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Symmetry non-restoration at high temperature and supersymmetry

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Abstract

We analyse the high-temperature behaviour of softly broken supersymmetric theories, taking into account the role played by effective non-renormalizable terms generated by the decoupling of superheavy degrees of freedom or the Planck scale physics. It turns out that discrete or continuous symmetries, spontaneously broken at intermediate scales, may never be restored, at least up to temperatures of the cutoff scale. There are a few interesting differences from the usual non-restoration in the non-supersymmetric theories case where one needs at least two Higgs fields and non-restoration takes place for a range of parameters only. We show that with non-renormalizable interactions taken into account the non-restoration can occur for any nonzero range of parameters, even for a single Higgs field. We show that such theories in general solve the cosmological domain wall problem, since the thermal production of the dangerous domain walls is enormously suppressed.

1. Introduction

The study of field theories at finite temperature [1,2], motivated by questions related to the cosmological evolution of the Universe, has revealed a close analogy with many condensed matter systems. In the considered cosmological scenarios, the broken symmetries of the effective gauge field theory that describe particle interactions (SM or GUT) are typically restored at high temperatures, in the same way as the rotational invariance of the ferromagnet is restored by rising its temperature. In many cases, however, as in the case of a certain ferroelectric crystal known as Seignette salt, the gauge models exhibit a high temperature symmetry non-restoration [2,3]. It turns out that for a certain range of parameter space this effect is presented in many minimal realistic particle physics models with spontaneously broken discrete and continuous symmetries [4–6]. Needless to say that such a behaviour would lead to a different picture of the hot Universe and have a direct relevance for the solution [4,5] of some problems of the standard big bang cosmology such as the domain wall [7] and the monopole [8] problems.

In the modern view the effective gauge theory that describes particle interactions below the Planck scale is a softly broken supersymmetric theory resulting from spontaneously broken supergravity or superstrings. It has been argued [9] that supersymmetric theories exhibit global and gauge symmetry restoration at high temperatures, in contrast ordinary non-supersymmetric theories in which both types of

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high-temperature behaviour are possible. This is a consequence of the more constrained nature of supersymmetric models in which all matter interactions, Yukawa as well as scalar, are determined by the superpotential. Note that the above is independent of the strength of supersymmetry breaking provided that it is soft.

The decoupling of superheavy particles as we cross their mass threshold implies that the effective theory valid at lower energies receives knowledge of the existence of these particles only through non-renormalizable interactions of the light fields suppressed by inverse powers of the superheavy mass scale. In an analogous fashion, the effective theory resulting from supergravity or superstrings below the Planck scale M_P displays an infinity of non-renormalizable interactions suppressed by the inverse powers of the Planck mass resulting from integrating out of the heavy modes at M_P. In both cases the theory below the superheavy scale is described by an effective superpotential that contains non-renormalizable interactions of the light fields [10,11]. These interactions acquire particular importance in the case of fields with vanishing renormalizable interactions, such as moduli fields. The finite-temperature corrections to such a theory can be computed in the standard fashion as long as the temperature (Θ) stays below the cutoff scale. The scalar potential will be modified by the field-dependent terms quadratic in Θ, while the higher powers of the temperature will be suppressed by inverse powers of the cutoff and therefore be negligible. In the present letter we point out that the inevitable presence of the non-renormalizable interaction in the effective field theory below M_P can imply that the high-temperature phase, in a class of supersymmetric models, is the one with a broken symmetry. The existing proofs [9] that the globally supersymmetric theories always possess a symmetric high-temperature ground state, are not valid when non-renormalizable superpotentials are allowed.

It is interesting to note that the high-temperature behaviour of such theories exhibits certain differences from the previously studied non-supersymmetric models with high-temperature symmetry non-restoration. For example, in conventional cases the symmetry non-restoration was observed exclusively in a system with more than one Higgs field and only in a certain range of the parameters. In contrast, in the case of supersymmetric theories with non-renormalizable terms included, the symmetry non-restoration may occur for a single Higgs field and for all (nonzero) values of the theory parameters (compatible with a symmetry).

