hep-ph/9703374

IOA-05-97

$b-\tau$ UNIFICATION AND NEUTRINO MASSES IN $SU(5)$ EXTENSIONS OF THE MSSM WITH RADIATIVE ELECTROWEAK BREAKING.

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Abstract

We make a complete analysis of the Yukawa coupling unification in $SU(5)$ extensions of the MSSM in the framework of the radiative symmetry breaking scenario. Both logarithmic and finite threshold corrections of sparticles have been included in the determination of the gauge and Yukawa couplings at M_Z. The effect of the heavy masses of each model in the renormalization group equations is also included. We find that in the minimal $SU(5)$ model $b-\tau$ Yukawa unification can be achieved for too large a value of α_s. On the other hand the *Peccei-Quinn* version of the *Missing Doublet* model, with the effect of the right handed neutrino also included, exhibits $b-\tau$ unification in excellent agreement with all low energy experimental data. Unification of all Yukawa couplings is also discussed.

March 1997

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I. INTRODUCTION

Softly broken supersymmetry [1], possibly resulting from an underlying superstring framework, can lead to $SU(2)_L \times U(1)_Y$ gauge symmetry breaking through radiative corrections for a certain range of values of the existing parameters [2] [3] [4]. A most appealing feature of the Minimal Supersymmetric Standard Model (MSSM) and its extensions is that it exhibits gauge coupling unification [5] in good agreement with the low energy values of the three gauge couplings, known to the present experimental accuracy, within the bounds set by the stability of the proton. The Higgs boson running mass-squared matrix, although positive definite at very high energies, when radiatively corrected develops at low energies a negative mass-squared eigenvalue that triggers electroweak symmetry breaking. Running parameters are studied in the framework of the Renormalization Group equations.

In the present article we complete the analysis of electroweak symmetry breaking in the MSSM and various $SU(5)$ extensions of it [6] [7], focusing on fermion masses. Yukawa coupling relations, such as the $b-\tau$ Yukawa coupling equality at high energies in relation to the experimental $\frac{m_b}{m_\tau}$ value, give an extra constraint on models in addition to existing strong constraints like the experimental value of $\alpha_s(M_Z)$. The input parameters of our analysis, apart from the standard low energy inputs (α_{EM}, G_F, M_Z, quark and lepton masses) are the soft breaking parameters (M_o, $M_{1/2}$, A_o), the ratio of the two v.e.vs $tan\beta(M_Z)$, the sign of the Higgs mixing parameter μ and the superheavy particle masses (M_{H_c}, M_Σ,...) entering through their thresholds in the Renormalization Group equations. On the other hand, our output includes the strong coupling $\alpha_s(M_Z)$, the complete sparticle spectrum as well as other quantities, like the unification scale M_{GUT}, etc. In this analysis, for obvious reasons, we shall concentrate on that part of parameter space that leads to α_s compatible with experiment. The experimental values of third generation fermion masses will be introduced as input while the high energy values of Yukawa couplings will be treated as an output.

We have included both logarithmic and finite one loop corrections (see Ref. [7] for more details) in the calculation of $sin^2\theta(M_Z)|_{\overline{DR}}$ that determines the $\hat{g}_1|_{\overline{DR}}$ and $\hat{g}_2|_{\overline{DR}}$ gauge coupling values which are taken as boundary conditions at M_Z. Yukawa couplings are determined with the same next to leading order accuracy as we employed in our analysis for the gauge couplings. We extract the $\overline{DR}$ values of the Yukawa couplings at M_Z from the corresponding pole masses of quarks and leptons [8] as we describe below.

The fermion masses, defined as the poles of the corresponding propagators, are related to the $\overline{DR}$ masses, $\hat{m}_f(Q)$, by the self energies, $\Sigma_f(\not{p})$, as

$$m_f^{pole} = \hat{m}_f(Q) + \Re e\Sigma_f(m_f^{pole}) \tag{1}$$

where $-i\Sigma_f(m_f^{pole}) = -i(\Sigma_1 + m_f^{pole}\Sigma_\gamma)$ is the one loop self energy on shell of quarks or leptons. Our results for self energies agree with those of references [9] [10]. In the case of the *Peccei-Quinn* version of the *Missing Doublet Model* (MDM+PQ) the presence of the right handed neutrino of mass M_R gives rise to a mass for the left handed neutrino $m_\nu = \frac{Y_{\nu_\tau}^2 v^2 sin\beta}{M_R}$ via the see-saw mechanism [11]. The calculated one loop neutrino self energy correction is denoted by Σ_ν. From equation (1) we arrive at the values of Yukawa couplings at M_Z

$$\hat{Y}_b(M_Z)|_{\overline{DR}} = \frac{m_b^{pole} - \Re e\Sigma_b(m_b^{pole})|_{\overline{DR}}}{\hat{v}(M_Z)|_{\overline{DR}}cos\beta(M_Z)} \tag{2}$$

$$\hat{Y}_\tau(M_Z)|_{\overline{DR}} = \frac{m_\tau^{pole} - \Re e\Sigma_\tau(m_\tau^{pole})|_{\overline{DR}}}{\hat{v}(M_Z)|_{\overline{DR}}cos\beta(M_Z)} \tag{3}$$

$$\hat{Y}_t(M_Z)|_{\overline{DR}} = \frac{m_t^{pole} - \Re e\Sigma_t(m_t^{pole})|_{\overline{DR}}}{\hat{v}(M_Z)|_{\overline{DR}}sin\beta(M_Z)} \tag{4}$$

$$\hat{Y}_\nu(M_Z)|_{\overline{DR}} = \sqrt{\frac{M_R(m_{\nu_\tau}^{pole} - \Re e\Sigma_{\nu_\tau}(m_{\nu_\tau}^{pole})|_{\overline{DR}})}{\hat{v}^2(M_Z)|_{\overline{DR}}sin^2\beta}} \tag{5}$$

where all hats mean running couplings or masses evaluated in the $\overline{DR}$ scheme. The running vacuum expectation value is given by the following formula,

$$M_Z^2 + \Re e\Pi_{ZZ}^T(M_Z^2) = \frac{1}{2}(\hat{g}_1^2(M_Z) + \hat{g}_2^2(M_Z))\hat{v}^2(M_Z) \tag{6}$$

where M_Z^2 refers to the experimental Z-boson mass and Π_{ZZ}^T is the real transverse part of the Z-boson one loop corrections at M_Z.

