## CONSEQUENCES OF A DUAL RESONANCE EXCLUSIVE KERNEL FOR HADRONIC CORRELATIONS

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The exclusive kernel of a previously proposed integral equation (IE) is expressed in terms of DRM amplitudes in the central region. The  $|B_6/B_5|$  relative normalization is fitted to four-prong NAL data. The IE is then solved numerically to compute the inclusive correlated density  $R_{cc}(y)$ . It is also predicted that  $R_{+-}: R_{++}: R_{--} = f\rho_c: \rho_+: \rho_-$ ,  $f \approx 2$ , at y = 0.

New production mechanisms, as well as old models, have been proposed and applied lately, mainly to explain the strong short range component, most probably in coexistence with a weak long-range one, discovered in inclusive twoparticle rapidity correlations [1, 2]. In implementing the dual resonance model (DRM) at the inclusive level one usually takes advantage of the Mueller's constraction: the n-particle inclusive density  $\rho_n$  is proportional to the appropriate discontinuities of a  $B_{2n+4}$  amplitude. The complexity however, and the ill-behaved asymptoticity of a  $B_N$ 's integral representation, give rise to serious computation problems as soon as *n* exceeds one. Thus, the  $\rho_2 \sim$  disc.  $B_8$  has been treated only qualitatively as far as we know, or calculated to order  $1/s_{12}$  for large  $s_{12}$  in the central region (CR), obtaining very reasonable behavior nevertheless [3].

In this letter we investigate quantitatively some consequences of the DRM for pair inclusive correlations, in the framework of an integral equation scheme (IES) proposed earlier [4]. The *n*-particle inclusive correlation density  $R_n(s; p_1,..., p_n)$  is the solution of n-1 coupled linear integral equations, which involve in their kernels the  $\rho_1$ , and scaling combinations of  $B_N$ 's for  $N \approx 4, 5,..., 4+n$ , [see eq. (3)].  $B_N$  stands for a generic N-point amplitude in a broad class of models. The DRM's  $B_N$ , restricted into a kinematic region  $D^*$ , belong into the class **\*\***. Two remarks are in order: i) Strictly within the IES (no Mueller's hypothesis), the  $\rho_1$  has to be expressed in terms of all the available exclusive channels integrated in the usual way. In our computations it is considered constant and equal to the experimental plateau value, estimated to be  $\rho_1 = \sigma_I^{-1}(d\sigma/dy) = 1.04 \pm 0.07$  at  $p_L = 205 \text{ GeV}/c$ [5, 6, 11]. ii) Given the  $\rho_1$ , the maximum-point  $B_N$ needed to determine  $R_n$  is,  $B_{n+4}$  in the IES,  $B_{2n+4}$  in the generalized optical theorem approach. Of course, the price one pays for this reduction is at least the complexity of the integral equations one has now to solve.

For n = 2, the simplest IE, i.e. no quantum numbers or transverse momenta couplings included, is the following

$$R_{2}(y_{3}, y_{4}) = g_{2}(y_{3}, y_{4}) + \int dy' \rho_{1}(y')g_{2}(y_{3}, y')R_{2}(y', y_{4}).$$
(1)

 $y_3$  and  $y_4$  are the rapidities of the secondary particles, and

$$R_2(y_3, y_4) = [\rho_2(y_3, y_4) / \rho_1(y_3) \rho_1(y_4)] - 1, \quad (2)$$

is the usual inclusive correlated density. The exclusive kernel (correlation) is given by

<sup>\*</sup> D is characterized by the existence of two leading momenta  $p_i$  and  $p_j$  such the  $(p_{\alpha} + p_l)^2$ , l = 1, 2, ..., N,  $l \neq \alpha$ ,  $\alpha = i, j$ , is large enough for the "potential" between  $\alpha$  and l to be no more complicated than two-body. Each secondary is arbitrarily correlated to the others. In this note D is taken to be the CR.

