

Testing Λ CDM with the Growth Function $\delta(a)$: Current Constraints

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We have compiled a dataset consisting of 22 datapoints at a redshift range (0.15,3.8) which can be used to constrain the linear perturbation growth rate $f = \frac{d \ln \delta}{d \ln a}$. Five of these data-points constrain directly the growth rate f through either redshift distortions or change of the power spectrum with redshift. The rest of the datapoints constrain f indirectly through the rms mass fluctuation $\sigma_8(z)$ inferred from Ly- α at various redshifts. Our analysis tests the consistency of the Λ CDM model and leads to a constraint of the Wang-Steinhardt growth index γ (defined from $f = \Omega_m(a)^\gamma$) as $\gamma = 0.67^{+0.20}_{-0.17}$. This result is clearly consistent at 1σ with the value $\gamma = \frac{6}{11} = 0.545$ predicted by Λ CDM. We also apply our analysis on a new null test of Λ CDM which is similar to the one recently proposed by Chiba and Nakamura (arXiv:0708.3877) but does not involve derivatives of the expansion rate $H(z)$. This also leads to the fact that Λ CDM provides an excellent fit to the current linear growth data.

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I. INTRODUCTION

Most of the current observational evidence for the accelerating expansion of the universe and the existence of dark energy comes from geometrical tests that measure directly the integral of the expansion rate of the universe $H(z)$ at various redshifts. Such tests include measurements of the luminosity distance by using standard candles like type Ia supernovae (SnIa) [1] and measurement of the angular luminosity distance using standard rulers (last scattering horizon scale [2], baryon acoustic oscillations peak [3] etc). Even though these tests are presently the most accurate probe of dark energy, the mere determination of the expansion rate $H(z)$ is not able to provide significant insight into the properties of dark energy and distinguish it from models that attribute the accelerating expansion to modifications of general relativity. The additional observational input that is required is the growth function $\delta(z) \equiv \frac{\delta\rho}{\rho}(z)$ of the linear matter density contrast as a function of redshift. The combination of the observed functions $H(z)$ and $\delta(z)$ can provide significant insight into the properties of dark energy (e.g. sound speed, existence of anisotropic stress etc) or even distinguish it from modified gravity theories[4].

The observational description of the expansion rate $H(z)$ is usually made through the use of parameterizations of the effective equation of state of dark energy $w(z) \equiv \frac{p_{de}(z)}{\rho_{de}(z)}$ which is related to $H(z)$ as

$$w(z) = \frac{\frac{2}{3}(1+z)\frac{d \ln H}{dz} - 1}{1 - \left(\frac{H_0}{H}\right)^2 \Omega_{0m}(1+z)^3} \quad (1.1)$$

The most commonly used such parametrization is the CPL parametrization [5, 6]

$$w(z) = w_0 + w_1 \frac{z}{1+z} \quad (1.2)$$

The cosmological constant (Λ CDM) corresponds to parameter values $w_0 = -1$, $w_1 = 0$ and is not only consistent with all current geometrical tests but it is also

favoured [2, 7] by most of them compared to other parameter values.

The corresponding parametrization of the linear growth function $\delta(z)$ can be made efficiently by introducing a growth index γ defined by

$$\frac{d \ln \delta(a)}{d \ln a} = \Omega_m(a)^\gamma \quad (1.3)$$

where $a = \frac{1}{1+z}$ is the scale factor and

$$\Omega_m(a) \equiv \frac{H_0^2 \Omega_{0m} a^{-3}}{H^2(a)} \quad (1.4)$$

This parametrization was originally introduced by Wang and Steinhardt [8] (see also [9] for more recent discussions) and was shown to provide an excellent fit to $\frac{d \ln \delta(a)}{d \ln a}$ corresponding to various general relativistic cosmological models for specific values of γ . In particular, it was shown that for dark energy with slowly varying $w(z) \simeq w_0$ the parameter γ in a flat universe is

$$\gamma = \frac{3(w_0 - 1)}{6w_0 - 5} \quad (1.5)$$

which for the Λ CDM case ($w = -1$) reduces to $\gamma = \frac{6}{11}$. It is therefore clear that the observational determination of the growth index γ can be used to test Λ CDM.

