Bounds on *R*-parity violating couplings at the weak scale and at the GUT scale

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We analyze bounds on trilinear *R*-parity violating couplings at the unification scale by renormalizing the weak scale bounds. We derive unification scale upper bounds upon the couplings which are broadly independent of the fermion mass texture assumed. The *R*-parity violating couplings are factors of 2–5 more severely bounded at the unification scale than at the electroweak scale. In the presence of quark mixing, a few of the bounds are orders of magnitude stronger than their weak scale counterparts due to new *R*-parity violating operators being induced in the renormalization between high and low scales. These induced bounds are fermion mass texture dependent. New bounds upon the weak scale couplings are obtained by the requirement of perturbativity between the weak and unification scales. A comprehensive set of the latest limits is included. [S0556-2821(99)04819-5]

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I. INTRODUCTION

When constructing the most general supersymmetric version of the standard model (SM) there are baryon- and lepton-number violating operators in the superpotential. These lead to rapid proton decay in disagreement with the strict experimental bounds [1]. Therefore, an extra symmetry beyond the SM gauge symmetry, $G_{\mathrm{SM}}=SU(3)\times SU(2)\times U(1)$, must be imposed to protect the proton. In most cases the discrete multiplicative symmetry, *R* parity (R_p) [2], is chosen. This prohibits all baryon- and lepton-number violating operators with mass dimension less or equal to 4 and leads to the minimal set of couplings consistent with the data. The resulting model is denoted the minimal supersymmetric standard model (MSSM) [3]. However, the choice of R_p is *ad hoc*. There are other symmetries which are theoretically equally well motivated [4] and which also prohibit rapid proton decay, e.g. both baryon-parity and lepton-parity. Baryon-parity even prohibits the dangerous dimension 5 operators [5]. For both baryon and lepton parity, R_p is violated ($\not{R}_p$).

There is at present no direct experimental evidence for supersymmetry and in particular no evidence for R_p or $\not{R}_p$ [1]. Theoretical models are our best guide. Ultimately we expect the weak-scale theory to be embedded in a more fundamental unified theory formulated at a significantly higher energy scale which should also be the origin of R_p or $\not{R}_p$. There is an extensive list of models with R_p [6]. However, $\not{R}_p$ grand unified models have been constructed for the gauge groups $SU(5)$ [7–11], $SU(5)\times U(1)$ [12,8,9], E_6 [13] and $SO(10)$ [8], as well, and there are also string models of $\not{R}_p$ [14]. At present no model is clearly preferred.

Grand unified theories (GUTs) typically make predictions for ratios of Yukawa couplings, e.g. m_b/m_τ [15]. If the GUT is extended to include a family symmetry for example via the Frogatt-Nielsen mechanism [16], a prediction is obtained for the order of magnitude of the Higgs Yukawa couplings. Since the quantum numbers are fixed, these predictions can be extended to the $\not{R}_p$-Yukawa couplings: see for example [17–19]. In string theories the Yukawa couplings are also in principle calculable.

When constructing an $\not{R}_p$ model at high energy, it is essential that it is consistent with all experimental bounds on baryon- and lepton-number violation. There are empirical bounds on all of the $\not{R}_p$-Yukawa couplings [20–23], some of which are quite strict. However, these bounds are all determined at the weak scale. They can therefore not be directly compared to the predictions of the unified models, which are at the GUT scale (M_{GUT}) or higher. There are at present no bounds for $\not{R}_p$ couplings at the GUT scale. In order to compare the unification predictions with the data we must employ the renormalization group equations (RGEs) for the $\not{R}_p$-Yukawa couplings. These equations have recently been given up to two-loop order with the full $\not{R}_p$ flavor structure in [24]. The effect of running the couplings from the weak scale to the GUT scale can be substantial [25,24].

It is the purpose of this paper to first update the weak-scale bounds on $\not{R}_p$ couplings and then to translate these bounds in a model independent way into GUT scale bounds.[1] For this we employ the full one-loop RGEs of the $\not{R}_p$-MSSM [24].[2] In order to obtain the GUT-scale bounds we assume a single coupling at the GUT scale in the current eigenstate basis. After running the RGEs, we obtain a set of couplings

[1]We do not discuss the bounds on the bilinear coupling of the superpotential term $\kappa_i L_i H_2$, since this analysis needs knowledge of the μ parameter possibly combined with the radiative electroweak symmetry breaking scenario, and we postpone it to a forthcoming article.

[2]Since we allow for the fully generated flavor structure of the $\not{R}_p$ couplings, a full calculation at two loops using the equations of [24] would be too complicated.

at the weak scale, both from the flavor structure of the RGEs and from the rotation into the mass eigenstate basis.[3] We compare this set with the existing weak-scale bounds, including bounds on products of couplings. We also include perturbativity bounds where they are more stringent than the empirical ones. The bounds on the induced couplings often lead to significantly stronger bounds on the GUT-scale couplings.

II. LOW ENERGY BOUNDS

The first systematic study of low-energy bounds on the R-parity violating Yukawa couplings was performed in [20]. Since then, updates have been performed in [22,21]. More recently there was a very nice thorough update of all the bounds on the lepton-number violating couplings performed in [27]. We present in Table I an updated version of the strongest bound at two standard deviations (2 sigma) on each coupling, respectively. For the lepton-number violating couplings we update the results from [27] using the more recent data compiled by the particle data group [1]. The main difference from [27] is due to the improved data on the tau lepton parameters. In the case of atomic parity violation we have made use of new experimental data [28] which is not yet included in [1] and which leads to a new value of Q_W. This differs from the standard model value by 2.5 sigma. Thus we quote a 3 sigma bound. We do not include the recent bounds obtained from R_b [29]. Though they are the best bounds at 1 sigma, they are very weak at 2 sigma.

In Table II we present a compilation of the bounds on the product of two couplings. We have updated the bound from the decay $K^+ \to \pi^+ \nu \bar{\nu}$ [30] with the new data in [1,31]. We have then translated this bound into a bound on the product of two couplings. In [30] the assumption was explicitly made that at the weak scale there is only one dominant coupling in the quark current basis. As described below this is not necessarily true for our studies.

