Calculation of the top quark mass in the flipped SU(5)×U(1) superstring model

G.K. Leontaris, J. Rizos and K. Tamvakis

Physics Department, University of Ioannina, GR-45110 Ioannina, Greece

Received 23 July 1990

We present a complete renormalization group calculation of the top-quark mass in the SU(5)×U(1) superstring model. We solve the coupled renormalization group equations for the gauge and Yukawa couplings in the two-loop approximation and obtain the top-quark mass as a function of two parameters of the model which could be chosen to be ratios of singlet VEVs associated with the surplus $[U(1)]^4$ breaking. We obtain a heavy top-quark with 150 GeV $\leqslant m_t <$ 200 GeV, for most part of the parameter space, while lower values are possible only in a very small extremal region. We also compute the allowed range of unification parameters $[M_X, \sin^2\theta_w, \alpha_3(M_W)]$ in the presence of a heavy top-quark.

Despite the fact that the standard model has been remarkably successful in describing low-energy phenomena, it has been considered unsatisfactory from a theoretical point of view since it contains many free parameters, such as fermion masses and mixing angles, which are a posteriori fixed by experiment. This has been the main motivation for Grand Unification. GUT's succeeded in predicting successful parameters relations, but within these theories there are still many free parameters. Among them, the top quark mass, since the top quark is the only fermion that still escapes detection, is virtually unconstrained by experiment to lie in a broad range between 60 and 180 GeV. In supersymmetric theories that arise as the field theory limit of superstrings [1] one can calculate the Yukawa couplings at the unification scale in terms of the gauge coupling. It is possible then, following the renormalization group flow, to predict the values of the top quark mass and the other free parameters of the standard model. The only "parameters" on which the fermion masses, calculated in this way, depend on, are VEVs of the various fields of the theory that are in principle determined by the vacuum. Since superstring theory is presently the only framework that promises unification of all forces including gravity, it is very important to obtain the detailed predictions of particular superstring models for the fermion masses. These predictions, apart from serving as a useful guide to experiments for the not yet observed, are a good test for the viability of the, in principle, many specific superstring models [2].

A presently popular model, constructed in the framework of the fermionic formulation of four dimensional superstrings [3], is the flipped SU(5) ×U(1) [4,5] superstring model [6]. The exact gauge group of the model at the superunification scale is $SU(5)\times U(1)'\times U(1)^4\times SO(10)\times SO(6)$. This is successively broken to $SU(5)\times U(1)'\times SO(6)\times SO(10)$ and to $[SU(3)_C\times SU(2)_L\times U(1)_Y]\times SO(6)\times SO(10)$. The "hidden" gauge group $SO(6)\times SO(10)$, which stays unbroken and becomes strong and confining [6,7] at some intermediate scale, is not directly relevant for low-energy physics. The gauge symmetry breaking of the model has been studied in refs. [6,8] and in ref. [9]. The model possesses two pairs of massless Higgs pentaplets. Only one generation of quarks and leptons, interpreted as the top-generation, obtains mass at the level of the cubic superpotential while the other two generations obtains naturally suppressed masses from non-renormalizable superpotential terms [6,9].

The Yukawa couplings of the top-generation at the unification scale [#1] M_X are [#2] [9]

$$\lambda_t^2=2g^2(1-\bar\chi^2),\quad \lambda_b^2=2g^2(\chi^2+\nu^2)\sin^2\psi,$$
$$\lambda_\tau^2=2g^2(\chi^2+\nu^2)\cos^2\psi. \tag{1}$$

The appearing parameters are defined as

$$\chi\equiv\Phi_{45}(\Phi_{23}^2+\bar\Phi_{31}^2+\Phi_{45}^2)^{-1/2},$$
$$\bar\chi\equiv\bar\Phi_{45}(\bar\Phi_{23}^2+\Phi_{31}^2+\bar\Phi_{45}^2)^{-1/2}, \tag{2}$$
$$\psi=\tan^{-1}\left(\frac{\chi\bar\Phi_{31}-\nu\Phi_{23}}{\chi\Phi_{23}+\nu\bar\Phi_{31}}\right), \tag{3}$$