The observational determination of the growth index γ requires knowledge not only of $\delta(z)$ but also of $H(z)$ and Ω_{0m} (see equation (1.4)). It is possible however to construct more direct tests of Λ CDM which require knowledge only of $\delta(z)$ and $H(z)$. Such a null test of Λ CDM was recently proposed in Ref. [10] where it was suggested that the validity of Λ CDM requires the following equality of observables

$$\frac{(H(z)^2/H_0^2)'}{(1+z)^2\delta'(0)^2} \int_0^\infty \frac{\delta(z)\delta'(z)}{(1+z)} dz + 1 = 0 \quad (1.6)$$

where $' \equiv \frac{d}{dz}$. In fact, as discussed section III, there is an improved version of this null test that does not involve

derivative of the observable $H(z)$ and therefore it is less prone to observational errors.

Both of the Λ CDM tests discussed above (the growth index γ and the null test) require observational determination of $\delta(z)$. There are several observational approaches that can lead to the determination of $\delta(z)$. For example, redshift distortions of galaxy power spectra [11], the rms mass fluctuation $\sigma_8(z)$ inferred from galaxy and $Ly - \alpha$ surveys at various redshifts [12]-[13], weak lensing statistics [14], baryon acoustic oscillations [15], X-ray luminous galaxy clusters [16], Integrated Sachs-Wolfe (ISW) effect [17] etc. Unfortunately, the currently available data are limited in number and accuracy and come mainly from the first two categories. They involve significant error bars and non-trivial assumptions that hinder a reliable determination of $\delta(z)$. In addition, a large part of the available data are at high redshifts ($z > 1$) where Λ CDM is degenerate with most other dark energy models since dark energy is subdominant compared to matter at high redshifts in most models.

Nevertheless, it is still instructive to consider the presently available data to investigate the possible weak constraints that can be imposed on Λ CDM. Such a task serves two purposes

1. It can be used as a *paradigm* for the time when more accurate data will be available
2. It can provide constraints from a *dynamical* test which are orthogonal and completely independent from the usual geometrical tests.

Thus, in what follows we use a wide range of presently available data on both redshift distortions and $\sigma_8(z)$ to determine the observed growth index γ and test Λ CDM in two ways

1. Comparing the measured value of γ with the Λ CDM prediction $\gamma = \frac{6}{11}$
2. Implementing a new null test that exploits the consistency between $H(z)$ and $\delta(z)$ in the context of Λ CDM.

The structure of this paper is the following: In the next section we present the dataset we have compiled and use it to determine the best fit value of γ under the assumption of a Λ CDM background. In section III we derive a new null test for Λ CDM and apply it using the fit performed in section II. Finally, in section IV we conclude, summarize and present the future prospects of the present study.

II. FITTING THE GROWTH INDEX

According to general relativity, the equations that determine the evolution of the density contrast δ in a flat background consisting of matter with density ρ_m and

dark energy with $\rho_{de} = \frac{p_{de}}{w}$ are of the form

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_m\delta \quad (2.1)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_{de}) \quad (2.2)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -8\pi Gw\rho_{de} \quad (2.3)$$

It is straightforward to change variables in eq. (2.1) from t to $\ln a$ ($\frac{d}{dt} = H\frac{d}{d\ln a}$) to obtain [8]

$$(\ln \delta)'' + (\ln \delta)'^2 + (\ln \delta)' \left[\frac{1}{2} - \frac{3}{2}w(1 - \Omega_m(a)) \right] = \frac{3}{2}\Omega_m(a) \quad (2.4)$$

where we used (2.3) and

$$\Omega_m(a) \equiv \frac{\rho_m(a)}{\rho_m(a) + \rho_{de}(a)} \quad (2.5)$$

as in (1.4). A further change of variables from $\ln a$ to $\Omega_m(a)$ can be made by considering the differential of eq. (2.5) and using energy conservation ($d\rho = -3(\rho+p)d\ln a$) leads to

$$d\Omega_m = 3w\Omega_m(1 - \Omega_m)d\ln a \quad (2.6)$$

Using (2.6) in (2.4) we get [8]

