SUPER-COSMOLOGY

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Cosmological implications of supersymmetric theories are discussed. Monopoles are shown to be suppressed A new mechanism for the creation of the baryon asymmetry of the universe is presented requiring a nucleon lifetime of the order of 10^{31} yr and $P \to \bar{\nu}_\mu K^+, \mu^+ K^0$ as the main decay mode

Grand unified theories of the strong and electroweak interactions [1] have successfully confronted many questions that had previously remained unanswered. Nevertheless, the enormous difference in scales of the unified and the electroweak symmetry requires exceedingly accurate cancellations among the parameters of the theory. This problem, known as the gauge hierarchy problem, finds a natural solution, at least in its technical aspects, if we incorporate in GUTs the notion of global supersymmetry [2]. In GUTs, supersymmetry is the only symmetry that can forbid scalar masses, thus keeping dimensional parameters of the electroweak sector sufficiently small. Supersymmetric grand unified models have attracted a great deal of attention recently. They are endowed with very interesting new features as well as new problems.

In this paper we concentrate on the cosmological implications of supersymmetric GUTs. Ordinary GUTs have been remarkably successful in unifying the standard Big Bang cosmology with particle physics at very high energies and thus providing for the first time a unified view of cosmos. For instance, an outstanding success of GUTs is the creation of the baryon asymmetry of the universe, a long-standing puzzle. Supersymmetric grand unified theories can equally well be married with cosmology and although some of their features are different, they are equally successful in supplying solutions. It is remarkable that they naturally lead to the solution of problems that have resisted plausible treatment within ordinary GUTs. We find that monopoles are naturally suppressed. We provide three different mechanisms for generating the desirable baryon to photon ratio. In one of them the proton decays preferably to $\bar{\nu}_\mu K^+, \mu^+ K^0$ at a rate of 10^{31} yr approximately.

A fundamental characteristic of global supersymmetry is that the vacuum energy is zero. Spontaneous breaking of supersymmetry gives a positive value to the energy to all orders in perturbation theory. Thus, in supersymmetric theories we have $E_{\text{vac}} \geqslant 0$. At finite temperature supersymmetry is broken [3]. Physically this results from the different statistics obeyed by bosons and fermions. The energy is still positive although the effective potential can easily be negative [‡1]. As we mentioned in the introduction the primary reason for incorporating supersymmetry in unified theories is that the technical aspect of the hierarchy problem is solved if supersymmetry is good down to M_W (or at most a few TeV). Down to the scale of breaking of supersymmetry (which we shall denote by M_S) the different phases of the theory have vacuum energy differences of the order T^4 [‡2]. (Remember that all vacua are degenerate of zero energy in the "cold" theory.) It is already evident that since the energy density is dominantly thermal, no intermediate de Sitter phase will appear as required by the inflationary scenario [4].

‡1 At very high temperatures $V_{\text{eff}} \simeq V_{\text{eff}}(0) - |c|T^4 + |c'|T^2$ The potential can be negative although $E \simeq V_{\text{eff}}(0) - |c'|T^2 + 3|c|T^4 > 0$

‡2 Intermediate mass, long-lived particles might provide terms $\sim mT^3$.

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The breaking of supersymmetry at M_S can be achieved in different ways which we classify in three categories. Supersymmetry can be broken:

(1) Spontaneously, with an F term as has been realized in the inverse hierarchy model of Witten [5].

(2) Spontaneously, with a D term of a U(1) group which develops an expectation value perturbatively or non-perturbatively [6].

(3) Explicitly by the introduction of soft non-supersymmetric interactions [7]. The behaviour of Witten's model at finite temperature has been examined by various authors [8]. Their conclusions stop at the statement that the phase transition delays down to M_S, something that renders the baryon asymmetry scenario defunct. It is also hard to draw conclusions about monopole suppression in such a scheme. Independently of these problems the only semi-realistic version of the model is not asymptotically free [9], putting in question its use in a perturbative framework. In examining the cosmological implications of supersymmetric unified theories little difference will arise whether we realize breaking schemes (2) and (3), so we can be quite general without referring specifically to the type of breaking.

