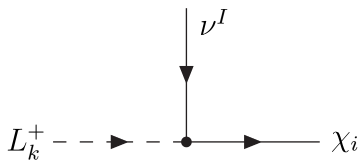
One can check that the coefficient of $|Y_\nu|^2|H_2^2|^2$ is precisely $m_N^2\mathcal{I}(M^2, m_N^2)$, where $\mathcal{I}$ is the integral appearing in eq. (B.10) after $\overline{\rm DR}$ subtraction [59].

Appendix C Feynman rules

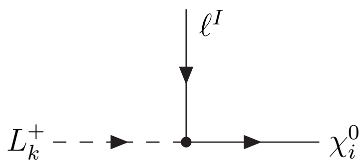
We exhibit here the relevant Feynman rules for the calculation of $\ell \to \ell'\gamma$ presented in Section 4.3. These rules are based on four-component fermion notation (see Appendix A) and employ the conventions of Ref. [7] for sfermion, chargino and neutralino masses and mixing matrices. The neutrinos ν^I are (self-conjugate) Majorana fermions [cf. eq. (A.9)]. In the basis defined in Section 2 we obtain:



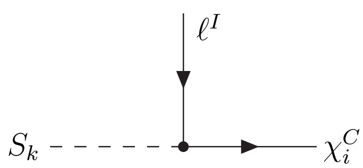
$$\frac{i}{2}\Big[(g_1 Z_N^{1i} - g_2 Z_N^{2i})(\mathcal{Z}_{\tilde\nu}^{Jk} - i\mathcal{Z}_{\tilde\nu}^{(J+3)k})U_{MNS}^{JI}\,P_L + (g_1 Z_N^{1i*} - g_2 Z_N^{2i*})(\mathcal{Z}_{\tilde\nu}^{Jk} + i\mathcal{Z}_{\tilde\nu}^{(J+3)k})U_{MNS}^{JI*}\,P_R\Big]\ , \tag{C.1}$$



$$-i\left(g_2 Z_L^{Jk} Z_-^{1i} - Y_\ell{}^J Z_L^{(J+3)k} Z_-^{2i}\right) U_{\rm MNS}^{JI}\,P_L\ , \tag{C.2}$$



$$i\left[\left(\frac{g_2}{\sqrt{2}c_W} Z_L^{Ik}(Z_N^{1i}s_W + Z_N^{2i}c_W) - Y_\ell^I Z_L^{(I+3)k} Z_N^{3i}\right) P_L + \left(-g_1\sqrt{2}\, Z_L^{(I+3)k} Z_N^{1i*} - Y_\ell^I Z_L^{Ik} Z_N^{3i*}\right) P_R\right]\ , \tag{C.3}$$



$$-\frac{i}{\sqrt{2}}\left[g_2 Z_+^{1i}(\mathcal{Z}_{\tilde\nu}^{Ik} - i\mathcal{Z}_{\tilde\nu}^{(I+3)k})\,P_L - Y_\ell^I Z_-^{2i*}(\mathcal{Z}_{\tilde\nu}^{Ik} - i\mathcal{Z}_{\tilde\nu}^{(I+3)k})\,P_R\right]\ . \tag{C.4}$$

Appendix D Order of magnitude estimates for contributions to one-loop neutrino masses

In this appendix, we estimate the order of magnitude of the one-loop contributions to the neutrino masses due to the graphs of fig. 2(a) and (b), and the corresponding graphs (not shown) in which the light sneutrinos [heavy neutrinos] in graph (a) [(b)] are replaced by heavy sneutrinos [light neutrinos].

