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A note on brane cosmology

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Abstract

We derive a new class of time-dependent solutions for the Randall–Sundrum model by patching together isometries broken by the brane. Solutions generated by generalized boosts along the fifth dimension are associated with localized gravity and lead to an effective Friedman equation on the brane with a scale factor exhibiting power law or exponential behaviour. The effective energy-density on the brane depends linearly on the brane tension. © 2001 Published by Elsevier Science B.V.

The large gap between the electroweak scale and the gravitational Planck scale has motivated the quest of a theory with a higher dimensional spacetime in which our world corresponds to a four-dimensional hypersurface. Although the idea that the world might correspond to a topological defect embedded in a higher dimensional spacetime is not new [1], string theory provides many examples of such hypersurfaces or *branes* and, therefore, strong motivation for their exploration. In a class of such higher dimensional models Standard Model Physics is confined on the brane while gravity propagates in the bulk. Large compact extra dimensions can lower the fundamental gravitational scale down to the TeV range [2]. The extra space, however, does not have to be compact but just highly curved. Such is the Randall–Sundrum spacetime [3,4] characterized by a graviton which, although depending on the transverse coordinate, it is sharply localized on the brane. The RS metric $ds_5^2 = e^{-2\kappa|y|} g_{\mu\nu}\, dx^\mu\, dx^\nu + dy^2$ corresponds to gluing together two regions of the AdS_5 space ($y > 0$ and $y < 0$) such that to posse a Z_2 symmetry. Smooth generalizations of such a geometry can be constructed with the help of a scalar field that gives rise to the brane as a kink-like soliton while, in addition to gravitons, other relevant degrees of freedom can be localized on the brane [5]. An additional important possibility that seems to be open in brane models is the possibility of explaining the smallness of the observed cosmological constant. Branes allow for an interplay between higher dimensional and four-dimensional cosmological constant contributions. Such *self-tuning* behaviour [6–11] arises also in the case that the brane is realized as a scalar soliton [10]. An interesting as well as important issue is whether higher dimensional models lead to a cosmological evolution compatible with standard Big Bang cosmology or extensions of it like inflationary models. The cosmological implications of higher dimensional models have been studied by a number of authors [12–25]. A general feature of brane cosmology is that in the Friedman-like equations on the brane the energy density of the brane, naively corresponding to the brane tension, appears quadratically [21] in contrast to the Friedman equations of the standard cosmol-

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ogy where it appears linearly. In the Randall–Sundrum framework, however, this quadratic contribution in the right hand side of the Friedman equation is compensated by the linear bulk energy density coming from a bulk cosmological constant. Exact compensation gives the well known Randall–Sundrum static flat solution. Dynamical evolution is possible in the form of exponential expansion when the above two contributions do not cancel, leaving an effective four-dimensional energy density.

In the present Letter we investigate the time evolution on the brane in the framework of the Randall–Sundrum AdS_5 spacetime looking for time-dependent solutions of Einstein's equations in the full five-dimensional space. We consider Lorentz transformations as a prototype of isometries that are broken by the presence of the brane and replace them with their Z_2-invariant analogues. The metric generated by such a transformation satisfies Einstein's equations with an effective energy-momentum tensor consisting of the contributions of the brane and a bulk cosmological constant. The corresponding cosmological evolution on the brane is that of exponential expansion. Next, we consider a generalization of the above metric that leads to a general time-dependence of the four-dimensional scale factor that includes standard time evolution like $a(\tau) \propto \tau^{1/2}$. These solutions exist for specific time-dependent bulk energy densities. In all cases the four-dimensional scale factor satisfies an effective Friedman equation that features the effective energy density on the brane calculated by summing the bulk energy density contribution and the brane tension. The general quadratic dependence on the brane tension is replaced by a linear one due to the relation between the Randall–Sundrum curvature parameter and the brane tension $\kappa = \xi/24M_5^3$ necessary for the existence of solutions.

