THE HIGGS SECTOR IN SUSY GUTS

C. KOUNNAS, A.B. LAHANAS, D.V. NANOPOULOS and K. TAMVAKIS
CERN, Geneva, Switzerland

We study the predictions ($\sin^2\theta_W$, M_X, m_b/m_τ) of supersymmetric GUTs with an enlarged Higgs sector of an intermediate mass. We find that colour octets, colour triplets or SU(3) × SU(2) singlets, of an intermediate mass result in greatly improved predictions in contrast to the minimal case. A mechanism for the generation of gluino masses via colour octets is also proposed.

1. Motivation. Since the advent of unified gauge theories, the nature of their Higgs sector remains shrouded with a bit of mystery. Apart from their role in the spontaneous breakdown of gauge symmetries, scalar fields are probably the least understood part of the theory. The notorious gauge hierarchy problem encountered in GUTs is exclusively associated with scalar boson masses. A cure to this problem, at least in the technical sense, is found in supersymmetric GUTs [1,2] due to the improved renormalization properties of supersymmetric theories [3]. The purpose of this note is to explore further the Higgs sector in supersymmetric GUTs and study its consequences on the various phenomenological predictions [$\sin^2\theta_W$, (m_b/m_τ)].

A few words of motivation are in order. Due to the increase in the number of fundamental particles that follow supersymmetrization, the standard phenomenological predictions of grand unified theories slightly change [4,5]. For example, the grand unification scale increases by one or two orders of magnitude making gauge boson mediated proton decay unobservable [+1]. Proton decay is still possible in Higgs mediated channels, nevertheless its rate, as well as its dominant decay modes, depend on the *details of the Higgs sector.* Similarly, as we shall see in some detail, the electroweak mixing angle is greatly influenced by the content and structure of the Higgs sector. As it was observed by two of the present authors (D.V.N. and K.T.) in a previous paper [7], colour triplets of an intermediate mass provide us with a value of $\sin^2\theta_W$ in perfect agreement with experiment in contrast with the usual case [5]. In this paper we undertake a more systematic investigation of the influence of various Higgs representations of intermediate mass on $\sin^2\theta_W$. It should be stressed that a non-trivial Higgs sector naturally leads to new physics below M_X with far reaching implications like $B - L$ violation [8] or gluino masses on which we shall briefly comment.

2. Renormalization group analysis. In order to begin our renormalization group analysis of the various phenomenological predictions of SUSY GUTs, let us first write down the renormalization group equations for the three gauge couplings of supersymmetric SU(3) × SU(2) × U(1). In addition to the SU(3) × SU(2) × U(1) gauge supermultiplets and the ordinary quark—lepton supermultiplets, we have included arbitrary numbers N_i of the following chiral superfields:

[+1] There exist, however, exceptions. See ref. [6].

$N_0 \rightarrow (1, 1, 1)$ (colour isosinglet), $N_D \rightarrow (1, 2, -1/2)$ (isodoublets),

$N_3 \rightarrow (1, 3, 0)$ (isotriplets), $N_T^{(1)} \rightarrow (3, 1, 1/3)$ (colour triplets),

$N_T^{(2)} \rightarrow (3, 1, -2/3)$ (colour triplets), $N_8 \rightarrow (8, 1, 0)$ (colour octets),

$N_{TD}^{(1)} \rightarrow (3, 2, 1/6)$ (colour triplets isodoublets),

$N_{TD}^{(2)} \rightarrow (3, 2, 5/6)$ (colour triplets isodoublets) .