$$3w\Omega_m(1 - \Omega_m)\frac{df}{d\Omega_m} + f^2 + f \left[\frac{1}{2} - \frac{3}{2}w(1 - \Omega_m) \right] = \frac{3}{2}\Omega_m \quad (2.7)$$

where we have set

$$f \equiv \frac{d\ln \delta}{d\ln a} \quad (2.8)$$

Using the ansatz

$$f = \Omega_m^{\gamma(\Omega_m)} \quad (2.9)$$

in eq. (2.7) and expanding around $\Omega_m = 1$ (good approximation especially at $z \gtrsim 1$) we find to lowest order

$$\gamma = \frac{3(w-1)}{6w-5} \quad (2.10)$$

which reduces to $\gamma = \frac{6}{11}$ for Λ CDM ($w = -1$).

Equations (2.9) and (2.10) provide excellent approximations to the numerically obtained form of $f(z)$. This is demonstrated in Fig. 1 where we plot the numerically obtained solution of eq. (2.4) for the normalized growth

$$g(z) \equiv \frac{\delta(z)}{\delta(0)} \quad (2.11)$$

in the case of Λ CDM ($\Omega_{0m} = 0.3$) along with the corresponding approximate result

$$g(z) = e^{\int_1^{\frac{1}{1+z}} \Omega_m(a)^\gamma \frac{da}{a}} \quad (2.12)$$

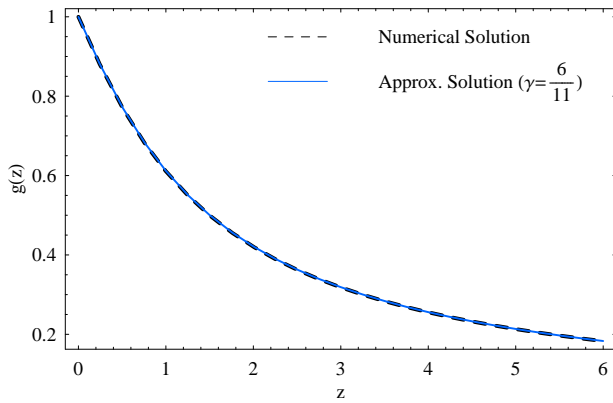


FIG. 1: The numerically obtained solution of eq. (2.4) for the normalized growth of eq. (2.11) in the case of Λ CDM ($\Omega_{0m} = 0.3$) (black dashed line) along with the corresponding approximate result with $\gamma = \frac{6}{11}$ obtained from eq. (2.13) (blue continuous line). The agreement between the two approaches is excellent.

with $\gamma = \frac{6}{11}$ obtained from

$$f(\Omega_{0m}, \gamma, a) = a \frac{d\delta/da}{\delta} = \Omega_m(a)^\gamma \quad (2.13)$$

The difference between the two approaches is less than 0.1%.

Despite of the impressive agreement between numerical result and analytical approximation, there has been a recent attempt [18] to improve further on the analytical approximation by considering an expansion of γ in redshift space up to first order in z

$$\gamma = \gamma_0 + \gamma'_0 z \quad (2.14)$$

where γ_0 and γ'_0 are constants. Using (2.14) in (2.9) and (2.7) it may be shown that [18]

$$\gamma'_0 = \left(\frac{1}{\ln \Omega_m} \right) \cdot \left[\Omega_m^{\gamma_0} + 3(\gamma_0 - \frac{1}{2})w_0(1 - \Omega_m) - \frac{3}{2}\Omega_m^{1-\gamma_0} + \frac{1}{2} \right] \quad (2.15)$$

which for $\gamma_0 = \frac{6}{11}$, $w_0 = -1$, $\Omega_m = 0.3$ becomes $\gamma'_0 = -0.012$. Since γ'_0 is very small for Λ CDM we set it to zero in what follows and assume $\gamma = \gamma_0 = \text{constant}$. In the Appendix however, we generalize our fit using equation (2.14) and show that the introduction of the new parameter γ'_0 increases significantly the errors and the allowed parameter region at 1σ .