2. The role of the non-renormalizable couplings

Consider a gauge theory with a set of the chiral superfields Φ^i in various representations of the gauge group G with their matter interactions described by a superpotential $W(\Phi)$. Supersymmetry will be assumed to be broken by the usual soft terms in the scalar potential

$$m^2|\Phi^i|^2 + m_{ik}^2\Phi^i\Phi^k + c_{ikl}\Phi^i\Phi^k\Phi^l + \text{h.c.} \tag{1}$$

as well as the gaugino masses $\frac{1}{2}M_a\lambda^a\lambda^a$ + h.c. The lowest-order temperature corrections can be put in the form

$$\Delta V = \frac{\Theta^2}{24}\mathrm{Tr}[M_s^2 + M_f^\dagger M_f + 3M_v^2], \tag{2}$$

where M_s, M_f and M_v are scalar, fermion and gauge boson mass matrices respectively. Since, with the above assumed soft SUSY breaking, the contribution to the supertrace comes out to be field-independent, we may put these corrections in the form

$$\Delta V = \frac{\Theta^2}{16}\mathrm{Tr}[M_s^2 + 3M_v^2] = \frac{\Theta^2}{8}\sum\left(\left|\frac{\partial^2 W}{\partial\Phi^i\partial\Phi^k}\right|^2 + 4g_a^2\Phi_i^*(T^aT^a)_k^i\Phi^k\right), \tag{3}$$

up to field-independent terms. Note that no field-dependent SUSY-breaking term contributes. A priori there is no reason why the global minimum of the full $\Theta \neq 0$ potential should be the symmetric one. Restricting oneself to the renormalizable terms only, however, always leads to a symmetric high-temperature ground state [9].

In order to illustrate that this need not be the case when non-renormalizable terms are included, we consider the simplest possible example of a single superfield Φ transforming under a discrete Z_2-symmetry $\Phi \to -\Phi$. In general the superpotential may contain an infinity of even power terms compatible with the symmetry. For simplicity we restrict ourselves to the lowest possible non-renormalizable coupling. So the model is described by the superpotential

$$W(\Phi) = -\frac{1}{2}\mu\Phi^2 + \frac{\Phi^4}{4!M}, \tag{4}$$

where M has to be understood as some large mass $\sim M_P$. This model could come about from a renormalizable model described by the superpotential

$$W(\Phi) = -\frac{1}{2}\mu\Phi^2 + M_X X^2 + \lambda X\Phi^2 \tag{5}$$

when the field X is integrated out. Notice that by field redefinition, any complex phase of the parameters can be simply absorbed in the overall phase of the superpotential and thus cannot affect the symmetry properties of the potential minima. So for definiteness we assume both μ and M to be real and positive. At $\Theta = 0$, we have a pair of non-symmetric minima at the intermediate scale $\Phi = \pm\sqrt{6\mu M}$ degenerate with the symmetric minimum at the origin. The $\Theta \neq 0$ potential (for $\Theta \gg 0$) reads

$$V = |\Phi|^2 \left|-\mu + \frac{\Phi^2}{6M}\right|^2 + \frac{\Theta^2}{8}\left|-\mu + \frac{\Phi^2}{2M}\right|^2. \tag{6}$$

The equation for the extremum is

$$\Phi\left(-\mu + \frac{\Phi^2}{2M}\right)\left(\frac{\Theta^2}{8M} - \mu + \frac{\Phi^2}{6M}\right) = 0. \tag{7}$$

Note that Φ is real in this equation. This equation has three solutions $\Phi = 0, \Phi^2 = 2M\mu$ and $\Phi^2 = 6M\mu$, but the third one exists as far as $\Theta^2 < 8M\mu$. Thus, at high temperature $\Theta > \Theta_c = \sqrt{8M\mu}$, above the intermediate scale but still below M_P, the only extrema are $\Phi^2 = 0$ and $\Phi = \pm\sqrt{2M\mu}$, the second of which was a saddle point at $\Theta = 0$. The determinants of the curvature matrix at these points can be easily computed and are equal to $\mu^2(\mu^2 - \Theta^4/64M^2)$ and $\mu^2(\Theta^4/16M^2 - 16\mu^2/9)$ respectively. Therefore, we see that above Θ_c the symmetric minimum becomes unstable (saddle point) and the only minimum of the theory is the one with a broken symmetry. Note that the intermediate scale can be quite high if μ is not very much smaller than M.

