A POSSIBLE SOLUTION TO THE PROBLEM OF COSMIC DOMAIN WALLS

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In a grand unified model, with radiatively induced symmetry breaking, unwanted discrete symmetries leading to cosmic domains could be dynamically broken, before the phase transition is completed.

Some spontaneously broken gauge theories can give rise to vacuum structures such as monopoles, strings and domains [1]. Grand unified theories [2] describe the evolution of the early Universe as a succession of phase transitions, during which the occurrence of such cosmic singularities is possible. Their formation is determined by the degeneracies of the vacuum manifold. These objects can be stable and lead to observable effects.

Cosmological monopole production has been discussed by many authors [3], who estimated that the number of monopoles is too large to be compatible with the observational limits. Possible resolutions of this monopole problem have been suggested [3,4]. Cosmic strings have also been discussed [1,5], in particular their possible relevance to the problem of galaxy formation. However, the problem of galaxy formation certainly is much more complicated, apparently requiring isothermal, rather than adiabatic, perturbations [6], which could have arisen from inhomogeneities (e.g. shear [7]). These could be of a quantum gravitational origin.

In this letter, we concentrate on the formation of domain walls and their subsequent evolution in the simplest grand unified model [SU(5)] [8], with radiatively induced (Coleman–Weinberg [9]) type of symmetry breaking. A domain wall with the size of a horizon would have a mass exceeding by many orders of magnitude the estimated mass of the Universe [1]. Thus, the domain walls are irreconcilable with the present state of the Universe [1]. Their occurrence in certain grand unified models renders these models inapplicable, unless they are accompanied by a mechanism for the disappearance of the walls.

Consider a theory with internal group G. The vacuum expectation value of the Higgs field lies on some orbit of G. If G_0 is the subgroup of transformations that leaves the vacuum expectation value unchanged, i.e. the subgroup of unbroken symmetries, the possible domain structures are fixed by the topology of the coset space G/G_0. If it is not connected, domain walls will be necessarily generated.

To be specific, let us consider SU(5). The minimal way to break SU(5) down to SU(3) × SU(2) × U(1) is realized with one real adjoint Higgs field Φ. Φ develops a non-vanishing vacuum expectation value in the direction

$$\langle\Phi\rangle = \phi \, \mathrm{diag}[1, 1, 1, -\tfrac{3}{2}, -\tfrac{3}{2}] \qquad (1)$$

up to a global SU(5) transformation. The effective potential is a function of the SU(5) invariants

$$\mathrm{Tr}(\Phi^2) , \quad \mathrm{Tr}(\Phi^4) , \quad \mathrm{Tr}(\Phi^3) .$$

If we impose the discrete symmetry $\Phi \to -\Phi$, the cubic term will not be present. For example, in the case that we allow only for dimensionless couplings (Coleman–Weinberg) no cubic super-renormalizable coupling is present and the discrete symmetry is forced upon us. Let us now look closer at the phase transition in the particular case that we have the above reflection symmetry. The nature and speed of the phase transition, of course, will depend on the presence and magnitude of a tree negative mass term. However, in general it will be of first order and will proceed by the formation of bubbles. The Higgs fields inside the bubbles will have non-zero vacuum expectation value and will point to a direction fixed by (1) up to a global gauge transformation. Since no global gauge transformation can take Φ to $-\Phi$, some of the bubbles will have Φ pointing in one direction and the rest in the opposite direction. When the transition will have completed itself, we shall be left with domains of the asymmetric phase separated by walls of the normal phase. All the above is of course, oversimplified but the essential point is that there will be two distinct but similar ordered phases. This is a direct consequence of the fact that the vacuum manifold, in the above example, had two connected components.

In the conjectured superunification scheme [10], $N = 8$ supergravity, valid near the Planck mass, gives rise to a composite SU(8) symmetry [11], which breaks directly to SU(5) [‡1]. Virtually nothing is known about the super GUT dynamics. Nevertheless, we shall make the assumption that the resulting SU(5) theory is scale invariant at the classical level. This means in particular two things. First, that the potential is endowed with the discrete symmetry $\Phi \to -\Phi$ and this, as we saw, leads to domains at least for the minimal Higgs sector [12]. Second, that fermions different from the ordinary ones could have survived massless down to the grand unification mass scale. Then, if we take a 5 and a $\bar{5}$ of these fermions [‡2], coupled to the adjoint 24 Higgs according to

$$-G_Y \bar{\psi}_i \Phi_{ij} \psi_j \,, \tag{2}$$

the lagrangian has the discrete symmetry

$$\Phi \to -\Phi \,, \quad \psi_i \to \gamma_5 \psi_i \,. \tag{3}$$

The one-loop effective potential, including the fermion contribution, is [‡3]

$$V(\Phi) = A\phi^4 [\ln(\phi^2/\langle\phi\rangle^2) - \tfrac{1}{2}] \tag{4}$$

and has the discrete symmetry $\phi \to -\phi$.

Higher orders in perturbation theory will respect this symmetry and so will the temperature corrections. The vacuum manifold of this theory will have two connected components (and therefore will lead to domains) unless the discrete symmetry is broken dynamically before the breaking of SU(5).

