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THE COLEMAN-WEINBERG MECHANISM IN EARLY COSMOLOGY

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The SU(5) → SU(3) × SU(2) × U(1) phase transition is examined in the case of zero Higgs bare mass. We find that, depending on assumptions made, two scenarios are most likely to occur. Either the transition is rapidly completed into the SU(3) × SU(2) × U(1) phase, or the Universe supercools down to temperatures where the theory gets strongly coupled. In the latter case non-perturbative effects come into play.

1. Introduction

Grand unified theories and their cosmological implications in the framework of standard hot big bang cosmology have recently drawn a lot of attention. Among the many problems not well understood is the dynamics of the symmetry-breaking phase transition GUT → SU(3) × SU(2) × U(1). Even in the case of the simplest GUT, the many existing free parameters allow for different scenarios with different possible consequences on the various relevant questions (monopoles, entropy generation, etc.).

It has been suggested that when the symmetry breaking is driven by radiative corrections, i.e., the elementary Higgs scalars have zero bare mass, large gauge hierarchies seem most natural. Although no theory of this type which gives a large hierarchy without some unnatural adjustment in the Higgs sector is known, it is certainly interesting enough to investigate the consequences of the Coleman-Weinberg type of potential [1]. It is the purpose of this paper to elucidate the problem of the symmetry-breaking phase transition in the case of the Coleman-Weinberg potential. We restrict ourselves to the simplest GUT SU(5) [2, 3] but believe that more complicated models would show similar qualitative behaviour.

As is known from the study of the Coleman-Weinberg mechanism in the Weinberg-Salam phase transition [4–6], the scale invariance of the classical potential forces all these transitions to be strongly first order and consequently very slow. In the case of the electroweak phase transition the Universe would supercool to absurdly low temperatures if it were not forced to the broken phase by the dynamical breakdown of chiral symmetry [4]. Transitions of this kind proceed with the formation of bubbles of true vacuum inside an expanding Universe. The standard procedure is to calculate the bubble formation rate, carrying over to finite temperature the work of Callan and Coleman [7]. For a strictly Coleman-Weinberg potential the progress of the phase transition depends entirely on the bubble nucleation rate since the false vacuum never ceases to exist as a local minimum of the potential. This is not the case when the symmetry is broken at the tree level [8, 9].

We examine the possibility that although at zero temperature the parameters are constrained so that SU(5) breaks down to SU(3) × SU(2) × U(1), the Universe passes through an intermediate SU(4) × U(1) phase. This could happen because for a range of the Higgs parameters the supercooled transition to SU(4) × U(1) is faster than the transition to SU(3) × SU(2) × U(1). Whether or not the Universe passes through this intermediate phase, together with the amount of supercooling achieved, has considerable consequences on the monopole abundance problem [10–12].

Depending on the assumptions made, two scenarios emerge. In the first the transition goes directly to the 3-2-1 phase with moderate supercooling. In the second the Universe cools down to a temperature at which SU(5) gets strongly coupled. In this case, we argue that a condensate forms which drives the phase transition.

2. The Higgs potential

The one-loop effective potential of the adjoint Higgs $\underset{\sim}{\Phi}$ is

$$V(\underset{\sim}{\Phi}) = \lambda_M\left[\mathrm{Tr}(\underset{\sim}{\Phi}^4) - \tfrac{7}{30}(\mathrm{Tr}\,\underset{\sim}{\Phi}^2)^2\right] + \lambda_C\left[\tfrac{13}{20}(\mathrm{Tr}\,\underset{\sim}{\Phi}^2)^2 - \mathrm{Tr}(\underset{\sim}{\Phi}^4)\right] + \frac{3}{64\pi^2}\sum_i M_i^4 \ln\left[\frac{M_i^2}{\mu^2}\right]. \tag{1}$$

We have neglected the scalar loops, anticipating that the Higgs self-couplings are of order g^4 [3]. For $\mathrm{Tr}(\underset{\sim}{\Phi}^2)$ fixed, the potential is known to have extrema for the critical orbits

$$\underset{\sim}{\Phi} = \phi U^+ \mathrm{Diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2})U,$$
$$\underset{\sim}{\Phi} = \hat{\phi} V^+ \mathrm{Diag}(1,1,1,1,-4)V,$$

where U and V are arbitrary SU(5) transformations and ϕ and $\hat{\phi}$ scalars. The first choice breaks SU(5) down to SU(3) × SU(2) × U(1) (3-2-1) and the second down to SU(4) × U(1) (4-1). The masses of the heavy gauge bosons are correspondingly

$$M_i^2 = \tfrac{25}{8}g^2\phi^2, \qquad i = 1,\ldots,12,$$
$$M_i^2 = \tfrac{25}{2}g^2\hat{\phi}^2, \qquad i = 1,\ldots,8.$$

