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A Pati–Salam model from branes

G.K. Leontaris [a], J. Rizos [b,1]

[a] *Physics Department, University of Ioannina, Ioannina, GR45110, Greece*
[b] *Centre de Physique Théorique, Ecole Polytechnique, F-91128 Palaiseau, France*

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Abstract

We explore the possibility of embedding the Pati–Salam model in the context of Type I brane models. We study a generic model with $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry and matter fields compatible with a Type I brane configuration. Examining the anomaly cancellation conditions of the surplus abelian symmetries we find an alternative hypercharge embedding that is compatible with a low string/brane scale of the order of 5–7 TeV, when the $U(4)_C$ and $U(2)_R$ brane stack couplings are equal. Proton stability is assured as baryon number is associated to a global symmetry remnant of the broken abelian factors. It is also shown that this scenario can accommodate an extra low energy abelian symmetry that can be associated to lepton number. The issue of fermion and especially neutrino masses is also discussed.

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1. Introduction

It has been recently realized that in Type I string theories the string scale is not necessarily of the order of the Planck mass, as it happens in the case of heterotic models, but it can be much lower depending on the compactification volume [1]. Furthermore, the discovery of D-branes [2], solitonic objects of Type I string theory, has revolutionized the string-theory viewpoint of our world. This includes the possibility that we are living on a p-dimensional hyper-surface, a $D(p-1)$ brane embedded in the 10-dimensional string theory. The rest, $10-p$ transverse dimensions constitute the so called bulk space. The gauge interactions, mediated by open strings, restrict their action in the brane, while gravitational interactions, mediated by closed strings, can propagate in the full 10-dimensional theory. These developments have reinforced expectations that some string radii can be brought down to the TeV range [3], energy accessible to the future accelerators, and that string theory could account for the stabilization of hierarchy without invoking supersymmetry [4].

Furthermore, new techniques have been developed for the construction of Type I models [5], including the D-brane configurations, based on Type IIB orientifolds [6]. Various models, basically variations of the Standard Model or it's left–right symmetric extensions, have been constructed [7,8], using these methods. Although some of these models are characterised as semi-realistic, from the phenomenological point of view, the structure of Type I string vacua is very rich to permit a complete classification. Hence, model building endeavour needs to be carried on until we reach a phenomenologically satisfactory vacuum.

E-mail address: george.leontaris@cern.ch (G.K. Leontaris).
[1] On leave from Physics Department, University of Ioannina, GR45110 Ioannina, Greece.

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One is tempted to adopt a bottom-up approach [9, 10], that is, to search for effective low energy models compatible with low unification and check their generic phenomenological properties [11–14] and the minimal conditions for phenomenological viability, before proceeding to explicit realizations in the context of string theory. Low scale unified models based on gauge symmetries beyond that of the Standard Model (SM) face several problems. Proton decay is usually the most serious obstruction when lowering the unification mass below the traditional grand unified scale $M_{\rm GUT} \sim 10^{16}$ GeV, due to the existence of gauge-mediated baryon number violating dimension-six operators. In addition, one needs to understand how a rapid convergence of the gauge couplings can occur in an energy region much shorter than the traditional M_Z–$M_{\rm GUT}$ of old unified models [15].

Rapid gauge-boson mediated proton decay excludes a wide range of gauge groups beyond the SM, however, examples of models which can in principle avoid this problem do exist. A natural candidate is the Pati–Salam (PS) model [16], originally proposed as a model of low unification scale. This model has been successfully reproduced and studied in the context of heterotic string theory [17].

With regard to the problem of coupling unification, there are various proposals in the literature [4,9,18]. One possibility is to assume power law running of the gauge couplings [18] and obtain full coupling unification at a low scale. An alternative scenario is based on the observation that the different collections of D-branes (associated with the extended gauge group factors) have not necessarily equal gauge couplings. The low energy electroweak data could then be reproduced by considering the usual logarithmic coupling evolution while assuming equality of two (instead of three) gauge couplings at the string scale [9].

In this work, we search for D-brane configurations where the left–right PS gauge symmetry is embedded. Since supersymmetry can be broken at the string/brane level [19] we are going to explore non-supersymmetric versions of the Pati–Salam model. We derive a generic D-brane configuration fermion and higgs spectrum and show that all the SM particles and the necessary Yukawa couplings for fermion masses are present. We address the problems of anomaly cancellation, hypercharge embedding, proton decay and gauge coupling unification. Our analysis shows that all these problems find natural solutions and that the non-supersymmetric Pati–Salam model is compatible with intermediate/low scale D-brane scenarios.

2. Particle assignment

A single D-brane carries a $U(1)$ gauge symmetry which is the result of the reduction of the ten-dimensional Yang–Mills theory. Therefore, a stack of n parallel, almost coincident D-branes gives rise to a $U(n)$ gauge theory where the gauge bosons correspond to open strings having both their ends attached to some of the branes of the stack [20]. For the embedding of the PS model we consider brane configurations of three different stacks containing 4-2-2 branes respectively, which give rise to a $U(4)_C \times U(2)_L \times U(2)_R$ or equivalently $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_L \times U(1)_R$ gauge symmetry.

