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Bounding the MSSM Higgs sector from above with the Tevatron's $B_s \to \mu^+\mu^-$

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Abstract

The discovery potential of the Tevatron CDF for the rare B-decay $B_s \to \mu^+\mu^-$ is analysed. We find that with an integrated luminosity of 2 fb^{-1}, and using CDF as the example detector, a 5σ combined discovery reach of the Tevatron is possible if the branching ratio for $B_s \to \mu^+\mu^-$ is $(1.7 \pm 0.46) \times 10^{-7}$. Such a possible signal for the decay $B_s \to \mu^+\mu^-$ will invite large $\tan\beta$ values and set an upper bound on the heaviest mass of the MSSM Higgs sector in a complete analogy to the upper bound of the lightest observable supersymmetric particle set from the excess over the SM prediction of the muon anomalous magnetic moment. If for example, the decay $B_s \to \mu^+\mu^-$ is found at Tevatron with Br$(B_s \to \mu^+\mu^-) = 2 \times 10^{-7}$ then the heaviest Higgs boson mass in the MSSM should be less than 790 GeV for $\tan\beta \lesssim 50$ provided that the CKM matrix is the only source for (s)quark flavour changing processes.
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1. Introduction

In the Minimal Supersymmetric Standard Model (MSSM) the Higgs bosons are *not* scalar superpartners of known Standard Model (SM) fermions. Apart from a naturally light Higgs boson, the rest of the Higgs scalars may well have masses at very high energies despite fine tuning matters. We argue here that if the Tevatron observes $B_s \to \mu^+\mu^-$, the heaviest Higgs boson mass of the MSSM is bounded from above to

be less than 1 TeV for $\tan\beta \lesssim 50$, provided that the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1] is the only source for (s)quark flavour changing neutral current (FCNC) processes and CP violation.

The published Tevatron/CDF physics results with luminosity 171 pb^{-1} is the bound on $B_s \to \mu^+\mu^-$ [2]

$$\text{Br}(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-7}, \quad \text{at 90\% C.L.} \quad (1.1)$$

The predicted ratio for Br$(B_s \to \mu^+\mu^-)$ in the SM is $(3.8 \pm 1.0) \times 10^{-9}$ [3]. The observable $B_s \to \mu^+\mu^-$ makes itself special in the MSSM where it is enhanced by orders of magnitude. This enhancement is due com-

pletely to the MSSM neutral part of the Higgs sector. In particular, $\mathrm{Br}(B_s \to \mu^+\mu^-)$ is proportional to the sixth power of $\tan\beta$ and inverse proportional to the fourth power of the CP-odd (or CP-even) Higgs boson mass, M_A [4]. The idea here is that, since the parameter $\tan\beta$ is theoretically[1] bounded from above and the amplitude $B_s \to \mu^+\mu^-$ exhibits a non-decoupling behaviour in the large SUSY mass limit, any observation of $B_s \to \mu^+\mu^-$ at the Tevatron will set an upper bound on the neutral heavy Higgs masses. We present a quantitative analysis of this upper bound in the MSSM under the assumptions of conserved R-parity symmetry and no extra source for FCNC's other than the CKM matrix itself.

Although the rare decay $B_s \to \mu^+\mu^-$, and the SUSY contribution to the muon anomalous magnetic moment δa_μ, go hand in hand in the minimal Supergravity scenario [17] this is not true in a general unconstrained MSSM which we explore here. It is, however, possible, and has been shown in Ref. [30], that following an argument along the lines of the previous paragraph but for δa_μ, one could set an upper mass bound on the lightest observable supersymmetric particle. For complementarity purposes, we repeat the analysis of Ref. [30] with the up-to-date (and persistently non-zero) measurement of δa_μ.

2. Discovery potential for $B_s \to \mu^+\mu^-$ at Tevatron

In order for $B_{s,d} \to \mu^+\mu^-$ to actually be observable at the Tevatron it would need to appear as a clear, unambiguous signal above background events. Because this is an important signature, potentially indicating unequivocally the existence of beyond the SM physics, a requirement that any observed signal must be at least five standard deviations above the level of background fluctuations would likely be imposed by the experimenters. Consequently, a study of the discovery potential of any future experiment requires that we estimate the amount of background in the experiment as the data increases. Because a detailed description of the background levels, and their uncertainties, exists for the CDF Collaboration, what follows will

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[1] Large $\tan\beta \gtrsim 50$–60 values lead to non-perturbative evolution of the top or bottom Yukawa couplings at higher energies.

draw heavily from their observed background levels [2,31,32].