In both cases the potential reads

$$V(\phi)=\left(\frac{5625}{1024}\right)\frac{c_1 g^4}{\pi^2}\phi^4\left(\ln\left(\frac{\phi^2}{\phi_0^2}\right)-\tfrac{1}{2}\right). \tag{2}$$

The constant c_1 is unity in the first case and $\frac{32}{3}$ in the second case. In the second case one has to read $\hat{\phi}(\hat{\phi}_0)$ instead of $\phi(\phi_0)$. $\phi_0(\hat{\phi}_0)$ is the minimum of the SU(3) × SU(2) × U(1) [SU(4) × U(1)] potential. The minima are related to the couplings via

$$\phi_0^2=\frac{8\mu^2}{25g^2}\exp\left[-\frac{64\pi^2\lambda_C}{15g^4}+\tfrac{11}{3}\right],$$

$$\hat{\phi}_0^2=\frac{2\mu^2}{25g^2}\exp\left[-\frac{128\pi^2\lambda_M}{45g^4}+\tfrac{11}{3}\right].$$

In order to ensure a deeper minimum in the SU(3) × SU(2) × U(1) direction, we must have

$$\sigma\equiv\hat{\phi}_0/\phi_0\leqslant\left(\tfrac{3}{32}\right)^{1/4}. \tag{3}$$

Note that the investigation of the gauge hierarchy problem suggests that [3] $\lambda_C(\mu)\simeq 0$ and $\lambda_M(\mu)\simeq 0.03$ at $\mu=2\cdot 10^{14}$ GeV. This corresponds to $\sigma\simeq 5\cdot 10^{-3}$, a number very close to zero.

At non-zero temperature the potential is modified by the addition of the free energy of the gauge bosons [13] (we neglect the contribution of the Higgs bosons). Thus we have

$$V(\phi,T)=V(\phi)+\frac{c_2 18T^4}{\pi^2}\int_0^\infty \mathrm{d}x\, x^2\ln\left\{\frac{1-\exp\left(-\sqrt{x^2+25g^2c_3\phi^2/8T^2}\right)}{1-\mathrm{e}^{-x}}\right\}. \tag{4}$$

The constants appearing in (4) are $c_2=c_3=1$ for SU(3) × SU(2) × U(1) and $c_2=\frac{2}{3}$, $c_3=4$ for SU(4) × U(1). The free energy has been normalized so that the full potential is zero at the origin. Expressing all the dimensional quantities in units of ϕ_0, the potential can be written as

$$V(\phi,T)=\left(\frac{5625}{1024}\right)\left(\frac{g^2}{\pi}\right)^2\phi^4\left(\ln\phi^2-\tfrac{1}{2}\right)$$

$$+\frac{18T^4}{\pi^2}\int_0^\infty \mathrm{d}x\, x^2\ln\left\{\frac{1-\exp\left(-\sqrt{x^2+25g^2\phi^2/8T^2}\right)}{1-\mathrm{e}^{-x}}\right\} \tag{5}$$

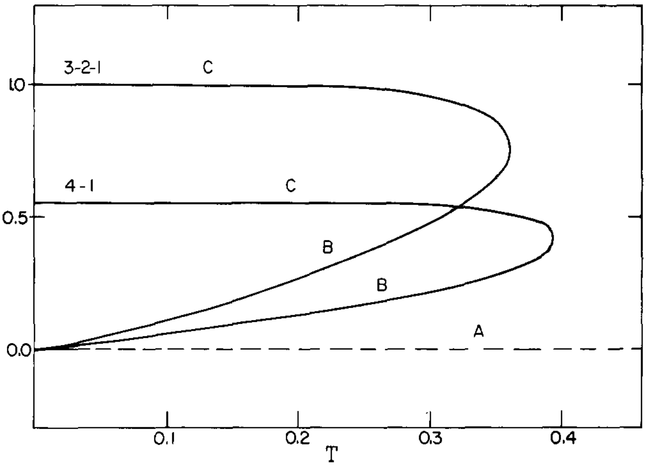

Fig. 1. Plot of the maximum (B) and minima (A and C) of the potential in the 3-2-1 and the 4-1 cases. In the second case, the curve is shown for $\sigma=(\frac{3}{32})^{1/4}$.

in the 3-2-1 direction, and

$$V(\phi,T)=\left(\frac{1875}{32}\right)\left(\frac{g^2}{\pi}\right)^2\phi^4\left(\ln\left(\frac{\phi^2}{\sigma^2}\right)-\tfrac{1}{2}\right)$$

$$+\frac{12T^4}{\pi^2}\int_0^\infty \mathrm{d}x\,x^2\ln\left\{\frac{1-\exp\left(-\sqrt{x^2+25g^2\phi^2/2T^2}\right)}{1-\mathrm{e}^{-x}}\right\} \tag{6}$$

for the 4-1 case.