Following the pictorial representation of Fig. 1 it is not difficult to see that the possible states arising from strings with both their ends on two distinct sets of branes can accommodate the fermions of the

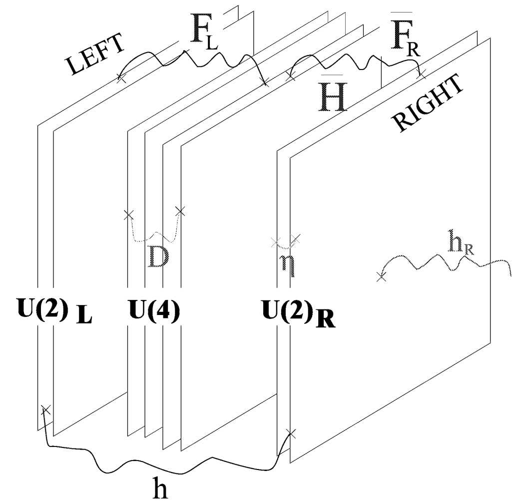

Fig. 1. Assignment of the Standard Model particles in a D-brane scenario with gauge group $U(4)_C \times U(2)_L \times U(2)_R$. The standard model particles are assigned to $F_L = Q + L$, $\overline{F}_R = u^c + d^c + e^c + \nu^c$ and the electroweak Higgs to $h = H_u + H_d$. They are all represented by strings having both their ends attached to two different branes. The PS breaking Higgs scalars ($\overline{H}$) are similar to $\overline{F}_R$. In gray we represent particles whose presence is not required in all versions of the model. These are the extra scalar triplets $D = \tilde{d} + \tilde{d}^c$, the right-handed doublets h_R and the singlet η.

SM as well as the necessary Higgs particles to break the gauge symmetry[21]. For example, an open string with one end on the $U(4)$ brane and the other end on the $U(2)_L$ brane transforms as $(\mathbf{4}, \mathbf{2}_L)$ whilst is a singlet under $SU(2)_R$. Thus, under the PS group the corresponding state is written as $(\mathbf{4}, \mathbf{2}, \mathbf{1})$. Due to the decompositions under the chains $U(n) \to SU(n) \times U(1)$ $(n = 4, 2, 2)$ all such states carry charges under three surplus $U(1)$ factors. Normalizing appropriately [2] these charges are $+1, -1$ for the vector/vector-bar representation of $SU(n)$, and thus, the standard model particle assignments are

$$\begin{aligned} F_L &= (\mathbf{4}, \mathbf{2}, \mathbf{1}, +1, \alpha_L, 0) = Q\big(\mathbf{3}, \mathbf{2}, \tfrac{1}{6}\big) + L\big(\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}\big), \\ \overline{F}_R &= (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, -1, 0, \alpha_R) \\ &= u^c\big(\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}\big) + d^c\big(\bar{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}\big) \\ &\quad + e^c(\mathbf{1}, \mathbf{1}, 1) + \nu^c(\mathbf{1}, \mathbf{1}, 0), \end{aligned} \tag{1}$$

where $\alpha_L = \pm 1$, $\alpha_R = \pm 1$ depending on the $U(1)_L$, $U(1)_R$ charges of $\mathbf{2}_L$, $\mathbf{2}_R$. The electroweak breaking scalar doublets can arise from the bi-doublet

$$\begin{aligned} h &= (\mathbf{1}, \mathbf{2}, \mathbf{2}, 0, -\alpha_L, -\alpha_R) \\ &= H_u\big(\mathbf{1}, \mathbf{2}, +\tfrac{1}{2}\big) + H_d\big(\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}\big), \end{aligned} \tag{2}$$

where we have chosen the $U(1)_{L,R}$ charges so that the Yukawa term $F_L \overline{F}_R h$ which provides with masses all fermions, is allowed. The PS breaking Higgs scalar particles are

$$\begin{aligned} \overline{H} &= (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, -1, 0, \gamma) \\ &= u^c_H\big(\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}\big) + d^c_H\big(\bar{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}\big) \\ &\quad + e^c_H(\mathbf{1}, \mathbf{1}, 1) + \nu^c_H(\mathbf{1}, \mathbf{1}, 0). \end{aligned} \tag{3}$$

Without loss of generality we can choose $\alpha_L = \alpha_R = 1$ which is equivalent to measuring left (right) $SU(2)_{L(R)}$ vector representation $U(1)_{L(R)}$-charges in $\alpha_L(\alpha_R)$ “units”, respectively.

Additional states can arise from strings having both their ends at the same brane. Among them one finds the $SU(4)$ sextet

$$D(\mathbf{6}, \mathbf{1}, \mathbf{1}, +2, 0, 0) = \tilde{d}^c\big(\bar{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}\big) + \tilde{d}\big(\mathbf{3}, \mathbf{1}, -\tfrac{1}{3}\big) \tag{4}$$

[2] We assume the $U(n) \sim SU(n) \times U(1)$ generators $T_a, a = 1, \ldots, n^2$, to be normalized as $\operatorname{tr} T_a T_b = \frac{1}{2}\delta_{ab}$ and the $SU(n)$ coupling constant to be $\sqrt{2n}$ times the $U(1)$ coupling constant.

