27 February 1997

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 394 (1997) 323–328

The μ-term in effective supergravity theories ⋆

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Received 10 November 1996
Editor: R. Gatto

Abstract

The Higgs mixing term coefficient $\mu_{\rm eff}$ is calculated in the scalar potential in supergravity theories with string origin, in a model independent approach. A general low energy effective expression is derived, where new contributions are included which depend on the modular weights $q_{1,2}$ of the Higgs superfields, the moduli and derivative terms. We find that in a class of models obtained in the case of compactifications of the heterotic superstring, the derivative terms are identically zero. Further, the total $\mu_{\rm eff}$-term vanishes identically if the sum of the two modular weights $q_1 + q_2$ is equal to two. Subleading μ-corrections, in the presence of intermediate gauge symmetries predicted in viable string scenarios, are also discussed.

In the minimal supersymmetric standard model (MSSM), non-zero masses for the quarks and leptons require the existence of two higgs superfields H_1, H_2. In the effective superpotential of the model, one of the higgs doublets couples to the up-type quarks, while the second higgs provides with masses the down-type quarks and charged leptons. If only these terms were present in the effective superpotential, the latter is invariant under a Peccei-Quinn (PQ) symmetry [1] which finally implies the existence of a higgs boson, the 'electroweak axion', with zero bare mass [2]. A way to avoid an unacceptably low mass for the axion in the MSSM, is to introduce a mixing term $\mu H_1 H_2$ [3,4] where the mass parameter μ should be of the order of the electroweak scale. The value of μ could be related to the gravitino mass $m_{3/2}$ or arise from the vacuum expectation value of a scalar component of a singlet field $\phi H_1 H_2 \rightarrow \langle\phi\rangle H_1 H_2$ [3]. Nevertheless, the introduction of an explicit μ-term in the theory generates a new hierarchy problem, since one has to introduce a new scale in the theory, associated with this mixing term.

In the context of the $N = 1$ effective supergravity theories, which emerge as a low energy limit of a superstring theory, it is possible to obtain an induced higgs mixing term [5,6] due to the effects of a hidden sector. From the point of view of string theories, the above features can be found in models with a gauge group G containing both an observable and a hidden sector. In general, the observable part has a rank larger than that of the MSSM symmetry. Usually, G breaks [3] down to the Standard Model (SM)-gauge group at an intermediate scale M_X, some two orders of magnitude below the string scale. A new mixing term for the Higgs fields responsible for the intermediate symmetry

⋆ Work partially supported by C.E.C Project CHRX-CT93-0319, the TMR- program ERB FMR XCT 960090, and ΠΕΝΕΔ 1170/95.

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PII S0370-2693(97)00021-X

breaking may also appear. In addition, induced mixing terms from intermediate symmetry effects are possible for the standard higgs doublets.

In this letter, we derive the μ-mixing terms in an effective supergravity theory with the generic stringy features described above. Taking into account these features and symmetries from the string, one obtains an $N = 1$ supergravity with the following ingredients: A real gauge invariant Kähler potential $\mathcal{K}$ which depends on the chiral superfields and moduli which are exact flat directions of the scalar potential, and a superpotential $\mathcal{W}$ which is a holomorphic function of the chiral superfields Q_i. The second derivatives of the Kähler potential determine the kinetic terms of the various fields in the chiral supermultiplets, while the Yukawa couplings appearing in the superpotential are subject to string constraints. The Kähler function $\mathcal{G}$ is defined [8]

$$\mathcal{G}(z,\bar{z}) = \mathcal{K}(z,\bar{z}) + \log|\mathcal{W}(z)|^2 \tag{1}$$

Denoting $z = (\Phi, Q)$, where Φ stands for the dilaton field S and other moduli T_i, U_i while Q stands for the chiral superfields, the superpotential $\mathcal{W}(z)$, at the tree level, is given by

$$\mathcal{W}(\Phi,Q) = k_0 + \frac{1}{2!}\mu_{ij}(\Phi)Q_iQ_j + \frac{1}{3!}\lambda_{ijk}(\Phi)Q_iQ_jQ_k + \cdots \tag{2}$$

where k_0 is a model dependent constant [9] and $\{\cdots\}$ stand for possible non-renormalizable contributions. Terms bilinear in the fields Q_i refer in fact to a higgs mixing term. At the perturbative level, due to the analyticity of the superpotential and the presence of a PQ-symmetry, the parameters μ and λ do not depend on the dilaton field S. Non-perturbative effects however, may allow dilaton contributions to the superpotential of the form $\propto e^{-8\pi^2 S}$, thus breaking the original PQ - symmetry which allows only $S + \bar{S}$ dilaton combinations.

