Constraints on linear-negative potentials in quintessence and phantom models from recent supernova data

L. Perivolaropoulos

Department of Physics, University of Ioannina, Greece

(Dated: February 2, 2008)

We study quintessence and phantom field theory models based on linear-negative potentials of the form $V(\phi) = s\ \phi$. We investigate the predicted redshift dependence of the equation of state parameter $w(z$ for a wide range of slopes s in both quintessence and phantom models. We use the gold dataset of 157 SnIa and place constraints on the allowed range of slopes s. We find $s = 0 \pm 1.6$ for quintessence and $s = \pm 0.7 \pm 1$ for phantom models (the range is at the 2σ level and the units of s are in $\sqrt{3}M_p H_0^2 \simeq 10^{-38} eV^3$ where M_p is the Planck mass). In both cases the best fit is very close to $s \simeq 0$ corresponding to a cosmological constant. We also show that specific model independent parametrizations of $w(z)$ which allow crossing of the phantom divide line $w = -1$ (hereafter PDL) provide significantly better fits to the data. Unfortunately such crossings are not allowed in any phantom or quintessence single field model minimally coupled to gravity. Mixed models (coupled phantom-quintessence fields) can in principle lead to a $w(z)$ crossing the PDL but a preliminary investigation indicates that this does not happen for natural initial conditions.

I. INTRODUCTION

Recent observations have indicated that the universe has entered a phase of accelerating expansion (the scale factor obeys $\ddot{a} > 0$) and that the total amount of clustered matter in the universe is not sufficient for its small average spatial curvature. This converging observational evidence comes from a diverse set of cosmological data which includes observations of type Ia supernovae [1, 2], large scale redshift surveys [3] and measurements of the cosmic microwave background (CMB) temperature fluctuations spectrum [4]. The observed accelerating expansion and flatness of the universe, requires either a modified theory of gravity[5] or, in the context of standard general relativity, the existence of a smooth energy component with negative pressure termed 'dark energy'[6]. This component is usually described by an equation of state parameter $w \equiv \frac{p}{\rho}$ (the ratio of the homogeneous dark energy pressure p over the energy density ρ). For cosmic acceleration, a value of $w < -\frac{1}{3}$ is required as indicated by the Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad (1.1)$$

Even though the cosmological constant remains a viable candidate for dark energy, current observational bounds [7, 8] on the value of the dark energy equation of state parameter $w(t_0)$ at the present time t_0 (corresponding to a redshift $z = 0$) yield $w(z = 0) \leq -1$ with $\frac{dw}{dz}|_{z=0} > 0$ at best fit.

The role of dark energy can be played by any physical field with positive energy and negative pressure which violates the strong energy condition $\rho + 3p > 0$ ($w > -\frac{1}{3}$). Quintessence scalar fields[9] with positive kinetic term ($-1 < w < -\frac{1}{3}$) violate the strong energy condition but not the dominant energy condition $\rho + p > 0$. Their energy density scales down with the cosmic expansion

and so does the cosmic acceleration rate. Phantom fields[10] with negative kinetic term ($w < -1$) violate the strong energy condition, the dominant energy condition and maybe physically unstable. However, they are also consistent with current cosmological data and according to recent studies[7, 8] they maybe favored over their quintessence counterparts.

Homogeneous quintessence or phantom scalar fields are described by Lagrangians of the form

$$\mathcal{L} = \pm\frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad (1.2)$$

where the upper (lower) sign corresponds to a quintessence (phantom) field in equation (1.2) and in what follows. The corresponding equation of state parameter is

$$w = \frac{p}{\rho} = \frac{\pm\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\pm\frac{1}{2}\dot{\phi}^2 + V(\phi)} \qquad (1.3)$$

For quintessence (phantom) models with $V(\phi) > 0$ ($V(\phi) < 0$) the parameter w remains in the range $-1 < w < 1$. For an arbitrary sign of $V(\phi)$ the above restriction does not apply but it is still impossible for w to cross the phantom divide line (hereafter PDL) $w = -1$ in a continous manner. The reason is that for $w = -1$ a zero kinetic term $\pm\dot{\phi}^2$ is required and the continous transition from $w < -1$ to $w > -1$ (or vice versa) would require a change of sign of the kinetic term. The sign of this term however is fixed in both quintessence and phantom models. This difficulty in crossing the PDL $w = -1$ could play an important role in identifying the correct model for dark energy in view of the fact that data favor $w \simeq -1$ and furthermore parametrizations of $w(z)$ where the PDL is crossed appear to be favored over the cosmological constant $w = -1$ (see section III and Refs [7, 8]).

In view of the above described problem it is interesting to consider the available quintessence and phantom scalar field models and compare the consistency with data of the predicted forms of $w(z)$ among themselves and with arbitrary parametrizations of $w(z)$ that cross the PDL. This is the main goal of the present study.

We focus on a particular class of scalar field potentials of the form

$$V(\phi) = s\,\phi \qquad (1.4)$$

where we have followed Ref. [11] and set $\phi = 0$ at $V = 0$. As discussed in section II (see also Ref. [11]) the field may be assumed to be frozen ($\dot{\phi} = 0$) at early times due to the large cosmic friction $H(t)$. It has been argued [12] that such a potential is favored by anthropic principle considerations because galaxy formation is possible only in regions where $V(\phi)$ is in a narrow range around $V = 0$ and in such a range any potential is well approximated by a linear function. In addition such a potential can provide a potential solution to the cosmic coincidence problem[13].