(see Fig. 1), which can be used to provide masses to the Higgs remnants (one d-like triplet) of the PS breaking Higgs mechanism (see Section 6). Further, one may generate a $U(1)_R$ charged singlet

$$\eta = (\mathbf{1}, \mathbf{1}, \mathbf{1}, 0, 0, +2) \tag{5}$$

which, as will become clear later (see Section 4), can be used for breaking an additional abelian symmetry. Possible states include also strings having one end attached to a brane and the other in the bulk while among them we find the $SU(2)_R$ doublet

$$h_R = (\mathbf{1}, \mathbf{1}, \mathbf{2}; 0, 0, +1), \tag{6}$$

which will be also used later for an alternative breaking of an additional abelian symmetry.

3. Anomalies

An essential difference between heterotic and Type I effective string theories is the number of potentially anomalous abelian factors. In the heterotic case only one such factor is allowed with rather tight restrictions on the form of its mixed anomalies, due to their relation with the dilaton multiplet. Type I theory is more tolerant, many anomalous abelian factors can be present and their cancellation is achieved through a generalized Green–Schwarz mechanism [24] which utilizes the axion fields of the Ramond–Ramond sector [22,23], providing masses to the corresponding anomalous gauge bosons. However, in Type I models, unlike the heterotic string case, gauge boson masses are fixed by undetermined vacuum expectation values and therefore the $U(1)$ gauge bosons may be light. Another important characteristic of Type I abelian factors is that their breaking leaves behind global symmetries, that can be useful for phenomenology.

As can be seen from the fermion charge assignments (1), the abelian gauge group factors have mixed anomalies with $SU(4)$, $SU(2)_L$ and $SU(2)_R$. We present these anomalies in matrix form

$$A = \begin{pmatrix} 0 & 3 & 3 \\ 6 & 6 & 0 \\ -6 & 0 & 6 \end{pmatrix}, \tag{7}$$

where its lines correspond to the abelian factors $U(1)_C$, $U(1)_L$, $U(1)_R$ and its columns to the non-abelian groups $SU(4)$, $SU(2)_L$, $SU(2)_R$. From the point of view of the low energy theory, it is crucial to examine whether there are any combinations of anomaly free abelian generators. This would imply the existence of additional unbroken $U(1)$ factors at low energies which may result to interesting phenomenology. For example, the existence of $U(1)$ factors offers the possibility to define the hypercharge generator in various ways provided that the fermion and electroweak breaking Higgs particles acquire the standard hypercharge assignments. We find that there exists only one non-anomalous combination

$$\mathcal{H} = T_C - T_L + T_R, \tag{8}$$

which also has the advantage of being free from gravitational anomalies as both, $\mathrm{trace}(\mathcal{H}) = 0$ and $\mathrm{trace}(\mathcal{H}^3) = 0$.

One may wonder about the existence of such additional anomaly-free abelian symmetry (on top of B–L and Y). The reason is that none of the SM fermions is charged under this symmetry. Actually, the only states potentially charged under $U(1)_{\mathcal{H}}$ are the PS breaking Higgs scalars $\overline{H}$ (and the scalars h_R, D, η). Later on, we will associate the value of the parameter γ, which determines the PS breaking Higgs charges with the symmetry breaking pattern and discuss the possibility of survival of $U(1)_{\mathcal{H}}$ at low energies.

Thus, at this stage, assuming that all anomalous abelian combinations will break, we are left with an effective theory with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_{\mathcal{H}}$.

4. Symmetry breaking and the hypercharge generator

We next analyse the pattern of symmetry breaking. The Higgs scalar $\overline{H}$, (provided that an appropriate potential exists), will acquire a non-zero vev and break the original symmetry down to SM augmented by a $U(1)$ factor. Subsequently, the electroweak symmetry breaking occurs via non-vanishing vevs of the H_u, H_d Higgs particles. Since the bi-doublet h, and consequently H_u, H_d are neutral under $U(1)_{\mathcal{H}}$, there will be always a leftover abelian combination whose structure is completely determined by the $\overline{H}$ charge $(\gamma - 1)$ (see relations (3), (8)). Thus, the hypercharge generator will be, in general, a linear combination of the usual PS generator and the additional abelian gauge factor $U(1)_{\mathcal{H}}$:

$$Y = \frac{1}{2} Q_{B-L} + \frac{1}{2} Q_{3R} + c Q_{\mathcal{H}}, \tag{9}$$

where c is to be determined by the symmetry breaking. The PS breaking Higgs particles, $\overline{H}$, contain two potential SM singlets with $U(1)_{B-L} \times U(1)_{3R} \times U(1)_{\mathcal{H}}$ charges

$$\begin{aligned} N_+ &= (+1, +1, -1 + \gamma) \\ N_- &= (+1, -1, -1 + \gamma). \end{aligned} \tag{10}$$