In the following we will assume that the Kähler potential $\mathcal{K}$ can be expanded in inverse powers of the $(S+\bar{S})$ fields with a tree level piece $K_0(T,\bar{T})$ which takes the following general form: [5,6]:

$$\begin{aligned} K_0(T,\bar{T}) =& -\log(S+\bar{S}) - \Sigma_n h_n \log(T_n + \bar{T}_n) \\ &+ \mathcal{Z}_{i\bar{j}}Q_i\bar{Q}_{\bar{j}} + (\tfrac{1}{2}\mathcal{M}_{ij}(T,\bar{T})Q_iQ_j + \text{c.c.}) \end{aligned} \tag{3}$$

where for simplicity T represents all kinds of moduli except from the dilaton field. Modular symmetries and Kähler transformations may be applied to obtain the transformation properties of the tree level matrices $\mathcal{Z}$ and $\mathcal{M}$ as well as of the chiral fields and the superpotential [6,7,10]. We will soon see that one of the main sources of the induced μ-term in the superpotential is the matrix $\mathcal{M}$ appearing in the Kähler function.

In order to calculate the relevant contributions, we need the inverse Kähler metric $G^{-1}_{I\bar{J}}$, which in the basis chosen has the block diagonal form $(\mathcal{K}^{-1}_{S\bar{S}}, \mathcal{K}^{-1}_{i\bar{j}})$ where the subscripts denote differentiation with respect to the fields z_I while $\mathcal{K}^{-1}_{i\bar{j}}$ is a $N{+}2 \otimes N{+}2$ matrix with the indices i,j taking the values $0,1,\ldots,N+1$ for the fields $T_{1,\ldots,N}, H_1, H_2$, respectively. In particular, in the simplest case of the presence of only one modulus T, $\mathcal{K}^{-1}_{i\bar{j}}$ is given by

$$\begin{aligned} &\mathcal{K}^{-1}_{i\bar{j}} \\ &= \frac{1}{\Delta}\begin{pmatrix} K_{1\bar{1}}K_{2\bar{2}} & -K_{0\bar{1}}K_{2\bar{2}} & -K_{0\bar{2}}K_{1\bar{1}} \\ -K_{1\bar{0}}K_{2\bar{2}} & K_{0\bar{0}}K_{2\bar{2}} - K_{0\bar{2}}K_{2\bar{0}} & K_{1\bar{0}}K_{0\bar{2}} \\ -K_{2\bar{0}}K_{1\bar{1}} & K_{0\bar{1}}K_{2\bar{0}} & K_{0\bar{0}}K_{1\bar{1}} - K_{0\bar{1}}K_{1\bar{0}} \end{pmatrix} \end{aligned} \tag{4}$$

where Δ denotes the determinant of the $\mathcal{K}_{i\bar{j}}$ matrix. The extension to N-moduli, is straightforward. To proceed further, we find it convenient to define the following covariant μ-derivatives:

$$\mathcal{D}_T\tilde{\mu}_{ij} = \partial_T\mu_{ij} + \mathcal{W}\partial_T\mathcal{M}_{ij} \tag{5}$$

$$\mathcal{D}_T\bar{\tilde{\mu}}_{\bar{i}\bar{j}} = \partial_T\bar{\mu}_{\bar{i}\bar{j}} + \bar{\mathcal{W}}\partial_T\bar{\mathcal{M}}_{\bar{i}\bar{j}} \tag{6}$$

and the combination

$$\tilde{\mu}_{ij} = \mu_{ij} + \mathcal{W}\mathcal{M}_{ij} \tag{7}$$

The part of the effective scalar potential related to the supersymmetry breaking effects is given by