Our goal in this section is to fit the parameter γ using observational data and compare it with the value $\gamma = \frac{6}{11}$ of Λ CDM. The most useful currently available data that can be used to constrain $\delta(z)$ (and γ) involve the redshift distortion parameter β [19] observed through the anisotropic pattern of galactic redshifts on cluster scales

TABLE I: The currently available data for the parameters β and b at various redshifts along with the inferred growth rates and references. Notice that Ref. [23] only reports the growth rate and not the β and b parameters since the growth rate was obtained directly from the change of power spectrum $Ly - \alpha$ forest data in SDSS at various redshift slices.

z	β	b	f_{obs}	Ref.
0.15	0.49 ± 0.09	1.04 ± 0.11	0.51 ± 0.11	[11],[24]
0.35	0.31 ± 0.04	2.25 ± 0.08	0.70 ± 0.18	[25]
0.55	0.45 ± 0.05	1.66 ± 0.35	0.75 ± 0.18	[26]
1.4	$0.60^{+0.14}_{-0.11}$	1.5 ± 0.20	0.90 ± 0.24	[27]
3.0	—	—	1.46 ± 0.29	[23]

(for a pedagogical discussion see [20]). The parameter β is related to the growth rate f as

$$\beta = \frac{d \ln \delta / d \ln a}{b} = \frac{f}{b} \quad (2.16)$$

where b is the bias factor connecting total matter perturbations δ and galaxy perturbations δ_g ($b = \frac{\delta_g}{\delta}$).

The currently available data for the parameters β and b at various redshifts are shown in Table I along with the inferred growth rates and references. This is an expanded version of the dataset used in Ref. [21] where a similar analysis was performed using a different parametrization suitable for modified gravity models. Notice that the Refs. of Table I have assumed Λ CDM (with $\Omega_{0m} = 0.3$) when converting redshifts to distances for the power spectra and therefore their use to test models different from Λ CDM may not be reliable. In addition, as pointed out in Ref. [22], the points of Table I obtained from Refs [25], [26], [27] correspond to measurements of $\beta(z)$ but the estimate of the bias b is made by comparing numerically simulated power spectra with the observed ones. Since the numerically simulated power spectra have assumed a Λ CDM cosmology, the resulting $f_{obs} = \beta b$ should be interpreted carefully can only be used to test the consistency of Λ CDM cosmology. Given this ambiguity we have evaluated the best fit value of the index γ both with and without the three points discussed above (see equations (2.25) and (2.26)).

In the same Table we also show the growth rate obtained in Ref. [23] which does not rely on β but is based on a different strategy, namely by finding directly the change of power spectrum $Ly - \alpha$ forest data in SDSS at various redshift slices. This point (in addition to the first one) has been used previously by other authors (see Ref. [21]) in a similar context.

Using the data of Table I we can perform a maximum likelihood analysis in order to find γ and check its consistency with the Λ CDM value $\frac{6}{11}$. We thus construct

$$\chi_f^2(\Omega_{0m}, \gamma) = \sum_i \left[\frac{f_{obs}(z_i) - f_{th}(z_i, \gamma)}{\sigma_{f_{obs}}} \right]^2 \quad (2.17)$$

where f_{obs} and $\sigma_{f_{obs}}$ are obtained from Table I while $f_{th}(z_i, \gamma)$ is obtained from eq. (2.9).

An alternative observational probe of the growth function $\delta(z)$ is the redshift dependence of the rms mass fluctuation $\sigma_8(z)$ defined by

$$\sigma^2(R, z) = \int_0^\infty W^2(kR) \Delta^2(k, z) \frac{dk}{k} \quad (2.18)$$

with

$$W(kR) = 3 \left(\frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2} \right) r \quad (2.19)$$

$$\Delta^2(kz) = 4\pi k^3 P_\delta(k, z) \quad (2.20)$$

with $R = 8h^{-1}Mpc$ and $P_\delta(k, z)$ the mass power spectrum at redshift z . The function $\sigma_8(z)$ is connected with $\delta(z)$ as

