

# Attempts at Explaining the NuTeV Observation of Di-Muon Events

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## Abstract

The NuTeV Collaboration has observed an excess in their di-muon channel, possibly corresponding to a long-lived neutral particle with only weak interactions and which decays to muon pairs. We show that this can *not* be explained by pair production of neutralinos in the target followed by their decay far downstream in the detector via a  $LLE$  R-parity violating (RPV) operator, as suggested in the literature. In the parameter region allowed by LEP the event rate is far too small. We propose instead a new neutralino production method via  $B$ -mesons, which can fully explain the observation. This is analogous to neutrino production via  $\pi$ -mesons. This model can be completely tested and thus also possibly excluded with NOMAD data. If it is excluded, the NuTeV observation is most likely not due to physics beyond the Standard Model. Our model can also be tested at the current and future  $B$ -factories. This opens up a new way of testing a long-lived neutralino LSP at fixed-target experiments and thus a possibility to close the gap between collider and cosmological tests of R-parity violation. We also discuss a possible explanation in terms of a neutral heavy lepton mixing with the Standard Model neutrinos. The flavour structure of the observation can be accounted for but the production rate is far too low.

## 1 Introduction

In supersymmetry [1] with broken R-parity [2, 3] the MSSM superpotential is extended by

$$W_{\mathcal{R}p} = \lambda_{ijk}\epsilon_{ab}L_i^a L_j^b \bar{E}_k + \lambda'_{ijk}\epsilon_{ab}L_i^a Q_j^b \bar{D}_k + \lambda''_{ijk}\epsilon_{\alpha\beta\gamma}\bar{U}_i^\alpha \bar{D}_j^\beta \bar{D}_k^\gamma + \kappa_i\epsilon_{ab}L_i^a H_u^b. \quad (1)$$

Here  $L, Q$  ( $\bar{E}, \bar{U}, \bar{D}$ ) are the lepton and quark doublet (singlet) left-handed chiral superfields, respectively.  $\lambda, \lambda', \lambda''$  are dimensionless coupling constants,  $i, j, k = 1, 2, 3$  are generation indices.  $a, b = 1, 2$  and  $\alpha, \beta, \gamma = 1, 2, 3$  are  $SU(2)_L$  and  $SU(3)_c$  gauge indices, respectively. The main phenomenological changes to the MSSM are that the lightest supersymmetric particle (LSP) is no longer stable and supersymmetric particles can be

produced singly at colliders. Through resonance production the couplings  $(\lambda, \lambda', \lambda'')$  can be probed down to about  $10^{-3}$  before the production cross section becomes too small [4–8]. If we consider MSSM supersymmetric pair production with a neutralino LSP then we can typically probe couplings down to  $10^{-5}$  or  $10^{-6}$  [9–12]. For smaller couplings the LSP decays outside the detector and we retrieve the MSSM signatures at colliders. Cosmologically one can exclude lifetimes for the LSP between  $1\text{ s} < \tau_{\chi_1^0} < 10^{17}\text{ yr}$  [13], which corresponds to couplings  $10^{-22} < (\lambda, \lambda', \lambda'') < 10^{-10}$ . This leaves a gap in experimental sensitivity to the R-parity violating couplings<sup>1</sup>  $10^{-10} < (\lambda, \lambda', \lambda'') < 10^{-6}$  [2]. Fixed-target experiments with remote detectors can probe significantly longer lifetimes than collider experiments and are thus an ideal environment for closing this gap in sensitivity [14].

The NuTeV Collaboration has searched for long-lived neutral particles ( $N^0$ ) with mass  $M_{N^0} \geq 2.2\text{ GeV}$  and small interaction rates with ordinary matter [15–17]. They look for the decay of the neutral particles in a detector which is 1.4 km downstream from the production point. They observe 3  $\mu\mu$  events where they only expect to see a background of  $0.069 \pm 0.010$  events. The probability that this is a fluctuation of this specific channel is about  $8 \cdot 10^{-5}$ , which corresponds to about  $4.6\sigma$ . The probability for a fluctuation of this magnitude into any of the di-lepton channels is about  $3\sigma$ .

The NuTeV experiment considered in detail the possibility that this discrepancy is due to a  $N^0$  which decays into a three-body final state. In Ref. [16] several kinematic distributions of the di-muon events were checked against the hypothesis of a  $N^0$  with mass 5 GeV: the transverse mass-, invariant di-muon mass- and the missing  $p_T$ -distributions all agree well with the  $N^0$  hypothesis. The distribution in the energy asymmetry  $A_E \equiv |E_1 - E_2|/(E_1 + E_2)$  of the three events ( $E_1$  and  $E_2$  are the two observed muon energies in each event) shows a low probability for the  $N^0$  hypothesis. Thus three out of four distributions work very well and as NuTeV does, we consider it worthwhile to investigate whether this observation could be due to new physics. It is the purpose of this letter to consider two possible models which could explain the observation: (i) a light neutralino which decays via R-parity violation and (ii) a neutral heavy lepton (NHL) mixing with the Standard Model neutrinos.

A search for R-parity violating neutralino decays at NuTeV has been proposed in Ref. [14] and the couplings  $\lambda_{122}$  and  $\lambda_{133}$  were discussed. In Ref. [16,17] NuTeV themselves mention the possibility of R-parity violating neutralino decays as a solution to the observed discrepancy, without looking at any specific couplings. In Ref. [15] NuTeV searched for the neutralino of a very specific model [18]. This neutralino was very light and decayed via  $L_1 L_2 \bar{E}_1$  or  $L_1 L_3 \bar{E}_1$  to an  $ee$  final state. Certain supersymmetric parameter ranges were excluded assuming neutralino pair-production.

Here we show that the simple scenarios discussed in the literature can not lead to an excess at NuTeV, since the decisive supersymmetric parameter range to get a significant neutralino production cross section has been excluded by LEP. We propose instead the production of light neutralinos via  $B$ -mesons which could give a measurable excess. We briefly present the two possible models and then discuss them quantitatively.

In section 5 we show that the production rate for neutral heavy leptons is also too low and does not lead to a viable explanation.

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<sup>1</sup>These coupling values have been determined for a photino LSP of  $M_{\chi_1^0} = \mathcal{O}(50)\text{ GeV}$  and scalar fermion masses of  $M_{\tilde{f}} = \mathcal{O}(100\text{ GeV})$ .

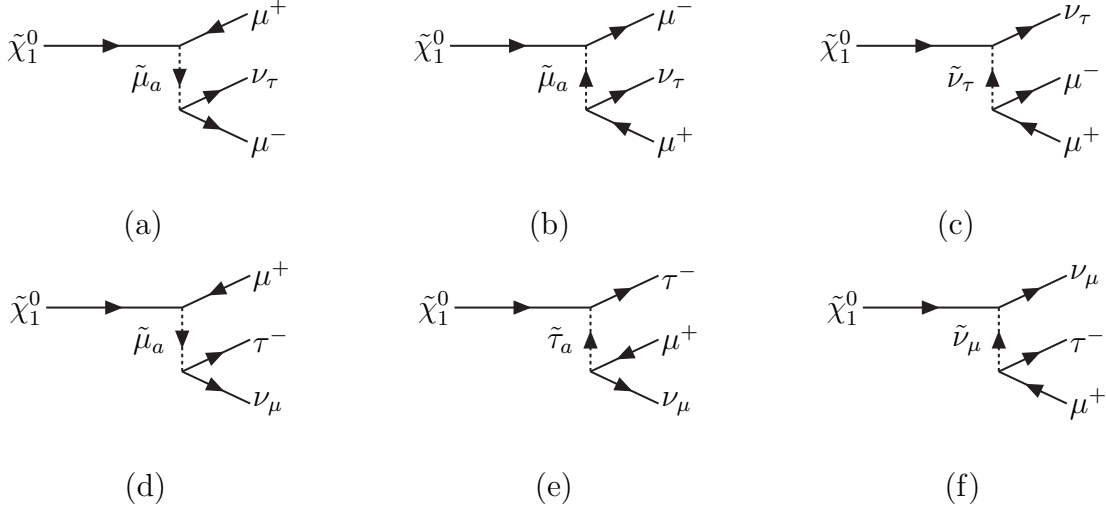


Figure 1: Neutralino decays through the R-parity violating coupling  $\lambda_{232}$ . Diagrams (a-c) give rise to di-muon events while the diagrams (d-f) to tau-muon ones. The index  $a=1,2$  denotes the mass eigenstate of the slepton.