Let us consider the five-dimensional anti-de-Sitter space AdS_5 with metric

$$ds^2 = e^{-2\kappa y}\left(-dt^2 + dx_\perp^2 + dx_{||}^2\right) = \frac{1}{\kappa^2 x_{||}^2} d\sigma^2, \qquad (1)$$

where the notation

$$x_{||} \equiv \frac{e^{\kappa y}}{\kappa},$$

$$dx_\perp^2 = \delta_{ij}\, dx^i\, dx^j, \quad i, j = 1, 2, 3, \qquad (2)$$

has been introduced. AdS_5 is maximally symmetric and its Riemann tensor satisfies

$$R_{\mu\nu\kappa\lambda} = \kappa(g_{\mu\kappa}g_{\nu\lambda} - g_{\nu\kappa}g_{\mu\lambda}), \quad \mu, \ldots = 0, \ldots, 4.$$

The metric $d\sigma^2$ and the $x_{||} =$ const slices of the full space are invariant under the *boosts*

$$t' = \frac{1}{\sqrt{1-\beta^2}}\left(t + \beta x_{||}\right), \qquad (3)$$

$$x'_{||} = \frac{1}{\sqrt{1-\beta^2}}\left(x_{||} + \beta t\right), \qquad (4)$$

$$x'^i_\perp = x^i_\perp. \qquad (5)$$

The total metric (1) is then written in the new boosted coordinates as

$$ds^2 = \frac{1}{\kappa^2 {x'_{||}}^2}\left(-dt'^2 + dx'^2_\perp + dx'^2_{||}\right)$$
$$= \frac{1-\beta^2}{\kappa^2 (x_{||} + \beta t)^2}\left(-dt^2 + dx_\perp^2 + dx_{||}^2\right). \qquad (6)$$

In terms of the original coordinates we have, up to a constant

$$ds^2 = \frac{1}{(e^{\kappa y} + \beta t)^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right)$$
$$+ \frac{e^{2\kappa y}}{(e^{\kappa y} + \beta t)^2} dy^2. \qquad (7)$$

This metric still describes AdS_5 just in another coordinate system, i.e., the boosted one. As a result, we would the same physics as the original metric if no discontinuities exist, since the boosts are just a set of general coordinate transformations. In the Randall–Sundrum model, however, where a brane sits at $y = 0$, we can have the above boosts for $y > 0$ and their analogues for $y < 0$ such that no global coordinate transformation to exists and describing different physics. Introducing conformal time $e^{-\beta\tau}\, d\tau = -dt$ and imposing the Z_2 symmetry $y \to -y$ we can set the metric (7) in the form

$$ds^2 = -\frac{e^{-2\beta\tau}\, d\tau^2}{(e^{\kappa|y|} + e^{-\beta\tau} - 1)^2} + \frac{\delta_{ij}\, dx^i\, dx^j}{(e^{\kappa|y|} + e^{-\beta\tau} - 1)^2}$$
$$+ \frac{e^{2\kappa|y|}\, dy^2}{(e^{\kappa|y|} + e^{-\beta\tau} - 1)^2}. \qquad (8)$$

When the parameter β vanishes, this metric is the static Randall–Sundrum metric

$$ds^2_{\mathrm{RS}} = e^{-2\kappa|y|} g_{\mu\nu}\, dx^\mu\, dx^\nu + dy^2.$$

Dynamical solutions with the same static limit has been constructed also in [26,27]. It is clear that if the *right* ($y > 0$) or *left* ($y < 0$) metric given (8) was extended to the whole spacetime, it would provide a solution of Einstein's equations with a cosmological constant. The existence of the brane, mathematically represented by the presence of the absolute value $|y|$, will induce extra $\delta(y)$ and constant terms coming from $|y|''$ and $(|y|')^2$. Such terms are exactly the terms expected from a bulk cosmological constant and a 3-brane term in the action. Thus, the metric (8) should be compatible with the action

$$\mathcal{S} = \int d^5x\,\sqrt{-g}\,\{2M_5^3 R - \Lambda_B\} - \int d^5x\,\sqrt{-\det(g_{\mu\nu})}\,\xi\,\delta(y), \tag{9}$$

which leads to Einstein's equations

$$G_{MN} = R_{MN} - \frac{1}{2}g_{MN}R = -g_{MN}\frac{\Lambda_B}{4M_5^3} - g_{\mu\nu}\delta^\mu_M\delta^\nu_N\frac{\xi}{4M_5^3}\frac{\delta(y)}{b} \equiv \frac{1}{4M_5^3}T_{MN}. \tag{10}$$