For quintessence models the scalar field behavior has been studied extensively [11, 14, 15, 16] and shown to lead to a future collapse of the scale factor (termed 'cosmic doomsday') due to the eventual evolution of the scalar field towards negative values of the potential where the gravity of the field is attractive. Such a doomsday however does not occur in the corresponding phantom models because the scalar field moves towards higher values of the potential where the field gravity is repulsive and leads to faster acceleration of the scale factor (see Figure 2 below) and eventually to a Big Rip singularity [17]. Thus w evolves towards values less than -1 for phantom models. One of the goals of this paper is to compare the consistency with SnIa data of this phantom behavior of $w(z)$ with the corresponding consistency of the quintessence behavior where w evolves towards values larger than -1.

The structure of the paper is the following: In the next section we solve numerically the field equations for phantom and quintessence models coupled to the Friedman equation and derive the cosmological evolution of the scalar field, the scale factor and the equation of state parameter w for several values of the potential slope s. In section III we fit the derived Hubble parameter to the SnIa Gold dataset [2] and obtain constraints for the potential slope s for both phantom and quintessence models. The quality of fit of these models is also compared to the quality of fit of arbitrary $w(z)$ parametrizations that can cross the PDL. Finally in section IV we summarize our results and state the main questions that emerge from them. In a preliminary effort to address some of these issues we show the evolution of $w(z)$ in a mixed quintessence + phantom model where the dark energy consists of a mixture of interacting phantom and quintessence fields.

II. PHANTOM AND QUINTESSENCE FIELD DYNAMICS

In order to study in some detail the scalar field dynamics, we consider the coupled Friedman-Robertson-Walker (FRW) and the scalar field equation

$$\frac{\ddot{a}}{a} = \mp\frac{1}{3M_p^2}(\dot{\phi}^2 + s\,\phi) - \frac{\Omega_{0m}H_0^2}{2a^3} \qquad (2.1)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - s = 0 \qquad (2.2)$$

where $M_p = (8\pi G)^{-1/2}$ is the Planck mass and we have assumed a potential of the form

$$V(\phi) = \mp s\,\phi \qquad (2.3)$$

where the upper (lower) sign corresponds to quintessence (phantom) models. By setting

$$\begin{aligned} H_0 t &\rightarrow t \\ \frac{\phi}{\sqrt{3}M_p} &\rightarrow \phi \\ \frac{s}{\sqrt{3}M_pH_0^2} &\rightarrow s \end{aligned} \qquad (2.4)$$

equation (2.1) may be written in rescaled form as

$$\frac{\ddot{a}}{a} = \mp(\dot{\phi}^2 + s\,\phi) - \frac{\Omega_{0m}}{2a^3} \qquad (2.5)$$

while the scalar field equation (2.2) remains unchanged. It is now straightforward to solve the system numerically [18] using the following initial conditions ($t \rightarrow t_i \simeq 0$)

$$\begin{aligned} a(t_i) &= (\frac{9\Omega_{0m}}{4})^{1/3}\,t_i^{2/3} \\ \dot{\phi}(t_i) &= 0 \\ \phi(t_i) &= \phi_i \end{aligned} \qquad (2.6)$$

since the universe is matter dominated at early times and an inflationary phase would redshift the gradient and velocity of the scalar field while the large cosmic friction $\frac{\dot{a}}{a}$ would freeze it at early times after inflation. The value of ϕ_i is chosen for each value of the slope s such that $\Omega_{0\phi} = \pm\dot{\phi}^2(t_0) + V(\phi(t_0)) = 1 - \Omega_{0m}$ at the present time t_0 (defined by $a(t_0) = H(t_0) = 1$). In what follows we have assumed a prior of $\Omega_{0m} = 0.3$. According to the numerical solution the scalar field is almost frozen at early times (when matter dominates) due to the large cosmic friction $H(t) \simeq \frac{2}{3t}$. At approximately the present time when the matter density drops and the field potential begins to dominate, the lower friction allows the field to move down (up) the potential for quintessence (phantom) models (see Figure 1).

