Cosmological Effects of Radion Oscillations

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We show that the redshift of pressureless matter density due to the expansion of the universe generically induces small oscillations in the stabilized radius of extra dimensions (the radion field). The frequency of these oscillations is proportional to the mass of the radion and can have interesting cosmological consequences. For very low radion masses m_b ($m_b \sim 10 - 100\ H_0 \simeq 10^{-32}\ eV$) these low frequency oscillations lead to oscillations in the expansion rate of the universe. The occurrence of acceleration periods could naturally lead to a resolution of the coincidence problem, without need of dark energy. Even though this scenario for low radion mass is consistent with several observational tests it has difficulty to meet fifth force constraints. If viewed as an effective Brans-Dicke theory it predicts $\omega = -1 + \frac{1}{D}$ (D is the number of extra dimensions), while experiments on scales larger than $1mm$ imply $\omega > 2500$. By deriving the generalized Newtonian potential corresponding to a massive toroidally compact radion we demonstrate that Newtonian gravity is modified only on scales smaller than m_b^{-1}. Thus, these constraints do not apply for $m_b > 10^{-3}\ eV$ (high frequency oscillations) corresponding to scales less than the current experiments ($0.3mm$). Even though these high frequency oscillations can not resolve the coincidence problem they provide a natural mechanism for dark matter generation. This type of dark matter has many similarities with the axion.

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I. INTRODUCTION

Cosmological theories with submillimeter dimensions have recently been the focus of several studies [1, 2, 3, 4, 5] (see also Ref.[6] for a recent review). These theories were originally proposed to provide a novel solution to the hierarchy problem by postulating that the fundamental Planck mass M_* is close to the TeV scale [3, 7, 8]. This is possible in theories with extra dimensions [3] because Gauss's law relates the Planck scales of the $4 + D$-dimensional theory M_* and the long distance 4-dimensional theory M_{pl} by

$$M_{pl}^2 = b_0^D M_*^{D+2} \qquad (1.1)$$

where b_0 is the present stabilized size of the extra dimensions. The size of the extra dimensions $b(t)$ (the radion field) is usually assumed to be stabilized to its present value b_0 by a 'radion stabilizing potential' $V(b)$. The early evolution of the radion has been well studied [1, 4, 9, 10] and has been shown to generate an inflationary era with observationally consistent phenomenology (see also Ref.[11] for earlier studies).

Studies of late evolution of the radion around its potential minimum b_0 have mainly focused on the resolution of the moduli problem [1, 10, 12, 13]: *How do we dilute the energy in radion oscillations which at the end of inflation is high enough to overclose the universe?* Here we assume that the radion is stabilized at b_0 either by a short period of late inflation or by some other mechanism. The oscillations we consider are mainly induced by redshifting matter at late times. They can lead to interesting late time cosmological effects through the coupling of $b(t)$ with the scale factor of the universe $a(t)$. These effects will be the focus of this paper. In what follows we will show that

- The redshift of matter ($\rho(t) \sim a^{-3}$) generically produces oscillations of the radion $b(t)$ around its (local) minimum b_0. This distinguishes the radion from an ordinary minimally coupled to gravity scalar field.

- For low radion masses $\mathcal{O}(10 - 100\ H_0)$ these oscillations lead to accelerating and decelerating periods of the expansion factor $a(t)$ and thus to a possible resolution of the coincidence problem [16]. However, oscillations in this frequency regime may be strongly constrained by classical tests of general relativity and by Casimir force measurements [17, 18].

- For higher radion masses ($m > 10^{-3}\ eV$) radion oscillations can not resolve the coincidence problem but they are consistent with classical gravity tests and their energy density redshifts like a^{-3}. They can therefore play the role of dark matter.