with $\nu\equiv\langle h_0\rangle/\langle h_0'\rangle$. Obviously, $0<\chi,\bar\chi<1$. In what follows we shall simplify our analysis assuming that the isodoublet VEVs $\langle h_0\rangle$ and $\langle\bar h_0\rangle$ are much smaller than $\langle h_0'\rangle$ and $\langle\bar h_0'\rangle$ and, therefore, can be neglected. This amounts to $\nu=0$ and $\psi=\tan^{-1}(\bar\Phi_{31}/\Phi_{23})$. In this simplified case [#3], there are four parameters, namely χ, $\bar\chi$, ψ and the isodoublet VEV ratio

$$k\equiv\frac{\langle\bar h_0'\rangle}{\langle h_0'\rangle}. \tag{4}$$

These parameters are not all free since the low-energy values of the b-quark and τ-lepton masses are fixed by experiment. Thus, there are only two free parameters. These can be chosen to be χ and $\bar\chi$ or χ and k or otherwise. The low-energy masses of b and τ are

$$m_b^2=\frac{\lambda_b^2(\mu)}{2\sqrt{2}\,G_F}\frac{1}{k^2+1},\quad m_\tau^2=\frac{\lambda_\tau^2(\mu)}{2\sqrt{2}\,G_F}\frac{1}{k^2+1},$$

where μ is a scale in the neighborhood of M_W. On the other hand, the top quark mass is

$$m_t^2=\frac{\lambda_t^2(2m_t)}{2\sqrt{2}\,G_F}\frac{k^2}{k^2+1}. \tag{5}$$

In terms of that, $(m_t)_{\rm phys}=m_t[1+(4/3\pi)\alpha_3(2m_t)]$.

There is an interesting constraint satisfied by the parameters χ and $\bar\chi$. The $[\mathrm{U}(1)]^4$ D-flatness conditions [#4] [8,9]:

$$|\Phi_{45}|^2-|\bar\Phi_{45}|^2=\tfrac{1}{15}\xi, \tag{6a}$$
$$|\Phi_{31}|^2+|\bar\Phi_{23}|^2+|\bar\Phi_{45}|^2-(|\bar\Phi_{31}|^2+|\Phi_{23}|^2+|\Phi_{45}|^2)=\tfrac{2}{15}\xi, \tag{6b}$$

can be expressed in terms of χ and $\bar\chi$ and give

$$\Phi_{45}^2=\frac{\xi}{15}\frac{\chi^2(1+2\bar\chi^2)}{\chi^2-\bar\chi^2},$$
$$\bar\Phi_{45}^2=\frac{\xi}{15}\frac{\bar\chi^2(1+2\chi^2)}{\chi^2-\bar\chi^2}. \tag{6c}$$

It is evident from (6) that we have [9]

$$\chi>\bar\chi\quad\text{for }\xi>0,$$
$$\chi<\bar\chi\quad\text{for }\xi<0.$$

The renormalization group equations for the gauge couplings in the two-loop approximation are given by the general formula

$$\frac{d\alpha_i}{dt}=\frac{\alpha_i^2}{2\pi}\left(b_i+\frac{1}{4\pi}\sum_j b_{ij}\alpha_j-\frac{Y_i}{4\pi}\right), \tag{7}$$

with $\alpha_i=g_i^2/4\pi$ and $t=\ln\mu$. With b_i, b_{ij} we have symbolized the beta-function coefficients in the one-loop and two-loop approximation respectively. α_i stands for α_Y, α_2 and α_3 below the unification scale [#5]. The coefficients have been calculated to be [11]

$$b_i=\begin{pmatrix}0\\-6\\-9\end{pmatrix}+\begin{pmatrix}2\\2\\2\end{pmatrix}N_G+\begin{pmatrix}\frac{3}{10}\\\frac{1}{2}\\0\end{pmatrix}N_2+\begin{pmatrix}\frac{1}{5}\\0\\\frac{1}{2}\end{pmatrix}N_3+\begin{pmatrix}\frac{1}{10}\\\frac{3}{2}\\1\end{pmatrix}N_{32}, \tag{8a}$$

[#1] In what follows we shall ignore the negligible effect introduced by the difference of the unification and the superunification scales.

[#2] Note that form (1) of the Yukawa couplings, although derived following ref. [9], is general. It also covers the alternative top generation choice of ref. [6].

[#3] For non-zero ν the analysis is more complicated and will not be carried out here.

[#4] $\xi=(90g^2/96\pi^2)\sqrt{2\alpha'}$ is the anomalous U(1) D-term scale. Although ξ is by our definition positive, we prefer to be more general in order to circumvent a sign dispute that has arisen in the calculation of the anomalous D-term [10].