$$\sigma_8(z) = \frac{\delta(z)}{\delta(0)} \sigma_8(z=0) \quad (2.21)$$

which implies

$$s_{th}(z_1, z_2) \equiv \frac{\sigma_8(z_1)}{\sigma_8(z_2)} = \frac{\delta(z_1)}{\delta(z_2)} = \frac{e^{\int_1^{1+z_1} \Omega_m^\gamma(a) \frac{da}{a}}}{e^{\int_1^{1+z_2} \Omega_m^\gamma(a) \frac{da}{a}}} \quad (2.22)$$

where we made use of eq. (2.12). Most of the currently available datapoints $\sigma_8(z_i)$ originate from the observed redshift evolution of the flux power spectrum of $Ly - \alpha$ forest [12, 13, 28]. These datapoints are shown in Table II along with the corresponding reference sources. Notice that the data from Ref. [28] were obtained using the normalized by a growth factor δ and in our analysis we taken this into account. These data are not as useful as the redshift distortion factors for the determination of γ for two reasons

1. The rms fluctuation $\sigma_8(z)$ is not connected directly with the growth rate $f(z)$. Instead, it is related with $f(z)$ through the integral of eq. (2.12).
2. Most of the $Ly - \alpha$ σ_8 data appear at high redshifts where Λ CDM is degenerate with most other dark energy models.

Using the data of Table II we construct the corresponding χ_s^2 defined as

$$\chi_s^2(\Omega_{0m}, \gamma) = \sum_i \left[\frac{s_{obs}(z_i, z_{i+1}) - s_{th}(z_i, z_{i+1})}{\sigma_{s_{obs}, i}} \right]^2 \quad (2.23)$$

where $\sigma_{s_{obs}, i}$ is derived by error propagation from the corresponding 1σ errors of $\sigma_8(z_i)$ and $\sigma_8(z_{i+1})$ while $s_{th}(z_i, z_{i+1})$ is defined in eq. (2.22). We can thus construct the combined $\chi_{tot}^2(\Omega_{0m}, \gamma)$ as

$$\chi_{tot}^2(\Omega_{0m}, \gamma) \equiv \chi_f^2(\Omega_{0m}, \gamma) + \chi_s^2(\Omega_{0m}, \gamma) \quad (2.24)$$

TABLE II: The currently available data for the rms fluctuation $\sigma_8(z)$ at various redshifts and references. Notice that the data from Ref. [28] were obtained using the normalized by a growth factor δ and in our analysis we took this into account.

z	σ_8	σ_{σ_8}	Ref.
2.125	0.95	0.17	[12]
2.72	0.92	0.17	
2.2	0.92	0.16	[13]
2.4	0.89	0.11	
2.6	0.98	0.13	
2.8	1.02	0.09	
3.0	0.94	0.08	
3.2	0.88	0.09	
3.4	0.87	0.12	
3.6	0.95	0.16	
3.8	0.90	0.17	
0.35	0.55	0.10	[28]
0.6	0.62	0.12	
0.8	0.71	0.11	
1.0	0.69	0.14	
1.2	0.75	0.14	
1.65	0.92	0.20	

Notice however that all Refs report their data points having assumed a Λ CDM model when converting redshifts to distances with $\Omega_{0m} = 0.3$ (except [27] that used $\Omega_{0m} = 0.25$ and [12, 13] that used $\Omega_{0m} = 0.26$).

Setting $\Omega_{0m} = 0.3$ and minimizing χ_{tot}^2 with respect to γ we find

$$\gamma = 0.674_{-0.169}^{+0.195} \quad (2.25)$$

which differs somewhat from the corresponding result of Ref. [21] because we have used a broader dataset, a different parametrization for f and we have assumed Λ CDM as our fiducial model thus avoiding the marginalization of the parameter w_0 . The result (2.25) indicates that the Λ CDM value of $\gamma = \frac{6}{11} = 0.545$ is well within 1σ from the best fit and is clearly consistent with data. The imposed constraints however are rather weak and even a flat model with matter only (SCDM) predicting $\gamma = 0.6$ (set $w = 0$ in eq. (2.10)) is consistent with the data. Also, this result indicates that if it was only the value of f_{obs} and the measured β that would be required then it could have been obtained trivially from the Λ CDM f_{obs} (which is analytically known) and the measured β without need for a simulation. In that case we would also have found almost perfect agreement with Λ CDM. Instead we find a somewhat larger value of γ . Performing the same analysis but by excluding the three datapoints at $z = 0.35, 0.55$ and 1.4 yields a slightly different value