When the minimum of the scalar potential occurs for either $N_+ = \langle e^c_H \rangle$, or $N_- = \langle \nu^c_H \rangle$ different than zero, the gauge symmetry breaks to the SM times an additional abelian factor. Extra abelian factors, although in principle consistent with low energy data [25], necessitate a breaking mechanism. An interesting property of the model presented here is that the appropriate scalar fields, which can break these extra abelian factors, are naturally generated in the D-brane scenario. These are the singlet field η (5) and the the right-handed doublet (6)

$$\begin{aligned} h_R(\mathbf{1}, \mathbf{1}, \mathbf{2}, 0, 0, +1) = {} & h^+_R(\mathbf{1}, \mathbf{1}, 0, +1, +1) \\ & + h^-_R(\mathbf{1}, \mathbf{1}, 0, -1, +1). \end{aligned}$$

Depending on the value of γ there are two possible breaking patterns, which are presented in Table 1. The

Table 1
The two symmetry breaking patterns of $SU(4) \times SU(2)_R \times U(1)_{\mathcal{H}}$ and the corresponding PS Higgs vevs, N_+, N_-, their right chirality $(\gamma = \pm 1)$, the resulting hypercharge generator, the leftover abelian factor and the scalar fields that can break this extra abelian factor

	c	vev	γ	Y	Additional $U(1)$	vevs that break additional $U(1)$
1	0	N_-	$+1$	$\frac{1}{2} Q_{B-L} + \frac{1}{2} Q_{3R}$	$Q_{\mathcal{H}}$	$\langle \eta \rangle$
2	$\frac{1}{2}$	N_+	-1	$\frac{1}{2} Q_{B-L} + \frac{1}{2} Q_{3R} + \frac{1}{2} Q_{\mathcal{H}}$	$\frac{1}{2} Q_{B-L} - \frac{1}{2} Q_{3R}$	$\langle h^-_R \rangle$

$U(1)_{\mathcal{H}}$-charge of the Higgs field $\overline{H}$ in the two cases is zero and -2, respectively. For $\gamma = +1$ (case 1 in Table 1) assuming a non-zero vev for N_-, the surviving abelian factors are of the form $\frac{1}{2}Q_{B-L} + \frac{1}{2}Q_{3R} + zQ_{\mathcal{H}}$ (where z is an arbitrary parameter). We are free to choose the hypercharge generator as traditionally (putting $z = 0$) and leave $\mathcal{H}$ as the surplus abelian factor. This is also dictated by the fact that the additional Higgs field η has the right charges to break completely $U(1)_{\mathcal{H}}$. The last breaking can in principle happen to a scale which can be lower than the PS breaking scale and can lead to a model with an additional $U(1)$ symmetry at low energies. For $\gamma = -1$ (case 2 in Table 1), provided N_+ develops a vev, the surviving abelian factors have the generic form $(2z - \frac{1}{2})Q_{B-L} + \frac{1}{2}Q_{3R} + zQ_{\mathcal{H}}$. Assuming vevs for h_R^-, the only unbroken combination left is $Y = \frac{1}{2}Q_{B-L} + \frac{1}{2}Q_{3R} + \frac{1}{2}Q_{\mathcal{H}}$. This is a novel hypercharge embedding which as discussed earlier does not affect the fermion and the electroweak Higgs charges. Again the extra $U(1)$ breaking scale is not necessarily associated with the M_R symmetry breaking scale but can be lower. The breaking of the additional $U(1)_{\mathcal{H}}$ symmetry implies the existence of a new Z'-boson. This is a very interesting prediction since recent analyses show compatibility of electroweak data with the existence of an additional gauge boson with mass of a few hundred GeV [25].

5. Gauge coupling running and the weak mixing angle

Our main aim in this section is to ensure that the above constructions imply the correct values for the weak mixing angle and the gauge couplings at M_Z. Moreover, it would be of particular interest if the present D-brane model is compatible with a low energy unification scale. The one loop renormalization group equations are of the form

$$\frac{1}{\alpha(\mu_2)} = \frac{1}{\alpha(\mu_1)} - \frac{b_i}{2\pi}\log\left(\frac{\mu_2}{\mu_1}\right)$$

and in our analysis we will assume two different energy regions $(\mu_1, \mu_2) = \{(M_Z, M_R), (M_R, M_U)\}$, where M_R is the $U(4) \times U(2)_R$ breaking scale and M_U the string scale. For simplicity, we will also assume that the additional $U(1)$ breaks at the same scale as the PS symmetry, that is $M_R = M_{Z'}$. The beta functions are b_3, b_2, b_Y for the first interval and $b_4, b_L, b_R, b_{\mathcal{H}}$ for the second in a self-explanatory notation. The matching conditions at M_R assuming properly normalized generators (all group generators $(T_a)$are normalized according to $\mathrm{tr}(T_a T_b) = \frac{1}{2}\delta_{ab}$), are

$$\frac{1}{\alpha_Y(M_R)} = \frac{2}{3}\frac{1}{\alpha_4(M_R)} + \frac{1}{\alpha_R(M_R)} + c^2\frac{1}{\alpha_{\mathcal{H}}(M_R)}$$

and

$$\alpha_3(M_R) = \alpha_4(M_R).$$

Moreover, at M_U we have

$$\frac{1}{\alpha_{\mathcal{H}}(M_U)} = \frac{8}{\alpha_4(M_U)} + \frac{4}{\alpha_R(M_U)} + \frac{4}{\alpha_L(M_U)}.$$