[#5] The unification scale is defined by $\alpha_3(M_X)=\alpha_2(M_X)=\alpha_X$ while the superunification scale by $\alpha_5(M_{SU})=\alpha_1(M_{SU})$.

$$b_{ij}=\begin{pmatrix}0&0&0\\0&-24&0\\0&0&-54\end{pmatrix}+\begin{pmatrix}\frac{38}{15}&\frac{6}{5}&\frac{88}{15}\\\frac{2}{5}&14&8\\\frac{11}{15}&3&\frac{68}{3}\end{pmatrix}N_G$$

$$+\begin{pmatrix}\frac{9}{50}&\frac{9}{10}&0\\\frac{3}{10}&\frac{7}{2}&0\\0&0&0\end{pmatrix}N_2+\begin{pmatrix}\frac{4}{75}&0&\frac{16}{15}\\0&0&0\\\frac{2}{15}&0&\frac{17}{3}\end{pmatrix}N_3$$

$$+\begin{pmatrix}\frac{1}{150}&\frac{3}{10}&\frac{8}{15}\\\frac{1}{10}&\frac{21}{2}&8\\\frac{1}{15}&3&\frac{34}{3}\end{pmatrix}N_{32}\,. \qquad (8b)$$

The index i stands for Y, 2, 3. We have denoted with N_G the number of generations while N_2 is the number of weak isodoublets and N_3 the number of colour triplets. N_{32} is the number of triplet–doublet representations none of which is present here but have been included for completeness. The term Y_i stands for the contribution of the Yukawa couplings at two loops. The only non-negligible contribution comes from the couplings of the top and bottom quarks and the τ-lepton. Their contribution is

$$Y_3=4(\alpha_t+\alpha_b),\quad Y_2=6(\alpha_t+\alpha_b)+2\alpha_\tau\,,$$
$$Y_1=\tfrac{1}{5}(26\alpha_t+14\alpha_b+18\alpha_\tau)\,. \qquad (9)$$

For the specific particle content of the flipped SU(5)×U(1) superstring model, we have $N_G=3$, $N_2=4$, $N_3=2$ and $N_{32}=0$. We have not included in the light spectrum the unwanted states f_4, $\bar{\ell}_4^c$ together with a linear combination of $\bar{f}_i$, ℓ_i^c, in the established notation [6,8,9], which presumably acquire an intermediate mass $\leqslant O(\Phi^3/M^3)$ not very far from M_X. The difference of this intermediate scale from M_X introduces a rather small error. Above M_X, the spectrum consists of $N_5=4$ pentaplets and $N_G=5$ complete "generations". The coefficients above M_X are ($i=1$, 5)

$$b_i=\begin{pmatrix}0\\-15\end{pmatrix}+\begin{pmatrix}2\\2\end{pmatrix}N_G+\begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\end{pmatrix}N_5\,,$$

$$b_{ij}=\begin{pmatrix}0&0\\0&-150\end{pmatrix}+\begin{pmatrix}\frac{13}{5}&\frac{72}{5}\\\frac{3}{5}&\frac{232}{5}\end{pmatrix}N_G+\begin{pmatrix}\frac{1}{5}&\frac{24}{5}\\\frac{1}{5}&\frac{49}{5}\end{pmatrix}N_5\,. \qquad (10)$$

In addition to the renormalization group equations for the gauge couplings (7), we also have the renormalization group equations for the Yukawa couplings, which are, below M_X,

$$\frac{d\alpha_b}{dt}=\frac{\alpha_b}{\pi}\left(3\alpha_b+\tfrac{1}{2}\alpha_t+\tfrac{1}{2}\alpha_\tau-\tfrac{8}{3}\alpha_3-\tfrac{3}{2}\alpha_2-\tfrac{7}{30}\alpha_Y\right),$$

$$\frac{d\alpha_t}{dt}=\frac{\alpha_t}{\pi}\left(3\alpha_t+\tfrac{1}{2}\alpha_b-\tfrac{8}{3}\alpha_3-\tfrac{3}{2}\alpha_2-\tfrac{13}{30}\alpha_Y\right),$$

$$\frac{d\alpha_\tau}{dt}=\frac{\alpha_\tau}{\pi}\left(\tfrac{3}{2}\alpha_b+2\alpha_\tau-\tfrac{3}{2}\alpha_2-\tfrac{9}{10}\alpha_Y\right), \qquad (11)$$

with $\alpha_{b,t,\tau}\equiv(\lambda^2_{b,t,\tau})/4\pi$.