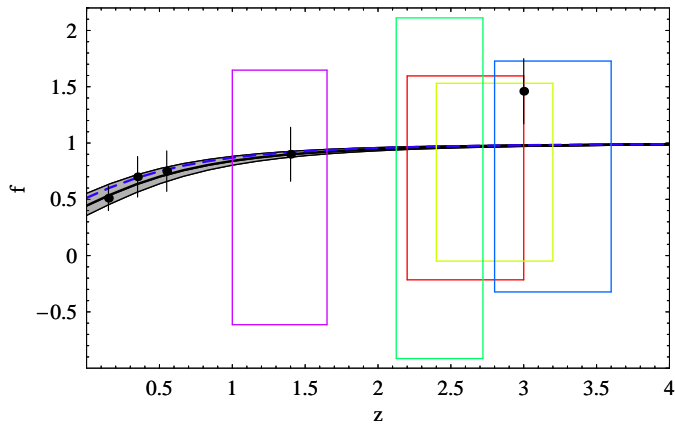


FIG. 2: The cosmological data for the growth rate $f(z)$ along with the best theoretical fit $f = \Omega_m(z)^\gamma$ with $\Omega_{0m} = 0.3$ (black continuous line) and the corresponding 1σ errors (shaded region). The errorboxes on f are obtained using the ratios at the specific redshifts. Clearly, the best fit shows a minor difference from Λ CDM (blue dashed line) only at low redshifts.

for

$$\gamma = 0.73 \pm 0.23 \quad (2.26)$$

but again well within 1σ from the Λ CDM value.

Ignoring the $\sigma_8(z)$ data of Table II leads to a negligible change in the best fit of all 22 points to $\gamma = 0.663 \pm 0.2$. This is consistent with the above discussion on the usefulness of these data. Alternatively, assuming the Λ CDM value for γ ($\gamma = \frac{6}{11}$) and minimizing with respect to Ω_{0m} we find $\Omega_{0m} = 0.24_{-0.07}^{+0.09}$.

The cosmological data for the growth rate $f(z)$ are shown in Fig.2 along with the best theoretical fit $f = \Omega_m(z)^\gamma$ with $\Omega_{0m} = 0.3$ and the corresponding 1σ errors (shaded region). In the same plot we show (dashed line) the corresponding $f_{\Lambda\text{CDM}}(z)$ obtained by solving numerically eq. (2.4) for $w = -1$ and $\Omega_{0m} = 0.3$. Clearly, the best fit (continuous line) shows a minor difference from Λ CDM (dashed line) only at low redshifts. We therefore conclude that Λ CDM is consistent with current data for the growth factor $\delta(z)$ and the consistency is maximized for $\Omega_{0m} = 0.24_{-0.07}^{+0.09}$.

III. ALTERNATIVE TESTS OF Λ CDM USING $\delta(z)$

The test of Λ CDM discussed in the previous section requires prior knowledge of the parameter Ω_{0m} since the growth function data are not accurate enough to produce a simultaneous fit for both Ω_{0m} and γ with reasonably small errors. Thus it is useful to derive a test that depends only on the observables $H(z)$ and $\delta(z)$. Such a consistency test has been recently discussed in Ref. [10] and involves both $H(z)$, $\delta(z)$ and their derivatives (see