## 2 The $R_p$ Violating Model

The heavy neutral particle we consider is the lightest neutralino  $\chi_1^0$ , which we also assume to be the LSP. In the notation of [9], the neutralino decays as  $\chi_1^0 \rightarrow \mathcal{O}_{R_p}$ , where  $\mathcal{O}_{R_p}$  is the dominant R-parity violating operator. Only two operators give a di-muon signature:  $\lambda_{2i2}\epsilon_{ab}L_2^aL_i^b\bar{E}_2$ ,  $i = 1, 3$ . For  $i = 1$  the neutralino will decay with equal probability to  $e\mu\nu$  and  $\mu\mu\nu$ . No  $e\mu\nu$ -events are observed, we therefore propose one dominant R-parity violating operator:

$$\mathcal{O}_{R_p} = \epsilon_{ab}\lambda_{232}L_\mu^aL_\tau^b\bar{E}_\mu. \quad (2)$$

For later reference we quote the experimental bound on this operator [19]

$$\lambda_{232} < 0.070 \left( \frac{m_{\tilde{\mu}_R}}{100 \text{ GeV}} \right), \quad (2\sigma). \quad (3)$$

The operator in Eq.(2) corresponds to the two neutralino decay modes (Fig.1)

$$\chi_1^0 \rightarrow \begin{cases} \mu_L^- \mu_R^+ \nu_\tau, \\ \tau_L^- \mu_R^+ \nu_\mu, \end{cases} \quad (4)$$

as well as their complex conjugate, since the neutralino is a Majorana spinor. We shall show below that for a light neutralino the  $\tau\mu$  decays are sufficiently phase space suppressed to give an expectation below one event. For the light neutralino production we shall consider two possibilities:

1. Pair production of the neutralinos [20] which proceeds via (a) s-channel  $Z^0$  boson exchange and (b) t-channel squark exchange.
2. Single neutralino production in the decay of bottom hadrons. The bottom hadrons are formed following the production of a  $b\bar{b}$  pair. These hadrons can then decay via the R-parity couplings  $\lambda'_{i13}$ , ( $i = 1, 2, 3$ ). We will only consider the decays of the  $B_d^0$  and  $B^+$  via R-parity violation (Fig.2)

$$B_d^0 \longrightarrow \bar{\nu}_i \tilde{\chi}_1^0, \quad (5)$$

$$B^+ \longrightarrow \ell_i^+ \tilde{\chi}_1^0. \quad (6)$$

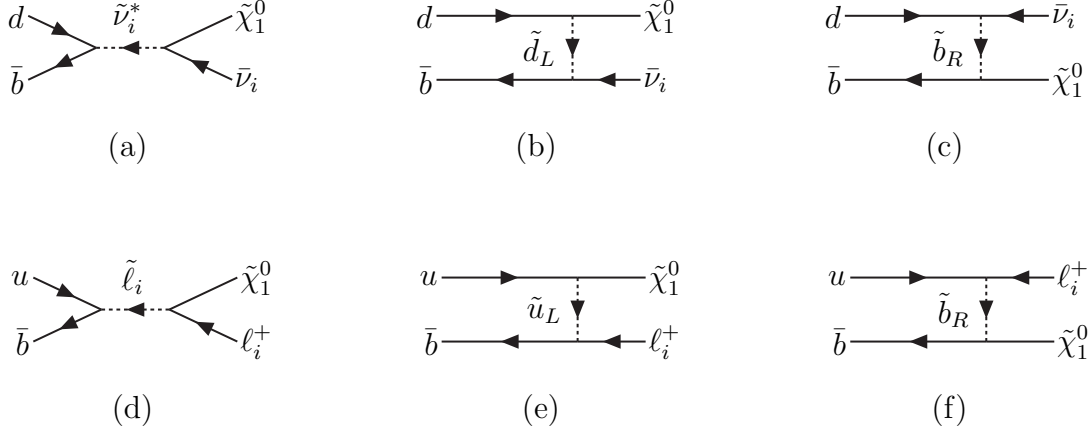


Figure 2: Neutralino production in  $B$ -meson decays : (a-c)  $B_d^0 \longrightarrow \bar{\nu}_i \tilde{\chi}_1^0$ , and (d-f)  $B^+ \longrightarrow \ell_i^+ \tilde{\chi}_1^0$ .

This mechanism allows one to produce light neutralinos via a strong interaction process and is analogous to the production of neutrino beams via  $\pi$ 's and  $K$ 's (and  $D$ 's). A related mechanism was discussed in the context of the Karmen time anomaly [18, 21].

For later reference we present the experimental bounds on the  $\lambda'_{i13}$  at  $2\sigma$  [3, 19]

$$\lambda'_{113} < 0.021 \frac{m_{\tilde{b}_R}}{100 \text{ GeV}}, \quad \lambda'_{213} < 0.059 \frac{m_{\tilde{b}_R}}{100 \text{ GeV}}, \quad \lambda'_{313} < 0.11 \frac{m_{\tilde{b}_R}}{100 \text{ GeV}}. \quad (7)$$

### 3 Quantitative Analysis

As discussed by the NuTeV experiment, the mass of the  $N^0$  is roughly 5 GeV. The constraints on a very light neutralino were discussed in detail in Ref. [18]. We expect them to mainly carry over to the present mass region [22]. In order to get a  $M_{\tilde{\chi}_1^0} = \mathcal{O}(5 \text{ GeV})$  neutralino and avoid the LEP bounds we must consider the case, where the electroweak gaugino masses  $M_1, M_2$  are independent parameters. In Fig. 3 we show the MSSM parameter space which corresponds to  $M_{\tilde{\chi}_1^0} = (5 \pm 0.5) \text{ GeV}$  for two values of  $\tan\beta$  and  $\text{sgn}\mu$ . The composition of the neutralino is more than 99% bino, provided the lightest chargino mass is greater than 100 GeV.

The dominant bino-nature of the LSP has immediate implications for pair production of neutralinos. The bino does not couple to the  $Z^0$  boson and thus the s-channel pair-production of the bino is negligible. This only leaves the t-channel production which is proportional to  $M_{\tilde{q}}^{-4}$  and thus strongly suppressed. We shall quantify this below.

In both cases neutralino production is followed by the decay. The matrix elements for the decay via  $\mathcal{R}_p$  were given in [11, 23]. As the neutralino in our model will be much lighter than the sleptons ( $M_{\tilde{\ell}} \gtrsim 90 \text{ GeV}$  from LEP) it is sufficient to neglect the momentum flow through the slepton propagators. For a purely bino neutralino in this limit the spin averaged matrix element is given by

$$|\overline{\mathcal{M}}|^2(\tilde{\chi}_1^0 \rightarrow \bar{\nu}_i \ell_j^+ \ell_k^-) = \frac{g^2 \lambda_{ijk}^2}{4} \left[ \frac{Y_{\nu_i}^2}{M_{\tilde{\nu}_i}^4} \left( m_{\ell_j \ell_k}^2 - m_{\ell_j}^2 - m_{\ell_k}^2 \right) \left( M_{\tilde{\chi}_1^0}^2 - m_{\ell_j \ell_k}^2 \right) - 2 \frac{Y_{\nu_i} Y_{\ell_j L}}{M_{\tilde{\nu}_i}^2 M_{\ell_j L}^2} \left( m_{\nu_i \ell_k}^2 m_{\ell_j \ell_k}^2 - M_{\tilde{\chi}_1^0}^2 m_{\ell_k}^2 \right) \right] \quad (8)$$

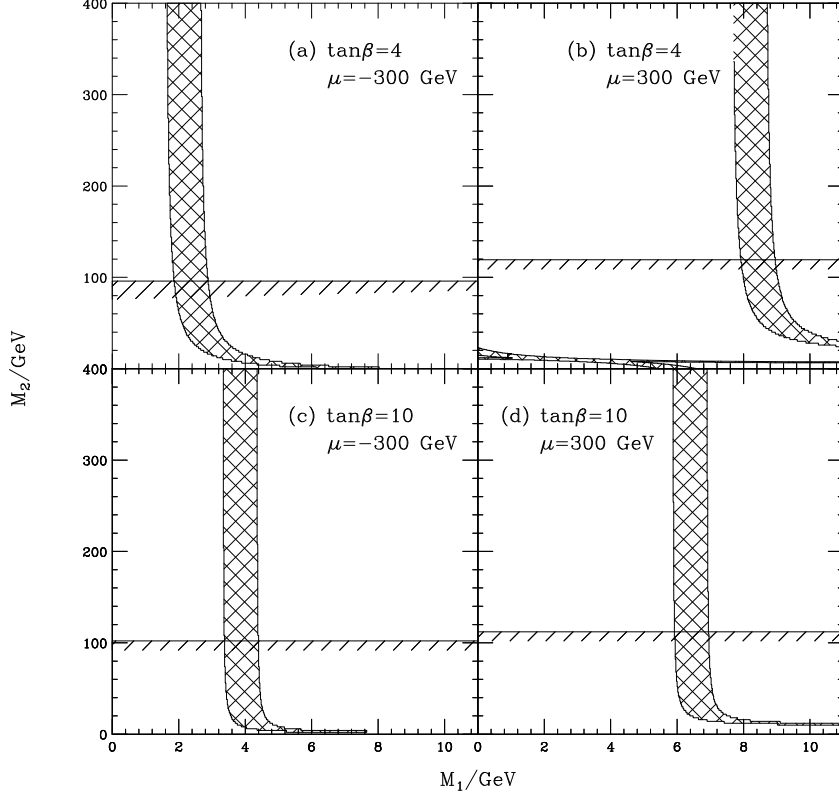


Figure 3: Solutions in  $(M_1, M_2, \mu, \tan \beta)$  giving  $4.5 \text{ GeV} \leq M_{\tilde{\chi}_1^0} \leq 5.5 \text{ GeV}$  in the cross-hatched region. Points below the horizontal hatched line are excluded by the requirement that  $M_{\tilde{\chi}_1^+} > 100 \text{ GeV}$ .