A class of 'oscillating physics' models with features similar to those discussed here has been developed in the

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context of non-minimally coupled scalar fields (Brans-Dicke theories) in an effort to explain the apparent periodicity in pencil beam galaxy redshift surveys [19] (low frequency oscillations of the Brans-Dicke scalar [20, 21, 22]) or to resolve the discrepancy between measurements of $\Omega_m \simeq 0.1 - 0.3$ and the inflationary prediction of $\Omega_{tot} = 1$ (high frequency oscillations of the Brans-Dicke scalar [23, 24, 25]). The case of radion oscillations discussed here corresponds to a very specific type of Brans-Dicke theory where the parameters of the theory are uniquely determined by the number of extra dimensions D.

The structure of this paper is the following: In section II we derive the cosmological equations assuming D toroidally compact extra dimensions stabilized by a potential and obtain an approximate analytic solution for small oscillations around the potential minimum. We also establish the connection with Brans-Dicke theory and derive the effective values of the Brans-Dicke parameters as a function of D. In section III we focus on the case of low radion mass and discuss the basic cosmological features of the model. The corresponding features for large radion masses are discussed in section IV. Finally, in section V we conclude and summarize the main remaining open issues.

II. RADION COSMOLOGY

We consider a flat $4 + D$ dimensional spacetime $R^1 \times R^3 \times T^D$ with toroidally compact D extra dimensions. The metric may be written as

$$g_{MN} = diag[1, -a^2(t)\tilde{g}_{ij}, -b^2(t)\tilde{g}_{mn}] \qquad (2.1)$$

where M, N run from 0 to $D+3$; i, j run from 1 to 3 and m, n run from 4 to $D + 3$. Also $a(t)$ is the scale of the non-compact 3-dimensional flat space (the scale factor of the universe) and $b(t)$ is the radius of the compactified toroidal space (the radion field).

The nonzero components of the energy-momentum tensor are given by

$$\begin{aligned} T_{00} &= \rho_{tot}g_{00} \\ T_{ij} &= -p_a g_{ij} \end{aligned} \qquad (2.2)$$

$$T_{mn} = -p_b g_{mn}$$

The energy density ρ_{tot} and the pressures p_a, p_b are derivable from the internal energy $\mathcal{U} = \mathcal{U}(a, b)$ as

$$\rho_{tot} = \frac{\mathcal{U}}{\mathcal{V}}, \quad p_a = -\frac{a\partial\mathcal{U}/\partial a}{3\mathcal{V}}, \quad p_b = -\frac{b\partial\mathcal{U}/\partial b}{D\mathcal{V}} \qquad (2.3)$$

where $\mathcal{V} = a^3(t)\Omega_D b^D(t)$ is the volume of the $(D + 3)$-space $(\Omega_D = 2\pi^{\frac{D+1}{2}}/\Gamma(\frac{D+1}{2}))$.

Consider now an internal energy of the form

$$\mathcal{U} = a^3\left(V(b) + \rho\right) \qquad (2.4)$$

$V(b)$ in equation (2.4) is the radion potential which can produce [1] sufficient primordial inflation to solve the horizon, flatness, homogeneity, and monopole problems and can stabilize b at

$$b_0 = \left(\frac{M_{Pl}^2}{M_*^{D+2}}\right)^{1/D} \qquad (2.5)$$

with a vanishing cosmological constant. The energy density $\rho = \rho(a)$ in (2.4) is due to mater and radiation.

The action for Einstein gravity in $N = D + 4$ dimensions is

$$S = \int d^N x \sqrt{-g^{(N)}}\left(\frac{1}{16\pi\overline{G}}\mathcal{R}[g] + \mathcal{L}_{\text{matter}}\right) \qquad (2.6)$$

The generalized Einstein equations are

$$R_{MN} = 8\pi\overline{G}\left(T_{MN} - \frac{T^K{}_K}{D+2}g_{MN}\right) \qquad (2.7)$$

where $\overline{G}$ is the $(4+D)$ dimensional gravitational constant. The gravitational coupling $16\pi G = \frac{1}{b_0^D M_*^{D+2}}$ is weak because b_0 is much greater than the $(4 + D)$-dimensional Planck length $\frac{1}{M_*}$. Thus, the hierarchy problem is 'transferred' to the large extra dimensions.