III. FRAMEWORK AND NUMERICAL INPUTS

The chiral superfields of the MSSM have the following $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers:

$$L:\left(1,2,-\frac{1}{2}\right), \quad \bar{E}:(1,1,1), \quad Q:\left(3,2,\frac{1}{6}\right),$$
$$\bar{D}:\left(3,1,-\frac{1}{3}\right), \quad H_1:\left(1,2,-\frac{1}{2}\right), \quad H_2:\left(1,2,\frac{1}{2}\right),$$
$$\bar{U}:\left(3,1,\frac{2}{3}\right). \tag{1}$$

We write the $\not{R}_p$-MSSM superpotential as

$$W = \epsilon_{ab}\Big[(\mathbf{Y}_E)_{ij} L_i^a H_1^b \bar{E}_j + (\mathbf{Y}_D)_{ij} Q_i^{ax} H_1^b \bar{D}_{jx} + (\mathbf{Y}_U)_{ij} Q_i^{ax} H_2^b \bar{U}_{jx} + \frac{1}{2}\lambda_{ijk} L_i^a L_j^b \bar{E}_k + \lambda'_{ijk} L_i^a Q_j^{xb} \bar{D}_{kx} + \mu H_1^a H_2^b + \kappa^i L_i^a H_2^b\Big] + \frac{1}{2}\epsilon_{xyz}\lambda''_{ijk} \bar{U}_i^x \bar{D}_j^y \bar{D}_k^z. \tag{2}$$

We denote an $SU(3)$ color index of the fundamental representation by $x,y,z = 1,2,3$. The $SU(2)_L$ fundamental representation indices are denoted by $a,b,c = 1,2$ and the generation indices by $i,j,k = 1,2,3$. We have introduced the three 3×3 matrices

$$\mathbf{Y}_E, \quad \mathbf{Y}_D, \quad \mathbf{Y}_U, \tag{3}$$

for the R_p conserving Yukawa couplings.

The boundary values of the running dimensional reduction scheme ($\overline{\mathrm{DR}}$) gauge couplings $g_1(M_Z)$ and $g_2(M_Z)$ can be determined in terms of the modified minimal subtraction scheme ($\overline{\mathrm{MS}}$) values of $\alpha_{EM}^{-1}(M_Z) = 127.9$ and $\sin^2\theta_W(M_Z) = 0.2315$. M_{GUT} is found by the condition $\alpha_1(M_{\mathrm{GUT}}) = \alpha_2(M_{\mathrm{GUT}})$. Because above M_Z we work to one loop order only, $M_{\mathrm{GUT}} = 2.1\times 10^{16}$ GeV is independent of any Yukawa couplings. The relation $\alpha_3(M_{\mathrm{GUT}}) = \alpha_2(M_{\mathrm{GUT}})$ is used to fix the strong coupling constant[4] $\alpha_3(M_Z) = 0.118$.

We use the following experimentally determined fermion mass parameters[5] (in GeV):

$$m_b(m_b) = 4.25, \quad m_t^{\mathrm{pole}} = 175, \quad m_\tau(m_\tau) = 1.777, \tag{4}$$
$$m_s = 0.12, \quad m_c(m_c) = 1.25, \quad m_\mu = 0.105,$$
$$m_d = 0.006, \quad m_u = 0.003, \quad m_e = 0.000511,$$

where m_i are listed in the $\overline{\mathrm{MS}}$ renormalization scheme except for the pole mass of the top quark, m_t^{pole}. The masses of the fermions are determined to 3 loops in QCD and 2 loops in QED [32] in the $\overline{\mathrm{MS}}$ scheme and at the scale M_Z. They are then converted into $\overline{\mathrm{DR}}$ diagonal Yukawa couplings using

$$h_{d,s,b,e,\mu,\tau}(M_Z) = \frac{m_{d,s,b,e,\mu,\tau}(M_Z)}{\sqrt{2} v \cos\beta},$$

[3]For a detailed discussion of the basis dependence of the $\not{R}_p$ couplings see [26].

[4]Note that the extracted value of $\alpha_3(M_Z)$ at one-loop accuracy and without sparticle splitting threshold effects is in excellent agreement with the experimental data. The R-parity violating couplings do not affect the running α_3 at one loop accuracy but they do at the two-loop level [24]. However, the effects are small ($\lesssim 2\%$) for λ, λ', $\lambda'' \lesssim 0.9$.

[5]For quarks and leptons with masses less than 1 GeV, their running masses have been determined at the scale $Q = 1$ GeV. As we go down to $Q = 1$ GeV from $Q = M_Z$ we decouple quarks or leptons when $m(Q) = Q$. In the case of the top quark, only QCD corrections have been taken into the calculation of its running mass from the pole mass listed here.

TABLE I. Latest 2σ limits on the magnitudes of weak scale trilinear R-parity violating couplings from indirect decays and perturbativity. The dependence on the relevant superparticle mass is shown explicitly. When the perturbativity bounds are more stringent than the empirical bounds for masses $m_{\tilde{l},\tilde{q}}=1$ TeV, then we display them in parentheses. Where a bound without parentheses has no explicit mass dependence shown, the mass dependence was too complicated to detail here and a degenerate sparticle spectrum of 100 GeV is assumed.