$$\begin{aligned}
& + \frac{Y_{\ell_j L}^2}{M_{\tilde{\ell}_j L}^4} (m_{\nu_i \ell_k}^2 - m_{\ell_k}^2) \left( M_{\tilde{\chi}_1^0}^2 + m_{\ell_j}^2 - m_{\nu_i \ell_k}^2 \right) + 2 \frac{Y_{\nu_i} Y_{\ell_k R}}{M_{\tilde{\nu}_i}^2 M_{\tilde{\ell}_k R}^2} \left( m_{\nu_i \ell_j}^2 m_{\ell_j \ell_k}^2 - M_{\tilde{\chi}_1^0}^2 m_{\ell_j}^2 \right) \\
& + \frac{Y_{\ell_k R}^2}{M_{\tilde{\ell}_k R}^4} (m_{\nu_i \ell_j}^2 - m_{\ell_j}^2) \left( M_{\tilde{\chi}_1^0}^2 + m_{\ell_k}^2 - m_{\nu_i \ell_j}^2 \right) + 2 \frac{Y_{\ell_j L} Y_{\ell_k R}}{M_{\tilde{\ell}_j L}^2 M_{\tilde{\ell}_k R}^2} \left( m_{\nu_i \ell_j}^2 m_{\nu_i \ell_k}^2 - m_{\ell_j}^2 m_{\ell_k}^2 \right) \Big].
\end{aligned}$$

Here  $Y_f$  is the hypercharge of the field  $f$  and  $m_{f_i f_j} = (f_i + f_j)^2$  is the invariant mass of the  $f_i, f_j$  pair of fields. This matrix element can be simplified by assuming a common sfermion mass,  $M_{\tilde{f}}$ , and by putting in explicit values for the couplings

$$|\overline{\mathcal{M}}|^2(\tilde{\chi}_1^0 \rightarrow \bar{\nu}_i \ell_j^+ \ell_k^-) = \frac{9g'^2 \lambda_{ijk}^2}{4M_{\tilde{f}}^4} \left( M_{\tilde{\chi}_1^0}^2 + m_{\ell_k}^2 - m_{\nu_i \ell_j}^2 \right) \left( m_{\nu_i \ell_j}^2 - m_{\ell_j}^2 \right). \quad (9)$$

In the analysis of Ref. [16, 17] the model for the heavy neutral lepton decay studied was based on a weak decay matrix element [24]

$$\mathcal{M}(N^0 \rightarrow \nu_i \ell_j^- \ell_k^+) = \frac{G_F}{\sqrt{2}} \bar{u}_{N^0} \gamma^\mu (1 - \gamma_5) u_{\ell_j} \bar{\nu}_{\ell_i} \gamma_\mu (1 - \gamma_5) u_{\nu_i}. \quad (10)$$

If we compute the squared amplitude and average over the spin of the incoming heavy lepton we obtain

$$|\overline{\mathcal{M}}|^2(N^0 \rightarrow \nu_i \ell_j^- \ell_k^+) = 16G_F^2 \left( m_{N^0}^2 + m_{\ell_k}^2 - m_{\nu_i \ell_j}^2 \right) \left( m_{\nu_i \ell_j}^2 - m_{\ell_j}^2 \right). \quad (11)$$

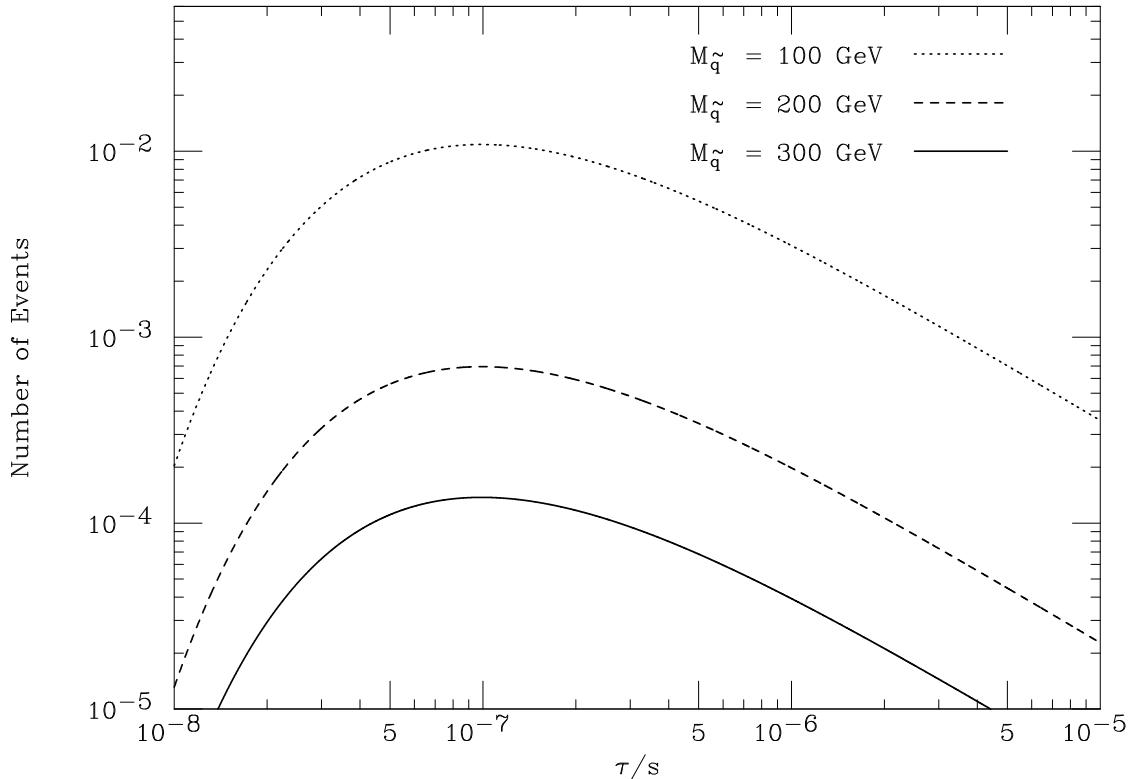


Figure 4: Number of Events in the NuTeV detector for neutralino pair production as a function of the neutralino lifetime.

So the distribution of the decay products from the R-parity violating decay will be exactly the same as the weak decay matrix element studied in [16, 17] and therefore this model has exactly the same problem with the energy asymmetry  $A_E$  as that discussed in [16, 17].

### 3.1 Neutralino Pair Production

We simulated neutralino pair production using HERWIG 6.2 [25–27].<sup>2</sup> This allows us to simulate the production cross section with the correct momentum spectrum for the neutralinos and to determine whether they can decay within the NuTeV detector. Those events where the neutralino could decay in the detector were weighted with the probability that the neutralino decayed in the detector, for a given lifetime

$$\mathcal{P} \approx \exp\left\{-\frac{\ell}{\beta\gamma c\tau_{\chi^0}}\right\} \frac{\Delta x}{\beta\gamma c\tau_{\chi^0}}, \quad (12)$$

where  $\ell = 1.4$  km is the distance target-detector,  $\Delta x = 35$  m is the length of the detector,  $\beta c$  is the speed of the neutralino and  $\tau_{\chi^0}$  is its lifetime. The neutralino was furthermore decayed with the full RPV matrix element [23, 29]. We then applied the NuTeV kinematic cuts [17] on the neutralino decay products. We required that the neutralinos decay within the fiducial volume<sup>3</sup> ( $2.54 \times 2.54 \times 28$ ) m<sup>3</sup> of the NuTeV detector at

<sup>2</sup> One modification to HERWIG was made in that we used the average of the central and higher gluon parton distribution functions from the leading-order fit of [28]. This will become the default in the next release of HERWIG.

<sup>3</sup>In the original version of our paper this number was smaller as found in [46]. We thank T. Adams for drawing our attention to the corrected value in the published version [17].