$$V_F = e^{\mathcal{G}}\left(\mathcal{G}_I\mathcal{G}^{-1}_{I\bar{J}}\mathcal{G}_J - 3\right) + \cdots \tag{8}$$

where $\{\cdots\}$ represent D-term contributions. Assuming a form of the Kähler potential and the superpotential dictated from modular symmetries, we can now define through (8) the boundary conditions for soft mass terms as well as the induced higgs mixing. As stressed in the introduction, any low energy effective supersymmetric field theory contains in its massless spectrum at least two higgs fields associated with the standard two doublets of the MSSM and the existence of a higgs mixing term, – μ-term – in theories of two higgs doublets is necessary. In effective quantum field theories arising from the heterotic string, the form of the Kähler potential may provide such terms in the effective superpotential.

Thus, in the Kähler function we will assume the existence of a higgs mixing term of the form $\mathcal{M}_{ij}H_iH_j$ where $\mathcal{M}$ depends on the moduli $(T_n, \bar{T}_n)$. An explicit higgs mixing term (the μ-term) may also exist in the original superpotential of the model. The most general form of the tree level superpotential arising in the theories under consideration, has been written in Eq. (2). As explained above, we will restrict our analysis in cases where the tree level Yukawa couplings of the superpotential $\mathcal{W}$ are functions of the moduli T_n, i.e., $\mu_{ij}(T), \lambda_{ijk}(T)$ but at the tree level, they do not depend on the dilaton S. For a more involved situation however, in a final example we will allow the possibility of the existence of an 'unobservable' phase $\varphi(T,\bar{T})$ for the case of the μ-tree level term, which could in principle depend on T and $\bar{T}$ moduli. Such a phase can be justified from the transformation properties under modular invariance of the physical mixing mass in certain compactifications of the heterotic string theory [5]. Then, we will soon see that due to the presence of induced μ contributions involving the derivatives of higgs mixing mass terms, such a phase will manifest itself in the effective μ-term. Finally, due to the possible existence of the intermediate symmetry breaking, new threshold effects can arise and in principle should not be ignored.

Under the above assumptions, we calculate the quantities involved in the effective potential including also terms proportional to the vacuum expectation values (vevs) of the higgs. The various kinds of derivatives which can arise are the following:

$$G_0 = -\frac{h}{\tau} - \sum_i \frac{q_i}{\tau^{1+q_i}} H_i\bar{H}_{\bar{i}} + \tfrac{1}{2}\mathcal{W}^{-1}\mathcal{D}_T\tilde{\mu}_{ij}H_iH_j + \tfrac{1}{2}\bar{\mathcal{W}}^{-1}\mathcal{D}_T\bar{\tilde{\mu}}_{\bar{i}\bar{j}}\bar{H}_{\bar{i}}\bar{H}_{\bar{j}} \tag{9}$$

$$G_{\bar{0}} = -\frac{h}{\tau} - \sum_i \frac{q_i}{\tau^{1+q_i}} H_i\bar{H}_{\bar{i}} + \tfrac{1}{2}\mathcal{W}^{-1}\mathcal{D}_{\bar{T}}\tilde{\mu}_{ij}H_iH_j + \tfrac{1}{2}\bar{\mathcal{W}}^{-1}\mathcal{D}_{\bar{T}}\bar{\tilde{\mu}}_{\bar{i}\bar{j}}\bar{H}_{\bar{i}}\bar{H}_{\bar{j}} \tag{10}$$

$$G_i = \tau^{-q_i}\bar{H}_{\bar{i}} + \mathcal{W}^{-1}\tilde{\mu}_{ij}H_j \tag{11}$$

$$G_{\bar{j}} = \tau^{-q_j}H_j + \bar{\mathcal{W}}^{-1}\bar{\tilde{\mu}}_{\bar{i}\bar{j}}\bar{H}_{\bar{i}} \tag{12}$$

where $\tau = T + \bar{T}$ and q_i are the modular weights of the corresponding higgs field. To obtain the inverse metric we also need the elements of $K_{I\bar{J}}$ matrix which are given by