To estimate the number of signal events and precision obtained in a hypothetical future experiment, we assume that, with no changes in selection criteria for the events from [2], the number of background events in the region of interest around the $B_{s,d}$ masses will be of the following form:

$$N_{\mathrm{bkg}} = \sigma_{\mathrm{bkg}} x, \qquad (2.1)$$

where $x = \int \mathcal{L}\,dt$ and σ_{bkg} is the equivalent cross section of the background in the mass region of interest. We set the discovery level for the number of $B_{s,d} \to \mu^+\mu^-$ candidates (N_{sig}) to be $N_{\mathrm{sig}} \geqslant 5\sqrt{N_{\mathrm{bkg}}}$. Using these relations we present the data in the form

$$\sigma_{B_{s,d}} \mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$$
$$= \frac{N_{\mathrm{sig}}}{2\epsilon\alpha x} \simeq \frac{5\sqrt{\sigma_{\mathrm{bkg}} \pm \xi(x)}}{2\epsilon\alpha\sqrt{x}}, \qquad (2.2)$$

where $\epsilon\alpha$ is the efficiency of the entire selection process multiplied by the detector acceptance. The uncertainty in $\epsilon\alpha$ is independent of the integrated luminosity so this uncertainty has been included as part of the constant error on $\sigma_{\mathrm{bkg}}\xi(x)$ represents the fact that all experiments have uncertainties and that often these uncertainties are based in part on the data themselves. As a result we expect the precision on part of the background estimate will improve as the data collects. The factor of two accounts for the fact that the experiment does not distinguish between the charge conjugate decays.

The advantage of using equation (2.2) is that all the quantities on the right are obtained from a single experimental analysis. In contrast, on the left side of the equation, $\mathrm{Br}(B_{s,d} \to \mu^+\mu^-)$ is the quantity of interest and $\sigma_{B_{s,d}} = \frac{f_{s,d}}{f_u}\sigma(B^+)$ depends on B meson fragmentation measurements from other experiments and a cross section measurement in a different decay mode (and for this Letter, at a different $\sqrt{s}$). So Eq. (2.2) allows the reader to more easily adjust estimates of experimental reach for different integrated luminosities, fragmentation estimates, and B meson or b quark cross section measurements.

Henceforth we will focus on the decay $\mathrm{Br}(B_s \to \mu^+\mu^-)$ only. In 171 pb^{-1} the CDF experiment reports $N_{\mathrm{bkg}} = 1.05 \pm 0.31$ events. The largest uncertainty is actually luminosity dependent because it comes from
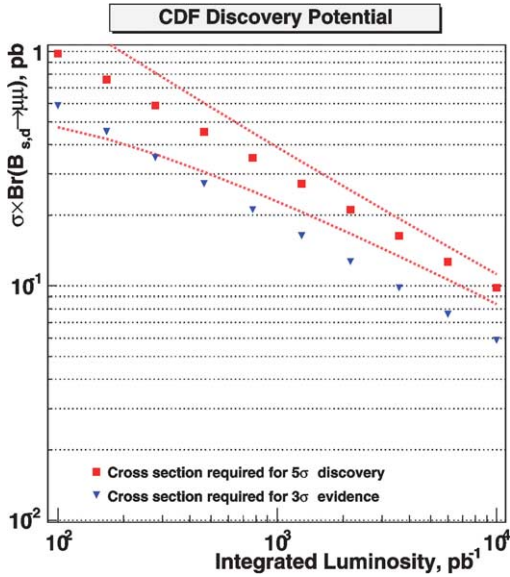
Fig. 1. Tevatron's CDF discovery potential for the product of the production cross section times the branching ratio $[\sigma(p\bar{p} \to B_{s,d} + X)\, \mathrm{Br}(B_s \to \mu^+\mu^-)]$ vs. integrated luminosity. The squares (triangles) are the center points for 5σ (3σ) observation. The dashed lines delineate the uncertainty in the discovery cross section (both statistical and systematic) as a function of integrated luminosity.