Although we do not commit ourselves to a particular grand unifying group, it is obvious that the above chosen representations are all members of the $\mathbf{5}, \mathbf{10}, \mathbf{24}$-SU(5) representations. The three gauge couplings are

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_G} - \frac{b_3}{2\pi} \ln\left(\frac{M_x}{\mu}\right), \quad \frac{\sin^2\theta_W(\mu)}{\alpha(\mu)} = \frac{1}{\alpha_G} - \frac{b_2}{2\pi} \ln\left(\frac{M_x}{\mu}\right), \quad \frac{3}{5}\frac{\cos^2\theta_W(\mu)}{\alpha(\mu)} = \frac{1}{\alpha_G} - \frac{b_1}{2\pi} \ln\left(\frac{M_x}{\mu}\right) , \tag{1}$$

where α_G is the common coupling at M_x and

$$b_3 = 9 - 2N_G - \tfrac{1}{2}(N_T^{(1)} + N_T^{(2)}) - (N_{TD}^{(1)} + 2N_{TD}^{(2)}) - 3N_8 ,$$

$$b_2 = 6 - 2N_G - \tfrac{1}{2}N_D - (\tfrac{3}{2}N_{TD}^{(1)} + 3N_{TD}^{(2)}) - 2N_3,$$

$$b_1 = -2N_G - (\tfrac{1}{5}N_T^{(1)} + \tfrac{4}{5}N_T^{(2)}) - (\tfrac{1}{10}N_{TD}^{(1)} + 5N_{TD}^{(2)}) - \tfrac{3}{10}N_D - \tfrac{3}{5}N_0 , \tag{2}$$

where N_G is the number of generations [+2]. In what follows we shall assume that global supersymmetry remains effectively unbroken down to energies of order M_W. It should be remembered, however, that the primordial scale of supersymmetry breaking could be much higher. As is the case in most models that realize supersymmetry breaking at a high scale [9], the tree of radiative mass splittings within most SU(3) × SU(2) × U(1) non-singlet supermultiplets are of order M_W [9].

The numbers N_i are not constants but change when we pass through various intermediate thresholds $\Lambda, \Lambda', ...$

$$M_W < ... < \Lambda < \Lambda' < ... < M_x .$$

For simplicity we shall assume the existence of only one such intermediate scale Λ. The generalization to more than one is straightforward. Denoting by N_i^+ the number of times the representation (i) appears for $\mu > \Lambda$ and N_i^- the corresponding number for $\mu < \Lambda$, we define

$$\Delta N_i \equiv N_i^+ - N_i^- , \quad \Delta b_k \equiv \sum_i C_i^k \Delta N_i = b_k^+ - b_k^- .$$

The equations for M_x, $\sin^2\theta_W$ and α_G can easily be derived in the case of an intermediate scale Λ. They are [7] [+3]

$$\ln(M_x/\mu) = [2\pi(3/8\alpha(\mu) - 1/\alpha_3(\mu)) + \Delta(b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1) \ln(\Lambda/\mu)]/(b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1)_+ , \tag{3}$$

[+2] In what follows we shall restrict ourselves to the case of three generations.
[+3] $(b_i)_+ = b_i$ for $\mu > \Lambda$.

$$\sin^2\theta_W(\mu) = \tfrac{5}{8}[\tfrac{3}{5} - \tfrac{3}{8}(b_2 - b_1)_+/(b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1)_+] + \tfrac{5}{8}[(b_2 - b_1)_+/(b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1)_+]\,\alpha(\mu)/\alpha_2(\mu)\,,$$

$$-\tfrac{5}{8}[(\alpha(\mu)/2\pi)\ln(\Lambda/\mu)/(b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1)_+]\,\det\begin{vmatrix} (b_2 - b_1)_+ & \Delta(b_2 - b_1) \\ (b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1)_+ & \Delta(b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1) \end{vmatrix},$$

$$1/\alpha_G = \{[1/\alpha(M_W)]\,(\tfrac{3}{8}b_3)_+ - [1/\alpha_3(M_W)]\,(\tfrac{3}{8}b_2 + \tfrac{5}{8}b_1)_+\}/(b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1)_+$$

$$-[(1/2\pi)\ln(\Lambda/M_W)/(b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1)_+]\,\det\begin{vmatrix} \Delta b_3 & b_{3+} \\ \Delta(b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1) & (b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1)_+ \end{vmatrix}. \qquad\text{(3 cont'd)}$$

Let us next study how each of the new chiral supermultiplets influence the values of the above three parameters.

