BROKEN GLOBAL SYMMETRIES IN SUPERSYMMETRIC THEORIES

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We show that the superpartners of real Goldstone bosons, resulting from global U(1) symmetries broken at superlarge energies M_X, obtain, in gauge theories with spontaneously broken supersymmetry (at energies $m \gtrsim m_W$), at most masses of $O(m^2/M_X)$ at the tree level.

The Peccei–Quinn solution to the strong *CP* problem [1] requires invariance under a global U(1) symmetry. The spontaneous breaking of this global symmetry leads to a Goldstone boson (the axion) which obtains a very small mass through the Adler–Bell–Jackiw anomaly. Experimental evidence, as well as astrophysical considerations [2], imply that the scale at which the U(1) symmetry breaks must be at least 10^9 GeV. Recently, Dine et al. [3] pointed out that it is natural to break the U(1) symmetry at the grand unification scale $M_X \sim 10^{15}$ GeV. This implies a very small mass for the axion $(M_W/M_X)\Lambda_{QCD} \sim 10^{-8}$ eV as well as a tiny coupling $m_W/M_X \sim 10^{-13}$ which renders the axion "invisible". The invisibility, however, hinges strongly on the pseudoscalar nature of the axion. A light scalar particle of the same mass would contribute to long-range gravitational interactions at 10^8 $\text{eV}^{-1} \sim 20$ m despite the tremendously small couplings.

Supersymmetry [4] holds out some hope for the solution of the gauge hierarchy problem [5] in grand unified theories (GUTs). It might be tempting to combine the PQ idea with supersymmetry [6]. But supersymmetric GUTs with a built-in PQ symmetry broken at M_X exhibit, in addition to the axion, a fermion and a scalar boson with analogous couplings. Since supersymmetry must be broken at a mass scale $m \gtrsim m_W$ one might expect the scalar boson (scalar axion) and the fermion (axino) to get masses $\gtrsim m_W$ and thus be irrelevant at low energies. In this note we will show, in the context of a general supersymmetric gauge theory, that the superpartners of the invisible axion, instead of obtaining a mass of order m at the tree level, get at most a mass $m^2/M_X \sim 10^{-2}$ eV. Their couplings to light fermions and Higgs' are related by supersymmetry to those of the axion and are also $\sim 10^{-13}$. Our conclusions apply in general for the superpartners of Goldstone bosons resulting from the breaking of global symmetries at M_X.

Consider a general supersymmetry gauge theory characterized by a gauge supermultiplet $(A_\mu^\alpha, \lambda^\alpha, D_\alpha)$ and left-handed matter supermultiplets (ϕ^i, ψ_L^i, F_i) transforming under various representations of the gauge group. The matter interactions are defined by the gauge invariant function $\mathcal{W}$ (superspace potential). The various F terms are obtained from $\mathcal{W}$ by differentiation

$$F_i = \partial\mathcal{W}/\partial\phi^i, \quad F^{*i} = (\partial\mathcal{W}/\partial\phi^i)^*, \quad F_{ij} = \partial^2\mathcal{W}/\partial\phi^i\partial\phi^j, \dots . \tag{1}$$

The bosonic potential is given by

$$V = F^{*k}F_k + \tfrac{1}{4}D_\alpha D_\alpha , \tag{2}$$

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where $D_\alpha = e_\alpha \Sigma_\phi \phi^* T_\alpha \phi$ (T_α is the representation of the generator corresponding to the direction α. The extremization condition is

$$\partial V/\partial \phi^i = F_{ik} F^{*k} + \tfrac{1}{2}(D_\alpha)_i D_\alpha = 0 \,. \tag{3}$$

The boson mass matrix squared and the fermion mass matrix are, in a (ϕ^i, ϕ_i^*) and a $(\psi_L^i, \lambda^\alpha)$ basis, given by

$$\mathcal{M}_B^2 = \left[\begin{array}{c|c} F_{ik}F^{*kj} + \tfrac{1}{2}D_{\alpha i}D_\alpha{}^j + \tfrac{1}{2}D_{\alpha i}{}^j D_\alpha & F^{*ijk}F_k + \tfrac{1}{2}D_\alpha{}^i D_\alpha{}^j \\ \hline F_{ijk}F^{*k} + \tfrac{1}{2}D_{\alpha i}D_{\alpha j} & F^{*ik}F_{kj} + \tfrac{1}{2}D_\alpha{}^i D_{\alpha j} + \tfrac{1}{2}D_{\alpha j}{}^i D_\alpha \end{array}\right], \quad \mathcal{M}_F = \begin{bmatrix} F_{ij} & D_{\alpha j} \\ D_{\alpha i} & 0 \end{bmatrix}, \tag{4}$$

respectively.