The minimum at non-zero temperature does not occur at $\phi_0(\hat{\phi}_0)$ anymore. The local extrema of the potential are plotted in fig. 1 as a function of the temperature*. At very high temperatures there is only one minimum, at $\phi=0$. Below some critical temperature there is a maximum B and two local minima C away from the origin, and A at the origin. This plot is indicative of a strongly first-order phase transition [14–16]. The barrier (B) ceases to exist only at zero temperature. What happens as the Universe cools down is illustrated by fig. 2 in terms of the 3-2-1 potential.

There is perhaps a distinction to be made between a Coleman-Weinberg type of potential and potentials for which the symmetry breaking is driven by a negative squared mass term. In the former case, the metastability survives down to zero temperature and the duration of the transition cannot be read from the order

* All numerical results presented are obtained using $g^2/4\pi=\frac{1}{42}$ and $M_{X,Y}=6\cdot10^{14}$ GeV.

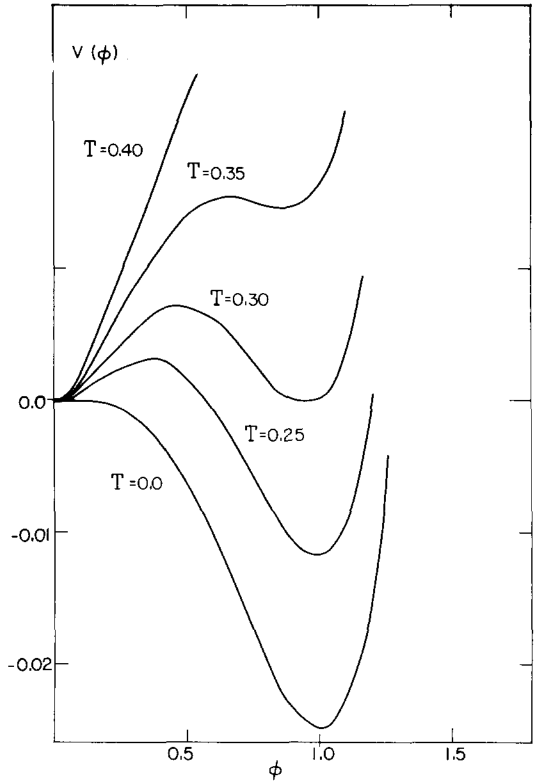

Fig. 2. Plot of the 3-2-1 potential for various typical temperatures.

parameter plot. One has to calculate the rate at which the bubbles of the broken phase fill the Universe as a function of temperature. The duration of the transition is determined by the temperature at which the rate becomes significant.

Another problem is to know whether the Universe goes directly into the 3-2-1 phase or if it passes through our intermediate 4-1 phase. Even if the 3-2-1 minimum is deeper at $T=0$ it need not be so at non-zero temperature. In fig. 3 we have plotted the value of the potential at the minimum as a function of temperature at different values of σ. One can see that for most values of σ the 3-2-1 minimum is indeed deeper and the $SU(3)\times SU(2)\times U(1)$ critical temperature ($T_c = 0.3014$) is higher than the $SU(4)\times U(1)$ one ($\hat{T}_c = 0.6028\ \sigma$). For example, with the choices of ref. [3], the $SU(4)\times U(1)$ critical temperature comes out $\hat{T}_c \simeq 0.0028$. Of course, since we are talking about a strongly first-order phase transition, what is important is not the critical temperature, but the temperature at which the Universe goes into the broken phase. But again, as the calculation will show, in the relevant range of parameters the transition proceeds faster in the 3-2-1 direction.

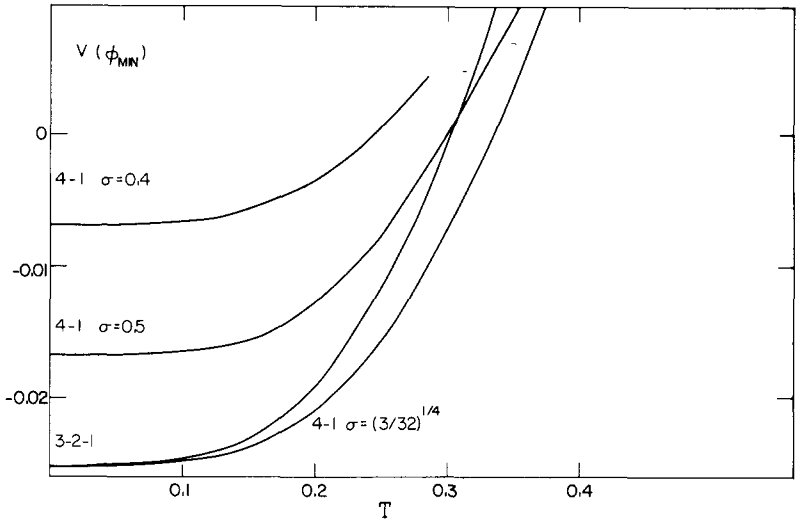

Fig. 3. Value of the potential at minimum.