Using the metric ansatz (2.1) and the energy-momentum components (2.2), it is straightforward to obtain the following set of evolution equations for the scale factors a and b from equations (2.7).

$$\begin{aligned} 6\frac{\dot{a}^2}{a^2} + D(D-1)\frac{\dot{b}^2}{b^2} + 6D\frac{\dot{a}}{a}\frac{\dot{b}}{b} &= \frac{V + \rho}{M_*^{D+2}b^D} \\ \frac{\ddot{b}}{b} + (D-1)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} &= \frac{1}{M_*^{D+2}b^D}\left(\frac{2V}{D+2} - \frac{b}{D(D+2)}\frac{\partial V}{\partial b} + \frac{\rho - 3p_a}{2(D+2)}\right) \\ \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + D\frac{\dot{a}}{a}\frac{\dot{b}}{b} &= \frac{1}{M_*^{D+2}b^D}\left(\frac{b}{2(D+2)}\frac{\partial V}{\partial b} - \frac{D-2}{2(D+2)}V + \frac{\rho + (D-1)p_a}{2(D+2)}\right) \end{aligned} \qquad (2.8)$$

In the picture of Ref. [3] (ADD) and [8] (RS), the matter-radiation energy density is assumed to be localized on the brane corresponding to the $a(t)$ scale factor. This localized energy density in general distorts the geometry of the compactified D-dimensional space (the bulk), but as far as the overall properties and the evolution of the radion are concerned, it is correct to treat the energy density on the wall as just being averaged over the whole space as done on the RHS of these equations. This assumption is consistent with the results of references [4, 12] where it was shown that the coupling of the radion field to the energy momentum tensor is given generically through the trace $(\rho - 3p_a)$ plus terms involving the stabilizing potential $V(b)$.

Equations (2.8) have been analyzed in the context of inflation (away from the stabilization point b_0) in [1] (ADD model).

Similar equations [4] arise in the context of Randall-Sundrum (RS) models [8], where the hierarchy problem is solved using extra dimensions of smaller sizes at the expense of introducing a non-flat background metric along the extra coordinates and a pair of branes whose distance $b(t)$ is stabilized at b_0 by the potential $V(b)$.

From equations (2.8) it is clear that radiation ($p_{rad} = \frac{1}{3}\rho_{rad}$) has no effect on the dynamics of the radion. This is not the case however for matter. Redshifting matter plays the role of a driving force and can induce radion oscillations during both the matter and radiation eras. These oscillations backreact on the scale factor $a(t)$ through the effective Friedman equations (2.8) and can induce interesting cosmological effects. In what follows we study these cosmological effects in the presence of redshifting matter and radiation neglecting any other possible existence leading to equations of state with $\rho - 3p_a \neq 0$.

To study the evolution of the system of the scale factor $a(t)$ coupled to the radion $b(t)$, we focus on the first two equations of the system (2.8) (the last one, given the energy-momentum conservation and the equation of state, is not independent). The potential $V(b)$ may be written to lowest order in $\delta \equiv \frac{b-b_0}{b_0}$ as

$$V = \rho_{\rm v}\left(\frac{b}{b_0} - 1\right)^2 = \rho_{\rm v}\delta^2 \qquad (2.9)$$

In order to write this system in dimensionless form we define the following dimensionless quantities

$$\bar{b} = \frac{b}{b_0} \qquad (2.10)$$

$$\bar{a} = \frac{a}{a_0} \qquad (2.11)$$

$$\overline{m}^2 = \frac{\rho_{\rm v}}{\rho_{om}} \qquad (2.12)$$

$$\bar{t} = t\left(\frac{\rho_{om}}{6M_*^{D+2}b_0^D}\right)^{1/2} \qquad (2.13)$$