<sup>\*\*</sup> The DRM can be partially unitarized by complexifying the Regge trajectories. This methos is followed here. On the other hand the functional boostrap method [4] that generates the IES, presupposes weak leading-particle couplings to the secondaries. Thus, it also treats unitarity in an approximate way. It is assumed that the two approximations are not drastically different.

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Fig. 1. Dual tree diagrams in the central region. They determine, along with  $\rho_1$ , the two-body exclusive kernel, eq. (3). Wavy lines signify Regge asymptotic limits.

$$g_{2}(y_{3}, y_{4}) = [|B_{6}(y_{3}, y_{4})/B_{5}(y_{3})|^{2} - |B_{5}(y_{4})/B_{4}(Y)|^{2}]/\rho_{1}(y_{4}).$$
(3)

The relevant amplitudes are shown in fig. 1. The incident and leading particles are singled out in the derivation of (1). Consequently, only the  $(12 \sim 34 \sim 56)$  and  $(12 \sim 43 \sim 56)$  orderings are considered. Given the first, the other is taken into account trivially by virtue of the translation invariance in rapidity.

This property originates from the short range order character of the DRM proven lately to hold at the planar-unitary level in general [7]. As a result, the eq. (1) assumes a convolution form that suggests immediately an integral transform technique for its solution, which is exhibited next.

$$R_{2}(y) = \frac{2\gamma^{2}}{\pi\rho_{1}} \int_{0}^{\infty} dk \, I(k) \cos ky / [1 - 2\gamma^{2} I(k)] .$$
 (4)

y denotes  $|y_3 - y_4|$ , and  $\gamma$  is the relative normalization of  $|B_6|$  to  $|B_5|$ , not precisely known theoretically due to the well known satellite and ghost-killing factor ambiquities; it is fitted to "semi-exclusive" data, as explained below. I(k) is the Fourier transform of the quantity  $\rho_1 g_2 \equiv K(y')$ .

$$I(k) = \int_{0}^{\infty} K(y') \cos ky' \, dy' \,.$$
 (5)

From the double-Regge limit expression for  $B_5$  [e.g. 8], the Koba-Nielsen formula [9] for  $B_6$ , and an insignificant modification into the Toller angle dependence (details will be given in a future communication), we find

$$K(y) = \gamma^{2} \{ \exp(-2\alpha_{12}y) \\ \times |B(-\alpha_{13}, -\alpha_{34}) F(a, b; c; s_{34}/\kappa)|^{2} \\ -|\Gamma(-\alpha_{12}) \psi(-\alpha_{12}, 1; \kappa)|^{2} \}.$$
(6)

 $F(\psi)$  is the Gauss' (confluent) hypergeometric function with  $a = -\alpha_{12}$ ,  $b = -\alpha_{13}$  and  $c = -\alpha_{12} - \alpha_{34}$ . The notation is further explained by fig. 1. We have checked that indeed K(y) vanishes like  $\exp(-y)$  for large y, in agreement with conclusions in ref. [3]. For the DRM to be phenomenologically applied, its zero-width divergence problems have to be dealt with first. To this effect, the standard prescription is followed, and  $s \rightarrow |s| \exp(-i\epsilon)$  is substituted into the eq. (6) with  $\epsilon = \Gamma_{\rho}/E_{\rho} \approx 0.16$ .  $\Gamma_{\rho}$  is the width of the  $\rho$  resonance. The  $\rho$  is chosen as a typical hadron. For simplicity and definiteness we consider  $\kappa_1 = \kappa_2 = \kappa = -(m^2 + \langle p_T \rangle^2)$ [8],  $\alpha_{12} = \alpha_{14} = \alpha_{13} = \alpha_0 - \langle p_T \rangle^2$  and  $\alpha_{34} = \alpha_0 + s_{34}$ , with  $m = m_{\rho}$  and  $\langle p_T \rangle = 0.4$  GeV/c.