3. Bubble dynamics

The rate of bubble nucleation is obtained solving the Euler-Lagrange equations for the euclidean theory with the boundary condition that the fields approach the false vacuum at infinity. The tunnelling probability per unit four-volume is given by [7]

$$\gamma = A\mathrm{e}^{-S_4} \tag{7}$$

where S_4 is the euclidean action corresponding to the tunnelling solution (the "bounce"). A has the dimensions of a mass to the fourth power. Its determination would require taking into account quantum fluctuations. We will take for A some typical mass of the problem to the fourth power [e.g., T_c^4, ϕ_0^4 or $|V(\phi_0)|$].

Since the extrema of the potential lie in the 3-2-1 and 4-1 directions, we have two possible classes of solutions interpreted as describing tunnelling towards these two broken vacua. The solution with the least action usually has the maximum symmetry. At zero temperature this means a solution with O(4) invariance.

At finite temperature we must consider the temperature-dependent potential and impose $1/T$ periodicity in time. The solution of highest symmetry is O(3) invariant (and periodic in time). Finding it seems a formidable task and therefore we have to make approximations. If the space extension of the bubble is far bigger than the period $1/T$, then the solution will be approximately static in imaginary time. Thus, assuming that only static configurations are important, we demand only [4] O(3) invariance. Up to a fixed gauge transformation, the solution is diagonal for all space time. So we are led to the one-dimensional problem

$$\frac{d^2\phi}{d\rho^2}+\frac{2}{\rho}\frac{d\phi}{d\rho}=\frac{1}{C}\frac{\partial V(\phi,T)}{\partial\phi}. \tag{8}$$

With boundary conditions

$$\lim_{\rho\to\infty}\phi(\rho)=0, \qquad \left.\frac{d\phi}{d\rho}\right|_{\rho=0}=0, \tag{9}$$

C is $\frac{15}{2}$ for $SU(3)\times SU(2)\times U(1)$ and 20 for $SU(4)\times U(1)$. The action is

$$S_4=\frac{S_3}{T}=\frac{4\pi}{T}\int_0^\infty d\rho\,\rho^2\left[\tfrac{1}{2}C\left(\frac{d\phi}{d\rho}\right)^2+V(\phi,T)\right]. \tag{10}$$

This problem can be easily solved numerically. Nevertheless, following Witten [4], one can obtain an explicit solution under the following approximations. First, keep only the first term of the small ϕ expansion of the vector meson free energy, i.e.,

$$(c_2c_3)(\tfrac{75}{16})g^2\phi^2T^2.$$

Second, replace ϕ in the argument of the logarithm of the potential by some typical value, i.e.,

$$\frac{\phi^2}{\phi_0^2}\sim\frac{T^2}{M_X^2},$$

where M_X is the mass of the superheavy gauge bosons. As the argument of the logarithm is large, we can also neglect the $-\frac{1}{2}$ term. The approximate potential then reads

$$V(\phi,T)\simeq c_1\left(\frac{5625}{1024}\right)\left(\frac{g^2}{\pi}\right)^2\phi^4\ln\left(\frac{8T^2}{25g^2\sigma^2c_3}\right)+\tfrac{75}{16}c_2c_3g^2T^2\phi^2, \tag{11}$$

i.e., a ϕ^4 potential with negative coupling. The action of the three-dimensional bounce associated with this potential is known [17]. Substituting the coefficients appearing in eq. (11) we obtain

$$S_4=\frac{S_3}{T}=2.63\left(\frac{c_2c_3C^3}{c_1^2}\right)^{1/2}\left(\frac{\pi^2}{g^3}\right)\left(\ln\left(\frac{25g^2\sigma^2c_3}{8T^2}\right)\right)^{-1}. \tag{12}$$

Our results for the action are plotted in fig. 4 as a function of the temperature. S_4

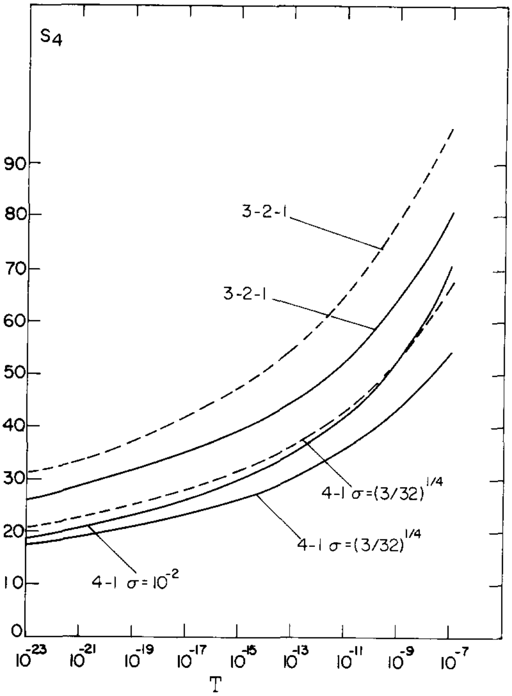

Fig. 4. The action of the SU(5) → SU(3) ⊗ SU(2) ⊗ U(1) and SU(4) ⊗ U(1) bounces using a non-running coupling constant. Dashed curves are obtained using Witten's approximation.