where a_0 is the scale factor at the present time $t = t_0$. Expanding equations (2.8) around the stabilization point b_0 ($V'(b_0) = 0$) the radion mass can be read off from the corresponding linearized equation of motion (see e.g. Ref. [1]) and is given in terms of the second derivative of the potential (2.9). Up to a constant factor of order one, the radion mass is

$$\begin{aligned} m_b{}^2 &= \frac{b_0^2 V''}{M_*^{D+2}b_0^D} \equiv \frac{b_0^2 V''}{M_{Pl}^2} \\ &= \frac{\rho_{\rm v}}{M_{Pl}{}^2} = \frac{\overline{m}^2\rho_{om}}{M_{Pl}{}^2} \simeq (10^{-33}\overline{m})^2 \ eV^2 \end{aligned} \qquad (2.14)$$

(ρ_{om} is the matter density at the present time t_0). In order to simplify our notation in what follows, we will omit the bar from the above dimensionless quantities (except $\overline{m}$). Thus, the system (2.8) may be written in dimensionless form as

$$\frac{\dot{a}^2}{a^2} + \frac{D(D-1)}{6}\frac{\dot{b}^2}{b^2} + D\frac{\dot{a}}{a}\frac{\dot{b}}{b} = \frac{\overline{m}^2(b-1)^2 + \frac{1}{a^3}}{b^D} \qquad (2.15)$$

$$\frac{\ddot{b}}{b} + (D-1)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} = \frac{3}{(D+2)b^D}\left[4\overline{m}^2(b-1)^2 - \frac{4\overline{m}^2 b(b-1)}{D} + \frac{1}{a^3}\right] \qquad (2.16)$$

This set of equations is similar to the cosmological field equations of a Brans-Dicke theory for specific parameter values. To identify these parameter values we compare the dimensionally reduced form of (2.6)

$$S = \int d^4x \frac{\sqrt{-g^{(4)}}}{16\pi G}\left[b^D \mathcal{R}^{(4)} + D(D-1)b^{D-2}\dot{b}^2 - V_b(b)\right] \qquad (2.17)$$

with the 4-dimensional Brans-Dicke theory of a dynami-

cal Planck mass

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G}\left[F(\phi)\mathcal{R} - Z(\phi)\partial_\mu\phi\partial^\mu\phi - V_\phi(\phi)\right] \quad (2.18)$$

The identification that needs to be made is

$$F(\phi) = b^D \quad (2.19)$$
$$-Z(\phi)\dot{\phi}^2 = D(D-1)b^{D-2}\dot{b}^2 \quad (2.20)$$

In the parameterization $\phi = b^D$ we obtain

$$F(\phi) = \phi \quad (2.21)$$
$$Z(\phi) = (-1 + \tfrac{1}{D})/\phi \quad (2.22)$$

which implies

$$\omega \equiv \frac{ZF}{F'^2} = -1 + \frac{1}{D} \quad (2.23)$$

Other parameterizations (e.g. $\phi = b$ or $Z(\phi) = -1$) are easily checked to give identical values for ω.

The existence of a finite value of ω implies a Yukawa interaction (fifth force) modification of Newton's law [24]

$$V(r) = -G\frac{M}{r}\left(1 + \frac{1}{3 + 2\omega}e^{-m_\phi r}\right) \quad (2.24)$$

where $m_b = \frac{1}{D(D+2)}m_\phi$, as can be verified by comparing the linearized equations of motion for the radion (see eq.(2.8)) with the equations of motion for a massive Brans-Dicke scalar.