In fig. 2, the exclusive density  $|B_6/B_4|^2$  is fitted to preliminary NAL data [10] on  $\sigma_{cc}(y)/\sigma_{o}(Y)$  for two values of the intercept  $\alpha_0$ , and the normalization parameter is fixed ( $\gamma \approx 0.09$ ).  $\sigma_{cc}(y) \equiv d\sigma_{cc}/dy$ ,  $y = |y_3 - y_4|$ , is the semi-exclusive ("semi", in the sense of undetected neutrals) two-particle cross section for the reaction  $pp \rightarrow pp + ch(3) + ch(4)$ , with the ch(3, 4) particles produced in the CR.  $\sigma_0$  accounts for those produced particles that do not populate the CR [4]. Consequently it should be roughly equal to the total diffractive cross section  $\sigma \approx \sigma_d^T$ , attributed the value  $(5.62 \pm 0.30)$  mb at  $p_{\rm L} = 205 \, {\rm GeV}/c$  [11]. This argument is supported by the fact that  $d\sigma_n$  (diffractive)/dy attains its maximum near  $y = \pm \log \sqrt{s}$ for small n, and practically vanishes for  $n \ge 8$  at NAL energies [11, 12].



Fig. 2. The exclusive density  $|B_6/B_4|^2$  multiplied by the total diffractive cross-section, fitted to preliminary NAL data on  $d\sigma_{\rm CC}/dy$  [10] (see the text for further explanations). The model-calculated errorbars correspond to the experimental error in  $\sigma_{\rm d}^{\rm T}$ . The arbitrarily normalized ISR data at  $\sqrt{s} = 23$  GeV (error estimates omitted), were extracted from the semi – exclusive correlation  $R_m$ , m = 4.5.  $d\sigma_{\rm CC}(\eta_1 = 0, \eta_2)/d\eta_2 \propto \sigma_{\rm n}^{-1}(d\sigma_n/d\eta_2)[R_m(\eta_1 = 0, \eta_2) + 1], 1 \le n \le 5$  [ref. [13], and paper no 441 in ref. [1]].

Results of our computation based on eq. (4) are shown and further explained in fig. 3. The "twocomponent" ( $\alpha_0 = 0.5$ ) curve seems to keep closer to the data. Thus, on might conclude that the DRM's inability to describe diffraction, at least at the planar diagram level, is not remedied by the IE scheme. It is encouraging though that the effective trajectory has to be *raised* for the DRM-IES to "fake" diffractive effects. If the  $\rho_1$  and  $R_2$  were normalized to  $\sigma_T$ instead of  $\sigma_1$ ,  $\sigma_0$  would be given by  $\sigma_d plus \sigma_{el}$ . The former trend is then met again, as it is found that an intercept  $\alpha_0 \approx 0.9$  is needed to correlate the data in figs. 2 and 3 fairly.

Finally, isospin effects can be studied in this approach. The eq. (1) assumes a matrix form in charge indices (eq. A(10), second paper in ref. [4]) and the matrix equation corresponding to eq. (4) allows predictions to be made about  $R_{++}(y)$ ,  $R_{+-}(y)$  and



Fig. 3. DRM – IES' prediction [eq. (4)] for the correlation density  $R_2(y)$  in comparison with NAL and ISR data on  $R_{cc}(y_1 \approx 0, y_2)$ . The solid line corresponds to  $\alpha_0 = 0.6$ . The dashed line is the predicted curve for  $\alpha_0 = 0.5$  plus a diffractive model-dependent term (dashed-dotted curve), taken from ref. [14] after reduction by a factor of 1.5 to account for the lower value of  $\sigma_d^T$  used in our calculation. The errorbars, for fixed  $\gamma$ , correspond to errors in  $\sigma_d^T$  and  $\rho_1$ . At y = 0 the diffractive component accounts for about 20% of the total correlation, in rough agreement with findings in ref. [14].

 $R_{-}(y)$ , in an obvious notation for the detected hadrons' charge. Necessarily no further details are given here. We predict

$$R_{+-}(0): R_{++}(0): R_{--}(0) = f\rho_{c}: \rho_{+}: \rho_{-}.$$
(7)

f is a DRM-dependent factor. For the chosen parametrization we find  $f \approx 2$ . The ratio  $R_{++}: R_{--}$  is model independent in the framework of the IES.

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