PHYSICAL REVIEW B VOLUME 58, NUMBER 5 1 AUGUST 1998-I

Superconductivity-induced Anderson localization

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(Received 17 February 1998)

We have studied the effect of a random superconducting order parameter on the localization of quasiparticles, by numerical finite-size scaling of the Bogoliubov–de Gennes tight-binding Hamiltonian. Anderson localization is obtained in $d=2$ and a mobility edge where the states localize is observed in $d=3$. The critical behavior and localization exponent are universal within error bars both for real and complex random order parameter. Experimentally these results imply a suppression of the electronic contribution to thermal transport from states above the bulk energy gap. [S0163-1829(98)03229-9]

During the past few years phase-coherent transport in hybrid superconducting structures has emerged as a new field of study, bringing together the hitherto separate areas of superconductivity and mesoscopic physics. Recent experiments have revealed a variety of unexpected phenomena, including zero-bias anomalies,[1–3] re-entrant and long-range behavior,[4] and phase-periodic transport.[5–9] These experiments can all be described by combining traditional quasiclassical Green's-function techniques with boundary conditions derived initially by Zaitsev[10] and simplified by Kuprianov and Lukichev[11] or alternatively by generalized current-voltage relations[12] based on a multiple-scattering approach to phase-coherent transport. The latter approach focuses attention on Andreev scattering,[13] whereby an electron can coherently evolve into a hole and vice versa, without phase breaking.

The aim of this paper is to address a new phenomenon, not describable by quasiclassical techniques, namely the onset of quasiparticle Anderson localization due to spatial fluctuations in a superconducting order parameter. In contrast with all of the above experiments, where the superconducting order parameter $\Delta(r)$ is typically homogeneous, there are many situations in which $\Delta(r)$ varies randomly in space, even though the underlying normal potential is perfectly ordered. One example is provided by the melting of a flux lattice[14,15] in an otherwise perfectly crystalline high-T_c superconductor. Another should occur in anisotropic superconductors, where by analogy with ^{3}He-A, disordered textures can arise when an anisotropic phase is nucleated from a more symmetric phase such as ^{3}He-B. In the first of these examples, the order parameter is not quenched. Nevertheless, close to the melting curve, the time scale for changes in $\Delta(r)$ can be made arbitrarily long and therefore in the spirit of the Born-Oppenheimer approximation, it is reasonable to freeze the disorder and when necessary, treat any temporal fluctuations as a contribution to the inelastic-scattering lifetime.

In one dimension, it is straightforward to demonstrate[16] that fluctuations in $\Delta(r)$ alone can localize the excitations, even at energies high above the bulk energy gap. However, localization in strictly one dimension is of little interest experimentally and therefore in this paper, we address the question of whether or not superconductivity-induced Anderson localization occurs in higher dimensions. Early analytic work[17,18] suggested that in the presence of time-reversal symmetry, states of energy $E=0$ are localized for dimensions $d\leqslant 2$, while in the absence of time-reversal symmetry such states are localized in all dimensions. However calculations using a numerical finite-size scaling approach[19] were inconclusive and to date there has been no experimental confirmation of these predictions. In this paper we provide firm numerical evidence for superconductivity-induced Anderson localization in $d=2$ and 3 dimensions and compute the exponent ν controlling the divergence of the localization length ξ at the mobility edge in $d=3$.

To address the question of superconductivity-induced Anderson localization, we analyze the tight-binding Bogoliubov–de Gennes equations

$$E\psi_i(E)=\epsilon_i\psi_i(E)-\gamma\sum_j \psi_j(E)+\Delta_i\phi_i(E),$$
$$E\phi_i(E)=-\epsilon_i\phi_i(E)+\gamma^*\sum_j \phi_j(E)+\Delta_i^*\psi_i(E), \qquad (1)$$

where $\psi_i(E)$ [$\phi_i(E)$] indicates the particle (hole) wave function of energy E on site i and j sums over the neighbors of i. Since only scaling behavior near a critical point is of interest, we examine the simplest possible model of a system with no normal disorder, but a spatially fluctuating order parameter, obtained by choosing ϵ_i equal to a constant ϵ_0 for all sites i and to set the energy scale, choose $\gamma=1$. Two models of disorder will be examined. In model 1 (which preserves time-reversal symmetry), we choose $\Delta_i=\Delta_0[1+\delta\Delta_i]$ and in model 2 (which breaks time-reversal symmetry), we choose $\Delta_i=\Delta_0[(1+\delta\Delta_i)+\imath(1+\delta\Delta_i')]$, where $\delta\Delta_i$ and $\delta\Delta_i'$ are random numbers uniformly distributed between $-\delta\Delta$ and $+\delta\Delta$. In what follows, we choose $\epsilon_0=0$.