Solving the RGE system together with the matching conditions, we derive the formulae for the low energy quantities as functions of the brane couplings $(\alpha_4, \alpha_R, \alpha_L)$, the beta function coefficients and the scales M_U, M_R:

$$\begin{aligned}\sin^2\theta_W(M_Z) = {} & \frac{3}{8(1+6c^2)} \\ & \times\Bigg[1 + \frac{\alpha_{\mathrm{em}}(M_Z)}{6\pi} \\ & \quad\times\Bigg\{\Big(-2b_4 - 3c^2 b_H + \big(5+48c^2\big)b_L \\ & \qquad - 3b_R\Big)\ln\left(\frac{\mu}{M_R}\right) \\ & \qquad + \Big(\big(5+48c^2\big)b_2 - 3b_Y\Big)\log\left(\frac{M_R}{M_Z}\right) \\ & \qquad - 6\pi\left(\frac{2(1+12c^2)}{3\alpha_4} - \frac{5+36c^2}{3\alpha_L}\right. \\ & \qquad\quad \left.+ \frac{(1+4c^2)}{\alpha_R}\right)\Bigg\}\Bigg], \end{aligned} \tag{11}$$

$$\begin{aligned}\frac{1}{\alpha_3(M_Z)} = {} & \frac{3}{8(1+6c^2)} \\ & \times\Bigg[\frac{1}{\alpha_{\mathrm{em}}(M_Z)} - \frac{1}{2\pi}\Big(-2\big(1+8c^2\big)b_4 \\ & \quad + c^2 b_H + b_L + b_R\Big)\ln\left(\frac{\mu}{M_R}\right) \\ & \quad - \frac{1}{6\pi}\Big(3b_2 - 8\big(1+6c^2\big)b_3 + 3b_Y\Big)\ln\left(\frac{M_R}{M_Z}\right) \\ & \quad + \big(1+4c^2\big)\left(\frac{2}{\alpha_4} - \frac{1}{\alpha_L} - \frac{1}{\alpha_R}\right)\Bigg]. \end{aligned} \tag{12}$$

Assuming coincident brane stacks, $\alpha_4 = \alpha_R = \alpha_L$, as in the case of *grand unification*, the last term in both equations vanishes and we can calculate M_R and M_U using low energy data. As expected we obtain $M_R \sim 10^{12}$, $M_U \sim 10^{16}$ GeV for $c = 0$ (assuming minimal matter content). The choice $c = 1/2$ is not possible in this case, since it requires $M_R < M_Z$.

As already noted, since the various groups live in different brane-stacks, the initial values of the gauge couplings is not necessarily the same. It is thus tempting to explore this possibility in order to obtain low energy string/brane scale. However, in order not to loose predictive power we shall choose two of the three (brane) couplings to be equal. We call this scheme *petite unification*[3] as opposed to *grand unification* where all couplings are equal. Thus, in the present *petite unification* scenario we end up with three distinct cases, namely $\alpha_L = \alpha_R \neq \alpha_4$, $\alpha_L = \alpha_4 \neq \alpha_R$ and $\alpha_R = \alpha_4 \neq \alpha_L$.

For $\alpha_R = \alpha_4$ the string/brane scale M_U is given by

$$\log\frac{M_U}{M_Z} = \frac{B_1}{3B_2}\log\frac{M_R}{M_Z} + 2\pi\big[3\big(\big(1+4c^2\big)\sin^2\theta_W - 1\big)\alpha_3(M_Z) + \big(5+36c^2\big)\alpha_{\rm em}(M_Z)\big] \times \big[3B_2\alpha_{\rm em}(M_Z)\alpha_3(M_Z)\big]^{-1}, \tag{13}$$

where

$$B_1 = -5b_3 - 3c^2(4b_2 + 12b_3 - 12b_4 + b_H - 4b_L) + 3(b_4 - b_R + b_Y)$$

and $B_2 = -b_R + (1+12c^2)b_4 - c^2(b_H - 4b_L)$. The beta functions depend on the details of the model particle spectrum. Following the analysis of Section 5, we have two possibilities for the hypercharge embedding: (i) $c = 0$ where we assume that the number of extra singlets (η) is $n_1 > 0$ and the number of right-handed doublets (h_R) is $n_2 = 0$ and (ii) $c = 1/2$ where $n_1 = 0$ and $n_2 > 0$. Furthermore, motivated by the analysis of Section 6, with regard to the Higgs remnant triplet masses, we are going to consider two subcases for each embedding: $n_6 = 0, n_1 = 1$ or $n_6 = 1, n_1 = 1$ for the case (i) where n_6 is the number of sextets (D), and $n_6 = 0, n_2 = 1$ or $n_6 = 1, n_2 = 2$ for the case (ii). For the minimal scenario where we have three generations and only one PS breaking Higgs multiplet $n_H = 1$, substituting the beta functions we get

$$B_1 = 2n_6 - 32(1+c)^2(-1+2c) - \frac{n_2}{2} - c^2(n_1 + 2n_2)$$

and

$$B_2 = -\frac{n_2}{6} - \frac{23 + 4c^2(103+16c) - 2n_6 + n_h}{3} - \frac{c^2(n_1 + 2n_2 - 4n_h)}{3},$$

where n_h is the number of bi-doublets (h).