Let us now take the superpotential to be invariant under global U(1) transformations

$$\phi^k \to \exp(\mathrm{i}\alpha Q_k)\phi^k \,. \tag{5}$$

Goldstone's theorem for the spontaneous breaking of the global U(1) symmetry can be written as

$$\chi \cdot Q \cdot \mathcal{M}_B^2 = 0 \,, \tag{6}$$

where χ stands for the vacuum expectation values of (ϕ^i, ϕ_i^*). The corresponding Goldstone boson is given by

$$G = N^{-1}(Q_i\langle\phi^i\rangle, -Q_i\langle\phi_i^*\rangle) \,. \tag{7}$$

The orthogonal state is

$$\eta = N^{-1}(Q_i\langle\phi^i\rangle, Q_i\langle\phi_i^*\rangle) \,. \tag{8}$$

The breaking of the gauge symmetries will also give Goldstone bosons which are, however, going to be absorbed by the gauge fields; they are

$$G^\alpha = N'^{-1}(\langle\phi^{ri}\rangle(Q^\alpha{}_r)_i{}^j, \langle\phi^{*r}{}_i\rangle(Q^\alpha{}_r)^i{}_j) \,. \tag{9}$$

Q_r^α denotes the α gauge group generator in the representation r. In terms of the D term derivaties, (9) can be written as

$$G^\alpha = (N'e\alpha)^{-1}(D_\alpha{}^j, D_{\alpha j}) \,. \tag{10}$$

The normalization of the global U(1) charge is carried out by demanding that G and η are both orthogonal to G^α or equivalently to the gauge directions. That implies

$$\langle\phi^i\rangle Q_i D_{\alpha i} = 0 \,, \quad \langle\phi_i^*\rangle Q_i D_\alpha{}^i = 0 \,. \tag{11,12}$$

Goldstone's theorem, together with (11) and (12), gives

$$F_{ik}F^{*kj}\langle\phi_j^*\rangle Q_j + \tfrac{1}{2}D_\alpha(D_\alpha)_i{}^j\langle\phi_j^*\rangle Q_j - F_{ijk}F^{*k}\langle\phi^j\rangle Q_j = 0 \,. \tag{13}$$

Acting with η on $\mathcal{M}_B^2$ gives, after using (13),

$$\mathcal{M}_B^2\eta = 2N^{-1}\langle\phi^j\rangle Q_j F_{ijk}F^{*k} \,. \tag{14}$$

Similarly, we obtain for the fermions $\Psi_L = N^{-1}(\langle\phi^i\rangle Q_i)$ and $\Psi_R = -N^{-1}(\langle\phi_c^*\rangle Q_i)$:

$$\mathcal{M}_F\Psi_L = N^{-1}F_{ij}Q_j\langle\phi^j\rangle \,, \quad \mathcal{M}_F\Psi_R = -N^{-1}F^{*ij}Q_j\langle\phi_j^*\rangle. \tag{15}$$

From U(1) invariance of the potential we can derive the identities

$$F_{ki}Q_i\langle\phi^i\rangle = -Q_kF_k \,, \quad F_{kij}Q_i\langle\phi^i\rangle = -(Q_k + Q_j)F_{jk} \,. \tag{16,17}$$

Using (16) we can cast (15) into the form

$$(\mathcal{M}_F\Psi_L)_j = -N^{-1}Q_jF_j \,. \tag{18}$$

In the case of unbroken supersymmetry ($F_j = 0, D_\alpha = 0$) (14) and (18) ensure that the full Goldstone supermultiplet (i.e., the bosons G and η, and the fermion Ψ) is massless as expected. If supersymmetry were broken by a D term of some $\widetilde{\mathrm{U}}(1)$ gauge group while all F's remained zero, then again the full supermultiplet would remain massless at the tree level, since the right-hand sides of (14) and (18) do not involve any D terms. Therefore, the only case that allows for tree level masses is when at least one of the F_k's is different from zero. It is also evident in (18) that if the fields that break supersymmetry through their auxiliary partners F have zero global U(1) charge, the fermion would remain massless.