goes very slowly to zero as the temperature goes to zero. The action for the tunnelling to 4-1 is a slowly decreasing function of σ. These two facts are related since S_4 is a function of T/σ. One also sees that for most values of σ, S_4 is bigger for the transition to 3-2-1 than for the transition to 4-1, although the minimum in the 3-2-1 direction is deeper. This fact would favour the transition to 4-1. Note finally that Witten's approximation agrees within 10% with the exact results★. However, this error is exponentiated in the bubble nucleation rate.

The phase transition develops in an expanding Universe. The standard assumption is that the Universe is homogeneous and isotropic and thus described by the Robertson-Walker metric. The expansion is governed by Einstein's equation

$$\left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{8\pi}{3M_P^2}\right)\rho - \frac{K}{R^2}, \tag{13}$$

where $K = -1, 0, 1$ corresponds to an open, flat or closed Universe. The energy

★ The approximation in the temperature dependent part of the potential is responsible for most of the difference from the exact results. The approximation of the logarithm has virtually no effect.

density is given by

$$\rho = \left(\tfrac{1}{30}\pi^2\right)\eta T^4 + \rho_0 . \tag{14}$$

Here η is the effective number of degrees of freedom at $\phi = 0$ [$\eta = 160.75$ in the standard 3 family SU(5) model] and ρ_0 is the vacuum energy, i.e., the minimum value of $|V(3\text{-}2\text{-}1)|$ at $T = 0$. (Remember that we have normalized the potential to be zero at the false vacuum.)

At very high temperatures, the temperature term in (14) dominates and the Universe is effectively an ideal gas of massless particles. Einstein's equations can be solved for a flat Universe under the assumption of adiabatic expansion and give

$$R(t) \sim t^{1/2}. \tag{15}$$

However, when $T \ll T_c$ the vacuum energy density dominates* and we derive an exponential rate of expansion

$$R(t) \sim \exp\left\{ \frac{\sqrt{\frac{8}{3}\pi\rho_0}}{M_P} t \right\}. \tag{16}$$

Let us now consider the rate at which bubbles of the true vacuum fill an expanding Universe characterized by eq. (14). Following Guth and Tye [12], the probability that a given point will remain in the symmetric phase at temperature T is given by

$$P(T) = \exp\left\{ -d \int_T^{T_c} \frac{\mathrm{d}x\, \gamma(x)}{x^4 g(x)} \left(\int_T^x \frac{\mathrm{d}y}{g(y)} \right)^3 \right\}. \tag{17}$$

The function g is

$$g(T) = \sqrt{\frac{\rho(T)}{\rho_0}}$$

and the constant

$$d = \frac{3}{16\pi} \left(\frac{M_P^4}{\rho_0^2} \right). \tag{18}$$

We have computed $P(T)$ under the two extreme assumptions that the Universe goes entirely into the 3-2-1 or into the 4-1 phase. Taking for definiteness the factor A

* The vacuum density is roughly $\rho = 53(T^4 + 0.06 T_c^4)$.

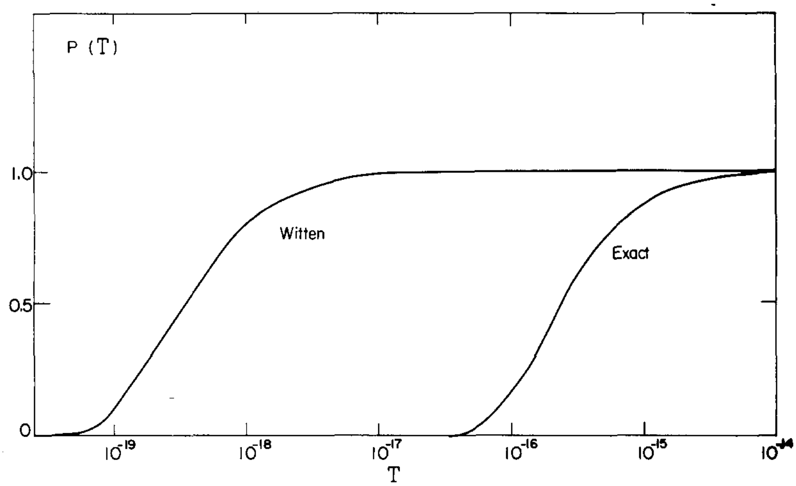

Fig. 5. $P(T)$ for the $SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$ transition using a non-running coupling constant and $A = (T_c)^4$.