For the case (i) which corresponds to the standard Hypercharge embedding ($c = 0$) and assuming *petite unification* we can obtain various values for string scale M_U depending on the unification condition. These cases have been analyzed and the basic results are presented in Table 2. One easily concludes that in all cases $M_U \gtrsim 10^{10}$ GeV. This embedding is thus compatible with branes but not with low scale string scenarios.

For the case (ii) that is $c = 1/2$, we remark that $B_1 = 0$, $B_2 = -134/3$ for the first subcase (no sextets, one right-handed doublet) and $B_1 = -1$, $B_2 = -45$ for the second (one sextet, one right-handed doublet). Hence, the string scale M_U depends either very weakly on M_R or it does not depend at all (at the one loop). In addition, the string scale is independent of the number of bi-doublets (and thus electroweak doublets). This is actually a consequence of the combination of the PS symmetry with the new hypercharge embedding (9) considered here which allows us to obtain generic results for the string/brane scale. Substituting the electroweak data [28] and taking into account the strong coupling uncertainties we obtain the combined range (includes both subcases which differ slightly)

$$M_U = (5.1\text{–}6.5)\ \text{TeV}. \tag{14}$$

The non-coinciding brane coupling ratio depends slightly on M_R and n_h and lies in the range

$$\frac{\alpha_L}{\alpha_4} = 0.4\text{–}0.5 \tag{15}$$

for $M_Z < M_R < M_U$ and $n_h = 1$–3. The absolute coupling values are $\alpha_4 = \alpha_R \sim 0.07$, $\alpha_L \sim 0.03$ so we are safely in the perturbative regime. In addition, the low string/brane scale obtained in (14) is compatible with

[3] For the introduction of this term see [27].

Table 2
Limits on the brane scale M_U, the intermediate scale M_R and the independent coupling ratio for various petite unification conditions and the two hypercharge embeddings ($Y = \frac{1}{2} Q_{B-L} + \frac{1}{2} Q_{3R} + c Q_{\mathcal{H}}$). In the calculations we have taken into account the combined limits for two cases of minimal spectrum (we examine the cases $n_H = 1$, $n_6 = 0, 1$, $n_1 = 1$ for $c = 0$, and $n_6 = 0$, $n_2 = 1$ or $n_6 = 1$, $n_2 = 2$ for $c = 1/2$. In addition we took $n_h = 3$ whenever that results depend on n_h) and incorporated the strong coupling uncertainties

Case	Petite unification condition	M_U	M_R	Remaining coupling ratio
$c=0$	$\alpha_R = \alpha_L$	$> 2 \times 10^{12}$	$< 2 \times 10^{12}$	> 0.8
$c=0$	$\alpha_4 = \alpha_L$	$> 6.1 \times 10^{9}$	$> 10^{2}$	> 0.4
$c=0$	$\alpha_4 = \alpha_R$	$> 6.8 \times 10^{13}$	$< 6.8 \times 10^{13}$	> 0.8
$c=1/2$	$\alpha_R = \alpha_L$	< 11	< 11	< 0.15
$c=1/2$	$\alpha_4 = \alpha_L$	–	–	–
$c=1/2$	$\alpha_4 = \alpha_R$	5.1×10^3–6.5×10^3	10^2–6.5×10^3	0.4–0.5

current limits from four-fermion interactions [29]. We also notice that for $c = 1/2$, $\alpha_4 = \alpha_L$ is impossible (since at $\mu \sim 10^{10}$ GeV α_R develops negative values) while $\alpha_R = \alpha_L$ yields a unification scale of 7 GeV, which is obviously excluded. The above results are also summarized in Table 2.

It is interesting to observe that this alternative hypercharge embedding appears also in the framework of heterotic PS model [17] where it can account for the disappearance of fractionally charged particles. Of course in the heterotic context non-standard hypercharge embeddings are not useful for unification due to the tight heterotic coupling relations.

6. Proton stability, neutrino masses and all that

One of the most serious problems of SM extensions is proton decay. In traditional GUTs it can be suppressed due to the high unification scale. However, such a suppression is not possible in low string scale models, considered in the previous section. In general there are three modes for proton decay (i) the gauge-mediated proton decay (ii) the Higgs mediated and (iii) higher dimension baryon number violating operators. In the PS model in particular, the exotic $SU(4)$ gauge bosons $(\mathbf{3}, \mathbf{1}, +\frac{2}{3})$, $(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$ carry both baryonic and leptonic quantum numbers but they are known not to mediate proton decay, due to the absence of di-quark coupling [15,16]. These particles can only contribute to semi-leptonic processes, like β-decay which leads to the bound $M_R > g_4 \cdot 10^3$ GeV.

Higher dimension baryon number violating operators are expected to be present in any GUT model embedded in string theory. They are suppressed by a factor $1/M_U^{d-4}$, where d is the dimension of the relative operator [26]. In order to be safe with current proton decay limits, one has to prevent the appearance of such operators up to a dimension as high as $d \sim 18$. This suppression would look natural only in the case it could be associated with a symmetry (gauged or global).