The two parameters appearing in these equations, namely the *brane tension* ξ and the *bulk cosmological constant* Λ_B should be matched with the two parameters of the metric κ and β.

Using the metric ansatz

$$ds^2 = -n^2(\tau, y)\,d\tau^2 + a^2(\tau, y)\gamma_{ij}\,dx^i\,dx^j + b^2(\tau, y)\,dy^2 \tag{11}$$

with γ_{ij} the metric of a three-dimensional maximally symmetric space, [1] we can compute components of Einstein's tensor G_{AB}. They are [16,22]

$$G_{00} = 3\left\{\frac{\dot a}{a}\left(\frac{\dot a}{a} + \frac{\dot b}{b}\right) - \frac{n^2}{b^2}\left(\frac{a''}{a} + \frac{a'}{a}\left(\frac{a'}{a} - \frac{b'}{b}\right)\right) + k\frac{n^2}{a^2}\right\}, \tag{12}$$

$$G_{ij} = \frac{a^2}{b^2}\gamma_{ij}\left\{\frac{a'}{a}\left(\frac{a'}{a} + 2\frac{n'}{n}\right) - \frac{b'}{b}\left(\frac{n'}{n} + 2\frac{a'}{a}\right) + 2\frac{a''}{a} + \frac{n''}{n}\right\} + \frac{a^2}{n^2}\gamma_{ij}\left\{\frac{\dot a}{a}\left(-\frac{\dot a}{a} + 2\frac{\dot n}{n}\right) - 2\frac{\ddot a}{a} + \frac{\dot b}{b}\left(-2\frac{\dot a}{a} + \frac{\dot n}{n}\right) - \frac{\ddot b}{b}\right\} - k\gamma_{ij}, \tag{13}$$

$$G_{05} = 3\left(\frac{n'}{n}\frac{\dot a}{a} + \frac{a'}{a}\frac{\dot b}{b} - \frac{\dot a'}{a}\right), \tag{14}$$

$$G_{55} = 3\left\{\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{b^2}{n^2}\left(\frac{\dot a}{a}\left(\frac{\dot a}{a} - \frac{\dot n}{n}\right) + \frac{\ddot a}{a}\right) - k\frac{b^2}{a^2}\right\}. \tag{15}$$

Substituting the boosted expressions (9), (10) and (11) of the metric components, we can easily see that they are indeed a solution to Einstein's equations for the case $k = 0$. We also determine the solution parameters in terms of the parameters of the action Λ_B and ξ. Their relations are

$$\xi = 24M_5^3\kappa, \tag{16}$$

$$\Lambda_B = -24M_5^3\left(\kappa^2 - \beta^2\right). \tag{17}$$

Notice that for $\beta = 0$, when the metric coincides with the static Randall–Sundrum metric, these relations are the well-known relations that relate the brane tension to the curvature parameter and express the fine-tuning between the bulk cosmological constant and the brane tension in order to have a flat solution. This last relation however in the case of non-vanishing β should not be interpreted as a fine-tuning since there is a continuous range of values of $\kappa^2 + \frac{\Lambda_B}{24M_5^3} \geqslant 0$ corresponding to solutions.

The four-dimensional metric corresponding to our solution is

$$ds_4^2 = -d\tau^2 + a_0^2(\tau)\,d\vec{x}^{\,2} \tag{18}$$

with

$$a_0(\tau) = e^{\beta\tau}. \tag{19}$$

[1] $\gamma_{rr} = (1 - kr^2)^{-1}$, $\gamma_{\theta\theta} = r^2$, $\gamma_{\phi\phi} = r^2\sin^2\theta$.