(a) Colour triplets. In the case that we have only colour triplets contributing, apart from the usual isodoublets, the set of the equations (2) becomes

$$b_3 = 9 - 2N_G - \tfrac{1}{2}(N_T^{(1)} + N_T^{(2)})\,, \quad b_2 = 6 - 2N_G - \tfrac{1}{2}N_D\,, \quad b_1 = -2N_G - (\tfrac{1}{5}N_T^{(1)} + \tfrac{4}{5}N_T^{(2)}) - \tfrac{3}{10}N_D\,. \qquad (4)$$

The charge $\tfrac{1}{3}$ triplets $N_T^{(1)}$ are members of an SU(5) pentaplet while the charge$-\tfrac{2}{3}$ triplets $N_T^{(2)}$ belong to the **10** representation. There is a lower mass limit of $O(10^{10}$ GeV) for the first variety from proton decay via dimension six operators [10], while the mass of the second variety could be much lower since they do not mediate directly proton decay [8]. In terms of (3) we have

$$b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1 = \tfrac{3}{8}(18 + N_T - N_T^{(1)})\,, \quad b_2 - b_1 = 6 + \tfrac{1}{5}(N_T^{(1)} - N_D) + \tfrac{4}{5}N_T^{(2)}\,. \qquad (5)$$

As it is already known from a previous analysis, only the difference $N_D - N_T^{(1)}$ appears in M_x and $\sin^2\theta_W$. As one should expect an unsplit full pentaplet should not have any influence on the mixing angle or the unification scale. For the minimal case $N_D = 2$ and $N_T^{(1)} = 0$ [*4], we obtain

$$\ln(M_x/M_W) = \tfrac{1}{10}\pi[1/\alpha(M_W) - 8/3\alpha_3(M_W)]\,,$$

$$\sin^2\theta_W(M_W) = \tfrac{3}{8} - (N_T^{(2)} + 7)^+/40 + [\alpha(M_W)/\alpha_3(M_W)]\,(N_T^{(2)} + 7)^+/15 + [\alpha(M_W)/4\pi]\,\Delta N_T^{(2)}\ln(\Lambda/M_W)\,. \qquad (6)$$

The unification point M_x is independent of the number of $\tfrac{2}{3}$ triplets, while the mixing angle decreases with their number $N_T^{(2)}$. For $(N_T^{(2)})_+ = 2$ and $(N_T^{(2)})_- = 0$, we obtain [$\Lambda_{\overline{MS}} = 0.1$ GeV, $\alpha(M_W)^{-1} = 127.56$ and $\alpha_3(M_W) = 0.101$]

$$M_x/M_W \simeq (6.3) \times 10^{13}\,, \quad \sin^2\theta_W(M_W) = 0.197 + (0.125) \times 10^{-2}\ln(\Lambda/M_W)\,.$$

For $\Lambda/M_W \simeq 10^8$, we obtain, $\sin^2\theta_W(M_W) \simeq 0.220$, while for $\Lambda/M_W \simeq 10^4$ we get $\sin^2\theta_W(M_W) \simeq 0.208$.

We see that colour triplets belonging to the **10** representation of SU(5) which are kept "light" have a significant influence on the mixing angle causing it to come out smaller than the value it has in the minimal case and thus, be in perfect agreement with experiment.

(b) SU(3) × SU(2) singlets. Although this interesting case has been pointed out in the literature [6], it is worth while to mention it here for reasons of completeness. Colour isosinglets are members of the **10** representation of SU(5). Their presence modifies the minimal b functions according to

$$b_3 = 9 - 2N_G\,, \quad b_2 = 6 - 2N_G - \tfrac{1}{2}N_D\,, \quad b_1 = -2N_G - \tfrac{3}{10}N_D - \tfrac{3}{5}N_0\,.$$

[*4] The case of "light" 1/3 triplets ($\gtrsim 10^{10}$ GeV) as it has been emphasized elsewhere [7] leads to $\sin^2\theta_W \simeq 0.22$ in sharp contrast with the minimal case for which $\sin^2\theta_W \simeq 0.236$ a value a bit too large. Even in the case of $N_D = 4$ the value obtained is not in disagreement with experiment.