The standard PS model contains B–L as a gauged symmetry, but this is not enough to avoid proton decay. Already at sixth order, B–L conserving operators (e.g., $QQQL$ originating from F_L^4) lead to baryon number violation. Furthermore, the spontaneous breaking of B–L leads to additional operators suppressed only by M_R/M_U. Fortunately, the current $U(4)_C \times U(2)_L \times U(2)_R$ extension of the PS model, incorporates the required $U(1)$ combination which corresponds to the baryon number itself. Indeed, as can be seen from (1), (2), $Q_C = 3B + L$ and thus

$$B = \frac{Q_C + Q_{B\text{–}L}}{4} \tag{16}$$

is a global symmetry of the theory (see discussion in the beginning of Section 3), which ensures the stability of the proton. Note that this symmetry survives the PS breaking as the ν_H^c has zero baryon number.

There are general no-go theorems [30] against the survival of global symmetries in the context of string theory at least at the perturbative level. They are expected to be violated since black holes can absorb charged particles but they cannot possess global charges themselves, due to “no-hair” theorems. However, there are arguments that in the context of Type I and Type IIB string vacua the assumptions of these no-go theorems can be evaded as the Fayet–Iliopoulos term associated with the anomalous $U(1)$ can be set to zero [22]. Moreover, global symmetries, as the baryon number, are expected to be violated due to non-perturbative phenomena (instantons). Of course, this violation is expected to be suppressed and may not be sufficient for standard baryogenesis scenarios. However, in the brane-world models we can use some alternative higher-dimensional mechanisms for the generation of baryon asymmetry [31].

Higgs mediated operators are inversely proportional to the Higgs remnant masses and could be dangerous for low string scale models. In the models discussed here the only Higgs light remnants are the triplets d^c_H. These triplets are assigned with baryon number under (16), thus, all their couplings with ordinary matter are baryon conserving. However, it is desirable that d^c_H triplet scalars receive masses (heavier than the proton) since if they stay light enough, proton can still decay to them (through baryon conserving processes). There are two possible scenarios for generating masses for these scalars. The first is to assume that the scalar potential — the details of which are not known since this is to be provided directly from string/brane theory — will eventually have some minimum which apart from symmetry breaking could also provide (M_R) masses for these scalars. The second is to introduce some extra scalar particles, namely the triplets originating from the sextet D (see (4)) which will mix with d^c_H and thus provide masses for them. We may assume a scalar potential of the form

$$\begin{aligned}\mathcal{V} &= \rho^2 DD^\dagger + \lambda \overline{H}\,\overline{H} D + \text{c.c.}\\ &= \rho^2\big(\tilde{d}^c\tilde{d}^{c\dagger} + \tilde{d}\tilde{d}^\dagger\big)\\ &\quad + \lambda\big(d^c_H\tilde{d}\langle\nu^c_H\rangle + d^{c\dagger}_H\tilde{d}^\dagger\langle\nu^c_H\rangle^\dagger\big) + \cdots,\end{aligned} \tag{17}$$

where ρ and λ are appropriate combinations of vevs. In the case $c = 1/2$ and in the lowest order, $\rho^2 = \langle HH^\dagger\rangle \sim M_R^2$, while $\lambda = \langle h_{Ri}h_{Rj}\rangle/M_U \sim M^2_{Z'}/M_R$. Note that due to $SU(2)$ antisymmetry at least two different h_R fields are necessary in order to obtain a non-vanishing coupling. This superpotential provides triplet masses of the order $m_1 \sim M_Z^2/M_R$, $m_{2,3} \sim M_R$. In the case $c = 0$ one can assume similarly $\rho^2 = \langle HH^\dagger\rangle$ and $\lambda = \langle\eta\rangle$.

Baryon number is not the only global symmetry left from the anomalous $U(1)$ breaking. As easily seen by the particle assignments (1)–(3) the lepton number corresponds to the combination

$$\mathcal{L} = \frac{Q_C - 3Q_{B-L}}{4}. \tag{18}$$

In the case of the baryon number all Higgs fields are neutral under it and the symmetry remains exact at the perturbative theory level. On the contrary the ν^c_H has lepton number (although h, h_R, η are neutral) and it will thus break $\mathcal{L}$ spontaneously and give rise to a massless Goldstone boson. One possible solution to this problem is discussed in [9] where a deviation from the orientifold point (along a direction that conserves baryon number) is considered. Furthermore, one may note that the correct lepton number for all fermions and electroweak Higgs fields is reproduced by a more general formula $\mathcal{L}' = kQ_C - \frac{3}{4}Q_{B-L} + \big(k - \frac{1}{4}\big)(Q_R - Q_L)$ where k is an arbitrary number, and (18) corresponds to the particular case $k = 1/4$. This alternative definition preserves the fermion charges but can give different PS Higgs charges. In the case $c = 1/2$ and choosing $k = 0$ we have

$$\mathcal{L}' = -\frac{3}{4}Q_{B-L} - \frac{1}{4}Q_R + \frac{1}{4}Q_L, \tag{19}$$

which renders ν^c_H neutral. Thus, lepton number is not broken at the level of PS symmetry (M_R), but at the $M_{Z'}$ scale as the right-handed doublets (h_R), utilized in this case for the additional $U(1)$ breaking, are charged. This leads to the interesting possibility that the lepton number breaking is associated to the breaking of an additional abelian symmetry.