ijk	$\lambda_{ijk}(M_Z)$[a]	$\lambda'_{ijk}(M_Z)$[b]	$\lambda''_{ijk}(M_Z)$[c]
111	-	$5.2\times 10^{-4}\times f(\tilde{m})$	-
112	-	$0.021\times\frac{m_{\tilde{s}_R}}{100\text{ GeV}}$	$2\times 10^{-9}\left(\frac{m_{\tilde{q}}}{100\text{ GeV}}\frac{.3\text{ GeV}}{\tilde{\Lambda}}\right)^{5/2}$
113	-	$0.021\times\frac{m_{\tilde{b}_R}}{100\text{ GeV}}$	10^{-4}
121	$0.049\times\frac{m_{\tilde{e}_R}}{100\text{ GeV}}$	$0.043\times\frac{m_{\tilde{d}_R}}{100\text{ GeV}}$	$2\times 10^{-9}\left(\frac{m_{\tilde{q}}}{100\text{ GeV}}\frac{.3\text{ GeV}}{\tilde{\Lambda}}\right)^{5/2}$
122	$0.049\times\frac{m_{\tilde{\mu}_R}}{100\text{ GeV}}$	$0.043\times\frac{m_{\tilde{s}_R}}{100\text{ GeV}}$	-
123	$0.049\times\frac{m_{\tilde{\tau}_R}}{100\text{ GeV}}$	$0.043\times\frac{m_{\tilde{b}_R}}{100\text{ GeV}}$	(1.23)
131	$0.062\times\frac{m_{\tilde{e}_R}}{100\text{ GeV}}$	$0.019\times\frac{m_{\tilde{t}_L}}{100\text{ GeV}}$	10^{-4}
132	$0.062\times\frac{m_{\tilde{\mu}_R}}{100\text{ GeV}}$	$0.28\times\frac{m_{\tilde{t}_L}}{100\text{ GeV}}$ (1.04)	(1.23)
133	$0.0060\sqrt{m_{\tilde{\tau}}/100\text{ GeV}}$	$1.4\times 10^{-3}\sqrt{m_{\tilde{b}}/100\text{ GeV}}$	-
211	$0.049\times\frac{m_{\tilde{e}_R}}{100\text{ GeV}}$	$0.059\times\frac{m_{\tilde{d}_R}}{100\text{ GeV}}$	-
212	$0.049\times\frac{m_{\tilde{\mu}_R}}{100\text{ GeV}}$	$0.059\times\frac{m_{\tilde{s}_R}}{100\text{ GeV}}$	(1.23)
213	$0.049\times\frac{m_{\tilde{\tau}_R}}{100\text{ GeV}}$	$0.059\times\frac{m_{\tilde{b}_R}}{100\text{ GeV}}$	(1.23)
221	-	$0.18\times\frac{m_{\tilde{s}_R}}{100\text{ GeV}}$ (1.12)	(1.23)
222	-	$0.21\times\frac{m_{\tilde{s}_R}}{100\text{ GeV}}$ (1.12)	-
223	-	$0.21\times\frac{m_{\tilde{b}_R}}{100\text{ GeV}}$ (1.12)	(1.23)
231	$0.070\times\frac{m_{\tilde{e}_R}}{100\text{ GeV}}$	$0.18\times\frac{m_{\tilde{b}_L}}{100\text{ GeV}}$ (1.12)	(1.23)
232	$0.070\times\frac{m_{\tilde{\mu}_R}}{100\text{ GeV}}$	0.56 (1.04)	(1.23)
233	$0.070\times\frac{m_{\tilde{\tau}_R}}{100\text{ GeV}}$	$0.15\sqrt{m_{\tilde{b}}/100\text{ GeV}}$	-
311	$0.062\times\frac{m_{\tilde{e}_R}}{100\text{ GeV}}$	$0.11\times\frac{m_{\tilde{d}_R}}{100\text{ GeV}}$ (1.12)	-
312	$0.062\times\frac{m_{\tilde{\mu}_R}}{100\text{ GeV}}$	$0.11\times\frac{m_{\tilde{s}_R}}{100\text{ GeV}}$ (1.12)	0.50 (1.00)
313	$0.0060\sqrt{m_{\tilde{\tau}}/100\text{ GeV}}$	$0.11\times\frac{m_{\tilde{b}_R}}{100\text{ GeV}}$ (1.12)	0.50 (1.00)

TABLE I. *(Continued).*

ijk	$\lambda_{ijk}(M_Z)$[a]	$\lambda'_{ijk}(M_Z)$[b]	$\lambda''_{ijk}(M_Z)$[c]
321	$0.070\times\frac{m_{\tilde{e}_R}}{100\,\text{GeV}}$	$0.52\times\frac{m_{\tilde{d}_R}}{100\,\text{GeV}}$ (1.12)	0.50 (1.00)
322	$0.070\times\frac{m_{\tilde{\mu}_R}}{100\,\text{GeV}}$	$0.52\times\frac{m_{\tilde{s}_R}}{100\,\text{GeV}}$ (1.12)	-
323	$0.070\times\frac{m_{\tilde{\tau}_R}}{100\,\text{GeV}}$	$0.52\times\frac{m_{\tilde{b}_R}}{100\,\text{GeV}}$ (1.12)	0.50 (1.00)
331	-	0.45 (1.04)	0.50 (1.00)
332	-	0.45 (1.04)	0.50 (1.00)
333	-	0.45 (1.04)	-

[a]Updated bounds from Refs. [27,21]. Bounds on $\lambda_{121},\lambda_{122},\lambda_{123}$ have been obtained from charged current universality [20]. Bounds on $\lambda_{131},\lambda_{132},\lambda_{231},\lambda_{232}$ and λ_{233} have been derived from [20] measurements of $R_\tau=\Gamma(\tau\to e\nu\overline{\nu})/\Gamma(\tau\to\mu\nu\overline{\nu})$ and $R_{\tau\mu}=\Gamma(\tau\to\mu\nu\overline{\nu})/\Gamma(\mu\to e\nu\overline{\nu})$ [1]. The bound on λ_{133} [35] has been obtained from the experimental limit on the electron neutrino mass [1].

[b]Bounds on $\lambda'_{112},\lambda'_{113},\lambda'_{121},\lambda'_{122}$, and λ'_{123} have been obtained from charged current universality [20]. The bound on λ'_{111} has been derived from neutrino-less double beta decay [36,37,38] where $f(\tilde{m}) =(m_{\tilde{e}}/100\text{ GeV})^2\times(m_{\tilde{\chi}^0}/100\text{ GeV})^{1/2}$, and on λ'_{131} from atomic parity violation [20,28]. This latter bound is at the 3σ level, since the data disagree with the standard model at the 2.5σ level [28]. The bound on λ'_{132} comes from the forward-backward asymmetry in e^+e^- collisions [20]. Bounds on $\lambda'_{133},\lambda'_{233}$ have been obtained from bounds on the neutrino masses [35] and on $\lambda'_{211},\lambda'_{212},\lambda'_{213}$ from $R_\pi=\Gamma(\pi\to e\nu)/\Gamma(\pi\to\mu\nu)$ [20,27]. Bounds on $\lambda'_{221},\lambda'_{231}$ come from ν_μ deep inelastic scattering [20,27] and on $\lambda'_{222},\lambda'_{223}$ from the D-meson decays [27], $D\to Kl\nu$. The bounds without parentheses on $\lambda'_{232},\lambda'_{331},\lambda'_{332},\lambda'_{333}$ have been derived from $R_l=\Gamma(Z\to\text{had})/\Gamma(Z\to l\overline{l})$ for $m_{\tilde{q}}=100\text{ GeV}$ [39] and on $\lambda'_{311},\lambda'_{312},\lambda'_{313}$ from $R_{\tau\pi}=\Gamma(\tau\to\pi\nu_\tau)/\Gamma(\pi\to\mu\nu_\mu)$ [20,27]. The bounds on the couplings $\lambda'_{321},\lambda'_{322}$ and λ'_{323} have been derived from D_s decays [27], i.e., $R_{D_s}=\Gamma(D_s\to\tau\nu_\tau)/\Gamma(D_s\to\mu\nu_\mu)$. There are also bounds on λ'_{3j3} from R_b [29] but these are weak at 2σ level and thus not displayed.