Table 1
Tabulated are discovery branching ratios for CDF at three different integrated luminosities. The uncertainties are from the background estimate in [2]

$\int \mathcal{L}\,dt$, fb^{-1}	$\sigma_{B_s}\,\mathrm{Br}(B_s \to \mu^+\mu^-)$, pb	$\mathrm{Br}(B_s \to \mu^+\mu^-) \times 10^{-7}$
0.5	0.44 ± 0.14	4.8 ± 1.7
2.0	0.22 ± 0.05	2.4 ± 0.65
10.0	0.098 ± 0.014	1.1 ± 0.24

Taking the 5σ values from Fig. 1 for 0.5, 2.0, and 10 fb^{-1} and using[2] $\frac{f_s}{f_u} = \frac{0.100}{0.391}$ and $\sigma(B+) = 3.6 \pm 0.6$ μb [33] we obtain the results in Table 1 for the branching ratio $\mathrm{Br}(B_s \to \mu^+\mu^-)$. The analysis above makes use of the detailed reports on this decay from the CDF experiment alone. In order to obtain a Tevatron estimate one would need to include the DØ data as well. There is a preliminary estimate for an upper limit of $\mathrm{Br}(B_s \to \mu^+\mu^-) = 4.6 \times 10^{-7}$ at 95% C.L. on DØ's public analysis web site [34].

It is difficult to account for all of the differences between the two experiments, some of which improve and some of which detract from the relative sensitivity between the two experiments, but it is clear that a limit on this decay mode similar to CDFs has been obtained. Consequently we felt justified in lowering our final estimate of Tevatron reach by a factor of $\sqrt{2}$ but present only the CDF-derived values in the Letter because of a better understanding of that detector by one of the authors.

3. Bounding the MSSM Higgs and SUSY sectors from above

As we stated in the introduction, excesses of the experimental data over the SM expectation for observables like the muon anomalous magnetic moment α_μ, and the rare B-meson decay $B_s \to \mu^+\mu^-$, would result in bounding from above the SUSY and the Higgs sectors, respectively. In this section, we quantify these statements.

At large $\tan\beta$, the leading supersymmetric contribution to $\mathrm{Br}(B_s \to \mu^+\mu^-)$ is depicted in Fig. 2. Its

the statistical fluctuations of 14 events that remain in the unblinded region after some of the final analysis cuts were relaxed. N_{bkg} can be converted into an equivalent background cross section using

$$\sigma_{\mathrm{bkg}} \pm \xi(x) = N_{\mathrm{bkg}}/x_0$$

$$= 6.14 \times 10^{-3} \pm 2.9 \times 10^{-4} \pm \frac{0.023}{\sqrt{x}} \text{ pb}.$$

For this Letter $x_0 = 171$ pb^{-1} while $\epsilon\alpha = 0.020 \pm 0.002$.

Fig. 1 shows how the $\sigma\,\mathrm{Br}(B_s \to \mu^+\mu^-)$ would scale with increasing integrated luminosity given that this decay is sufficiently prolific to allow a discovery using parameters from the CDF experiment. The uncertainty band shown here reflects the uncertainty in the background estimate and takes into account the additional statistical uncertainty that would be added in quadrature if this decay was seen at the 5σ level.

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[2] f_x is the probability that a b-quark fragments into a given B meson. The most recent results from the Particle Data Group are used. http://pdg.lbl.gov/.
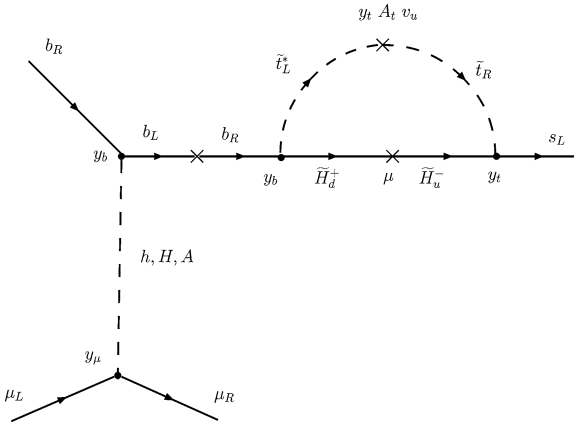
Fig. 2. The leading supersymmetric contribution to $B_s \to \mu^+\mu^-$ at large $\tan\beta$. "Crosses" and "dots" indicate mass insertions and Yukawa vertices, respectively, and their corresponding interaction parameters are explicitly given.

amplitude depends mainly on 6 beyond the SM parameters, namely, the CP-odd Higgs mass M_A (or the CP-even Higgs boson mass because $M_A \simeq M_H$), the ratio of the vacuum expectation values, $\tan\beta$, the soft supersymmetry breaking trilinear coupling A_t and the two scalar superpartners of the top quark masses, $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$. It is well known [4], that the self energy diagram of Fig. 2, enhances the ratio $\mathrm{Br}(B_s \to \mu^+\mu^-)$ in the MSSM by many orders of magnitude. It is worth making a parenthesis here in order to understand why this is so.