This exponential expansion can be described with an *effective Friedman equation*

$$\left(\frac{\dot a_0}{a_0}\right)^2 = \frac{8\pi G}{3}\rho_{\rm eff}. \tag{20}$$

This can be easily seen considering that[2] $8\pi G = \frac{1}{4M_P^2} = \frac{\kappa}{4M_5^3}$. Then, the effective energy density should satisfy

$$\left(\frac{\dot a_0}{a_0}\right)^2 = \beta^2 = \frac{\kappa}{12M_5^3}\rho_{\rm eff}. \tag{21}$$

In view of the parameter relations (18) and (19), the effective four-dimensional energy density is

$$\rho_{\rm eff} = \frac{1}{2}\left(\frac{\Lambda_B}{\kappa} + \xi\right) = \frac{1}{2}\left(\rho_B^{(4)} + \rho_b^{(4)}\right) \tag{22}$$

being the average of the bulk contribution to the four-dimensional energy density[3] $\Lambda_B \times \kappa^{-1}$ and the naive four-dimensional energy density ξ. Note that in the static Randall–Sundrum case these two terms cancel to give an exactly vanishing $\rho_{\rm eff}$. It is worth comparing the above effective Friedman equation with the analogous general equation [21] derived for any constant bulk energy–momentum

$$\left(\frac{\dot a_0}{a_0}\right)^2 = \frac{1}{24M_5^3}\rho_B + \frac{1}{(24M_5^3)^2}\rho_b^2 - \frac{k}{a_0^2} + \frac{\mathcal{C}}{a_0^4}. \tag{23}$$

The two equations coincide for $k = \mathcal{C} = 0$ due to the relation $\rho_b = \xi = 24M_5^3\kappa$ which transforms the quadratic dependence on the brane tension into a linear one, namely

$$\begin{aligned}\left(\frac{\dot a_0}{a_0}\right)^2 &= \frac{\kappa}{24M_5^3}\left(\rho_B^{(4)} + \rho_b^{(4)}\right)\\ &= \frac{\kappa}{24M_5^3}\left(\frac{\rho_B^{(5)}}{\kappa} + \frac{(\rho_b^{(4)})^2}{24M_5^3\kappa}\right)\\ &= \frac{1}{24M_5^3}\left(\rho_B^{(5)} + \frac{(\rho_b^{(4)})^2}{24M_5^3}\right).\end{aligned}$$

It seems, therefore, misleading to interpret the above equation as indicating a quadratic dependence on the brane energy density.

The next step in the quest for time-dependent solutions compatible with standard cosmology would be to replace the exponential in the metric ansatz with a general function of time $a_0(\tau)$. Introducing the generalized trial solutions

$$a(\tau, y) = \frac{a_0(\tau)}{a_0(\tau)(e^{\kappa|y|} - 1) + 1}, \tag{24}$$

$$n(\tau, y) = \frac{1}{a_0(\tau)(e^{\kappa|y|} - 1) + 1} = \frac{a}{a_0}, \tag{25}$$

$$b(\tau, y) = \frac{a_0(\tau)e^{\kappa|y|}}{a_0(\tau)(e^{\kappa|y|} - 1) + 1} = e^{\kappa|y|}a, \tag{26}$$

we obtain, for $k = 0$, the Einstein tensors

$$G_{05} = 0, \tag{27}$$

$$G_{00} = g_{00}\left\{6\kappa^2 - 6\left(\frac{\dot a_0}{a_0}\right)^2 - 6\kappa\frac{\delta(y)}{b_0}\right\}, \tag{28}$$

$$G_{55} = g_{55}\left\{6\kappa^2 - 6\left(\frac{\dot a_0}{a_0}\right)^2 - \frac{3}{n(\tau, y)}\left(\frac{\ddot a_0}{a_0} - \left(\frac{\dot a_0}{a_0}\right)^2\right)\right\}, \tag{29}$$

$$G_{ij} = g_{ij}\left\{6\kappa^2 - 6\left(\frac{\dot a_0}{a_0}\right)^2 - 6\kappa\frac{\delta(y)}{b_0} - \frac{3}{n(\tau, y)}\left(\frac{\ddot a_0}{a_0} - \left(\frac{\dot a_0}{a_0}\right)^2\right)\right\}. \tag{30}$$