The unifying gauge group G will, in general, have a set of subgroups G_i with broken gauge symmetry. Supersymmetry constrains the different vacua corresponding to G_i and G to be all degenerate of zero energy (at $T = 0$). It is also possible that instead of multiminima we could have only one minimum with broken unifying gauge symmetry [that would be of course SU(3) × SU(2) × U(1) invariant]. In the special case of a monominimum, supersymmetric GUTs would look like ordinary GUTs. The more standard case of multiminima appears more interesting. Let us consider for definiteness supersymmetric SU(5) with a supermultiplet Φ in the adjoint representation. All other superfields are irrelevant in considering the breaking of the unifying gauge symmetry. A superpotential for the model is

$$W = \tfrac{1}{3}\lambda\, \mathrm{tr}[\Phi^3] + \tfrac{1}{2}M\, \mathrm{tr}[\Phi^2] \,. \tag{1}$$

The potential is

$$V = \mathrm{tr}\,|M\Phi + \lambda(\Phi^2 - \tfrac{1}{5}\,\mathrm{tr}\,\Phi^2)|^2 + \tfrac{1}{2}e^2\, \mathrm{tr}([\Phi, \Phi^+])^2 \,. \tag{2}$$

The model is characterized by the three degenerate supersymmetric minima (fig. 1)

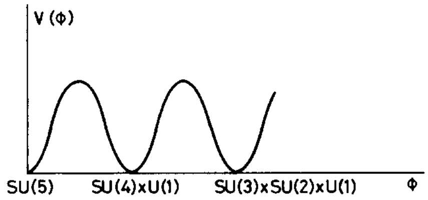

Fig 1 The effective potential at zero temperature.

$\langle\Phi\rangle = 0$

(SU(5)-symmetric),

$\langle\Phi\rangle = \tilde{\phi}\ \mathrm{diag}(1, 1, 1, 1, -4)$

(SU(4) × U(1)-symmetric),

$\langle\Phi\rangle = \phi\ \mathrm{diag}(2, 2, 2, -3, -3)$

(SU(3) × SU(2) × U(1)-symmetric),

where $\tilde{\phi} = M/3\lambda$ and $\phi = M/\lambda$. When we turn on the temperature, supersymmetry is broken and the different vacua are shifted by amounts proportional to the temperature. Approximately at high temperatures, the potential will be (fig. 2).

$$V(\phi, T) \simeq V(\phi) + c_1\phi^2 T^2 + c_2 T^4 \,. \tag{3}$$

The coefficient c_2 is calculated from

$$c_2 = -\tfrac{1}{90}\pi^2(N_B + \tfrac{7}{8}N_F) \,,$$

where N_B and N_F are the number of helicity states of light bosons and fermions. Considering only gauge particles and the quark–lepton supermultiplets it is equal to $-\frac{23}{8}\pi^2$, $-\frac{61}{24}\pi^2$ and $-\frac{19}{8}\pi^2$ for the SU(5), SU(4) × U(1) and SU(3) × SU(2) × U(1) phases correspondingly. A general formula for c_1 is

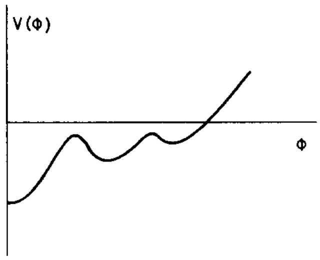

Fig 2. The effective potential at very high temperature

$$c_1 = \frac{1}{24}\,\{\mathrm{tr}[M_S^2] + \mathrm{tr}[M_F^2] + 3\,\mathrm{tr}[M_V^2]\}\,. \qquad (4)$$

The global minimum is always the SU(5) symmetric one, at the origin. For any temperature higher than the scale of supersymmetry breaking M_S, the phases with broken gauge symmetry correspond only to local minima, characterized by larger vacuum energy. A naive scenario would be that the universe cools down to M_S in the symmetric phase and there (hopefully)

$$\underset{\mathrm{SU(5)}}{V(M_S)} - \underset{\mathrm{SU(3)\times SU(2)\times U(1)}}{V(M_S)} > 0\,.$$