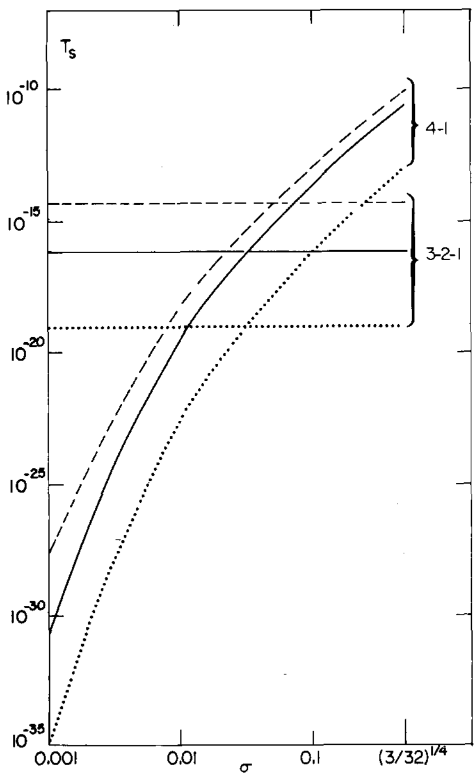

Fig. 6. Plot of the transition temperature versus σ. Non-dashed curves correspond to $A = (T_c)^4$ in eq. (7). Dashed curves are obtained using $A = (\phi_0)^4 [(\hat{\phi}_0)^4$ in the 4-1 case]. Curves with $A = |V(\phi_0)|$ lie between the plotted ones. Dotted curves refer to Witten's approximation and $A = (T_c)^4$.

in eq. (7) to be T_c^4, we have $d(T_c)^4 \simeq e^{39}$ in the first case and $\simeq e^{42+4\ln\sigma}$ in the second case. Concentrating on the first case, the transition will occur roughly at $S_4 \sim \ln(dT_c^4) \simeq 39$. Hence we anticipate ~ 15 orders of magnitude of supercooling (see fig. 4).

$P(T)$ is plotted as a function of T in fig. 5 for the 3-2-1 phase transition. The transition takes place between temperatures of $10^{-15}\ \phi_0$ and $10^{-16}\ \phi_0$. Witten's approximation in the determination of S_4 gives temperatures lower by three orders of magnitude. Results for the transition temperature T_S, defined as the temperature for which $P(T_S) \simeq 10\%$, are plotted for the 3-2-1 and 4-1 cases in fig. 6. Our main results can be read on this curve.

(i) For $0 < \sigma \lesssim 10^{-2}$, the Universe goes directly into the SU(3)×SU(2)×U(1) phase. It supercools for about fifteen orders of magnitude before the transition takes place. As already mentioned this range of σ includes the case of ref. [3] where an analysis of the hierarchy problem indicates that σ is indeed very small.

(ii) For $10^{-1} \lesssim \sigma < (\frac{3}{32})^{1/4}$, the Universe goes first into an SU(4)×U(1) phase. It will undergo at smaller temperatures a second phase transition to SU(3)×SU(2)×U(1). Since this range of values for σ is not theoretically preferred [3], the tedious study of this second phase transition is not necessary. A supercooled intermediate phase scenario would certainly lead to much lower final transition temperatures.

(iii) Finally for $10^{-2} \lesssim \sigma \lesssim 10^{-1}$ there is a competition between the direct transition 5 → 3-2-1 and the two-step one 5 → 4-1 → 3-2-1.

4. Improvement of the model

The calculation above has been done under some drastic approximations which are in some sense mandatory in order to keep the amount of work needed within reasonable limits.

(i) we have not used a renormalization group improved potential;

(ii) we have not calculated the prefactor A of eq. (7) (a formidable task) but only tried to estimate its order of magnitude;

(iii) we have made the O(3) approximation to the bounce.

The renormalization group improvement of the potential [eqs. (5) and (6)] is a very hard task due to the presence of two mass scales T and ϕ_0. We are in a situation where none of them is negligible as compared to the other. In a recent paper Sher [18] uses the ansatz

$$\frac{g^2}{4\pi} = \frac{12\pi}{40\ln(T^2/\Lambda^2)}, \tag{19}$$

with Λ determined from the condition

$$\alpha_{\mathrm{GUT}} = \frac{12\pi}{40\ln(M_X^2/\Lambda^2)}, \tag{20}$$

and fixed $\phi_0(\hat{\phi}_0)$, i.e.,

$$\Lambda = M_X e^{-6\pi/40\alpha_{GUT}} \simeq 1.5 \cdot 10^6\ \text{GeV}. \tag{21}$$

This ansatz has the advantage of leading to simple computation.