Using equation (2.23) this becomes

$$V_{m_b}(r) = -G\frac{M}{r}\left(1 + \frac{D}{D+2}e^{-m_b r}\right) \quad (2.25)$$

The Yukawa interaction of equation (2.25) is due purely to radion dynamics while geometrical effects have been integrated out by the dimensional reduction of equation (2.17) ($b_0 \to 0$). The Yukawa interaction induced purely by geometrical effects (fixed b_0, $m_b \to \infty$) has been studied in Ref.[27]. The resulting modified Newtonian potential in that case is

$$V_{b_0}(r) = -G\frac{M}{r}\left(1 + 2De^{-r/b_0}\right) \quad (2.26)$$

Combining equations (2.25) and (2.26) we obtain the total modified Newtonian potential for a massive toroidally compact radion

$$V_{tot}(r) = -G\frac{M}{r}\left(1 + \frac{D}{D+2}e^{-m_b r} + 2De^{-r/b_0}\right) \quad (2.27)$$

Corresponding modified Newtonian potentials can be obtained for alternative compactification schemes (spherical and Calabi-Yau) thus generalizing the corresponding results of Ref.[27].

Fifth force tests on scales larger than $1mm$ (solar system and terrestrial) imply a constraint [17] $\omega > 2500$ for the massless ($V(\phi) = 0$) Brans-Dicke model. On scales $r \ll m_b^{-1}$ a massive Brans-Dicke model behaves like the original massless theory. On scales larger than the range of spatial fluctuations ($r \gg m_b^{-1}$) the Brans-Dicke scalar dynamics are frozen by the potential $V(\phi)$ and the model behaves like Einstein gravity. Therefore fifth force experiments on these scales do not constrain the massive Brans-Dicke theories and the constraint $\omega > 2500$ is not applicable provided $m_b^{-1} < 1mm$ (minimum scale of experimental constrains). A separate requirement for consistency with fifth force experiments, coming from the geometric effects of toroidal extra dimensions in equation (2.27) is $b_0 < 1mm$.

Since we are interested in small radion oscillations, it is convenient to linearize the system (2.15), (2.16) setting $\delta = b - 1$. Keeping only the dominant terms we obtain the system

$$\frac{\dot{a}^2}{a^2} = \frac{1}{a^3} - D\frac{\dot{a}}{a}\dot{\delta} \quad (2.28)$$
$$\ddot{\delta} + 3\frac{\dot{a}}{a}\dot{\delta} = -m^2\,\delta + \frac{3}{D+2}\frac{1}{a^3} \quad (2.29)$$

where

$$m^2 \equiv \frac{12\overline{m}^2}{(D+2)D} \quad (2.30)$$

The solution of this system is well approximated by

$$\delta \simeq \delta_0\frac{\cos(mt+\theta)}{t} \simeq \frac{3}{2}\delta_0\frac{\cos(mt+\theta)}{a^{3/2}} \quad (2.31)$$
$$\frac{\dot{a}}{a} \equiv H = -\frac{D}{2}\dot{\delta} + \left[\left(\frac{D}{2}\dot{\delta}\right)^2 + \frac{1}{a^3}\right]^{1/2} \quad (2.32)$$

where δ_0 is determined by the initial conditions at the time t_i when the oscillations start. For radion oscillations induced purely by the redshifting matter we keep only the particular solution of the inhomogeneous differential equation (2.29) and find

$$\delta_0 \simeq -\frac{4\pi}{3(D+2)m}, \quad \theta = 0 \quad (2.33)$$

In the following section we briefly discuss the cosmological effects of this solution for very low radion mass.

III.   LOW RADION MASS

For $m \sim \mathcal{O}(10 - 100\,H_0) \simeq 10^{-32}\,eV$ the fifth force terrestrial and solar system constrains apply ($\omega > 2500$) and due to equation (2.23) the model is not consistent with them at the classical level. A possible way out of

this constraint are modifications of the effective Brans-Dicke parameters due to quantum effects [26].

Nevertheless, the model has several interesting features which are worth discussing before proceeding to the phenomenologically more relevant case of large m.

For radion masses comparable to H_0, the dominant corrections come from the linear term in $\dot{\delta}$ which does not average out to zero. Thus we obtain

$$H = \overline{H} - \frac{D}{2}\dot{\delta} \simeq \overline{H}\left(1 + \xi\, m\, \sin(mt + \theta)\right) \qquad (3.1)$$

where $\overline{H} = \frac{1}{a^{3/2}}$ is the unperturbed Hubble parameter and ξ, θ are constants depending on the initial conditions of the radion oscillations and the number of extra dimensions D ($\xi \equiv 3\delta_0 D/4$).