[c]The indirect bounds on λ''_{ijk} existing in the literature are on λ''_{112} from double nucleon decay [33] [$\tilde{\Lambda}$ is a hadronic scale and it can be varied from 0.003 to 1 GeV and $(m_{\tilde{q}}/\tilde{\Lambda}\text{ GeV})^{5/2}$ from 2×10^{11} to 10^5 for $m_{\tilde{q}}=100\text{ GeV}$] and on λ''_{113} from neutron oscillations [40,33] for $m_{\tilde{q}}=100\text{ GeV}$. For $m_{\tilde{q}}=200$ (600) GeV the bound on λ''_{113} is 0.002 (0.1). The bound on λ''_{3jk} has been derived from $R_l=\Gamma(Z\to\text{had})/\Gamma(Z\to l\overline{l})$ at 1σ for $\tilde{m}=100\text{ GeV}$ [41] and, for heavy squark masses, is not more stringent than the perturbativity bound, which is displayed in the parentheses.

$$h_{u,c,t}(M_Z)=\frac{m_{u,c,t}(M_Z)}{\sqrt{2}v\sin\beta}, \tag{5}$$

where $v=246\text{ GeV}$ is the standard model Higgs vacuum expectation value (VEV), and $\tan\beta=v_2/v_1$ is the ratio of the two MSSM Higgs VEVs. As an example study, throughout most of the paper we set $\tan\beta=5$. We briefly discuss the case of $\tan\beta=35$ at the end.

We use central values of the mixing angles in the ''standard'' parametrization of V_{CKM} detailed in Ref. [1]:

$$s_{12}=0.2195,\quad s_{23}=0.039,\quad s_{13}=0.0031. \tag{6}$$

We initially set the CP-violating phase $\delta_{13}=0$ but later we examine $\delta_{13}=\pi/2$ to see if including CP violation affects the GUT scale bounds. Once one has allowed CP violation in the R_p conserving couplings there does not seem any compelling theoretical reason to ban it from the $\not{R}_p$ couplings. We are mainly interested in showing that the inclusion of CP violation does not change the GUT scale bounds rather than determining induced phases in weak scale $\not{R}_p$ couplings. We therefore assume for simplicity that CP violation is negligible in the GUT scale $\not{R}_p$ coupling.

For the purposes of the calculations we assume the entire MSSM spectrum to be at the scale of the top quark mass, m_t, and furthermore we assume a desert between m_t and M_{GUT}.

IV. NUMERICAL PROCEDURE

To obtain $\mathbf{Y}_U(M_Z)$ and $\mathbf{Y}_D(M_Z)$, assumptions have to be made about the Yukawa matrices in the weak eigenbasis. To start with, we assume that the mixing occurs only within the down quark sector, and that the Yukawa matrices are Hermitian. We later also consider the other extreme case where the mixing only occurs in the up quark sector. With the definition of $\mathbf{Y}_D,\mathbf{Y}_U$ in Eq. (2) and the mixing fully in the down quark sector, we obtain

$$\mathbf{Y}_D(M_Z)=V^*_{\text{CKM}}\mathbf{Y}_{D_{\text{diag}}}(M_Z)V^T_{\text{CKM}}. \tag{7}$$

TABLE II. Current relevant upper limits on the values of products of weak scale *R*-parity violating couplings for $\tilde{m}=100$ GeV.

$\lvert\lambda_{1j1}\lambda_{1j2}\rvert$	$7\ \times 10^{-7}$ [a]
$\lvert\lambda_{231}\lambda_{131}\rvert$	$7\ \times 10^{-7}$ [b]
$\lvert\lambda_{231}\lambda_{232}\rvert$	5.3×10^{-6} [c]
$\lvert\lambda_{232}\lambda_{132}\rvert$	8.4×10^{-6} [d]
$\lvert\lambda_{233}\lambda_{133}\rvert$	1.7×10^{-5} [e]
$\lvert\lambda_{122}\lambda'_{211}\rvert$	4.0×10^{-8} [f]
$\lvert\lambda_{132}\lambda'_{311}\rvert$	4.0×10^{-8} [g]
$\lvert\lambda_{121}\lambda'_{111}\rvert$	4.0×10^{-8} [h]
$\lvert\lambda_{231}\lambda'_{311}\rvert$	4.0×10^{-8} [i]
$\lvert\lambda'_{i1k}\lambda'_{i2k}\rvert$	2.2×10^{-5} [j]
$\lvert\lambda'_{i12}\lambda'_{i21}\rvert$	10^{-9} [k]
$\mathrm{Im}\,\lambda'_{i12}\lambda'^{*}_{i21}$	$8\ \times 10^{-12}$ [l]
$\lvert\lambda'_{113}\lambda'_{131}\rvert$	$3\ \times 10^{-8}$ [m]
$\lvert\lambda'_{i13}\lambda'_{i31}\rvert$	$8\ \times 10^{-8}$ [n]
$\lvert\lambda'_{1k1}\lambda'_{2k2}\rvert$	$8\ \times 10^{-7}$ [o]
$\lvert\lambda'_{1k1}\lambda'_{2k1}\rvert$	8.0×10^{-8} [p]
$\lvert\lambda'_{11j}\lambda'_{21j}\rvert$	8.5×10^{-8} [q]
$\lvert\lambda'_{22k}\lambda'_{11k}\rvert$	$4\ \times 10^{-7}$ [r]
$\lvert\lambda'_{21k}\lambda'_{12k}\rvert$	4.3×10^{-7} [s]
$\lvert\lambda'_{22k}\lambda'_{12k}\rvert$ $(k=2,3)$	2.1×10^{-6} [t]
$\lvert\lambda'_{221}\lambda'_{131}\rvert$	2.0×10^{-6} [u]
$\lvert\lambda'_{23k}\lambda'_{11k}\rvert$	2.1×10^{-6} [v]
$\lvert\lambda'_{ij1}\lambda'_{ij2}\rvert$ $(j\neq 3)$	6.1×10^{-6} [w]
$\lvert\lambda'_{i31}\lambda'_{i32}\rvert$	1.6×10^{-5} [x]
$\lvert\lambda'_{i31}\lambda'_{i12}\rvert$	2.4×10^{-5} [y]
$\lvert\lambda''_{i32}\lambda''_{i21}\rvert$	7.6×10^{-3} [z]
$\lvert\lambda''_{i31}\lambda''_{i21}\rvert$	6.2×10^{-3} [aa]
$\lvert\lambda''_{232}\lambda''_{132}\rvert$	2.5×10^{-3} [bb]
$\lvert\lambda''_{332}\lambda''_{331}\rvert$	4.8×10^{-4} [cc]