Let us now set up our calculation of the top-quark mass. We first start by choosing fixed values for $\sin^2\theta_w(M_W)$, $\alpha_3(M_W)$, $m_b(M_W)$ and $m_\tau(M_W)$ [#6] within the range allowed by experiment. Our initial values are, respectively, 0.230, 0.122, 5 GeV and 1.8 GeV. As we shall see later, modification of these initial values within the range allowed by experiment does not change our results in any essential way. As we mentioned earlier, the experimental constraint on m_b and m_τ reduces the number of free parameters χ, $\bar{\chi}$, k and ψ into one pair which can be taken to be χ, $\bar{\chi}$ with $k=k(\chi,\bar{\chi})$ and $\psi=\psi(\chi,\bar{\chi})$. Equally well, we could take $\bar{\chi}$, k to be our parameters while $\chi=\chi(\bar{\chi},k)$ and $\psi=\psi(\bar{\chi},k)$. The expressions (1) of the Yukawa couplings in terms of these parameters are true at the superunification scale at which $\alpha_5=\alpha_1$. From all previous renormalization group analyses [4,12] however, we know that this scale is not very far from M_X, i.e. the scale at which $\alpha_3=\alpha_2$. Thus, it will not make much of a difference to take these expressions to be true at M_X. From another point of view, we could take the expressions (1) to hold at M_X while they would still be true for slightly shifted values of the parameters, say χ and $\bar{\chi}$, corresponding to a given value of m_t. This would just modify slightly the correspondence of the parameter values to the predicted top-quark mass but it would not modify their range or the range of the predicted top-quark mass values. We proceed now to the calculation of the top quark mass. In practice we would like to know the values of k, m_t etc. for a sufficient number of $(\chi_i,\bar{\chi}_j)$ pairs. For a given $(\chi_i,\bar{\chi}_j)$ pair we determine $k(\chi_i,\bar{\chi}_j)$ and $m_t(\chi_i,\bar{\chi}_j)$ in the following way: We start by putting particular values k', $\lambda'_t(m_W)$ to the test, and determine M'_X and $(\chi'_i,\bar{\chi}'_j)$ through the evolution of the coupled differential system (8)–(11). We vary successively k', $\lambda'_t(m_W)$ until we obtain $\chi'_i=\chi_i$ and $\bar{\chi}'_j=\bar{\chi}_j$. Repeating this procedure for a sufficient number of $(\chi_i\bar{\chi}_j)$ pairs

[#6] $0.222<\sin^2\theta_w<0.234$, $0.107<\alpha_3(M_W)<0.137$.

we find essentially the desired parameters k, λ_t, M_X, α_X as a function of the $(\chi, \bar{\chi})$ pair. In each case separately we check also for the existence of the superunification scale in order for the corresponding solution to be acceptable. Having obtained the above values, we solve successively eq. (5) and determine the top mass $m_t = m_t(\chi, \bar{\chi})$.

Selected values for m_t, at a scale $\mu = 2m_t$, as well as M_X and α_X are shown in table 1, in terms of the values for χ, $\bar{\chi}$ and k to which they correspond [#7]. In fig. 1 we plot the contours $m_t(\chi, \bar{\chi}) =$ constant while in fig. 2 one can read off the corresponding values of $k(\chi, \bar{\chi})$, for any particular $m_t(\chi, \bar{\chi})$ of fig. 1. Values for $m_t < 180$ GeV can be obtained for $\chi > \bar{\chi} > 0.80$, while values $m_t < 140$ GeV are possible for $\chi > \bar{\chi} > 0.95$ but they correspond to values of $k > O(30)$. We have also seen that m_t can be arbitrarily lowered for sufficiently small χ in the region $\bar{\chi} < \chi < 0.01$ corresponding to $k < 1$. [#8]. Extremal values of χ are technically natural, despite the fact that they imply that some singlet VEVs are much smaller than the others, since the D-flatness conditions (5a) and (5b) are not modified by the inclusion of the SU(5)×U(1)-

[#7] The Higgs VEVs ratio k is in principle calculable from the details of $SU(2)_L \times U(1)_Y$ breaking and the way that the supersymmetry breaking is fed to the low-energy matter fields through radiative corrections. A determination of k would make the m_t parameter space one dimensional and would sharpen dramatically the predictability of the model.