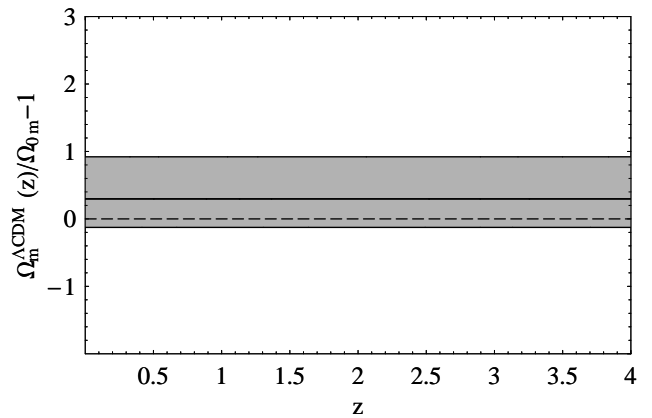


FIG. 3: The 1σ range (shaded region) of the lhs of eq. (3.6) (black continuous line). Notice that by construction it is independent of the redshift z since we have assumed that the geometric part of eq. (3.6) ($H(z)$) behaves like Λ CDM ($\Omega_{0m} = 0.3$). The value 0 corresponding to Λ CDM for both $H(z)$ and $\delta(z)$ (dashed line) is clearly well within 1σ from the best fit (continuous line). This is to be expected since the range of γ in eq. (2.25) includes the value $\gamma = \frac{6}{11}$.

eq. (1.6)). Here we derive an improved version of this test that is independent of the derivative of $H(z)$ and therefore it is less subject to observational errors. We start from eq. (2.1) and change variables from t to a to obtain

$$\frac{dH(a)^2}{da} + 2 \left(\frac{3}{a} + \frac{\delta''}{\delta'} \right) H^2 = \frac{3\Omega_{0m}H_0^2\delta}{a^5\delta'} \quad (3.1)$$

where $' \equiv \frac{d}{da}$. The solution to (3.1) is [29]

$$\frac{H^2}{H_0^2} = \frac{3\Omega_{0m}}{a^6\delta'(a)^2} \int_0^a a\delta(a)\delta'(a)da \quad (3.2)$$

which may be expressed in redshift space as

$$\frac{H^2}{H_0^2} = -\frac{3\Omega_{0m}(1+z)^2}{\delta'(z)^2} \int_z^\infty \frac{\delta(z)\delta'(z)}{1+z} dz \quad (3.3)$$

and setting $z = 0$ we find

$$\Omega_{0m} = -\frac{1}{3}\delta'(0)^2 \left[\int_0^\infty \frac{\delta(z)\delta'(z)}{1+z} dz \right]^{-1} \quad (3.4)$$

In order to promote eq. (3.4) to a null test for Λ CDM we must express Ω_{0m} in terms of geometrical observables like $H(z)$.

In the context of Λ CDM we have

$$\Omega_m^{\Lambda\text{CDM}}(z) = \left[\left(\frac{H(z)}{H_0} \right)^2 - 1 \right] \frac{1}{(1+z)^3 - 1} \quad (3.5)$$

leading to $\Omega_m^{\Lambda\text{CDM}}(z) = \Omega_{0m}$ when $H(z)$ has the Λ CDM

form. By dividing (3.5) with (3.4) we have

$$\frac{\Omega_m^{\Lambda\text{CDM}}(z)}{\Omega_{0m}} - 1 = - \frac{3 \left(\frac{H(z)^2}{H_0^2} - 1 \right) \int_0^\infty \frac{\delta(z)\delta'(z)}{1+z} dz}{[(1+z)^3 - 1] \delta'(0)^2} - 1 = 0 \quad (3.6)$$

where the last equality should hold only if ΛCDM is a valid theory. Using (2.12), (2.13) and (2.25) in (3.6) we can find the 1σ range of the lhs of eq. (3.6) which is shown in Fig. 3 (shaded region). Notice that by construction it is independent of the redshift z since we have assumed that the geometric part of eq. (3.6) ($H(z)$) behaves like ΛCDM ($\Omega_{0m} = 0.3$). The value 0 corresponding to ΛCDM is clearly well within 1σ as shown in Fig. 3. This is to be expected since the range of γ in eq. (2.25) includes the value $\gamma = \frac{6}{11}$. Had we allowed for a more general form of $H(z)$ then the 1σ range of eq. (3.6) would be redshift dependent. However, as discussed in section II, in that case the allowed range of γ would be less reliable since the surveys leading to the data of Tables I and II convert redshifts to distances using ΛCDM with $\Omega_{0m} = 0.3$. Therefore, these datapoints can only be used to test ΛCDM .