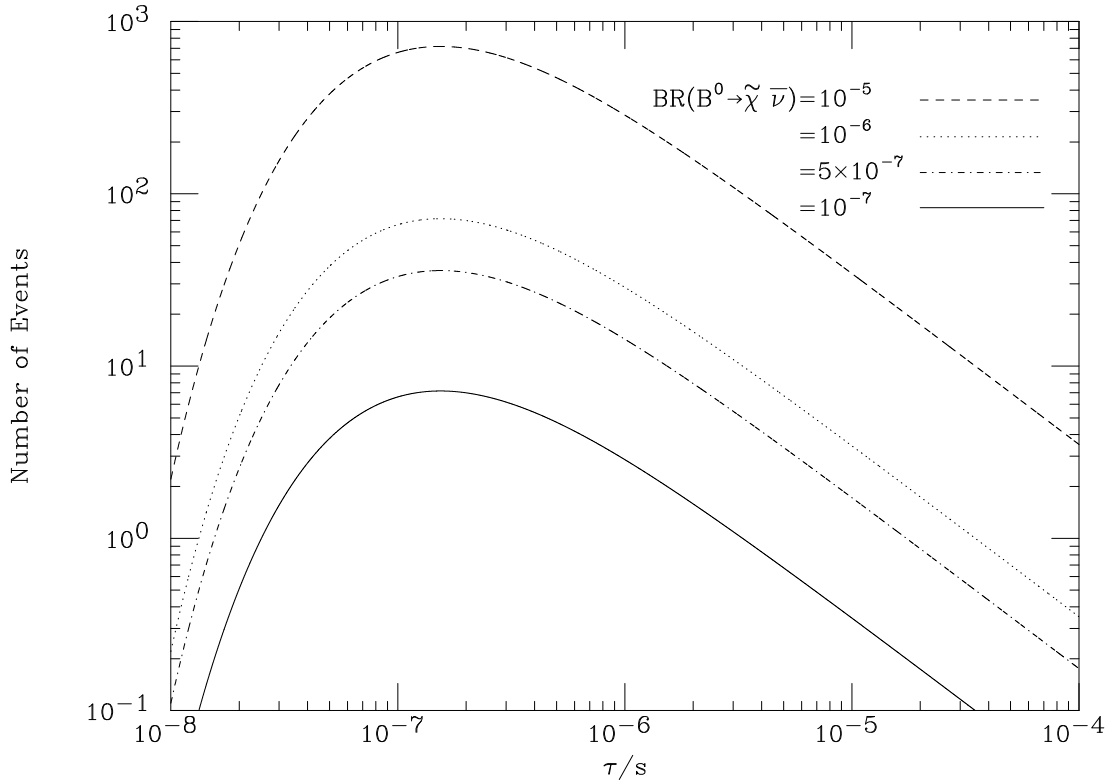


Figure 5: Number of events in the NuTeV detector for neutralino production in  $B$ -meson decays as a function of the neutralino lifetime.

a distance 1.4 km downstream of the production target. The muons produced in the neutralino decay were required to have energy,  $E_\mu > 2.2$  GeV, and the transverse mass,  $m_T = |P_T| + \sqrt{P_T^2 + M_V^2} > 2.2$  GeV, as in Ref. [15–17]. Here  $P_T$  and  $M_T$  are the transverse momentum and mass of the visible decay products, respectively.

As the production of a bino only occurs via  $t$ -channel squark exchange the cross section will depend on the (assumed degenerate) squark mass as  $\sim 1/M_{\tilde{q}}^4$ . The number of events which would be observed in the NuTeV detector are given in Fig. 4 as a function of the lifetime of the neutralino. Given the current limits on the squark mass from both LEP [30–33] and the Tevatron [34, 35] it is impossible, for any neutralino lifetime, to get sufficient events to explain the NuTeV results via neutralino pair production. In Ref. [14] the LEP constraints on the MSSM parameter space were not taken into account.

### 3.2 Neutralino Production in $B$ -meson Decays

As with the neutralino pair production we used HERWIG to simulate  $b\bar{b}$  production. One of the produced  $B$  mesons was then forced at random to decay via RPV. The overall normalization was properly taken into account. The partial widths for the decays of the  $B_0$  and  $B^+$  via RPV are given by

$$\begin{aligned}
 \Gamma(B_d^0 \rightarrow \bar{\nu}_i \tilde{\chi}_1^0) &= \frac{\lambda_{i13}^2 f_B^2 m_{B^0}^2 p_{cm}}{16\pi(m_d + m_b)^2} \left[ \frac{L_{\nu_i}}{M_{\tilde{\nu}_i}^2} - \frac{L_d}{2M_{\tilde{d}_L}^2} - \frac{R_b^*}{2M_{\tilde{b}_R}^2} \right]^2 (M_{B^0}^2 - M_{\tilde{\chi}_1^0}^2), \\
 &= \frac{9\lambda_{i13}^2 g'^2 f_B^2 m_{B^0}^2 p_{cm}}{256\pi(m_d + m_b)^2 M_f^4} (M_{B^0}^2 - M_{\tilde{\chi}_1^0}^2), \tag{13}
 \end{aligned}$$

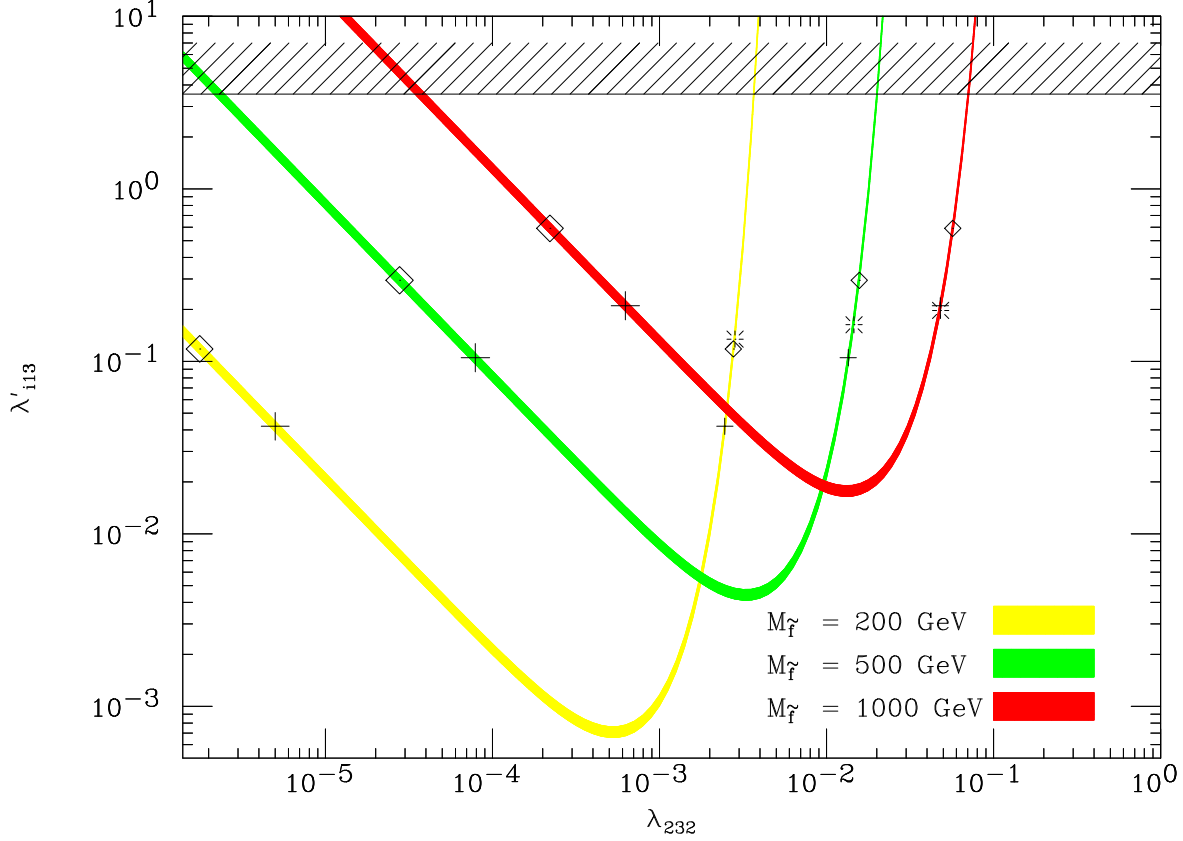


Figure 6: Regions in  $\lambda_{232}, \lambda'_{i13}$  parameter space in which we would expect  $3 \pm 1$  events to be observed in the NuTeV detector. The limits from [3,19] on the couplings  $\lambda'_{113}$  (crosses) and  $\lambda'_{213}$  (diamonds) allow solutions between the two points for each of the masses shown. The region above the stars is ruled out for the coupling  $\lambda'_{213}$  by the limit on the product of the couplings  $\lambda_{232}\lambda'_{213}$  from the limit on the branching ratio  $B^+ \rightarrow \mu^+\nu$  [36]. The hatched region shows the experimental bound on the coupling  $\lambda'_{i13}$  from perturbativity. The corresponding limits on the coupling  $\lambda_{232}$  from both low energy experiments [3,19] and perturbativity are not shown as our solutions do not extend into this region.