The leading contribution in $\mathrm{Br}(B_s \to \mu^+\mu^-)$, comes from the amplitude in Fig. 2. Its calculation can be sketched with the following steps

$$\mathcal{A}(b_R \to s_L \bar{\mu}_R \mu_L)$$
$$\sim y_b \frac{1}{m_b} y_b V_{tb} \mu y_t A_t v_u y_t$$
$$\times V_{ts}^* f(\mu, m_{\tilde{t}_L}, m_{\tilde{t}_R}) \frac{1}{M_A^2} y_\mu \qquad (3.1)$$

$$\hookrightarrow V_{tb} V_{ts}^* y_\mu y_b y_t^2 \frac{\tan\beta}{M_A^2} \left(\frac{\mu A_t}{m_{\tilde{t}}^2} \right) \qquad (3.2)$$

$$\hookrightarrow V_{tb} V_{ts}^* m_\mu m_b m_t^2 \frac{\tan^3\beta}{M_A^2}, \qquad (3.3)$$

where V is the CKM matrix. The function $f(\mu, m_{\tilde{t}_L}, m_{\tilde{t}_R})$ is a loop factor which, in the limit of heavy squark masses, behaves like $1/m_{\tilde{t}}^2$. $m_{\tilde{t}}^2$ denotes the

geometric mean squark mass, i.e., $m_{\tilde{t}}^2 = m_{\tilde{t}_L} m_{\tilde{t}_R}$. This limit is taken when passing from Eq. (3.1) to Eq. (3.2). The approximation symbol "$\sim$" stands for constant, $\mathcal{O}(1)$, numerical factors irrelevant to our point here.

Eq. (3.2), shows that the amplitude is proportional to one generic (i.e., not originated from the conversion of a Yukawa coupling to a quark or lepton mass) $\tan\beta$ parameter. The other two powers of $\tan\beta$ arise in Eq. (3.3) from the fact that the Higgs field mediating the amplitude is the heavy one and at large $\tan\beta$, is $(y_{b,\mu} \propto m_{b,\mu} \tan\beta)$. The reader should also notice that the light Higgs boson coupling to the bottom quarks or to the leptons is *not* proportional to $\tan\beta$ and thus leads to subdominant contributions. It is very interesting to note that, in the SUSY decoupling limit, $\mu \sim A_t \sim m_{\tilde{t}} \to \infty$ in Eq. (3.2), the "Higgs penguin" *does not* decouple from the MSSM. This non-decoupling property makes the rare decay $B_s \to \mu^+\mu^-$ visible even for multi-TeV superparticle spectrum.

To summarise, squaring the amplitude in (3.3) we obtain

$$\mathrm{Br}(B_s \to \mu^+\mu^-) \propto \frac{\tan^6\beta}{M_A^4}. \qquad (3.4)$$

Thus given an upper "theoretical" bound for $\tan\beta$ or measuring $\tan\beta$ by other means, an observation of the decay $B_s \to \mu^+\mu^-$ at Tevatron such as the one we described in Table 1, will set an upper bound on M_A.

In our numerical analysis, we make use of two independent calculations for $\mathrm{Br}(B_s \to \mu^+\mu^-)$. The first contains the 1-loop calculation, including corrections after the electroweak symmetry breaking [9] with the method of $\tan\beta$ resummation as described[3] in [17]. Our second calculation is based on the effective Lagrangian technique [6,11,13,14]. In particular, we follow the work [13] where a general resummed effective Lagrangian for Higgs-mediated FCNC interactions in the MSSM has been derived, without restrictions to particular assumptions that rely on the quark-Yukawa structure of the theory or CP conservation. For large $\tan\beta$ the two calculations are in good agreement.