[#8] Our results are for $\xi > 0$. In the case of $\xi < 0$ [10], as can be seen in figs. 1 and 2, a "lighter" top quark is possible.

Table 1
Selected values for k, m_t, M_X, M_{SU}, a_X for $\sin^2\theta_w = 0.230$, $a_3 = 0.122$.

χ	$\bar{\chi}$	k	m_t	M_X	M_{SU}	a_X
0.05	0.05	5.5	197	1.59×10^{16}	2.29×10^{16}	0.0562
0.50	0.05	32	192	1.68×10^{16}	2.33×10^{16}	0.0560
0.95	0.05	36	188	1.76×10^{16}	2.36×10^{16}	0.0557
0.05	0.10	5.5	197	1.58×10^{16}	2.29×10^{16}	0.0562
0.50	0.10	32	192	1.68×10^{16}	2.33×10^{16}	0.0560
0.95	0.10	36	188	1.76×10^{16}	2.36×10^{16}	0.0557
0.05	0.20	5.6	197	1.58×10^{16}	2.29×10^{16}	0.0562
0.50	0.20	32	192	1.68×10^{16}	2.32×10^{16}	0.0560
0.95	0.20	36	187	1.76×10^{16}	2.35×10^{16}	0.0557
0.05	0.30	5.6	197	1.58×10^{16}	2.28×10^{16}	0.0562
0.50	0.30	32	192	1.67×10^{16}	2.32×10^{16}	0.0560
0.95	0.30	36	187	1.76×10^{16}	2.35×10^{16}	0.0558
0.05	0.40	5.6	196	1.57×10^{16}	2.28×10^{16}	0.0562
0.50	0.40	32	191	1.67×10^{16}	2.32×10^{16}	0.0560
0.95	0.40	36	186	1.76×10^{16}	2.35×10^{16}	0.0558
0.05	0.50	5.6	196	1.57×10^{16}	2.28×10^{16}	0.0563
0.50	0.50	32	190	1.66×10^{16}	2.32×10^{16}	0.0560
0.95	0.50	37	185	1.75×10^{16}	2.34×10^{16}	0.0558
0.05	0.60	5.7	194	1.56×10^{16}	2.27×10^{16}	0.0563
0.50	0.60	33	189	1.65×10^{16}	2.31×10^{16}	0.0560
0.95	0.60	37	184	1.74×10^{16}	2.34×10^{16}	0.0558
0.05	0.70	5.7	193	1.54×10^{16}	2.26×10^{16}	0.0563
0.50	0.70	33	187	1.64×10^{16}	2.30×10^{16}	0.0561
0.95	0.70	37	182	1.73×10^{16}	2.33×10^{16}	0.0559
0.05	0.80	5.8	189	1.52×10^{16}	2.24×10^{16}	0.0564
0.50	0.80	33	183	1.62×10^{16}	2.28×10^{16}	0.0561
0.95	0.80	37	177	1.71×10^{16}	2.32×10^{16}	0.0559
0.05	0.90	6.0	179	1.49×10^{16}	2.22×10^{16}	0.0565
0.50	0.90	34	172	1.59×10^{16}	2.26×10^{16}	0.0562
0.95	0.90	37	165	1.68×10^{16}	2.29×10^{16}	0.0560
0.05	0.95	6.1	163	1.46×10^{16}	2.20×10^{16}	0.0566
0.50	0.95	34	155	1.56×10^{16}	2.24×10^{16}	0.0563
0.95	0.95	38	147	1.65×10^{16}	2.27×10^{16}	0.0561

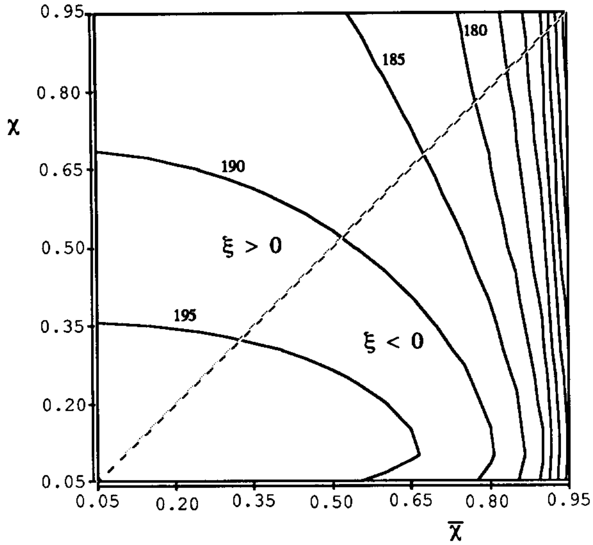

Fig. 1. Plot of the contours $m_t(\chi, \bar{\chi})$ = constant.