IV. CONCLUSION

We have compiled a dataset consisting of redshift distortion factors $\beta(z)$ and rms mass fluctuations $\sigma_8(z)$ at various redshifts obtained from galaxy and $Ly - \alpha$ forest redshift surveys. Using this dataset we have obtained the best fit form of the linear growth function $\delta(z)$ using the Wang and Steinhardt parametrization (2.13) with $\Omega_{0m} = 0.3$. We have found a best fit value of γ as $\gamma = 0.674_{-0.169}^{+0.195}$, a range which includes the ΛCDM value $\gamma = \frac{6}{11}$. Thus ΛCDM is in excellent agreement with current linear growth data. We have reached the same conclusion by applying a generalized version of the null test of Ref. [10].

The combination of geometrical ($H(z)$) and dynamical ($\delta(z)$) observables used here to test ΛCDM could also be used to test modified gravity theories which can not be

easily tested by using geometrical tests alone. However, in that case, care should be taken to reanalyze the power spectra using the proper form of $H(z)$ when converting redshifts to distances.

Nevertheless, given the current uncertainties in the growth rate observations, data of much better quality will be needed in order to distinguish between ΛCDM and modified gravity theories. These data will most likely come from large scale weak lensing surveys like DUNE, which is expected to measure the the equation of state of dark energy to a precision better than 5% [30].

Numerical Analysis: The mathematica files with the numerical analysis of this study may be found at <http://nesseris.physics.uoi.gr/growth/growth.htm> or may be sent by e-mail upon request.

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V. APPENDIX

It is straightforward to generalize our analysis of section II using (2.14) in (2.13), (2.17), (2.22). In that case we have two parameters to fit and we obtain

$$\gamma_0 = 0.77 \pm 0.29 \quad (5.1)$$

$$\gamma'_0 = -0.38 \pm 0.85 \quad (5.2)$$

Clearly, the 1σ error region is significantly increased in that case (see Fig. 4) due to the introduction of the additional parameter and ΛCDM ($\gamma_0 = \frac{6}{11}$, $\gamma'_0 \simeq 0$) remains well within 1σ from the best fit.

In order to avoid the increased error region we may also set $\gamma_0 = \frac{6}{11}$ and fit γ'_0 only. In this case we get

$$\gamma'_0 = 0.17 \pm 0.54 \quad (5.3)$$

which is again consistent with ΛCDM with more reasonable errors.

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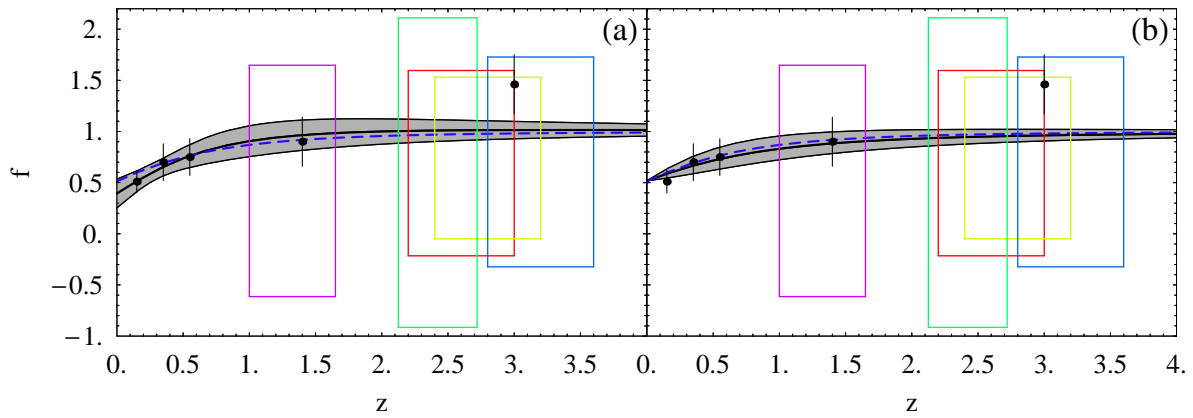


FIG. 4: Same as Fig. 2 with a generalized parametrization (2.14) where either both parameters γ_0, γ'_0 are allowed to vary (a) or only γ'_0 is allowed to vary while γ_0 is fixed to its Λ CDM value (b).

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