For a given set of pole masses m_t^{pole}, m_b^{pole}, m_τ^{pole} we define the $\overline{DR}$ Yukawa couplings at M_Z. Then we use the 2-loop Renormalization Group equations to run up to the scale M_{GUT}, where $\hat{g}_1$ and $\hat{g}_2$ meet. We impose the unification condition

$$g_{GUT} = \hat{g}_1(M_{GUT}) = \hat{g}_2(M_{GUT}) = \hat{g}_3(M_{GUT}) \tag{7}$$

and run down to M_Z. The thresholds of the heavy masses, both in gauge and Yukawa couplings, are treated with by using the well known method of the step function approximation [12] [6]. We have assumed universal boundary conditions for the soft breaking parameters A_o, M_o, $M_{1/2}$. The whole procedure is iterated until convergence is reached, imposing the constraints of radiative symmetry breaking, the proton decay bound, the experimental bounds on supersymmetric particles and the perturbativity of the Yukawa and gauge couplings up to M_P.

The value of the bottom quark pole mass can be determined indirectly through its influence on hadron properties. In Table I we display the pole mass of the bottom quark as it is extracted from experiment in different processes following various techniques [13]. (For the determination of the pole mass of the bottom quark from QCD momentum sum rules for the Υ system see the recent article of Ref. [14].)

TABLE I

Description	$m_b(GeV)$
Lattice computation of Υ spectroscopy	5 ± 0.2
$e^+e^- \to b$ hadrons cross sections	4.827 ± 0.007
Heavy quark effective theory	4.61 ± 0.05
Υ system	4.60 ± 0.02

Table I: Values of the bottom quark pole mass

As a characteristic mean value of the bottom quark mass we shall choose below $m_b = 4.9GeV$. The experimental mass of τ-lepton is $m_\tau = 1.777GeV$. In addition, we adopt for the mass of the top quark the average experimental value of $D\emptyset$ and CDF experiment $m_t = 180GeV$. We also take the τ-neutrino to be in the cosmologically interesting domain $m_{\nu_\tau} = 3 - 10eV$. When we depart from the above values we indicate so.

The unification of the Yukawa couplings in the MSSM or in some supersymmetric Grand Unified Models has been studied in [8] [15] [16]. The presence of right-handed neutrinos as the only thresholds in the grand desert is studied in [17] while some of their consequences in a supersymmetric $SO(10)$ model in [18]. In this article we examine, in a higher level accuracy framework, the ratios of the Yukawa couplings at the Unification scale M_{GUT} in the minimal $SU(5)$ and the Peccei-Quinn version of the Missing Doublet Model, in which the right-handed neutrino is included.

II. A LITTLE BIT MORE ABOUT THE MINIMAL SU(5) MODEL

In this section we apply the preceeding formalism on the minimal supersymmetric $SU(5)$ model. We start with the superpotential [19],

$$\mathcal{W} = \frac{1}{2} M_1 Tr(\Sigma^2) + \frac{1}{3}\lambda_1 Tr(\Sigma^3) + M_2 \overline{H} H + \lambda_2 \overline{H} \Sigma H$$
$$+ \sqrt{2} Y^{ij}_{(d)} \Psi_i \phi_j \overline{H} - \frac{1}{4} Y^{ij}_{(u)} \Psi_i \Psi_j H \qquad i,j = 1..3 \tag{8}$$

where the superfields Σ, H, $\overline{H}$, ϕ, Ψ belong to the representations **24**, **5**, $\overline{\mathbf{5}}$, $\overline{\mathbf{5}}$, and **10** respectively. The $SU(5)$ breaking occurs when the Σ superfield develops a vacuum expectation value in the direction $< \Sigma > \equiv V\, Diag(2,2,2,-3,-3)$. In the resulting effective $SU(3) \times SU(2) \times U(1)$ theory we get the following heavy particles with quantum numbers:

$$M_{H_c}^{(3,1,-\frac{1}{3})} = M_{\overline{H}_c}^{(3,1,\frac{1}{3})} = 5\lambda_2 V \tag{9}$$

$$M_\Omega^{(8,1,0)} = M_\omega^{(1,3,0)} \equiv M_\Sigma = 5\lambda_1 V \quad , \quad M_S^{(1,1,0)} = \frac{M_\Sigma}{5} \tag{10}$$

$$M_V = 5g_5 V \tag{11}$$

In addition to the RGE's for the gauge couplings [7], we display here the RGE's for the Yukawa couplings, treating the superheavy thresholds in the step function approximation

$$16\pi^2 \frac{dY_b}{dt} = Y_b\{6Y_b^2 + Y_t^2 + Y_\tau^2 + 3Y_b^2\Theta(\overline{H}_c) + 2Y_t^2\Theta(H_c) + \frac{3}{2}\lambda_2^2\Theta(\omega)$$
$$+ \frac{3}{10}\lambda_2^2\Theta(S) + 3\lambda_2^2\Theta(V) - (\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) - 8g_5^2\Theta(V)\} \tag{12}$$

$$16\pi^2 \frac{dY_\tau}{dt} = Y_\tau\{4Y_\tau^2 + 3Y_b^2 + \frac{3}{2}\lambda_2^2\Theta(\omega) + \frac{3}{10}\lambda_2^2\Theta(S) + 3\lambda_2^2\Theta(V)$$
$$+ 3Y_b^2\Theta(\overline{H}_c) + 3Y_t^2\Theta(\overline{H}_c) - (\frac{9}{5}g_1^2 + 3g_2^2) - 12g_5^2\Theta(V)\} \tag{13}$$

$$16\pi^2 \frac{dY_t}{dt} = Y_t\{6Y_t^2 + Y_b^2 + 3Y_b^2\Theta(\overline{H}_c) + 3Y_t^2\Theta(H_c) + \frac{3}{10}\lambda_2^2\Theta(S)$$
$$+ \frac{3}{2}\lambda_2^2\Theta(\omega) + 3\lambda_2^2\Theta(V) - (\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) - 10g_5^2\Theta(V)\} \tag{14}$$

$$16\pi^2 \frac{d\lambda_1}{dt} = 3\lambda_1\Theta(\omega)\{\frac{6}{5}\lambda_1^2\Theta(S) + 3\lambda_1^2\Theta(V) + \lambda_2^2 - 4g_2^2 - 6g_5^2\Theta(V)\} \tag{15}$$

$$16\pi^2 \frac{d\lambda_2}{dt} = \lambda_2\Theta(\omega)\{3Y_b^2 + Y_\tau^2 + 3Y_t^2 + 3\lambda_2^2\Theta(\omega) + \frac{3}{5}\lambda_2^2\Theta(S) + 6\lambda_2^2\Theta(V)$$
$$+ \frac{6}{5}\lambda_1^2\Theta(S) + 3\lambda_1^2\Theta(V) + \lambda_2^2 - (\frac{3}{5}g_1^2 + 7g_2^2) - 12g_5^2\Theta(V)\} \tag{16}$$

where $t = ln\frac{Q}{M_{GUT}}$. We have denoted $\Theta(x) \equiv \Theta(Q^2 - M_x^2)$. Note that $M_{GUT} = max\{M_V, M_{H_c}, M_\Sigma\}$ which we take equal to M_V as it turns out to be in all relevant cases [7].