Thus, in the minimal case of $N_D = 2$, we are led to $b_3 - \frac{3}{8}b_2 - \frac{5}{8}b_1 = \frac{5}{8}(20 + N_0)$ and $b_2 - b_1 = \frac{1}{5}(28 + 3N_0)$. The resulting equations for the mixing angle and the unification scale are

$$\ln(M_x/M_W) = \{2\pi[1/\alpha(M_W) - 8/3\alpha_3(M_W)] + N_0 \ln(\Lambda/M_W)\}/(20 + N_0)$$

$$\sin^2\theta_W(M_W) = 4/(20 + N_0) + \tfrac{1}{3}[\alpha(M_W)/\alpha_3(M_W)] \, (28 + 3N_0)/(20 + N_0) + [\alpha(M_W)/\pi] \, [2N_0/(20 + N_0)] \, \ln(\Lambda/M_W)$$
$$(7)$$

For $N_0 = 2$ (and $\Lambda_{\overline{MS}} \simeq 0.1$ GeV) we obtain

$$\ln(M_x/M_W) \simeq 28.89 + 0.09 \ln(\Lambda/M_W) \, , \quad \sin^2\theta_W \simeq 0.222 + 0.454 \times 10^{-3} \ln(\Lambda/M_W) \, ,$$

which for $\Lambda/M_W \simeq 10^4$ gives $M_x/M_W \simeq 8.13 \times 10^{12}$ and $\sin^2\theta_W \simeq 0.226$, while for $\Lambda/M_W \simeq 10^8$ gives $M_x/M_W \simeq 1.9 \times 10^{13}$ and $\sin^2\theta_W \simeq 0.230$. Of course, the singlet could be left massless down to M_W in which case we would obtain $M_x/M_W \simeq 3.5 \times 10^{12}$ and $\sin^2\theta_W \simeq 0.222$ in perfect accordance with the predictions of standard non-symmetric SU(5) [6].

(c) Coloured isodoublets. These fields are either members of **10** or the adjoint **24** representations of SU(5). In both cases they tend to give a bad value for the mixing angle. For example in the case of $N_D = 2$ we have

$$b_2 - b_1 = \tfrac{28}{5} - \tfrac{7}{5}N_{TD}^{(1)} + 2N_{TD}^{(2)}, \quad b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1 = \tfrac{15}{2} - \tfrac{1}{4}N_{TD}^{(1)} + N_{TD}^{(2)} \, .$$

For $N_{TD}^{(1)} = 2$ and $N_{TD}^{(2)} = 0$ ($\Lambda_{\overline{MS}} = 0.1$ GeV)

$$\ln(M_x/M_W) = 7^{-1}[2\pi(3/8\alpha - 1/\alpha_3) - \tfrac{1}{2}\ln(\Lambda/M_W)] \simeq 34.05 - 0.07 \ln(\Lambda/M_W) \, ,$$

$$\sin^2\theta_W \simeq 0.300 - 2.08 \times 10^{-3} \ln(\Lambda/M_W) \, .$$

It is evident from the last equation that we can obtain acceptable values for $\sin^2\theta_W$ only for $\Lambda \simeq M_x$. Thus, the coloured isodoublets arising from **10** are harmful to the mixing angle if they are kept light.

For $N_{TD}^{(1)} = 0$ and $N_{TD}^{(2)} = 2$ ($\Lambda_{\overline{MS}} \simeq 0.1$ GeV) we obtain

$$\ln(M_x/M_W) = (\tfrac{19}{2})^{-1}[2\pi(3/8\alpha - 1/\alpha_3) + 2\ln(\Lambda/M_W)] = 25.09 + 0.21 \ln(\Lambda/M_W) \, ,$$

$$\sin^2\theta_W \simeq 0.150 + 1.54 \times 10^{-3} \ln(\Lambda/M_W) \, .$$

It is evident that only for $\Lambda \simeq M_x$ can we get a reasonable value for $\sin^2\theta_W$. Thus, coloured isodoublets of both kinds are harmful to the mixing angle and should be avoided.