$$K_{0\bar{0}} = \frac{h}{\tau^2} + \sum_i \frac{q_i(q_i+1)}{\tau^{2+q_i}} H_i\bar{H}_{\bar{i}} + \tfrac{1}{2}\partial_{\bar{T}}\left[\mathcal{W}^{-1}\mathcal{D}_T\mu_{ij}H_iH_j + \bar{\mathcal{W}}^{-1}\mathcal{D}_T\bar{\mu}_{ij}\bar{H}_{\bar{i}}\bar{H}_{\bar{j}}\right] \tag{13}$$

$$K_{0\bar{i}} = -\frac{q_i}{\tau^{1+q_i}}H_i + \bar{\mathcal{W}}^{-1}\left(\partial_T\bar{\tilde{\mu}}_{\bar{i}\bar{j}} - \tfrac{1}{2}\bar{\mathcal{W}}^{-1}\bar{\tilde{\mu}}_{\bar{i}\bar{j}}\partial_T\bar{\mu}_{\bar{k}\bar{\ell}}\bar{H}_{\bar{k}}\bar{H}_{\bar{\ell}}\right)\bar{H}_{\bar{j}} \tag{14}$$

$$K_{i\bar{j}} = \frac{1}{\tau^{q_i}}\delta_{i\bar{j}} \tag{15}$$

while the determinant is given by

$$\begin{aligned}\Delta = \frac{1}{\tau^{2+q_1+q_2}}\Big[& h + \sum_i q_i(q_i+1)\tau^{-q_i}H_i\bar{H}_{\bar{i}} \\ &+ \tfrac{1}{2}\tau^2\partial_{\bar{T}}\left(W^{-1}\mathcal{D}_T\mu_{ij}H_iH_j + \bar{W}^{-1}\mathcal{D}_T\bar{\mu}_{\bar{i}\bar{j}}\bar{H}_{\bar{i}}\bar{H}_{\bar{j}}\right) \\ &+ \tau^{q_2}\left(-q_2\tau^{-q_2}H_2 + \tau\partial_T\left(\bar{\mathcal{W}}^{-1}\bar{\tilde{\mu}}_{\bar{i}\bar{2}}\bar{H}_{\bar{i}}\right)\right) \\ &\times\left(-q_2\tau^{-q_2}\bar{H}_{\bar{2}} + \tau\partial_{\bar{T}}\left(\mathcal{W}^{-1}\tilde{\mu}_{2j}H_j\right)\right) \\ &+ \tau^{q_1}\left(-q_1\tau^{-q_1}H_1 + \tau\partial_T\left(\bar{\mathcal{W}}^{-1}\bar{\tilde{\mu}}_{\bar{i}\bar{1}}\bar{H}_{\bar{i}}\right)\right) \\ &\times\left(-q_1\tau^{-q_1}\bar{H}_{\bar{1}} + \tau\partial_{\bar{T}}\left(\mathcal{W}^{-1}\tilde{\mu}_{1j}H_j\right)\right)\Big]\end{aligned} \tag{16}$$

In the presence of higgs fields with vevs not very far from the unification point, there are in principle, numerous mixing terms arising from all combinations of light and heavy higgs fields through the quantity $G_J G^{-1}_{J\bar{I}} G_I$. However, in the following we will assume that the intermediate gauge symmetry breaks down to the standard gauge group at a scale at least one or two orders of magnitude bellow M_{string}. The possible vev-dependent μ-contributions depend quadratically on these vevs, thus they are rather suppressed. On the contrary, there exist vev-independent contributions which are of the order of the gravitino mass $m_{3/2}$, and/or the possibly existing explicit μ-mass term of the tree level superpotential. Obviously, since these terms are independent of the large higgs vev's, it turns out that they are present even in the absence of any intermediate symmetry.