In Figure 1 we have plotted the ratio of bottom and tau Yukawa couplings at M_{GUT} as a function of $tan\beta$ and the input high energy mass M_Σ. We select as input soft breaking masses at M_{GUT}, so as to give consistent low energy results within the experimentally acceptable region. We obtain that the $b-\tau$ unification in the minimal SU(5) is in agreement with the pole mass of the bottom quark extracted from the Lattice computation of Υ spectroscopy as it is shown in Table I. Nevertheless, even in this case the output $\overline{MS}$ value of $\alpha_s(M_Z)$ is quite large ($\alpha_s(M_Z) > 0.127$) compared with its average experimental value 0.118 ± 0.003 [13]. Both the proton decay bound and the theoretical perturbativity bound on the top Yukawa coupling ($Y_t < 1.5$ at M_{GUT}) strongly constrain the Yukawa unification. We have also seen that Yukawa unification is sensitive through α_s to the logarithmic and finite self-energy corrections of Yukawa couplings as well as to the analogous corrections to the Weinberg angle at M_Z. As it is obvious from Figure 1, decreasing the value of M_Σ destroys (for the lower values of m_b) or restores (for higher values of m_b) the picture of Yukawa unification in contrast to the MSSM case, in which all the high energy particles have been decoupled from the renormalization group equations. In Ref. [20] the hope was expressed that lowering the value of M_Σ might result in the unification of gauge couplings at a scale close to the so-called string scale $M_{string} = 5 \times 10^{17} GeV$. This can be the case only if $m_b \geq 5GeV$, as shown in Figure 1, but again with too large a value for α_s [7]. A proposed mechanism to reconcile the value of α_s with the experimental data is the one which goes through the high energy threshold of colour triplet mass M_{H_c}. Lowering this value down to $10^{15} GeV$ we obtain values of $\alpha_s(M_Z)$ which are in good agreement with the experimental ones. This cannot be the case however in minimal SU(5) where we encounter the bound $M_{H_c} \geq 1.8 \times 10^{16}$ GeV due to the proton decay constraint. This problem is solved in the *Peccei-Quinn* version of the *Missing Doublet Model* where the proton decay rate is suppressed due to the presence of the intermediate scale $10^{12} GeV$. Putting aside the problem of large α_s in the minimal $SU(5)$ model and varying M_{H_c} in the region $(1.8-3) \times 10^{16} GeV$ we obtain values for the $b-\tau$ Yukawa ratio, inside the shaded region of Figure 1.

III. THE PECCEI-QUINN VERSION OF THE MISSING DOUBLET MODEL

In order to avoid the numerical fine-tuning required in the minimal $SU(5)$ model we can replace the adjoint Higgs representation by the **75** representation which couples the Higgs pentaplets to an extra pair of Higgses Θ and $\overline{\Theta}$ in the $\mathbf{50}+\overline{\mathbf{50}}$ representation respectively that contains no isodoublets. This extension of the minimal $SU(5)$ is known as the *Missing Doublet* model [21]. The superpotential of this model is

$$\mathcal{W} = M_1 Tr(\Sigma^2) + \frac{1}{3}\lambda_1 Tr(\Sigma^3) + \lambda_2 H\Sigma\Theta + \overline{\lambda}_2\overline{H}\Sigma\overline{\Theta} + M_2\overline{\Theta}\Theta$$
$$-\frac{1}{4}Y^{ij}_{(u)}\Psi_i\Psi_j H + \sqrt{2}Y^{ij}_{(d)}\Psi_i\phi_j\overline{H} \qquad i,j=1..3 \tag{17}$$

As it has been shown in Ref. [7] this model built in such a way, although it predicts small values of $\alpha_s(M_Z)$, leaves a narrow window in the parametric space (especially in the $tan\beta$) to work with, because of the bound which comes from the proton decay stability. This problem which is also present in the minimal $SU(5)$, albeit with large values of $\alpha_s(M_Z)$, provided strong motivation to construct versions of $SU(5)$ with a *Peccei-Quinn* symmetry [22] that naturally suppresses $D=5$ operators by a factor proportional to the ratio of the *Peccei-Quinn* breaking scale over the GUT scale [23]. If we promote the symmetry by an extra global *Peccei-Quinn* $U(1)$ factor the terms responsible for Higgs masses must be replaced by

$$\lambda_2 H\Sigma\Theta + \overline{\lambda}_2\overline{H}\Sigma\overline{\Theta} + \lambda_2' H'\Sigma\Theta' + \overline{\lambda}_2'\overline{H}'\Sigma\overline{\Theta}'$$
$$+ M_2\Theta\overline{\Theta}' + M_2'\Theta'\overline{\Theta} + \lambda_3 P\overline{H}'H'$$

We have introduced a new set of chiral superfields,$H'(\mathbf{50})$, $\overline{H}'(\overline{\mathbf{50}})$, $\Theta'(\mathbf{50})$, $\overline{\Theta}'(\overline{\mathbf{50}})$ and P is a gauge singlet superfield. The charges under the *Peccei-Quinn symmetry* are $\Psi(1)$, $\phi(-1/2)$, $H(-2)$, $\overline{H}(-1/2)$, $\Theta(2)$, $\overline{\Theta}(1/2)$, $\Theta'(-2)$, $\overline{\Theta}'(-1/2)$, $\overline{H}'(1/2)$, $H'(2)$, $P(-5)$. The *Peccei-Quinn* symmetry is broken * when the gauge singlet P develops a vacuum expectation value

*This is achieved through a suitable sector involving singlets [24].

at an intermediate energy $< P > \equiv \frac{M_{H_f'}}{\lambda_3} \sim 10^{10} - 10^{12} GeV$. The model is naturally extended by introducing the three right-handed neutrino superfields N_i^c adding to the superpotential the following Yukawa interactions:

$$\mathcal{W}_N = Y_\nu N^c \phi H + \frac{1}{2}\lambda_4 N^c N^c P \tag{18}$$

These terms induce Majorana masses for the right-handed neutrino multiplets N_i^c and these masses are proportional to the intermediate breaking scale since $M_R = \frac{\lambda_4}{\lambda_3} M_{H_f'} \sim O(10^{11} - 10^{12}) GeV$. Finally, the terms incorporated in $\mathcal{W}_N$ induce through the see-saw mechanism [11] very small masses for the neutrinos. The spectrum obtained after the breaking of the $SU(5)$ gauge group can be read from Table II where we have used the following reasonable assumptions for the parameters in the superpotential of this model:

$$\lambda_2 \simeq \overline{\lambda}_2 \simeq \lambda_2' \simeq \overline{\lambda}_2' \qquad M_2 \simeq M_2' \sim 10^{18} GeV \tag{19}$$