What about higher-order couplings? In general, the system may include an infinite number of non-renormalizable terms compatible with the symmetry. In such a case the superpotential becomes

$$W(\Phi) = -\frac{1}{2}\mu\Phi^2 + \frac{\Phi^{2n}}{2n(2n-1)M_{(n)}^{2n-3}}, \tag{8}$$

where the sum over $n > 1$ is assumed. The high-temperature potential now is

$$V = |\Phi|^2 \left|-\mu + \frac{\Phi^{2n-2}}{(2n-1)M_{(n)}^{2n-3}}\right|^2 + \frac{\Theta^2}{8}\left|-\mu + \frac{\Phi^{2n}}{M_{(n)}^{2n-3}}\right|^2. \tag{9}$$

To see that above a certain temperature $\Theta \gg \sqrt{M\mu}$ the minimum with $\Phi \neq 0$ is the ground state, let us simply show that at this temperature there always is a state with $\Phi \neq 0$, which has an energy lower than the one with unbroken discrete symmetry. For this notice that the second term has at least one minimum (zero of the polynomial $-\mu + (\Phi^{2n}/M_{(n)}^{2n-3})$) with $\Phi = \hat{\Phi} \sim \sqrt{M\mu}$. At this point the second term vanishes (by definition) and the energy is given by the first term, which is of the order of

$$\sim |\hat{\Phi}|^2\mu^2, \tag{10}$$

whereas the energy of the symmetric state $\Phi = 0$ is given by the second term and is equal to

$$\frac{\Theta^2}{8}\mu^2. \tag{11}$$

Thus, above a certain temperature ($\gg \sqrt{M\mu}$) the state with a broken symmetry is the lowest-energy state.

We can easily generalize the previous toy model into a two-field $U(1)$ gauge model with a superpotential

$$W(\Phi, \bar{\Phi}) = -\mu(\Phi\bar{\Phi}) + \frac{(\Phi\bar{\Phi})^2}{4M}. \tag{12}$$

The potential now is

$$V = \frac{\Theta^2}{8}\left[2\left|-\mu + \frac{\Phi\bar{\Phi}}{M}\right|^2 + (|\Phi|^4 + |\bar{\Phi}|^4)/4M^2 + 4g^2(|\Phi|^2 + |\bar{\Phi}|^2)\right] + (|\Phi|^2 + |\bar{\Phi}|^2)\left|-\mu + \frac{\Phi\bar{\Phi}}{2M}\right|^2 + \frac{g^2}{2}(|\Phi|^2 - |\bar{\Phi}|^2)^2. \tag{13}$$

Again, the $\Theta = 0$ vacua are $\Phi = \bar{\Phi} = 0$ and $|\Phi| = |\bar{\Phi}| = \sqrt{2M\mu}$. For $\Theta \neq 0$ the potential is minimized by $|\Phi| = |\bar{\Phi}| = v$ where either $v = 0$ or

$$v^4 + v^2(-8M\mu/3 + 5\theta^2/12) + 4(M\mu)^2/3 + (-M\mu + 2g^2M^2)\Theta^2/3 = 0 \tag{14}$$

Clearly, the gauge term favours the symmetric minimum so non-restoration can take place only if $g^2 < \mu/2M$. Now at high temperatures ($\gg \sqrt{M\mu}$), above the intermediate mass scale, except the symmetric extremum at the origin, we have the solution $|\Phi| = |\bar{\Phi}| = \sqrt{4M\mu/5}$. The curvature matrix at the origin has eigenvalues

$$\mu^2\left(1 \pm \frac{\Theta^2}{8M\mu}\right) + \frac{g^2}{2}\Theta^2, \tag{15}$$

indicating that the origin gets destabilized for $\Theta > \sqrt{8M\mu}$. Thus, we conclude that at high temperature the broken phase lies lower and is the ground state of the system.

Let us now discuss an example with R-symmetry non-restoration at high temperature. Consider the simplest model with a discrete R-symmetry, under which the superpotential changes sign $W \longrightarrow -W$, and a single superfield with the same transformation properties $\Phi \longrightarrow -\Phi$. Then, the most general superpotential is simply a polynomial with only odd powers Φ^{2n+1} included. For simplicity we consider only the case with $n < 3$. Thus, the superpotential becomes

$$W = \mu^2\Phi - \frac{h}{3}\Phi^3 + \frac{\Phi^5}{5M^2}. \tag{16}$$

Since we are interested in zero-temperature breaking of R symmetry at scales much below M, we assume, as before, $\mu \ll M$. Note that for $h \sim 1$ the theory still admits the zero-temperature ground state with $\Phi \sim M$. So we choose $h \sim \mu/M$ and for convenience write it as $h = \lambda\mu/M$, where λ is a parameter of order 1. Now the only zero temperature supersymmetric ground states are those with the R symmetry spontaneously broken at the intermediate scale

$$\Phi_\pm^2 = \frac{\lambda M\mu}{2}\left(1 \pm \sqrt{1 - \frac{4}{\lambda^2}}\right). \tag{17}$$