(d) Isotriplets. These fields can arise from the **24** representation of SU(5). They lead to

$$b_3 = 9 - 2N_G \, , \quad b_2 = 6 - 2N_G - \tfrac{1}{2}N_D - 2N_3, \quad b_1 = -2N_G - \tfrac{3}{10}N_D \, .$$

In the case of $N_D = 2$, we have

$$b_2 - b_1 = \tfrac{28}{5} - 2N_3 \, , \quad b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1 = \tfrac{15}{2} + \tfrac{3}{4}N_3 \, .$$

For $N_3 = 1$ we obtain

$$\ln(M_x/M_W) = (\tfrac{33}{4})^{-1}[2\pi(3/8\alpha - 1/\alpha_3) + \tfrac{3}{4}\ln(\Lambda/M_W)] \simeq 28.89 + 0.09 \ln(\Lambda/M_W) \, ,$$

$$\sin^2\theta_W \simeq 0.293 - 0.014 \ln(\Lambda/M_W)$$

Taking $\Lambda/M_W \simeq 10^2$, we get $\sin^2\theta_W(M_W) \simeq 0.228$ and $M_x/M_W \simeq 5.3 \times 10^{12}$. The choice $\Lambda/M_W \simeq 10^3$ leads to $\sin^2\theta_W \simeq 0.196$ which is too small. Hence, although the influence of the isotriplet could be helpful in obtain-

ing a mixing angle in agreement with experiment, Λ is forced to be quite close to M_W [+5]

(e) Colour octets. Let us finally examine what happens when we include particles in the octet representation of SU(3). They can arise as members of a **24** supermultiplet of SU(5). The b functions are modified according to

$$b_3 = 9 - 2N_G - 3N_8, \quad b_2 = 6 - 2N_G - \tfrac{1}{2}N_D, \quad b_1 = -2N_G - \tfrac{3}{10}N_D.$$

Thus, for $N_D = 2$, we obtain

$$b_2 - b_1 = \tfrac{28}{5}, \quad b_3 - \tfrac{3}{8}b_2 - \tfrac{5}{8}b_1 = \tfrac{15}{2} - 3N_8.$$

These equations lead to the following, in the case of $N_8 = 1$

$$\ln(M_x/M_W) \simeq (\tfrac{9}{2})^{-1}[2\pi(3/8\alpha - 1/\alpha_3) - 3\ln(\Lambda/M_W)] \simeq 52.96 - 0.666\ln(\Lambda/M_W),$$

$$\sin^2\theta_W \simeq 0.143 + 0.291 \times 10^{-2}\ln(\Lambda/M_W).$$

Choosing $\Lambda/M_W \simeq 10^{12}$, we obtain $\sin^2\theta_W \simeq 0.223$ and $M_x/M_W \simeq 1.02 \times 10^{15}$. For $\Lambda/M_W \simeq 10^{11}$ we are led to $\sin^2\theta_W \simeq 0.217$ and $M_x/M_W \simeq 4.7 \times 10^{15}$. Hence, with the colour octet kept as "light" as 10^{13}–10^{14} GeV, we are led easily to a mixing angle in comfortable agreement with the current experimental value.

Summarizing the renormalization group analysis, we conclude that coloured triplets, as well as colour octets of an intermediate mass, can lead to a mixing angle very close to the currently measured value.

Perhaps at this point it would be appropriate to have a look at another prediction of GUTs, namely to the b quark to τ lepton mass ratio. Naturally we shall restrict ourselves only to those representations that were not proved harmful to the mixing angle. Colour triplets have been studied elsewhere and were found to have almost no influence on the successful ordinary GUTs predictions [7]. SU(3) × SU(2) singlets will have obviously almost no influence since the U(1) subgroup is known to contribute very little to the ratio. For the case of colour octets we have listed in table 1, along with the predictions for $\sin^2\theta_W$ and M_x, the values obtained for the ratio as a function of the intermediate scale Λ. It is evident that colour octets tend to spoil the mass ratio. Thus, colour triplets and SU(3) × SU(2) singlets emerge from our analysis as unique possibilities of an enriched Higgs sector of an intermediate mass with improved phenomenological predictions.