In the following we present first the vev-independent contributions and show their origin. It is enough for the moment to concentrate on the SM-higgs doublets. First we approximate $\Delta \sim \frac{h}{\tau^{2+q_1+q_2}}$, while we assume a single pair of higgs fields. In this case we will simplify our notation by the replacement $\mu_{ij} \to \mu_{12}$ or even simply μ. Starting from the diagonal terms $G_{\bar{I}} G^{-1}_{I\bar{I}} G_I$, where $I = T, H_1, H_2$, we obtain for $I = T$,

$$G_{\bar{T}} G^{-1}_{T\bar{T}} G_T \to -\frac{1}{\mathcal{W}} (T+\bar{T}) \{D_T + D_{\bar{T}}\} \tilde{\mu}_{12} \tag{17}$$

while two more contributions result from the diagonal terms with respect to the derivatives of the two higgs fields $H_{i=1,2}$, namely

$$\sum_i G_{\bar{i}} G^{-1}_{i\bar{i}} G_i \to 2\frac{\tilde{\mu}_{12}}{\mathcal{W}} \tag{18}$$

The terms in Eqs. (17), (18) are the same with those obtained in previous works [5,6] and constitute the Yukawa coupling of the corresponding fermion mass. Now, in the scalar potential the corresponding soft parameter receives additional contributions from off-diagonal terms $G_{\bar{I}} G^{-1}_{I\bar{J}} G_J$ where $I \neq J$. In particular, it can be easily seen that these contributions are obtained from the two terms $G_{\bar{T}} G^{-1}_{T\bar{i}} G_i$, $i = 1, 2$. Two types of terms may arise here. The first one is directly proportional to the combination of $\tilde{\mu}$ multiplied by the modular weight q_i of the corresponding higgs field H_i. There is a second term common in both ($i = 1, 2$) terms which depends on the properties of the quantity $\tilde{\mu}$ under differentiation with respect to the moduli. More explicitly for the first of the two contributions we have

$$\sum_i G_{\bar{T}} G^{-1}_{T\bar{i}} G_i \to -(q_1 + q_2)\frac{\tilde{\mu}_{12}}{\mathcal{W}} \tag{19}$$

where the normalization of the fields has been taken into account. The second type is proportional to the covariant μ-derivative, i.e.,

$$\frac{2}{\mathcal{W}} \left[(T+\bar{T}) D_{\bar{T}}\right] \tilde{\mu}_{12} \tag{20}$$

A third contribution similar to the second is also possible, however this is proportional to the $\mu_{12} v_1 v_2 \partial_T \mathcal{M}_{12}$, where v_i is the vev of the corresponding higgs, and is assumed to be small. The remarkable fact however, is that even if the higgs vevs are sent to zero and no intermediate scale exists, these new mixing terms from off-diagonal $K^{-1}_{I\bar{J}}$-elements contribute substantially to the μ-term. In particular in the large radius limit, i.e., for large values of the moduli $T > 1$, the contribution (20) might be significant as it is proportional to $T + \bar{T} = 2\,\mathrm{Re}\,T$ and should not be ignored. We may conclude that, although the analysis above is done for rather general effective supergravity models, the parameters entering the μ-formula are rather constrained. Indeed, starting from the second term in (19), it is a remarkable fact that only the sum of the two Higgs modular weights $q = q_1 + q_2$ enters in the μ expression. Although the q_i themselves are model dependent, the value of q however, could be constrained from general requirements. For example, certain constraints can be put on q [7,11] from the transformation properties of the superpotential terms.

Let us now collect the above contributions into an effective higgs mixing mass term. For practical purposes it is useful to simplify the above formulae and keep the leading terms. With the definition,

$$\mu_{\rm sim}(T,\bar{T}) \equiv \frac{\mu}{c} + \mathcal{M} \tag{21}$$

with c being a numerical value associated with the vacuum expectation value of the superpotential,

$$c = \langle |W| \rangle = e^{-\langle \mathcal{K} \rangle/2} m_{3/2} \tag{22}$$

we summarize our results in the following simple formulae:

$$\begin{aligned}
G_{\bar{0}} K^{-1}_{0\bar{0}} G_0 &\to -h\tau^{-(1+q)}(\partial_T + \partial_{\bar{T}})\mu_{\rm sim} \\
G_{\bar{1}} K^{-1}_{1\bar{1}} G_1 + G_{\bar{2}} K^{-1}_{2\bar{2}} G_2 &\to 2h\tau^{-(2+q)}\mu_{\rm sim} \\
G_{\bar{0}} K^{-1}_{0\bar{1}} G_1 + G_{\bar{0}} K^{-1}_{0\bar{2}} G_2 &\to -qh\tau^{-(2+q)}\mu_{\rm sim} \\
G_{\bar{1}} K^{-1}_{1\bar{0}} G_0 + G_{\bar{2}} K^{-1}_{2\bar{0}} G_0 &\to 2h\tau^{-(1+q)}\partial_{\bar{T}}\mu_{\rm sim} \\
G_{\bar{1}} K^{-1}_{1\bar{2}} G_2, \quad G_{\bar{2}} K^{-1}_{2\bar{1}} G_1 &\to 0
\end{aligned} \tag{23}$$