[a] From $\mu\to 3e$ [42].
[b] From $\mu\to 3e$ [42].
[c] From μTi$\to e$Ti at one loop [43].
[d] From μTi$\to e$Ti at one loop [43].
[e] From μTi$\to e$Ti at one loop [43].
[f] From μTi$\to e$Ti at tree level [43].
[g] From μTi$\to e$Ti at tree level [43].
[h] From μTi$\to e$Ti at tree level [43].
[i] From μTi$\to e$Ti at tree level [43].
[j] From $K\to\pi\nu\bar{\nu}$ [30]. Also $\lvert\lambda'_{i11}\lambda'_{i21}\rvert\sim 10^{-6}$ from ϵ'/ϵ [44].
[k] From Δm_K [11].
[l] From ϵ_K [11].
[m] From Δm_B [38].
[n] From Δm_B [11].
[o] From $K_L\to\mu e$ [11].
[p] From μTi$\to e$Ti at tree level [43].
[q] From μTi$\to e$Ti at tree level [43].
[r] From μTi$\to e$Ti at tree level [43].
[s] From μTi$\to e$Ti at tree level [43].
[t] From μTi$\to e$Ti at tree level [43].
[u] From μTi$\to e$Ti at tree level [43].
[v] From μTi$\to e$Ti at tree level [43].
[w] From K and B systems [45].
[x] From K and B systems [45].
[y] From K and B systems [45].
[z] From non-leptonic decays of heavy quark mesons, $B^+\to\overline{K}^0+K^+$ [46].
[aa] From non-leptonic decays of heavy quark mesons, $\Gamma(B^+\to\tilde{K}^0+\pi^+)/\Gamma(B^+\to J/\psi+K^+)$ [46].
[bb] From non-leptonic decays of heavy quark mesons [46].
[cc] From the contribution of K-$\overline{K}$ mixing to the K_L-K_S mass difference [47].

$Y_{D_{\rm diag}}(M_Z)$ is the diagonal matrix with $h_d(M_Z)$, $h_s(M_z)$, and $h_b(M_Z)$ along the diagonal. Thus $\mathbf{Y}_D(M_Z)$ is determined uniquely in terms of its eigenvalues and the CKM matrix, and all of the R_p-conserving couplings are defined at $\mu=M_Z$ in the $\overline{\rm DR}$ scheme. Because the data on neutrino oscillations are controversial, we do not include mixing of the charged leptons: i.e., $\mathbf{Y}_E(M_Z)$ is set by its eigenvalues, the charged lepton masses evaluated at M_Z.

To begin, the system of all gauge couplings and all the Higgs Yukawa couplings is evolved to $M_{\rm GUT}$ using the one-loop RGEs of the $R\!\!\!/_p$-MSSM [25,24]. At the GUT scale, we then add only one non-zero (and real) $R\!\!\!/_p$ coupling. This coupling is in the weak current eigenbasis. All of the dimensionless couplings, now including the $R\!\!\!/_p$ coupling, are then evolved down to M_Z. In the process more than one non-zero $R\!\!\!/_p$ coupling is generated. The Higgs Yukawa couplings evaluated at M_Z in general lead to incorrect fermion masses, so they are reset, as in Eqs. (4), (5). The system of couplings is then re-evolved up to $M_{\rm GUT}$ now including the $R\!\!\!/_p$ couplings. At $M_{\rm GUT}$, the $R\!\!\!/_p$ couplings can differ from their initial values at $M_{\rm GUT}$ and are reset. The process is iterated until the system converges.

The $R\!\!\!/_p$ couplings thus obtained at the scale M_Z are valid in the weak eigenbasis. For comparison with experiment, the quark superfields must be rotated to the quark mass eigenbasis. To do this, we follow the procedure of Ref. [30]. If we assume all the Cabibbo-Kobayashi-Maskawa (CKM) mixing is in the down quark sector only, we obtain the $R\!\!\!/_p$ interactions

$$\mathcal{W}_{R\!\!\!/_p} \supset \lambda'_{ijk}(V^\dagger_{\rm CKM})_{mk}[N_i(V_{\rm CKM})_{jl}D_l - E_iU_j]\overline{D}_m + \frac{1}{2}\lambda''_{ijk}(V^\dagger_{\rm CKM})_{mj}(V^\dagger_{\rm CKM})_{nk}\overline{U}_i\overline{D}_m\overline{D}_n. \tag{8}$$

All superfields written in Eq. (8) are in the quark mass eigenbasis, contrary to those in Eq. (2). The λ' terms have been expanded into two SU(2) components containing $Q_i \equiv (U_i, D_i)$ and $L_i \equiv (N_i, E_i)$. Referring to Eq. (8), we define the rotation of the couplings to the quark mass basis (denoted with a tilde):

$$\tilde{\lambda}'_{ijk} = \lambda'_{ijm}(V^*_{\rm CKM})_{mk}, \tag{9}$$

$$\tilde{\lambda}''_{ijk} = \lambda''_{imn}(V^*_{\rm CKM})_{mj}(V^*_{\rm CKM})_{nk}. \tag{10}$$

As shown in Ref. [30], several $R\!\!\!/_p$ interactions [as implied by Eqs. (9), (10)] result in flavor changing neutral current (FCNC). Upper bounds may then be obtained upon $\tilde{\lambda}'$ and $\tilde{\lambda}''$ from FCNC data. Thus, starting with a dominant $R\!\!\!/_p$ coupling in the weak eigenbasis at the GUT scale, we evolve $\lambda'_{ijk}, \lambda''_{ijk}$ to the electroweak scale, causing some of the $R\!\!\!/_p$ couplings to become non-zero through RG evolution. At the electroweak scale, this system of $R\!\!\!/_p$ couplings is rotated into the quark mass basis using Eqs. (9), (10).