TABLE II		
75	50	Colour Triplets
$M_{S_8}(8,3,0) \equiv M_\Sigma$	$M(6,1,\frac{4}{3}) = 4M_2$	$M_{H_c} \simeq M_{H_c'} \simeq 32\lambda_2^2 \frac{V^2}{M_2}$
$M_S(1,1,0) = 0.4 M_\Sigma$	$M(8,2,\frac{1}{2}) = 8M_2$	$M_{\Theta_c} \simeq M_{\Theta_c'} \simeq 8M_2$
$M_{S_6}(6,2,\frac{5}{6}) = M_{\overline{S}_6}(\overline{6},2,-\frac{5}{6}) = 0.4 M_\Sigma$	$M(\overline{6},3,-\frac{1}{3}) = 4M_2$	Doublets
$M_{\mathcal{O}}(8,1,0) = 0.2 M_\Sigma$	$M(\overline{3},2,-\frac{7}{6}) = 8M_2$	$M_{H_f} = 0$
$M_{S_3}(3,1,\frac{5}{3}) = M_{\overline{S}_3}(\overline{3},1,-\frac{5}{3}) = 0.8 M_\Sigma$	$M(1,1,-2) = 8M_2$	$M_{H_f'} = \lambda_3 < P > \simeq 10^{11} GeV$
$M_\Sigma \equiv 20 M_1 = -\frac{40}{3}\lambda_1 V$		$M_V = \sqrt{6} g_5 V$

Table II: The spectrum of the MDM+PQ model

The running of the Yukawa couplings at the one loop level is given by the following renormalization group equations,

$$16\pi^2\frac{dY_b}{dt} = Y_b\{6Y_b^2 + Y_t^2 + Y_\tau^2 + \frac{3}{2}Y_b^2\Theta(H_c) + \frac{3}{2}Y_b^2\Theta(H_c') + Y_t^2\Theta(H_c)$$

$$+ Y_t^2\Theta(H_c') + Y_\nu^2\Theta(H_c)\Theta(R) - (\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) - 8g_5^2\Theta(V)\} \quad (20)$$

$$16\pi^2\frac{dY_\tau}{dt} = Y_\tau\{4Y_\tau^2 + 3Y_b^2 + \frac{3}{2}Y_b^2\Theta(H_c) + \frac{3}{2}Y_b^2\Theta(H_c') + \frac{3}{2}Y_t^2\Theta(H_c)$$

$$+ \frac{3}{2}Y_t^2\Theta(H_c') + Y_\nu^2\Theta(R) - (\frac{9}{5}g_1^2 + 3g_2^2) - 12g_5^2\Theta(V)\} \quad (21)$$

$$16\pi^2\frac{dY_t}{dt} = Y_t\{6Y_t^2 + Y_b^2 + \frac{3}{2}Y_b^2\Theta(H_c) + \frac{3}{2}Y_b^2\Theta(H_c') + \frac{3}{2}Y_t^2\Theta(H_c)$$

$$+ \frac{3}{2}Y_t^2\Theta(H_c') + Y_\nu^2\Theta(R) - (\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) - 10g_5^2\Theta(V)\} \quad (22)$$

$$16\pi^2\frac{dY_\nu}{dt} = Y_\nu\Theta(R)\{Y_\tau^2 + 3Y_t^2 + 2Y_\nu^2 + \frac{3}{2}Y_b^2\Theta(H_c) + \frac{3}{2}Y_b^2\Theta(H_c')$$

$$+ 2Y_\nu^2\Theta(R) + 3Y_\nu^2\Theta(H_c) - (\frac{3}{5}g_1^2 + 3g_2^2)\} \quad (23)$$

$$16\pi^2\frac{d\lambda_1}{dt} = 3\lambda_1\Theta(\mathcal{O})\{\frac{4000}{81}\lambda_1^2 + \frac{1152}{29}\lambda_1^2\Theta(S) + 144\lambda_1^2\Theta(S_6) + \frac{128}{3}\lambda_1^2\Theta(S_3)$$

$$+ \frac{160}{3}\lambda_1^2\Theta(S_8) + \frac{1744}{27}\lambda_1^2\Theta(V) - 6g_3^2 - 10g_5^2\Theta(V)\} \quad (24)$$

Note that $\Theta(R)$ stands for $\Theta(Q^2 - M_R^2)$ and that again we consider cases with $M_V = M_{GUT}$.

We have taken for all figures arranged below the input soft breaking masses at M_{GUT} to be: $A_o = 400GeV$, $M_o = 300GeV$ and $M_{1/2} = 300GeV$. Also the superheavy mass M_Σ is taken to be $M_\Sigma = 10^{16}GeV$. These values are indicative values corresponding to acceptable α_s [7].

In Figure 2 we have plotted the ratio of the $b-\tau$ Yukawa couplings at M_{GUT} as a function of $tan\beta(M_Z)$ when we vary the pole mass of the bottom quark in the range allowed by Table I. It is clear in the case of the MDM+PQ model we get $b-\tau$ Yukawa unification both for $\mu > 0$

or $\mu < 0^\dagger$ together with the excellent agreement of the value of $\alpha_s(M_Z)$ as this is compared with the experimental average value 0.118 ± 0.003. In the case of $\mu > 0$ for large $\tan\beta$ there are significant finite threshold corrections in the bottom and tau Yukawa couplings which arise from the chargino/squark or gluino/squark loops and they are proportional to $\mu tan\beta$. This fact [15] supports $b-\tau$ Yukawa unification in the large value region of $tan\beta$ for $\mu > 0$. Note also that the upper bound in the input parameter $tan\beta(M_Z) \leq 36$ does *not* come from the bound of the proton decay rate as in the case of the minimal $SU(5)$ model, but from the requirement of the radiative symmetry breaking. This is a special feature of the *Peccei-Quinn* version of the *Missing Doublet Model* and is valid for all figures arranged here.

In Figure 3 we display the ratio of both $b-\tau$ and $t-b$ Yukawa couplings as the top pole mass is varied inside the region $m_t^{pole} = 182 \pm 8 GeV$ in the MDM+PQ model. The $\frac{Y_b}{Y_\tau}$ ratio then takes values in the regions $0.91 - 1.09$ for $\mu < 0$ and $1.00 - 1.11$ for $\mu > 0$ respectively. Note also that as can be seen in Fig.3c and Fig.3d the $t-b$ ratio never reaches unity due to the constraint of the radiative symmetry breaking.