Our calculation of supersymmetric contributions to muon anomalous magnetic moment δa_μ, follows the

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[3] Basically, one has to replace the two bottom Yukawa couplings y_b, in Fig. 2 with the effective $y_b/(1 + \Delta m_b)$, where $\Delta m_b \propto \mu \tan\beta$ depends on the gluino and sbottom masses [35]. Notice that in Eq. (3.2) only one bottom Yukawa coupling survives at the end.

work in Ref [36]. Recently, two-loop $\tan\beta$-enhanced corrections have been carried out [37]. These corrections account for $\delta a_\mu \lesssim 1 \times 10^{-10}$ if the sparticles are heavier than approximately 400 GeV and they are neglected in our analysis here. Finally, for the Higgs mass bound we use the numerical code from [38].

We use a high statistics scan over the unconstrained MSSM[4] parameter space in the region where the soft breaking masses are below 2.5 TeV. We assume that the FCNC couplings originate only from the CKM matrix. Furthermore, we assume that the Lightest Supersymmetric Particle (LSP) is stable and neutral.[5] That means the LSP is either a neutralino or sneutrino. For each set of outputs we define the Lightest Observable Supersymmetric Particle (LOSP) as the second lightest SUSY particle or the third if the first two LSPs are both neutralino and sneutrino. Finally, we scan only over the positive sign of the μ-parameter consistent with the bound from $b \to s\gamma$. In any case, results for $\mathrm{Br}(B_s \to \mu^+\mu^-) \sim 10^{-7}$, are independent of this assumption (see Eq. (3.2)). The unification assumption $M_1 = M_2/2$ has been assumed for simplicity. We have imposed the current experimental bounds from the direct SUSY searches at LEP and Tevatron [39]. For the light Higgs boson we have set a rather conservative constraint $M_h \geqslant 90$ GeV since we work in the large $\tan\beta$ regime.

Our results are summarised in Fig. 3. Fig. 3(a) displays the ratio $\mathrm{Br}(B_s \to \mu^+\mu^-)$ with respect to the CP-odd Higgs mass, M_A. Clearly, $\mathrm{Br}(B_s \to \mu^+\mu^-)$ can be three or even four orders of magnitude bigger than the SM expectation and certainly within the 5σ Tevatron/CDF reach as shown in the figure, for 2 fb^{-1} (see Table 1 for other choices of luminosity). Even for very large $M_A \sim 1$ TeV, the ratio can still be enhanced by an order of magnitude relative to its SM prediction. However, as M_A increases, $\mathrm{Br}(B_s \to \mu^+\mu^-)$ cannot be bigger than certain values. The envelope contour in

Fig. 3(a) is well approximated with

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = 5 \times 10^{-7} \left(\frac{\tan\beta}{50}\right)^6 \left(\frac{650\,\mathrm{GeV}}{M_A}\right)^4$$
$$+ 1.0 \times 10^{-8}. \qquad (3.5)$$

We have checked that Eq. (3.5) fits well for larger values of $\tan\beta$. Now, from Eq. (2.2), if CDF with 2 fb^{-1} (10 fb^{-1}) detects 17 (88) signal events for the $B_s \to \mu^+\mu^-$ that means that $\mathrm{Br}(B_s \to \mu^+\mu^-) \approx 2 \times 10^{-7}$ and the heaviest Higgs boson mass will be less than 790 GeV for *all* $\tan\beta \lesssim 50$.

A more "natural" prediction for $\mathrm{Br}(B_s \to \mu^+\mu^-)$ is approximated with the inner curve

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = 5 \times 10^{-7} \left(\frac{\tan\beta}{50}\right)^6 \left(\frac{300\,\mathrm{GeV}}{M_A}\right)^4$$
$$+ 5.0 \times 10^{-9}, \qquad (3.6)$$

and represents the statistically populated area of points. Using Fig. 3(a) and our analysis shown in Table 1 for 5σ $B_s \to \mu^+\mu^-$ discovery at Tevatron/CDF, we conclude that for $\tan\beta \leqslant 50$,

$$M_A \lesssim 660\ (305)\ \mathrm{GeV} \quad \text{with } x = 0.5\ \mathrm{fb}^{-1},$$
$$M_A \lesssim 790\ (360)\ \mathrm{GeV} \quad \text{with } x = 2\ \mathrm{fb}^{-1},$$
$$M_A \lesssim 970\ (440)\ \mathrm{GeV} \quad \text{with } x = 10\ \mathrm{fb}^{-1}, \qquad (3.7)$$

where the numbers in parenthesis fall into Eq. (3.6). Further uncertainties due to the B_s decay constant f_{B_s}, are also reduced when calculating the M_A, as compared to the ones involved in the calculation of $\mathrm{Br}(B_s \to \mu^+\mu^-)$, since it is $\sqrt{f_{B_s}}$ vs. $f_{B_s}^2$. An error of 5–10% on M_A can still be present but can be further reduced once ΔM_s is measured, possibly at Tevatron, along the lines of the last paper in Ref. [3]. Eq. (3.7) is the main result of our Letter.