Then, the phase transition SU(5) → SU(3) × SU(2) × U(1) would proceed. Such a turn of events, if it could be true, would be disastrous for any appreciable baryon creation. Fortunately, things cannot happen this way in a realistic GUT. For example, the supersymmetric SU(5) model we considered (supplied in addition with three generations of quark–lepton supermultiplets and a pair of 5 and $\bar{5}$ Higgs supermultiplets), is asymptotically free. As we come to lower energies (equivalently, lower temperatures) the SU(5) gauge interactions become strong long before we approach M_S. Strong gauge forces completely invalidate the naive scenario [10]. The scale at which the SU(5) coupling becomes strong is easily calculated from

$$\alpha^{-1}(T) \simeq \alpha^{-1}(M) + (b/2\pi)\ln(T/M)\,,$$

$$b = \tfrac{55}{3} - \tfrac{2}{3}(5 + \tfrac{1}{2}\cdot 3 + 3\cdot\tfrac{3}{2} + \tfrac{1}{2}N_H) - \tfrac{1}{6}(3 + 3\cdot 3 + N_H)$$

$$= 9 - \tfrac{1}{2}N_H\,. \qquad (5)$$

With $M \simeq 10^{16}$–10^{17} GeV and $\alpha(M) \simeq 1/24$ we obtain, for $N_H = 2$, $\Lambda_{SU(5)} \simeq 10^9$ GeV. Thus, as we approach the range of temperatures 10^{10}–10^9 GeV we should inevitably expect strong-coupling phenomena which will drastically alter the picture. Unfortunately, present computational ability in strong-coupling phenomena is quite limited. Nevertheless let us try to understand qualitatively what might happen [‡3]. As the gauge interactions increase in strength there will be a tendency to enter a "confining phase" in which, as we cool down, massive states will be created and the number of light degrees of freedom will drastically decrease. At the same range of temperatures SU(4) × U(1) and SU(3) × SU(2) × U(1) are still weakly coupled and possess a large number of light degrees of freedom. Hence it seems quite probable that the energy density of the SU(5) phase will decrease since light particles will be confined at such "low" temperatures thus shifting the value of the symmetric minimum higher than the broken minima (fig. 3a). A rapid phase transition to a broken phase should then be expected. If the transition goes towards the SU(4) × U(1) phase new strong-coupling phenomena would unavoidably occur around ~10^6 GeV where $\alpha_{SU(4)} \sim O(1)$ and we will end up in the SU(3) × SU(2) × U(1) phase (fig. 3b). It is interesting that in supersymmetric GUTs the final asymptotically free group has $\alpha_{SU(3)} \sim O(1)$ at much lower energies than SU(4) and SU(5). In a sense, the phase with the smallest unbroken symmetry wins. Thus, in this scenario we end up in SU(3) × SU(2) × U(1) in a rather unique way. We find this property rather remarkable and certainly this is not the case in traditional GUTs. Notice that there is no intermediate period with exponential expansion, due to the vanishing of the cosmological constant, and no appreciable reheating. Thus, we do not have the usual problems associated with "slowly" growing bubbles in an exponentially expanding universe [4].

‡3 It is very unlikely that supersymmetry would break in the strong-coupling region. See ref [11].

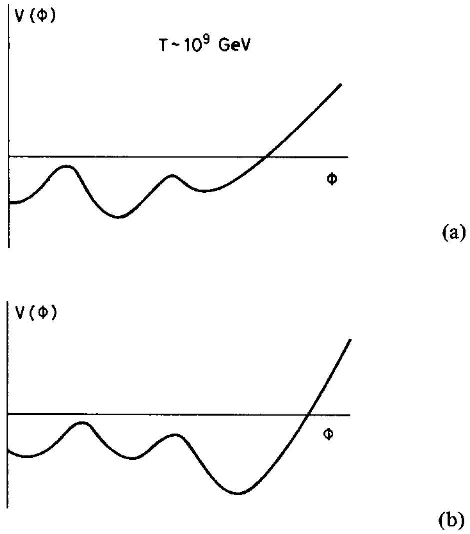

Fig. 3 The effective potential at intermediate temperatures.