Let us focus now on the phase transition. The temperature corrections [13] to the potential will induce at high temperature a term approximately

$$\tfrac{5}{8} g^2 T^2 \,\mathrm{Tr}(\Phi^2) \,. \tag{5}$$

This term has the well-known effect of stabilizing the wrong (symmetric) vacuum and of delaying the phase transition. If a large negative mass term were present at tree level, the phase transition would approximately go below a temperature T_c at which the two terms would cancel. In such a case, since the perturbative potential has the discrete symmetry $\Phi \to -\Phi$, two degenerate asymmetric vacua would exist and therefore domains with the Higgs vacuum expectation value pointing in opposite directions would form. In the case of a strictly Coleman–Weinberg potential, however, such as the one under consideration, the transition proceeds very slowly through tunnelling and the system cools to lower and lower temperatures with most of it still in the symmetric phase [14]. As the system supercools, the running gauge coupling

$$\frac{g^2(T)}{4\pi} \sim \frac{2\pi}{b \ln(T/\Lambda)} \tag{6}$$

becomes stronger and stronger, since the theory is asymptotically free [‡4]. Due to the very slow tunnelling process, the phase transformation will not have been completed even at temperatures $\approx 10^7$ GeV. Below these temperatures, we start entering the strong coupling regime where new phenomena, non-perturbative in nature, start becoming important.

SU(5) instantons, for instance, mediate a determinantal interaction between fermions, as in QCD, which can give a non-vanishing expectation value to the SU(5) singlet $\bar{\psi}_i \psi_i$. The instanton induced fermion mass is, of course, zero at very high energies (temperatures) but eventually grows as we approach the infrared region. A typical instanton induced fermion mass has to be (very roughly)

$$m(T) \sim T c_5 [2\pi/\alpha(T)]^{10} \exp[-2\pi/\alpha(T)] \,, \tag{7}$$

where $c_5 \approx 7.5 \times 10^{-4}$ [15].

It is immediately evident that the discrete symmetry $\Phi \to -\Phi$ will be broken. Cubic terms like

$$G_Y^3 m(T) \,\mathrm{Tr}(\Phi^3) \tag{8}$$

‡1 One could also conceive other ways of enlarging the SU(5) theory.

‡2 The asymptotic freedom of the theory is unspoiled.

‡3 $A = (3/64\pi^2)(\frac{25}{8})^2 g^4 - (1/64\pi^2)(\frac{105}{8}) G_Y^4$.

‡4 The scale parameter Λ is determined by the requirement that $g^2(M_X)/4\pi \approx 1/42$. Thus, $\Lambda \approx 10^5 - 10^6$ GeV for the above discussed theory ($b \approx 12$).

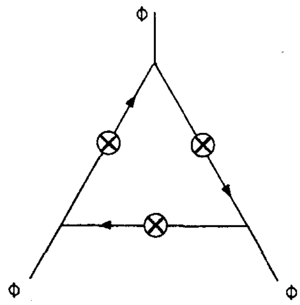

Fig. 1.

violating this symmetry will be dynamically generated (see fig. 1). No such terms could be generated in perturbation theory unless we had massive fermions to begin with, in which case we would have no discrete symmetry. The singlet $F^{\alpha}_{\mu\nu}F^{\alpha}_{\mu\nu}$ gets also a non-vanishing vacuum expectation value with important consequences on the phase transition, as we have studied elsewhere [16]. When the phase transition is finally completed near $T \sim O(1)\,\Lambda$ the fermions get masses of order M_X due to their direct couplings to Φ.

Let us examine now the cosmological consequences of the dynamical breaking of the unwanted discrete symmetry, as far as domains are concerned. A thin wall separating two domains will have surface energy density (roughly) [17]

$$\sigma \simeq \frac{(\text{typical mass})^3}{(\text{typical coupling})^2} \simeq g\langle\phi\rangle^3 \,. \tag{9}$$

The time at which the wall would collapse is [17]

$$t_c \sim 1/G\sigma = M_P/\sigma \,. \tag{10}$$

M_P is the Planck mass. The domains have to disappear at times t_1 smaller than their collapse time t_c, i.e.

$$t_1 < t_c \,. \tag{11}$$

Since

$$t_1/t_c \sim G\sigma t_1 \sim \sigma t_1^{-1}/(Gt_1^2)^{-1} = \frac{\text{energy density of walls}}{\text{energy density of the Universe}}\,, \tag{12}$$

the previous inequality is equivalent to demanding that the Universe never becomes domain-wall dominated. Now, if we have a small initial bias $\epsilon(t)$ between the vacua (as the cubic term induced by instantons), the walls will disappear when the energy difference $\epsilon(t)$ becomes dynamically important, i.e. it becomes comparable to the energy density of the walls,

$$\epsilon(t_1)\,t_1 \sim \sigma t_1^{-2} \,. \tag{13}$$

In our specific example of SU(5)

$$\epsilon(T) \sim G_Y^3\, m(T)\langle\phi\rangle^3 \,. \tag{14}$$

Then, inequality (11) implies

$$\epsilon(T) > G\sigma^2 \,. \tag{15}$$

Using the instanton induced mass as a typical non-perturbative contribution, we get, assuming $G_Y \sim O(g)$

$$Tc_5[2\pi/\alpha(T)]^{10}\alpha^2(T)\exp[-2\pi/\alpha(T)] \gtrsim M_X^3/M_P^2 \,. \tag{16}$$

This can be satisfied at temperatures $T_1 \sim O(1)\,\Lambda$.