Now let us assume that the global U(1) symmetry is spontaneously broken at a scale M_X much larger than the scale of supersymmetry breaking m, as in the case of the grand unified axion. From (18) we see immediately that the fermionic partner has a mass at most m^2/M_X, since the normalization factor $N \sim \mathrm{O}(M_\mathrm{X})$. The scalar partner requires a little more thought. Consider the boson mass matrix and neglect all D terms, since they do not play a role as shown above. Assume that we have diagonalized the FF^* pieces of $\mathcal{M}_\mathrm{B}^2$. Then $\mathcal{M}_\mathrm{B}^2$ takes the form (we take all vev's real for simplicity)

$$\left[\begin{array}{cccc|cccc} \frac{m^4}{M^2} & & & & & & & \\ & \lambda_1 & & 0 & & & & \\ & & \ddots & & & & B & \\ & 0 & & \lambda_n & & & & \\ \hline & & & & \frac{m^4}{M^2} & & & \\ & & B & & & \lambda_1 & & 0 \\ & & & & & & \ddots & \\ & & & & & 0 & & \lambda_n \end{array}\right] \equiv \begin{bmatrix} A & B \\ B & A \end{bmatrix}, \tag{19}$$

where B is a matrix of at most order m^2. In the limit $m^2 \to 0$, (1000,...; 0,...), (0,...; 1,..., 0) are eigenvectors to the eigenvalue zero; any linear combination of them is also such an eigenvector. As m departs from zero, the two vectors are not eigenvectors to the eigenvalue zero any more. However, the total matrix has a zero eigenvalue (corresponding to the Goldstone boson). Let it be $(v; v')$ where v, v' are vectors of the length corresponding to the size of the blocks in (19). Obviously $A\bar{v} + B\bar{v}' = 0$ and $A\bar{v}' + B\bar{v} = 0$. This yields $(A - B)(\bar{v} - \bar{v}') = 0$, and $(A + B) \times (\bar{v} - \bar{v}') = 0$. If $\bar{v} = \bar{v}'$, then, for $\bar{v} \neq 0$, $(A + B)$ has a zero eigenvalue. If $\bar{v} = -\bar{v}$, then $(A - B)$ has a zero eigenvalue. If $v \neq \pm v'$, both $A \pm B$ have a zero eigenvalue. But using $\mathrm{Rank}\begin{pmatrix} A & B \\ B & A\end{pmatrix} = \mathrm{Rank}\begin{pmatrix} A+B & B-A \\ A+B & A-B\end{pmatrix}$ one sees that $\mathcal{M}_\mathrm{B}$ would have the eigenvalue zero twice, which we exclude [‡1]. Let us then take $\bar{v} = -\bar{v}$ [this corresponds to the vector G of eq. (7)]. $\bar{v}$ is then the eigenvector with eigenvalue zero of $A - B$. As $B \to 0$ ($m^2 \to 0$), (1, 0,...) is such an eigenvector, and we will expect $\bar{v}$ to have the form $(1, 0, \ldots) + (m^2/M^2)\,\boldsymbol{\omega}$ where $\boldsymbol{\omega}$ is a normalized vector. If $(A - B)\bar{v} = 0$, then $(A + B)\bar{v} = 2B\bar{v} \approx m^2/M^2$ if one also uses the bounds on B. Consider now the orthogonal vector to $(\bar{v}, -\bar{v})$, namely $\tilde{\boldsymbol{\eta}} = (\bar{v}, \bar{v})$, which corresponds to η of eq. (8). Then $\mathcal{M}_\mathrm{B}^2 \cdot \tilde{\boldsymbol{\eta}} \sim m^4/M^2$. If we expand η in terms of the eigenvectors $\boldsymbol{\varepsilon}_i$ of $\mathcal{M}_\mathrm{B}^2$, $[\tilde{\boldsymbol{\eta}} = x_i \boldsymbol{\varepsilon}_i, (x_1^2 + x_2^2 + \ldots)^{1/2} = 1]$. Then $\mathcal{M}_\mathrm{B}^2 \cdot \tilde{\boldsymbol{\eta}} = \lambda_i x_i \boldsymbol{\varepsilon}_i$ [‡2], which implies that the x_i are negligible if $\lambda_i \sim M^2$, and since $\tilde{\boldsymbol{\eta}}$ is approximately orthogonal to the zero eigenvalue eigenvector, the remaining x_i is of order one and the corresponding eigenvalue is $\approx m^4/M^2$. The alert reader might object that this argument only holds if none of the λ_i in (19) goes to zero with m^2, for then we have degenerate eigenvalues ≈ 0. However, in the limit $m^2 \to 0$, the eigenvectors (1, ..., 0) and (0, ..., 1, ..., 0) (where the 1 stands in a position where a $\lambda_i = 0$) are distinguished by their couplings to the lagrangian and no degeneracy need be feared.