Let us come now to the problem of superheavy magnetic monopoles and see whether it is still a problem in supersymmetric GUTs. As usual the early universe is described with a Robertson–Walker metric with zero curvature. The scale factor R obeys

$$(\dot{R}/R)^2 = \tfrac{8}{3}\pi M_P^{-2}\rho \,, \tag{6}$$

ρ is the energy density. A universe cooling down in any of the supersymmetric phases will, in general have a density of the form

$$\rho \simeq c_2 T^4 + mT^3 \,. \tag{7}$$

The second term accounts for the presence of semi-superheavy degrees of freedom which have not completely decoupled. The time-temperature law is approximately

$$t \simeq [M_P/2(\tfrac{8}{3}\pi c_2)^{1/2}]\, T^{-2} \,, \tag{8}$$

or

$$t \simeq (1/6\pi)^{1/2}(M_P/m^{1/2})T^{-3/2} \,, \tag{9}$$

depending on the temperature region.

Assuming that the phase transition proceeds through the creation of bubbles, the number of monopoles will be roughly comparable to the number of bubbles. The probability for the creation of a bubble per unit volume per unit time must approximately be on dimensional grounds

$$\sim hT^4\, \theta(T_c - T) \,,$$

where h is a dimensionless constant and T_c is the temperature at which the transition starts. It is then easy to compute the number of monopoles to be [12]

$$N_m \sim cT^3 h^{3/4} \,, \tag{10}$$

or

$$N_m \sim c' T^3 h^{3/4} \,, \tag{11}$$

depending on the time–temperature law. The temperature at which the transition is completed is found to be [12]

$$1/T^* \simeq 1/T_c + \mathrm{O}(10)/h^{1/4} M_P \,, \tag{12}$$

in the first case, and

$$1/T^* \simeq 1/T_c + \mathrm{O}(1) m/h^{1/4} M_P^2 \,, \tag{13}$$

in the second case.

Since $T_c \simeq T^* \simeq 10^9$–$10^{10}$ GeV, eq. (12) implies $h^{1/4} \simeq 10^{-8}$–10^{-9}. This leads to

$$N_m/T^3 \sim 10^{-27} \,,$$

which is within the bound [12]. No appreaciable reheating takes place. Similarly we obtain monopole suppression from (13). It should be noted that the experimental bound on the number of monopoles also provides an upper bound on T^* or equivalently in our scenario on T_c around 10^{10} GeV. It is remarkable that $\Lambda_{SU(5)}$ coincides with this limit which in higher groups may be problematic. It is amusing that the monopole density, although it satisfies the present experimental bound, is not exceedingly smaller, thus leaving open the possibility that it could help us understand the missing mass of the universe.

The next problem we shall turn to is that of the creation of the baryon asymmetry. The ingredients of the standard scenario ‡4 are baryon number, *CP*- and *C*-violating interactions on the one hand and the condition of non-equilibrium on the other. In order to obtain a quantitatively sound prediction, the first ingredient must be of the right magnitude and the second must be added at the right time. The particles needed could in principle be superheavy gauge bosons ‡5, Higgs bosons or superheavy fermions. It is assumed that soon after the Big Bang the baryon number violating interactions come into equilibrium so that any primordal asymmetry was washed out. A new baryon asymmetry can be generated if the expansion rate of the universe is sufficiently rapid compared to the interaction rate so that they drop out of equilibrium. This leads to the condition

$$M_X \gtrsim \tfrac{1}{10}\alpha_X M_P \,, \tag{14}$$

where M_X is the mass of the interaction particles and α_X their coupling to light particles (M_P is the Planck mass).

The baryon to photon ratio arising from the decays of Higgs bosons can be summarized by the approximate formula

$$N_B/N_\gamma \simeq 10^{-2}(\Delta B)_H \,. \tag{15}$$

In order to obtain a non-zero ΔB at least two Higgs multiplets are needed.

In the supersymmetric scenario the phase transition is postponed down to temperatures of order 10^{10}–10^9 GeV. Above this temperature region, SU(5) is un-

‡4 For a review of cosmological implications of GUTs, see Nanopoulos [1]

‡5 Superheavy gauge bosons are less important for various reasons, see Nanopoulos [1].