At high Θ the potential becomes

$$V = \frac{1}{M^4}\left[|\mu^2M^2 - \lambda\mu M\Phi^2 + \Phi^5|^2 + 2\Theta^2|\Phi|^2|\Phi^2 - \lambda\mu M/2|^2\right]. \tag{18}$$

We see that the second, Θ-dependent, term has two degenerated minima at any temperature

$$\Phi = 0, \quad \Phi^2 = \frac{\lambda}{2}\mu M; \tag{19}$$

whether the minimum with broken symmetry is a lowest one is decided by the first term (zero-temperature potential), which splits the energies of the above solutions, and this is simply a matter of the choice of parameters. To provide a simple existence proof let us choose $\lambda = 2$. Then, the first term has a minimum $\Phi_\pm = \pm\sqrt{M\mu}$, which coincides with the minimum of the Θ-dependent term and is thus a true ground state of the system! The energy difference between this minimum and the one with an unbroken symmetry is μ^4. So we see that in a range of parameters the R symmetry is never restored.

The above examples are sufficient to make our point. The discussion of what happens in more realistic theories, such as MSSM or GUTs, is left for a future article [13].

3. Application: the domain wall problem

It has been shown [4,5] that the symmetry non-restoration at high temperature may provide a natural solution to the domain wall [7] and the monopole problems [8], which are grave difficulties for the standard cosmological scenario. Here we will address the issue of the domain wall problem in the context of the models of spontaneously broken discrete symmetries induced by the effective non-renormalizable couplings in the superpotential. As we have seen, these systems exhibit high-temperature symmetry non-restoration with characteristic features. The crucial point is that the order parameter does not necessarily grow with temperature, but may become frozen. This goes against with previously studied cases (with symmetries being broken by renormalizable interactions) in which the order parameter to temperature ratio remains constant at high temperature, so that the thermal production of the domain walls is enormously suppressed [4]. This ensures that the absence of the phase transition suffices to solve the domain wall problem. In our case this question needs an additional study. Let us consider again a simple prototype model with the superpotential (4). As was pointed out, this system at $\Theta = 0$ has five extrema: three of them ($\Phi = 0$ and $\Phi'_\pm = \pm\sqrt{6M\mu}$) are the degenerate local minima, and the other two ($\Phi_\pm = \pm\sqrt{2M\mu}$) are saddle points. At $\Theta \neq 0$ the points $\Phi'_\pm$ get displaced:

$$\Phi'_\pm = \pm\sqrt{6\left(M\mu - \frac{\Theta^2}{8}\right)} \tag{20}$$

and they disappear at $\Theta > \Theta_c = \sqrt{8M\mu}$. Above this critical temperature the theory has two degenerate minima $\Phi_\pm = \pm\sqrt{2M\mu}$ and one saddle point $\Phi_0 = 0$. Thus, the symmetry is never restored above Θ_c. However, for the study of the domain wall formation, we need to consider the evolution of the system in the interval $0 < \Theta < \Theta_c$. The evolution goes as follows: Φ_0 is a local minimum in the interval $0 < \Theta < \Theta_c$, whereas $\Phi_\pm$ becomes a minimum only above $\Theta > \sqrt{\frac{2}{3}}\Theta_c$ (for $\Theta < \sqrt{\frac{2}{3}}\Theta_c$, the $\Phi_\pm$ is a saddle point). The third pair, $\Phi'_\pm$, is a local minimum in the interval $\Theta < \sqrt{\frac{2}{3}}\Theta_c$. Then, for $\Theta_c > \Theta > \sqrt{\frac{2}{3}}\Theta_c$ it becomes a saddle point and, finally, disappears above Θ_c. The important message is that at $\Theta = \sqrt{\frac{2}{3}}\Theta_c$ the extrema $\Phi_\pm$ and $\Phi'_\pm$ are coincident and represent an unstable turning point. Thus for $\Theta = \sqrt{\frac{2}{3}}\Theta_c$ the theory has no stable point with broken symmetry and, therefore, in some interval $\Theta \sim \sqrt{\frac{2}{3}}\Theta_c$ the symmetry is inevitably restored. In the cosmological context this would lead to a restoration of the symmetry at $\Theta = 0$ (since during the cooling the system would be trapped in the symmetric minimum). In order to have a discrete symmetry spontaneously broken at zero temperature, we can assume the soft SUSY-breaking negative mass term $-m^2|\Phi|^2$, possibly radiatively generated. Then, the broken phase will be stable for any Θ provided $|m| > \mu$. This avoids a troublesome phase transition, with a discrete symmetry breaking, for all the temperatures, and thus domain walls are never formed by the Kibble mechanism [12].