Adding all the above terms and dividing by the determinant $h\tau^{-(2+q)}$ we arrive at our final result for the leading part of the low energy coefficient of the effective μ-term,

$$\frac{\mu_{\rm eff}}{m_{3/2}} = \left\{1 - \tilde{q} - {\rm Re}\,T(\partial_T - \partial_{\bar{T}})\right\}\mu_{\rm sim}(T,\bar{T}) \qquad (24)$$

with $\tilde{q} = \frac{q_1+q_2}{2}$. This formula can be further simplified in models where μ and $\mathcal{M}$ parameters are having simple and well defined properties under the modular transformations. Consider in particular the case where μ is a constant whilst $\mathcal{M}(T,\bar{T})$ has a scaling property under the T and $\bar{T}$ derivatives [5], i.e.,

$$(T+\bar{T})\partial_{T/\bar{T}}\mathcal{M}(T,\bar{T}) = \mathcal{M}(T,\bar{T})$$

In this case, the derivative term in (24) vanishes and the formula takes the simple form

$$\mu'_{\rm eff} = (1-\tilde{q})(e^{\langle\mathcal{K}\rangle/2}\mu + m_{3/2}\mathcal{M}) \qquad (25)$$

We should point out here, that under the above assumptions we can see from (25) that there exists a possibility where the presence of the higgs mixing term $\mathcal{M}$ in the Kähler function does not imply an effective μ-term in the low energy potential, namely when $q_1+q_2=2$. In fact, as we will see in our example, this is the case of a class of string models obtained in the (2,2) compactifications of the heterotic superstring.

As we have explained above, in the case of the intermediate symmetry additional terms can play a role in the mixing of the higgs fields involved in the symmetry breaking. The sub-leading μ-contributions are proportional to $v_i v_j$ and have the highest negative power of τ. We note that such terms can come also from the expansion of Δ. Indeed, from the sub-leading terms of the matrix (4) and the leading, vev-independent, term of Δ, we get the following contribution to the the μ-term (for simplicity we assume $v_i = v_j = v$)

$$\tau^{-\tilde{q}}h^{-1}2(q_1q_2+\tilde{q})v^2$$

while expanding the Δ and taking the sub-leading terms we get

$$\tau^{-\tilde{q}}h^{-1}2(-\tilde{q})v^2$$

Adding up we get the total sub-leading contribution

$$\tau^{-\tilde{q}}h^{-1}2q_1q_2v^2 \qquad (26)$$

For example if $\tilde{q} \equiv (q_1+q_2)/2$ is around 3, then this contribution is a 10% correction to the leading term, assuming $\tau \sim 0.1$ and $v \sim 0.01$, while they are suppressed in the large radius limit.

As an application of the above procedure, we consider the general form of the Kähler function

$$K(T,\bar{T},Q_i,\bar{Q}_i) = -\sum_i h_i \log\left[\prod_n (T_n+\bar{T}_n)^{q_i^n} - \hat{Q}_i\hat{\bar{Q}}_i\right] \qquad (27)$$

where the $\hat{Q}_i$ denote fields which, in general, correspond to linear combinations of the eigenstates. Expanding the logarithm in terms of the eigenstates, one gets

$$K(T,\bar{T},Q_i,\bar{Q}_i) = -\sum_{n,i} h_i q_i^n \log\left(T_n+\bar{T}_n\right) + \mathcal{Z}_{i\bar{j}}Q_i\bar{Q}_i + \tfrac{1}{2}\mathcal{M}_{ij}Q_iQ_j + \cdots \qquad (28)$$

where the matrices $\mathcal{Z}$, $\mathcal{M}$ are proportional to $\prod_n(T_n+\bar{T}_n)^{-q_i^n}$.