broken and the 5 and $\bar{5}$ of Higgs multiplets are nearly massless ($\ll M_X$). The lower mass limit [13] for the coloured triplets, compatible with a proton lifetime greater than 10^{31} yr, is around 10^9-10^{10} GeV coinciding with the temperature region around $\Lambda_{SU(5)}$. The decays of the Higgs bosons that will provide us with a non-vanishing baryon asymmetry must occur out of equilibrium at a temperature smaller than or equal to their mass in order to ensure that the decay products will not recombine. Recombination and thermalization would wash out the baryon asymmetry. In our scenario this is automatically satisfied with coloured triplet Higgs's of mass 10^9-10^{10}GeV. No thermalization occurs since during the phase transition we are out of equilibrium. The lightness of the Higgs's and the out of equilibrium condition ensure that we obtain the appropriate number of particles

$$N_H/N_\gamma \sim (10^9-10^{10})\ \mathrm{GeV}/M_H\ . \tag{16}$$

Nevertheless it is well known that one Higgs supermultiplet ‡6 is not enough to obtain a non-vanishing baryon asymmetry. Additional Higgs multiplets can be introduced, but they must not contain light doublets because that would lead to too large a value for $\sin^2\theta_W$. However, we can arrange things so that only one linear combination of the doublets is light [14]. Thus we introduce an additional chiral supermultiplet (a 5 and a $\bar{5}$) with a positive mass squared that will get no expectation value. Its couplings to quarks and leptons will be similar to those of the ordinary Higgs and the natural value for its mass would be of order $\Lambda_{SU(5)}$ ‡7. The baryon asymmetry due to the Higgs's would arise from diagrams of the type shown in fig. 4 and would be roughly

$$\Delta B \propto \mathrm{Im}\{\mathrm{tr}(a_1 b_2 b_1^\dagger a_2^\dagger)/\mathrm{tr}(a_1 b_1^\dagger)\}\ . \tag{17}$$

Normally most of the baryon asymmetry we shall obtain will come from the decays of the light Higgs triplet with mass of $O(\Lambda_{SU(5)})$ since most of the heavy Higgs's will be produced at high temperatures under conditions of equilibrium and the baryon asymmetry

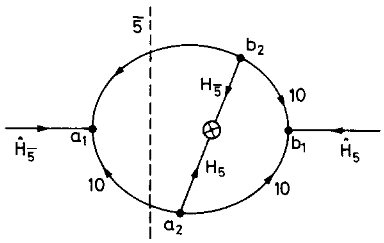

Fig 4. A diagram contributing to the baryon asymmetry.

generated through their decays will be washed out. Nevertheless, it is possible to have modes carrying a non-vanishing baryon number that do not thermalize [15]. This becomes possible from the approximate conservation of a global quantum number. The baryon asymmetry generated through these modes will not be washed out by thermal effects but will survive to low temperatures. This scenario seems to favour reducible light fermion representations, i.e., SU(5).

The alert reader has probably already noticed that our low-mass Higgs supermultiplet that was introduced out of necessity in order to obtain a non-vanishing baryon asymmetry has a drastic effect in the decay modes of the proton. The dominant decay mode of the proton is $p \to \bar{\nu}_\mu K^+$ involving second-generation particles, with all the characteristics of Higgs mediated decay [16]. The proton life-time will be the usual $\tau_p \sim 10^{31}$ yr. Here we have a case of a supersymmetric GUT where a substantial baryon asymmetry forces the proton to decay to strange particles in variance to usual supersymmetric GUTs where either proton decay is postponed or the proton decays at a rate 10^{31} yr with main decay modes consisting of $\bar{\nu}_\tau K^+$ [17]. It is amusing to point out that the existence of light ($\sim 10^{10}$ GeV) Higgs's makes $\sin^2\theta \simeq 0.218$ agree with experiment and puts it in sharp contrast with usual supersymmetric GUTs [17].

In addition to the mechanism of baryon production presented above, there is another independent way to obtain the desirable baryon to photon ratio, namely by introducing superheavy fermions [18]. This mechanism can be carried over naturally to supersymmetric GUTs by introducing new superheavy supermultiplets. A type of superheavy fermions employed in standard GUTs decay mainly to three light fermions through the exchange of a superheavy Higgs boson (fig. 5). The new supermultiplet is in a real representation of

‡6 By one Higgs supermultiplet we really mean two, a 5 and a $\bar{5}$.

‡7 It is amusing to note that the out of equilibrium condition (14) can be satisfied with Higgs bosons of mass 10^9-10^{10} GeV provided their couplings are $10^{-9}-10^{-8}$, i.e., as if they coupled to the lightest generation. We can ignore it of course since we are off equilibrium anyhow.