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[2] The four-dimensional Planck mass is determined to be $M_P^2 = M_5^3/\kappa$. Using standard time we have

$$ds_4^2 = a^2(t,0)\left(-dt^2 + d\vec x^{\,2}\right) = g_{\mu\nu}(t)\,dx^\mu\,dx^\nu$$

and

$$\sqrt{-G} = \sqrt{-g}\,b(t,y)\left(a(t,y)/a(t,0)\right)^4.$$

Since,

$$R[G] = \frac{a^2(t,y)}{a^2(t,0)}R^{(4)}[g] + \cdots,$$

performing the y-integration, we obtain

$$\begin{aligned}\mathcal{S} &= 2M_5^3\int d^4x\int dy\,\sqrt{-G}\,R\\ &= 2M_5^3\int d^4x\,\sqrt{-g}\,a^{-2}(t,0)\left\{\int dy\,a^2(t,y)b(t,y)\right\}R^{(4)} + \cdots\\ &= \frac{2M_5^3}{\kappa}\int d^4x\,\sqrt{-g}\,R^{(4)}[g] + \cdots,\end{aligned}$$

from which we read off

$$M_P^2 = M_5^3/\kappa.$$

[3] Note that κ^{-1} is the effective size of the fifth dimension.

These, through Einstein's equations, can be matched with a conserved energy–momentum tensor

$$T_N^M = \mathrm{Diag}(-\rho, p, p, p, p_T) \tag{31}$$

with components

$$\rho = 24M_5^3\left\{\kappa\frac{\delta(y)}{b_0} - \kappa^2 + \left(\frac{\dot a_0}{a_0}\right)^2\right\}, \tag{32}$$

$$p = -\rho - \frac{12M_5^3}{n(\tau, y)}\left\{\frac{\ddot a_0}{a_0} - \left(\frac{\dot a_0}{a_0}\right)^2\right\}; \tag{33}$$

p_T equals to the bulk part of p. Note that the bulk energy density is only time-dependent while the bulk pressure has also a space-dependence. Notice that there is no time-dependence introduced on the brane tension which is again $\xi = 24M_5^3\kappa$. It is easy to see that the content of the previous section can be easily recovered from the above energy–momentum tensor which becomes constant.

Let as now introduce a power law time-dependence in the scale factor

$$a_0(\tau) = C\tau^\gamma \tag{34}$$

The 5D metric then turns out to be

$$ds^2 = -\frac{d\tau^2}{C^2\tau^{2\gamma}(e^{\kappa|y|} - 1) + 1} + \frac{C^2\tau^{2\gamma}\delta_{ij}\,dx^i\,dx^j}{C^2\tau^{2\gamma}(e^{\kappa|y|} - 1) + 1} + \frac{C^2\tau^{2\gamma}e^{\kappa|y|}\,dy^2}{C^2\tau^{2\gamma}(e^{\kappa|y|} - 1) + 1}, \tag{35}$$

whereas the corresponding densities are

$$\rho(\tau) = 24M_5^3\left\{\kappa\frac{\delta(y)}{b_0} - \kappa^2 + \frac{\gamma^2}{\tau^2}\right\}, \tag{36}$$

$$p(\tau, y) = -\rho(\tau) + 12M_5^3\frac{\gamma}{\tau^2}\left\{\frac{1}{n(\tau, y)} - 2\gamma\right\}; \tag{37}$$

p_T is given by the bulk-part of p.

A physical explanation for the spatial dependence of the pressure is given by the fact that the force along the fifth dimension takes the form

$$f(\tau, y) \equiv -\frac{\partial p}{\partial y} = 12M_5^3 C\gamma\tau^{\gamma-2}\kappa\,\mathrm{sign}(y)\,e^{\kappa|y|} \tag{38}$$

representing a force directed towards the brane and increasing with distance from it. A bulk pressure density of this form is apparently required to sustain the brane at its given position.