The resulting system of non-zero $\tilde{\lambda}'$ and $\tilde{\lambda}''$ couplings valid at the electroweak scale is then checked against the bounds summarized (together with their sources) in Tables I, II. Almost all of the bounds depend on the sparticle masses. The $R\!\!\!/_p$ GUT scale coupling is varied until the couplings generated at M_Z just pass the low-energy bounds. The value of the $R\!\!\!/_p$ GUT scale coupling at this point is then an upper bound upon the non-zero R-parity violating GUT scale coupling. These bounds are summarized in Table III.

V. CASE STUDIES

Here, we detail the results of the above procedure for various cases. Initially, we present the bounds on GUT scale $R\!\!\!/_p$ couplings for a simplified case in which there is no *CP* violating phase and zero mixing, i.e. $V_{\rm CKM}=1$. The results are displayed in Tables I, III. The perturbativity bounds upon λ''_{ijk} presented in Table I are in full agreement with those given by Ref. [33]. A two-loop calculation alters the perturbativity bounds by up to 10% [24]. In this case, there are no bounds caused by inducing new non-zero $R\!\!\!/_p$ couplings in the renormalization; a GUT scale bound is obtained by renormalizing the empirical bound on the dominant low energy coupling. The explicit dependence upon the sparticle masses in Table III has been demonstrated numerically. It is valid because of an approximate linear relation between GUT and weak scale $R\!\!\!/_p$ couplings, valid in the limit that they are small. This mass dependence is incorrect for cases where the bound multiplied by a sparticle mass $\tilde{m}/100\,{\rm GeV}$ is large, i.e., greater than 0.6. In those cases one can use the perturbativity bound. As can be seen from Table III, bounds on $\lambda_{ijk}(M_{\rm GUT})$ are approximately twice as severe than those on $\lambda_{ijk}(M_Z)$, whereas those on $\lambda'_{ijk}, \lambda''_{ijk}(M_{\rm GUT})$ are 3–5 times as severe as their weak scale counterparts.

Next, we examine the effects of quark mixing by assuming that it occurs in the Hermitian $\mathbf{Y}_D$ given by Eq. (7). Here, we set the *CP* violating phase to zero. The results are displayed in Table IV without parentheses. Obviously the bounds upon $\lambda_{ijk}(M_{\rm GUT})$ in Table IV are identical to those in Table III, because the weak and mass bases of the leptons have been assumed to be identical. When the bounds on $\lambda'_{ijk}(M_{\rm GUT})$, $\lambda''_{ijk}(M_{\rm GUT})$ including quark mixing effects are compared to those without mixing in Table III, we see a remarkable difference for many of the couplings. Many of them are an order of magnitude more stringent when quark mixing has been taken into account. The λ''_{123} GUT scale coupling is essentially unbounded in Table III (or bounded by the limit of perturbative believability), whereas in Table IV the bound becomes strengthened by an incredible seven orders of magnitude. λ''_{113} becomes more constrained by a factor of 500. For the λ'_{ijk} in Table III that had the strongest bound being that of perturbativity (for heavy sparticle masses), down quark mixing effects imply that the empirical bounds are the strongest.

In order to check the robustness of the bounds under changes in the assumed R-parity conserving texture, we now perform the analagous analysis for the case of mixing in a Hermitian $\mathbf{Y}_U$ only. For this case, Eq. (7) becomes replaced by

$$\mathbf{Y}_U(M_Z) = V^T_{\rm CKM}\mathbf{Y}_{U_{\rm diag}}(M_Z)V^*_{\rm CKM}, \tag{11}$$

with $\mathbf{Y}_D(M_Z) = \mathbf{Y}_{D_{\rm diag}}$. The superpotential terms in Eq. (8) become

TABLE III. Bounds on the trilinear *R*-parity violating couplings at the GUT scale which are in agreement with the low energy experimental bounds of Tables I and II. The dependence of the superparticle masses is shown explicitly, except where it is too complicated and $\tilde{m}=100\,\text{GeV}$ is assumed. ** indicates that the strongest bound is the one where the couplings are small enough to use perturbation theory, for example 3.5. The input value of tan β has been chosen to be tan $\beta(M_Z)=5$.