$$\begin{aligned}
\Gamma(B^+ \rightarrow \ell_i^+ \tilde{\chi}_i^0) &= \frac{\lambda_{i13}^2 f_B^2 m_{B^+}^2 p_{cm}}{8\pi(m_u + m_b)^2} \left[ \frac{L_{\ell_i}}{M_{\tilde{\ell}_i}^2} - \frac{L_u}{2M_{\tilde{u}_L}^2} - \frac{R_b^*}{2M_{\tilde{b}_R}^2} \right]^2 \left( M_{B^+}^2 - m_{\ell_i}^2 - M_{\tilde{\chi}_i^0}^2 \right), \\
&= \frac{9\lambda_{i13}^2 g'^2 f_B^2 m_{B^+}^2 p_{cm}}{128\pi(m_u + m_b)^2 M_{\tilde{f}}^4} \left( M_{B^+}^2 - m_{\ell_i}^2 - M_{\tilde{\chi}_i^0}^2 \right), \quad (14)
\end{aligned}$$

where  $p_{cm}$  is the momentum of the decay products in the rest frame of the decaying meson,  $m_{u,d,b}$  are the up, down and bottom quark masses respectively,  $m_{B^0}$  is the  $B^0$  mass,  $m_{B^+}$  is the  $B^+$  mass. Here  $L_f = -g'Y_{fL}/2$  for the left-handed fermions and  $R_f = g'Y_{fR}/2$  for the right-handed fermions.  $f_B$  is the pseudo-scalar decay constant for  $B$  decays,  $M_{\tilde{\chi}_1^0}$  is the lightest neutralino mass,  $M_{\tilde{d}_L}$  is the left down squark mass,  $M_{\tilde{u}_L}$  is the left up squark mass, and  $M_{\tilde{b}_R}$  is the right bottom squark mass. In Eqns. (13,14) we have assumed that the sfermions have a common mass  $M_{\tilde{f}}$ . The pseudo-scalar decay constant for the  $B$  system has not been measured experimentally and must be taken from lattice QCD. We have used the value

$$f_B = 204 \pm 30 \text{ MeV}, \quad (15)$$



from Ref. [37] where we have added the errors in quadrature. The branching ratio for the decay  $B^0 \rightarrow \tilde{\chi}_1^0 \bar{\nu}$  was taken as an input and the branching ratio for  $B^+ \rightarrow \tilde{\chi}_1^0 \ell^+$  calculated from it using the above results. The same cuts were applied as in the previous section. The number of events which would be observed in the detector is shown in Fig. 5. This shows that even for branching ratios below  $10^{-7}$  there is a significant range of neutralino lifetimes for which there are enough events to explain the NuTeV results. The present experimental upper limit on the branching ratio of the purely muonic decay is  $Br(B^\pm \rightarrow \mu^\pm \nu_\mu) < 2.1 \cdot 10^{-5}$  [36].

Using the results for the RPV branching ratios of the B mesons and the neutralino lifetime we can find regions in  $(\lambda_{232}, \lambda'_{113})$  parameter space, for a given sfermion mass, in which there are  $3 \pm 1$  events inside the NuTeV detector, this is shown in Fig. 6. We have included the low-energy bounds Eq.(7). In the case of the coupling  $\lambda'_{213}$  there is also a bound on the product of the couplings  $\lambda_{232} \cdot \lambda'_{213}$  from the limits on the branching ratios for  $B^0 \rightarrow \tau^- \mu^+$  and  $B^+ \rightarrow \mu^+ \nu$  [36], the latter giving the stricter bound

$$\frac{\lambda'_{213}{}^2 \lambda_{232}{}^2 f_B^2 m_{B^+}^5}{32\pi M_{\tilde{f}}^4 (m_b + m_u)^2 \Gamma_{B^+}} \left(1 - \frac{m_\mu^2}{m_{B^+}^2}\right)^2 \leq 2.1 \times 10^{-5}, \quad (16)$$

Here  $\Gamma_{B^+}$  is the total width for the  $B^+$ . This gives

$$\lambda'_{213} \lambda_{232} \leq 3.8 \times 10^{-4} \left(\frac{M_{\tilde{f}}}{200 \text{ GeV}}\right)^2. \quad (17)$$

In Fig. 6 we see that for every value of  $\lambda'_{i13}$  there are two solutions in  $\lambda_{232}$ , except for a minimum value of  $\lambda'_{i13}$ , below which there are no solutions. This can be understood as follows. The maximum fraction of neutralinos decays in the distant detector for a lifetime  $\tau = \beta c \gamma / \ell$ , *i.e.* when the decay length corresponds to the flight length, the distance between the production target and the detector. This optimised lifetime corresponds numerically to

$$\lambda_{232} = 5.3 \cdot 10^{-4} \left(\frac{M_{\tilde{f}}}{200 \text{ GeV}}\right)^2. \quad (18)$$

This requires the minimum production rate and thus the minimum value of  $\lambda'_{i13}$ , which is the dip in the curves in Fig. 6. For larger values of  $\lambda'_{i13}$  the neutralino production is increased. We can then tune the lifetime of the neutralino such that the decay length is either shorter or longer than the flight length, yielding the two solutions shown in the figure.

### 3.3 $\tau$ -Decays

As discussed in section 2, in our model the neutralino can decay to  $\mu\tau\nu$  as well as  $\mu\mu\nu$ . Using the calculation of Eq.(9) we can compute the branching ratios  $Br_{\mu\mu} \equiv Br(\tilde{\chi}_1^0 \rightarrow \mu^\pm \mu^\mp \nu_\tau)$  and  $Br_{\mu\tau} \equiv Br(\tilde{\chi}_1^0 \rightarrow \tau^\pm \mu^\mp \nu_\mu)$ , which are displayed in Fig. 7. For neutralino masses above 10-15 GeV the two decays have practically equal branching ratios. However, when the neutralino mass is close to the  $\tau$ -mass,  $Br_{\mu\tau}$  is phase space suppressed. For  $M_{\tilde{\chi}_1^0} = 5 \text{ GeV}$  we have  $Br_{\mu\tau} = 0.287$ . In obtaining Fig. 7 the sfermions have been assumed to be degenerate and left/right stau mixing has been neglected.<sup>4</sup> In princi-

<sup>4</sup>In models where the scalar masses are unified at the GUT scale the running of the masses to low scales forces the right stau to be lighter than the left stau. For low  $\tan\beta$  it is a good approximation to neglect left/right stau mixing. For large values of  $\tan\beta$  the right stau becomes much lighter, but this does not contribute to the decay. It is thus a conservative assumption to require degenerate scalar fermion masses.

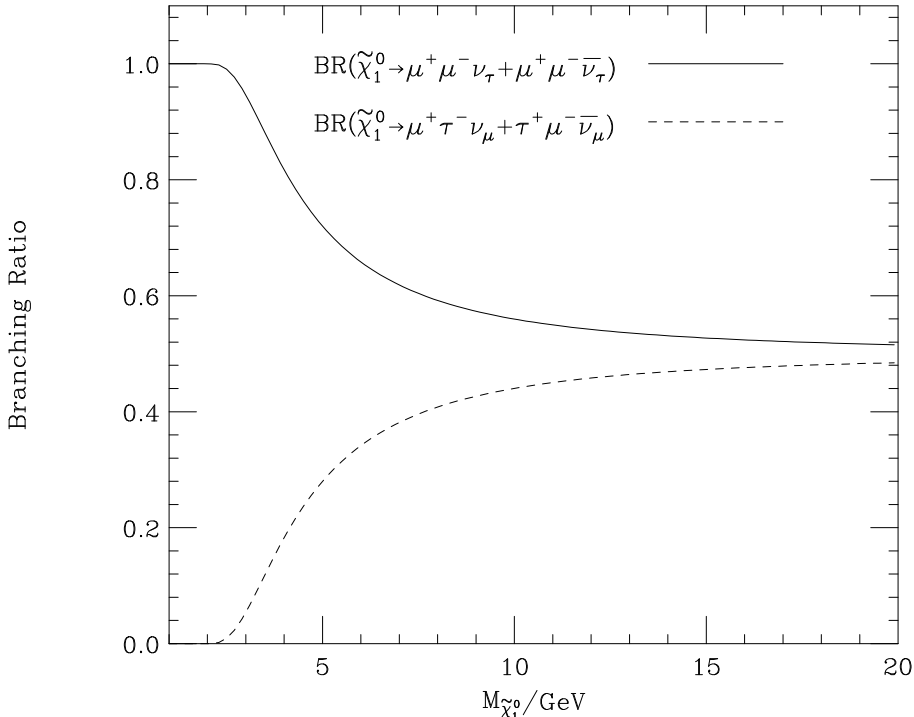


Figure 7: Branching ratios for the decay of a purely bino lightest neutralino via the RPV coupling  $\lambda_{232}$ . The sfermions have been assumed to be degenerate and light/right stau mixing has been neglected.