The current BNL [40] experimental result on the muon anomalous magnetic moment, shows a 2.7σ difference relative to the SM expectation. The deviation from the SM value $\delta a_\mu = \alpha_\mu^{\mathrm{exp}} - \alpha_\mu^{\mathrm{SM}}$ at 90% C.L. reads [41]

$$12.9 \times 10^{-10} \lesssim \delta a_\mu \lesssim 36.0 \times 10^{-10}, \qquad (3.8)$$

with a central value $\delta a_\mu = 24.5 \times 10^{-10}$ [41]. A possible excess can originate from six supersymmetric mass parameters, namely, the gaugino masses M_1 and M_2, the Higgsino mass parameter μ, two supersymmetric scalar partner masses of the muons, $m_{\tilde{\mu}_L}$ and $m_{\tilde{\mu}_R}$, and

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[4] Unconstrained MSSM means here that we impose no restrictions on the boundary conditions for squark and slepton masses or trilinear soft SUSY breaking couplings at the GUT scale. We do not also require radiative electroweak symmetry breaking.

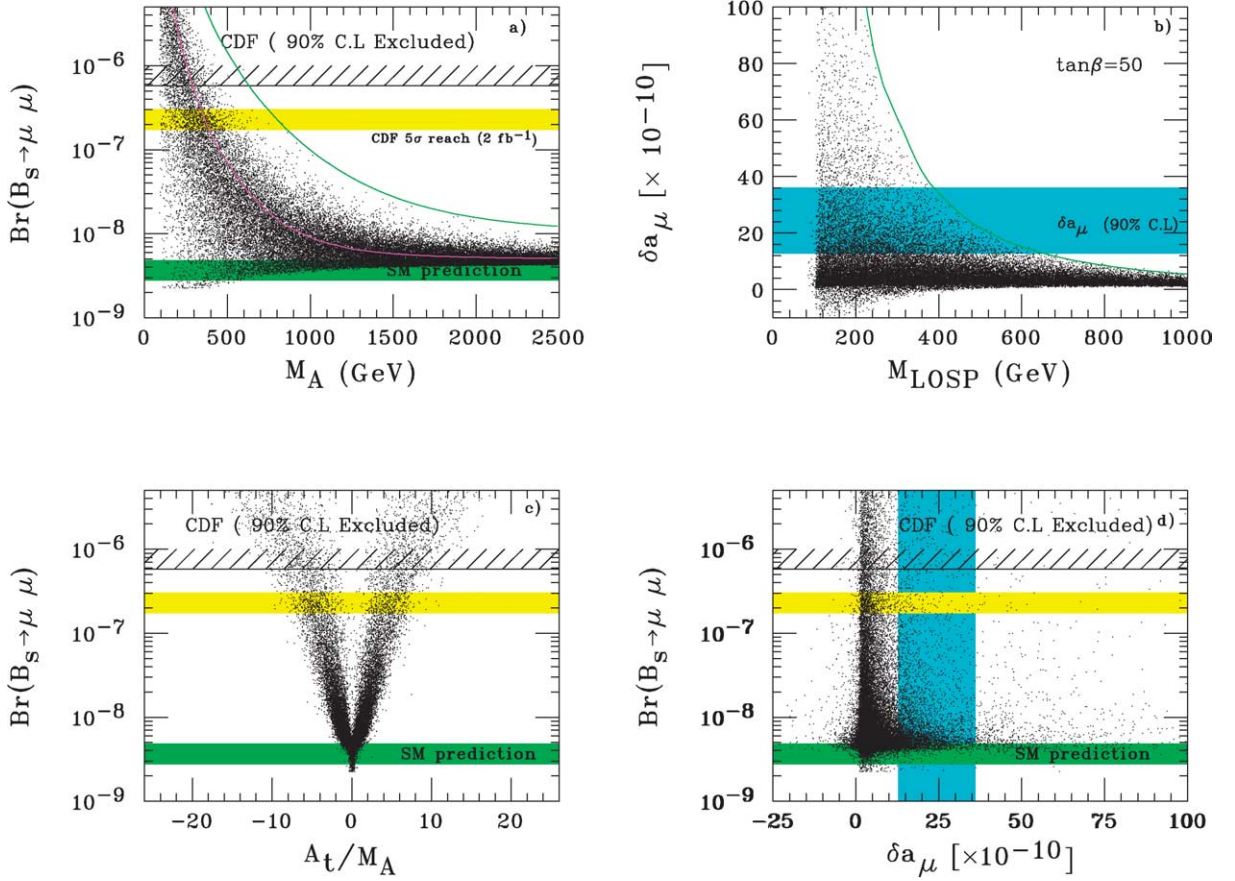[5] This is only necessary for the R-parity conserved (we assume here) models.