For

$$m\, \xi \geq \frac{1}{2} \qquad (3.2)$$

equation (3.1) implies periodically accelerating Hubble expansion. At maximum acceleration the expansion factor is

$$a(t) \sim t^{2(1+m\xi)/3} \qquad (3.3)$$

For radion oscillations induced purely by redshifting matter (the source term $\frac{1}{a^3}$) we may use (2.33) to obtain

$$m\, \xi \simeq \frac{\pi D}{(D+2)} > \frac{1}{2} \qquad (3.4)$$

and therefore these oscillations are sufficient to produce periods of accelerating expansion.

The periods of acceleration of the scale factor consist a very interesting feature for the following reasons:

1. *No Dark Energy:* They can potentially explain the acceleration of the scale factor observed in the recent SNe Ia data [28] without the requirement of any form of dark energy. Alternative attempts to achieve accelerating expansion without the need of dark energy can be found in [15, 29].

2. *Coincidence problem:* They can resolve the coincidence problem by inducing other accelerating and decelerating periods in the past. Alternative resolutions of the coincidence problem based on oscillating energy of minimally coupled scalars with oscillating potentials [30, 31] or quintessence induced by extra dimensions [26, 32] have also been recently proposed.

3. *Consistency:* They are consistent with nucleosynthesis constrains assuming that they start at $t_i \simeq m^{-1} > t_{eq}$ *i.e.* much later than the time of nucleosynthesis (this is the behavior expected for oscillating scalars in an expanding universe [33]). The

model is also consistent with observational constraints other than the fifth force [34]: The age of the universe is longer compared to the CDM model without cosmological constant (SCDM). Structure formation is mildly affected. The first Doppler peak of the cosmic background radiation is shifted only slightly and remains consistent with experimental results. The time dependence of Newton's constant constrained by measurements of spin-down rate of pulsars, imposes some restrictions on the parameter θ which are more difficult to meet for a *large number* of extra dimensions.

4. *Predictiveness:* They make the prediction of deceleration at high redshifts at a rate higher than that of non-oscillating models.

5. *Naturalness:* They are well motivated without the requirement of extra scalar fields or potentials.

6. *Periodicity of galaxy distribution:* The north-south pencil beam survey of Ref.[19] suggests an apparent periodicity in the galaxy distribution. The number of galaxies as a function of redshift seems to clump at regularly spaced intervals of $128h^{-1}Mpc$. Recent simulations [35] have indicated that this regularity has a priori probability less than 10^{-3} in CDM universes with or without a cosmological constant. This suggests a new cosmological puzzle. Low mass radion oscillations induce a modulation in the galaxy redshift count by the factor

$$\frac{dz}{dz_0} = \frac{H}{\overline{H}} = 1 + \xi\, m\, \sin(mt + \theta) \qquad (3.5)$$

Such oscillations of $\frac{dz}{dz_0}$ can explain the peaks in the survey of Ref.[19] provided that their amplitude is larger than $1/2$ [36]. This condition is identical with the condition (3.2) required to have periods of accelerated expansion for solving the coincidence problem. It may be shown [34] that the value of m required to induce a periodicity of $128h^{-1}Mpc$ for $D = 2$ is $m \simeq 100$ which corresponds to $m_b \simeq 10^{-31}eV$.

Despite these interesting features, the main drawback for this range of radion mass is the possible inconsistency of the model with fifth force constraints, as discussed above.