ple the NuTeV experiment can observe the  $(\mu\tau\nu)$ -modes through the decays:  $\tau^\pm \rightarrow e^\pm\nu\nu$  and  $\tau^\pm \rightarrow \pi^\pm(n \cdot \pi^0)\nu$ , which would lead to unobserved  $(e, \mu)$  and  $(\pi, \mu)$  events, respectively. Here  $(n \cdot \pi^0)$  indicates an additional  $n = 0, 1, 2, 3$  emitted neutral pions. Given the 3 observed  $(\mu, \mu)$  events one would expect the following number of events for  $M_{\tilde{\chi}_1^0} = 5$  GeV

$$N_{(e,\mu)} = 3 \cdot \frac{Br_{\mu\tau}}{1 - Br_{\mu\tau}} \cdot Br(\tau \rightarrow e\nu\nu) \approx 0.21 \quad (19)$$

$$N_{(\pi,\mu)} = 3 \cdot \frac{Br_{\mu\tau}}{1 - Br_{\mu\tau}} \cdot Br(\tau \rightarrow \pi(n \cdot \pi^0)\nu) \approx 0.56, \quad (20)$$

where we have used the  $\tau$  branching ratios from Ref. [36]. Thus the non-observation of  $(e, \mu)$ - and  $(\pi, \mu)$ -events is consistent. We note that some of the  $\tau \rightarrow \pi^\pm(n \cdot \pi^0)\nu$  decays would show extra activity in the detector and thus be rejected as pure  $\pi^\pm$  events. Therefore the above estimate is conservative [38].

## 4 Future Tests of the $R_p$ Violating Model

### 4.1 NOMAD

The NOMAD experiment [39–41] was a neutrino oscillation experiment at CERN which was dismantled in 1999. The data however are still on tape and could be used to test the current proposal. We modified our program to estimate the event rate at NOMAD. For this we used the following numbers [39–41]: distance target-detector:  $\ell = 835$  m, fiducial volume of the detector:  $V = (2.6 \times 2.6 \times 4)$  m<sup>3</sup>, target material: Beryllium, target density:

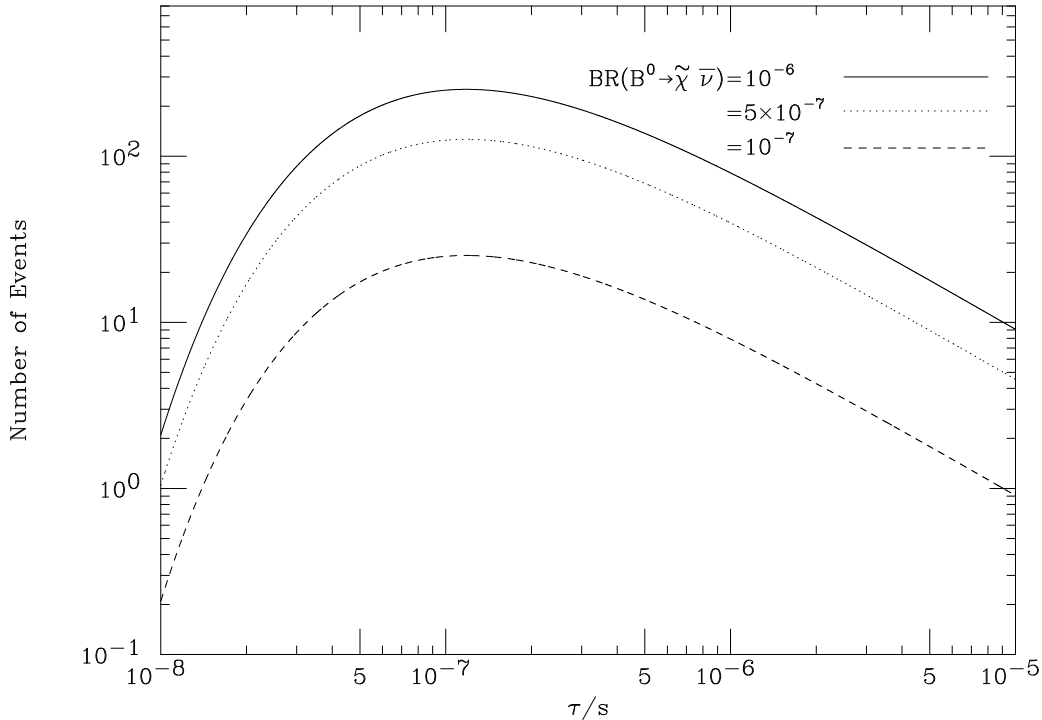


Figure 8: The predicted number of di-muon events at NOMAD as a function of the neutralino lifetime. We have used our model for neutralino production through  $B$ -meson decays. We indicate the prediction for three different branching ratios of the neutral  $B$ -meson decay to neutralinos as in Fig.5.

$\rho = 1.85 \text{ g/cm}^3$ , target length:  $d = 1.1 \text{ m}$ , proton beam energy:  $E = 450 \text{ GeV}$ , integrated number of protons:  $N_p = 4.1 \cdot 10^{19}$ . Using these numbers we show our prediction for the number of events at NOMAD in Fig.8. For the same  $B^0$ -meson branching ratio we obtain about an order of magnitude more events than at NuTeV. Thus our model can be *completely* tested by the NOMAD data!

The higher sensitivity at NOMAD can be understood as follows. The total  $b\bar{b}$  production cross section for collisions on  $Be$  at NOMAD is  $4.7 \text{ nb}$ , while for collisions on  $BeO$  at NuTeV it is  $94 \text{ nb}$ . The total integrated luminosities are  $5.58 \cdot 10^{11} \text{ nb}^{-1}$  (NOMAD) and  $6.189 \cdot 10^9 \text{ nb}^{-1}$  (NuTeV), respectively. Therefore the number of  $b\bar{b}$ -events are  $2.6 \cdot 10^{12}$  (NOMAD) and  $5.8 \cdot 10^{11}$  (NuTeV), respectively, *i.e.* about 4.5 times more at NOMAD. The NOMAD detector is closer and than the NuTeV detector and thus subtends a larger solid angle by about a factor of 3. The required neutralino lifetime is about the same because NOMAD is about half the distance but the energy is also about half. The NOMAD detector is about 8 times shorter but the Lorentz boost is only about half the NuTeV boost, so this corresponds to a factor of 4. All in all we would expect about a factor of 3.4 times more events at NOMAD than at NuTeV. Comparing Fig. 8 with Fig. 5 we see that this is close to what the full numerical simulation gives.

## 4.2 B-Factories

As outlined above, for neutralino production we are relying on a rare B-meson decay

$$B^\pm \rightarrow \mu^\pm + \chi_1^0, \quad (21)$$

$$B^0 \rightarrow \nu + \chi_1^0, \quad (22)$$

which can possibly be observed at a present or future B-factory. In the Standard Model we have the decay  $B^\pm \rightarrow \mu^\pm + \nu$  with a predicted branching ratio [42] of about  $3 \cdot 10^{-7}$ . This is probably just below visibility at BaBar [42].

The decay (21) differs from the Standard Model decay  $B^\pm \rightarrow \mu + \nu$  in the energy of the  $\mu$ , which is now only  $E_\mu = (M_{B^\pm}^2 + m_\mu^2 - M_\chi^2)/(2M_{B^\pm}) \approx 0.27$  GeV for  $M_\chi = 5$  GeV. We thus have a monochromatic muon with an order of magnitude less energy than in the Standard Model decay. This is a distinctive signature which we propose for investigation at BaBar and other B-factories. We presume this is very difficult due to many sources of soft muons as background. Also the efficiency for such soft muons is typically very low, only about 5% [42].

The decay (22) is invisible, with the neutralino decay far outside the detector at a B-factory. If we have a  $B^0$ - $\bar{B}^0$  system and could tag one of the mesons, via a conventional decay, then we would have an unexpected invisible decay on the opposite side. We propose this as a possible signature for investigation by the experimental collaborations.