Concerning the factor A, the expectation

$$A \simeq \phi_0^4 \qquad \left(\text{or } T_c^4 \text{ or } |V(\phi_0)|\right) \tag{22}$$

has been used extensively in the literature and seems very reasonable. It is, however, true that following the theoretical result

$$A \simeq \left(\frac{S_4}{2\pi}\right)^4 \left\{\frac{\det'(-\Box + V''(\phi))}{\det(-\Box + V''(0))}\right\}^{-1/2} \tag{23}$$

in the O(4) case [7], and

$$A \simeq T\left(\frac{S_4}{2\pi}\right)^{3/2} \left\{\frac{\det'(-\Box + V''(\phi))}{\det(-\Box + V''(0))}\right\}^{-1/2} \tag{24}$$

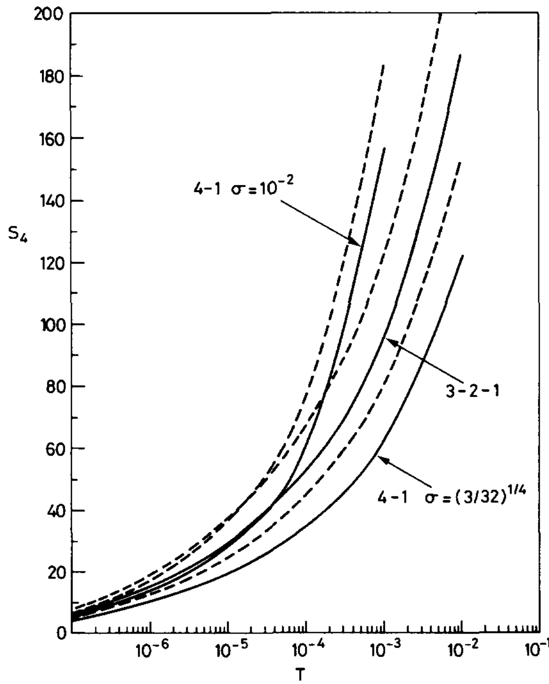

Fig. 7. Same as fig. 4 but with a running coupling constant given by eqs. (19) and (20).

in the O(3) case [19], one can deduce that [4]

$$A \simeq T^4 .$$

This is because in Witten's approximation (undoubtedly not a bad one on the qualitative level) $V(\phi)$ is given by eq. (11) where T is the only dimensionfull parameter. This result has dramatic effects on the transition temperature. However, it cannot be trusted for too small T as it gives a vanishing bubble nucleation probability in the limit $T \to 0$. But in this limit, the barrier goes away! In fact, this vanishing nucleation probability comes from the overcoming of the leading term (e^{-S_4}) behaviour by the order $\hbar$ correction (T^4). The semiclassical approximation clearly breaks down in the limit $T \to 0$. The thing to do in such a situation is to stop

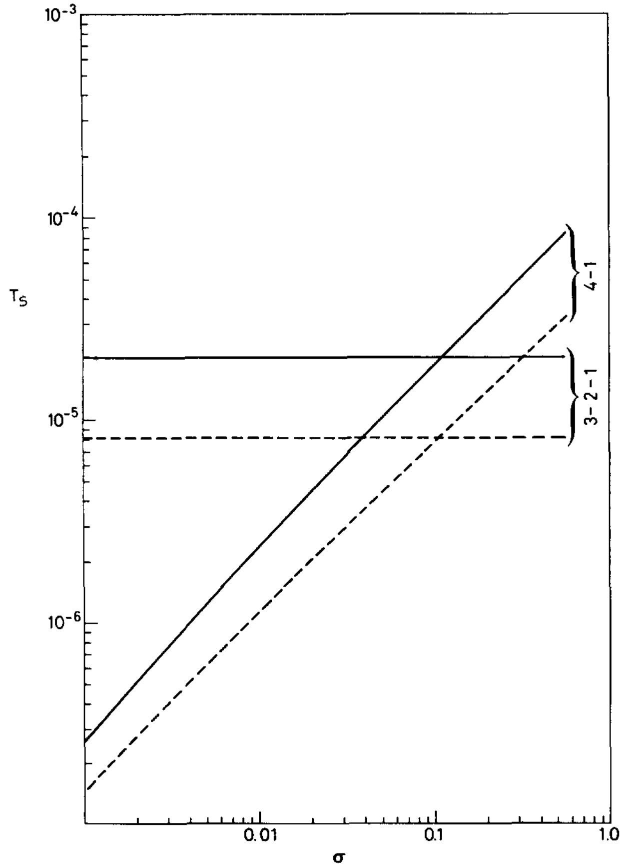

Fig. 8. Plot of the transition temperature versus σ using $A = (\phi_0)^4$, dashed curve refer to Witten's approximation.

the perturbative development before the first obviously crazy term, the $O(\hbar)$ term in our case. The prefactor A is thus truly undetermined.