Fig. 3. (a) The $\mathrm{Br}(B_s \to \mu^+\mu^-)$ vs. the CP-odd Higgs mass in the MSSM. The lower shaded band shows the SM prediction for $\mathrm{Br}(B_s \to \mu^+\mu^-)$. The 90% C.L. excluded area from Tevatron/CDF [2] is also displayed. Following our results from Table 1, the 5σ reach at $2\ \mathrm{fb}^{-1}$ for $\mathrm{Br}(B_s \to \mu^+\mu^-)$ is also depicted (upper shaded band). (b) The SUSY contribution to the muon anomalous magnetic moment, δa_μ versus the mass of the Lightest Observed Supersymmetric Particle (LOSP). Black points correspond to neutralino or sneutrino LSP. The shaded region is the allowed value on δa_μ at 90% C.L. [40,41]. (c) The $\mathrm{Br}(B_s \to \mu^+\mu^-)$ vs. the ratio of the soft SUSY breaking trilinear parameter with the CP-odd Higgs mass, A_t/M_A. (d) Combined predictions for δa_μ and $\mathrm{Br}(B_s \to \mu^+\mu^-)$ in the MSSM. For all the figures here we have chosen $\tan\beta = 50$ and $m_t = 178$ GeV.

$\tan\beta$. Then, in the mass insertion approximation, one may write this contribution simply as [30]

$$\delta a_\mu \simeq \frac{g_i^2}{16\pi^2} m_\mu^2 \mu M_i \tan\beta F, \qquad (3.9)$$

with $F \propto 1/M_{\mathrm{SUSY}}^4$ being a loop factor depending on the supersymmetric masses (charginos, neutralinos, smuons). Feng and Matchev in [30] used the excess on the muon anomalous magnetic moment to set an upper mass bound on the Lightest Observable Supersymmetric Particle (M_{LOSP}). We repeat their analysis in Fig. 3(b), where we plot the supersymmetric contribution δa_μ, versus the mass of the LOSP. The

current excess on δa_μ already sets an upper bound $M_{\mathrm{LOSP}} \lesssim 700$ GeV, on the LOSP mass. This is clear from the envelope solid contour in Fig. 3(b), which is described by the equation

$$\delta a_\mu = 18 \times 10^{-10} \frac{\tan\beta}{50} \left(\frac{550\ \mathrm{GeV}}{M_{\mathrm{LOSP}}}\right)^2. \qquad (3.10)$$

Thus the current central value [40][6] $\delta a_\mu = 24.5 \times 10^{-10}$ implies that the lightest observable supersymmetric particle should weigh less than 450 GeV. The analogy with the ratio $\mathrm{Br}(B_s \to \mu^+\mu^-)$ in Fig. 3(a) is obvious.

Notice that, from the Fig. 3(a) one cannot set a lower bound on the CP-odd Higgs mass from the Tevatron/CDF experimental bound alone. The parameter which can be excluded by CDF is the ratio of the soft SUSY breaking trilinear parameter with the CP-odd Higgs mass, A_t/M_A. This bound can be understood easily from Eq. (3.2) and can be derived from Fig. 3(c) to be

$$\frac{|A_t|}{M_A} \lesssim 12 \quad \text{for } \tan\beta \gtrsim 50. \tag{3.11}$$

Combining Eqs. (3.11) and (3.7) we arrive at the conclusion that any observation of $\mathrm{Br}(B_s \to \mu^+\mu^-)$ will not only bound the Higgs sector from above but will also set an upper bound to the top trilinear soft SUSY breaking coupling. A soft supersymmetry breaking parameter is bounded by a Higgs sector parameter!