## 5 Neutral Heavy Leptons

In [16, 17] the NuTeV collaboration also considered the possibility of a neutral heavy lepton (NHL) to explain their observation. Here a NHL,  $N_{iL}$ ,  $i = 1, 2, 3$  is considered as a primarily isosinglet field under  $SU(2)_L$  with a small admixture of the light Standard Model neutrinos. This is discussed for example in Refs. [43, 44]. We follow the notation of Ref. [43]. In general such a NHL has charged current (CC) and neutral current (NC) purely leptonic decays proceeding via a virtual  $W^\pm$  or  $Z^0$ -boson, respectively,

$$N_{iL} \rightarrow \ell_j^- + \ell_k^+ + \bar{\nu}_k, \quad (CC) \quad (23)$$

$$N_{iL} \rightarrow \nu_m + \ell_n^+ + \ell_n^-. \quad (NC) \quad (24)$$

For the NC-decay the charged leptons are from the same family, whereas for the CC-decay they can also be from different families. A given CC-leptonic decay is proportional to the mixing element  $|U_{jN_i}|^2$ . There is a corresponding NC-decay proportional to the same mixing element for  $m = j$ . For a given set of NHL masses and mixings, we typically would expect both NC- and CC-decays to occur.  $k, n = 1, 2, 3$  are free indices which all contribute to the decay rate, independent of the mixings.

NuTeV observe an excess of di-muon events. Assume we have one NHL,  $N_i$ , with mass  $M_{N_i} = 5$  GeV, and the other NHL's unobservably heavy. The di-muon events could occur through CC-decays with  $j = k = 2$  and the mixing element  $U_{2N_i}$  or through NC-decays with  $n = 2$  and the mixing elements  $U_{mN_i}$ ,  $m = 1, 2, 3$ . For  $j = k = n = 2$  we obtain di-muon events through both NC- and CC-decays.

If the CC-decays contribute, *i.e.*  $j = 2$ , we would expect there to be accompanying  $(e, \mu)$  events with similar probability, from the  $k = 1$  mode. For example for a non-vanishing  $|U_{2N_i}|^2$ , using the decay rates given in [44], we obtain the ratio of  $(e, \mu)$  to  $(\mu, \mu)$  events given by  $R_{e\mu/\mu\mu} \equiv \Gamma(N_2 \rightarrow e^+ \mu^- \nu_\mu)/\Gamma(N_2 \rightarrow \mu^+ \mu^- \nu_\mu)$ . We plot this as a function of the NHL-mass in Fig.9. From the plot we see that we would expect *more*  $(e\mu)$  events than  $(\mu, \mu)$ -events. This is excluded by the NuTeV non-observation of such events.

If the NC-decays contribute we can expect further  $(e, e)$  and  $(\tau, \tau)$  events. The latter are kinematically suppressed as in the  $R_p$  case above. A search for the  $(e, e)$ -modes has only been presented for low-energy electrons [15]. However, given a non-vanishing mixing

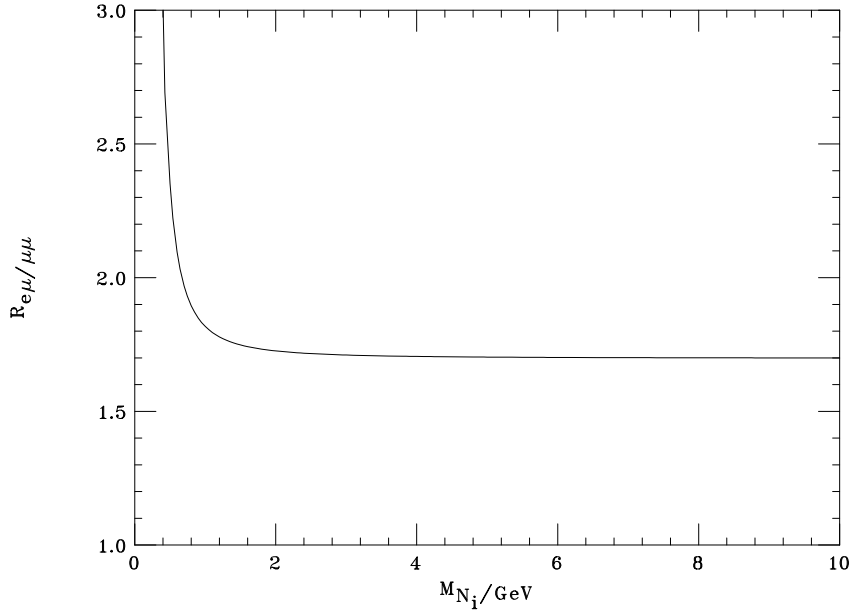


Figure 9: Ratio of  $(e, \mu)$ - to  $(\mu, \mu)$ -events,  $R_{e\mu/\mu\mu} \equiv \frac{\Gamma(N_2 \rightarrow e^+ \mu^- \nu_\mu)}{\Gamma(N_2 \rightarrow \mu^+ \mu^- \nu_\mu)}$  in the decay of a NHL,  $N_i$ , versus the mass of  $N_i$ .

element  $U_{mN_i}$  which gives the  $(\mu, \mu)$ -events via NC-decays we would expect further CC-decays:  $N_{iL} \rightarrow \ell_j^\pm + \ell_k^\mp + \nu_k$ ,  $k = 1, 2, 3$ . In particular, for  $j = 1, 2$  this leads again to  $(e, \mu)$  events which were not observed.

We have thus eliminated all cases except a special model, which we consider in more detail. Assume  $j = 3$  and  $U_{3N_i}$  is the only non-negligible mixing element. Furthermore, as above, assume  $M_{N_i} = 5$  GeV and the other NHL's are unobservably heavy. We then have the following decay modes

$$N_i \xrightarrow{\text{CC}} \{ \tau\tau\nu_\tau, \tau\mu\nu_\mu, \tau e\nu_e \}, \quad (25)$$

$$N_i \xrightarrow{\text{NC}} \{ \nu_\tau ee, \nu_\tau \mu\mu, \nu_\tau \tau\tau \}. \quad (26)$$

The  $\tau$  and  $\tau\tau$  decay modes are kinematically suppressed as in the  $\mathcal{R}_p$  case discussed above and the observed di-muon events are obtained from the NC-decay. This model has been studied by the NOMAD collaboration for  $M_{N_i} = 10 - 190$  MeV [41].

We now estimate the event rate for this model ( $j = 3$ ,  $U_{3N_i} \neq 0$ ). The production mechanism will go either via (CC or NC) Drell-Yan with the tau neutrino mixing with the  $N_i$  or via B-meson decays. We have computed the Drell-Yan production cross section to be  $\sigma_{DY} = \mathcal{O}(10^{-1} \text{ pb}) \cdot |U_{3N_i}|^2$ . The neutral current contribution to the NHL production is more than an order of magnitude smaller. The total integrated luminosity at NuTeV corresponds to  $\sim 6.2 \cdot 10^6 \text{ pb}^{-1}$  giving the number of produced  $N_i$ :  $N_{N_i}^{\text{prod}} \sim 4 \cdot 10^6 |U_{3N_i}|^2$ . Of these only about 1% fly in the direction of the detector [14], leaving us with  $N_{N_i} \sim 4 \cdot 10^4 |U_{3N_i}|^2$ . In order to estimate the total number of events we must combine this with the fraction of  $N_i$  which decay in the detector given by Eq. (12). The total event rate is proportional to

$$N_{ev} \approx N_{N_i} \exp \{ -a|U_{3N_i}|^2 \} \cdot b|U_{3N_i}|^2 = 4 \cdot 10^4 b |U_{3N_i}|^4 \exp \{ -a|U_{3N_i}|^2 \} \quad (27)$$

where  $a = \ell/(\beta\gamma c(\tau_{N_i}|U_{3N_i}|^2))$  and  $b = \Delta x/(\beta\gamma c\tau_{N_i}|U_{3N_i}|^2)$  from Eq. (12) are independent of  $|U_{3N_i}|$ . The event rate is maximal for  $|U_{3N_i}|^2 = 2/a$ . We obtain an upper limit on the

lifetime if we assume the NC decay is dominant. The latter we determine through the scaled muon lifetime

$$\tau_{N_i} < \tau_\mu \left( \frac{m_\mu}{M_{N_i}} \right)^5 |U_{3N_i}|^{-2} = 9 \cdot 10^{-15} s |U_{3N_i}|^{-2}. \quad (28)$$

We then obtain  $a = 5.2 \cdot 10^8 / \gamma$  and  $b = 1.3 \cdot 10^7 / \gamma$ . For  $\gamma = 10$ , for example, we obtain the maximal event rate for  $|U_{3N_i}| = 9 \cdot 10^{-5}$ , which is compatible with the independent bound  $\sum_i |U_{3N_i}|^2 < 0.016$  [45]. Following Eq. (12) the total fraction decaying in the detector is then roughly 1.1%. Combining this with the previous estimate of the number produced we get a total maximal number of events of about  $N_{ev}^{max} = 5 \cdot 10^{-7}$ , which is of course too small.