To investigate the importance of these effects, with particular emphasis on the comparison between the $5 \to 3$-2-1 and $5 \to 4$-1 transitions, we have repeated the computation using the ansatz (19) and (20). The results for the bounce action can be found in fig. 7. As compared with fig. 4, the action is smaller. As a consequence, the transition will occur at much higher temperatures. One can also see that the $5 \to 4$-1 bounce has a smaller action for a more limited range of values of σ. This could have been predicted from fig. 4 where the curve for 4-1 ($\sigma = 10^{-2}$) happens to be steeper than the 3-2-1 curve. So even if the 4-1 curve is under the 3-2-1 curve for small T it is above it for bigger T's.

In fig. 8 we have plotted the transition temperature versus σ, using $A = \phi_0^4$. As compared to the non-running approximation, the transition occurs at a temperature higher by 10 orders of magnitude! Note that the 3-2-1 final state is preferred for a slightly larger range of σ. The transition temperature is around 10^{10} GeV. Remember, however, that this was obtained by $A \simeq (\phi_0)^4$ and not T^4.

5. Condensation phenomena

Suppose now that one uses $A \simeq T^4$. In this case $P(T)$ as given by the formula (17) stays equal to 1 (with a ten digit precision) from $T = T_c$ down to $T = \Lambda \equiv 1.5 \cdot 10^6$ GeV. At this point formula (19) gives an infinite result for α. This reflects the breakdown of perturbation theory for $T \simeq \Lambda$ [20]. It is clear that the appearance of this breakdown is independent of Sher's ansatz and even of the fact that the potential is of the Coleman-Weinberg type.

Let us now speculate about what might happen for $T \lesssim \Lambda$. A plausible scenario is that the $\underset{\sim}{5}$ and $\underset{\sim}{10}$ of ordinary fermions give rise to condensates with expectation value of order Λ. The most attractive channel [21] is the condensate

$$\langle \psi^{\mathrm{T}}_{\alpha\beta} G \varepsilon^{\alpha\beta\gamma\delta\eta} \psi_{\gamma\delta} \rangle , \tag{25}$$

which breaks the symmetry down to SU(4).

We will now argue that this condensate may finally drive the transition. First of all, the fundamental of Higgs gets an expectation value of the order Λ (Λ is the only dimensional quantity available) through its Yukawa coupling

$$G H^{\alpha} (\psi^{\mathrm{T}} \varepsilon^{\alpha} \psi) + \mathrm{h.c.} \tag{26}$$

Once H has got its expectation value (as this is also a first-order phase transition, some new amount of supercooling is expected), the Φ potential is modified because of the ϕ-H coupling. This coupling has a totally negligible influence on the ϕ

potential close to its minimum, but it may greatly help the transition by decreasing the barrier (remember that the barrier is only of order T).

The direct couplings $|H^2|\mathrm{Tr}\,\phi^2$ and $H^+\phi^2 H$ are negligible as the temperature gets lowered. Note that the latter gives a positive mass squared to Φ in order for the 3-2-1 minimum to be deeper [22]. The hope is thus that the gauge loops give a negative mass squared to Φ. This is indeed the case for the one-loop term

$$\frac{6g^4(T)}{64\pi^2}\mathrm{Tr}\left\{\mathcal{M}_H^2\mathcal{M}_\phi^2\ln\left(\frac{\mathcal{M}_H^2+\mathcal{M}_\phi^2}{\sigma^2}\right)\right\}, \tag{27}$$

which is negative for small Φ.

It seems more likely that the five expectation value triggers Φ in the SU(4) ⊗ U(1) direction (slightly broken by $\langle H\rangle$), however a direct transition down to SU(3) ⊗ SU(2) ⊗ U(1) is possible. As we are in a strong coupling regime, the answer is hard to decide.

Finally, once the transition is completed, the vacuum energy is released and the Universe reheated. The condensate evaporates, and H loses its expectation value. If ϕ is SU(3) ⊗ SU(2) ⊗ U(1) symmetric, the Universe is in the true vacuum, however if it is SU(4) ⊗ U(1) symmetric, as is most probable, a new transition is required.

No further tumbling is possible as the fermions are in a real SU(4) representation. Perturbation theory breakdown at $T \simeq \Lambda_{SU(4)}$ only results in a transition into a confined SU(4) phase. The exit, if any, to the 3-2-1 phase thus seems extremely hard to describe.

6. Conclusions

We think that it is clear that no firm conclusion is possible due to the very big uncertainties we have encountered. Two opposite possibilities seem to emerge. Either the Universe goes into the 3-2-1 phase at temperatures high enough so that non-perturbative effects do not play a role or it cools down to temperatures so low that perturbative theory breaks down and a fermion condensate forms. We argue that this condensate would drive the transition of the Higgs field, presumably down to the SU(4) ⊗ U(1) phase. The exit to SU(3) ⊗ SU(2) ⊗ U(1) seems problematic.

Cosmological consequences of a Coleman-Weinberg type SU(5) potential are also investigated in ref. [23]. We disagree with their treatment of the SU(5) → SU(4) ⊗ U(1) transition.

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