In the above example, an energy difference between the two degenerate vacua was developed dynamically during the period of supercooling and thus the formation of domain walls was avoided. The fact that we considered SU(5) is not essential. Similar phenomena could be seen in other models. The case of SU(2) × U(1) phase transition with a Coleman–Weinberg potential contains features similar to our example, with the important difference that the quark vacuum expectation value that breaks the discrete symmetry is not a singlet and (happily) breaks the gauge group also at Λ_{QCD} [18], long before we enter the SU(2) infrared regime.

Although a lot of handwaving is inevitably involved in discussing the little known dynamics of first-order phase transitions, our example serves to show how unwanted discrete symmetries and their consequences could be avoided by the theory itself.

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References

[1] T.W.B. Kibble, J. Phys. A9 (1976) 1387; Phys. Rep. 67 (1980) 183, and references therein.
[2] For a review, see J. Ellis, in: Gauge theories and experiments at high energy, eds. K.C. Bowler and D.G. Sutherland (1980);
D.V. Nanopoulos, CERN preprint TH.2981 (1980);
P. Langacker, SLAC-PUB 2544 (1980), to be published in Phys. Rep.

[3] Ya.B. Zel'dovich and M.Y. Khlopov, Phys. Lett. 79B (1979) 239;
J.P. Preskill, Phys. Rev. Lett. 43 (1979) 1365;
M.B. Einhorn, D.L. Stein and D. Toussaint, Phys. Rev. D21 (1980) 3295.
[4] A.M. Guth and S.-H. Tye, Phys. Rev. Lett. 44 (1980) 631;
F. Bais and S. Rudaz, Nucl. Phys. B170 (1980) 507;
P. Langacker and S.-Y. Pi, Phys. Rev. Lett. 45 (1980) 1;
A.D. Linde, Phys. Lett. 96B (1980) 293;
A. Kennedy, G. Lazarides and Q. Shafi, Phys. Lett. 99B (1981) 38;
A. Guth and E. Weinberg, Phys. Rev. D23 (1981) 876;
M.B. Einhorn and K. Sato, Nucl. Phys. B180 (1981) 385.
[5] Ya.B. Zel'dovich, Mon. Not. R. Astron. Soc. 192 (1980) 663;
A. Vilenkin, Phys. Rev. Lett. 46 (1981) 1169; Phys. Rev. D, to be published;
A.E. Everett, SLAC-PUB-2697 (1981);
G. Lazarides, Q. Shafi and T.F. Walsh, CERN preprint TH.3118 (1981).
[6] For a review see: J. Silk, The cosmic microwave background radiation, U.C. Berkeley preprint (1980), presented at the Tenth Texas Symp. Relativistic astrophysics (Baltimore, 1980);
P.J.E. Peebles, The large-scale structure of the Universe (Princeton U.P., Princeton, 1980).
[7] J.R. Bond, E.W. Kolb and J. Silk, The generation of isothermal perturbations in the very early Universe, U.C. Berkeley preprint (1981).
[8] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
[9] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.
[10] J. Ellis, M.K. Gaillard, L. Maiani and B. Zumino, in: Unification of the fundamental particle interactions, eds. S. Ferrara, J. Ellis and P. van Nieuwenhuizen (Plenum, New York, 1980) p. 69;
J. Ellis, M.K. Gaillard and B. Zumino, Phys. Lett. 94B (1980) 343;
J.-P. Derendiger, S. Ferrara and C.A. Savoy, CERN preprint TH.3025 (1981).
[11] E. Cremmer and B. Julia, Phys. Lett. 80B (1978) 48.
[12] The associated domain walls problem has also been noticed by A. Guth and S.-Y. Pi, A. Guth, private communication.
[13] L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320;
S. Weinberg, Phys. Rev. D9 (1974) 3357;
A.D. Linde, Rep. Prog. Phys. 42 (1979) 389.
[14] L.F. Abbott, Nucl. Phys. B185 (1981) 233;
A. Billoire and K. Tamvakis, CERN preprint TH.3019 (1981), to be published;
G.P. Cook and K.T. Mahanthappa, Phys. Rev. D23 (1981) 1321;
M.A. Sher, UCSC preprint 81/139 (1981), to be published.
[15] D.J. Gross, R.D. Pisarski and L.G. Jaffe, Rev. Mod. Phys. 53 (1981) 43, and references therein.
[16] K. Tamvakis and C.E. Vayonakis, CERN preprint TH.3108 (1981).
[17] A. Vilenkin, Phys. Rev. D23 (1981) 852.
[18] E. Witten, Nucl. Phys. B177 (1981) 477.