When the potential energy dominates, the universe enters the present accelerating phase. As the field moves down (up), the potential energy becomes negative (more
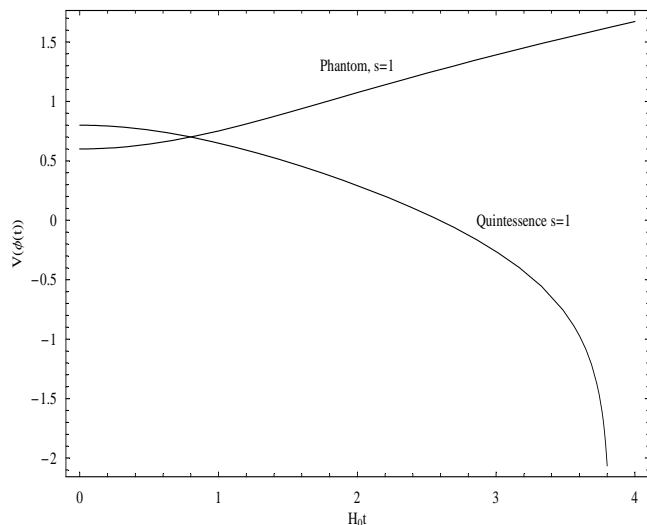
FIG. 1: The potential energy evolution for quintessence and phantom models with linear potential of slope $s = 1$. In this plot $t_0 = 0.96$.
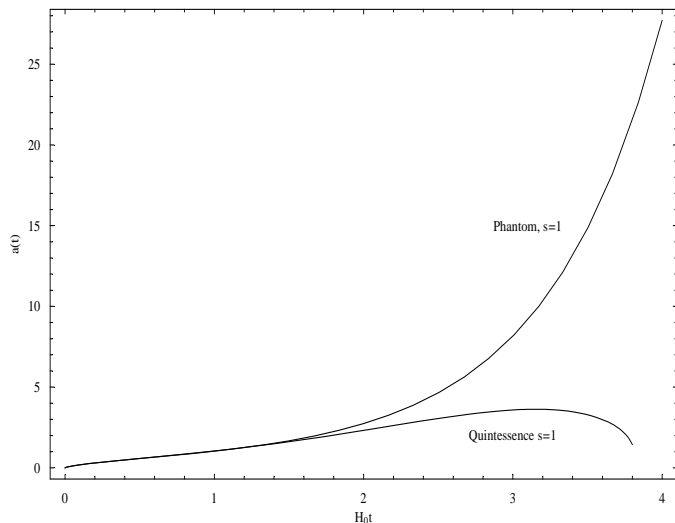

FIG. 2: The scale factor evolution for representative ($s = 1$) quintessence and phantom models with linear potential. The present time corresponds to $t_0 = 0.96$ as in Figure 1.

positive), the field gravity becomes attractive (more repulsive) and the scale factor begins to decelerate again (accelerate more rapidly) until the universe ends with a Big Crunch (Big Rip) (see Figure 2). Using the numerical solution of the system (2.5)-(2.2) we can also evaluate the redshift dependence of the equation of state parameter

$$w(z) = \frac{p}{\rho} = \frac{\pm\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\pm\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{\frac{1}{2}\dot{\phi}^2 + s\,\phi}{\frac{1}{2}\dot{\phi}^2 - s\,\phi} \qquad (2.7)$$

which is shown in Figure 3 in the redshift range $0 \leq z \leq 2$ for both quintessence and phantom models and for several values of the slope s.

In the case of quintessence $w(z)$ has been evaluated in Ref. [11] (see also [14, 15, 16]) with results in good
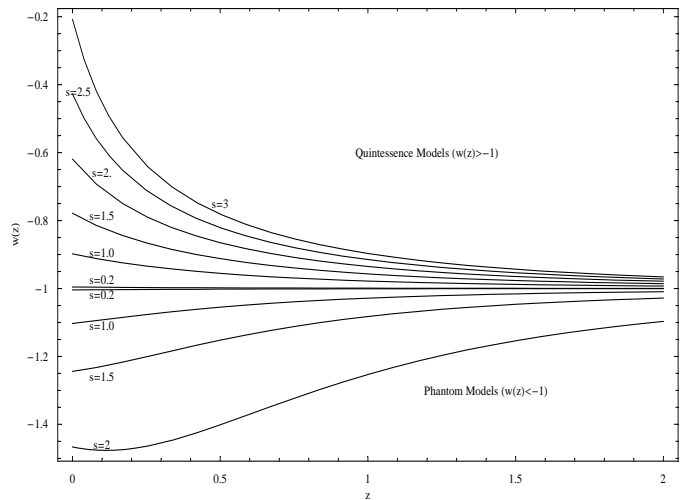

FIG. 3: The redshift evolution of the equation of state parameter $w(z)$ for phantom and quintessence models and for several values of the slope s.

agreement with the corresponding results presented here. As discussed in the introduction the PDL is not crossed for any value of s. Instead, $w(z)$ evolves towards larger (smaller) values of -1 for a quintessence (phantom) scalar field.

III. FIT TO THE GOLD DATASET

Having solved numerically the rescaled system (2.5),(2.2) for both quintessence and phantom models, it is straightforward to obtain the corresponding Hubble parameter $H(z; s) = \frac{\dot{a}}{a}(z; s)$ as a function of redshift. This may now be used to obtain the corresponding Hubble free luminosity distance

$$D_L^{th}(z; s) = (1 + z) \int_0^z dz' \frac{H_0}{H(z'; s)} \qquad (3.1)$$

Using the maximum likelihood technique [20] we can find the goodness of fit to the corresponding observed $D_L^{obs}(z_i)$ ($i = 1, ..., 157$) coming from the SnIa of the Gold dataset [2]. The observational data of the gold dataset are presented as the apparent magnitudes $m(z)$ of the SnIa with the corresponding redshifts z and 1σ errors $\sigma_{m(z)}$. The apparent magnitude is connected to $D_L(z)$ as

$$m(z; s) = \bar{M}(M, H_0) + 5log_{10}(D_L(z; s)) \qquad (3.2)$$

where $\bar{M}$ is the magnitude zero point offset and depends on the absolute magnitude M and on the present Hubble parameter H_0 as

$$\bar{M} = M + 5log_{10}(\frac{c\,H_0^{-1}}{Mpc}) + 25 \qquad (3.3)$$

The goodness of fit corresponding to any slope s is determined by the probability distribution of s $i.e.$

$$P(\bar{M}, s) = \mathcal{N}e^{-\chi^2(\bar{M}, s)/2} \qquad (3.4)$$

where

$$\chi^2(\bar{M}, s) = \sum_{i=1}^{157} \frac{(m^{obs}(z_i) - m^{th}(z_i; \bar{M}, s))^2}{\sigma^2_{m^{obs}(z_i)}} \qquad (3.5)$$

and $\mathcal{N}$ is a normalization factor. The parameter $\bar{M}$ is a nuisance parameter and can be marginalized (integrated out) leading to a new $\bar{\chi}^2$ defined as

$$\bar{\chi}^2 = -2ln \int_{-\infty}^{+\infty} e^{-\chi^2/2} d\bar{M} \qquad (3.6)$$