Our example is a generalization of the Kähler forms obtained [5] in (2,2) compactifications of the heterotic superstring. Nevertheless, it can be easily seen that as far as we work at the tree level approximation, the approximated Kähler potential has definite properties under the group of modular transforms [7] and it is the same in both cases. We will present here a specific example in order to see how a matrix $\mathcal{M}_{ij}$ may arise. Consider for example the case of two moduli T, U and the fields $Q = A+\bar{B}$, $\bar{Q} = \bar{A}+B$ of Ref. [5], where A, B belong to 27 and $\overline{27}$ of E_6. The Kähler function reads

$$K = -\log\left(T+\bar{T})(U+\bar{U}) - (A+\bar{B})(\bar{A}+B)\right) \qquad (29)$$

where A, B are identified with the higgs fields. Expanding in terms of the latter, one gets at first order the higgs mixing term $\mathcal{M}$

$$\mathcal{M} = \frac{1}{(U+\bar{U})(T+\bar{T})} \qquad (30)$$

which has the same form as in the general case above. Let us now return to our μ-formulae. Due to the properties of the Kähler function and assuming μ constant, we conclude that $\mu_{\rm eff}$ is given by (25). Moreover, in a class of models compactified on an orbifold the untwisted A, B fields associated with the higgses transform as modular forms of weight 1, thus the sum

q_1+q_2 is equal to two, and the total $\mu_{\rm eff}$ vanishes identically. Thus, even if the Kähler potential contains a higgs mixing term of the form (30), due to intriguing cancellations, it is not possible to generate an effective μ-term in the scalar potential within this class of orbifold string constructions unless an explicit- moduli dependent- μ term is present in the superpotential. This is the case of the particular model discussed in [5].

Furthermore, consider a more general case where the μ parameter of the superpotential depends on a phase factor of the form

$$\mu(T,\bar{T}) = \mu_0(T)\left(\frac{\imath cT + d}{-\imath c\bar{T} + d}\right)^{1/2} \tag{31}$$

while assume a scaling property for $\mu_0(T)$, i.e., $(T+\bar{T})\partial_T\mu_0(T) = \mu_0(T)$. The interesting point to note here is that this phase will have an observable effect through the derivative part in the $\mu_{\rm eff}$ formula (24). In fact this term will give a contribution

$$\begin{aligned}&(T+\bar{T})(\partial_T - \partial_{\bar{T}})\mu(T,\bar{T})\\ &= \left\{1 + 2\left(\frac{{\rm Re}\,T}{|\imath T + d/c|}\right)^2\right\}\mu(T,\bar{T})\end{aligned} \tag{32}$$

which is proportional to μ, up to a factor whose existence is due to the μ-phase.

In conclusion, in the context of effective supergravities characterised by properties of compactified heterotic string theories, we have derived a general form of the effective higgs mixing term $\mu_{\rm eff}$ of the low energy effective scalar potential. Using a gauge invariant form of the Kähler function, constrained by modular symmetries, we find additional contributions to $\mu_{\rm eff}$. To leading order, these are found to depend on a specific combination $\mu_{\rm sim}$ of the higgs mixing – moduli-dependent – matrix $\mathcal{M}$ of the Kähler potential and the possible μ-term coefficient of the superpotential as shown in formula (24). Thus, all possible sources can be classified in the following two categories: (i) a term directly proportional to this combination with a proportionality factor $1-\tilde{q}$ where $\tilde{q}$ is half the sum of the modular weights of the two higgs fields breaking the symmetry, and (ii) a derivative term on $\mu_{\rm sim}$ with respect to the moduli $T, \bar{T}$.

We discussed models with properties dictated by modular symmetries where some of these contributions vanish. We further examined cases corresponding to models with intermediate symmetry breaking scales which are not far from the string unification point. There, in addition to the above contributions there are vev-dependent terms which could be important in specific regions of the $\tilde{q}, T$, vev-parameters.

We would like to thank C. Kounnas for helpful discussions. N.D.T. wishes to thank the Theory Division of CERN for kind hospitality. G.K.L. would like to thank J.D. Vergados for a stimulating discussion and I. Antoniadis for useful comments.

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