3. Models. Let us next move to some examples of GUTs with an extended Higgs sector in order to illustrate how we can be led to models with the previously described chiral supermultiplet structure. It is sufficient to stay within SU(5).

[+5] It should also be kept in mind that isotriplets with an expectation value might spoil the relation $M_W = M_Z \cos\theta_W$.

Table 1
$\sin^2\theta_W$, M_x/M_W, m_b/m_τ for one colour octet of mass Λ.

Λ/M_W	$\sin^2\theta_W$	M_x/M_W	$\dfrac{(m_b/m_\tau)\text{SUSY}}{(m_b/m_\tau)\text{GUTs}}$
10^6	0.183	1.00×10^{19}	0.876
10^7	0.190	2.15×10^{18}	0.955
10^8	0.197	4.64×10^{17}	1.034
10^9	0.203	1.00×10^{17}	1.116
10^{10}	0.210	2.15×10^{16}	1.198
10^{11}	0.217	4.64×10^{15}	1.283
10^{12}	0.223	1.00×10^{15}	1.378

(a) The **10** *representation. It is alrea*dy known that because of anomalies, a supersymmetric SU(5) requires the introduction of a $\overline{\mathbf{10}}$ supermultiplet as well. Then a general superspace potential would be

$$W \sim m(\mathbf{10})(\overline{\mathbf{10}}) + \lambda(\overline{\mathbf{10}})(\mathbf{24})(\mathbf{10}) + f_{ij}Q_{\bar{5}}^{i}Q_{\bar{5}}^{j}(\mathbf{10}) + g(\mathbf{10})(\mathbf{10})(5_{\mathrm{H}}) + \dots . \tag{8}$$

The coupling to the quark–lepton supermultiplets $Q_{\bar{5}}$ because of antisymmetry mixes different generations. It is not dangerous since it does not mediate direct proton decay. Nevertheless, proton decay can occur via higher order tree graphs involving the $B - L$ violating coupling to the Higgs pentaplets. As it has been analyzed elsewhere, nucleon decay proceeds mainly via the channels $\mathrm{p} \to \mu^{-}\mathrm{K}^{+}\pi^{+}$, $\mathrm{n} \to \mu^{-}\mathrm{K}^{+}$ which conserve $B + L$ [8].

The mass matrix for the **10** in terms of the components $S = (1, 1, 1)$, $\varphi = (3, 1, \frac{1}{3})$ and $\eta = (3, 2, \frac{1}{6})$ is [for $\langle 24 \rangle$ $= (m'/\lambda)\,\mathrm{diag}(2, 2, 2, -3, -3)$]

$$W \sim (m - 3m')\,\bar{S}S + (m + 2m')\,\bar{\varphi}\varphi + (2m - m')\bar{\eta}\eta . \tag{9}$$

Taking $m \sim m', m'/\lambda \sim M_{\mathrm{x}}/g$, we always have the freedom to choose one of the three mass coefficients in (9) to be at an intermediate scale Λ by fine tuning the ratio m/m'. The other two masses will be both of order $O(M_{\mathrm{x}})$. Thus, depending on the adjustment we make, we are led to either SU(3) × SU(2) singlets or colour triplets or, finally, coloured isodoublets of intermediate mass [cases (b), (a), (c)].