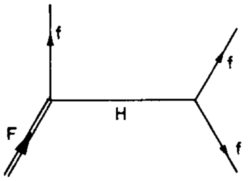

Fig. 5. Superheavy fermion (F) decay to three ordinary fermions (f).

the grand unifying group so that it can be given a mass m_F. The rate of a process as in fig. 5 would be

$$(\lambda^2/4\pi)^2\, m_F^5/(M_H)^4 \,, \tag{18}$$

m_F must not be too light because it might push $\Lambda_{SU(5)}$ to an unacceptable value, Let us choose $m_F \simeq 10^{15}$ GeV. In such a universe, two-body interactions mediated by superheavy bosons of mass M will remain in equilibrium down to a temperature

$$T_0 \simeq M[(1.6N^{1/2}/\alpha_G^2)M/M_P]^{1/3} \simeq 10^{16}\ \text{GeV}\,, \tag{19}$$

where N is the number of particle species. Since $T_0 > m_F$ it is safe to assume that $m_F/n_\gamma|_{T_0} \sim O(1)$. The temperature at which the new fermions decouple will be

$$T_F \sim \tfrac{1}{10}(\lambda^2/4\pi)^{4/3}(m_F/M)^3(MM_P^2)^{1/3}(n_F/n_\gamma|_{T_0})^{-1/3}\,, \tag{20}$$

which is approximately

$$T_F \sim (10^8 - 10^9)\ \text{GeV}\,.$$

Therefore

$$\frac{n_B}{n_\gamma} \simeq \frac{\tilde{n}_B}{n_\gamma}\left(\frac{T_F}{T_F'}\right)^3 \simeq \frac{n_F}{n_\gamma}\Big|_{T_0} (10^{-3} - 10^{-9})\left(\frac{T_F}{T_F'}\right)^3 .$$

The reheating temperature T_F' is at most an order of magnitude higher than T_F; therefore we obtain the desired baryon to photon ratio

$$n_B/n_\gamma \simeq 10^{-6} - 10^{-12}.$$

It is remarkable that during the period $m_F/3N \simeq 10^{13}$ GeV and $\Lambda_{SU(5)} \sim T_F$ the universe is "matter" dominated.

In conclusion let us summarize the main points of our scenario.

(1) The phase transition to the broken phase occurs at temperatures $10^{10} - 10^9$ GeV where the gauge coupling of SU(5) becomes strong.

(2) No intermediate de Sitter period and no appreciable reheating occur.

(3) Monopoles are naturally suppressed.

(4) The desirable baryon to photon ratio can arise:

(a) By giving the coloured triplets in 5 and $\bar{5}$ mass of order $10^9 - 10^{10}$ GeV and by introducing an extra 5 and $\bar{5}$ chiral supermultiplet of similar mass, does not get an expectation value and couples to quarks and leptons as the ordinary Higgs. In this scheme the proton would decay to $\bar{\nu}_\mu K^+$, $\mu^+ K^0$ at a rate of 10^{31} yr in contrast with the minimal supersymmetric SU(5);

(b) In the case when the mass of the fundamental two Higgs supermultiplets is of order M_X and not of order $\Lambda_{SU(5)}$, then the reducibility of the light fermions may also lead to baryon asymmetry [15]. The approximate SU(N) × SU(N) global symmetry (where N is the number of 10 and $\bar{5}$ supermultiplets) existing at low temperatures ($T \ll M_X$) can lead to non-thermalized modes generating a non-vanishing baryon asymmetry. Of course in this case proton decay is postponed;

(c) Independently of the above mechanism, by introducing new superheavy supermultiplets coupled to light fermions and Higgs's. This is the super-symmetric version of the superheavy fermions scenario [18].

It is remarkable that supersymmetry has forced on us a new dynamical scale of $(10^9 - 10^{10})$ GeV. This is the natural scale of SU(5). We stress again the fact that the existence of "light" superheavy Higgs bosons (10^{10} GeV) makes [19] $\sin^2\theta_W \simeq 0.218$ in full agreement with experiment and much smaller than the usual supersymmetric GUTs value ($\sin^2\theta_W \sim 0.236$). A detailed analysis will be presented elsewhere [19].

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