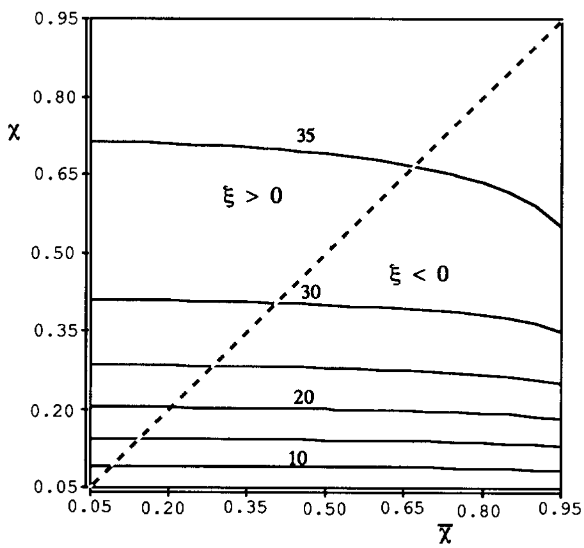

Fig. 2. Plot of the contours $k(\chi, \bar{\chi})$ = constant.

breaking VEVs which would introduce corrections of the same order. Note that these results are in agreement with what was anticipated on the basis of the inequalities obtained in ref. [9]. In fig. 3 we also plot the contours of the ratio m_b/m_τ $(\chi, \bar{\chi})$ at M_X.

For completeness we have also calculated the unification scale M_X, the top quark mass m_t and the cor-

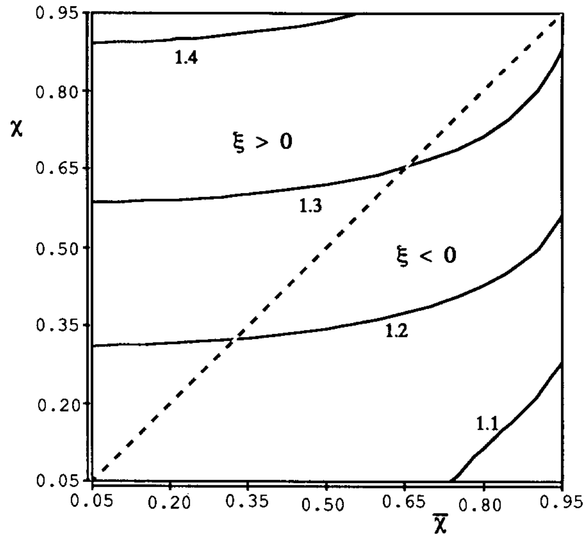

Fig. 3. Plot of the contours $(m_b/m_\tau)(M_X)(\chi, \bar{\chi})$ = constant.

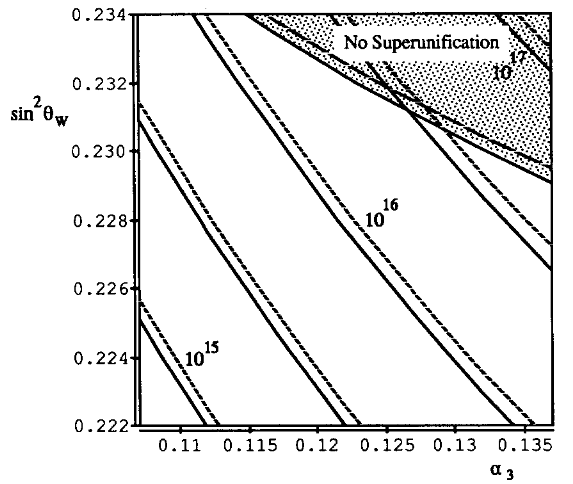

Fig. 4. Plot of contours $M_X(\sin^2\theta_w, \alpha_3)$, in the case $\chi=0.5$, $\bar{\chi}=0.1$ (solid lines) and in the case of negligible Yukawa couplings (dashed lines).

responding VEV ratio k, in terms of the low energy parameters $\sin^2\theta_w$, α_3 for fixed $\chi=0.5$, $\bar{\chi}=0.1$. The results for M_X are presented in fig. 4. The dashed lines correspond to the case where the Yukawa couplings are ignored. The allowed $\sin^2\theta_w$ and α_3 range is restricted from below from proton decay and from above from the existence of superunification. It is clear that comparing to the light top case, where the top coupling is neglected, the unification scale M_X is raised. For m_t and k we have found that they are modified no more than 5% for the values of table 1, for the whole range of acceptable values of $\sin^2\theta_w$ and α_3, depending mainly on α_3.