Using equations (3.5) and (3.6) it is straightforward to show (see Refs [8, 21]) that

$$\bar{\chi}^2(s) = \chi^2(\bar{M} = 0, s) - \frac{B(s)^2}{C} + \ln(C/2\pi) \qquad (3.7)$$

where

$$B(s) = \sum_{i=1}^{157} \frac{(m^{obs}(z_i) - m^{th}(z_i; \bar{M} = 0, s))}{\sigma^2_{m^{obs}(z_i)}} \qquad (3.8)$$

$$C = \sum_{i=1}^{157} \frac{1}{\sigma^2_{m^{obs}(z_i)}} \qquad (3.9)$$

Equivalent to marginalization is the minimization with respect to $\bar{M}$. It is straightforward to show [22] that χ^2 can be expanded in $\bar{M}$ as

$$\chi^2(s) = \chi^2(\bar{M} = 0, s) - 2\bar{M}B + \bar{M}^2 C \qquad (3.10)$$

which has a minimum for $\bar{M} = \frac{B}{C}$ at

$$\chi^2(s) = \chi^2(\bar{M} = 0, s) - \frac{B(s)^2}{C} \qquad (3.11)$$

Using (3.11) we can find the best fit value of s ($s = s_0$) as the value that minimizes $\chi^2(s)$ ($\chi^2(s_0) = \chi^2_{min}$). The 1σ error on s is determined by the relation [20]

$$\Delta\chi^2_{1\sigma} = \chi^2(s_{1\sigma}) - \chi^2_{min} = 1 \qquad (3.12)$$

i.e. s is in the range $[s_0, s_{1\sigma}]$ with 68% probability. Similarly the 2σ error (95.4% range) is determined by $\Delta\chi^2_{2\sigma} = 4$ and the 3σ error (99% range) by $\Delta\chi^2_{3\sigma} = 6.63$.

Figures 4 and 5 show plots of the differences $\Delta\chi^2(s) \equiv \chi^2(s) - \chi^2(s \simeq 0)$ with respect to the cosmological constant ($\chi^2(s \simeq 0) = 177.1$) for quintessence and phantom models with the 1σ, 2σ and 3σ ranges marked by dashed lines.

From Figures 4 and 5 it is clear that for both phantom and quintessence models the best fit is obtained for $s \simeq 0$ corresponding to a cosmological constant. For phantom models however the fit in the range $0 < s < 1$ is almost degenerate (all values of s in this range give essentially equally good fit). The 2σ range for quintessence is $s \simeq 0 \pm 1.6$ while for phantom fields the corresponding range is $s \simeq \pm 0.7 \pm 1$ (the best fit is at $s \simeq 0.7$ and the symmetry of the evolution with respect to $+s \rightarrow -s$
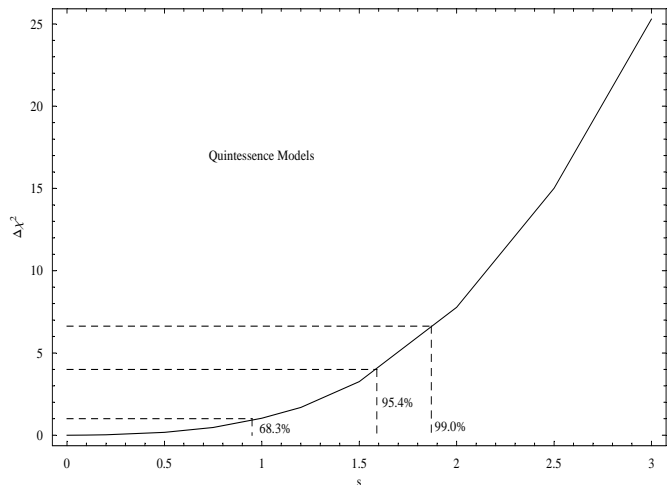

FIG. 4: The differences $\Delta\chi^2(s) \equiv \chi^2(s) - \chi^2(s \simeq 0)$ for quintessence models. The curve has been sampled at $s = 0$, 0.1, 0.2, 0.5, 0.75, 1.0, 1.2, 1.5, 2.0, 2.5, 3.0 and the corresponding points have been joined.
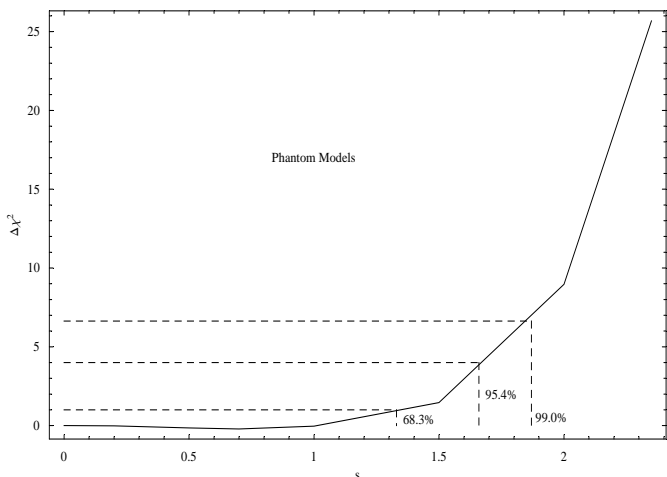