In the case of graph (a), the dominant contribution involves the light sneutrino–

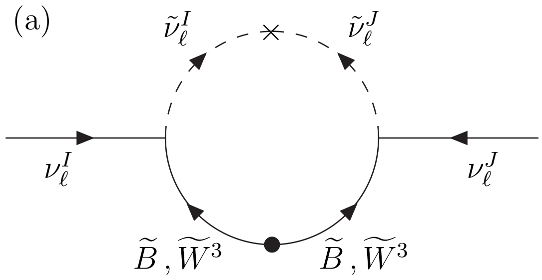


Figure 5: One-loop corrections to light neutrino masses. The $\times$ marks the location of the $\Delta L = 2$ transition. (a) The loop consisting of light sneutrinos and gauginos. The $\times$ indicates the location of light sneutrino–antisneutrino mixing, and the solid dot indicates a factor of the gaugino Majorana mass in the numerator of the fermion-number-violating gaugino propagator. (b) The loop consisting of the neutral Higgs field H_2^2 and a heavy neutrino. The $\times$ indicates the lepton-number-violating heavy neutrino propagator, which is proportional to $M\delta^{KL}$, and the solid dot indicates a mass insertion of the form $(H_2^{2\,*})^2$. The contributions of the corresponding graphs (not shown) in which the gauginos in (a) are replaced by the Higgsino $\widetilde{H}_2^2$, the light sneutrinos in (a) are replaced by heavy sneutrinos, and the heavy neutrinos in (b) are replaced by light neutrinos are all suppressed by an additional powers of $\mathcal{O}(vM^{-1})$ as explained in the text.

neutrino–gaugino interaction term[26] of eq. (5.5). We can estimate the leading contribution of this graph by replacing the internal lines by the interaction eigenstate fields that appear in eq. (5.5), as depicted in fig. 5. That is, we first replace the S_k with the $\widetilde{\nu}_\ell^I$, which must point *away from both* external vertices, as shown in fig. 5(a). The latter is possible only in the presence of light sneutrino–antisneutrino mixing, which is indicated by the $\times$ in fig. 5(a). Using the expected magnitudes of the model parameters given by eqs. (2.11) and (2.14), the $\times$ in fig. 5(a) produces a factor $\Delta m^2_{\tilde{\nu}_\ell} \sim \mathcal{O}(v^3 M^{-1})$. The neutralino line can be treated perturbatively. In the lowest order approximation, we take the neutralino to be a gaugino (either $\widetilde{B}$ or $\widetilde{W}^3$, with Majorana masses M_1 and M_2, respectively), and we treat the mixing of the gauginos with the neutral higgsino states ($\widetilde{H}_1^1$ and $\widetilde{H}_2^2$) as a perturbation. The corresponding gaugino propagators (with internal four-momentum q) shown in fig. 5(a) are fermion-number-violating propagators (indicated by the clashing arrows), and are given by $iM_k/(q^2 - M_k^2)$ for $k = 1, 2$. We denote the presence of the gaugino mass [which is of

[26]Of the three light sneutrino-neutrino-neutralino interactions of eq. (5.5), the two sneutrino-neutrino-higgsino interaction terms are suppressed by a factor of $\mathcal{O}(m_D M^{-1})$ relative to the sneutrino-neutrino-gaugino interaction, and can be neglected.

$\mathcal{O}(v)$] in the numerator by the solid dot in fig. 5(a). Not including this explicit factor of the gaugino mass, the loop in graph (a) then consists of two massive scalar propagators [with mass of $\mathcal{O}(v)$] and one fermion-number-violating propagator; hence the loop integral has a mass dimension of -2. Thus, the corresponding loop integral is of $\mathcal{O}(v^{-2})$. Combining the above results, the order of magnitude of the contribution of graph (a) is:

$$C_L \frac{v^3}{M} \cdot \frac{1}{v^2} \cdot v = C_L \frac{v^2}{M}\,, \tag{D.1}$$

which is indeed of order the tree-level neutrino mass multiplied by the product of the relevant vertex coupling constants and a typical loop factor of $1/16\pi^2$ (denoted by C_L above).