In conclusion, we calculated the top-quark mass for the flipped SU(5)×U(1) superstring model as a function of the Higgs VEVs ratio k and a parameter which is a ratio of SU(5)×U(1)-singlet VEVs. Our solution of the coupled renormalization group equations for the gauge and Yukawa couplings shows that m_t can take values everywhere in the allowed region of 100 to 180 GeV. A relatively "light" top-quark, i.e. with $m_t \sim 100$ GeV, can be obtained for extremal values of λ_t, λ_b at M_X. Nevertheless for most part of the parameter space, we obtain a heavy top-quark with mass 150 GeV$\leqslant m_t \leqslant$200 GeV. It can also be concluded that the required Higgs VEV ratio tends to be large, i.e. greater than O(10).

One of us (J.R.) would like to thank the University of Ioannina Research Commitee for financial support.

References

[1] J. Schwartz, Phys. Rep. 89 (1982) 223;
M.B. Green, Surv. High Energy Phys. 3 (1983) 127;
M.B. Green, J. Schwartz and E. Witten, Superstring theory (Cambridge U.P., Cambridge, 1987).

[2] P. Candedas, G.T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B 258 (1985) 46;
B.R. Green, K.H. Kirklin, P.J. Miron and G.G. Ross, Nucl. Phys. B 287 (1986) 667; B 292 (1987) 606;
L. Dixon, I.A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B 261 (1985) 678; B 274 (1986) 285;
L.E. Ibáñez, J. Mas, H.P. Nilles and F. Quevedo, Nucl. Phys. B 301 (1989) 157;
D. Bailin, A. Love and S. Thomas, Phys. Lett. B 194 (1987) 385; Nucl. Phys. B 298 (1988) 75;
K.S. Narain, Phys. Lett. B 169 (1986) 41;
K.S. Narain, M.H. Sarmadi and C. Vafa, Nucl. Phys. B 288 (1987) 551;
W. Lerche, D. Lüst and A.N. Schellekens, Nucl. Phys. B 287 (1987) 477;
H. Kawai, D.C. Lewellen and S.H. Tye, Phys. Rev. Lett. 57 (1986) 1982; Nucl. Phys. B 288 (1987) 1.

[3] I. Antoniadis, C. Bachas, C. Kounnas and P. Windey, Phys. Lett. B 17 (1986) 51;
I. Antoniadis, C. Bachas and C. Kounnas, Nucl. Phys. B 289 (1987) 87;
I. Antoniadis and C. Bachas, Nucl. Phys. B 298 (1988) 586.

[4] I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopoulos, Phys. Lett. B 194 (1987) 231; B 205 (1988) 459; B 208 (1988) 209.

[5] S.M. Barr, Phys. Lett. B 112 (1982) 219;
J.P. Dependinger, J.E. Kim and D.V. Nanopoulos, Phys. Lett. B 139 (1984) 170.

[6] I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopoulos, Phys. lett. B 231 (1989) 65.

[7] G.K. Leontaris, J. Rizos and K. Tamvakis, Phys. Lett. B 243 (1990) 220.

[8] J. Lopez and D.V. Nanopoulos, preprint CTP-TAMU-60/89.

[9] J. Rizos and K. Tamvakis, Retracing the phenomenology of the flipped SU(5)×U(1) superstring model, preprint IOA-241/90, Phys. Lett. B 251 (1990), to be published.

[10] M. Dine and C. Lee, Fermion masses and Fayet–Illiopoulos terms in string theory, preprint CCNY-89.

[11] J.E. Björkman and D.R.T. Jones, Nucl. Phys. B 259 (1985) 533.

[12] J. Ellis, J.S. Hagelin, S. Kelley and D.V. Nanopoulos, Nucl. Phys. B 311 (1988) 1.