FIG. 5: The differences $\Delta\chi^2 = \chi^2(s) - \chi^2(s \simeq 0)$ for phantom models. The curve has been sampled at $s = 0$, 0.2, 0.5, 0.7, 1.0, 1.5, 2.0, 2.35 and the corresponding points have been joined.

has been imposed). The value of χ^2 at best fit is $\chi^2_{min} = \chi^2(s \simeq 0) = 177.1$ for quintessence (identical to the value corresponding to a cosmological constant with $\Omega_\Lambda = 0.7$) and $\chi^2_{min} = \chi^2(s \simeq 0.7) = 176.9$ for phantom fields. Clearly both classes of models can not provide better fits than the cosmological constant ΛCDM.

We are thus faced with the following question: 'What are the particular features required by $w(z)$ for better fits to the SnIa data?'. To address this question we can use arbitrary parametrizations of $w(z)$ and identify the forms of $w(z)$ that best fit the data. This task has been undertaken by several authors [7, 8] and the best fit forms of $w(z)$ found had the following common properties:

- The value of $w(z = 0)$ at best fit was found to be in the range $-1 > w(z = 0) > -2$

- The function $w(z)$ at best fit was found to cross the PDL from below at least once with $\frac{dw}{dz} > 0$ in the range $0 < z < 1$.

This is demonstrated in Figure 6 where we plot $w(z)$ for two representatives of the field theory models studied here (quintessence with $s = 2$ and phantom with $s = 1.5$) superimposed with $w(z)$ for the best fits of two arbitrary parametrizations. The parametrizations considered are the following

- A linear ansatz

$$w(z) = w_0 + w_1\, z \qquad (3.13)$$

Using the equation[8]

$$w(z) = \frac{p_{DE}(z)}{\rho_{DE}(z)} = \frac{\frac{2}{3}(1+z)\frac{d\ln H}{dz} - 1}{1 - (\frac{H_0}{H})^2 \Omega_{0m}(1+z)^3} \qquad (3.14)$$

we can obtain the Hubble parameter $H(z)$ corresponding to the $w(z)$ of (3.13) as

$$H^2(z) = H_0^2[\Omega_{0m}(1+z)^3 +$$
$$(1 - \Omega_{0m})(1+z)^{3(1+w_0-w_1)}\, e^{3w_1 z}] \qquad (3.15)$$

This can now be used to obtain $D_L^{th}(z; w_0, w_1)$ from equation (3.1) and minimize the χ^2 obtained from the Gold dataset[2]. Using the Gold dataset, the best fit parameter values for this ansatz are[18] $(w_0, w_1) = (-1.4 \pm 0.1, 1.7 \pm 0.4)$ giving $\chi^2 = 174.3$ at the minimum (the errors are at the 1σ level).

- The ansatz

$$w(z) = w_0 + w_1 \frac{z}{1+z} \qquad (3.16)$$

which varies between w_0 at $z = 0$ and $w_0 + w_1$ at $z \to \infty$ with crossover at $z = 1$ where the two values contribute equally. The existence of such a crossover has the advantage that observations near it apply to a reduced parameter phase space, and hence the remaining parameter estimates are more sensitive[19]. The Hubble parameter corresponding to this ansatz is

$$H^2(z) = H_0^2[\Omega_{0m}(1+z)^3 +$$
$$(1 - \Omega_{0m})(1+z)^{3(1+w_0+w_1)}\, e^{3w_1(\frac{1}{1+z}-1)}] \qquad (3.17)$$

The best fit parameter values for this ansatz are[18] $(w_0, w_1) = (-1.6 \pm 0.1, 3.3 \pm 0.5)$ giving $\chi^2 = 173.9$ at the minimum (the errors are at the 1σ level).

These parametrizations were chosen for their relative simplicity and for leading to fairly good fits to the data relatively to other parametrizations (see also Ref. [8]). As seen in Figure 6 they share both of the properties
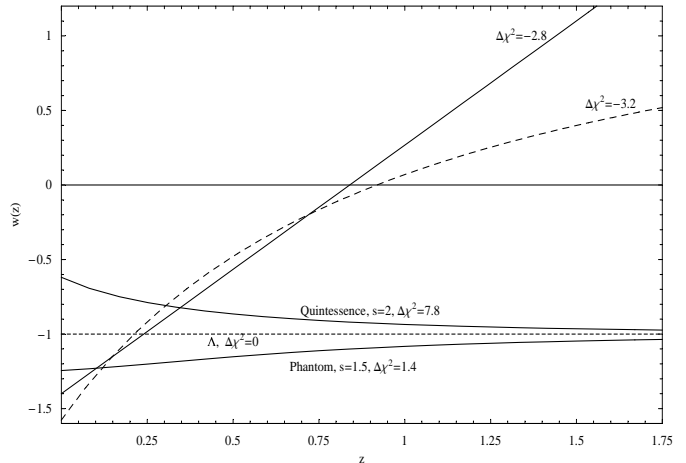

FIG. 6: The evolution of w for two representatives of the field theory models studied here superposed with two better fits obtained by arbitrary parametrization: a linear ansatz $w(z) = w_0 + w_1\, z$ and the ansatz $w(z) = w_0 + w_1 \frac{z}{1+z}$.

refered above ($w(z = 0) < -1$ and cross the PDL $w = -1$).