Now, what about their thermal production? First let us estimate when the domain walls would start to dominate the Universe assuming that there is at least one horizon size wall at any time (temperature). At $\Theta \gg \Theta_c$ the dominant contribution to the wall energy density comes from the second Θ-dependent term in the potential of Eq. (6). The corresponding wall solution (for the planar infinite wall) can be approximated by the kink and its energy density per unit surface is

$$\sigma \sim \frac{\Theta}{M}(M\mu)^{\frac{3}{2}}. \tag{21}$$

The thickness is

$$\delta \sim \frac{M}{\Theta}\frac{1}{\sqrt{M\mu}}. \tag{22}$$

The energy of an R-radius wall is then given by $E_R \sim R^2\sigma \sim R^2\frac{\Theta}{M}(M\mu)^{\frac{3}{2}}$ provided $R \gg \delta$. Walls start to dominate when their energy density overcomes that of the radiation. The corresponding temperature Θ_d can be found from

$$\sigma R_H^{-1} \sim \Theta_d^4, \tag{23}$$

where $R_H \sim M/\Theta_d^2$ is a horizon size in radiation dominated era. From (22) and (24), we get $\Theta_d \sim \mu\sqrt{\frac{\mu}{m}}$. However, expression (22) for σ is only valid until the temperature drops to $\sim \Theta_c$. Below, the first term in Eq. (6) starts to dominate, the wall tension becomes frozen and the surface energy density is $\sigma \sim \mu^2 M$. So, the horizon size walls (if present) would dominate at best around $\Theta_d \sim \mu$. Note that above $\Theta > \Theta_c$ the walls are wider than the horizon $\delta > R_H$. So strictly speaking, there is no wall inside the horizon, but rather the horizon could appear inside the wall. In the latter case, the energy density of the wall inside the horizon is simply a false vacuum ($\Phi = 0$) energy density $\sim \theta^2\mu^2$. This makes our upper limit on Θ_d even stronger, since now the condition that walls dominate reads

$$\Theta_d^2 \mu^2 > \Theta^4. \tag{24}$$

Thus, we conclude that the temperature at which infinite walls would dominate is about $\Theta_d \sim \mu$.

It is thus clear that we have to estimate the thermal production rate of those walls, which have a chance to survive and have a horizon size at temperature Θ_d. Such problematic walls are those that would have the size of the scales that enter the horizon at Θ_d. Thus, the dangerous size of the walls at any Θ is the one obtained by scaling $R_H(\Theta_d)$ back to the temperature Θ. In the radiation-dominated era all the length scales evolve as a scale factor $\sim \Theta^{-1}$ and thus, the comoving scale at temperature Θ is given by

$$R_w(\Theta) \sim \frac{M}{\Theta \Theta_d}. \tag{25}$$

It is not surprising that the suppression factor for walls of that size is enormous at any temperature. The thermal production rate is exponentially suppressed by the factor $e^{-E/\Theta}$. In our case

$$\frac{E}{\Theta} \sim \left(\frac{M}{\Theta}\right)^2 \sqrt{\frac{M}{\mu}}. \tag{26}$$

Even at temperatures $\sim M$ the formation rate is negligible. Thus, dangerous walls are never produced thermally (at least below the temperatures $\sim M_P$ where our estimates can be trusted).

4. Summary

In this letter we have studied a possible role of the non-renormalizable interactions in the thermal history of supersymmetric theories. Our results show that this role may be crucial, since the non-renormalizable couplings can prevent the internal symmetries from the restoration at arbitrarily high temperature (at least up to M_P). In contrast to previously observed cases, the order parameter does not necessarily grow with temperature and can become frozen. Also, it turns out that the symmetry non-restoration may take place in the case of a single Higgs superfield and for arbitrary values of the parameters. Our observations indicate that in SUSY theories the symmetries broken at intermediate scales by non-renormalizable terms, in general, have a tendency to non-restoration. These effects are expected to have important cosmological consequences. In particular we have shown that they may solve the cosmological domain wall problem in SUSY theories.

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