Proceeding next to Figure 4 we vary the mass of the colour triplet M_{H_c}(Fig4a,b) and the mass of the intermediate doublet $M_{H'_f}$ (Fig4c,d) for $\mu < 0$ and $\mu > 0$ keeping fixed the $m_t^{pole} = 180 GeV$ and $m_b^{pole} = 4.9 GeV$. Acceptable values for α_s are obtainable if we stay in the vicinity of $M_{H_c} = 10^{15} GeV$ and $M_{H'_f} = 10^{11} GeV$ [7]. For larger values of the colour triplet or smaller values of the intermediate doublet we get larger values for the strong coupling α_s completely ruled out by experiment. As can be seen in Figure 4 $b-\tau$ unification seems to prefer the case where $\mu < 0$ and low values of $tan\beta \simeq 5$. If we lower the m_b^{pole} down to $4.8 GeV$ we also get for $\mu > 0$ (this is shown in Fig.2) $b-\tau$ unification in the low and large $tan\beta$ regime.

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† We follow the conventions of Ref. [6]

In Figure 5 we consider the behavior of all ratios of Yukawa couplings. The Renormalization group equations for the three ratios $R_{t-\nu} \equiv \frac{Y_t}{Y_\nu}$, $R_{b-\tau} \equiv \frac{Y_b}{Y_\tau}$ and $R_{t-b} \equiv \frac{Y_t}{Y_b}$ are

$$\frac{dR_{t-\nu}}{dt} = R_{t-\nu}\{3(Y_t^2 - Y_\nu^2\Theta(R)) + (Y_b^2 - Y_\tau^2) + 3(Y_t^2 - Y_\nu^2)\Theta(H_c) - (\frac{4}{15}g_1^2 + \frac{16}{3}g_3^2) - 10g_5^2\Theta(V)\} \tag{25}$$

$$\frac{dR_{b-\tau}}{dt} = R_{b-\tau}\{3(Y_b^2 - Y_\tau^2) + (Y_t^2 - Y_\nu^2\Theta(R)) + (Y_\nu^2 - Y_t^2)\Theta(H_c) - (\frac{16}{3}g_3^2 - \frac{4}{3}g_1^2) + 4g_5^2\Theta(V)\} \tag{26}$$

$$\frac{dR_{t-b}}{dt} = R_{t-b}\{5(Y_t^2 - Y_b^2) - (Y_\tau^2 - Y_\nu^2\Theta(R)) + (Y_t^2 - Y_\nu^2)\Theta(H_c) - \frac{2}{5}g_1^2 - 2g_5^2\Theta(V)\} \tag{27}$$

It can be easily observed that the $SO(10)$-type condition $Y_t = Y_b = Y_\tau = Y_\nu$ in the limit of high energies where all thresholds theta functions are equal to 1 corresponds a zero of the beta functions for the ratios provided we neglect the contribution of gauge couplings. In contrast to that the $SU(5)$ condition $Y_b = Y_\tau$ leads to a zero of the $\beta_{R_{b-\tau}} = \frac{dR_{b-\tau}}{dt}$ function in the presence of gauge couplings. Note that neither of the above occurs in MSSM where the contributions of the extra fields imposed by the extended GUT symmetry are absent.

Figure 5 shows that the $SO(10)$-like Yukawa unification requires as expected large values of $tan\beta$ which are not allowed by the requirement of radiative symmetry breaking. Note however that this is not necessarily true in a realistic $SO(10)$ model. Although complete Yukawa unification is excluded $Y_b = Y_\tau$ and $Y_t = Y_\nu$ can be realized independently for a wide range of $tan\beta$ values. It should be noted that the neutrino pole-mass that corresponds to $t-\nu$ Yukawa coupling unification is approximately $m_{\nu_\tau}^{pole} = (100GeV)^2/M_R$. This, very roughly, corresponds to a low energy value of the top Yukawa coupling approximately twice that of the neutrino Yukawa coupling. For a cosmologically interesting neutrino of $m_{\nu_\tau}^{pole} = 10eV$ this fixes the lepton number-PQ-breaking scale to be $M_R = 10^{12}GeV$.

IV. BRIEF CONCLUSIONS

In this article we have studied in a 2-loop accuracy framework the unification of the Yukawa couplings in extensions of the MSSM which are based on the group $SU(5)$. Both logarithmic and finite threshold contributions of SUSY particles have been included in the calculation of the gauge and Yukawa couplings at M_Z. We have taken into account the effect of all heavy masses in the renormalization group for the gauge and Yukawa couplings. The models that we have examined are the *minimal* $SU(5)$ model and the *Peccei-Quinn* version of the *Missing Doublet* model. In the minimal $SU(5)$ model $b-\tau$ unification is possible for m_b values larger than $5GeV$ in the low $tan\beta$ regime but in all cases the encountered $\alpha_s(M_Z)$ values are too large to reconcile with experiment. Due to the bound of the proton decay stability we cannot go to the large $tan\beta$ regime in this model. On the other hand, in the *Peccei-Quinn* version of the *Missing Doublet* model, where the effect of the right handed neutrino is naturally included, we have found (Fig.2,3,4) that $b-\tau$ unification at M_{GUT} can be achieved both in the low ($\mu > 0$ and $\mu < 0$) and the large ($\mu > 0$) $tan\beta$ regime for α_s values compatible with the low energy experimental data. It is interesting to note that this is achieved for value of the intermediate *Peccei-Quinn* $b-\tau$ lepton number breaking scale in the cosmologically interesting neighborhood of $10^{12}GeV$. We have also considered whether all the Yukawa couplings are unified at the scale M_{GUT} (Fig.5). Only $b-\tau$ Yukawa coupling unification is possible within this model since the constraint of the radiative symmetry breaking forbids an $SO(10)$ like condition $Y_t = Y_b = Y_\tau = Y_\nu$ at M_{GUT}.

Acknowledgments

We would like to thank A. B. Lahanas and J. Rizos for useful discussions and comments. Both acknowledge support from the Program ΠΕΝΕΔ 95 of the Greek Ministry of Research and Technology.