Finally, in Fig. 3(d) we plot the two observables δa_μ and $\mathrm{Br}(B_s \to \mu^+\mu^-)$ together. Since both are saturated by different mass scales (δa_μ by the SUSY and $\mathrm{Br}(B_s \to \mu^+\mu^-)$ by the Higgs mass scale)), we do not expect any correlation between them in the general unconstrained MSSM. This is exactly what we obtain in Fig. 3(d). In fact, the statistical distribution of the points with large $\mathrm{Br}(B_s \to \mu^+\mu^-)$ and small δa_μ is wider than the other way around (we have scanned the parameter space uniformly). However, in the mSUGRA scenario, the SUSY mass M_{LOSP} and the CP-odd Higgs mass, M_A, are related (mainly) through the parameter $M_{1/2}$. The mSUGRA points in Fig. 3(d) lay along the diagonal in the δa_μ–$\mathrm{Br}(B_s \to \mu^+\mu^-)$ plane. As it was remarked in Ref. [17], a discovery of $\mathrm{Br}(B_s \to \mu^+\mu^-)$ at the Tevatron and a deviation on a_μ (as is currently the case [44]) on both observables with respect to their SM values will definitely favour some kind of mSUGRA scenario.

We have checked the robustness of our results presented in Fig. 3 against (i) a general scan in the multi-TeV region of soft supersymmetry breaking masses, (ii) inclusion of various supersymmetric phases, and (iii) the inclusion from the gluino loop corrections. The last two contributions have been implemented in the analysis of Ref. [13]. We find numerically that these contributions do not spoil the upper bound of Eq. (3.7). However, one expects different bounds in the case of non-trivial squark mass intergenerational mixings in the supersymmetry breaking sector. The effects of the latter will not be missed at B-factories, colliders, or other non-accelerator experiments; where strong constraints on them already exist.

We note in passing, that Tevatron [2] and B-factories [45] search for the B_d leptonic decay, $B_d \to \mu^+\mu^-$. Current limits obtained in the region (few $\times$ 10^{-7}). The branching ratio $\mathrm{Br}(B_d \to \mu^+\mu^-)$ is smaller than $\mathrm{Br}(B_s \to \mu^+\mu^-)$ by a factor $|V_{td}/V_{ts}|^2 \lesssim 0.06$. Any observation at Tevatron for $B_s \to \mu^+\mu^-$ is likely to be accompanied with an observation of $B_d \to \mu^+\mu^-$ at B-factories if the Higgs sector is responsible for making these leptonic rare decays visible.

4. Epilogue

We investigate the discovery potential of the Tevatron hadron collider, focusing mainly on the CDF experiment, for the rare decay $B_s \to \mu^+\mu^-$. We find that Tevatron can reach a 5σ (3σ) observation for B_s-cross section times $\mathrm{Br}(B_s \to \mu^+\mu^-)$ with luminosity given in Fig. 1. Based on these results and the knowledge of the $\sigma(B_s)$ from the Tevatron we show in Table 1 the 5σ discovery bands for $\mathrm{Br}(B_s \to \mu^+\mu^-)$ at several luminosities. For example, the Tevatron/CDF with integrated luminosity 2 fb^{-1}, can discover $B_s \to \mu^+\mu^-$ with a ratio $\mathrm{Br}(B_s \to \mu^+\mu^-) = (2.4 \pm 0.65) \times 10^{-7}$. If this is true, we show that in the MSSM, with the assumption that flavour changing processes originate exclusively from the CKM matrix, the Higgs sector is bounded from above with the heavy Higgs bosons weight no more than 790 GeV for $\tan\beta \lesssim 50$ (see Fig. 3(a) and Eq. (3.7)). The situation is completely analogous with the muon anomalous magnetic moment searches, where the current excess over the SM prediction seems to indicate an upper bound of

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[6] The value for δa_μ in Eq. (3.8) is the 1.28σ range (equivalent to 90% C.L.) of the value given (in Note added in proof) in Ref. [41]. New estimates on the light by light scattering [42], as well as QED α^4 contributions [43], have been taken into account when quoting this value.

700 GeV on the lightest observable supersymmetric particle.

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