For each $w(z)$ we have evaluated $\Delta\chi^2$ with respect to the cosmological constant $\chi^2_{\Lambda CDM} = 177.1$. The field theory models which do not cross the PDL have positive $\Delta\chi^2$ and are therefore worse fits than ΛCDM. For example a quintessence field with $s = 2$ gives $\Delta\chi^2 = 7.8$ while a phantom field with $s = 1.5$ gives $\Delta\chi^2 = 1.4$ (see Figure 6). In contrast, the arbitrary parametrizations which cross the PDL have negative $\Delta\chi^2$ and are therefore better fits than ΛCDM. In particular for the linear ansatz (3.13) we find $\Delta\chi^2 = -2.8$ while for the smoother ansatz of (3.16) we find $\Delta\chi^2 = -3.2$. These differences mean that the point $(w_0, w_1) = (-1, 0)$ corresponding to the cosmological constant from the viewpoint of these parametrizations, is worse at more than 1σ from the best fits obtained from these parametrizations. The problem with such $w(z)$ parametrizations is that it seems to be highly non-trivial to approximate their behavior using field theory models (even exotic ones).

IV. CONCLUSION - OUTLOOK

We have shown that phantom and quintessence field theory models have serious difficulty to exceed the quality of fit of a cosmological constant to the SnIa Gold dataset. In contrast, arbitrary parametrizations of the Hubble parameter and of $w(z)$ that cross the PDL can perform significantly better in fitting the SnIa redshift data. We are thus faced with the question: 'Which field theory models can mimic the behavior of the arbitrary parametrizations crossing the PDL?'

As shown in the introduction this type of behavior can not be achieved by either quintessence or phantom

fields. In principle[23] such behavior could be achieved by field theories involving combinations of quintessence and phantom fields (called 'quintom' in recent studies [24]). However, preliminary investigations have demonstrated that mimicking the $w(z)$ behavior indicated by the best fit parametrizations requires significant fine tuning even for quintom field theories. This can be demonstrated by studying the quintom Lagrangian

$$\mathcal{L} = \frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 - s\,\phi_1 - s\,\phi_2 - q\,\phi_1\,\phi_2 \qquad (4.1)$$

which leads to the rescaled dynamical equations

$$\frac{\ddot{a}}{a} = -\dot{\phi}_1^2 + \dot{\phi}_2^2 + s\,(\phi_1 + \phi_2) + q\,\phi_1\,\phi_2 - \frac{\Omega_{0m}}{2a^3} (4.2)$$

$$\ddot{\phi}_1 + 3\frac{\dot{a}}{a}\dot{\phi}_1 + s + q\phi_2 = 0 \qquad (4.3)$$

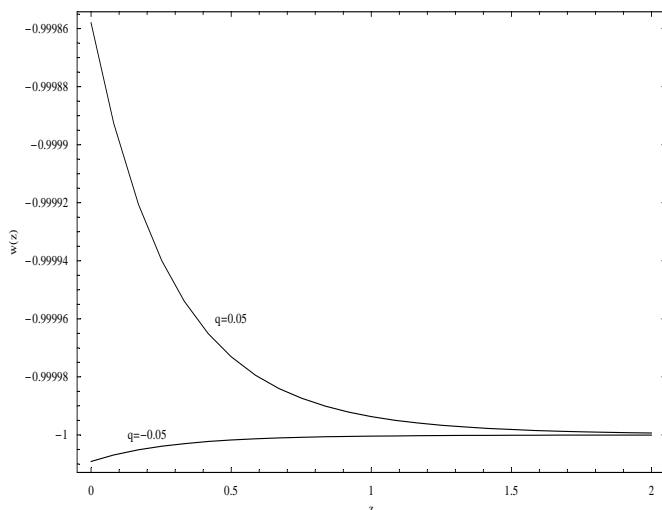$$\ddot{\phi}_2 + 3\frac{\dot{a}}{a}\dot{\phi}_2 - s - q\phi_1 = 0 \qquad (4.4)$$

FIG. 7: The quintom model of the Lagrangian (4.1) with natural initial conditions and $s = 0.2$ can mimic quintessence and phantom models by changing the sign of the coupling q.

Even though we have not explored thoroughly the parameter space of this quintom model we have shown that the natural initial conditions used for the analysis of quintessence and phantom systems presented above above do not lead to a crossing of the PDL in the way indicated by the best fit parametrizations. As shown in Figure 7 (obtained with $s = 0.2$) we can simply mimic either the quintessence or the phantom behavior of $w(z)$ by changing the sign of the interaction coupling q.

Alternative field theory models that can in principle lead to a $w(z)$ crossing the PDL with a single scalar field can be constructed by considering generalizations of the kinetic term of the Lagrangian in the form of k-essence[25, 26]. An example of such models includes a Lagrangian with a field dependent term multiplying the kinetic term in the form

$$\mathcal{L} = \frac{1}{2}f(\phi)\dot{\phi}^2 - V(\phi) \qquad (4.5)$$

It has been shown however[27] than even in such models transitions through $w = -1$ are physically implausible because they are either realized by a discrete set of trajectories in phase space or they are unstable with respect to cosmological perturbations.