For each model, we compute the transfer matrix T for a long strip ($d=2$) and a long bar ($d=3$) of length L sites and cross-section M^{d-1} sites, respectively, and identify the localization length ξ_M with the inverse of the corresponding smallest Lyapunov exponent. The results are, of course, sensitive to the chosen energy E and since, in the absence of disorder (i.e., $\delta\Delta=0$), there exists an energy gap at $E=0$, the usual choice of $E=0$ adopted in the absence of superconductivity is inappropriate. As a guide to a reasonable choice of E, we consider the related problem of a system with normal disorder but with a uniform order parameter. In this case ϵ_i is chosen randomly from a uniform probability distribution but $\Delta_i=\Delta_0$, for all i. As noted in Ref. 20 if $\psi_i^0(E_0)$ is a solution of the normal-state Schrödinger equation, namely $\epsilon_i\psi_i^0(E_0)-\Sigma_j\psi_j^0(E_0)=E_0\psi_i^0(E_0)$, then $\psi_i(E)$

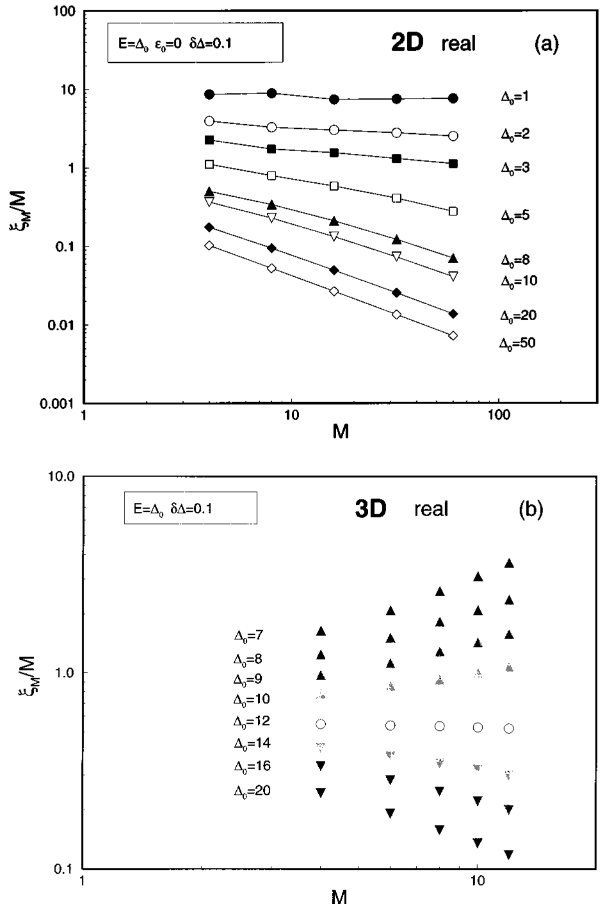

FIG. 1. (a) The ξ_M/M plotted as a function of the finite width M in two dimensions for $E=\Delta_0$, $\epsilon_0=0$, $\delta\Delta=0.1$, and various values of Δ_0. (b) As in (a) but in three dimensions where a critical point is indicated.

and $\phi_i(E)$ are each proportional to $\psi_i^0(E_0)$, where $E=\sqrt{[E_0^2+|\Delta_0|^2]}$. This means that if in the absence of superconductivity a state at energy E_0 is localized by normal disorder, then in the presence of a uniform order parameter Δ_0, quasiparticle states at energy E are localized with the same localization length. As a consequence all critical properties are unchanged, provided E_0 is replaced by E. In the normal-state problem the least localized states occur at $E_0=0$ and therefore in the presence of normal disorder and a uniform superconducting order parameter these states occur at $E=|\Delta_0|$. Of course, in what follows we are interested in the opposite limit of a spatially fluctuating order parameter with no normal disorder. Nevertheless, guided by the above observation we choose $E=\langle|\Delta_i|\rangle$, where $\langle|\Delta_i|\rangle$ is the ensemble averaged order parameter, which gives $E=\Delta_0$ for model 1 and $E=\sqrt{2}\Delta_0$ for model 2.