IV. HIGH RADION MASS

For $m_b > 10^{-3}eV >> \overline{H}_0$ ($m \gtrsim 10^{30}$), the constraint $\omega > 2500$ applies only for experiments on scales $r \lesssim 1mm$. Such experimental constraints are not available at present and therefore radion oscillations are experimentally allowed for this mass range. In this case,

the linear term in equation (2.32) can be ignored since it averages out to zero. Thus we obtain

$$H^2 \simeq \left[\frac{1}{a^3} + \left(\frac{D}{2}\dot{\delta}\right)^2\right]$$

$$\simeq \left(\frac{1}{a^3} + \frac{4\,\xi^2 m^2 \sin^2(mt+\theta)}{9\,t^2}\right) \qquad (4.1)$$

$$= \frac{1}{a^3}\left(1 + \frac{\Omega_{0r}}{\Omega_{0m}}\right) \qquad (4.2)$$

where in the last equation we have considered averaging over time, used (2.13) and

$$t_0^{-2} = \frac{9}{4}\overline{H}_0^2 = \frac{9\,\rho_c(t_0)}{4\,\rho_m(t_0)} = \frac{9}{4\Omega_{0m}} \qquad (4.3)$$

$$\Omega_{0r} \equiv \frac{1}{2}\xi^2 m^2 \qquad (4.4)$$

Therefore equation (4.2) implies that in this mass range the oscillating radion can play the role of a new dark matter component redshifting as a^{-3} with present relative energy density

$$\Omega_{0r} = \left(\frac{D}{2}\dot{\delta}(t_0)\right)^2 \qquad (4.5)$$

For radion potentials of the form $V(\delta) \sim \delta^{2n}$ with $n > 1$ the radion oscillation energy redshifts slower than a^{-3} and can result to accelerating expansion of the universe. This effect was studied in the context of a minimally coupled scalar field in Ref.[14] where the resulting dark energy was termed "Frustrated Cold Dark Matter".

For radion oscillations induced purely by redshifting matter (see eqs. (3.4) and (4.4)) we obtain Ω_{0r} of $\mathcal{O}(1)$ independent of m. Setting $\Omega_{0r} = \mathcal{O}(1)$ we find an amplitude of the radion oscillation

$$\delta = \frac{\dot{\delta}}{m} = \mathcal{O}(m^{-1}) \qquad (4.6)$$

For $m_b \gtrsim 10^{-3}eV$ we have

$$\frac{\delta G}{G} \simeq \delta \lesssim \mathcal{O}\left(10^{-30}\right) \qquad (4.7)$$

which is consistent with nucleosynthesis and is not likely to produce observable astrophysical or cosmological effects.

The high radion mass $(m_b > 10^{-3}eV)$ has therefore three important effects

- It allows for a significant contribution of the oscillating energy to the energy density of the universe.

- It suppresses the amplitude of the oscillations making them compatible with terrestrial measurements of G, nucleosynthesis constraints and stellar evolution.

- It confines the fifth force type effects to scales less than $\mathcal{O}(m_b^{-1}) \lesssim 1mm$ making them compatible with tests for intermediate range forces and solar system tests of general relativity [17, 18].

V. CONCLUSIONS - OPEN ISSUES

Our conclusions can be summarized as follows

- Radion oscillations are generically induced at late times by redshifting matter.

- For low radion masses $(m \simeq 10 - 100\ H_0)$ these oscillations could provide a solution to two important cosmological problems: *the coincidence problem* (why do we live at the special time when the universe's expansion begins to accelerate) and *the apparent periodicity of galaxy distribution* with spatial period $\simeq 128h^{-1}Mpc$. However, fifth force constraints based on solar system and terrestrial observations may not be consistent with this range of radion masses.

- For high radion masses $(m_b > 10^{-3}eV)$ radion oscillations are consistent with fifth force and other constraints and they can provide the source of a new type of dark matter which has many similarities with axions (they are both a result of oscillating scalars).

Open issues that require further study are the following :

- Can quantum effects modify the effective Brans-Dicke parameters of low mass radion oscillations, making them consistent with fifth force constraints while still allowing the resolution of the coincidence and galactic periodicity problem?

- What are the clustering properties of the oscillating radion dark matter?

- What experimental or observational tests could detect the low amplitude - high frequency of the radion and the corresponding Newton's constant G?

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