(b) The **24** *representation.* As is well known, in SU(5) we need a Higgs superfield in the adjoint representation in order to achieve the strong breaking. If in addition to that we introduce another Higgs superfield in the adjoint representation whose bosonic components do not obtain an expectation value, then the following interaction terms are possible

$$W \sim \tfrac{1}{2}m\,\mathrm{tr}(\Phi_{24}^{2}) + m'\,\mathrm{tr}(\Phi_{24}\Phi_{24}') + \tfrac{1}{3}\lambda\,\mathrm{tr}(\Phi_{24}^{3}) + \tfrac{1}{2}\lambda'\,\mathrm{tr}(\Phi_{24}^{2}\Phi_{24}') + \tfrac{1}{2}\lambda''\,\mathrm{tr}(\Phi_{24}\Phi_{24}'^{2}) + \tfrac{1}{3}\lambda'''\,\mathrm{tr}(\Phi_{24}'^{3}) + \dots , \tag{10}$$

where Φ_{24}' is the new adjoint superfield. Taking

$$\langle \Phi_{24} \rangle = V\,\mathrm{diag}(2, 2, 2, -3, -3) , \quad \langle \Phi_{24}' \rangle = \left(\begin{array}{c|c} \mathrm{O} & \mathrm{X} \\ \hline \mathrm{X}^{+} & \tau \end{array}\right) + c\left(\begin{array}{c|c} 2 & 0 \\ \hline 0 & 3 \end{array}\right)I$$

we end up with the following mass terms

$$W \sim \lambda V\{2\,\mathrm{tr}(\mathrm{O}^{2}) - \mathrm{tr}(\mathrm{XX}^{+}) - 3\,\mathrm{tr}(\tau^{2}) - 30c^{2}I^{2}\} + \tfrac{1}{2}m\{\mathrm{tr}(\mathrm{O}^{2}) + 2\,\mathrm{tr}(\mathrm{XX}^{+}) + \mathrm{tr}(\tau^{2}) + 30\,c^{2}I^{2}\}$$

$$= (2\lambda V + \tfrac{1}{2}m)\,\mathrm{tr}(\mathrm{O}^{2}) + (m - \lambda V)\,\mathrm{tr}(\mathrm{XX}^{+}) + (\tfrac{1}{2}m - 3\lambda V)\,\mathrm{tr}(\tau') + 30\,c^{2}(\tfrac{1}{2}m - \lambda V)I^{2} .$$

We are free to adjust any of the above four mass coefficients to be of $O(\Lambda)$ (where Λ is the desired intermediate scale). This way we can obtain "light" isotriplets, "light" octets or "light" coloured isodoublets.

It should be stressed that we cannot couple the adjoint to the quark–lepton supermultiplets and thus there is no lower mass limit for the members of Φ_{24}' coming from proton decay.

The alert reader has probably not missed the fact that asymptotic freedom of the strong coupling is reduced with the inclusion of coloured Higgs representation. Indeed

$$b_{3} = 9 - 2N_{\mathrm{G}} - \tfrac{1}{2}(N_{\mathrm{T}}^{(1)} + N_{\mathrm{T}}^{(2)}) - 3N_{8}$$

(for $N_{\mathrm{G}} = 3$) can become zero for $N_{\mathrm{T}}^{(1)} = 6, N_{\mathrm{T}}^{(2)} = N_{8} = 0$ or for $N_{\mathrm{T}}^{(2)} = 6, N_{\mathrm{T}}^{(1)} = N_{8} = 0$ or for $N_{\mathrm{T}}^{(1)} = N_{\mathrm{T}}^{(2)}$ $= 0$ and $N_{8} = 1$. It is interesting that with just one colour octet we have a vanishing colour β function for $\Lambda < \mu$ $< M_{\mathrm{x}}$. It might be tempting to speculate that unification with gravity in the framework of supergravity theory, which allegedly is characterized by a vanishing β function (probably), might require a vanishing β function below the unification point as well. This could be used in reverse to determine the type and number of additional Higgs representations. It is also amusing to note that a set of two **24** representations with a mass matrix of the type (where M_{P} is the Planck mass)
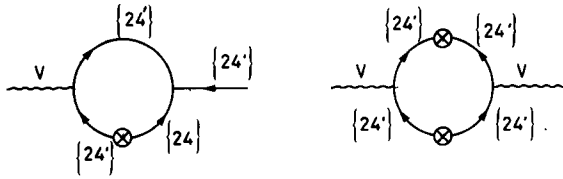
Fig. 1. Radiatively induced gluino mass.

$$\begin{pmatrix} 0 & \lambda M_x \\ \lambda M_x & M_P \end{pmatrix}$$

leads to an octet mass $\Lambda \sim (\lambda^2 M_x^2)/M_P \sim 10^{13}$ GeV (for a natural choice $\lambda \sim 10^{-1}$).