ijk	$\lvert\lambda_{ijk}(M_{GUT})\rvert$	$\lvert\lambda'_{ijk}(M_{GUT})\rvert$	$\lvert\lambda''_{ijk}(M_{GUT})\rvert$
111	-	$1.4\times10^{-4}\times f(\tilde{m})$	-
112	-	$0.0059\times\frac{m_{\tilde{s}_R}}{100\,\text{GeV}}$	$4\times10^{-9}\left(\frac{m_{\tilde{q}}}{100\,\text{GeV}}\,\frac{.3\,\text{GeV}}{\bar{\Lambda}}\right)^{5/2}$
113	-	$0.0059\times\frac{m_{\tilde{b}_R}}{100\,\text{GeV}}$	2×10^{-5} [a]
121	$0.032\times\frac{m_{\tilde{e}_R}}{100\,\text{GeV}}$	$0.012\times\frac{m_{\tilde{d}_R}}{100\,\text{GeV}}$	$4\times10^{-9}\left(\frac{m_{\tilde{q}}}{100\,\text{GeV}}\,\frac{.3\,\text{GeV}}{\tilde{\Lambda}}\right)^{5/2}$
122	$0.032\times\frac{m_{\tilde{\mu}_R}}{100\,\text{GeV}}$	$0.012\times\frac{m_{\tilde{s}_R}}{100\,\text{GeV}}$	-
123	$0.032\times\frac{m_{\tilde{\tau}_R}}{100\,\text{GeV}}$	$0.012\times\frac{m_{\tilde{b}_R}}{100\,\text{GeV}}$	(**)
131	$0.041\times\frac{m_{\tilde{e}_R}}{100\,\text{GeV}}$	$0.0060\times\frac{m_{\tilde{t}_L}}{100\,\text{GeV}}$	2×10^{-5} [a]
132	$0.041\times\frac{m_{\tilde{\mu}_R}}{100\,\text{GeV}}$	$0.091\times\frac{m_{\tilde{t}_L}}{100\,\text{GeV}}\,(1.65)$ [b]	(**)
133	$0.0039\sqrt{m_{\tilde{\tau}}/100\,\text{GeV}}$	$4.4\times10^{-4}\sqrt{m_{\tilde{b}}/100\,\text{GeV}}$	-
211	$0.032\times\frac{m_{\tilde{e}_R}}{100\,\text{GeV}}$	$0.016\times\frac{m_{\tilde{d}_R}}{100\,\text{GeV}}$	-
212	$0.032\times\frac{m_{\tilde{\mu}_R}}{100\,\text{GeV}}$	$0.016\times\frac{m_{\tilde{s}_R}}{100\,\text{GeV}}$	(**)
213	$0.032\times\frac{m_{\tilde{\tau}_R}}{100\,\text{GeV}}$	$0.016\times\frac{m_{\tilde{b}_R}}{100\,\text{GeV}}$	(**)
221	-	$0.051\times\frac{m_{\tilde{s}_R}}{100\,\text{GeV}}\,(**)$	(**)
222	-	$0.060\times\frac{m_{\tilde{s}_R}}{100\,\text{GeV}}\,(**)$	-
223	-	$0.060\times\frac{m_{\tilde{b}R}}{100\,\text{GeV}}\,(**)$	(**)
231	$0.046\times\frac{m_{\tilde{e}_R}}{100\,\text{GeV}}$	$0.057\times\frac{m_{\tilde{b}_L}}{100\,\text{GeV}}\,(**)$	(**)
232	$0.046\times\frac{m_{\tilde{\mu}_R}}{100\,\text{GeV}}$	0.20 (1.66)[b]	(**)
233	$0.046\times\frac{m_{\tilde{\tau}_R}}{100\,\text{GeV}}$	$0.048\sqrt{m_{\tilde{b}}/100\,\text{GeV}}$	-
311	$0.041\times\frac{m_{\tilde{e}_R}}{100\,\text{GeV}}$	$0.031\times\frac{m_{\tilde{d}_R}}{100\,\text{GeV}}\,(**)$	-
312	$0.041\times\frac{m_{\tilde{\mu}_R}}{100\,\text{GeV}}$	$0.031\times\frac{m_{\tilde{s}_R}}{100\,\text{GeV}}\,(**)$	0.16 (0.76)[b]
313	$0.0039\sqrt{m_{\tilde{\tau}/100\,\text{GeV}}}$	$0.031\times\frac{m_{\tilde{b}_R}}{100\,\text{GeV}}\,(**)$	0.16 (0.76)[b]

TABLE III. *(Continued).*

ijk	$\|\lambda_{ijk}(M_{GUT})\|$	$\|\lambda'_{ijk}(M_{GUT})\|$	$\|\lambda''_{ijk}(M_{GUT})\|$
321	$0.046\times\frac{m_{\tilde{e}_R}}{100\,\text{GeV}}$	$0.17\times\frac{m_{\tilde{d}_R}}{100\,\text{GeV}}$ [c] (**)	0.16 (0.76)[b]
322	$0.046\times\frac{m_{\tilde{\mu}_R}}{100\,\text{GeV}}$	$0.17\times\frac{m_{\tilde{s}_R}}{100\,\text{GeV}}$ [c] (**)	-
323	$0.046\times\frac{m_{\tilde{\tau}_R}}{100\,\text{GeV}}$	$0.17\times\frac{m_{\tilde{b}_R}}{100\,\text{GeV}}$ [c] (**)	0.16 (0.76)[b]
331	-	0.16 (1.66)[b]	0.16 (0.76)[b]
332	-	0.16 (1.66)[b]	0.16 (0.76)[b]
333	-	0.16 (1.66)[b]	-

[a]For $m_{\tilde{q}}=200(600)$ GeV the bound is $\lambda''_{113}=\lambda''_{131}\lesssim 4\times 10^{-4}(3\times 10^{-2})$.
[b]From perturbativity of the top Yukawa coupling.
[c]This bound can be used only for small departures of sparticle masses from the electroweak scale.

$$\mathcal{W}_{\not{R}_p}\supset\lambda'_{ijk}[N_iD_j-E_iU_l(V^{\dagger}_{\text{CKM}})_{jl}]\overline{D}_k + \frac{1}{2}\lambda''_{ijk}(V_{\text{CKM}})_{li}\overline{U}_l\overline{D}_j\overline{D}_k, \tag{12}$$

for superfields in the quark mass eigenbasis. This implies the rotation of $\not{R}_p$ couplings,

$$\tilde{\lambda}'_{ijk}=\lambda'_{ilk}(V^{*}_{\text{CKM}})_{jl} \tag{13}$$

TABLE IV. Basis dependent bounds on the trilinear *R*-parity violating couplings at the GUT scale with the mixing assumed in the down [up] quark sector. The value of $\tilde{m}=100\,\text{GeV}$ for squarks and sleptons is assumed. The input value of $\tan\beta$ and the hadronic scale $\tilde{\Lambda}$ have been chosen to be $\tan\beta(M_Z)=5$ and 300 MeV respectively.

