Comment on "Solitonlike Solutions of the Grad-Shafranov Equation"

In a recent publication the author claims construction of a new class of solitonlike solutions of the Grad-Shafranov equation in plane geometry. However, this construction is based on the mathematically erroneous (as shown below) choice $\nabla p = |\Psi|^2 \Psi \nabla \Psi$ for the pressure gradient with Ψ being an analytic continuation of the poloidal magnetic flux function in the complex plane. Therefore, the cubic Schrödinger equation considered by the author is irrelevant to the equilibrium problem and the Grad-Shafranov equation.

The equilibrium equations considered are [Eqs. (5) [1]]

$$(\nabla \Psi \times \nabla B_z) \cdot \hat{\mathbf{z}} = 0, \qquad \nabla p + \nabla^2 \Psi \nabla \Psi + B_z \nabla B_z = 0, \tag{1}$$

where B_z is the z component of the magnetic field; the functions p and B_z are constant on magnetic surfaces, i.e., $p = p(\Psi)$ and $B_z = B_z(\Psi)$. The following forms of p and B_z are then chosen [Eqs. (13) [1]]

$$B_z \nabla B_z = \alpha_0^2 \Psi \nabla \Psi, \tag{2}$$

$$\nabla p = \alpha_0^2 |\Psi|^2 \Psi \nabla \Psi, \tag{3}$$

and Eq. (1) is extended in the complex plane, thus leading to the cubic Schrödinger equation [Eqs. (14) [1]]

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\alpha_0^2 (1 + |\Psi|^2) \Psi. \tag{4}$$

A solitonlike solution of (4) is [Eqs. (15) [1]]

$$\Psi(x, y) = \Psi_p \,\mathrm{sech}(x/L) e^{-j(\alpha_0 + 1/2\alpha_0 y_0^2) y}. \tag{5}$$

However, relation (3) containing $|\Psi|^2$ is not permissible and leads to inconsistent results. An explicit proof follows. Taking the curl of (3) yields $\nabla |\Psi| \times \nabla \Psi = 0$, implying that Ψ depends only on $|\Psi|$:

$$\Psi = f(|\Psi|). \tag{6}$$

In order that the complex function Ψ be analytic, Eq. (6) and the Cauchy-Riemann conditions lead to $|\Psi| = \mathrm{const}$ and therefore $\Psi = \mathrm{const}$. Furthermore, even without imposing the analyticity requirement upon Ψ, by solely introducing the polar form of the latter, $\Psi = |\Psi| \exp(j\Theta(x, y))$, Eq. (6) implies that

$$\Theta = \Theta(|\Psi|). \tag{7}$$

Solution (5), however, is inconsistent with (7) [otherwise, it should hold that $x = x(y)$]. Therefore, (4) is irrelevant to the Grad-Shafranov equation; as a matter of fact, the real part of (5),

$$u(x, y) = \Psi_p \,\mathrm{sech}(x/L) \cos[(\alpha_0 + 1/2\alpha_0 y_0^2) y],$$

does not satisfy the respective Grad-Shafranov equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\alpha_0^2 (1 + u^2) u. \tag{8}$$

A mathematically legitimate choice for ∇p, instead of (3), could be

$$\nabla p = \alpha_0^2 \Psi^3 \nabla \Psi. \tag{9}$$

This leads to an equation of the form (8) for Ψ. Solving this equation in the complex plane, however, is a task not easier than that for real Ψ.

In conclusion, because of the mathematically erroneous choice (3) for an analytic continuation of Ψ, the cubic Schrödinger Eq. (4) considered by the author is irrelevant to the equilibrium problem (1) and to the Grad-Shafranov equation. Despite this unlucky situation we consider the idea of the author appealing and, hopefully, successful in the future if one exploits it in an appropriate setting.

Part of this work was conducted during a visit of one of the authors (G. N. T.) to the Max-Planck Institut für Plasmaphysik, Garching. The hospitality of that Institute is greatly appreciated. This work was performed under the Contract of Association ERB 5005 CT 99 0100 between the European Atomic Energy Community and the Hellenic Republic.

G. N. Throumoulopoulos,[1,*] K. Hizanidis,[2,†] and H. Tasso[3,‡]
[1]University of Ioannina
Association Euratom-Hellenic Republic
Department of Physics
GR 451 10 Ioannina, Greece
[2]National Technical University of Athens
School of Electrical and Computer Engineering
9 Iroon Polytehniou Street
Zografou Campus
157 73 Athens, Greece
[3]Max-Planck-Institut für Plasmaphysik
Euratom Association
D-85748 Garching bei München, Germany

Received 13 June 2003; published 18 June 2004
DOI: 10.1103/PhysRevLett.92.249501
PACS numbers: 52.55.–s, 52.30.Cv

*Electronic address: gthroum@cc.uoi.gr
†Electronic address: kyriakos@central.ntua.gr
‡Electronic address: het@ipp.mpg.de

[1] Giovanni Lapenta, Phys. Rev. Lett. **90**, 135005 (2003).