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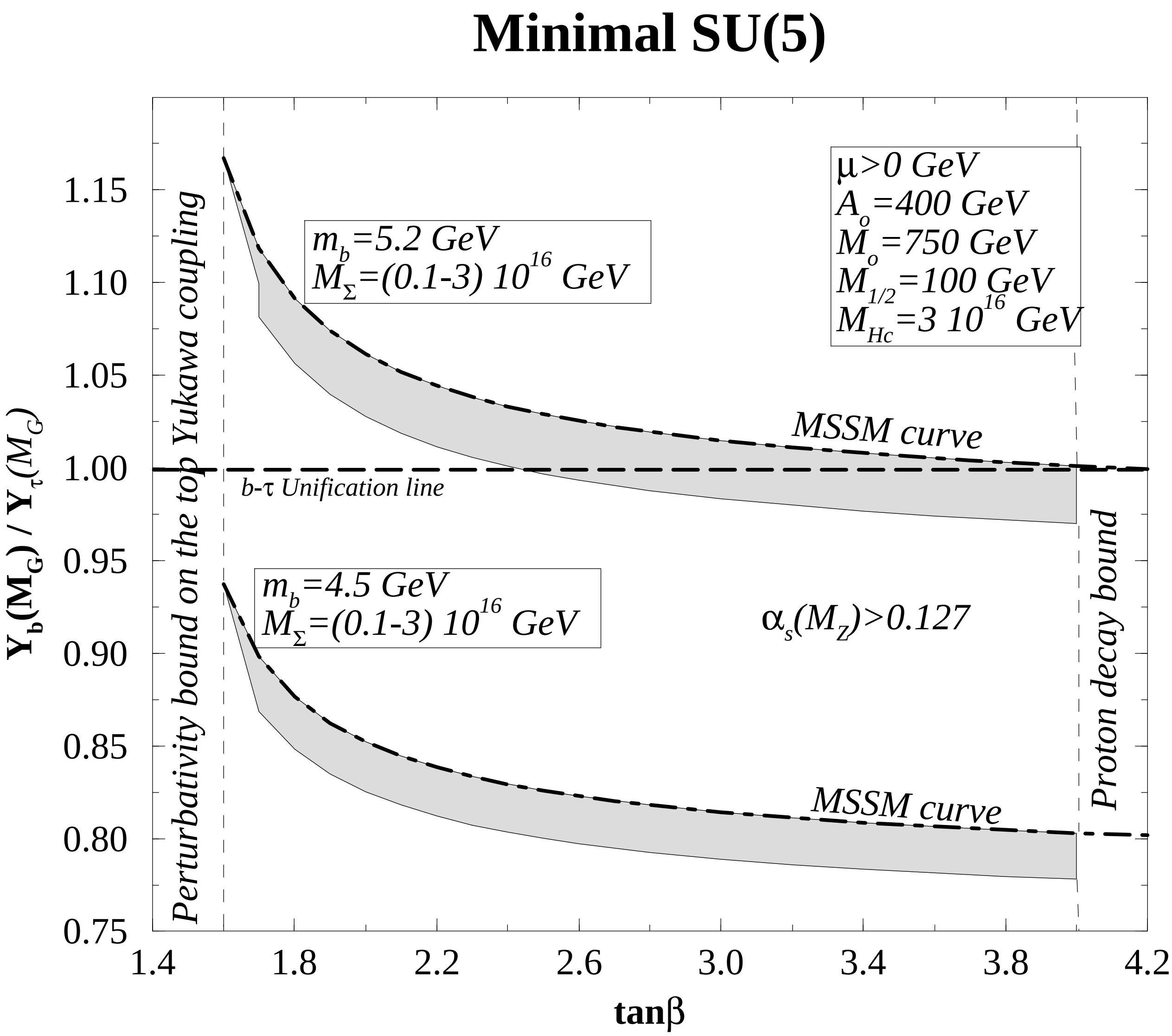

Figure 1: The allowed region for the ratio of bottom and tau Yukawa couplings at M_{GUT}. The input values of the tau lepton and top quark are $m_\tau = 1.777GeV$ and $m_t = 180GeV$ respectively. The extracted value of $\alpha_s(M_Z)$ is greater than .127 . If we still lower the M_Σ value down to $10^{14}GeV$, we observe quite large values of α_s. In the upper curve of the shaded regions the heavy particles M_Σ, M_{H_c} have been decoupled (MSSM curve).