The raw data for ξ_M/M versus M, for model 1 with $E=\Delta_0$, $\epsilon_0=0$, and $\delta\Delta=0.1$, are shown in Figs. 1(a) and 1(b) for two and three dimensions, respectively. The strength of disorder in the order parameter is $W=2\Delta_0\delta\Delta$, whose critical value is denoted W_c, and is varied by changing Δ_0, with fixed $\delta\Delta$. In two dimensions ξ_M/M decreases with increasing M indicating that all states are localized with $W_c=0$, whereas in three dimensions there is a crossover from localized to extended behavior at around $\Delta_0\approx 12$ which for the adopted value of $\delta\Delta=0.1$ corresponds to $W_c\approx 2.4$.

To quantify the critical behavior in three dimensions, we linearize the data about W_c by writing $\ln(\xi_M/M)=\alpha_M+\beta_M \ln W$ and obtain the coefficients α_M and β_M for various M. In terms of the fixed-point values $\ln(\xi_M/M)_c$ and $\ln W_c$, we note that $\alpha_M=\ln(\xi_M/M)_c-\beta_M \ln W_c$. Thus, a graph of

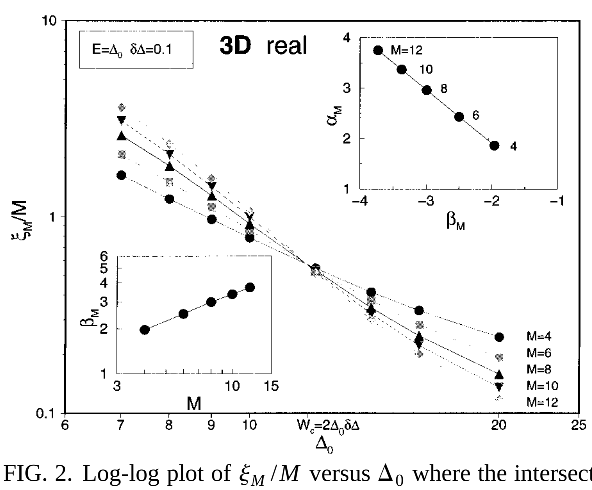

FIG. 2. Log-log plot of ξ_M/M versus Δ_0 where the intersection defines W_c and $(\xi_M/M)_c$. The upper-right inset shows the coefficients α_M versus β_M whose slope is $-\ln W_c$. The lower-left inset shows a log-log plot of β_M versus M which yields the value for the exponent $\nu=1.64\pm 0.06$.

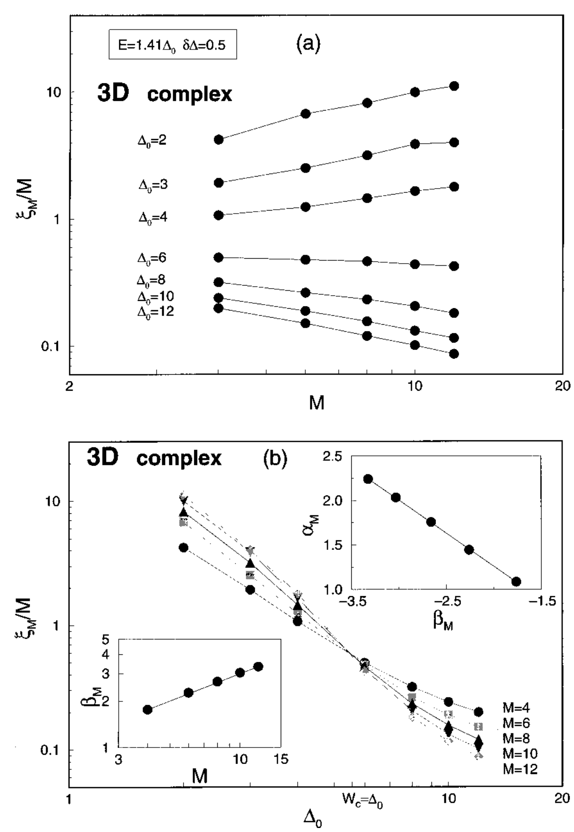

FIG. 3. (a) Log-log plot of ξ_M/M versus M for various values of Δ_0 when time-reversal symmetry is broken (model 2) which shows a crossover from extended to localized states. (b) Log-log plot of ξ_M/M versus Δ_0 for model 2 where the intersections define W_c and $(\xi_M/M)_c$. The upper-right inset shows the coefficients α_M versus β_M whose slope is $-\ln W_c$. The lower-left inset shows a log-log plot of β_M versus M which yields the value for the exponent $\nu=1.69\pm 0.06$.