ijk	$\|\lambda_{ijk}(M_{GUT})\|$	$\|\lambda'_{ijk}(M_{GUT})\|$	$\|\lambda''_{ijk}(M_{GUT})\|$
111	-	1.5×10^{-4} [1.5×10^{-4}]	-
112	-	6.7×10^{-4} [0.0059]	4.1×10^{-10} [4.1×10^{-10}]
113	-	0.0059 [0.0059]	1.1×10^{-8} [2×10^{-5}]
121	0.032	0.0015 [6.7×10^{-4}]	4.1×10^{-10} [4.1×10^{-10}]
122	0.032	0.0015 [0.012]	-
123	0.032	0.012 [0.012]	1.3×10^{-7} [0.028]
131	0.041	0.0027 [0.0060]	1.1×10^{-8} [2×10^{-5}]
132	0.041	0.0027 [0.091]	1.3×10^{-7} [0.028]
133	0.0039	4.4×10^{-4} [4.4×10^{-4}]	-
211	0.032	0.0015 [0.016]	-
212	0.032	0.0015 [0.016]	(**) [2.1×10^{-9}]
213	0.032	0.016 [0.016]	(**) [1.0×10^{-4}]
221	-	0.0015 [0.051]	(**) [2.1×10^{-9}]
222	-	0.0015 [0.060]	-
223	-	0.049 [0.060]	(**) [0.028]
231	0.046	0.0027 [0.057]	(**) [1.0×10^{-4}]
232	0.046	0.0028 [0.20]	(**) [0.028]
233	0.046	0.048 [0.048]	-
311	0.041	0.0015 [0.031]	-
312	0.041	0.0015 [0.031]	0.099 [1.5×10^{-7}]
313	0.0039	0.0031 [0.031]	0.015 [0.0075]
321	0.046	0.0015 [0.17]	0.099 [1.5×10^{-7}]
322	0.046	0.0015 [0.17]	-
323	0.046	0.049 [0.17]	0.015 [0.16]
331	-	0.0027 [0.16]	0.015 [0.0075]
332	-	0.0028 [0.16]	0.015 [0.16]
333	-	0.091 [0.16]	-

$$\tilde{\lambda}''_{ijk}=\lambda''_{ljk}(V_{\rm CKM})_{il}\,, \tag{14}$$

supplanting Eqs. (9), (10). The rest of the numerical procedure is identical to that outlined in the previous section.

Some of the bounds from mixing in the up quark sector (displayed in square brackets in Table IV) are again remarkably different to those without mixing in Table III. There is qualitatively less change in the λ'_{ijk} bounds from the inclusion of up-quark mixing than down quark mixing, but some of the λ''_{ijk} show an even larger strengthening effect. For example, λ''_{212}, instead of being bounded only by the perturbative limit, acquires an empirical bound of 2.1×10^{-9}, obviously very constraining upon relevant GUT models.

To see the effect of CP violation, we pick $\delta_{13}=\pi/2$ as an example and follow the above procedure for quark mixing in the down quark sector (and subsequently in the up quark sector). The bounds in Table IV remain unchanged by the addition of CP violation. While being the main purpose of this particular case study, we now briefly present results on the small phases picked up by the $\not{R}_p$ couplings in their renormalization from the GUT scale to the weak scale. The largest imaginary parts of couplings acquired occur when the dominant couplings are large. The induced imaginary part of these couplings at the weak scale is as large as $\sim10^{-3}$ for quark mixing either in the down quark or the up quark sector. For example, let us suppose we start with the case where the mixing is in the down quark sector and the dominant coupling at the GUT scale is $\lambda''_{212}=-\lambda''_{221}$ and is taken to be real. Then the renormalization down to electroweak scale induces non-zero and complex values for all of the other λ''_{ijk}. The largest imaginary component is obtained for λ''_{232} where $\mathrm{Im}\,\lambda''_{232}(M_Z)=-\mathrm{Im}\,\lambda''_{223}(M_Z)\simeq4\times10^{-3}$.

To investigate how sensitive the GUT scale bounds are to the free parameter $\tan\beta$, we performed another analysis with $\tan\beta=35$ and no mixing. For the case of the limits on λ_{ijk}, we find that the bound relaxes by up to 9%. In the cases of the λ'_{ijk} or λ''_{ijk} $\not{R}_p$ couplings we obtain a 30% or 6% weakening of the the bound respectively. Thus, to a 30% accuracy level, the bounds of the Table III are stable over a large range of $\tan\beta$. Of course there is a strong dependence of the perturbativity bounds in the regions $\tan\beta\lesssim3$ and $\tan\beta\gtrsim40$ upon the input value of $\tan\beta$ [24,34]. The bounds from these values of $\tan\beta$ are stronger than for $\tan\beta=5$ and so presenting the bounds for $\tan\beta=5$ yields a conservative estimate.

VI. SUMMARY

We have examined changes in empirical bounds on $\not{R}_p$ couplings as they are renormalized to the unification scale, working to one loop accuracy in perturbation theory but including all of the 45 trilinear supersymmetric $\not{R}_p$ couplings. The latest empirical bounds upon the couplings have been collated in Tables I, II. The bounds upon λ'_{ijk} presented in Table I in parentheses are new except for λ'_{333}, and are derived from the requirement of perturbativity below the unification scale. They are the most stringent bounds on these couplings depending upon the squark mass. We have demonstrated that at high energy, the empirical bounds upon the dominant $\not{R}_p$ couplings are more severe than the empirical bounds applied at M_Z and are displayed in Table III. The bounds are made stronger by a factor of 2–5 from their renormalization. These upper bounds are still applicable under changes in the CP-violating phase and the inclusion of quark mixing. They are also approximately stable (at the 30% level) to changes in the parameter $\tan\beta$. However, when quark mixing is included some of the limits become several orders of magnitude more severe than their weak scale counterparts due to new R-parity violating operators being induced in the renormalization between high and low scales. These very strong limits are dependent upon the fermion mass texture, as we have demonstrated by calculating them in the cases where the quark mixing is wholly within the down quark sector or wholly within the up quark sector. While a CP violating phase in the CKM matrix does not affect the bounds, the weak scale $\not{R}_p$ couplings acquire small imaginary components from the renormalization. The magnitudes of these phases are dependent upon the mass texture assumed. Since in general $\not{R}_p$ terms can be induced via nonrenormalizable operators in GUT or other unified models, this analysis is hopefully useful for their phenomenology and construction. A necessary condition upon any unified model is that it satisfy the upper bounds given in Table III. Stronger constraints arising from bounds upon induced couplings depend upon the fermion mass texture assumed and so must be checked on a case-by-case basis. The results presented here represent the most comprehensive collation of bounds upon trilinear supersymmetric $\not{R}_p$ couplings to date.

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