[‡1] The two eigenvectors would be of the form (v, v') and $(v, -v')$, and thus there would be a massless scalar.
[‡2] λ_i are the eigenvalues of the ε_i.

A different argument can be given if one considers the FGP mass relation [7] imposed by supersymmetry, i.e.,

$$\mathrm{tr}[\mathcal{M}_B^2] + 3\,\mathrm{tr}[\mathcal{M}_V^2] = 2\,\mathrm{tr}[\mathcal{M}_F \mathcal{M}_F^+] + (\tilde{e}\tilde{D}) \sum_i \tilde{Q}_i \,,$$

where $\mathcal{M}_V^2$ is the vector mass matrix squared

$$(\mathcal{M}_V^2)_{\alpha\beta} = \tfrac{1}{2}(D_\alpha{}^i D_{\beta i} + D_{\alpha i} D_\beta{}^i)\,,$$

and $\Sigma_i \tilde{Q}_i, \tilde{D}$ correspond to a possible factor $\tilde{\mathrm{U}}(1)$ gauge group. If we were to take the limit of $M_X \to \infty$, the extra zero in the right-hand side (that contains the fermion mass m^2/M_X) would have to be compensated for in the left-hand side by an extra zero in the boson mass matrix. Therefore, the scalar boson must necessarily have a mass m^2/M_X too. There can be no interference from the vectors since one could consider the gauge couplings to go to zero.

We have seen that the breaking of a global U(1) symmetry [‡3] at M_X in a gauge theory with supersymmetry breaking at m leaves the partners of a Goldstone boson with masses m^2/M_X at the tree level. We expect this behaviour to occur also for more general global symmetries. In the case of an anomalous U(1) (Peccei–Quinn symmetry) the invisible axion (a pseudoscalar) will obtain a tiny mass from the anomaly. Its partners, a scalar and a fermion, obtain bigger, but nevertheless very small, masses at tree level or order m^2/M_X, roughly $(10^{-2}$–$1)$ eV for typical supersymmetry breaking scales. Their couplings to light fermions are at most of order $f \sim m_W/M_X \sim 10^{-13}$. Radiative corrections from quark–squark loops will be small, i.e., at most of order $f^2 m_W \sim 10^{-17}$ eV. Nevertheless, it might be possible to find experimental signals of their existence through their couplings to Higgs'. The coupling of the scalar to gravitation would also be very pronounced if it happened to remain massless [‡4]. With a mass of 10^{-2} eV it will not influence macroscopic gravitational interactions, but it will play a role in astrophysics.

In conclusion, we would like to restate our result. In supersymmetric theories with a global symmetry broken at very high energy scales, the partners of Goldstone bosons remain light.

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‡3 This includes R-symmetries with minor modifications in the proof.
‡4 What we obtained is just an upper bound.

References

[1] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16 (1977) 1791;
S. Weinberg, Phys. Rev. Lett. 40 (1978) 223;
F.A. Wilczek, Phys. Rev. Lett. 40 (1978) 279.
[2] D.A. Dicus, E.W. Kolb, V.L. Teplitz and R.V. Wagoner, Phys. Rev. D18 (1978) 1829 and D22 (1980) 839;
K. Sato and H. Sato, Prog. Theor. Phys. 54 (1975) 1564.
[3] M. Dine, W. Fischler and M. Srednicki, Phys. Lett. 104B (1981) 199;
M. Wise, H. Georgi and S.L. Glashow, Phys. Rev. Lett. 47 (1981) 402;
R. Barbieri, R.N. Mohapatra, D.V. Nanopoulos and D. Wyler, Phys. Lett. 107B (1981) 80.
[4] J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39;
D.V. Volkov and V.P. Akulov, Phys. Lett. 46B (1973) 109.
[5] E. Witten, Nucl. Phys. B185 (1981) 513;
S. Dimopoulos and H. Georgi, HUTP-81/A022 (1981);
N. Sakai, Tohoku University preprint TU/81/225 (1981);
R.K. Kaul, Tata Institute preprint TIFR/TH/81-32 (1981).
[6] S. Raby and H.P. Nilles, SLAC preprint 2743 (1981), to be published in Nucl. Phys. B.
[7] S. Ferrara, L. Girardello and F. Palumbo, Phys. Rev. D20 (1979) 403;
S. Ferrara, CERN preprint TH-3190 (1981).