The Mathematica[28] file used for the numerical analysis and the production of the figures of the paper can be downloaded from [18] or sent by e-mail upon request.

[1] Riess A *et al.*, 1998 Astron. J. **116** 1009; Perlmutter S J *et al.*, 1999 Astroph. J. **517** 565; S. Perlmutter et al, Nature (London) **391**, 51(1998); Bull.Am.Astron.Soc**29**,1351(1997); P.Garnavich et al, Astrophys.J.**493**L53 (1998); B.P.Schmidt et al, Astrophys. J.**507**,46 (1998); Tonry, J L *et al.*, 2003 Astroph. J. **594** 1; Barris, B *et al.*, 2004 Astroph. J. **602** 571; Knop R *et al.*, 2003 Astroph. J. **598** 102;

[2] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astrophys. J. **607**, 665 (2004) [arXiv:astro-ph/0402512].

[3] N.A.Bahcall et al, Science**284**,1481 (1999); Percival, W.J., *et al.*, Mon. Not. Roy. Ast. Soc. **327**, 1297 (2001); M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004) [arXiv:astro-ph/0310723].

[4] D.N.Spergel et al, arXiv.astro-ph/0302209; D. Miller *et al.*, Astrophys. J. **524**, L1 (1999) [arXiv:astro-ph/9906421]; C. Bennett et al, astro-ph/0302207; P. de Bernardis *et al.* [Boomerang Collaboration], Nature **404**, 955 (2000) [arXiv:astro-ph/0004404];S. Hanany *et al.*, Astrophys. J. **545**, L5 (2000) [arXiv:astro-ph/0005123];T. J. Pearson *et al.*, Astrophys. J. **591**, 556 (2003) [arXiv:astro-ph/0205388];B. S. Mason *et al.*, Astrophys. J. **591**, 540 (2003) [arXiv:astro-ph/0205384];A. Benoit *et al.* [the Archeops Collaboration], Astron. Astrophys. **399**, L25 (2003) [arXiv:astro-ph/0210306].

[5] V. Sahni and Y. Shtanov, IJMP **D11**, 1515 (2002); JCAP **0311**, 014 (2003); L. Perivolaropoulos and C. Sourdis, Phys. Rev. D **66**, 084018 (2002) [arXiv:hep-ph/0204155]; C. Deffayet, G. Dvali and G. Gabadadze, Phys. Rev. **D65**, 044023 (2002); J. S. Alcaniz, Phys. Rev. D **65**, 123514 (2002). astro-ph/0202492; E. V. Linder, Phys. Rev. Lett. **90**, 091301 (2003); L. Perivolaropoulos, Phys. Rev. D **67**, 123516 (2003) [arXiv:hep-ph/0301237]; D.F.Torres, Phys. Rev. D**66**,043522(2002); B. Boisseau,

G. Esposito-Farese, D. Polarski and A. A. Starobinsky, Phys. Rev. Lett. **85**, 2236 (2000)[arXiv:gr-qc/0001066].

[6] M. S. Turner and M. White, Phys. Rev. **D56**, R4439 (1997); T. Padmanabhan, arXiv:astro-ph/0411044; P. H. Frampton, arXiv:astro-ph/0409166.

[7] U. Alam, V. Sahni, T. D. Saini and A. A. Starobinsky, Mon. Not. Roy. Astron. Soc. **354**, 275 (2004) [arXiv:astro-ph/0311364]; T. R. Choudhury, T. Padmanabhan, astro-ph/0311622; Y. Wang and P. Mukherjee, Astrophys. J. **606**, 654 (2004) [arXiv:astro-ph/0312192]; Huterer D and Cooray A, 2004 `astro-ph/0404062`; R. A. Daly and S. G. Djorgovski, Astrophys. J. **597**, 9 (2003) [arXiv:astro-ph/0305197]; J. S. Alcaniz, Phys. Rev. **D69**, 083521 (2004) astro-ph/0312424; J. A. S. Lima, J. V. Cunha and J. S. Alcaniz, Phys. Rev. D **68**, 023510 (2003) [arXiv:astro-ph/0303388]; A. Upadhye, M. Ishak and P. J. Steinhardt, arXiv:astro-ph/0411803.

[8] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **70**, 043531 (2004) [arXiv:astro-ph/0401556].

[9] B.Ratra and P.J.E.Peebles, Phys.Rev. D**37**,3406(1988); astro-ph/0207347; C.Wetterich, Nucl. Phys. B**302**,668(1988); P.G.Ferreira and M.Joyce, Phys. Rev. D.**58**,023503(1998); P.Brax and J.Martin, Phys. Rev. D.**61**,103502(2000); L.A.Urena-Lopez and T.Matos, Phys. Rev. D.**62**,081302(2000); T.Barreiro, E.J.Copeland and N.J.Nunes, Phys. Rev. D.**61**,127301(2000); D.Wands, E.J.Copeland and A.R.Liddle, Phys. Rev. D.**57**,4686 (1998); A.R. Liddle and R.J. Scherrer, Phys. Rev. D **59**, 023509 (1998); R.R. Caldwell, R. Dave and P. Steinhardt, Phys. Rev. Lett **80**, 1582 (1998); I. Zlatev, L. Wang and P. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999); S. Dodelson, M. Kaplinghat and E. Stewart, Phys. Rev. Lett. **85**, 5276(2000); V.B.Johri, Phys. Rev. D.**63**,103504(2001); V.B.Johri, Class. Quant. Grav.**19**,5959 (2002); V.Sahni and L.Wang, Phys. Rev. D.**62**,103517(2000); M. Axenides and K. Dimopoulos, JCAP **0407**, 010 (2004)[arXiv:hep-ph/0401238]; M. Doran and J. Jaeckel Phys. Rev. D **66**, 043519 (2002) [arXiv:astro-ph/0203018].