α_M versus β_M yields $\ln(\xi_M/M)_c$, $-\ln W_c$ and hence the critical disorder W_c. The critical exponent ν for the divergence of the localization length ξ of the infinite system is obtained by substituting α_M into the first linear relation, which yields $\ln(\xi_M/M)=\ln(\xi_M/M)_c+\beta_M \ln(W/W_c)$. Moreover, near the critical point $\ln(W/W_c)\sim(W-W_c)/W_c$ and $\xi\sim|W-W_c|^{-\nu}$, so that $\ln(\xi_M/M)=\ln(\xi_M/M)_c\pm\xi^{-1/\nu}\beta_M$, where the + (−) sign refers to $W>W_c$ ($W<W_c$). The finite-size scaling requirement $\xi_M/M=f(\xi/M)$ immediately implies $\beta_M\sim M^{1/\nu}$, which permits the computation of the exponent ν.

Figure 2 shows a graph of $\ln(\xi_M/M)$ versus $\ln\Delta_0$, from which α_M and β_M for the chosen widths M can be extracted. The top-right inset shows the resulting plot of α_M versus β_M whose slope is $-\ln W_c$ and the corresponding intercept is $\ln(\xi_M/M)_c$. This yields $W_c=2.36\pm0.04$ which corresponds to $\Delta_0=11.73\pm0.12$ and $(\xi_M/M)_c=0.58\pm0.02$. The lower-left inset shows $\ln\beta_M$ versus $\ln M$ whose slope yields the critical exponent $\nu=1.64\pm0.06$.

For model 2, where time-reversal invariance is broken due to the presence of a complex order parameter, all states are localized in $d=2$. In contrast, Fig. 3(a) shows the corresponding plots of ξ_M/M versus M in three dimensions which clearly show a crossover from extended to localized behavior. Results from a more accurate calculation are presented in Fig. 3(b), where the upper-right figure yields $W_c=5.57\pm0.12$ and $(\xi_M/M)_c=0.58\pm0.02$. The lower-left inset shows $\ln\beta_M$ versus $\ln M$, the slope of which leads to the value for the exponent $\nu=1.69\pm0.06$. The errors in the calculation of ξ_M/M are monitored as a function of length L and chosen to be less than about 0.01 by taking long strips of lengths $L=250\,000$ and bars of $L=200\,000$ ($L=50\,000$) for the real (complex) case. The errors for W_c and ν are estimated from the corresponding least-square fits. We have also repeated our calculations by taking points closer to the critical value W_c, where the above analysis holds, with no significant change of our results.

The first important feature of the above calculation is the unambiguous prediction of superconductivity-induced quasiparticle localization in $d=2$ and the presence of a mobility edge in $d=3$. Localization arises from fluctuations in the superconducting order parameter alone, without the need for additional normal disorder. A second key result is the observation that for both models we find $(\xi_M/M)_c\sim0.58$ and $\nu\sim1.6$, which are remarkably close to the values reported for normal $d=3$ real systems,[21] and also consistent with reported data for ordinary disordered critical systems with and without time-reversal invariance.[22,23] Recently, slightly different scaling behavior is obtained with and without time reversal by an alternative data analysis based on polynomial fits.[24] Our study in $d=3$ cannot distinguish such small difference if it exists.

From an experimental point of view, it is worth noting that the absence of quasiparticle diffusion does not imply the vanishing of the electrical conductance, because Andreev scattering does not conserve quasiparticle charge. It does, however, imply a vanishing of the electronic contribution to thermal transport from certain states above the gap. In a clean superconductor at a finite temperature T, this varies as $\exp(-\Delta/k_bT)$, where Δ is the bulk energy gap. In contrast, in the presence of a fluctuation-induced quasiparticle mobility edge E_c, this will be replaced by $\exp(-E_c/k_bT)$. Thus, for example, the melting of a flux lattice in a high-temperature superconductor should be accompanied by an exponential change in the electronic contribution to the thermal conductance.

This work was supported by a TMR of the EU and for D.E.K. and S.N.E. partially by the Greek Secretariat of Science and Technology, who are also grateful for the hospitality at Lancaster where most of the work was carried out.

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