Apart from the low energy values of the Weinberg angle and the strong coupling, a consistent string model is also expected to reproduce the low energy fermion mass pattern. The PS symmetry implies unification of all Yukawa couplings. Thus for the heaviest generation, which is expected to receive mass at tree-level, we have $m_\tau = m_b$ at the brane scale. In an ordinary GUT, the observed low energy difference of the two running masses is attributed to the $SU(3)$-

contributions in m_b. In low energy unified models the range M_U–M_Z is too short to account for the $m_b - m_\tau$ difference, however, the required enhancement can be anticipated by the ratio of the gauge couplings given in (15). In addition, the rest of the fermion masses and mixings are expected to be easily reproduced due to the potential presence of extra Higgs doublets (which as shown above do not affect the string scale) and generation mixing.

For neutrino masses in particular, recent experimental explorations have shown that it is likely that a crucial role is played by the right-handed neutrino ν^c which is absent in the SM. In most extensions of the SM theory, ν^c receives a large mass of the order of the unification scale. Then, the see-saw mechanism is used to generate a tiny mass for the left-handed neutrino, which is compatible with experimental and astrophysical limits. In the context of a D-brane approach to SM one has to assume that ν^c will possibly arise as a gauged neutral fermion propagating in the bulk and explain the light neutrino mass by the smallness of the brane-bulk couplings, naturally suppressed by the bulk volume [12–14]. On the contrary, one important feature of the PS extension of the SM model (and left–right models in general), is that the right-handed neutrino lives on the brane as any other fermion of the SM. In addition, a Dirac neutrino mass term $L\nu^c\langle H_u\rangle$ is generated by the coupling $F_L\overline{F}_R h$ which cannot be forbidden as it also generates masses for all the SM fermions. A Majorana mass is also possible from an effective term $\kappa F_R F_R$ where κ an appropriate vev combination. These terms lead to the neutrino mass matrix

$$m_\nu = \begin{array}{c c} & \begin{array}{cc} \nu & \nu^c \end{array} \\ \begin{array}{c} \nu \\ \nu^c \end{array} & \begin{pmatrix} 0 & \langle H_u\rangle \\ \langle H_u\rangle & \kappa \end{pmatrix} \end{array} \tag{20}$$

with eigenvalues $m_{\text{light}} \sim \frac{\langle H_u\rangle^2}{\kappa}$, $m_{\text{heavy}} \sim \kappa$ assuming $\kappa > \langle H_u\rangle$. For the $c = 0$ model the simplest choice is $\kappa = \langle H^\dagger H^\dagger\rangle/M_U = M_R^2/M_U$ which gives adequately suppressed neutrino masses for $M_R \lesssim M_U \sim 10^{10}$ (see Table 2). For the $c = 1/2$ model $\kappa = \langle H^\dagger H^\dagger h_R h_R\rangle/M_U^3$ requires $M_U > 10^8$ in order to suppress enough the left-handed Majorana neutrino masses at an experimentally acceptable range. Hence, in this case a different mass generation mechanism must be employed. A possible solution applicable in general left–right symmetric model has been presented in [13]. The main idea is to consider a bulk right-handed neutrino that mixes only with the brane right-handed neutrino. An additional possibility would be to consider masses for the bulk neutrinos along the lines proposed in [14], as well as potentially unsuppressed gravitational matter interactions [32], and utilize a generalized see-saw mechanism (including the Kaluza–Klein excitations of bulk neutrinos) to reconcile the experimentally acceptable neutrino masses with a low string scale.

7. Conclusions

In this Letter we have explored a generic Pati–Salam like model based on an $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry, compatible with a D-brane configuration. We have found two consistent models one with the standard and one with an alternative hypercharge embedding. The former is compatible with the low energy data for an intermediate string scale of the order of 10^{10} GeV, while the later is shown to be compatible with the electroweak data for a string scale of the order of 5–7 TeV provided that the $U(4)_C$ and $U(2)_R$ brane sets have equal couplings ($\alpha_4 = \alpha_R$) while the $U(2)_L$ coupling is about a half of this value ($\alpha_L \sim \alpha_4/2$).

Both scenarios contain an extra abelian factor which can break at an acceptable scale by vevs of appropriate scalar fields incorporated in the models. In the low string scale case we have identified lepton number with a global symmetry of the theory whose breaking is associated with the breaking of the additional abelian factor.

Proton stability is assured, as an anomalous combination of the surplus abelian factors of the original gauge group is identified with the baryon number. This combination is to be broken by a generalized Green–Schwarz mechanism at the string level leaving behind baryon number as an exact global symmetry.

The right-handed neutrino is part of the non-trivial fermionic representations of the theory, while there can exist mechanisms which make the left-handed Majorana mass compatible with recent data. More particularly, in the case of intermediate string scale the lightness of the neutrino can be guaranteed by a see-saw mechanism at the brane level while in the case of a low energy string scale a generalized see-saw mechanism incorporating bulk sterile neutrinos and possibly bulk masses is required.

It would be interesting if the model presented here, and especially the variation with low string/brane scale, could find a direct realization in the context of Type I constructions [33].

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