[10] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002) [arXiv:astro-ph/9908168]; P. F. Gonzalez-Diaz, Phys. Lett. B **586**, 1 (2004) [arXiv:astro-ph/0312579]; P. Singh, M. Sami and N. Dadhich, Phys. Rev. D **68**, 023522 (2003) [arXiv:hep-th/0305110]. G. Calcagni, arXiv:gr-qc/0410111; V. B. Johri, Phys. Rev. D **70**, 041303 (2004) [arXiv:astro-ph/0311293]; V.K.Onemli and R.P.Woodard, Class. Quant. Grav.**19**, 4607 (2000); S. Hannestad and E. Mortsell, Phys. Rev. D**66**,063508(2002); S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D **68**, 023509 (2003) [arXiv:astro-ph/0301273]; P.H.Frampton, hep-th/0302007; P. F. Gonzalez-Diaz and C. L. Siguenza, Nucl. Phys. B **697**, 363 (2004) [arXiv:astro-ph/0407421]; G.W.Gibbons, hep-th/0302199;B.Mcinnes, astro-ph/0210321; L. P. Chimento and R. Lazkoz, Phys. Rev. Lett. **91**, 211301 (2003)[arXiv:gr-qc/0307111]; H. Stefancic,Phys. Lett. B **586**, 5 (2004) [arXiv:astro-ph/0310904];

H. Stefancic,Eur. Phys. J. C **36**, 523 (2004) [arXiv:astro-ph/0312484]; P. F. Gonzalez-Diaz, Phys. Rev. D **69**, 063522 (2004)[arXiv:hep-th/0401082]; V. K. Onemli and R. P. Woodard, Phys. Rev. D **70**, 107301 (2004) [arXiv:gr-qc/0406098]; T. Brunier, V. K. Onemli and R. P. Woodard, Class. Quant. Grav. **22**, 59 (2005) [arXiv:gr-qc/0408080]; E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Rev. D **70**, 043539 (2004) [arXiv:hep-th/0405034]; S. Nojiri and S. D. Odintsov, Phys. Lett. B **571**, 1 (2003) [arXiv:hep-th/0306212]; S. Nojiri and S. D. Odintsov, Phys. Lett. B **562**, 147 (2003) [arXiv:hep-th/0303117].

[11] J. Garriga, L. Pogosian and T. Vachaspati, Phys. Rev. D **69**, 063511 (2004) [arXiv:astro-ph/0311412].

[12] J. Garriga and A. Vilenkin, Phys. Rev. D **61**, 083502 (2000) [arXiv:astro-ph/9908115]; J. Garriga and A. Vilenkin, Phys. Rev. D **67**, 043503 (2003) [arXiv:astro-ph/0210358]; J. Garriga, A. Linde and A. Vilenkin, Phys. Rev. D **69**, 063521 (2004) [arXiv:hep-th/0310034].

[13] P. P. Avelino, arXiv:astro-ph/0411033.

[14] S. Dimopoulos and S. Thomas, Phys. Lett. B **573**, 13 (2003) [arXiv:hep-th/0307004].

[15] R. Kallosh, J. Kratochvil, A. Linde, E. V. Linder and M. Shmakova, JCAP **0310**, 015 (2003) [arXiv:astro-ph/0307185].

[16] Y. Wang, J. M. Kratochvil, A. Linde and M. Shmakova, arXiv:astro-ph/0409264.

[17] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003) [arXiv:astro-ph/0302506]; S. Nesseris and L. Perivolaropoulos, [arXiv:astro-ph/0410309].

[18] The mathematica file with the numerical analysis and the figures of the paper can be downloaded from http://leandros.physics.uoi.gr/negpot.htm

[19] E. V. Linder, Phys. Rev. D **68**, 083503 (2003) [arXiv:astro-ph/0212301].

[20] Press, W.H., et. al., 'Numerical Recipes', Cambridge University Press (1994).

[21] E. Di Pietro and J. F. Claeskens, Mon. Not. Roy. Astron. Soc. **341**, 1299 (2003) [arXiv:astro-ph/0207332].

[22] S. Fay, private communication.

[23] W. Hu, Phys. Rev. D **71**, 047301 (2005) [arXiv:astro-ph/0410680].

[24] Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, arXiv:astro-ph/0410654; B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B **607**, 35 (2005) [arXiv:astro-ph/0404224]; B. Feng, M. Li, Y. S. Piao and X. Zhang, arXiv:astro-ph/0407432.

[25] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. D **63**, 103510 (2001) [arXiv:astro-ph/0006373].

[26] A. Melchiorri, L. Mersini, C. J. Odman and M. Trodden, Phys. Rev. D **68**, 043509 (2003) [arXiv:astro-ph/0211522].

[27] A. Vikman, Phys. Rev. D **71**, 023515 (2005) [arXiv:astro-ph/0407107].

[28] http://www.wolfram.com/