Suppose we replace the light sneutrinos of graph (a) with heavy sneutrinos. In this case, the effect of heavy sneutrino–antisneutrino mixing is $\Delta m^2_{\tilde{\nu}_h} \sim \mathcal{O}(m_B^2) \sim \mathcal{O}(vM)$. From eq. (5.5), we see that there are potentially two contributions—one involving the gauginos and one involving the higgsino $\widetilde{H}_2^2$. In the case of the gaugino loop graph, each vertex introduces a $\mathcal{O}(vM^{-1})$ suppression. Thus, following the analysis above, we conclude that the order of magnitude of the heavy-sneutrino loop is suppressed by a factor of $\mathcal{O}(v^2M^{-2})$ as compared with the light-sneutrino loop. In the case of the loop graph involving $\widetilde{H}_2^2$, we note that there is no diagonal Majorana mass term for this higgsino field. Moreover, $\widetilde{H}_1^1$ does not couple to the external neutrinos, so we cannot use the off-diagonal Majorana mass term $\mu\widetilde{H}_1^1\widetilde{H}_2^2$ for the fermion-number-violating neutralino propagator. Therefore, the heavy-sneutrino loop can be neglected.

In the case of graph (b), the propagator of the heavy neutrino (with internal four-momentum q) is given by $iM\delta^{KL}/(q^2 - M^2)$, due to the presence of the lepton-number violating mass M (indicated by the $\times$). Since the loop integral is dimensionless, it naively appears that the resulting loop integral should be of $\mathcal{O}(M)$. However, an explicit computation of the graph of fig. 2(b) demonstrates that the coefficient of the leading $\mathcal{O}(M)$ term vanishes exactly after summing over the internal neutral Higgs and Goldstone states. The subleading term does not vanish and is of $\mathcal{O}(v^2M^{-1})$, which is the magnitude of the *light* neutrino mass. This cancellation can be easily understood by noting that the two vertices of fig. 2(b) arise from interactions of eq. (5.5) that involve H_2^2. Thus we replace the neutral Higgs and Goldstone lines of fig. 2(b) by the H_2^2 field [cf. eq. (5.6)]. According to the interaction Lagrangian of eq. (5.5), the H_2^2 field must point *into both* external vertices, as shown in fig. 5(b). This requires a mass insertion on the H_2^2 line of the form $(H_2^2)^2 + \text{H.c.}$ In fact, such a term exists in the MSSM Higgs potential [49] after shifting the neutral field $H_2^2 \to H_2^2 + v_2/\sqrt{2}$, which results in a term of the form $\frac{1}{4}m_Z^2 \sin^2\beta (H_2^2)^2 + \text{H.c.}$ Thus, in the mass insertion approximation, graph (b) consists of the lepton-number-violating heavy neutrino propagator, two massive scalar field lines[27] and an insertion of $\mathcal{O}(v^2)$. After extracting the factor of M from the numerator of the heavy neutrino propagator, the remaining loop integral now has a mass dimension of -2, which yields a result of $\mathcal{O}(M^{-2})$. Combining these result, the order of magnitude of the contribution of fig. 5(b) is given by:

$$C'_L \frac{1}{M^2} \cdot M \cdot v^2 = C'_L \frac{v^2}{M}\,, \tag{D.2}$$

which is again of order the tree-level neutrino mass multiplied by the product of the relevant vertex coupling constants and a typical loop factor (denoted above by C'_L). This result confirms our previous argument above. A careful evaluation of the leading behavior of the loop integral (in the limit of $M \gg v$) then reproduces the result obtained in eq. (5.8). Note that the factor of $\sin^2\beta \equiv v_2^2/v^2$ that arises in the mass insertion on the H_2^2 line cancels out a similar factor of v_2^2 that appears in $C'_L \propto Y_\nu^2$.

If the heavy neutrinos in fig. 5(b) are replaced by light neutrinos, the resulting contribution is suppressed by an additional factor of $\mathcal{O}(v^2 M^{-2})$ due to the suppression of the $\nu_\ell^I \nu_\ell^K H_2^2$ interaction of eq. (5.5).

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