It is interesting to note that the fermionic members of the octet could provide us with a gluino mass when supersymmetry is broken. The dominant contribution comes from the supergraphs of fig. 1. The resulting octet—gluino mass matrix would be

	ψ_8'	$\tilde{g}$
ψ_8'	M_1	$(\alpha_s/\pi)\,(M_s^2/M_1)$
$\tilde{g}$	$(\alpha_s/\pi)\,(M_s^2/M_1)$	$(\alpha_s/\pi)\,(M_s^2/M_1)$

M_1 is the octet mass and M_S is the scale of supersymmetry breaking. Hence one combination gets a mass of $O(M_1)$ and the orthogonal to it, a mass of $O(\alpha_s/\pi(M_S^2/M_1))$.

Since the issue of the gluino mass is quite independent of our previous analysis, let us try to see for what values of M_S and M_1 do we get a reasonable value of the mass. It is not difficult to see that only for a very heavy octet $(M_1 \sim M_P)$ and a large value for the primordial supersymmetry breaking $M_S \sim (M_W M_P)^{1/2}$ we get reasonable gluino masses of the order of $(\alpha_s/\pi)M_W$.

4. Conclusions. In conclusion, here are the main points of our analysis.

(1) Colour triplets of intermediate mass improve the predictions of the mixing angle. Proton decay via dimension-six operators proceeds mainly to $\bar{\nu}_\mu K^+$, $\mu^+ K^0$ in the case of charge $\frac{1}{3}$ triplets, while in the case of triplets coming from the **10** (charge $\frac{2}{3}$) it is dominated by the $B + L$ conserving mode $\mu^- \pi^+ K^+$, $\mu^- K^+$ [8]. Dimension-five operators can be avoided as it has been analyzed elsewhere [11] by the appropriate choice of Higgs couplings or by imposition of R symmetries.

(2) Colour octets of intermediate mass improve greatly the value of the mixing angle. They cannot mediate any baryon number violating interactions since they cannot couple to the quark—lepton supermultiplets. They tend to spoil a m_b/m_τ a little.

(3) Coloured isodoublets and isotriplets of intermediate mass tend to increase the value of the mixing angle.

(4) Light SU(3) × SU(2) singlets predict the same values of $\sin^2\theta_W$ and M_x as in ordinary GUTs.

(5) Finally, we have proposed a mechanism for the generation of radiative gluino masses through colour octets.

References

[1] D. Volkov and V.P. Akulov, Phys. Lett. 46B (1973) 109;
    J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39.
[2] E. Witten, Nucl. Phys. B188 (1981) 513;
    S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;
    N. Sakai, Z. Phys. C11 (1981) 153;
    R.K. Kaul, Phys. Lett. 109B (1982) 19.

[3] J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310;
S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1974) 413;
S. Ferrara and O. Piguet, Nucl. Phys. B93 (1975) 261.
[4] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24 (1981) 1681;
L. Ibañez and G.G. Ross, Phys. Lett. 105B (1981) 439.
[5] M.B. Einhorn and D.R.T. Jones, Nucl. Phys. B196 (1982) 475;
J. Ellis, D.V. Nanopolous and S. Rudaz, Nucl. Phys. B202 (1982).
[6] A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, MPI preprint, MPI-PAE/PTh 29/82 (May 1982).
[7] D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 113B (1982) 151.
[8] A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, CERN preprint TH. 3324 (June 1982).
[9] R. Barbieri, S. Ferrara and D.V. Nanopoulos, Z. Phys. C13 (1982) 267; Phys. Lett. 116B (1982) 16;
J. Ellis, L. Ibañez and G.G. Ross, Phys. Lett. 113B (1982) 283.
[10] J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Phys. Lett. 88B (1979) 320.
[11] D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 114B (1982) 235.