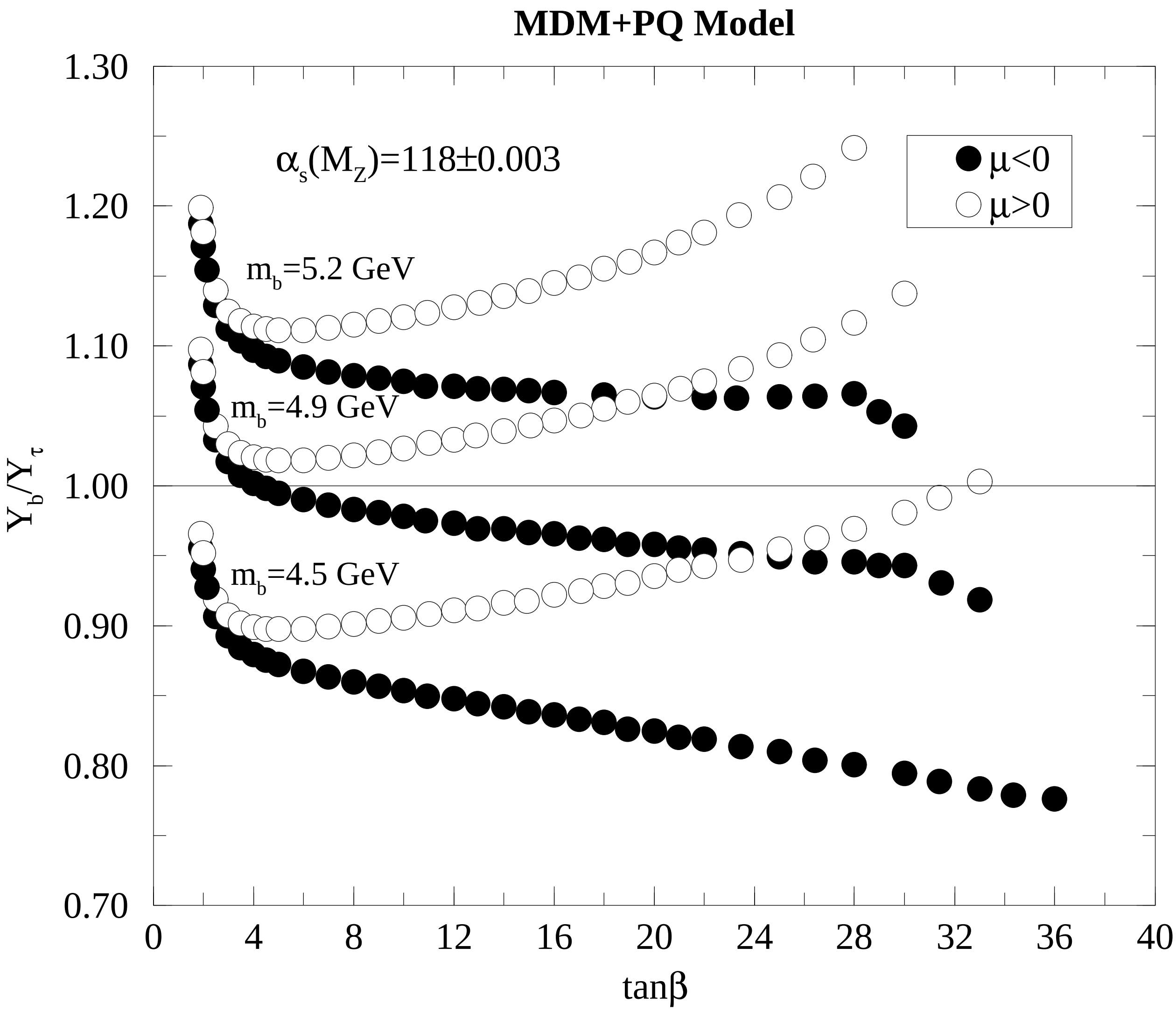

Figure 2: The ratio of the $b-\tau$ Yukawa couplings as a function of $\tan\beta$ in the MDM+PQ model when the m_b^{pole} is varied for $\mu < 0$ and $\mu > 0$. The fixed input parameters are $m_t^{pole} = 180GeV$, $M_{H_f'} = 10^{11}GeV$, $M_{H_c} = 10^{15}GeV$, $M_R = 10^{11}GeV$, $m_{\nu_\tau}^{pole} = 10eV$.

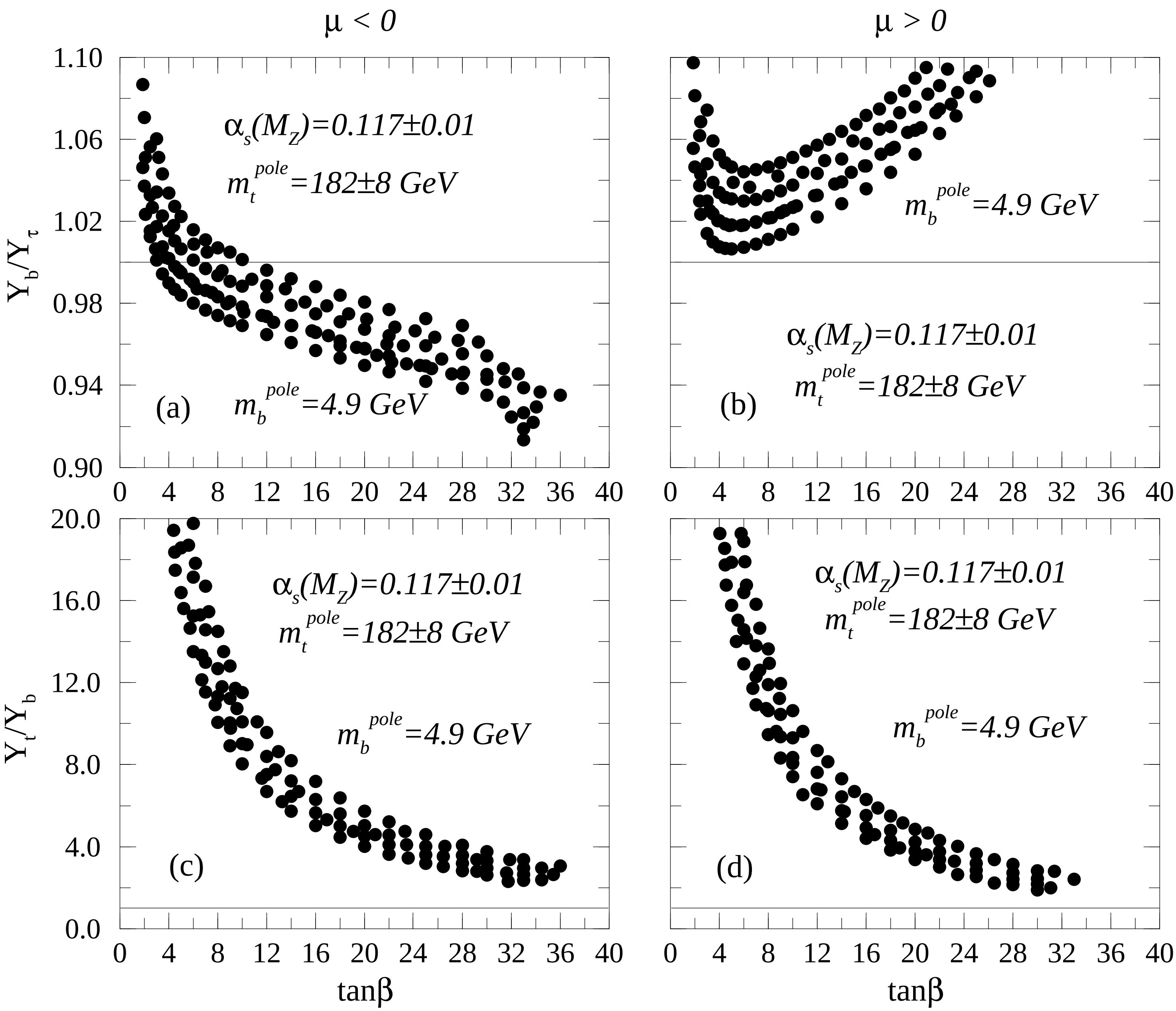

Figure 3: The ratio of the $b-\tau$ and $t-b$ Yukawa couplings at M_{GUT} as the top pole mass is varied. We keep the bottom pole mass, $m_b^{pole} = 4.9 GeV$ fixed. The horizontal line shows the $b-\tau$ or $t-b$ unification. The other inputs are similar to those displayed in the Figure 2.