The scale factor resulting from such an energy–momentum density satisfies an effective Friedman equation

$$\left(\frac{\dot a_0}{a_0}\right)^2 = \frac{\gamma^2}{\tau^2} = \frac{\kappa}{12M_5^3}\rho_{\mathrm{eff}}. \tag{39}$$

Thus,

$$\rho_{\mathrm{eff}} = \frac{\rho_B(\tau)}{2\kappa} + 12M_5^3\kappa = \frac{1}{2}\left(\frac{\rho_B(\tau)}{\kappa} + \xi\right) = \frac{1}{2}\big(\rho_B^{(4)}(\tau) + \rho_b^{(4)}\big), \tag{40}$$

in terms of the bulk part of the five-dimensional energy density ρ_B and the brane tension. Thus, again we have the same averaging formula as in the time-independent case and there is no quadratic dependence on the brane tension. Time-dependent perturbations on the brane tension will not modify the linear dependence. This is clear from

$$\left(\frac{\dot a_0}{a_0}\right)^2 = \frac{1}{24M_5^3}\left(\rho_B^{(5)} + \frac{(\rho_b^{(4)})^2}{24M_5^3}\right) = \frac{1}{24M_5^3}\left(\rho_B^{(5)} + \frac{(\rho_b^{(4)} + \delta\xi(\tau))^2}{24M_5^3}\right) \sim \frac{\kappa}{24M_5^3}\big(\rho_B^{(4)} + \rho_b^{(4)} + 2\delta\xi(\tau)\big) + \mathrm{O}\big(\delta\xi^2\big).$$

From the above analysis it is clear that both cases of inflation ($a_0(\tau) \propto e^{\beta\tau}$) and standard cosmological expansion ($a_0(\tau) \propto \tau^{1/2}$) could be described by a common suitable bulk energy density. Indeed, the bulk energy density

$$\rho_B(\tau) = 24M_5^3\left\{-\kappa^2 + \frac{\beta^2\kappa^{-1}}{1 + \beta^2\gamma^{-2}\tau^2}\right\} \tag{41}$$

corresponds to a scale factor

$$a_0(\tau) \propto e^{\gamma\sinh^{-1}\left(\frac{\beta}{\gamma}\tau\right)}. \tag{42}$$

Using that $\sinh^{-1}(x) \sim x$ for $x \ll 1$ and $\sinh^{-1}(x) \sim \ln 2x$ for $x \gg 1$, we get an early exponential expansion and a late power-law one

$$a_0(\tau) \propto \begin{cases} e^{\beta\tau}, & \tau \to 0, \\ \tau^\gamma, & \tau \to \infty. \end{cases} \tag{43}$$

A bulk energy density of this sort would describe a temporary inflationary phase succeeded by a phase of standard power law expansion.

Summarizing, we have studied a class of time-dependent solutions of Einstein's equations in the framework of a five-dimensional spacetime corresponding to the standard Randall–Sundrum AdS metric in the static case. We considered Lorentz transformations as a prototype of isometries that are broken by the presence of the brane and replace them with their Z_2-invariant analogues. The metric generated by such a transformation satisfies Einstein's equations with an effective energy–momentum tensor consisting of the contributions of the brane and a bulk cosmological constant. The corresponding cosmological evolution on the brane is that of exponential expansion. Next we studied a generalization of the above metric corresponding to a general time-dependence of the four-dimensional scale factor that includes standard time evolution like $a(\tau) \propto \tau^{1/2}$. These solutions exist for specific time-dependent bulk energy densities. In all cases the four-dimensional scale factor satisfies an effective Friedman equation that features the effective energy density on the brane calculated by summing the bulk energy density contribution and the brane tension. The general quadratic dependence on the brane tension is replaced by a linear one due to the relation between the Randall–Sundrum curvature parameter and the brane tension.

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