The reason this is so much smaller than in the supersymmetric model is that the lifetime of the NHL is typically much shorter. Thus the NHL's typically would decay well before the detector. We get the maximal number of events when the lifetime is approximately the flight time. For this we need a very small  $|U_{3N_i}|$ . Since we only have one parameter in this model this feeds into the cross section resulting in the very low rate. We do not expect the production via  $B$ -mesons to help. The branching ratio is suppressed compared to the SM decay branching ratio  $Br(B^+ \rightarrow \tau^+ N_i) = |U_{3N_i}|^2 Br(B^+ \rightarrow \tau^+ \nu_\tau) \approx 7 \cdot 10^{-5} |U_{3N_i}|^2$  and thus also too small.

## 6 Conclusions

We have reconsidered the NuTeV di-muon observation in the light of supersymmetry with broken R-parity and neutral heavy leptons. We have shown that it is not possible to obtain the observed event rate with pair production of light neutralinos or via the production of neutral heavy leptons. However, we have introduced a new production method of neutralinos via  $B$ -mesons. Due to the copious production of  $B$ -mesons in the fixed target collisions the observed di-muon event rate can be easily obtained for allowed values of the R-parity violating couplings.

The model we have proposed can be completely tested using current NOMAD data. We suspect this is true of any model one might propose. If the NOMAD search is negative our model is ruled out and the NuTeV observation is most likely not due to physics beyond the Standard Model.

It is worth pointing out that through this mechanism we have opened a new sensitivity range in the R-parity violating couplings. At colliders we can probe the range where the neutralino decays in the detector. For a photino neutralino this corresponds to [2]

$$\lambda > 5 \cdot 10^{-7} \sqrt{\gamma} \left( \frac{\tilde{m}}{200 \text{ GeV}} \right)^2 \left( \frac{100 \text{ GeV}}{M_{\tilde{\gamma}}} \right)^{5/2} = 9 \cdot 10^{-4} \sqrt{\gamma} \left( \frac{\tilde{m}}{200 \text{ GeV}} \right)^2 \left( \frac{5 \text{ GeV}}{M_{\tilde{\gamma}}} \right)^{5/2}. \quad (29)$$

Here we have substituted the light neutralino mass we are considering. For significant boost factors we thus can probe couplings at most down to  $10^{-3}$ . From Fig. 6 we see that for a 200 GeV sfermion we can probe couplings down to about  $5 \cdot 10^{-6}$ , which is more than two orders of magnitude smaller! It is thus worthwhile to study the production of neutralinos via mesons at fixed target experiments in more detail.

Before concluding we also note that one might worry that the lightest supersymmetric Higgs boson would decay dominantly to the two light neutralinos and thus be invisible. However, as with the  $Z^0$  boson, the Higgs does not couple to a Bino neutralino.

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## References

- [1] For a recent review see for example: S. P. Martin, hep-ph/9709356.
- [2] For a recent review see for example: H. Dreiner, hep-ph/9707435;
- [3] G. Bhattacharyya, Nucl. Phys. Proc. Suppl. **52A** (1997) 83 [hep-ph/9608415].
- [4] V. Barger, G. F. Giudice and T. Han, Phys. Rev. D **40** (1989) 2987.
- [5] J. Butterworth and H. Dreiner, Nucl. Phys. B **397** (1993) 3 [hep-ph/9211204].
- [6] H. Dreiner, P. Richardson and M. H. Seymour, Phys. Rev. D **63** (2001) 055008 [hep-ph/0007228].
- [7] J. Kalinowski, R. Ruckl, H. Spiesberger and P. M. Zerwas, Phys. Lett. B **406** (1997) 314 [hep-ph/9703436].
- [8] J. Erler, J. L. Feng and N. Polonsky, Phys. Rev. Lett. **78** (1997) 3063 [hep-ph/9612397].
- [9] H. Dreiner and G. G. Ross, Nucl. Phys. B **365** (1991) 597.
- [10] D. P. Roy, Phys. Lett. B **283** (1992) 270.
- [11] H. Dreiner and P. Morawitz, Nucl. Phys. B **503** (1997) 55 [hep-ph/9703279].
- [12] B. C. Allanach *et al.*, JHEP **0103** (2001) 048 [hep-ph/0102173].
- [13] J. Ellis, G. B. Gelmini, J. L. Lopez, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B **373** (1992) 399.
- [14] L. Borissov, J. M. Conrad and M. Shaevitz, hep-ph/0007195.
- [15] J. A. Formaggio *et al.* [NuTeV Collaboration], Phys. Rev. Lett. **84** (2000) 4043 [hep-ex/9912062].
- [16] T. Adams *et al.* [NuTeV Collaboration], hep-ex/0009007.
- [17] T. Adams *et al.* [NuTeV Collaboration], Phys. Rev. Lett. **87** (2001) 041801 [hep-ex/0104037].

- [18] D. Choudhury, H. Dreiner, P. Richardson and S. Sarkar, Phys. Rev. D **61** (2000) 095009 [hep-ph/9911365].
- [19] B. C. Allanach, A. Dedes and H. K. Dreiner, Phys. Rev. D **60** (1999) 075014 [hep-ph/9906209].
- [20] A. Bartl, H. Fraas and W. Majerotto, Nucl. Phys. B **278** (1986) 1.
- [21] D. Choudhury and S. Sarkar, Phys. Lett. B **374** (1996) 87 [hep-ph/9511357].
- [22] A. Dedes, H. Dreiner, P. Richardson, work in preparation.
- [23] H. Dreiner, P. Richardson and M. H. Seymour, JHEP **0004** (2000) 008 [hep-ph/9912407].
- [24] J. A. Formaggio, J. Conrad, M. Shaevitz, A. Vaitaitis and R. Drucker, Phys. Rev. D **57** (1998) 7037.
- [25] G. Corcella *et al.*, JHEP **0101** (2001) 010 [hep-ph/0011363].
- [26] G. Corcella *et al.*, hep-ph/9912396.
- [27] G. Marchesini, B. R. Webber, G. Abbiendi, I. G. Knowles, M. H. Seymour and L. Stanco, Comput. Phys. Commun. **67** (1992) 465.
- [28] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Phys. Lett. B **443** (1998) 301 [hep-ph/9808371].
- [29] P. Richardson, hep-ph/0101105.
- [30] R. Barate *et al.* [ALEPH Collaboration], Phys. Lett. B **499** (2001) 67 [hep-ex/0011047].
- [31] M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. B **471** (1999) 308 [hep-ex/9910020].
- [32] P. Abreu *et al.* [DELPHI Collaboration], Phys. Lett. B **496** (2000) 59 [hep-ex/0103034].
- [33] G. Abbiendi *et al.* [OPAL Collaboration], Phys. Lett. B **456** (1999) 95 [hep-ex/9903070].
- [34] F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **56** (1997) 1357.
- [35] B. Abbott *et al.* [D0 Collaboration], Phys. Rev. Lett. **83** (1999) 4937 [hep-ex/9902013].
- [36] D. E. Groom *et al.* [Particle Data Group Collaboration], Eur. Phys. J. C **15** (2000) 1.
- [37] A. Ali Khan *et al.* [CP-PACS Collaboration], hep-lat/0103020.
- [38] We thank L. Borissov, J. Conrad and M. Shaevitz for discussion of this point.
- [39] D. Autiero *et al.*, Nucl. Instrum. Meth. A **387** (1997) 352.
- [40] J. Altegoer *et al.* [NOMAD Collaboration], Nucl. Instrum. Meth. A **404** (1998) 96.



- [41] P. Astier *et al.* [NOMAD Collaboration], Phys. Lett. B **506** (2001) 27 [hep-ex/0101041].
- [42] P. F. Harrison and H. R. Quinn [BABAR Collaboration], SLAC-R-0504 *Papers from Workshop on Physics at an Asymmetric B Factory (BaBar Collaboration Meeting), Rome, Italy, 11-14 Nov 1996, Princeton, NJ, 17-20 Mar 1997, Orsay, France, 16-19 Jun 1997 and Pasadena, CA, 22-24 Sep 1997.*
- [43] M. Gronau, C. N. Leung and J. L. Rosner, Phys. Rev. D **29** (1984) 2539.
- [44] L. M. Johnson, D. W. McKay and T. Bolton, Phys. Rev. D **56** (1997) 2970 [hep-ph/9703333].
- [45] S. Bergmann and A. Kagan, Nucl. Phys. B **538** (1999) 368 [hep-ph/9803305].
- [46] Version 2 on the hep-ex arXiv of [17].