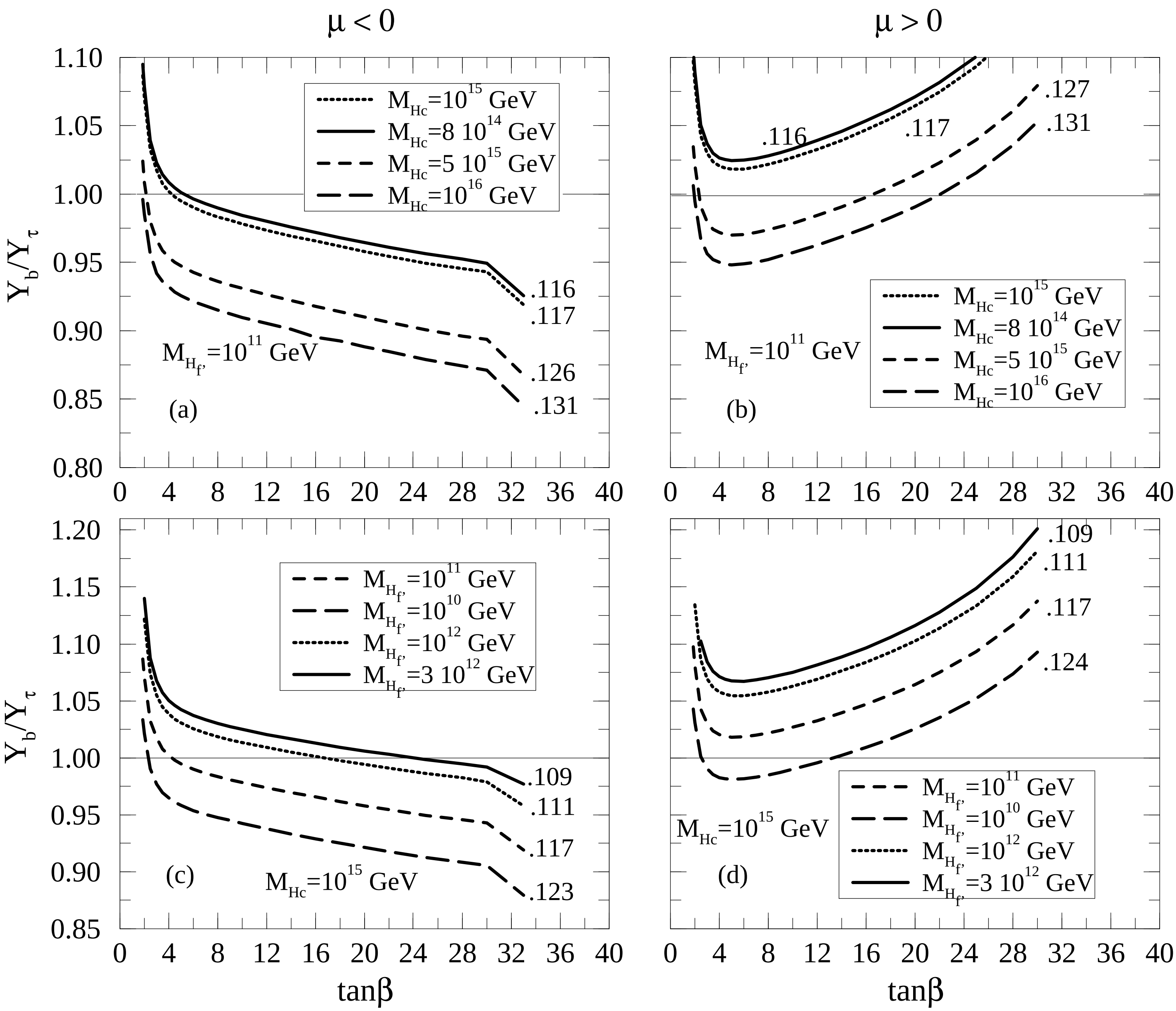

Figure 4: The effect of the variation of the M_{H_c} and $M_{H'_f}$, within the range allowed by proton stability in the $b-\tau$ unification and also in the extracted value of α_s. We take as inputs: $m_b^{pole} = 4.9GeV$ and $m_t^{pole} = 180GeV$.

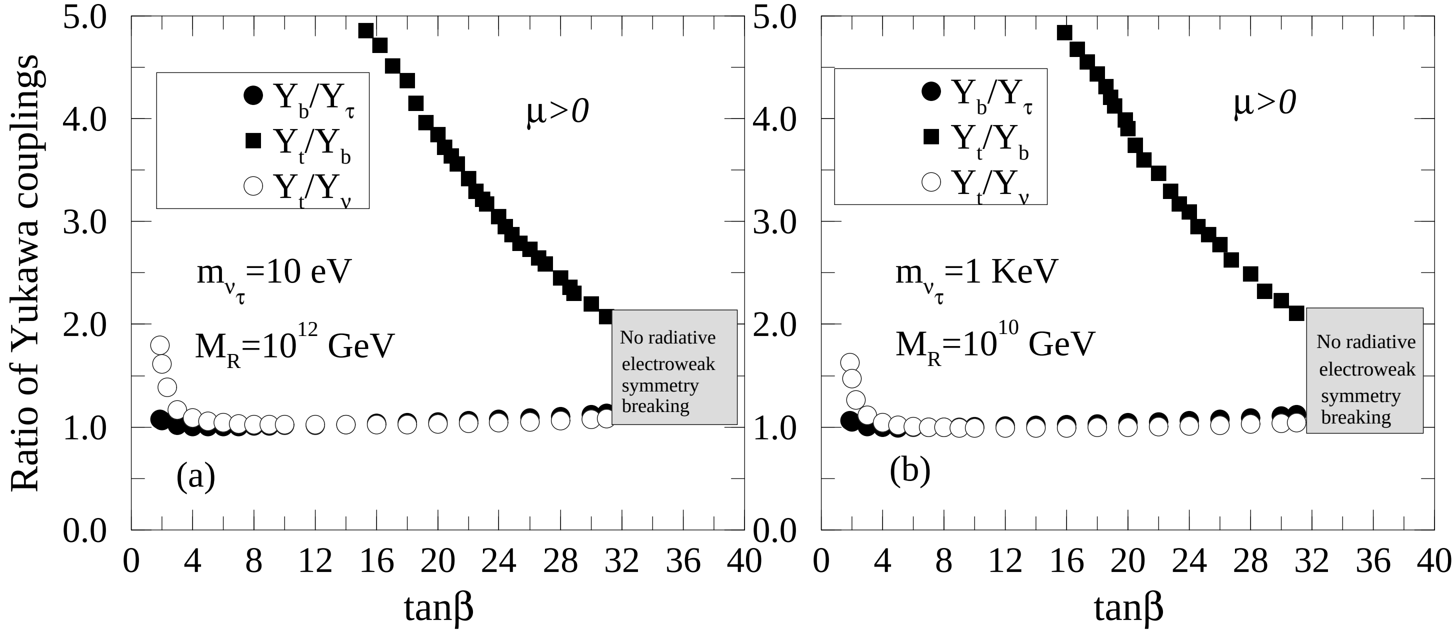

Figure 5: Unification of all Yukawa couplings in the MDM+PQ model with $m_b^{pole} = 4.9GeV$, $m_t^{pole} = 180GeV$ and the other input parameters displayed in Fig.2